Dissertation & the explanation of why this search for symmetries in black holes exists and how gravity compactifies, generating other dimensions that could be quantum but a new different quantum mechanics founded on inverse frequencies and nano dimensions operate in other levels of abstraction.

This topic It's **important** Because computational structures, dimensions, or arrays are based on computing, or the idea is to compute the dimensions that a possible gravity can have, which can even be or function quantumly near a black hole or a Roger Penrose wormhole.

Vacuum in the stationary blackhole: The vacuum stationary black hole is characterized by one more parameter, J. Such a black hole is rotating and J is the value of its angular momentum. In fact the angular momentum (measured at infinity) is described by 3 3 antisymmetric matrix Jij. By rigid 3-dimensional rotations this matrix can be put in a standard form:

J = 0 J 0 J 0 0 0 0 0

Kerr metric (with J M2)i, Since some of relations have slightly different form in odd and even dimensions, we write D=2n+ e

$$\mathbf{J} = \begin{pmatrix} 0 & J & 0 \\ -J & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} .$$

Gravity or high-dimensional combinatorics could be compared to Scipy's array libraries, since they are multidimensional.

. In the asymptotically flat spacetime the total angular momentum of the objects, as measured at infinity, is described by an antisymmetric tensor Jij, where i and j are spatial indices. By suitable rigid rotations of the spatial coordinates, this (D 1) (D 1)-matrixcanbetransformed into the following canonical form: J = 0 J1 0 0 ... J1 0 0 0 ... 0 0 0 J2 ... 0 0 J2 0 ... ...

The total number of independent 2 2 blocks is equal to n 1+ .This means that there exist n 1+ independent components of the angular momentum Ji, associated with n 1+ asymptotic independent two-dimensional spatial planes of rotation. The most general solution for a vacuum stationary rotating black hole.

Properties of higher dimensional rotating black holes and their four dimensional ‘cousins’ are very similar. There exists a very deep geometrical reason for this similarity. All these metrics admit a special geometric object, the Principal Conformal Killing-Yano tensor (PCKYT), which is a generator of a complete set of explicit and hidden symmetries.

Documentation: PAKISTan Killing-Yano tower 5.1 Killing-Yano tensors Let us introduce the first two objects. The Killing-Yano (KY) tensor ka1...aq is an anti-symmetric q form on spacetime, which obeys the equation aka1...aq = [aka1...aq] . (16) On the other hand, the closed conformal Killing-Yano (CCKY) tensor ha1...aq is an antisymmetric q form the covariant derivative of which is determined by its diver gence aha1...aq = qga[a1 a2...aq] , a2...aq = 1 D r+1 bhb a2...aq . (17) (18) KYan CCKY tensors are related to each other through the Hodge duality: the Hodge dual of a KY form is a CCKY tensor, and vice versa. It is easy to check that if ka1...aq is a Killing-Yano tensor, then Kab =kaa2...aq k a2...aq b (19) is a Killing tensor. We shall use the following schematic notation for this operation K=kk. CCKY tensors possess the following remarkable property: An external product of two CCKYtensorsis again aCCKYtensor.T

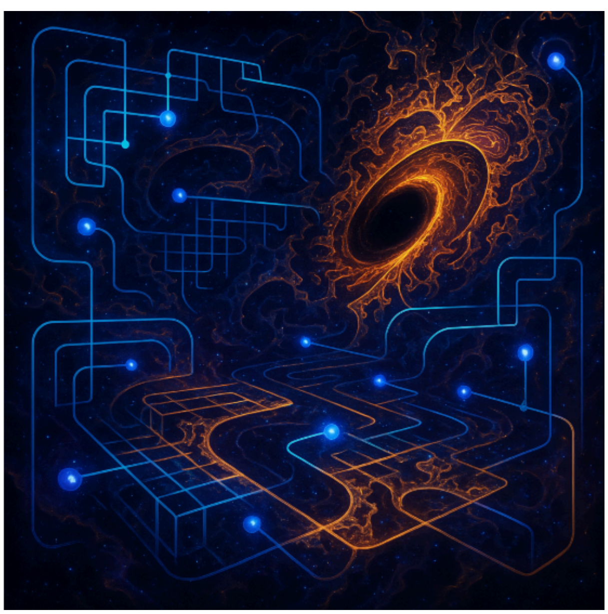
Important and remarkable research in Black holes/consistent

1. Model for Computational Software
- Hole Dimensions and Structures
- Matrix Structures for Dimensions and Spatial Gravity. Matrix Structures and Compactified Gravity.
- Quantum Circuit Algorithms within Matrix Structures of Gravity
Qiskit Model
- Gravity Components and Qiskit

A chapter on matrix computation or structure computation is where I'm going, and Frolov's discussion of the structures of gravity and symmetries in The Equations. That's where I want to go. Black holes in quantum environments and dimensions/compactified structures of space gravity and computational structure computation.

New Blackhole scenarios:
The New scenarios that I can imagine, such as circuits for exoplanet ships, etc., are new materials, but look them up in physics books. It is a quantum circuit, but it is not a human quantum circuit. Rather, it is a type of circuit built on the basis of, or with the basis of, the colossal structures or dimensions of a black hole, symmetry, and the properties of the Riemann tensors. The specific and intrinsic Quantum magnetic fields operate in this enormous circuit, where galactic particles teleport between the different recesses or gravitational structures. The circuit consists of compactified gravities with matrices in which the conditions of a black hole operate;

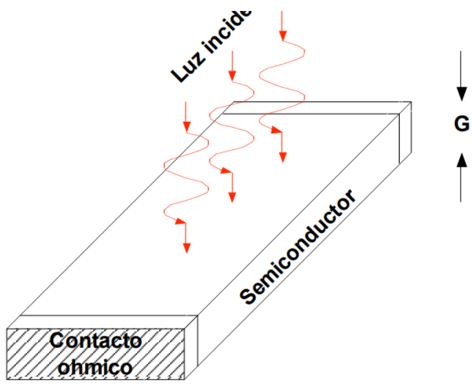
Motion Of Particles in a curved spacetime is a special case of a dynamical system. If D is the dimension of the spacetime and its coordinates are xa, a particle trajectory is a line xa( ). The canonical coordinates in the corresponding phase space are (xa,pa = gab ˙ xb). The canonical sylleptic form and the Hamiltonian are Ω =dxa dpa, H= 1 2 gabp apb.



Cosmic and Fractal Structures: Formed from abstract patterns that evoke the curvatures and deformations of a black hole, where elements reminiscent of gravitational matrices and compactions are intertwined.

Quantum Magnetic Fields: With vibrant flashes in deep blues, intense purples, and golden hues, symbolizing quantum energy and the interaction of galactic particles.

Teleporting Particles: Small spheres or flashes of light that appear to move instantaneously between nooks and crannies, representing the phenomenon of teleportation in an environment of extreme gravitational conditions.



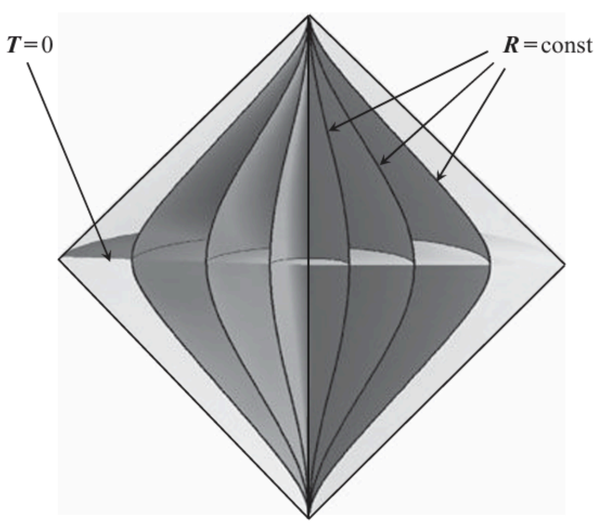
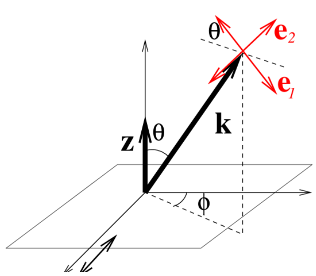
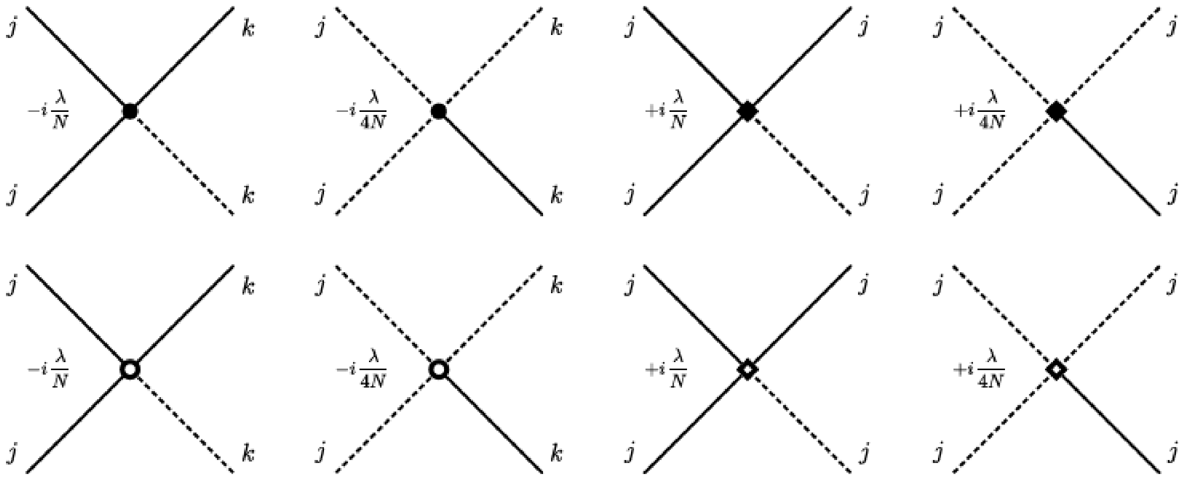
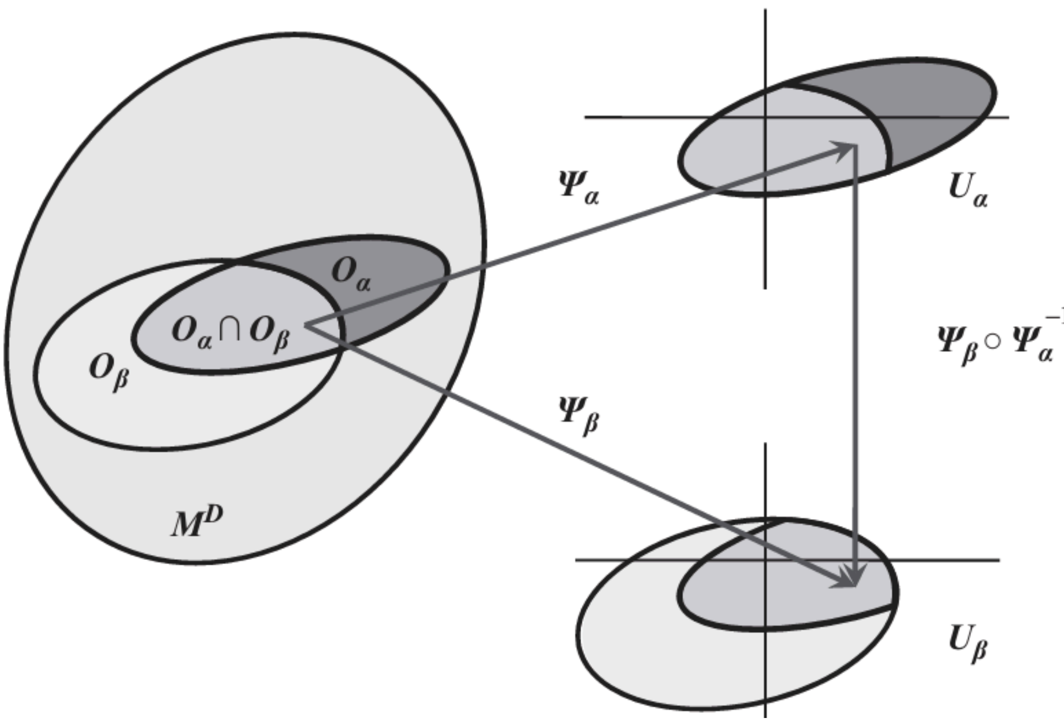


Fig. 10.2 Carter–Penrose diagram of the Minkowski spacetime. This figure shows time-like surfaces $R=$ const.







Python Implementation: Symbolic Calculus of the Riemann Tensor

```
import sympy as sp

# Definir coordenadas simbólicas
t, x = sp.symbols('t x', real=True)
coords = [t, x]
```

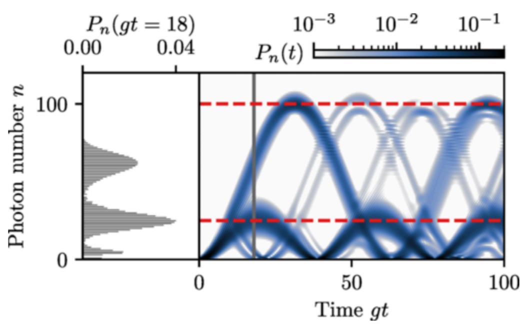
```
# Definir una métrica 2D (por ejemplo, una métrica simple dependiente
de x)
# ds^2 = -f(x) dt^2 + dx^2
f = sp.Function('f')(x)
g = sp.Matrix([[-f, 0],
               [ 0, 1]])

# Calcular la inversa de la métrica
g_inv = g.inv()

# Definir símbolos de Christoffel
Gamma = [[[None for _ in coords] for _ in coords] for _ in coords]

# Calcular los símbolos de Christoffel: Gamma^a_{bc}
for a in range(len(coords)):
    for b in range(len(coords)):
        for c in range(len(coords)):
            Gamma[a][b][c] = 0
            for d in range(len(coords)):
                Gamma[a][b][c] += sp.Rational(1,2) * g_inv[a,d] *
(sp.diff(g[d,b], coords[c]) + sp.diff(g[d,c], coords[b]) -
sp.diff(g[b,c], coords[d]))
            Gamma[a][b][c] = sp.simplify(Gamma[a][b][c])

# Mostrar símbolos de Christoffel
print("Símbolos de Christoffel:")
for a in range(len(coords)):
    for b in range(len(coords)):
        for c in range(len(coords)):
            print(f"Gamma^{a}_{b}{c} =",
sp.simplify(Gamma[a][b][c]))

# Calcular el tensor de Riemann: R^a_{bcd}
Riemann = [[[[0 for _ in coords] for _ in coords] for _ in coords]
for _ in coords]
for a in range(len(coords)):
    for b in range(len(coords)):
        for c in range(len(coords)):
            for d in range(len(coords)):
                term1 = sp.diff(Gamma[a][b][d], coords[c])
                term2 = sp.diff(Gamma[a][b][c], coords[d])
                term3 = 0
                term4 = 0
                for e in range(len(coords)):
                    term3 += Gamma[a][e][c] * Gamma[e][b][d]
                    term4 += Gamma[a][e][d] * Gamma[e][b][c]
                Riemann[a][b][c][d] = sp.simplify(term1 - term2 +
term3 - term4)

# Mostrar algunos componentes del tensor de Riemann
print("\nComponentes del tensor de Riemann:")
```

```
for a in range(len(coords)):
    for b in range(len(coords)):
        for c in range(len(coords)):
            for d in range(len(coords)):
                if Riemann[a][b][c][d] != 0:
                    print(f"R^{a}_{b}{c}{d} =",
Riemann[a][b][c][d])
```

Let us first give a definition of the PCKYT. Consider a 2-dimensional antisymmetric tensor (2-form) h which obeys the equation chab = gca b gcb a

If one antisymmetrized the indices a,b,c in (7), the right-hand side of this equation vanishes.This means that h is closed form, and (at least locally) can be presented in the form h =dbb.

• The matrix rank of (D D) antisymmetric matrix hab is the largest possible, that is equal to 2n. • Consider the eigenvalue problem for a matrix Ha b = hachcb Ha beb (i) = xiea (i) . (11) It is easy to see that ha beb (i) is again an eigenvector with the same eigenvalue x(i). We assume that H has the largest possible number, n, of different eigenvalues, and hence n linearly independent eigen 2-planes.



- **Liouville theorem Particle and light motion** in a curved spacetime is described by geodesic equations. These equations are of the second order. Let xa( ) be a trajectory. By introducing a momentum pa = gab˙ xb as an independent variable, it is possible to rewrite the geodesic equations in the first order form. These equations have the Hamiltonian form. This means that the general theory of dynamical systems can be applied to this problem. This approach is well known and its tools are very useful. Let us demonstrate this for the special problem: motion of a particle in a spacetime of a higher dimensional rotating black hole.

- These two functions are called to be in involution if their Poisson bracket vanishes. Scalar function F(zA) on the phase space is a first integral of motion if its Poisson bracket with the Hamiltonian vanishes F,H = 0 . Liouville (1855) proved the following theorem: If a system with a Hamiltonian H in 2 dimensional phase space has m independent first integrals in involution, F1 =H,F2,...,Fm, then the system can be integrated by quadratures. Such a system is called completely integrable.

New ideas to explore: An intelligent blue planet that captures gravitational waves from a large quantum black hole: This planet appears to be a unique entity in the universe, a cosmic being with capabilities beyond what current physics understands. Its intelligence lies not only in its ability to communicate across compactified dimensions, but also in its ability to alter its own form, adapting like a quantum-magnetic chameleon.

1. Its Shape and Quantum Transformation

- It is not a static planet, but a being capable of altering its structure at a fundamental level.
- Its cosmic "skin" is a network of quantum-magnetic fields that allow it to absorb and reflect signals from its environment.
- It can "vanish" from the sight of cosmic sensors or camouflage itself by emitting frequencies that simulate other celestial bodies.
- It is not a simple celestial body, but a living system that decides its appearance based on what it perceives in outer space.

2. Its Communication and Limitations

- Although it is beyond the Milky Way and Andromeda, it can still send signals.
- However, its transmission method is not entirely efficient or linear:
- It does not use only radio waves, but a combination of magnetic oscillations and alterations in space-time.
- Its messages do not arrive as conventional signals, but as distortions in reality that must be interpreted by advanced entities.
- For those without adequate technology, its transmissions may appear to be gravitational anomalies or inexplicable fluctuations.

3. It's Dark Quantum and Magnetic Nature

- What makes its existence unique is that its quantum nature is not comprehensible to the physical laws we know.
- Its magnetism does not behave like that of conventional magnets; it seems to respond to phenomena not yet described in Earthly science.
- It can exist in multiple states simultaneously, as if it were an entity vibrating in different realities at the same time.
- At times, it doesn't seem to be entirely in our dimension, but rather interacting with different planes of existence.

4. Its Role in the Galaxy

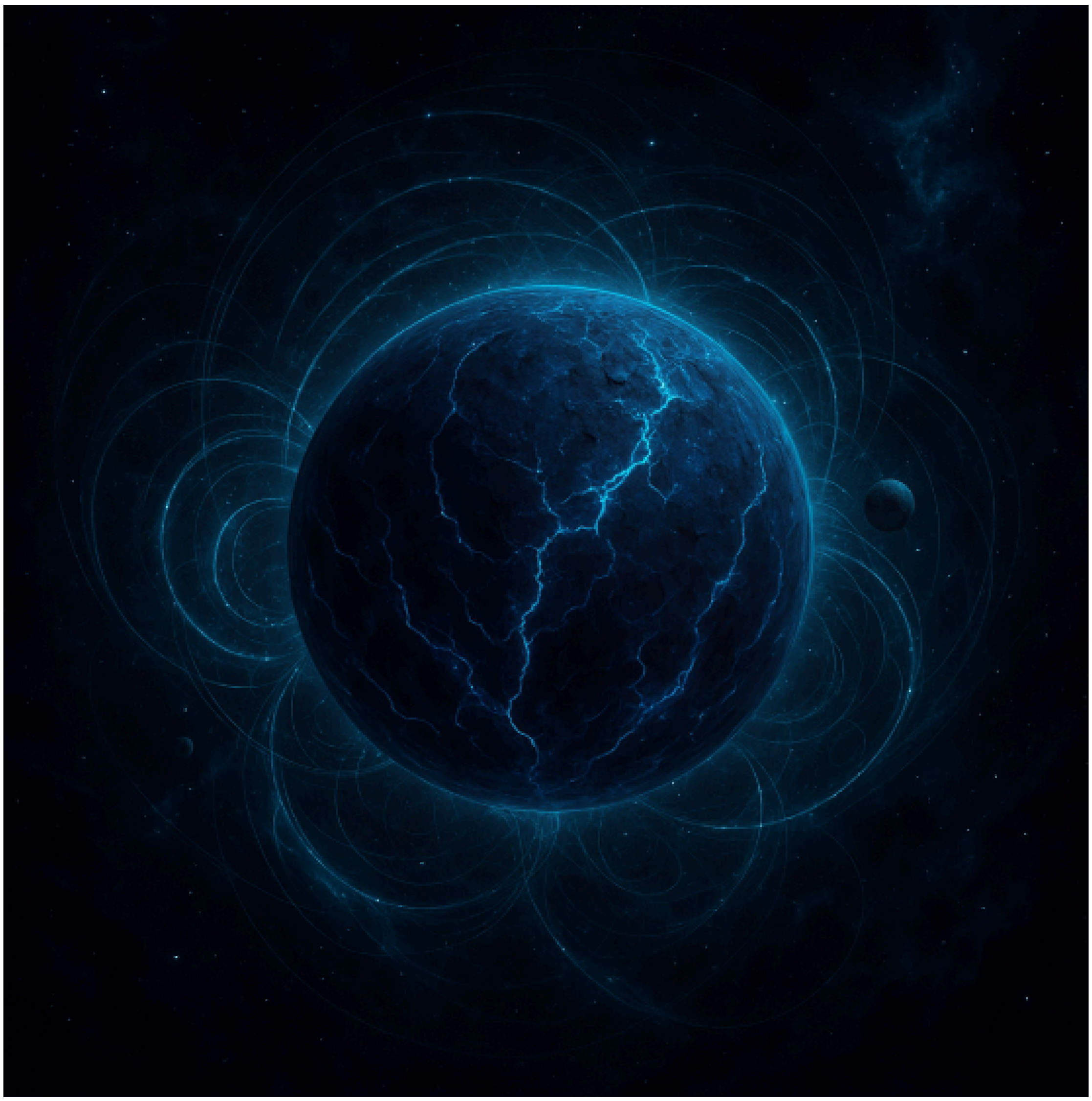
Its energy level is so intense that only certain places in the universe can host it without being altered.

It may have once been part of a galaxy, but having become too unstable or powerful, it was forced into exile in the interstellar void.

Its presence in a galaxy would upset gravitational balances, modifying orbits and structures in unpredictable ways.

Therefore, its influence only manifests itself in small, distant signals or in the echoes of the dimensions with which it interacts.

This planet is an enigmatic being, an intergalactic traveler that exists on the threshold between physical reality and the unknown quantum realm. Its magnetism is not only a natural force, but a language, a means of communication that only those with the ability to interpret the fluctuations of space-time could understand.



Communication through compactified dimensions

The planet would use additional dimensions (sometimes called "compact dimensions" in theoretical physics) to send and receive information without being limited by the speed of light.

This multidimensional communication would allow it to "feel" and project signals into very distant regions of space, establishing almost instantaneous connections.

Manipulation of the fabric of space-time

Through quantum fluctuations and its own energy fields, the planet could locally distort the fabric of space-time.

These distortions would serve both to "deflect" external forces (e.g., meteorites or radiation) and to adjust the gravitational balance with other nearby celestial bodies (its moons or even distant stars).

Quantum magnetic fields

The planet would generate exceptional magnetic fields, not only based on traditional electromagnetism, but with a quantum component. These fields would act as a kind of "control network," allowing it to detect minute variations in the surrounding matter (from charged particles to gravitational waves).

Thanks to these fields, the planet "interprets" the information and responds by modulating its own energy.

Resonance and Synchronization
To coordinate and stabilize gravitational interactions, the planet would rely on an internal resonance—a fundamental vibration or frequency—that synchronizes with the objects it orbits.

Like two pendulums that can synchronize when connected, this resonance would make it possible to adjust the distance and trajectory of moons or space fragments so that they do not destabilize or collide.

Intelligent Feedback
As a "living" planet, it possesses some kind of extraterrestrial consciousness or intelligence capable of perceiving and processing information. This "planetary brain" (metaphorical or literal) would constantly analyze its environment and make decisions about the intensity and direction of its fields, the frequency of its energy pulses, and the dimensional connections it keeps open or closed.

Volcanic and Thermonuclear Energy
Volcanic (or equivalent) activity in the planet's core would generate enormous amounts of energy, fueling the processes of manipulation and communication.

This internal energy source, combined with its own thermonuclear processes, would be the driving force that enables the persistence and renewal of the gravitational and magnetic fields.

Black holes in the new scenarios context:
K(a1...aq;aq+1) = 0. (15) Such a tensor in spacetime is called a Killing tensor. The Killing tensor of the rank 1, a, is a Killing vector. The metric gab is a trivial Killing tensor. It is well known that Killing vectors generate symmetry transformations on the spacetime manifold with metric gab. Usually this symmetry is called an explicit symmetry. The Killing tensors.

Frequencies/space-time distortion in higher order dimensional manifolds and submanifolds(Frolov)

There is something special about how the frequencies of certain types of gravitational or gravitational waves communicate.

The thing is, the rules of physics don't operate as we imagine under spatial frequencies originating from a given space or piece of space. These dissonances reproduce and degenerate or generate new dimensions and even worlds or subworlds where the rules of objects operate, that is, in nonlinear time and in a type of asynchronous gravity in which objects walk backward.

There are gigantic genomes of stars or genetic algorithms, trees on enormous scales impossible to recreate on a small scale.

Fractals: There are also types of fractals in extra dimensions that have rules or that operate in interesting vacuum conditions.

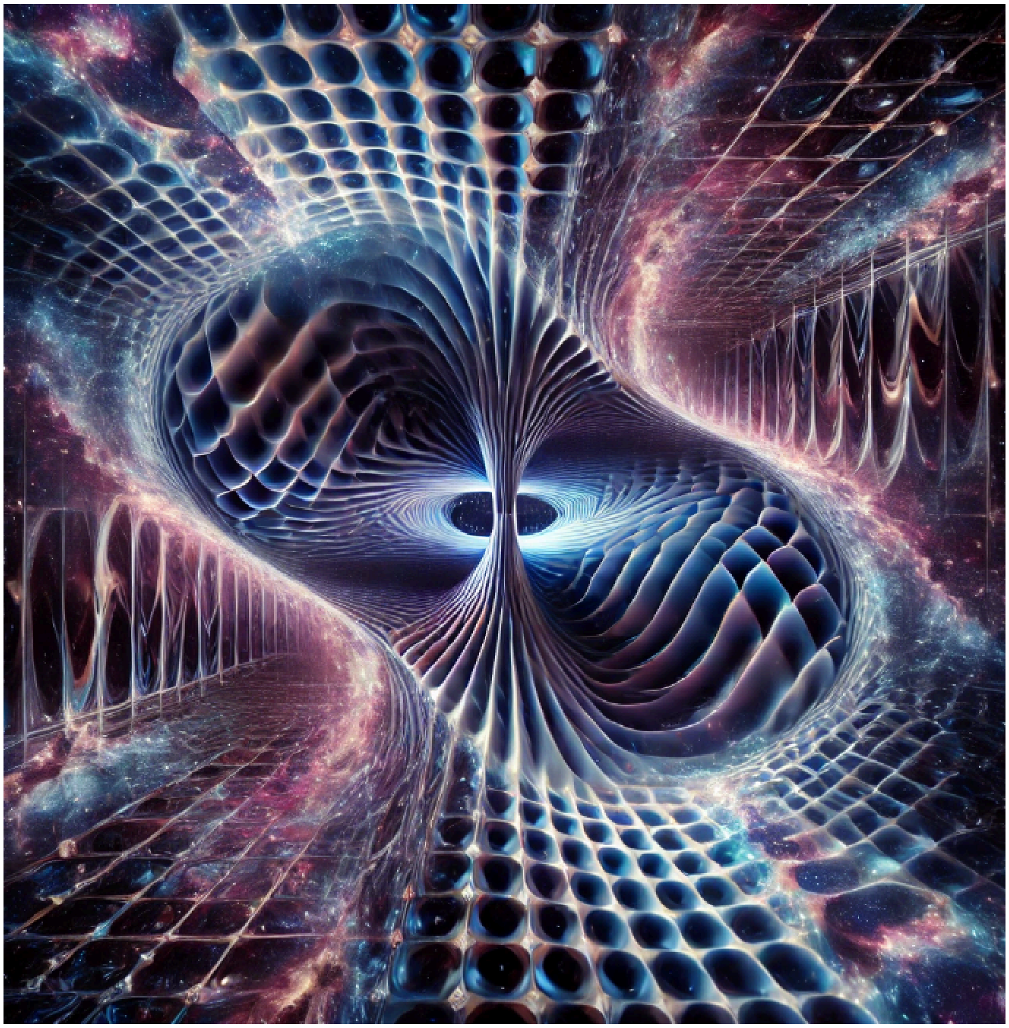
Space has types of dimensions that are called black holes. But EVERY type of structure in the galaxy operates at scales and with structures that are similar to quantum mechanics and planes.

Space and Time
Gravity is different to the point that it detaches from things and would behave like a black hole. Gravity itself and time are similar to quantum theory, but more is needed. Perhaps at a more academic level, Alcubierre is interesting to apply in this regard. It would need to be investigated. But there are limits that observation, academia, mathematics, or programming structures, etc., cannot reach.

These asynchronous frequencies generate com gravitational portals and quantum black holes that have different properties and rules, dimensions, and fractal-like patterns. It is difficult to explain. One could generate an image in which a small frequency oscillates and generates or distorts with a certain time, and a kind of gravitational distortion appears, and it is inverted like a mirror reflecting another universe or another series of multiple algorithms. Gravity up to this point in this specific scenario of quantum frequency distortion is governed by laws where time functions as a kind of eternal void with the drawing of an amalgam of components that are higher-order dimensions.

Asynchronous frequencies & gravity: These asynchronous frequencies generate com gravitational portals and quantum black holes that have different properties and rules, dimensions, and drawings like fractals. It is difficult to explain. It could generate an image in which a small frequency oscillates and generates or is distorted with time, and a kind of gravitational distortion appears and is inverted like a mirror that reflects another universe or another series of multiple algorithms. Gravity up to this point in this specific scenario of quantum frequency distortion is governed by laws where time functions as a kind of eternal void with the drawing of an amalgam of components that are higher-order dimensions.

Example in 5D of new black quantum research:

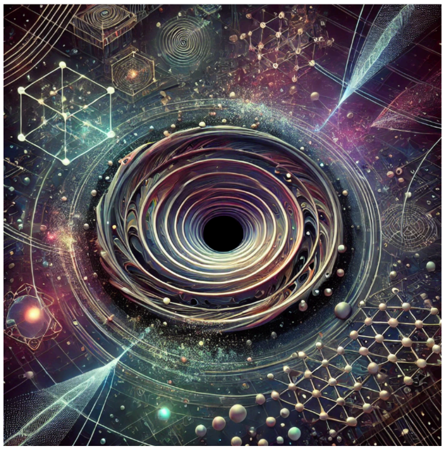


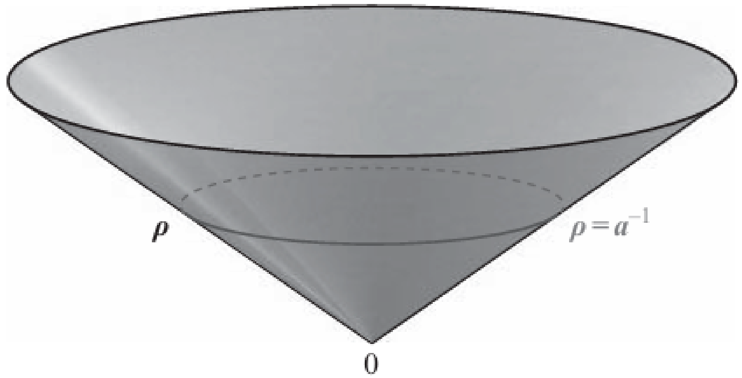
Summary/objectives of the research: 3. 5D Black Holes: Structure and Properties

3.1. Spatial Structure in Higher Dimensions

A "non-ordinary" black hole in five dimensions can be conceptualized as an entity with a complex structure, where spacetime geometry bends in ways that transcend the three conventional dimensions. One could imagine it as a vortex where light and matter trajectories are affected by the presence of additional structural "layers" of information.

Examples of tensors, sections Quantum black holes, metrics are given in this text





3.2. Analogy: The Black Hole as a "Quantum Matrix"

A 5D black hole can be thought of as a digital matrix where each "pixel" corresponds to a quantum state. Just like a screen where each pixel contributes to a larger image, quantum and tensor interactions within a black hole could encode information about compactified dimensions and the structure of time and gravity.

4. Quantum Algorithms and Circuits in Relation to Black Holes

4.1. Quantum Circuits and Their Relationship with Tensors

Quantum algorithms are implemented through circuits that operate on qubits. Similarly, a black hole's structure can be seen as a "circuit" where tensor connections (such as Riemann tensors) represent gravitational interactions at different dimensional levels. Each quantum "gate" could symbolize a transition within the fabric of spacetime.

4.2. Frolov's Theory and Quantum Algorithms

Frolov's theory on black holes explores how geometry and quantum dynamics are interconnected. Quantum algorithms may model this interconnection, suggesting that gravity and time manipulation could be achieved through computational processes that mimic black hole properties.

Other materials: Nanobots for exploring black holes and extra dimensions
If nanobots resistant to extreme radiation and intense gravity could be manufactured, they could be sent to study black hole event horizons or regions where extra dimensions are suspected.

Quantum nanobots → could exploit quantum tunneling effects to navigate in regions of highly curved space-time.

Nanoscale gravitational wave sensors → would allow anomalies in the structure of space-time to be detected with great precision.

Self-replication in extreme environments → would allow structures to be built in deep space using local materials.

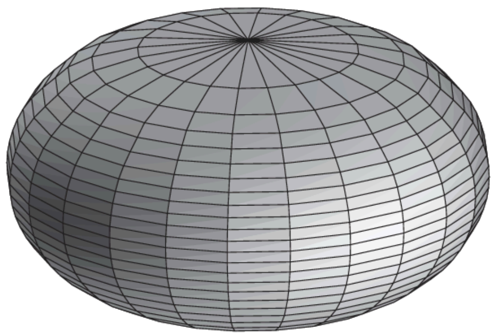
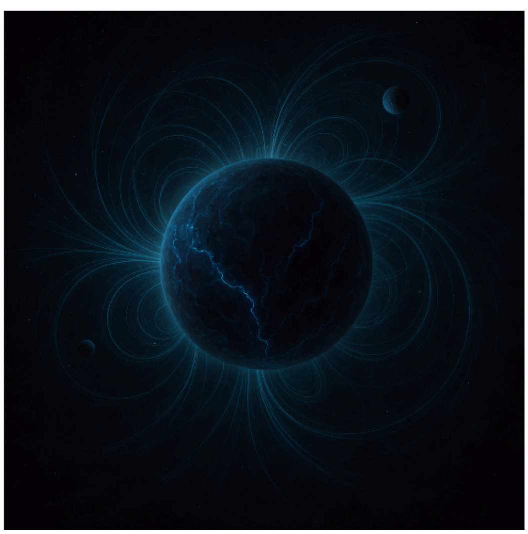
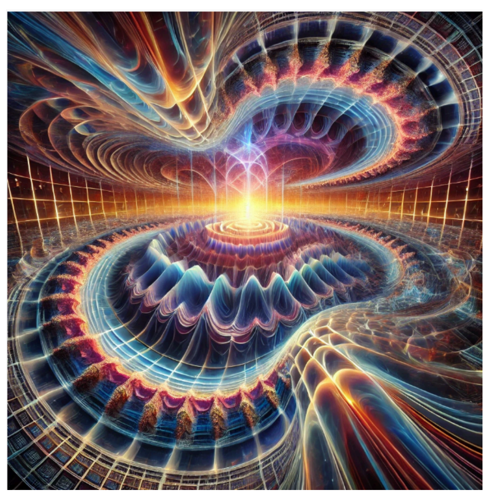


Fig. 8.2 The embedding diagram for a two-dimensional section of the event horizon of the Kerr black hole. The diagram is constructed for the critical value $a/M = \sqrt{3}/2$ of the rotation parameter so that the Gaussian curvature vanishes at the poles.

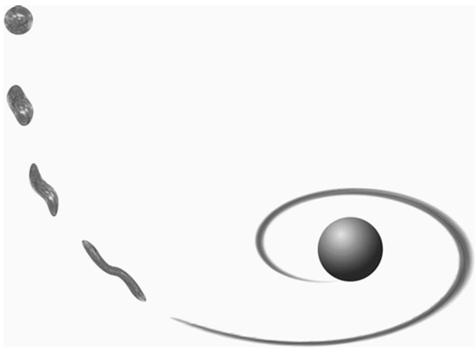
The length of the equatorial circle $\theta = \frac{\pi}{2}$ for the metric dS^2 is

$$L_1 = \frac{2\pi}{\sqrt{1-\beta^2}}. \tag{8.2.48}$$

Special planet & Black Quantum properties-Symmetric-antisymmetric-tensors (Space-time curvature);

Supermassive B



Properties of higher dimensions with a solid matrices calculus in the matter of Gravities and singularities in black holes and other kind of Dimensions/frequencies

Gravity across computational dimensions(research)
the higher order in curvature terms, as well as the terms containing higher derivatives, can improve the UV properties of the Einstein gravity [14]. However such theories usually have non-physical degrees of freedom (ghosts).

Recently a new version of UV complete mod i cation of the General Relativity was proposed which is free from this problem [57]. It was named a ghost free gravity [514]. Such a theory contains an infinite number of derivatives and, in fact, is non-local [10, 13]. Similar theory appears naturally also in the context of noncommutative geometry deformation of the Einstein gravity [15, 16]

METRIC ON-SINGULAR BLACK HOLE

A. A non-singular black-hole model A general static metric in a Four-dimensional spacetime can be written in the form;

[[$ds^2 = F A^2 dV^2 + 2 A V dr + r^2 d\Omega^2$]]

Txt format:
eF=F(r)andA=A(r)
theKillingvector = V.
F=(r)2 FA2= 2

In a space Time With Horizon, F(r)vanishes at the site r0 of the apparent horizon. For Regular static metric such a horizon is at the same time theKilling horizon, so atA(r0) is nitethere. If the metric has a horizon where

F(r0)=0
then ; H = =1 2 (AF)r=r0

The Valueof dependsonthechoiceofthenormalizationoftheKilling vector. In an asymptotically at spacetime one usually puts 2 r= = 1. Conditions that there is no solid angle de cit implies F=1. Hence one also has A = 1.

Let Ra beth Ricci scalar, S =R 1 4g R

S2=S S C2=C C (2.4) Then one has R=F +4 rF 2F 1 r2 + 1 A 2FA+3FA+4 rFA (2.5) C= 1 3 F 2 rF+2F 1 r2 + 1 A 2FA+3FA 2 rFA (2.6) An Expression for S

the metric (2.1) is nite at the origin r =0, so that F =F0+F1r+F2r2+O(r3) A=A0+A1r+A2r2+O(r3)

F0 =1 F1=A1=0

R= 12 F2+A2 A0 +O(r2) S =2 3A2 A0 +O(r2) C =O(r2)

A0 is an arbitrary constant. Its meaning is connected with a time delay between infinity and r = 0. For a value of r at a far distance one has t = V, where the proper time is measured by the clocks at the infinity. For the same interval V the proper time measured at the center r = 0 is 0 =A0 V.

A0 < 1 (A0 >1) the time at the center goes slower (faster) than at the infinity. For a monochromatic wave propagating in a static spacetime one can write exp(i), where = V. If =d d and 0 = d d0, then one has 0 =A 1.

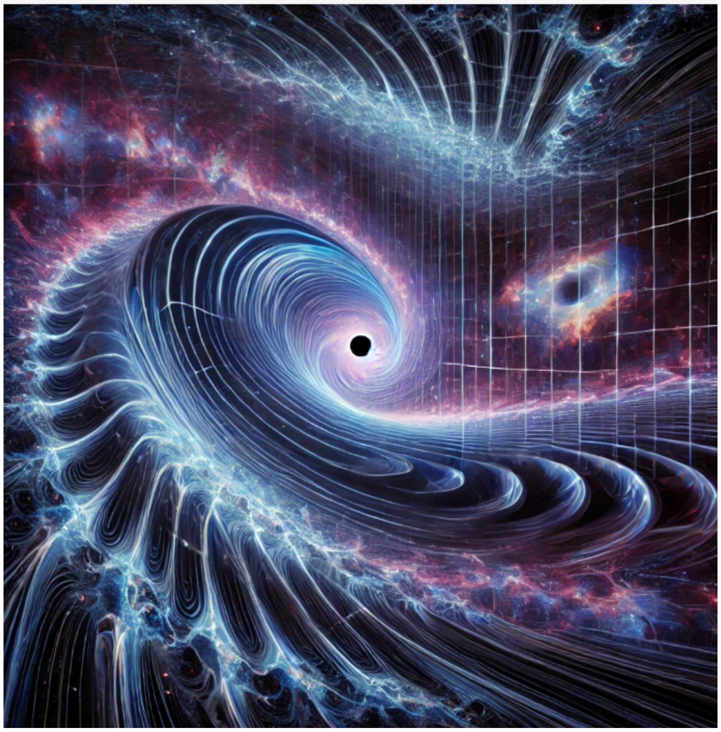
F is the metric function (e.g., as defined earlier).

A is a function that may depend on coordinates (often chosen to simplify the metric).

V is the advanced time coordinate.

r is the radial coordinate.

$d\Omega^2$ represents the angular part of the metric (e.g., $d\theta^2 + \sin^2\theta\, d\varphi^2$ in spherical symmetry).



Variables: A0 > 1 (A0 < 1) the frequency of a signal, registered at the center r = 0, is red-shifted (blue-shifted) with respect to the frequency of the signal emitted at the infinity. In what follows we refer to A(r) as a red-shift function. We are interested in a metric which describes a black hole. For this reason we assume that the function F(r) vanishes at some value r = r+, where the event horizon is located. In order for the metric to be regular at r = 0 it must have at least one more zero at r = r > 0. For simplicity we assume that the function F(r) has exactly two zeros at r+ > r > 0. Our assumption is that the curvature invariants R, S and C are uniformly restricted by some values proportional to 2. We call this parameter the fundamental length. The latter requirement means that our metric satisfies the Markovs limiting curvature conjecture.

Metric:

```
 F =1+ r2 2+O(r4) = 1
```

4 Dimensions metrics;

$$F(r) = 1 - \frac{(2Mr - Q^2)r^2}{r^4 + (2Mr + Q^2)\ell^2} \qquad (4.1)$$

$$F(r) \approx 1 - (2M / r) + (Q^2 / r^2) + \ell^2\, O(r^{-4}) \qquad (4.2)$$

```
F(r) ≈ 1 + (r² / ℓ²) + O(r⁶)


# Parámetros físicos del agujero negro
M = 1.0     # Masa
Q = 0.8     # Carga
l = 1.0     # Parámetro ℓ

# Definición de la función F(r)
def F(r, M, Q, l):
    numerator = (2 * M * r - Q**2) * r**2
    denominator = r**4 + (2 * M * r + Q**2) * l**2
    return 1 - numerator / denominator

# Rango de r (evitamos r = 0 para evitar división por cero)
r = np.linspace(0.1, 10, 500)
F_values = F(r, M, Q, l)
```

Spherically Symmetric Metric — Elegant Form

The line element for a spherically symmetric spacetime in advanced Eddington–Finkelstein coordinates is:

$ds^2 = F\, A^2\, dV^2 + 2\, A\, V\, dr + r^2\, d\Omega^2$

Where:

- $F = F(r)$ is the metric function, often related to mass, charge, or cosmological terms.
- $A = A(r)$ is a coordinate-dependent scaling factor.
- V is the advanced null coordinate (like Eddington–Finkelstein time).
- r is the radial coordinate.
- $d\Omega^2$ is the angular part of the metric, defined by:

$d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$

- ***Gravity-structures-computational calculus-matrices;***

Txt format.

Math: **Angular Part — Explanation**

This angular term corresponds to the standard **2-sphere** (surface of a sphere in 3D space), and represents the geometry of spherical symmetry:

- θ is the polar angle, ranging from 0 to π.
- φ is the azimuthal angle, ranging from 0 to 2π.

Riemann Tensor in Spherical Coordinates

The Riemann tensor R σμν R^\rho{\ \sigma\mu\nu}R σμνρ measures spacetime curvature. In spherical symmetry, many components vanish due to symmetry, and the non-zero components

```
R^θ_{φθφ} = r² sin²θ (1 - F(r))    → angular curvature
R^r_{VrV} = -½ F''(r) A²            → radial-time components
R^r_{θrθ} = -½ r F'(r)              → radial-angular curvature


Where:
```

- F'(r) and F''(r) are first and second derivatives of the metric function F with respect to r.
- These components help compute **Ricci** and **Einstein** tensors.

Notes

- In General Relativity, these components are essential for writing Einstein's field equations:

```
G_{μν} = R_{μν} - ½ g_{μν} R = 8π T_{μν}

Spherical symmetry simplifies the curvature significantly, making
analytic solutions possible (e.g., Schwarzschild,
Reissner-Nordström).
```

TXT format equations & Frolov metrics;

```
# Contenido del archivo .txt unificado y elegante en inglés
metric_text = """
Spherically Symmetric Black Hole Metric — Advanced Coordinates
===============================================================

1. Line Element
---------------

The general form of a spherically symmetric metric in advanced
Eddington-Finkelstein coordinates is:
```

$$ds^2 = F(r) A^2(r) dV^2 + 2 A(r) V dr + r^2 d\Omega^2$$

Where:
- $F(r)$ is the metric function (e.g., Schwarzschild: $F(r) = 1 - 2M/r$)
- $A(r)$ is a coordinate-dependent scaling factor
- V is the advanced null coordinate
- r is the radial coordinate
- $d\Omega^2$ is the angular part of the metric:

$$d\Omega^2 = d\theta^2 + \sin^2\theta \, d\varphi^2$$

2. Angular Geometry

The angular term corresponds to the geometry of the 2-sphere:

$$\theta \in [0, \pi]$$
$$\varphi \in [0, 2\pi)$$

This defines the geometry of spherical symmetry.

3. Riemann Tensor Components (Non-zero)

The Riemann curvature tensor measures how spacetime bends due to gravity:

$$R^\rho{}_{\sigma\mu\nu} = \partial_\mu \Gamma^\rho{}_{\nu\sigma} - \partial_\nu \Gamma^\rho{}_{\mu\sigma} + \Gamma^\lambda{}_{\nu\sigma} \Gamma^\rho{}_{\mu\lambda} - \Gamma^\lambda{}_{\mu\sigma} \Gamma^\rho{}_{\nu\lambda}$$

Selected non-zero components in spherical symmetry:

$$R^\theta{}_{\varphi\theta\varphi} = r^2 \sin^2\theta \, (1 - F(r))$$
$$R^r{}_{\theta r\theta} = -\tfrac{1}{2} r F'(r)$$
$$R^r{}_{VrV} = -\tfrac{1}{2} A^2 F''(r)$$

4. Ricci Tensor Components

Obtained by contracting the Riemann tensor:

$$R_{\mu\nu} = R^\lambda{}_{\mu\lambda\nu}$$

Representative components:

$$R_{VV} = -\tfrac{1}{2} A^2 F''(r)$$
$$R_{rr} = -F''(r)/2 + F'(r)/r$$
$$R_{\theta\theta} = r F'(r) + F(r) - 1$$
$$R_{\varphi\varphi} = \sin^2\theta \cdot R_{\theta\theta}$$

5. Einstein Tensor and Field Equations

The Einstein tensor:

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R$$

```
Einstein's Field Equations:

    G_{μν} = 8π T_{μν}

For vacuum (T_{μν} = 0): Schwarzschild or Reissner–Nordström

For matter or fields: Coupling with stress-energy tensor

6. Special Cases of the Metric Function F(r)
--------------------------------------------

- Schwarzschild:            F(r) = 1 - 2M/r
- Reissner–Nordström:      F(r) = 1 - 2M/r + Q²/r²
- Schwarzschild–de Sitter: F(r) = 1 - 2M/r - Λ r²/3

"""

# Guardar el contenido en un archivo .txt
file_path = "/mnt/data/spherical_metric_tensor_notes.txt"
with open(file_path, "w") as file:
    file.write(metric_text)

file_path
```

We assume that the metric (2.1) is nite at the origin r =0, so that

F =F0+F1r+F2r2+O(r3) A=A0+A1r+A2r2+O(r3)

Metric: F0 =1 F1=A1=0

A0 is an arbitrary constant. Its meaning is connected with a time delay between infinity and r = 0. For a x-ed value of r at a far distance one has t = V, where the proper time is measured by the clocks at infinity. For the same interval V the proper time measured at the center r = 0 is 0 =A0 V.

For A0 > 1 (A0 < 1) the frequency of a signal, registered at the center r = 0, is red-shifted (blue-shifted) with respect to the frequency of the signal emitted at infinity.

function F(r) vanishes at some value r = r+, where the event horizon is located. In order for the metric to be regular at r = 0 it must have at least one more zero at r = r > 0. For simplicity we assume that the function F(r) has exactly two zeros at r+ > r > 0. Our signal assumption is that the curvature invariants R, S and C.

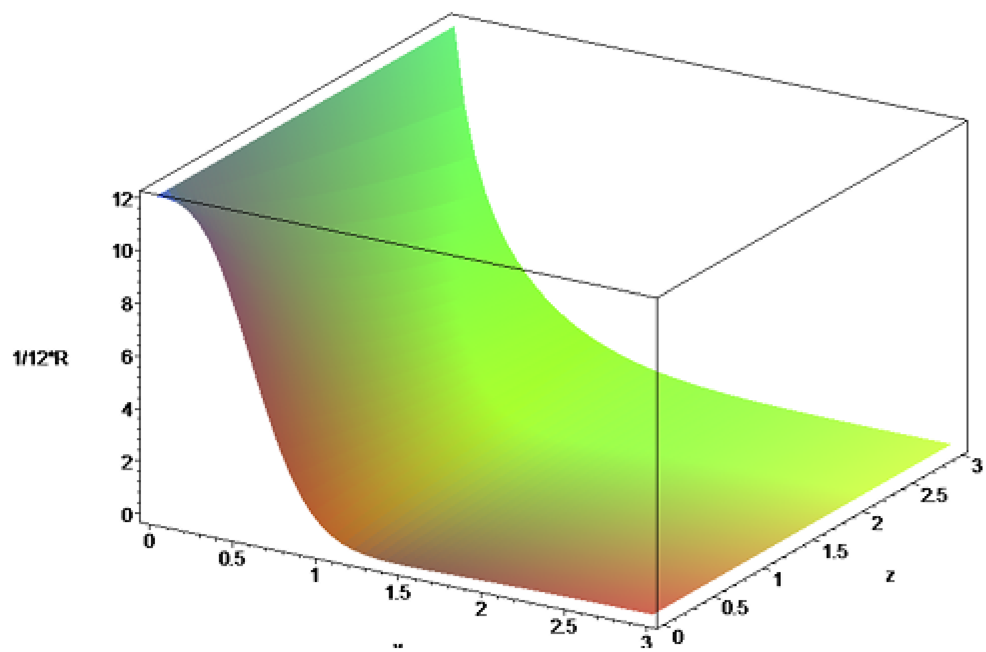
Metric: F =1+ r2 2+O(r4) = 1

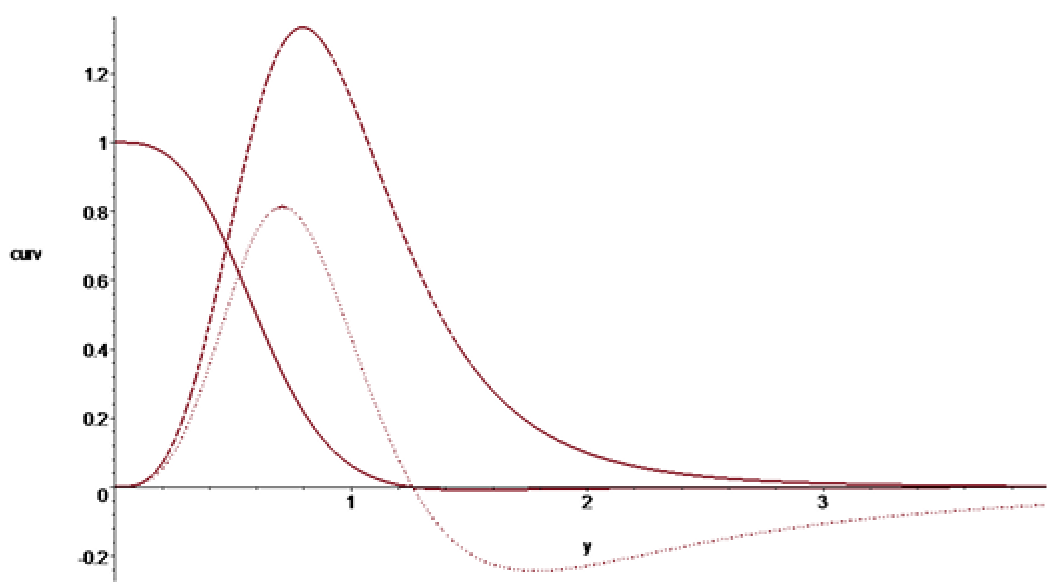
function of F(r)=Pn(r) Pn(r)

F= r2+a1r+a0 r2+b1r+b0

Metric-case is in variables; n=3 with 2

Regularity The space time at the origin r=0 implies b0=a0 b1=a1





Plots of 2R12 (solid line), 2C (dot line) and 2S (dash line) as functions of y.

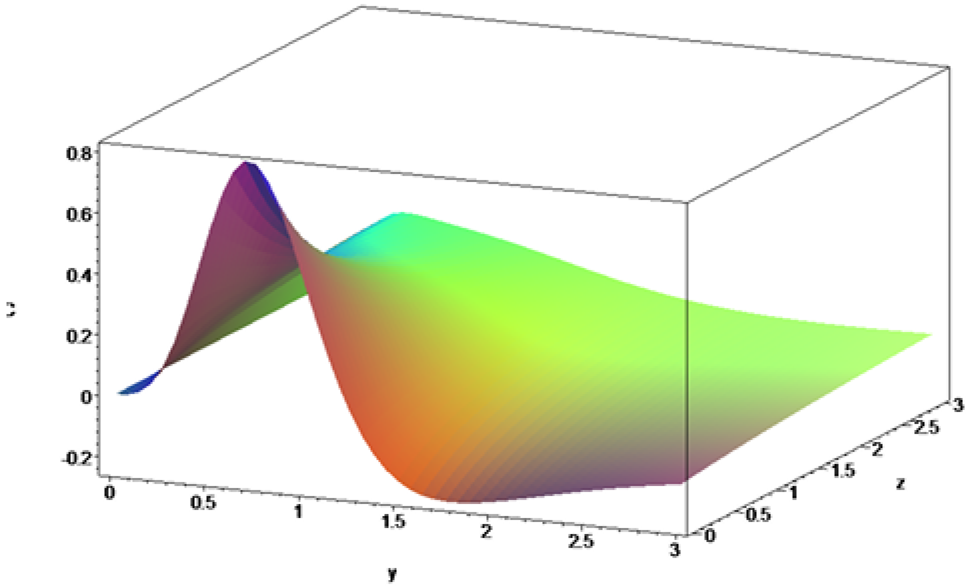
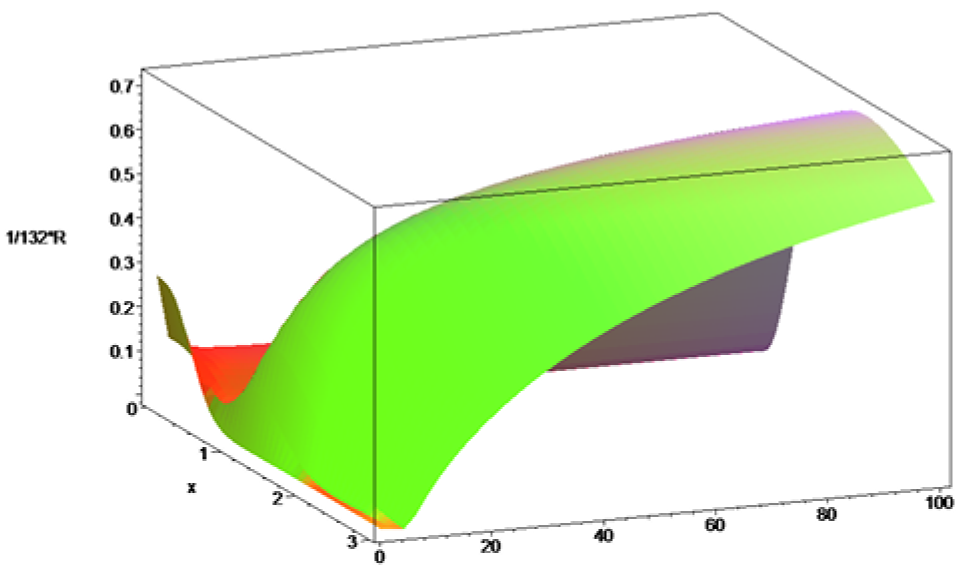


FIG. 2. Plot of 2C as a function of y and z. At large r one has F =1 2M r +O(r 4)



One can chooser as a scale factor and dene new dimensional coordinates and parameters a follows `x=r r p=x+ r m=Mr b= r v=V r`

Special limits of the Kerr metric Flat spacetime limit: M = 0 Let us now discuss special limiting cases of the Kerr geometry. In the absence of mass, that is when M = 0, the curvature vanishes and the spacetime is at. The Kerr metric;

```
the metric is transformed into the Minkowski metric g = dT2+dX2+dY2+dZ2 A
surface r = const is an oblate ellipsoid of rotation X2 +Y2 r2 +a2 + Z2 r2 =1

Metric; h =db=dT (XdX+YdY +ZdZ)+adX dY
```

All of these structures can be computationally solid for the next generations of physics formulations with Python language & Quantum computing across the solid states of galaxies.

```
Using the following notations for at space Killing vectors,
generators of the Poincare group: LX =Y Z Z Y LY =Z X X Z LZ=X Y Y
X PT = T PZ= Z
```

Here is what we described as multiple dimensions in a black hole metric;

```
kab = facfb c = Lab +a(Pa TLb Z +La ZPb T) +a2(Pa TPb T Pa ZPb Z)
```

Killing-vector

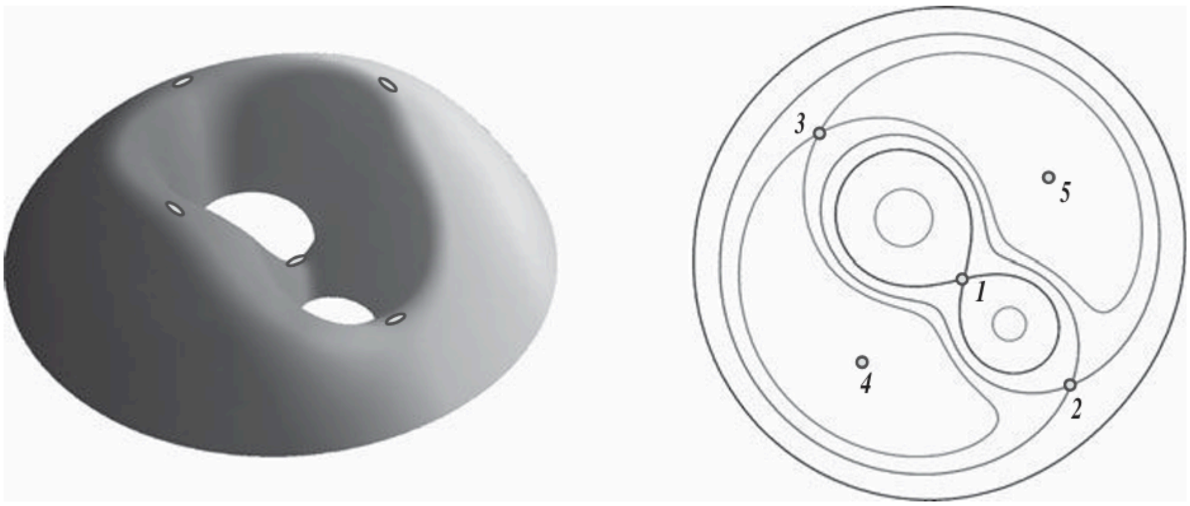
```
The Killing vector is ( ) = 1 3 (3.98) h =PT, while the secondary
Killing vector reads ( ) = k ( ) = a2PT +aLZ.
```

A rotating black hole is called extremal. The spatial distance to the horizon in the limit a M innately grows. It is interesting that some of the hidden symmetries in the vicinity of the horizon of extremal black holes become explicit. Two connected detects take place: the eigenvalues of the principal tensor become functionally dependent, and, besides t and , two new additional Killing vectors arise. Let us discuss the case of the extremal black hole in more detail. We start by noticing that in the extremal limit the function r, (3.8), which enters the Kerr metric.

```
Killing Yano quantities for the metric g (3.100): h =db=(1+z2)d dT
+2 zdz dT +zdz d f = z(1+z2)d dT +2dz dT +dz d The primary Killing
vector is = 1 3 h =

the Killing tensor kab = fa cf c b on does not produce a new
Killing vector, as one has ka b b = a
```

Binary system stars: Mass function for a binary system Letusfirstdiscusshowtodetermine the mass of objects in binary system.Considera non-relativistic two-bodyproblemwiththeLagrangian L=1 2M1˙r2 1+1 2M2˙r2 2−U(|r1−r2|). (1.6.1) Letusdenoter=r1−r2,and choose the center-of-mass coordinate system

M1r1+M2r2=0, r1= M2 M1+M2 r, r2=− M1 M1+M2 r.



An orbit is always planar;

SUMMARY

•An orbit with negative energy Eisa is a closed ellipse. Such Motion Periodic.

•The Major Semi-axis=α/2|E|.

U at the slice Z = 0.Points 1, 2,3,4,5 critical points of the potential where its gradient vanishes. Points 1, 2,and3are located on the X-axis, while the Y-axis is orthogonal to this direction. The right plot shows the equipotential surfaces (at Z = 0).

Black holes-physics/solid structures; • stellar-mass black holes with M ~ 3 − 30M ;
• (super) massive black holes with M ~ 105 − 109M ; Black Holes in Astrophysics and Cosmology 27
• intermediate-mass black holes with M ~ 103M ;

• primordial black holes withmass up to M ;

• micro-black holes.

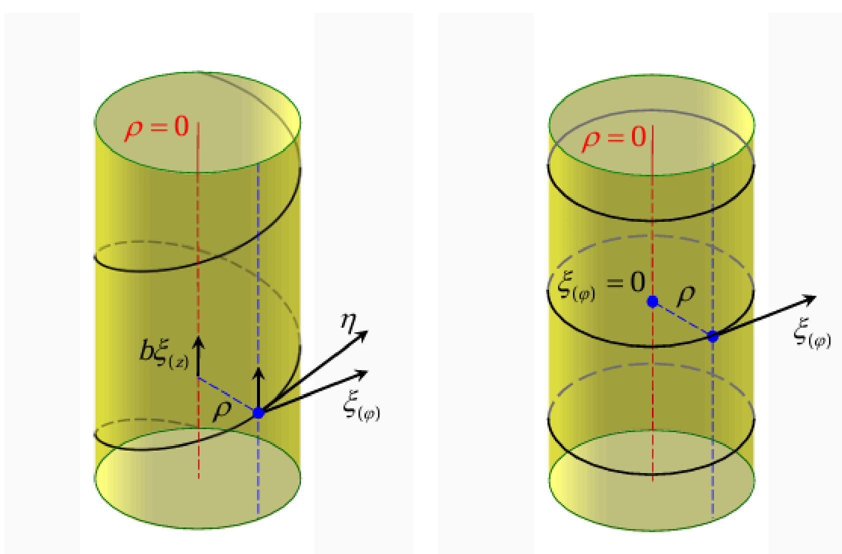
DEFINITION GM2 & dimensionless rotation in a blackhole;

An Astrophysical Black hole is uniquely specified by two parameters: the black hole mass M and its angular momentum, J. Instead of J one can use the dimensionless rotation parameter α = cJ/(GM2).

This parameter vanishes for non-rotating black holes and 1 for an extremely rotating one.For α>1a black hole does not exist. The corresponding formal solution in this case has a naked singularity.

• *A black hole has no rigid boundary surface.*Any matter, falling into a black hole, disappears into its abyss. In the case of a neutron star the interaction of the accreting matter with its surface produces specific radiation. There is no such (classical) emission from the black hole horizon. • The gravitational force grows infinitely at the black hole surface.As a result, there exists an innermost stable circular orbit.For Non-rotating black hole (α = 0) it lies at 6GM/c2,while for the extremely rotating one (α = 1) it liesatGM/c2. • The characteristic orbital frequencies are 2200 Hz(M/M)−1 and16 50Hz(M/M)−1 for the non-rotating and extremely rotating black holes, respectively. • All the mass and rotation multipole moments of the gravitational field of the rotating black hole are uniquely determined by its mass M and the rotation parameter α.

Figures 1.2



Killing vectors with closed and open orbits. Left figure shows the action of the symmetry with non-closed orbits.

```
The Killing vector ( ) in the plane TZ = const. In a general case,
however, a Killing vector eld may not have xed points. For example,
consider a Killing vector = ( ) + (Z). One nds 2 = 2 + 2 > 0

Coordinates:  coordinates (T Z) given by values (T 0Z) and (T 2 Z+2
). This can be reformulated in coordinates adapted to the Killing
vector . If we de ne =Z
```

```
 The vector = ( ) a2 ( )
```

The polynomial y has now two non trivially different roots y and the coordinate y runs between these roots, y (y+y). In this case one can find two candidates for the Killing vector with xed points: + and , with xed points at y = +y and y = y, respectively.

Dimension1:

```
 These2-forms generate both isometries of the metric according to 1
3 h= 1 3 f
```

The C-metric typically describes a pair of black holes moving in the opposite direction with constant acceleration caused either by a cosmic string of negative energy density between them or by two positive-energy strings pulling the black holes from infinity. As the string is present, the corresponding solution does not represent, strictly speaking, a regular isolated black hole.

Higher-dimensional KerrNUT(A)dS metrics

Coordinates naturally split into two sets: Killing coordinates k (k = 0 n 1+) associated with the explicit symmetries, and radial and longitudinal coordinates x (= 1 n) labeling the orbits of Killing symmetries.