1.0 Introduction

My SURG project is aimed at creating and implementing a mathematical model to improve precision of seismic data from Mars. Anticipated results from the project are applicable in studies to determine the origins and failure mechanisms of low-frequency quakes, particularly in regions of both Earth and Mars that do not have an abundance of seismic instruments for data collection.

The overall project centers around the broad research question: *"What mathematical model and implementation probabilistically incorporates different errors into faulting mechanisms of low-frequency quakes?"* Since one seismometer is used for data collection, we need to account for probability because the model used rotates seismic radiation patterns in an abstract space to have a view that a different seismometer would have otherwise had. Abstract rotation compounds the related error with a probability dependent on the model's geometry. My research this summer is thus set in two phases: one for creating the model and the other for testing and implementing it. It is a continuation of prior work I have done on the same topic.

2.0 Research Background

The vast majority of earthquakes and marsquakes ("earthquakes" that occur on Mars) represent shear failure mechanisms which radiate seismic waves in specific, calculable patterns called radiation patterns (Shearer 2019, 246-255). These patterns, in combination with the direction from and distance at which the seismic waves they produce are recorded, determine the relative amplitudes of their three constituent body waves: P, SV and SH waves (Shearer 2019, 42-49). Measurements of these body waves are used to constrain the sheer failure mechanism of the quake which paints a picture of stress distribution in the Martian crust. The stress distribution can give insight into geodynamic processes on Mars, a planet thought not to be as geologically active as Earth since it is smaller and colder.

In a study done by Madelyn Sita and Suzan van der Lee (Sita and van der Lee 2022), waveform data recorded from the InSight mission to Mars was used to determine the faulting mechanisms of their source quakes, which after accounting for the noise were represented visually as "beachballs" (Appendix B). Different beachball orientations correspond to different combinations of amplitudes of the three body waves that define the mechanism. I was involved in this study to more precisely define the mathematical model that generates these beachballs using methods in algebraic geometry – a more robust way to account for said noise by working with an abstraction of the data parameters in 3D space (see Mars Project Report for details). While the 3 waves need 3 different noise parameters, the previous model had averaged them out to the same parameter for ease of calculation using a sphere of errors. My model was aimed at considering each parameter separately as an ellipsoid of errors to get a more precise error distribution.

The new model, while more precise in theory, left open questions that I intend to tackle over the summer guided by the specific research question: *"How can analytic and physical errors be probabilistically incorporated into faulting mechanisms to account for variation in uncertainty of both seismic waveform data and its defining model?"* Analytic errors come from the model's geometry, while physical errors come from the waveform's noise parameters.

Broken down, this has two avenues of exploration: 1) Would the new model create a statistically significant difference compared to the previous model when tested with seismic data? This is a necessary next step to tie the model back to the original waveform data from which it was developed, and answering it gives a reliable success metric for the project. 2) Does it necessitate a revision of the underlying probability distribution initially used to approximate the noise? This avenue is a consequence of the model itself, since the method that led to it inherently generates a trigonometric probability distribution. Combining this with the Gaussian (normal) distribution that depends only on the noise may create a new aggregate distribution which is different enough that it cannot be ignored, but similar enough that accuracy is not sacrificed.

3.0 Methodology

This project is ideally going to happen in two phases, each set to meaningfully answer the two sub-questions earlier described. Phase 1 involves data testing to verify and quantify the improvement in precision caused by the new model. Before testing it on the actual data from the InSight mission that ties back to the broader study, I will perform two preliminary tests to ensure that the model works and is statistically viable. For the first test, I will use Python to generate data points representing waveform amplitudes (as vectors) that have similar fundamental properties to those of a normal fault, following which I will generate corresponding beachballs to compare against those from the previous model. The generated vectors act as simulations of faulting mechanisms of a normal fault. The improvement in precision will be verified if more beachballs from the new model can be accurately approximated to the reference mechanism than those from the previous one, and fewer inaccurate approximations are made in a similar fashion.

In order to further quantify this improvement, I will perform the second preliminary test with sample data from a seismic station on Earth which already has more accurate reference values contributed to by many seismometers to compare against, as opposed to Mars which has datasets recorded by the sole seismometer used in the InSight mission. The multiple stations on Earth provide access to the authoritative failure mechanisms available for earthquake measurements, so comparing the model's results against them is a better way to quantify its success. Moreover, performing a test on Earth data using a model based on Mars data verifies that the model is independent of the source planet, meaning it could be similarly applied to sparsely instrumented regions on earth that are not insusceptible to earthquakes (e.g. the East African Rift Valley, my home region). I will thus make comparisons between current and previous models by quantifying deviations of each model's approximation from the already reliable approximation used in the Earth station's seismic data.

Phase 2 will involve testing two versions of the new model on Mars data (sample data in Appendix A) in order to determine which one works better. The first version is the same model tested in phase 1, and the second is a revised version that combines the trigonometric and Gaussian probability distributions described by the second sub-question. Before testing, I will create the revised version of the new model using geostatistical methods, after which I will compare both versions against the current one used in Sita and Van der Lee's paper, as this is the only point of reference available. I will quantify deviations using a similar method as in the Earth data test, but this time with a focus on how large the difference in deviations is between the two versions. If this difference is statistically significant, I will adopt the second version as the final and accordingly produce a more reliable dataset for further application in the broader study.

4.0 Qualifications

I am applying for a summer research grant because I have been critically involved in the work leading up to it, for instance the abstraction techniques used to generate the defining model. I also submitted an abstract based on this work to the American Geophysical Union (AGU) for presentation at their Annual Fall Meeting in December 2022 – a presentation that concluded with the avenues for further research highlighted in this proposal. As such, I am in the best position to carry it forward, which the summer will avail me the time and, hopefully, the resources for.

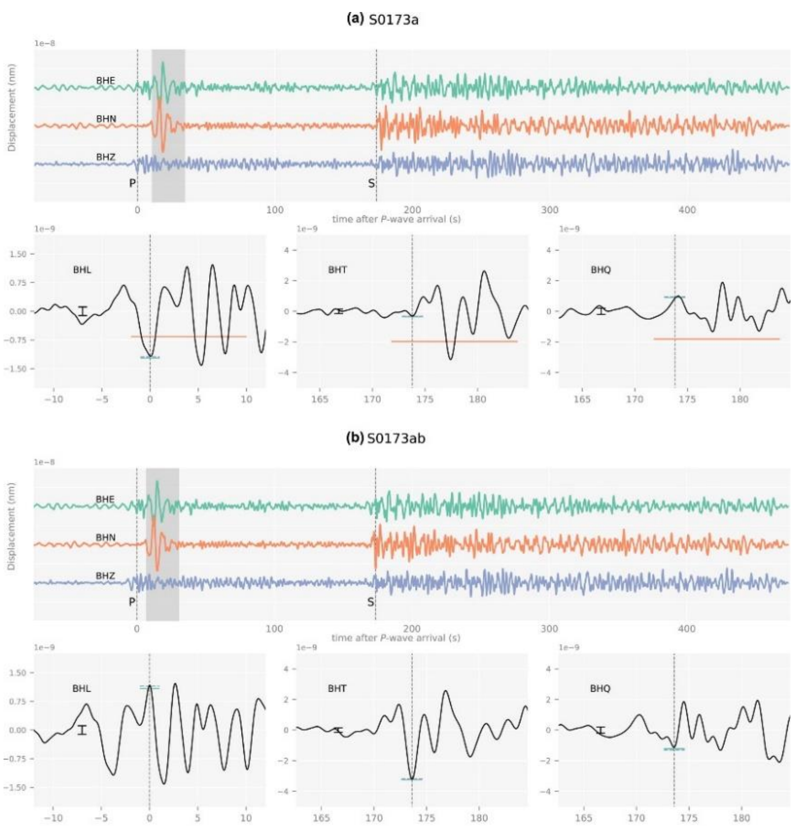
With regards to prerequisite skills, I will have taken all the relevant coursework related to vector calculus (necessary for the geometric abstraction used in the model), probability/statistics (for the relevant statistical analysis), and programming (used to generate and read simulated and actual data for testing). I also have modelling experience with discrete and continuous variables, got from coursework and prior research experience.

References

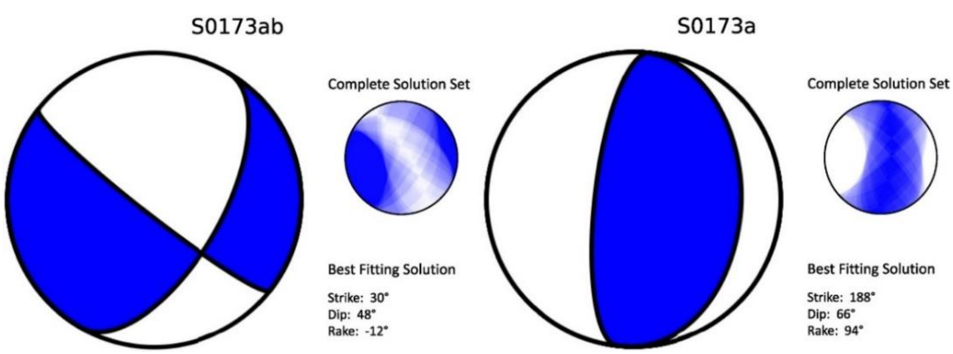
- Sita, Madelyn, and Suzan van der Lee. 2022. “Potential Volcano-Tectonic Origins and Faulting Mechanisms of Three Low-Frequency Marsquakes Detected by a Single InSight Seismometer.” *Journal of Geophysical Research: Planets* 127 (10). https://doi.org/10.1029/2022je007309.
- Shearer, Peter M. 2019. *Introduction to Seismology*. 3rd ed. London: Cambridge University Press.
- Agaba, Victor. 2022. “Tolerance Estimation of Low-Frequency Marsquakes using Ellipsoidal Trilateration.” *Mars Project Report*. https://drive.google.com/file/d/17W_ccV5KeqGfvP1gcub_Pmoa5xNrJu_V/view.

Appendices

Appendix A: Sample seismic data from which waveform amplitudes are extracted



Appendix B: Example of beachball visual



Appendix C: COVID-19 Contingency Plan

The bulk of my research involves mathematical analysis and computer/online work, which I do not need to be outdoors to conduct. In case restrictions related to COVID-19 are imposed, I will thus be able to carry out my research as intended from a safe distance.