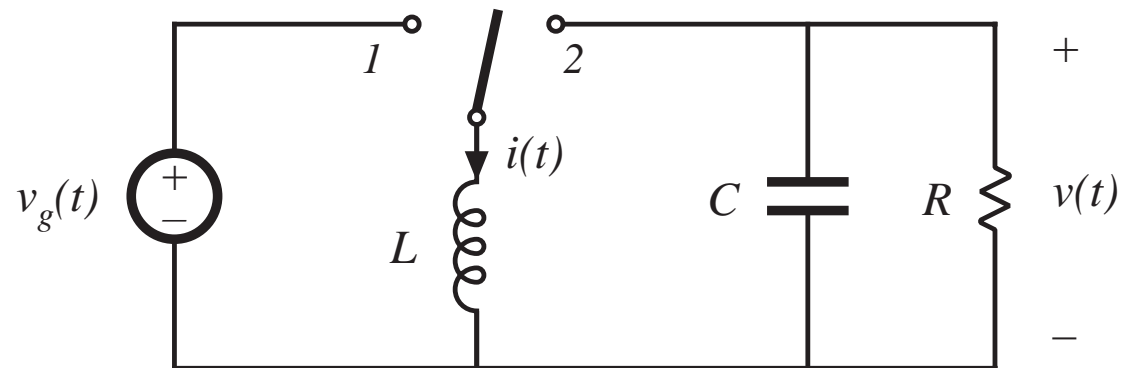


7.2. The basic ac modeling approach

Buck-boost converter example

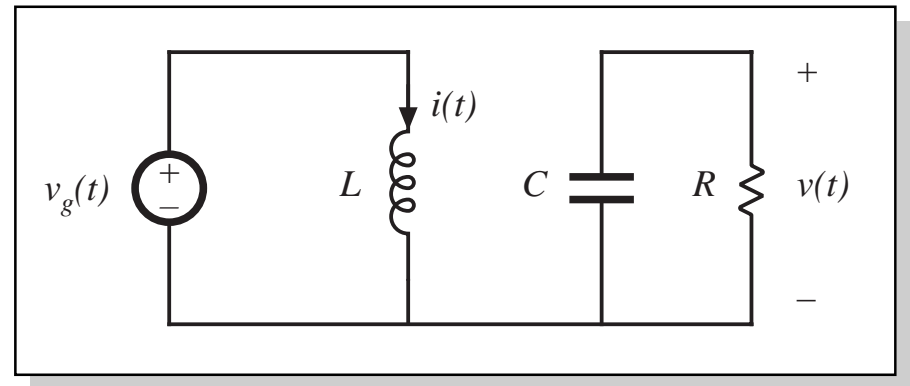


Switch in position 1

Inductor voltage and capacitor current are:

$$v_L(t) = L \frac{di(t)}{dt} = v_g(t)$$

$$i_C(t) = C \frac{dv(t)}{dt} = -\frac{v(t)}{R}$$



Small ripple approximation: replace waveforms with their low-frequency averaged values:

$$v_L(t) = L \frac{di(t)}{dt} \approx \left\langle v_g(t) \right\rangle_{T_s}$$

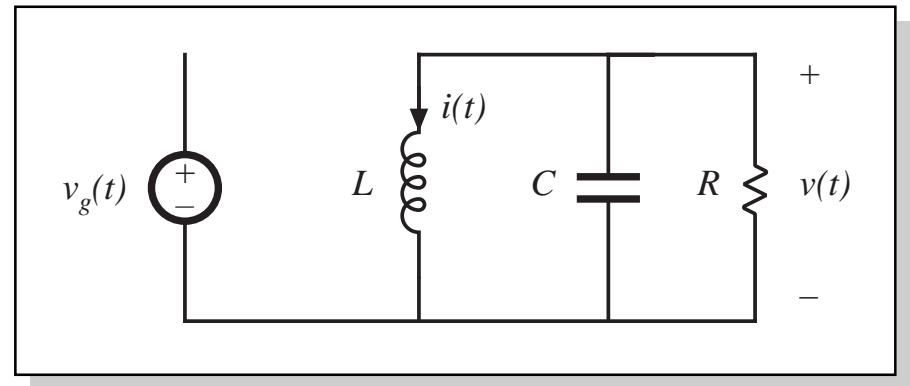
$$i_C(t) = C \frac{dv(t)}{dt} \approx -\frac{\left\langle v(t) \right\rangle_{T_s}}{R}$$

Switch in position 2

Inductor voltage and capacitor current are:

$$v_L(t) = L \frac{di(t)}{dt} = v(t)$$

$$i_C(t) = C \frac{dv(t)}{dt} = -i(t) - \frac{v(t)}{R}$$



Small ripple approximation: replace waveforms with their low-frequency averaged values:

$$v_L(t) = L \frac{di(t)}{dt} \approx \langle v(t) \rangle_{T_s}$$

$$i_C(t) = C \frac{dv(t)}{dt} \approx -\langle i(t) \rangle_{T_s} - \frac{\langle v(t) \rangle_{T_s}}{R}$$

7.2.1 Averaging the inductor waveforms

Inductor voltage waveform

Low-frequency average is found by evaluation of

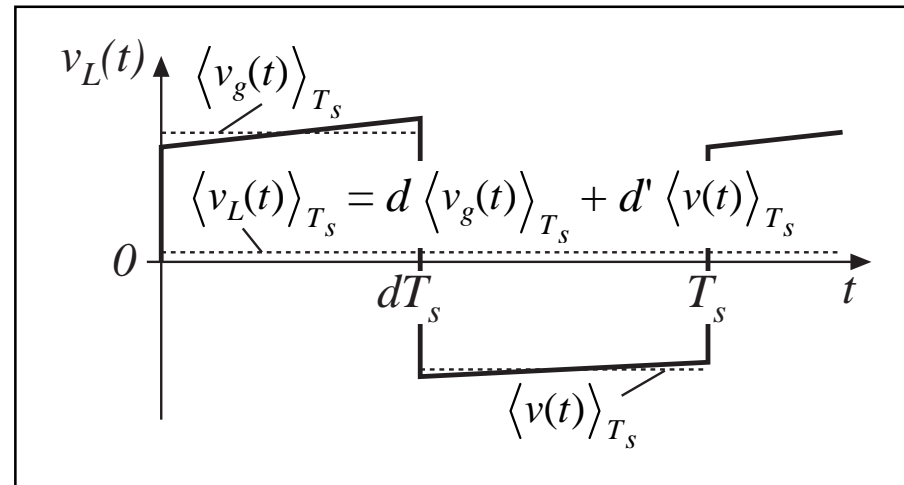
$$\langle x_L(t) \rangle_{T_s} = \frac{1}{T_s} \int_t^{t+T_s} x(\tau) d\tau$$

Average the inductor voltage in this manner:

$$\langle v_L(t) \rangle_{T_s} = \frac{1}{T_s} \int_t^{t+T_s} v_L(\tau) d\tau \approx d(t) \langle v_g(t) \rangle_{T_s} + d'(t) \langle v(t) \rangle_{T_s}$$

Insert into Eq. (7.2):

$$L \frac{d\langle i(t) \rangle_{T_s}}{dt} = d(t) \langle v_g(t) \rangle_{T_s} + d'(t) \langle v(t) \rangle_{T_s}$$



This equation describes how the low-frequency components of the inductor waveforms evolve in time.

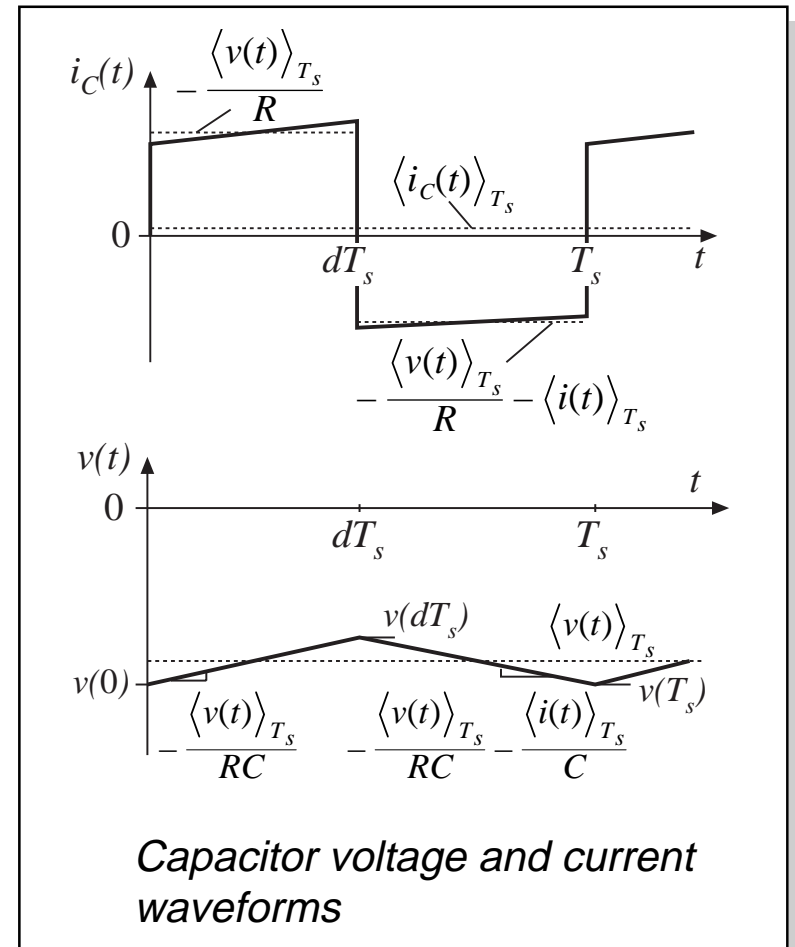
7.2.3 Averaging the capacitor waveforms

Average capacitor current:

$$\langle i_C(t) \rangle_{T_s} = d(t) \left(-\frac{\langle v(t) \rangle_{T_s}}{R} \right) + d'(t) \left(-\langle i(t) \rangle_{T_s} - \frac{\langle v(t) \rangle_{T_s}}{R} \right)$$

Collect terms, and equate to $C d\langle v \rangle_{T_s} / dt$:

$$C \frac{d\langle v(t) \rangle_{T_s}}{dt} = -d'(t) \langle i(t) \rangle_{T_s} - \frac{\langle v(t) \rangle_{T_s}}{R}$$



7.2.4 The average input current

We found in Chapter 3 that it was sometimes necessary to write an equation for the average converter input current, to derive a complete dc equivalent circuit model. It is likewise necessary to do this for the ac model.

Buck-boost input current waveform is

$$i_g(t) = \begin{cases} \langle i(t) \rangle_{T_s} & \text{during subinterval 1} \\ 0 & \text{during subinterval 2} \end{cases}$$

Average value:

$$\langle i_g(t) \rangle_{T_s} = d(t) \langle i(t) \rangle_{T_s}$$

