Single-seismometer Focal Mechanism and Uncertainty Estimation from Body Waves

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Introduction & Background

In a study done by Sita and van der Lee (2022), waveform data from NASA's InSight mission is used to determine the focal mechanisms of marsquakes. P- and S-wave displacement amplitudes are collected into a 3D vector

$$\mathbf{A} = (A^P, A^{SV}, A^{SH})^t$$

with corresponding noise levels estimated from seismograms recorded by a single station.

Assuming a double-couple source, the relationship between focal mechanism and observed amplitudes can be modeled using far-field body wave expressions. However, in data-sparse environments—such as Mars, where we often have only one seismometer and limited subsurface information—**relative amplitudes** become more practical to use than absolute ones.

Building on this foundation, our research introduces a mathematically robust and computationally efficient method for estimating source parameters (strike, dip, and rake). Our approach accounts for **asymmetric noise levels** across components and identifies all mechanisms that fit the data well.

Mathematical Model

Given a ray path's azimuth ϕ and take-off angles i, j of P and S waves respectively, we model the amplitudes as functions of the source mechanism's strike ψ, dip δ and rake λ, further accounting for the P and S velocities α_h and β_h at source depth h. The expressions are

$$A^P \sim \frac{s_R(3\cos(i) - 1) - q_R \sin(2i) - p_R \sin^2(i)}{\alpha_h^3}$$

$$A^{SV} \sim \frac{1.5 s_R \sin(2j) + q_R \cos(2j) + 0.5 p_R \sin(2j)}{\beta_h^3}$$

$$A^{SH} \sim \frac{q_L \cos(j) - p_L \sin(j)}{\beta_h^3}$$

$$s_R = 0.5 \sin(\lambda) \sin(2\delta)$$

$$q_R = \sin(\lambda)\cos(2\delta)\sin(\psi_r) + \cos(\lambda)\cos(\delta)\cos(\psi_r)$$

$$p_R = \cos(\lambda)\sin(\delta)\sin(2\psi_r) - 0.5\sin(\lambda)\sin(2\delta)\cos(2\psi_r)$$

$$p_L = 0.5\sin(\lambda)\sin(2\delta)\sin(2\psi_r) + \cos(\lambda)\sin(\delta)\cos(2\psi_r)$$

$$q_L = -\cos(\lambda)\cos(\delta)\sin(\psi_r) + \sin(\lambda)\cos(2\delta)\cos(\psi_r)$$

We use **cosine similarity** between observed and synthetic amplitude vectors as the misfit function.

$$\Phi(\boldsymbol{m}) = \frac{-\boldsymbol{A}_o^t \boldsymbol{A}_s}{\|\boldsymbol{A}_o\| \|\boldsymbol{A}_s\|}$$

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Visualizations

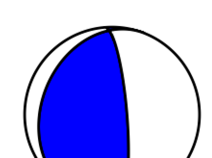

Figure 1: Synthetic focal mechanism to recover.

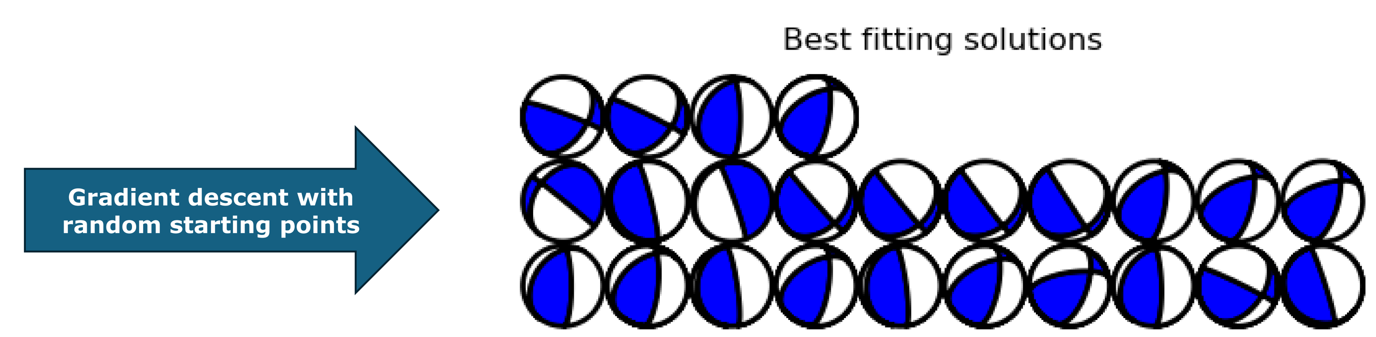

Figure 2: Complete set of best-fitting focal mechanisms on synthetic data.

Collection 1: Using algorithmic implementation on synthetic data.

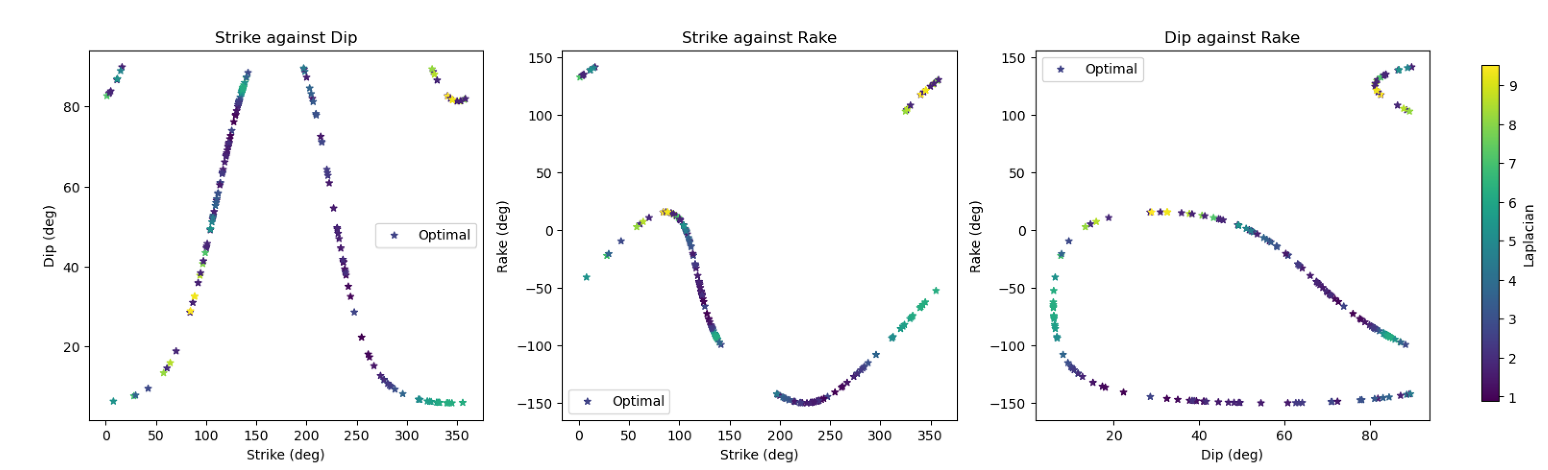

Figure 3: Complete set of best-fitting solutions in strike-dip-rake space. Both fault planes are included, so each solution appears with its mirror image. The Laplacian, estimated using finite differences, gives information about local curvature for each solution.

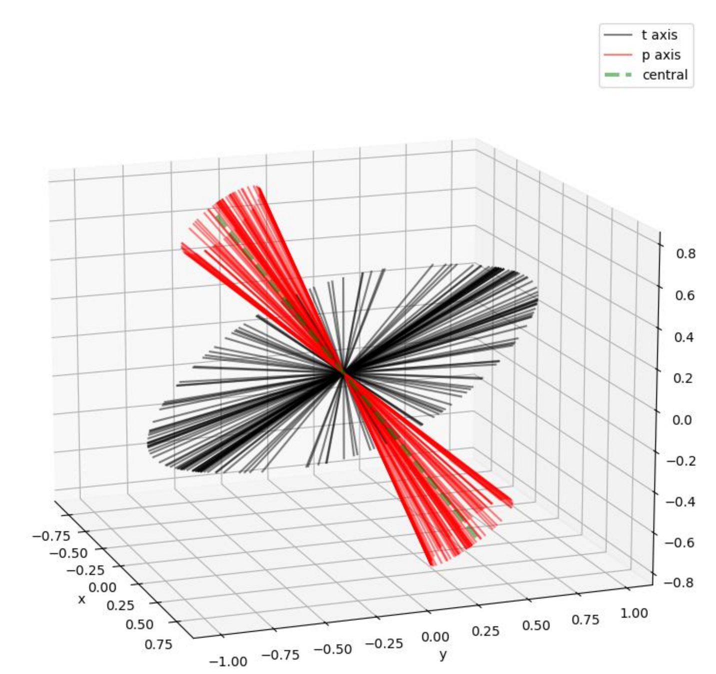

Figure 4: Complete set of best-fitting solutions visualized in T-axis (black) and P-axis (red) space. Each solution corresponds to exactly one pair of orthogonal axes. The P-axes sweep out a narrow cone.

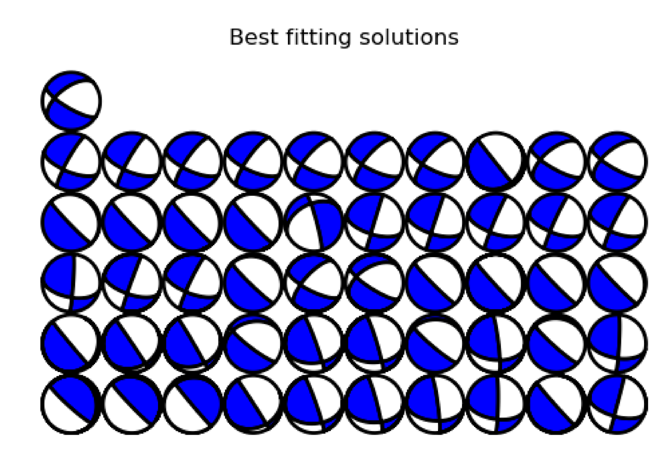

Figure 5: Complete set of best-fitting solutions visualized as beachballs.

Collection 2: Using algorithmic approach on earthquake data from MBAR, Uganda.

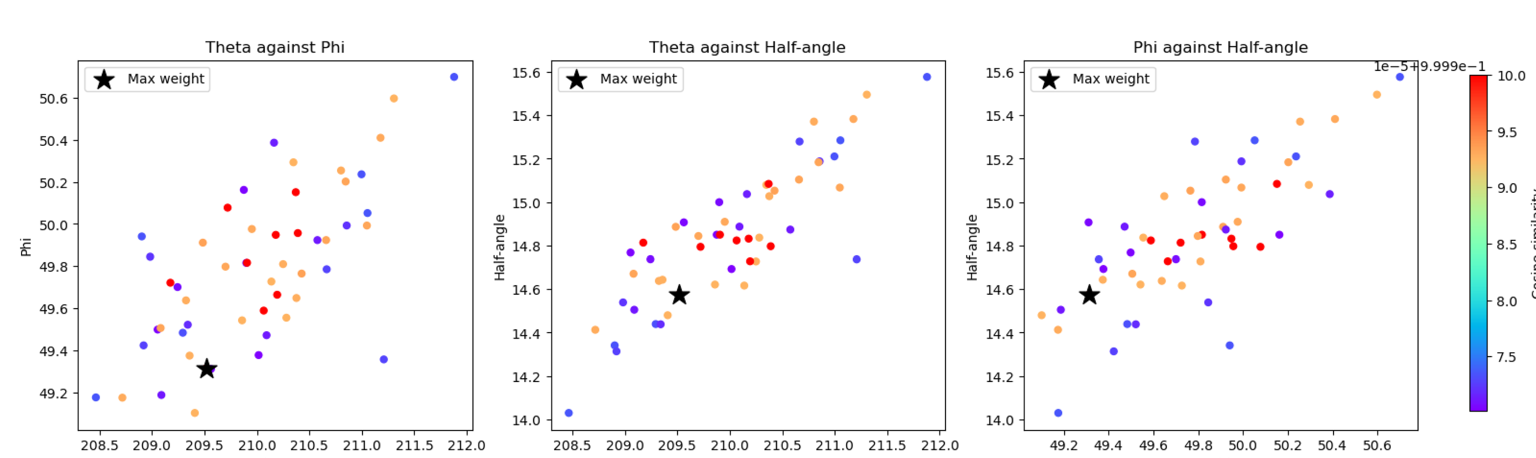

Figure 6: Distribution of acceptable mechanism classes accounting for asymmetric noise. Each point describes a "cone" of mechanisms using angular parameters. For higher signal-noise ratios, the effect of nonlinearity is minimized.

Collection 3: Using Monte Carlo experiment to observe distribution of all classes of mechanisms that produce acceptable amplitude vectors.

Algorithmic Approach

For a highly nonlinear multivariable misfit function whose landscape is difficult to visualize directly in 4D space, we use an algorithmic approach to parameter estimation.

Using a computational graph, we can calculate gradients by backpropagation. This makes **gradient descent** effective for locating local minima. By **random sampling** of starting points, we can trace out classes of best fitting solutions.

We found that the best-fitting mechanisms usually have a stress axis sweeping out a narrow cone. We can classify mechanisms using a three angles to describe the cone, an indicator of the axis.

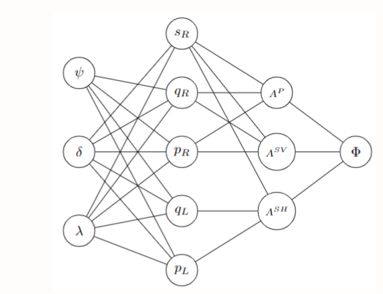

Figure 7: Computational graph for gradient calculations.

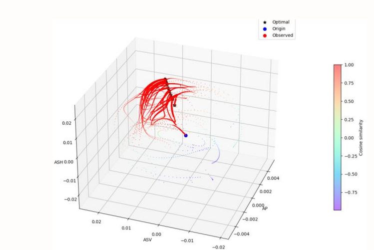

Figure 8: Demonstration of gradient descent. Optimal solutions produce amplitudes lying on a straight line.

Uncertainty Measurement

Using **Monte Carlo methods**, we can investigate the distribution of mechanism classes which produce acceptable amplitudes.

An amplitude vector $\boldsymbol{A}_s$ is acceptable if it is parallel to a vector in the error ellipsoid surrounding the observed vector $\boldsymbol{A}_o$.

Given noise matrix $\boldsymbol{\Sigma}$, the vector $\boldsymbol{A}_s$ must lie on the line segment between boundary points $\boldsymbol{A}_b$ defined by

$$\boldsymbol{A}_b = \left(1 - \frac{1}{C_{oo}}\right)\boldsymbol{A}_o \pm \left(\frac{1}{C_{oo}}\sqrt{\frac{C_{oo} - 1}{C_{oo}C_{ss} - C_{os}^2}}\right)(C_{oo}\boldsymbol{A}_s - C_{os}\boldsymbol{A}_o)$$

Constants of the form C_{xy} are matrix inner products given by

$$C_{xy} = \langle \boldsymbol{A}_x, \boldsymbol{A}_y \rangle_{\boldsymbol{\Sigma}^{-1}} = \boldsymbol{A}_x{}^t \boldsymbol{\Sigma}^{-1} \boldsymbol{A}_y$$

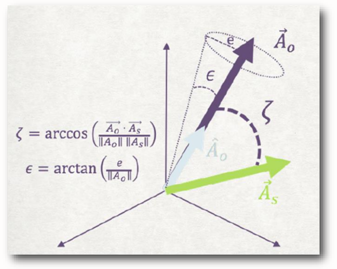

Figure 9: Old method for uncertainty quantification using spherical approximation does not account for asymmetry.

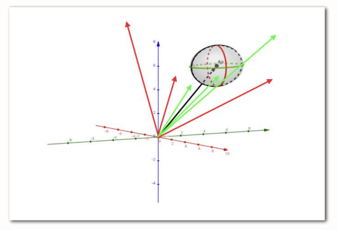

Figure 10: New method for uncertainty quantification accounts for asymmetry using noise ellipsoids.

Conclusion

When using a single seismometer and in data-sparse settings, our methodology is more reliable in:

- **Accuracy:** It can characterize entire classes of best-fitting mechanisms, which a grid search cannot do.
- **Efficiency:** Gradient descent is much faster than brute force inverse methods, and it makes use of the gradient information available.
- **Robustness:** It respects any asymmetries in measured noise levels for P and S waves without any major tradeoffs.

However, we have highly nonlinear transformations which make the method less reliable at low signal-noise ratios.

References:

- Sita, M., and S. van der Lee, Potential Volcano-Tectonic origins and Faulting Mechanisms of Three Low-Frequency Marsquakes Detected by a Single Insight Seismometer, JGR Planets, doi:10.1029/2022JE007309, 2022.
- Agaba, V., M. Sita, and S. van der lee, Tolerance Estimation of Marsquake Wave Amplitudes, AGU Fall Meeting, Chicago, 2022.