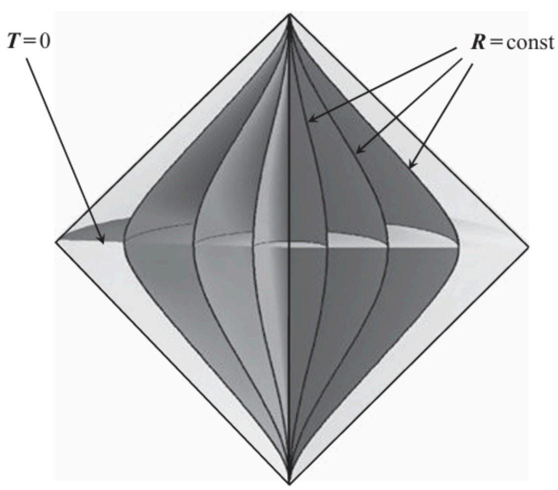
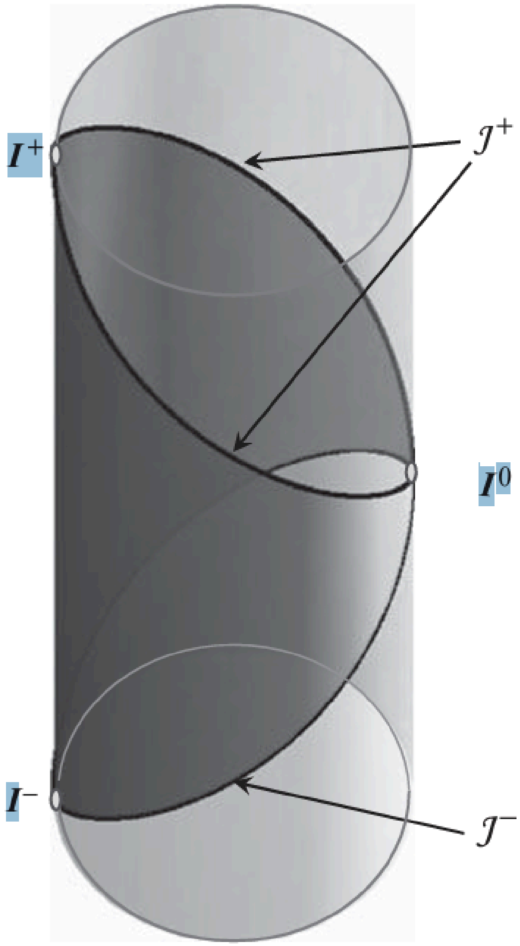
$d\omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$ is the metric on a unit sphere $S^2$. The radial coordinate R changesin the interval $[0,\infty)$.Letusmake the following changeof coordinates T $\pm$R=tan 1 .1.5) 2 ($\psi \pm\xi$). ( 10 The coordinates ($\psi,\xi$)change in the domain $0 \le\xi \le\pi$, $-\pi +\xi \le\psi \le\pi-\xi$. ( In these coordinatesthe interval ds2 takesthe form ds2 = −2d˜s2, .1.6) 10 d˜s2 =−dψ2 +dξ2 +sin2ξ dω2, ( .1.7) 10 where The conformal factor =2cos 1 2 (ψ +ξ)cos 1 .1.8) 2 (ψ −ξ). ( 10 vanishes at the 'boundary' of the Minkowski spacetime. This boundary consists ofthe followingpieces: I+ I Fig. 10.1 Carter–Penrose diagram ofthe Minkowski spacetime. 1. The past null infinity J−,where $\psi =-\pi +\xi$ and $0 <\xi<\pi$. 2. The futurenull infinity J+,where $\psi = \pi -\xi$ and $0 <\xi<\pi$.Theseboundariesarenull surfacesin the metric d˜s2. 3. The spatial infinity I0 where $\xi = \pi$. I0 4. The future, I+, and the past I− time-like infinities, where $\psi = \pi$ and$\psi =-\pi$,respectively. The coordinate transformation Eq. (10.1.5) is chosen so that it brings the points of infinity to the finite coordinate 'distance'. Points at infinity in the Minkowski spacetime correspondto $\psi + \xi$ and $\psi -\xi$ with the values $\pm\pi$.At these values, the metric ds2 becomes meaningless, but the metric d˜s2, conformal to ds2, isregular1.Atafixedvalue of ψ the metric Eq. (10.1.7) is d˜ l2 = dξ2 +sin2 ξ dω2. ( .1.9) 10 This is a metriconaunit 3D sphere S3. Hence Eq. (10.1.7) is the metric of a static Einstein universe. In fact the Minkowski spacetime is mapped onto a part ofthis space. Figure 10.1 shows a 2D-sector $\theta = \pi/2$,$\varphi = 0$,where the conformal metric is:

e Minkowski spacetime in the new coordinates we consider its 3D section Z = 0. On this section $\theta = \pi/2$ elements line elements ds2 =−dT2 +dR2 +R2dφ2 = 2d˜s2, d˜s2 =−dψ2 +dξ2 +sin2ξdφ2. ( 10.1.11) Figures10.2–10.4 showthe coordinate domain for ($\psi,\xi,\varphi$).We choose ξ as a radial coor dinate,and φ is the corresponding polar angle.Letusemphasize that these figures are not embedding diagrams. We use them to illustrate the coordinate different surfaces and lines of the originalMinkowski spacetime.Namely, Figure 10.2 shows surfaces of constant radius R of the Minkowski spacetime, Figure 10.3 shows surfaces of the fixed Minkowski time T,and Figure 10.4 shows a set of null cones passing through the center R = 0.





$I^+$
$\mathcal{J}^+$
$I^0$
$I^-$
$\mathcal{J}^-$