

1. a)

$$P(A, E) = P(E=1 | A=1) P(A)$$

$$P(E=1 | A=1) \cdot P(A=1) = P(A=1 | E=1) P(E=1)$$

$$P(E=1 | A=1) = \frac{P(A=1 | E=1) P(E=1)}{P(A)}$$

$$P(A=1 | E=1) = P(A=1 | E=1, B=0) P(B=0) \\ + P(A=1 | E=1, B=1) P(B=1)$$

$$P(A=1 | E=1) = 0,29066$$

$$P(A=1) = P(A=1 | E=0, B=0) P(E=0) P(B=0)$$

$$+ P(A=1 | E=1, B=0) P(E=1) P(B=0)$$

$$+ P(A=1 | E=0, B=1) P(E=0) P(B=1)$$

$$+ P(A=1 | E=1, B=1) P(E=1) P(B=1)$$

$$= 0,0025$$

(since $P(B)P(E) = P(B, E)$)

$$P(E=1 | A=1) = \frac{0,29066 \cdot 0,002}{0,0025} = 0,23$$

answer: 23%

$$\begin{aligned}
 1 \text{ b) } P(E=1 | A=1, B=0) &= \frac{P(A=1, B=0 | E=1) \cdot P(E=1)}{P(A=1, B=0)} \\
 &= \frac{P(A=1 | B=0, E=1) \cdot P(B=0) \cdot P(E=1)}{P(A=1 | B=0) \cdot P(B=0)}
 \end{aligned}$$

$$\begin{aligned}
 P(A=1 | B=0) &= P(A=1 | B=0, E=0) P(B=0) P(E=0) \\
 &\quad + P(A=1 | B=0, E=1) P(B=0) P(E=1)
 \end{aligned}$$

$$\begin{aligned}
 P(A=1 | B=0) &\approx 0,001 \cdot 0,999 \cdot 0,998 + 0,94 \cdot 0,001 \cdot 0,002 \\
 &= 0,0019
 \end{aligned}$$

$$P(E=1 | A=1, B=0) = \frac{P(A=1 | B=0, E=1) P(E=1)}{P(A=1 | B=0)} = \frac{0,94 \cdot 0,002}{0,0019}$$

$$\text{answer: } \frac{94}{95}$$

$$1c) P(A=1 | M=1) = \frac{P(M=1 | A=1) P(A=1)}{P(M=1)}$$

$$P(M=1) = P(M=1 | A=1) P(A=1) + P(M=1 | A=0) P(A=0) \\ = 0,7 \cdot 0,0025 + 0,01(1 - 0,0025)$$

$$P(A=1 | M=1) = \frac{0,7 \cdot 0,0025}{0,0117}$$

answer: 0,1496

$$1d) P(A=1 | M=0, J=0) = \frac{P(M=1, J=0 | A=1) P(A=1)}{P(M=1, J=0)} \\ = \frac{P(M=1 | A=1) P(J=0 | A=1) P(A=1)}{P(M=1) P(J=0)}$$

$$P(M=1) = P(M=1 | A=1) P(A=1) + P(M=1 | A=0) P(A=0) \\ = 0,7(1 - 0,0025) + 0,01 \cdot 0,0025 = 0,698$$

$$P(J=0) = 1 - (0,9(1 - 0,0015) + 0,05 \cdot 0,0025) = 0,101$$

$$P(A=1 | M=0, J=0) = \frac{0,7 \cdot 0,05 \cdot 0,0025}{0,698 \cdot 0,101}$$

answer: 0,0012

$$e) P(A=1 | M=0) = \frac{P(M=0 | A=1) P(A=1)}{P(M=0)}$$

$$P(M=0 | A=1) = 1 - P(M=1 | A=1)$$

answer: 0,3

$$f) P(A=1 | B=1, M=0) = \frac{P(A=1, B=1 | M=0)}{P(B=1)}$$

$P(A$

2a)

$$P(S_k = 1 \mid D = 0) = \frac{1}{2^k + (-1)^k}$$

$$P(D = 0 \mid S_1, \dots, S_k) = \frac{P(S_1, \dots, S_k \mid D = 0) P(D = 0)}{P(S_1, \dots, S_k)}$$

$$r_k = \frac{\frac{1}{2} \cdot \frac{1}{2^k + (-1)^k}}{\left(\frac{1}{2}\right)^k \cdot \frac{1}{2}} = \frac{1}{1 + \left(-\frac{1}{2}\right)^k}$$

The doctor's diagnosis depends on the day in the way that it matters if the k th day is even or odd. Is k even/odd

b) It becomes less certain.
Since the ratio converges to 1.
~~at~~

$$b) \quad P(Z=1 | X=0, Y=0) = 1 - (1-p_x)^0 (1-p_y)^0 = 0$$

$$P(Z=1 | X=0, Y=1) = 1 - (1-p_y) = p_y > 0$$

$$P(Z=1) < P(Z=1 | X=0, Y=1)$$

$$b) \quad P(Z=1 | X=1, Y=0) = 1 - (1-p_x) = p_x$$

$$P(Z=1 | X=0, Y=1) = p_y$$

$$p_x < p_y$$

$$c) \quad P(Z=1 | X=1, Y=0) = p_x$$

$$P(Z=1 | X=1, Y=1) = 1 - (1-p_x)(1-p_y) = p_y + p_x - p_x p_y$$

$$p_x < p_x + p_y - p_x p_y$$

$$p_x p_y < p_y, \text{ True since } p_x < 1$$

$$d) \quad P(X=1) < P(X=1 | Z=1)$$

$$f) \quad P(X=1 | Z=1) > P(X=1 | Y=1, Z=1)$$

$$g) \quad P(X=1)P(Y=1)P(Z=1) < P(X=1, Y=1, Z=1)$$

2.3

1. F

2. T

3. T

4. F

5. T

6. T

7. F

8. T

9. T

10. F

4

a) F

b) T

c) F

d) F

e) F

f) F

g) T

h) F

i) T

j) T