AULA 24/02/2021

AULA 03/03/2021

Armada em 1 dir: $\frac{l_y}{l_x} \geq 2$

Armada em cruz: $\frac{1}{2} < \frac{l_y}{l_x} < 2$

- h = 10cm
- piso = 1,5 kN/m²
- sobrecarga = 3,0 kN/m²

$\frac{l_y}{l_x} = 3$

1 - Cálculo da carga:

$p_d = 1,5 \times (25 \times 0,10 + 1,5 + 3)$

$p_d = 10,5\ kN/m^2$ → peso próprio ($P_{conc} \times h_{laje}$; 25 kN/m³, 0,10 m)

2 - Cálculo dos esforços: $p_d = 10,5\ kN/m$

$-\frac{10,5 \times 2^2}{8} = -5,25\ kN.m$

$\frac{10,5 \times 2^2}{14,22} = 2,96\ kN.m$

3 - Cálculo da flecha inicial

$W_0 = \frac{k p l^4}{384 D}$, k = 2

$D = \frac{E_{cs} \times h^3}{12(1-\nu^2)}$, $E_{cs} = 0,85 \times 21500 \left(\frac{f_{ck}+8}{10}\right)^{1/3}$

Assumindo $f_{ck} = 30\ MPa$

$E_{cs} = 0,85 \times 21500 \left(\frac{38}{10}\right)^{1/3} = 28518\ MPa$

$D = \frac{28518 \times 0,10^3}{12(1-0,04)} = 2,48 \frac{MN}{m^2} \times m^3 = MN \times m = 2476\ kN.m$

$W_0 = \frac{2 \times 10,5 \times 2^4}{384 \times 2476} = 3,534 \times 10^{-4}\ m = 0,35\ mm$

4 - Cálculo da reação

$\frac{5 p l_x}{8} = 13,125\ kN/m$; $\frac{3 p l_x}{8} = 7,875\ kN/m$

5 - Cálculo da área de aço:

M = 2,96 kN.m; $M_e = -5,25\ kN.m$; $b_w = 100\ cm$; h = 10cm

c = 2,5cm; $f_{ck} = 30\ MPa$; $f_{yk} = 500\ MPa$; φ 8mm

$d = 10 - 2,5 - \frac{3}{2} \times 0,8 = 6,3\ cm$ ($h - c - \frac{3}{2}\phi_L$)

$f_{cd} = \frac{3}{1,4} = 2,14\ kN/cm^2$

$f_{yd} = \frac{50}{1,15} = 43,48\ kN/cm^2$

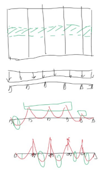
$KMD = \frac{M_d}{b_w \times d^2 \times f_{cd}}$; $A_s = \frac{M_d}{KZ \times d \times f_{yd}}$

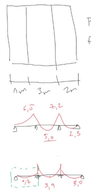
- Armadura positiva:

$KMD = \frac{296}{100 \times 6,3^2 \times 2,14} = 0,035$ → KZ = 0,9759; $A_s = \frac{296}{0,9759 \times 6,3 \times 43,48} = 1,107\ cm^2/m$

- Armadura negativa:

$KMD = \frac{525}{100 \times 6,3^2 \times 2,14} = 0,062$ → KZ = 0,9602; $A_s = \frac{525}{0,9602 \times 6,3 \times 43,48} = 1,996\ cm^2/m$





$p_d = 10,5\ kN/m^2$

$f_{ck} = 30\ MPa$

Momentos

Área

$M_e = 7,2\ kN.m/m$

M 5,0 kN.m/m

- Arm. pos:

$KMD = \frac{500}{100 \times 6,3^2 \times 2,14} = 0,059$

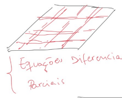
$A_s = \frac{500}{0,9634 \times 6,3 \times 43,48} = 1,89\ cm^2/m$

- Arm neg:

$KMD = \frac{720}{100 \times 6,3^2 \times 2,14} = 0,085$

$A_s = \frac{720}{0,9472 \times 6,3 \times 43,48} = 2,77\ cm^2/m$

AULA 10/03/2021

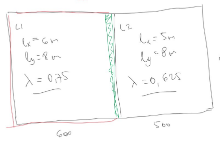


Equações Diferenciais Parciais

- Vigas rígidas
- Vigas flexíveis

$M_e = $ { 80% max mom. neg. ; média mom.

CASO 2



h = 10cm

$p_d = 10\ kN/m^2$

$f_{ck} = 30\ MPa$

↳ D = 2476 kN.m

$M = 0,001\ m_x\ p l_x^2$ [kN] [kN.m/m]

L1: λ = 0,75

$w_c = 2,70$ → $W_c = 0,001 \times 2,7 \times \frac{10 \times 6^4}{2476} = 0,014\ m$

$m_{xe} = -28,8$ → $M_{xe} = 0,001 \times (-28,8) \times 10 \times 6^2 = -10,368\ kN.m/m$

$m_x = 28,8$ → $M_x = 10,368\ kN.m/m$

$m_y = 46$ → $M_y = 0,001 \times 46 \times 10 \times 6^2 = 16,56\ kN.m/m$

$r_x = 246$ → $R_x = 0,001 \times 246 \times 10 \times 6 = 14,67\ kN/m$

$r_y = 201$ → $R_y = 12,06\ kN/m$

86% 10,4 = 8,3

$\frac{10,4 + 4,4}{2} = 5,9$

L2: λ = 0,625 (interpolar valores λ = 0,60 e λ = 0,65)

$w_c = 1,44$ → $W_c = 3,6 \times 10^{-3}\ m$

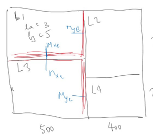
$m_{xe} = -17,3$ → $M_{xe} = -4,325\ kN.m/m$

$m_x = 17,3$ → $M_x = 4,325\ kN.m/m$

$m_y = 35,25$ → $M_y = 8,8125\ kN.m/m$

$r_x = 222$ → $R_x = 11,1\ kN/m$

$r_y = 138$ → $R_y = 6,9\ kN/m$



h = 8cm

$f_{ck} = 40\ MPa$

$p = 12\ kN/m^2$

$E_{cs} = 0,85 \times 21500 \left(\frac{40+8}{10}\right)^{1/3} = 30827\ MPa$

$D = \frac{30827 \times 0,08^3}{12 \times (1-0,2^2)} = 1370\ kN.m$

L1: λ = 0,6

$w_c = 0,9$ → $W_c = 0,001 \times 0,9 \times \frac{12 \times 3^4}{1370} = 6,4 \times 10^{-4}\ m$

$m_{xe} = -9,6$ → $M_{xe} = 0,001 \times (-9,6) \times 12 \times 3^2 = -1,04\ kN.m/m$

$m_{ye} = -26,6$ → $M_{ye} = 0,001 \times (-26,6) \times 12 \times 3^2 = -2,9\ kN.m/m$

$m_x = 9,6$ → $M_x = 1,04\ kN.m/m$

$m_y = 26,6$ → $M_y = 2,9\ kN.m/m$

$r_{xe} = 310$ → $R_{xe} = 0,001 \times 310 \times 12 \times 3 = 11,16\ kN/m$

$r_x = 221$ → $R_x = 7,96\ kN/m$

$r_{ye} = 67$ → $R_{ye} = 2,41\ kN/m$

$r_y = 48$ → $R_y = 1,73\ kN/m$

24/03/2021

Premissas de projeto:

h = 10cm → pp = 2,5 kN/m²

C30

- peso revestimento 1,5 kN/m²
- sobrecarga 3,0 kN/m²
- contrapiso (1cm / γ 23 kN/m³) = 0,23 kN/m²

$p_d = 1,5 \times (2,5 + 1,5 + 3,0 + 0,23) = 11,0\ kN/m^2$

$f_{ck} = 30\ MPa$

$f_{yk} = 500\ MPa$

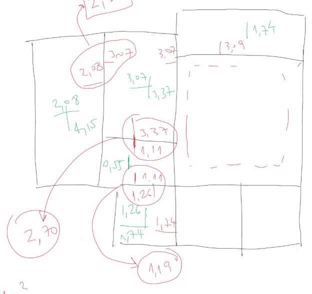
$E_{cs} = 0,85 \times 21500 \left(\frac{30+8}{10}\right)^{1/3} = 28518\ MPa$

$D = \frac{E_{cs} \times h^3}{12(1-\nu^2)} = \frac{28518000 \times 0,1^3}{12(1-0,2^2)} = 2476\ kN.m^2/m$

Laje	l_x	l_y	λ	
L1	3,0	4,45	0,67	cruz
L2	3,0	3,35	0,90	cruz
L3	1,10	3,0	0,37	1 dir
L4	2,0	2,35	0,85	cruz
L5	2,0	2,0	1.0	cruz
L6	3,5	5,5	0,64	cruz
L7	1,5	5,5	0,27	1dir
L8	2,0	3,35	0,60	cruz

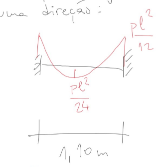
80% maior mom.

$\frac{m_1 + m_2}{2}$



Começando pelas lajes armadas em uma direção:

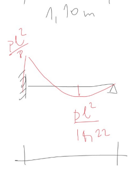
L3



$M = \frac{11 \times 1,1^2}{24} = 0,55\ kN.m/m$

$M^e = \frac{11 \times 1,1^2}{12} = 1,11\ kN.m/m$

L7



$M = \frac{11 \times 1,5^2}{14,22} = 1,74\ kN.m/m$

$M^e = \frac{11 \times 1,5^2}{8} = 3,09\ kN.m/m$

$x_1 \quad y_1$

$x \quad y$

$x_2 \quad y_2$

$\frac{x - x_2}{x_1 - x_2} = \frac{y - y_2}{y_1 - y_2}$

$$y = \frac{(y_1 - y_2)(x - x_2)}{(x_1 - x_2)} + y_2$$

Lajes armadas em cruz

L1 λ = 0,67, Caso 2, $l_y = 4,45m$, $l_x = 3m$

$m_{xe} = -21,0$ → $M^{xe} = 0,001 \times p \times l_x^2 \times m_{xe} = 0,001 \times 11 \times 3^2 \times 21 = 2,08\ kN.m/m$

$m_x = 21,0$ → $M_x = 2,08\ kN.m/m$

$m_y = 41,9$ → $M_y = 0,001 \times 11 \times 3^2 \times 41,9 = 4,15\ kN.m/m$

L2 λ = 0,9, Caso 5, $l_x = 3m$

$m_{xe} = -31$ → $M_{xe} = 0,001 \times 11 \times 3^2 \times 31 = 3,07\ kN.m/m$

$m_{ye} = -34$ → $M_{ye} = 3,37\ kN.m/m$

$m_x = 31$ → $M_x = 3,07\ kN.m/m$

$m_y = 34$ → $M_y = 3,37\ kN.m/m$

L4 λ = 0,85, Caso 4, $l_x = 2,0m$

$m_{xe} = -28,6$ → $M_{xe} = 1,26\ kN.m/m$

$m_{ye} = 39,6$ → $M_{ye} = 1,74\ kN.m/m$

$m_x = 28,6$ → $M_x = 1,26\ kN.m/m$

$m_y = 39,6$ → $M_y = 1,74\ kN.m/m$

$p_{d,ELS} = 7,23\ kN/m^2$

$w_c = 1,97$

$W_c = 0,001 \times w_c \times \frac{p l^4}{D} = 0,001 \times 1,97 \times \frac{7,23 \times 3^4}{2476}$

$W_c = 4,7 \times 10^{-4}\ m = 0,47\ mm$

$f_{yk} = 500\ MPa$ → $f_{yd} = \frac{500}{1,15} = 43,48\ kN/cm^2$

Adotar inicialmente φ = 8mm

$M_y = 4,15\ kN.m/m$

$M_x = 3,07\ kN.m/m$

$h = 10cm$ → $d = 10 - 0,8 - \frac{0,8}{2} - 2,5 = 6,3\ cm$

$f_{ck} = 30\ MPa$

↳ $f_{cd} = \frac{30}{1,4} = 21,43\ MPa$

M

y:

$KMD = \frac{M_d}{b_w \times d^2 \times f_{cd}} = \frac{415}{100 \times 6,3^2 \times 2,143} = 0,049$

$KZ = 0,9697$ → $A_{sy} = \frac{M_d}{KZ \times d \times f_{yd}} = \frac{415}{0,9697 \times 6,3 \times 43,48}$

$A_{sy} = 1,56\ cm^2/m$

$A_{sy,min} = 0,17 \times 100 \times 0,10 = 1,70\ cm^2/m$

↳ Adotar φ 5mm c.c 10cm