Computational Intelligence
Master AI
2020-2021

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Introduction to Fuzzy Computation

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Exam: 7/1/2021

FINAL = 35% Exam + 50% Pract + 5% LabExerFuzzy + 5% LabExerNN + 5% LabExerGA
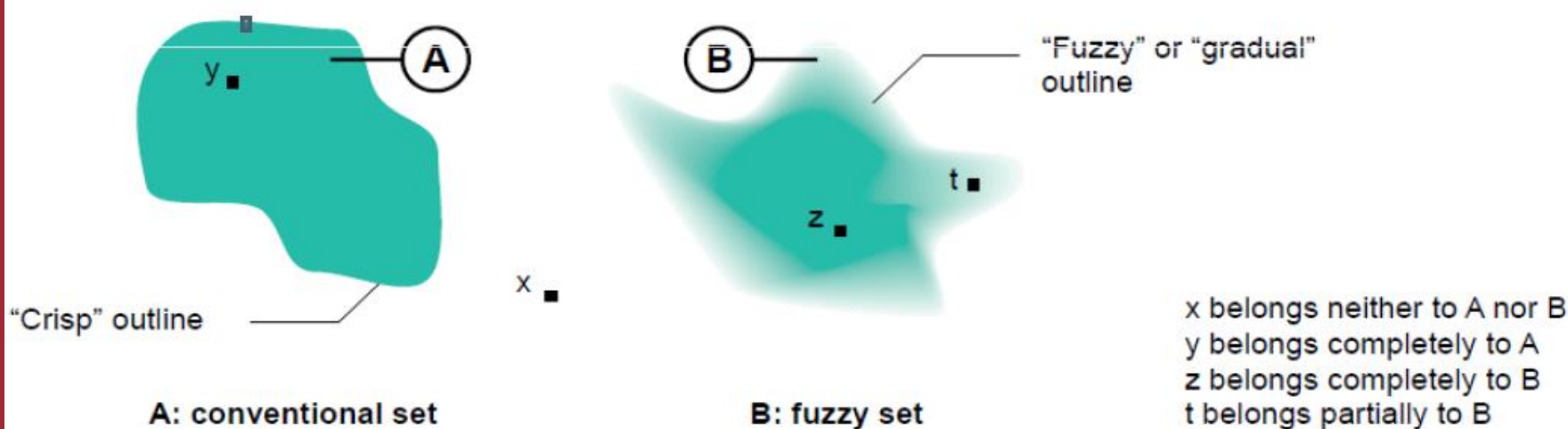
Introduction to Fuzzy Computation

- Fuzzy logic

- Fuzzy sets

- Fuzzy rules

- Fuzzy rule-based systems (Mamdani and Sugeno)

- Adaptive Neuro-Fuzzy Inference System (ANFIS)

- Classification and Regression Trees (CART)

- Extracting Fuzzy Models from Data

Fuzzy Logic vs. Fuzzy Arithmetic

- *Fuzzy logic* is the generalization of ordinary logic (Boolean) and allows truth values other than absolute truth and utter falsity.

- *Fuzzy arithmetic* is a formalism for making calculations with numerical quantities that are imprecisely known. These quantities are known as fuzzy numbers and are defined as fuzzy sets on the real numbers.

- Fuzzy logic is comparable to fuzzy arithmetic in the same way that logic is comparable to arithmetic. They don't have much in common in terms of the kinds of problems they're used to address.
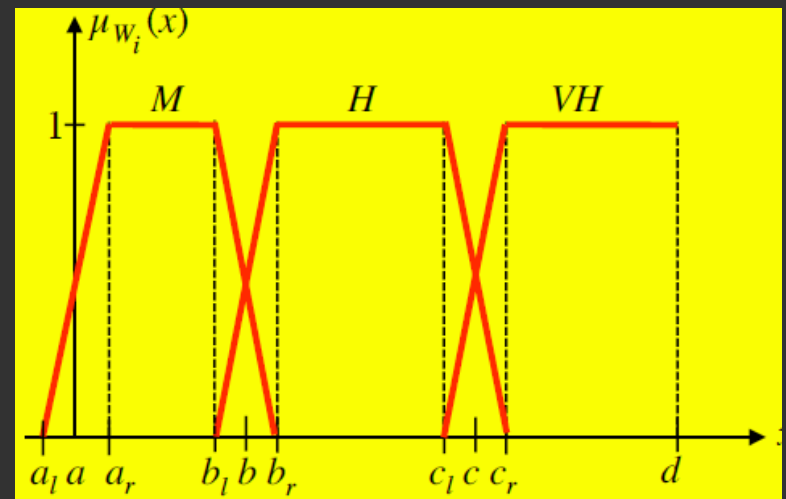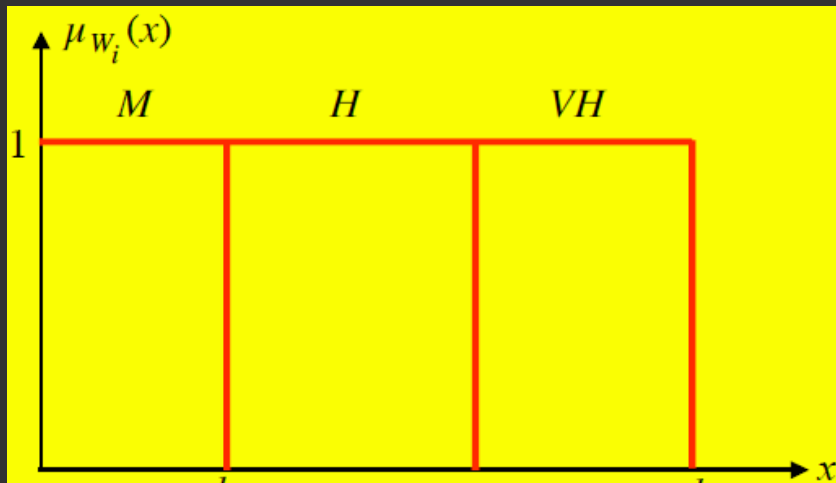
Fuzzy Logic

- Unlike Boolean logic (bivalent/Aristotelian), fuzzy logic is *multi-valued*
    - Fuzzy logic represents degrees of membership and degrees of truth
    - Things can be *part true* and *part false* at the same time

"Crisp" outline

A: conventional set

"Fuzzy" or "gradual" outline

B: fuzzy set

x belongs neither to A nor B
y belongs completely to A
z belongs completely to B
t belongs partially to B

Fuzzy Set Definition (Type-1)

Zero-order uncertainty partition (crisp set) vs.

first-order uncertainty partition (type-1 fuzzy set)

Fuzzy Logic

- *Heraclitus* (535-475 BC): things can be simultaneously *True* and *False*

- *Plato* (428-348 BC): there is a third region beyond *True* and *False*

- *Łukasiewicz* (1878-1956): many valued logic. First described three-valued logic, it was later generalized to n-valued (for all finite n)

- *Knuth* (1938): three-valued logic similar to *Łukasiewicz*. His insight, apparently missed by *Łukasiewicz*, was to use the integral range [-1, 0, +1] rather than [0, 1, 2]

Fuzzy Logic
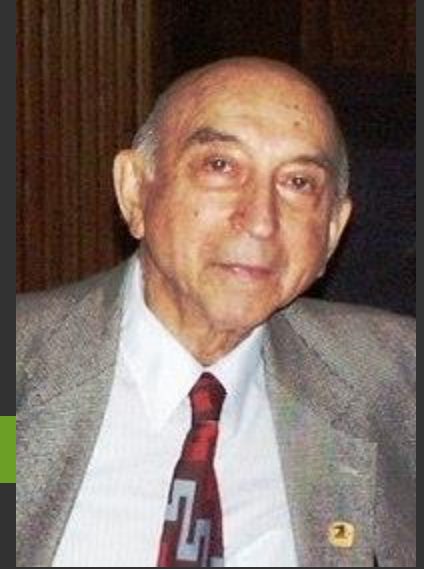
Lotfi A. Zadeh

- was a mathematician, computer scientist, electrical engineer, artificial intelligence researcher and professor emeritus at the University of California, Berkeley

- in his paper fuzzy sets (1965), proposed using a membership function operating on the domain of all possible values and new operations for the calculus of logic and showed that fuzzy logic was a generalization of Boolean logic.

- he also proposed fuzzy numbers as a special case of fuzzy sets, as well as the corresponding rules for consistent mathematical operations (fuzzy arithmetic).

Fuzzy Logic Characteristics

- Represent vagueness and imprecision of statements in natural language

- Knowledge is interpreted as a collection of elastic/fuzzy constraints on a set of variables.

- Inference is viewed as a process of propagation of elastic constraints

- Fuzzy logic allows decision making with estimated values under incomplete or uncertain information

- Exact reasoning is viewed as a limiting case of approximate reasoning

Fuzzy Set Definition (Type-1)

Let X be a nonempty set. A fuzzy set A in X is characterized by its membership function

$$\mu_A : X \rightarrow [0, 1]$$

and $\mu_A(x)$ is interpreted as the degree of membership of element x in the fuzzy set A for each $x \in X$.

It is clear that *A* is completely determined by the set of ordered pairs:
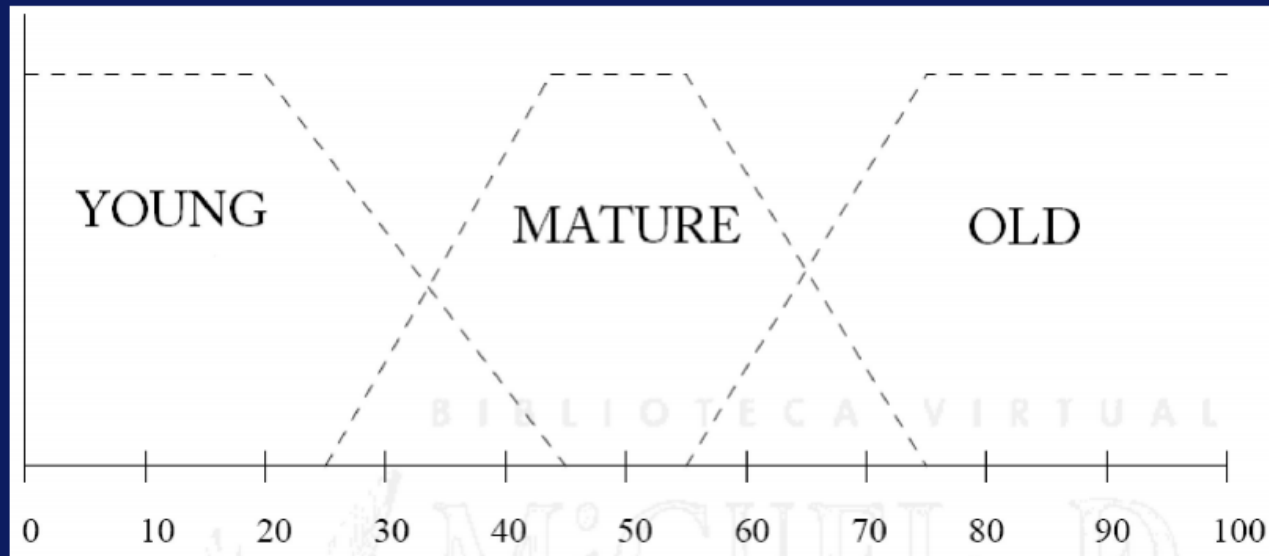
$$A = \{(x, \mu_A(x))/x \in X\}$$

Fuzzy Set: example

A = Set of tall people

Crisp set A

1.0

170   Heights (cm)

Fuzzy set A

1.0
.9
.5

170   180   Heights (cm)

Membership function
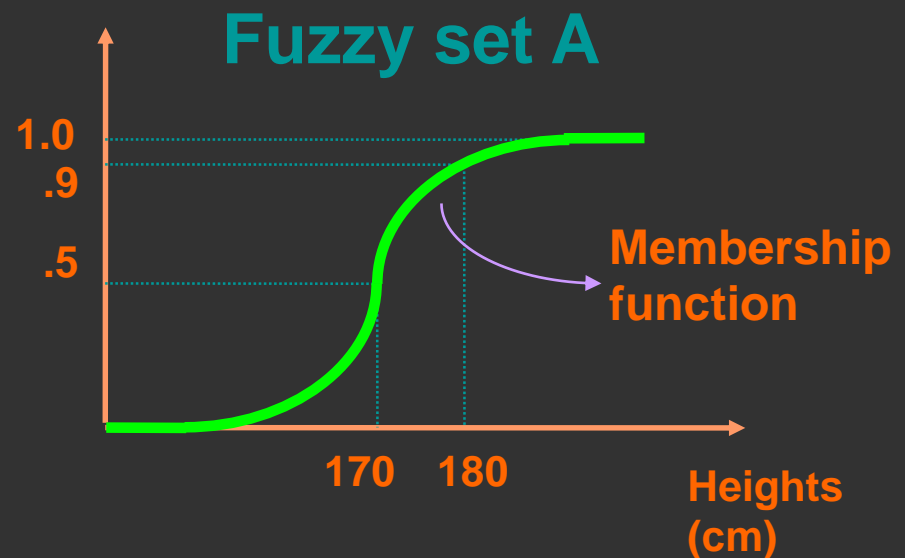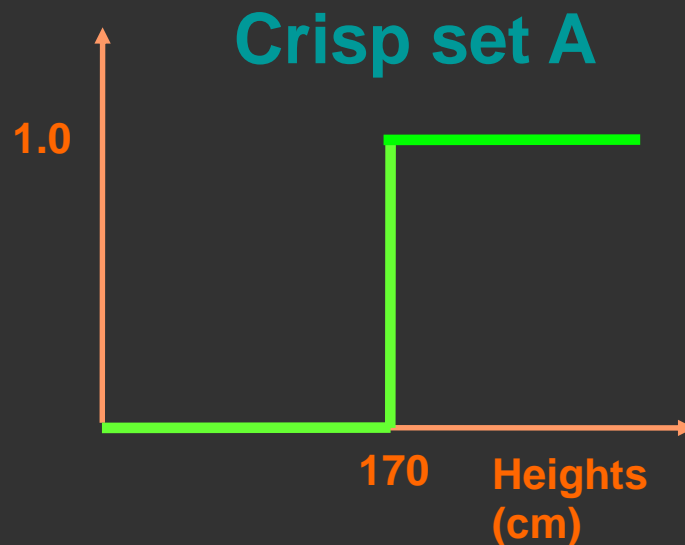
Fuzzy Set: example

- E = {0, ..., 100} (Age)
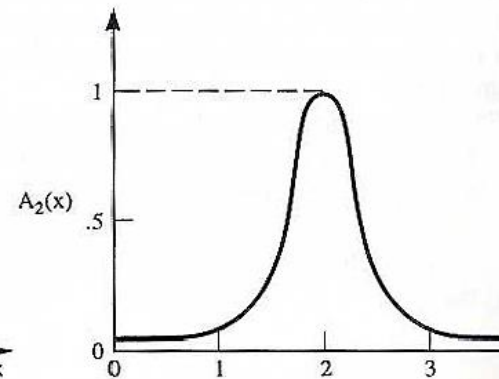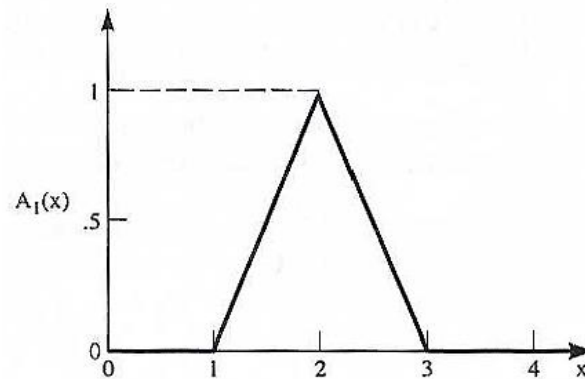- Fuzzy sets: Young, Mature, Old

Fuzzy Sets

Fuzzy sets representing the same concept may vary considerably.

Class of real numbers that are close to 2

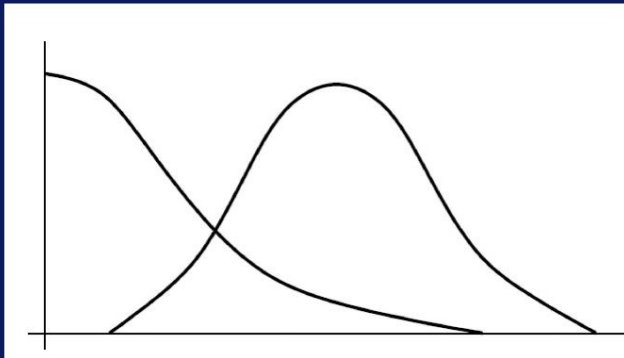The four fuzzy sets should possess the properties:

- $A_i(2)=1$ and $A_i(x)<1$, for all $x \neq 2$;

- A_i is symmetric with respect to $x=2$;
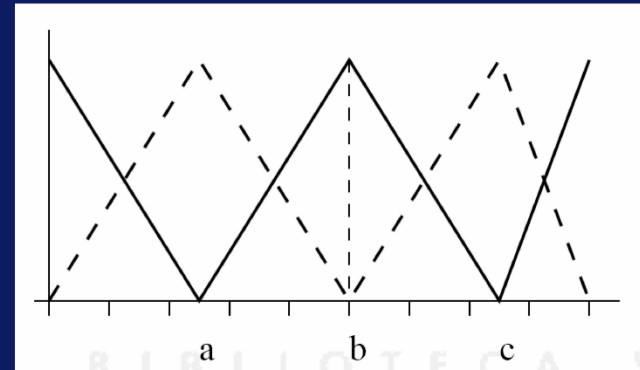
- A_i decreases monotonically from 1 to 0
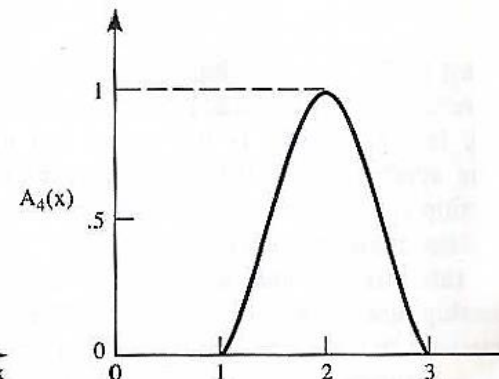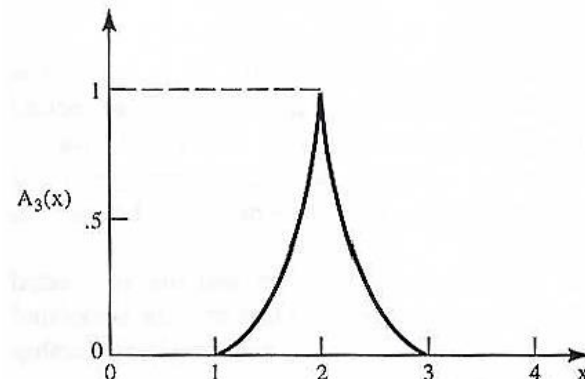
Examples of membership functions that may be used in different contexts for characterizing fuzzy sets of real numbers close to 2.
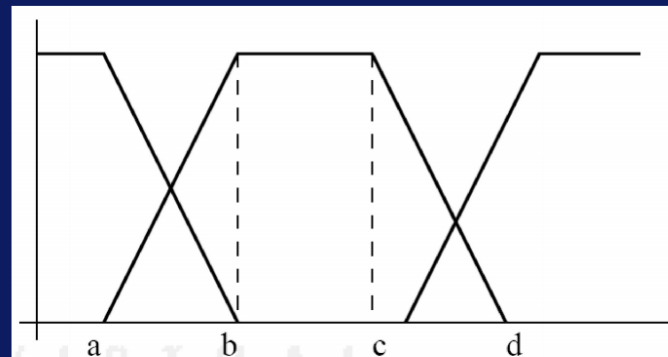
Types of Membership Functions
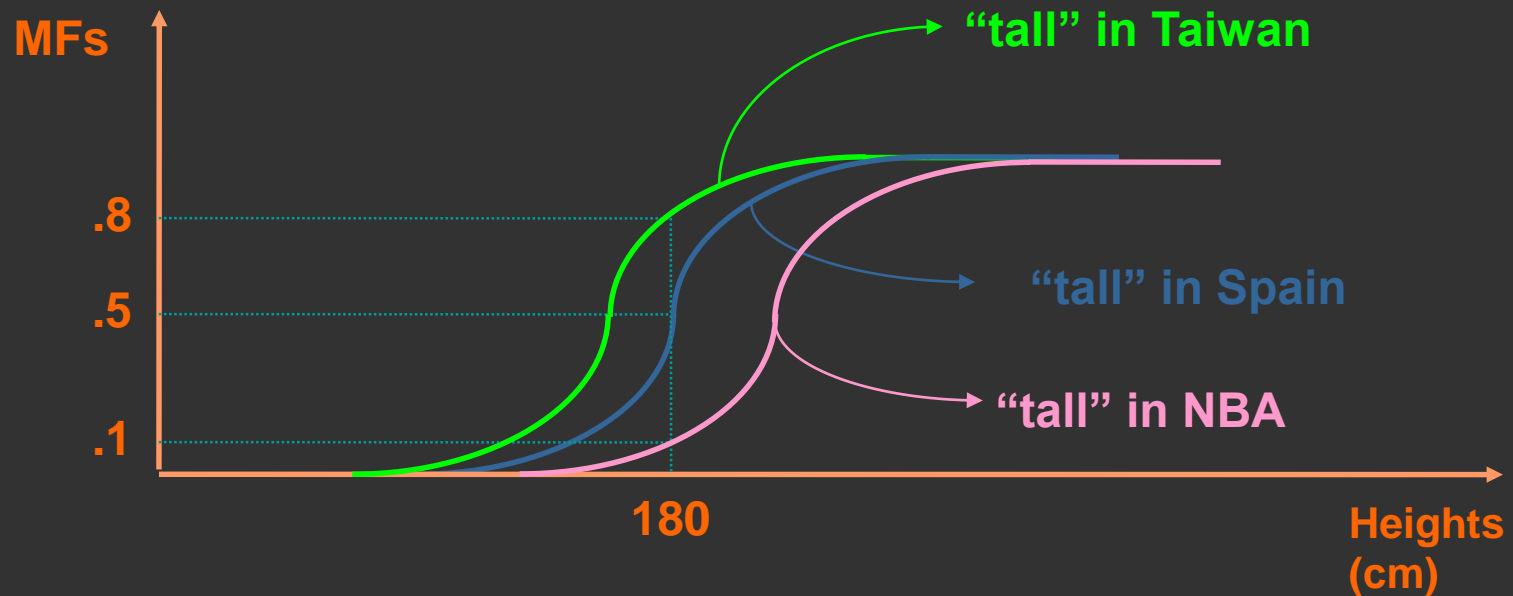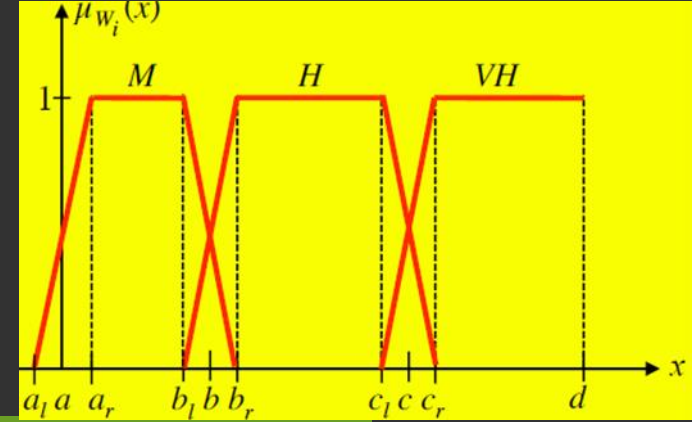
Gaussian

Triangular

Trapezoidal

Membership Functions

❑ About MFs

➢ Subjective measures (MFs represent distributions of possibility rather than probability)

➢ Not probability functions

Fuzzy Sets: Types

Thus far we introduced only one type of fuzzy set! First-order FS or Ordinary FS or Type-1 FS

Given a relevant universal set X, any arbitrary fuzzy set of this type (set A) is defined by a function of the form:
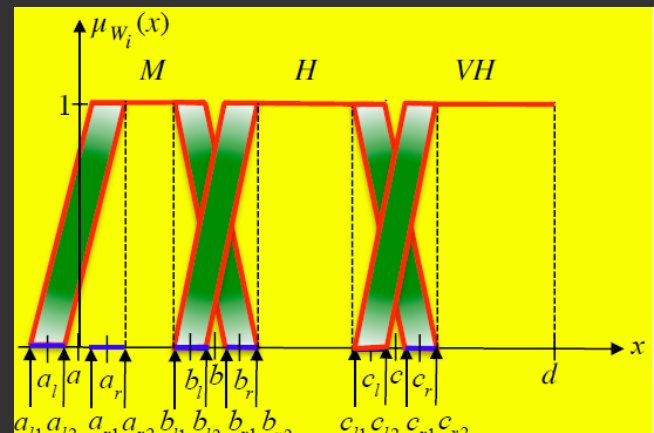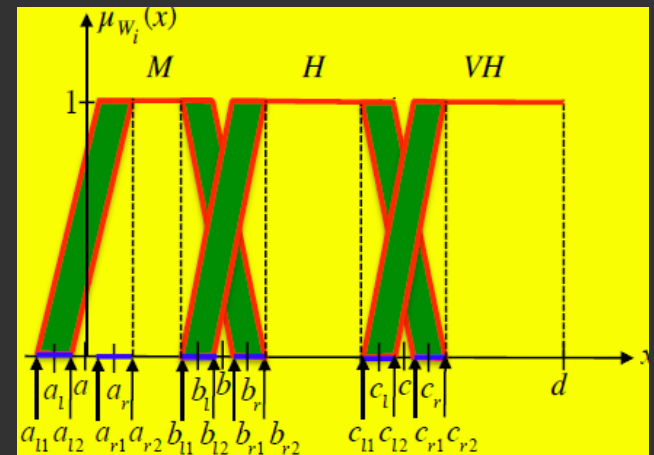
$$A : X \rightarrow [0, 1]$$ **Ordinary Fuzzy Sets**

- By far the most common in the literature and applications

- Their membership functions are often overly precise, i.e. they require that each element of the universal set be assigned a particular real number

- p. e. we may only be able to identify meaningful lower and upper bounds of membership grades for each element of the universal set. With OFS we should suppress the identification uncertainty by choosing for example the middle value or…

Fuzzy Sets: Types (Type-2)

- A second-order uncertainty partition of the real-variable, x, partitions it into overlapping intervals, where one is unsure about where the overlap begins and ends, so that the degree of membership in each region of overlap is an interval of real numbers that is a subset of x.

- A non-uniformly shaded footprint of uncertainty (FOU) denotes a non-uniform weighting of all of its points, and is called a non-uniformly weighted second-order uncertainty partition.
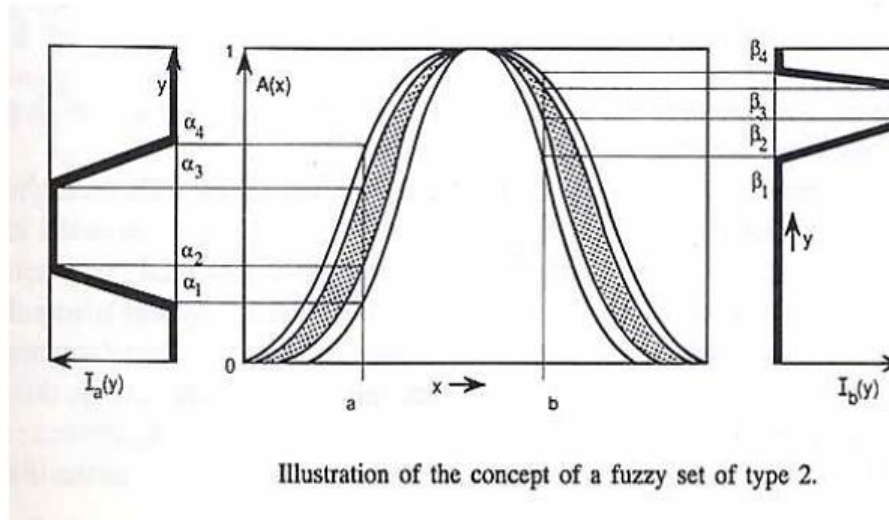
Fuzzy Sets: Types

Interval-valued fuzzy sets can further be generalized by allowing their intervals to be fuzzy.

Fuzzy sets of **type 2**
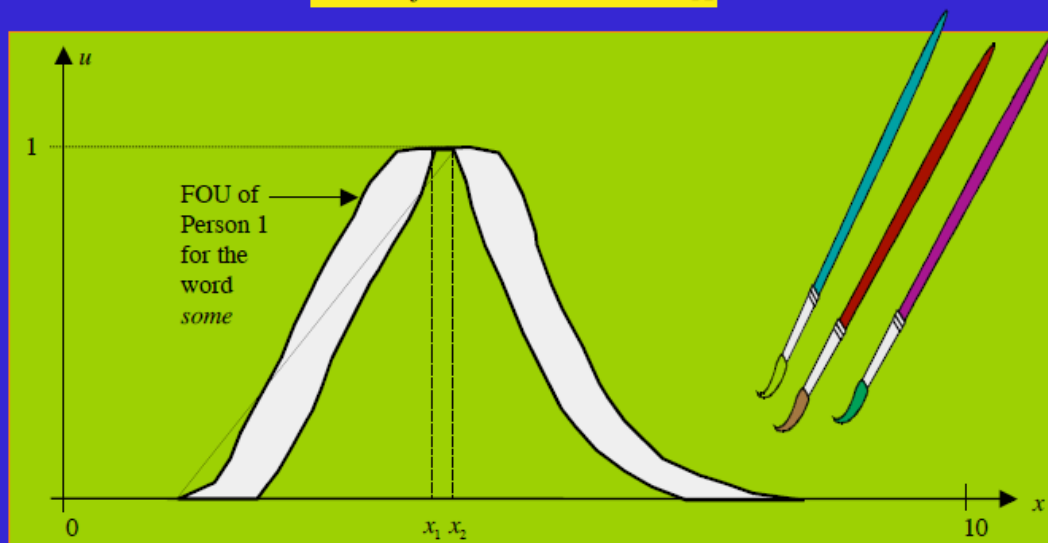
$$A : X \rightarrow \mathcal{F}([0, 1])$$

Fuzzy power set: Set of all ordinary fuzzy sets that can be defined within the universal set [0, 1]

Illustration of the concept of a fuzzy set of type 2.

Types 3, 4, etc...
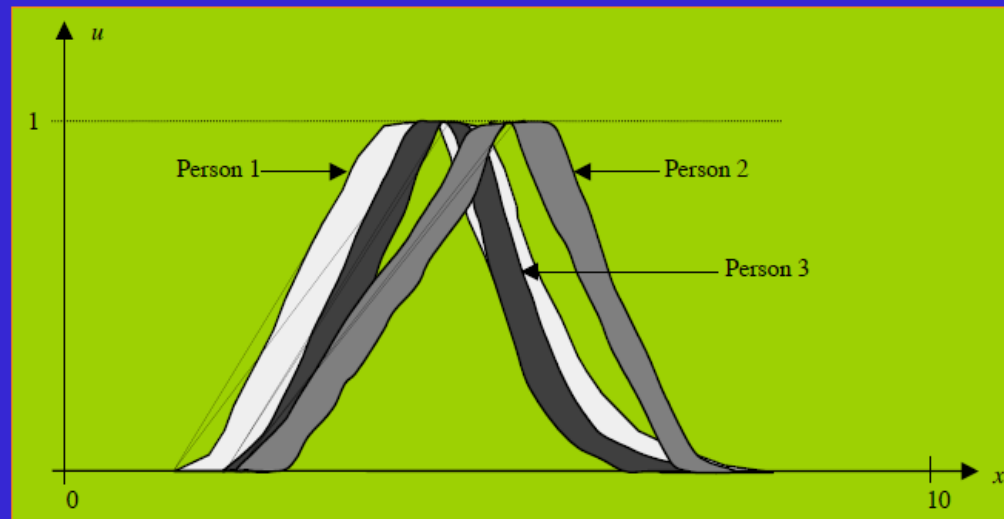
Examples Type-2 Fuzzy Set

- *Intra-uncertainty* about a word can be modeled using a type-2 *person fuzzy set* $\tilde{A}(p_j), j = 1, \ldots, n_{\tilde{A}}$

FOU: footprint of uncertainty

FOU of Person 1 for the word *some*

Examples Type-2 Fuzzy Set

- **Collect person MFs from a group of people**

Linguistic Variables

- A *linguistic variable* is a fuzzy variable
  - e.g. the fact "*NBA players are tall*" implies linguistic variable "*NBA players*" takes the linguistic value "*tall*"
  - Use linguistic variables to form *fuzzy rules*:

  IF   'project duration' is *long*
  THEN   'risk' is *high*

  IF   'risk' is *very high*
  THEN   'project funding' is *very low*

Fuzzy Rules

- A *fuzzy rule* is a conditional statement in the familiar form:

  IF x is A

  THEN y is B

  – x and y are linguistic variables

  – A and B are linguistic values determined by fuzzy sets on the universe of discourses X and Y, respectively.
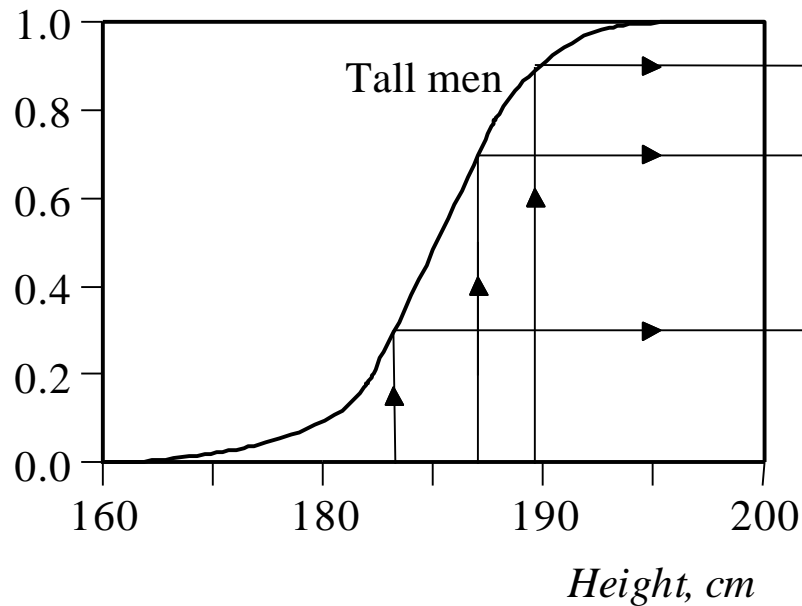
Fuzzy System

- A *fuzzy logic system* is a system that uses fuzzy rules, fuzzy logic, and fuzzy sets

- Many rules in a fuzzy logic system will <u>fire</u> to some extent

  - If the antecedent is true to some *degree of membership*, then the consequent is true to the <u>*same*</u> *degree* (Zadeh Extension Principle)
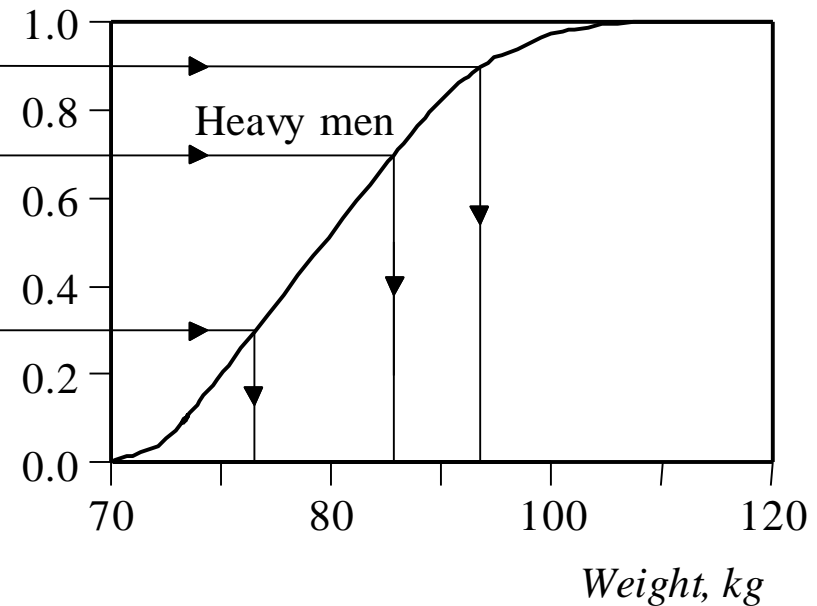
Fuzzy Rules

IF   *height* is *tall*

THEN *weight* is *heavy*

Extension Principle

Suppose that f is a function from X to Y, and A is a fuzzy set on X defined as:

$$A = \{(x_1, \mu_A(x_1)), (x_2, \mu_A(x_2)), . . , (x_n, \mu_A(x_n))\}$$

Then the extension principle states that the image of the fuzzy set A under the mapping f can be expressed as a fuzzy set $B \subseteq Y$.

$$B = f(A) = \{(y, \mu_B(y))\}, \quad \text{where } \mu_B(y) = \sup \{\mu_A(x) \mid x \in X, y = f(x)\}$$

The supremum (sup) function applies if there are two or more values of x that have the same value $f(x)$.

Extension Principle

• The extension principle generalizes an ordinary mapping of a function to a mapping between fuzzy sets.
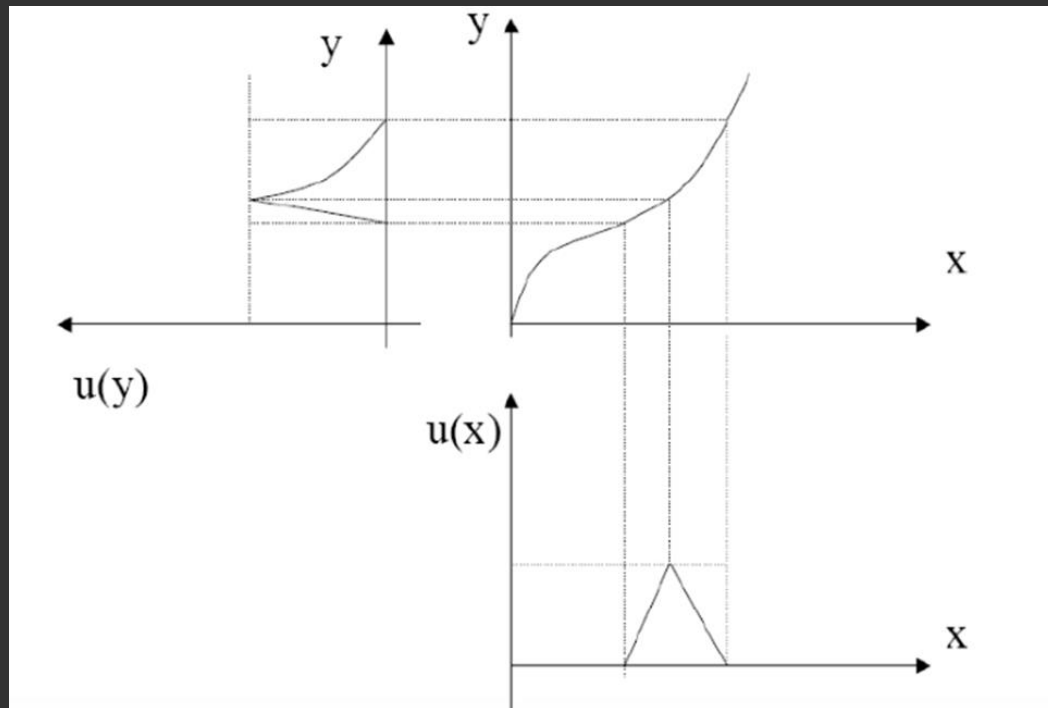
Fuzzy Rules

- Other examples (multiple antecedents):
  - e.g. IF '*project duration*' is *long*
    AND '*project staffing*' is *large*
    AND '*project funding*' is *inadequate*
    THEN *risk* is *high*

  - e.g. IF *service* is *excellent*
   OR *food* is *delicious*
    THEN *tip* is *generous*

Fuzzy Rule-Based Systems

Principal FRBS for engineering problems (i.e., dealing with real-valued inputs and outputs):

- *Mamdani*
- *Takagi-Sugeno-Kang*

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Fuzzy Inference: Mamdani

- *Ebrahim Mamdani* (1974), the Mamdani method for fuzzy inference is:
  1. Fuzzify the input variables;   2. Evaluate the rules
  3. Aggregate the rule outputs;   4. Defuzzify
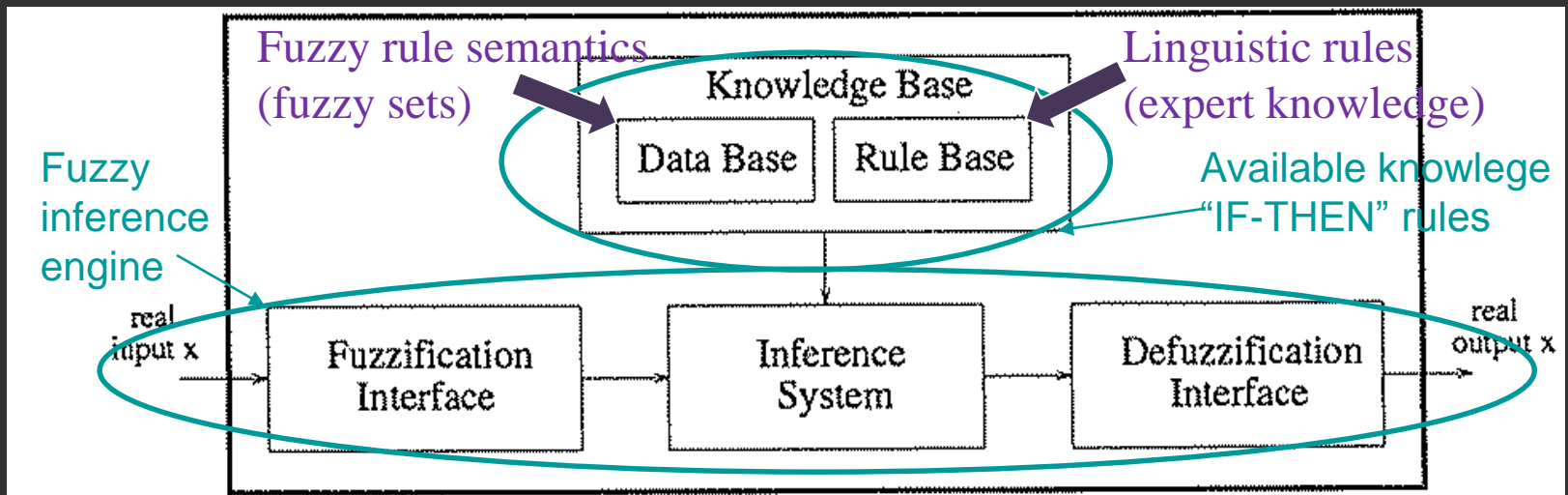
Fuzzy Inference: Mamdani

Imprecise Reasoning

- Classic Modus Ponens

$$\frac{\begin{array}{c} A \\ A \to B \end{array}}{B} \qquad\qquad \frac{\begin{array}{c} \neg A \\ A \to B \end{array}}{?}$$

- Imprecise:
  Generalized Modus Ponens

$$\frac{\begin{array}{c} \mu_{A'}(x) \\ A \to B \end{array}}{\mu_{B'}(y)} \qquad \frac{\begin{array}{c} IF\ X\ is\ A \quad THEN \quad Y\ is\ B \\ X\ is\ A' \end{array}}{Y\ is\ B'}$$

Fuzzy Inference: Mamdani

The membership function of the fuzzy set $B´$ is obtained by applying the *compositional rule of inference:*

$$B' = A' \circ R$$

When applied to the ith rule:

$$R_i : IF\ X_{i1}\ is\ A_{i1}\ and\ \ldots\ and\ X_{in}\ is\ A_{in}\ THEN\ Y\ is\ B_i,$$

Simplest expression CRI: $\mu_{B_i'}(y) = I(\mu_{A_i}(x_0), \mu_{B_i}(y))$

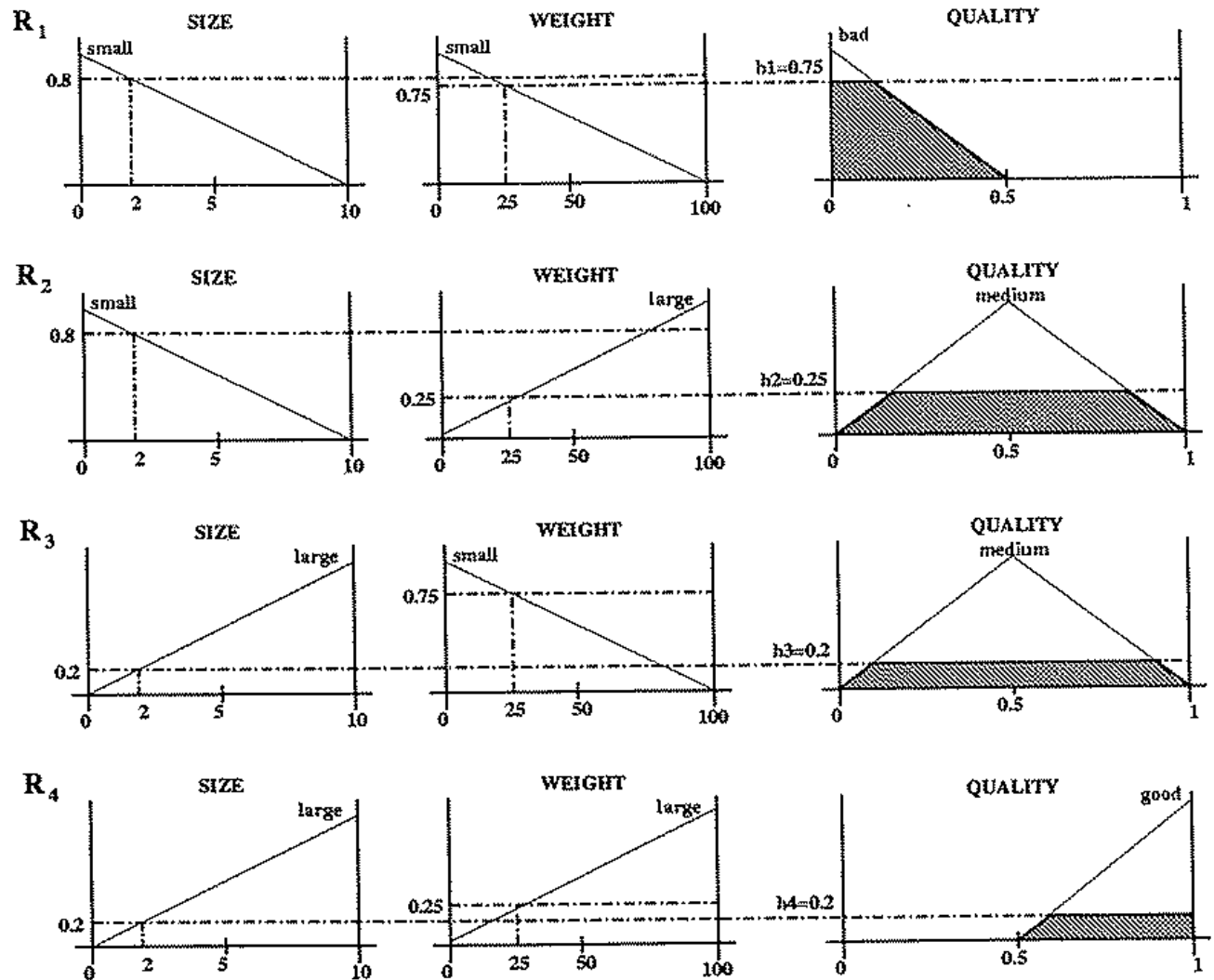where I is a fuzzy implication operator (common choice *minimum t-norm*)

Fuzzy Inference: Mamdani

Current system input = (2, 25)

Degree of firing or firing strength between the antecedent part of the i-th rule and the current inputs to the system. T-norm = conjunctive operators (minimum, algebraic product)
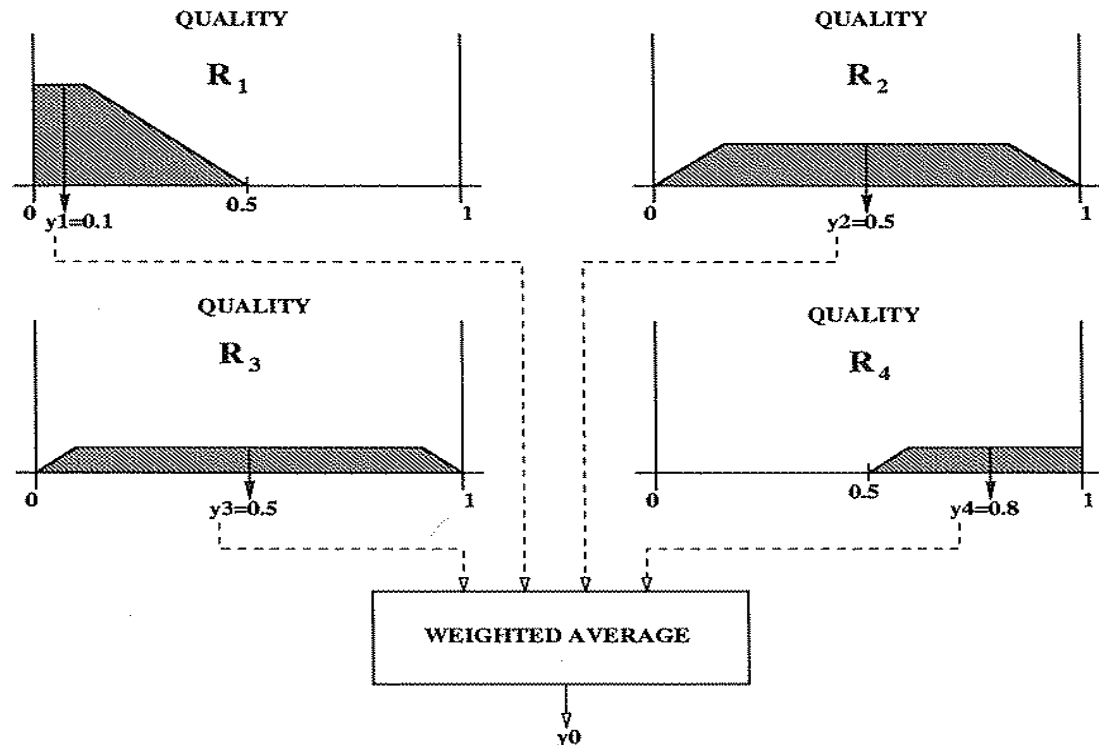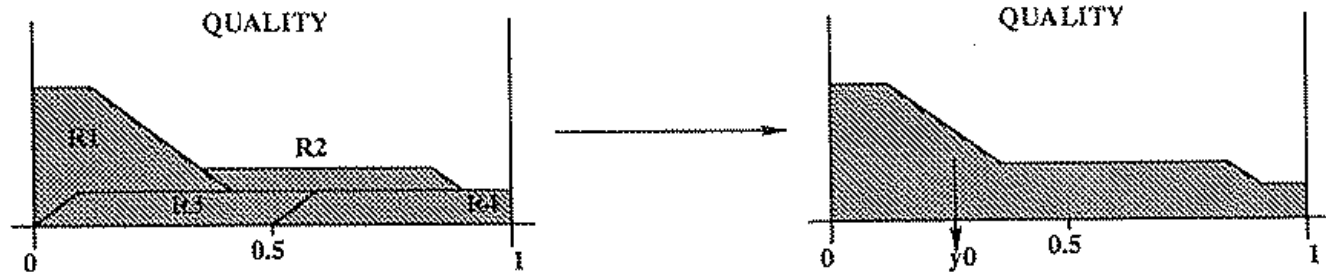
Mamdani – Defuzzification

First Aggregation then Inference - FATI
(center of maximums, center of gravity, etc.)

First Inference then Aggregation - FITA
(maximum value)

Mamdani – Example

4. Defuzzification

– Calculate the *center of gravity* (*cog*):

$$COG = \frac{\int_a^b \mu_A(x)\, x\, dx}{\int_a^b \mu_A(x)\, dx}$$

- An easier approximation (with some imprecision…):

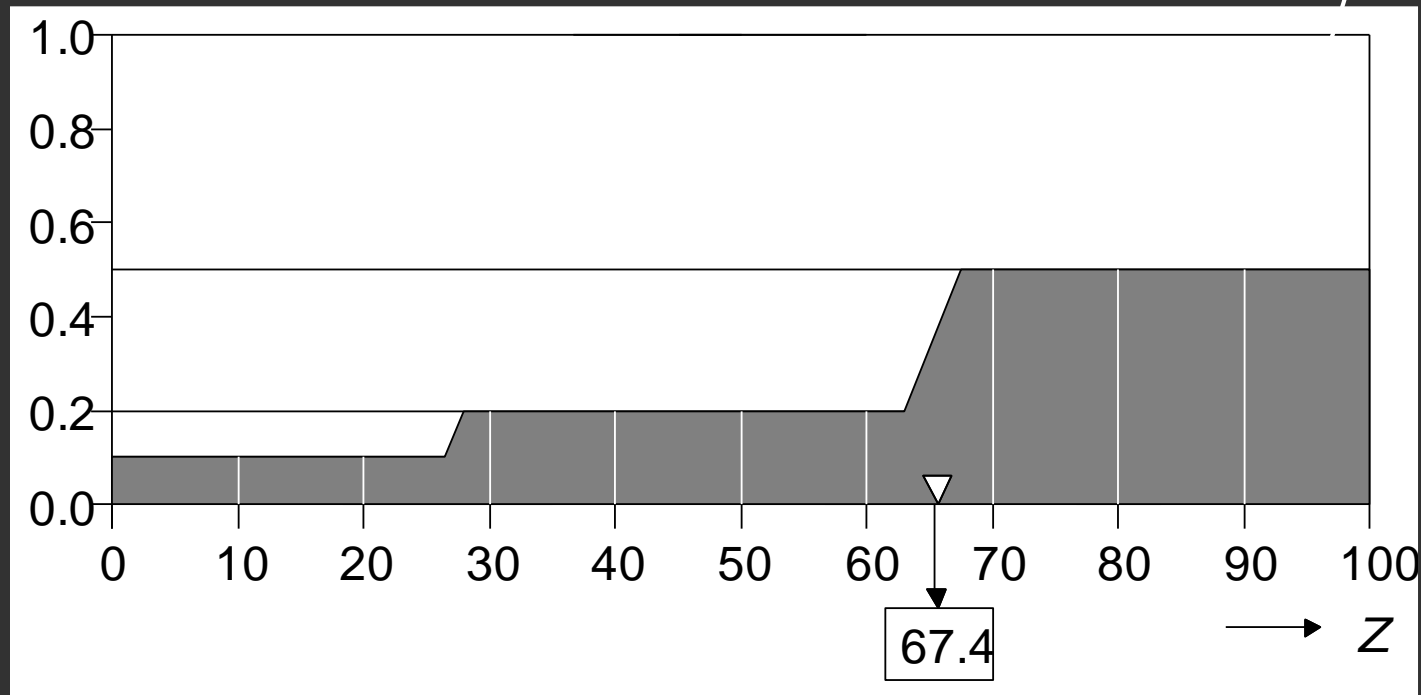$$y = \frac{\sum_{j=1}^{r} \mu_j \cdot s_j}{\sum_{j=1}^{r} \mu_j}$$

S_j is the center of gravity of set j

$$COG = \frac{(0+10+20) \times 0.1 + (30+40+50+60) \times 0.2 + (70+80+90+100) \times 0.5}{0.1+0.1+0.1+0.2+0.2+0.2+0.2+0.5+0.5+0.5+0.5} = 67.4$$

Mamdani – Example

4. Defuzzification
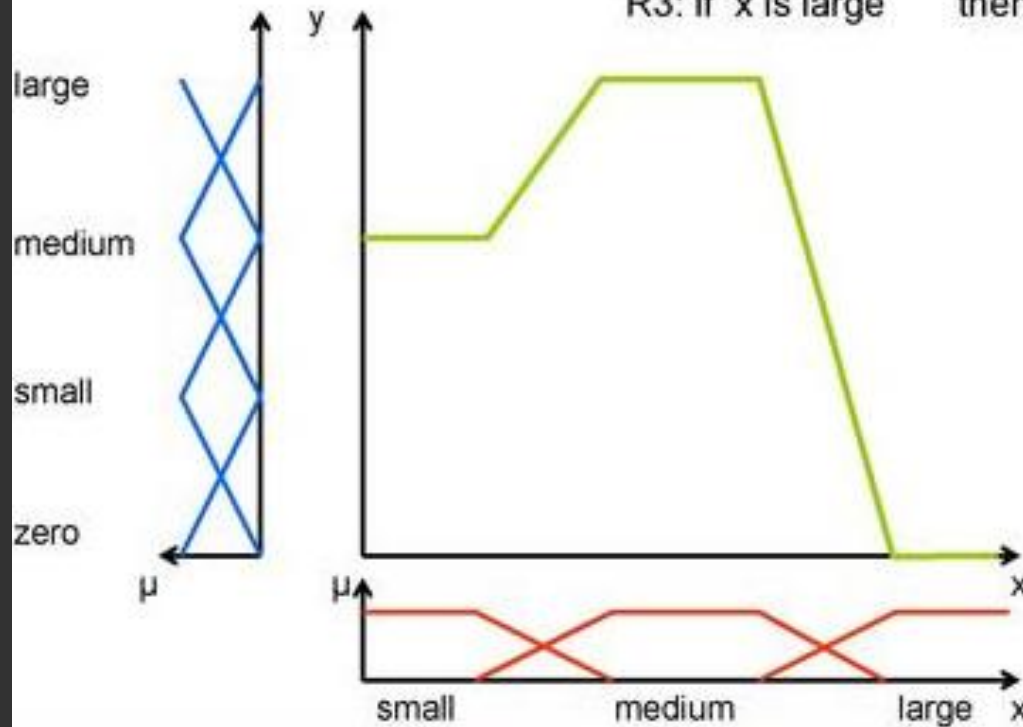
– Use a reasonable sampling of points

Mamdani: Visual example

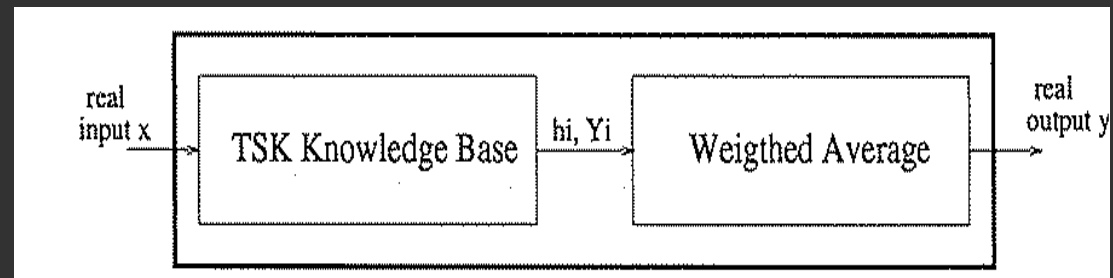Fuzzy Rule System (Mamdani)

R1: if x is small then y is medium
R2: if x is medium then y is large
R3: if x is large then y is zero

Fuzzy Inference: TSK

- Takagi-Sugeno-Kang (1985), the TSK method for fuzzy inference is:

$$IF\ X_1\ is\ A_1\ and\ \ldots\ and\ X_n\ is\ A_n$$
$$THEN\ Y = p_1 \cdot X_1 + \cdots + p_n \cdot X_n + p_0,$$

Polynomial function

Output TSK (m rules): $\dfrac{\sum_{i=1}^{m} h_i \cdot Y_i}{\sum_{i=1}^{m} h_i}$

$$h_i = T(\mu_{A_{i1}}(x_1), \ldots, \mu_{A_{in}}(x_n))$$

Matching degree between the antecedent part of the i-th rule and the current inputs to the system. T-norm = conjunctive operators (minimum, algebraic product)

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Zero-Order TSK FRBS

Takagi-Sugeno-Kang

If speed is low then resistance = 2
If speed is medium then resistance = 4
If speed is high then resistance = 8

MFs

low   **medium**   **high**

.8

.3
.1

2

Speed

Rule 1: h1 = .3; Y1 = 2
Rule 2: h2 = .8; Y2 = 4
Rule 3: h3 = .1; Y3 = 8

$$\text{Resistance} = \sum(h_i \ast Y_i) / \sum h_i$$
$$= (0.3\ast2+0.8\ast4+0.1\ast8)/(0.3+0.8+0.1)$$
$$= 3.83$$

First-Order TSK FRBS

Takagi-Sugeno-Kang

Rule base
If X is A1 and Y is B1 then Z = p1*x + q1*y + r1
If X is A2 and Y is B2 then Z = p2*x + q2*y + r2

Fuzzy reasoning

A_1

B_1 $h1$

X Y
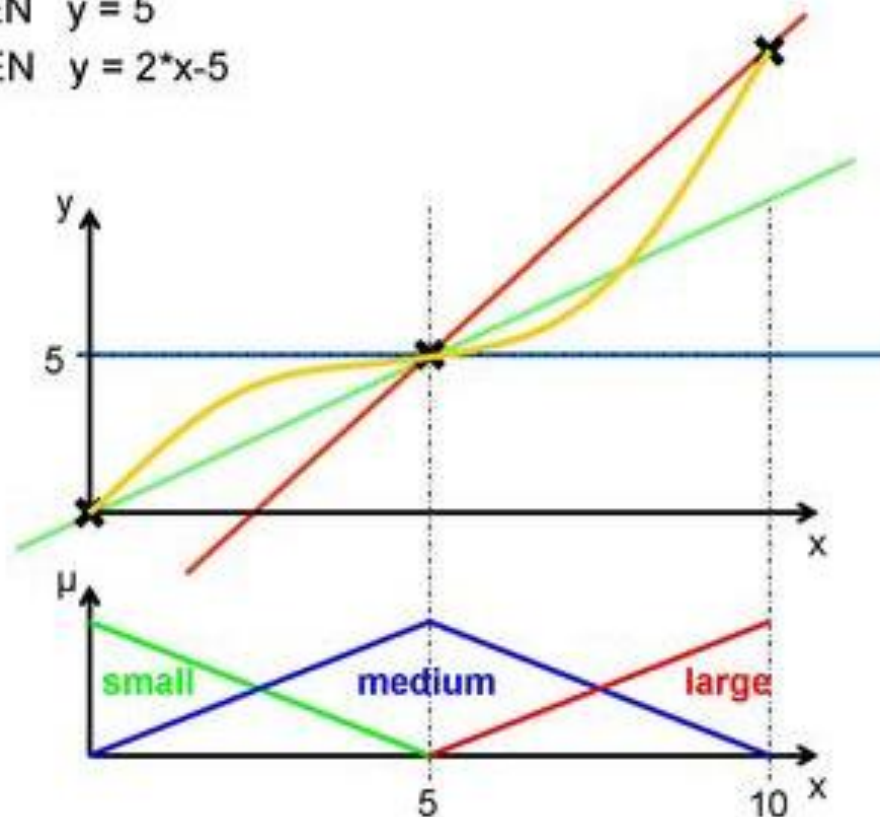
A_2

B_2 $h2$

X Y

$x=3$ $y=2$
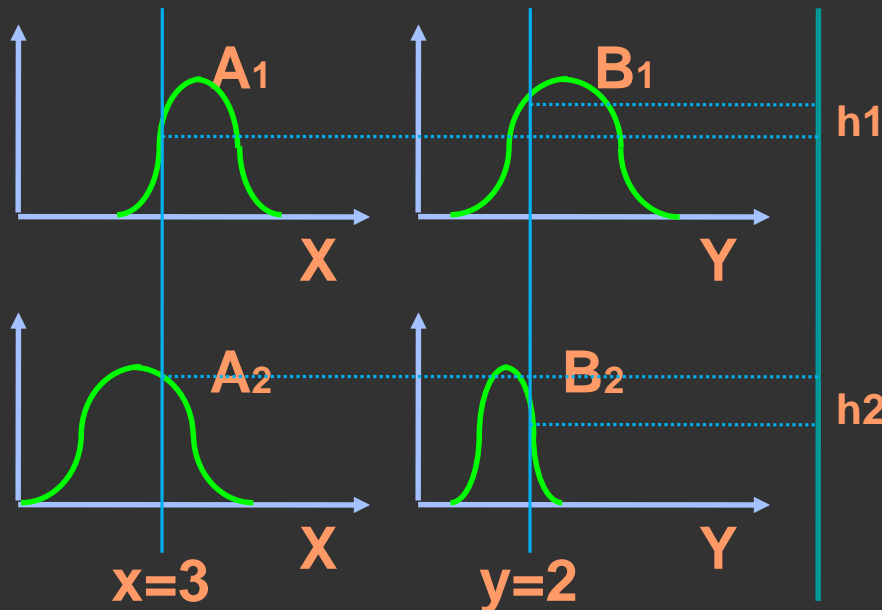
$Z_1 = p_1*3+q_1*2+r_1$

$Z_2 = p_2*3+q_2*2+r_2$

$$Z = \frac{h_1*Z_1+h_2*Z_2}{h_1+h_2}$$

TSK: Visual example

R1: IF x IS small   THEN   y = x
R2: IF x IS medium   THEN   y = 5
R3: IF x IS large   THEN   y = 2*x-5

$$y = \frac{\sum_{i=1}^{r} \mu_{R_i} \cdot y_i(\vec{x})}{\sum_{i=1}^{r} \mu_{R_i}}$$

Demo TSK

Fuzzy Juggling ball system

Matlab: juggler

Mamdani vs. TSK

Advantages of the TSK Method
- It is computationally efficient.
- It works well with linear techniques (e.g., PID control).
- It works well with optimization and adaptive techniques.
- It has guaranteed continuity of the output surface.
- It is well suited to mathematical analysis.

Advantages of the Mamdani Method
- It is intuitive.
- It has widespread acceptance.
- It is well suited to human input.

Some Misconceptions on Fuzzy Logic

Fuzzy logic is the same as imprecise logic

Fuzzy logic is not any less precise than any other form of logic: it is an organized and mathematical method of handling *inherently* imprecise concepts.

Some Misconceptions on Fuzzy Logic

Fuzzy logic is a new way of expressing probability

Fuzzy logic and probability refer to different kinds of uncertainty. Fuzzy logic is specifically designed to deal with **imprecision** of facts (fuzzy logic statements), while probability deals with **chances** of that happening *(but still considering the result to be precise)*

Example:

- ➢ P(A) = 0.5 means that A may be true or may be false
- ➢ A logical value of 0.5 means both true and false at the same time
- ➡ However, still is a point of controversy

Applications of Fuzzy Logic

- Why use *fuzzy systems*?
  - Apply fuzziness (*and therefore accuracy*) to linguistically defined terms and rules
  - Lack of crisp or concrete mathematical models exist

- When do you avoid *fuzzy systems*?
  - Traditional approaches produce acceptable results
  - Crisp or concrete mathematical models exist and are easily implemented

Applications of Fuzzy Logic

Application Areas of Fuzzy Logic

The Blow given table shows how famous companies using fuzzy logic in their products.

Product	Company	Fuzzy Logic
Anti-lock brakes	Nissan	Use fuzzy logic to controls brakes in hazardous cases depend on car speed, acceleration, wheel speed, and acceleration
Auto transmission	NOK/Nissan	Fuzzy logic is used to control the fuel injection and ignition based on throttle setting, cooling water temperature, RPM, etc.
Auto engine	Honda, Nissan	Use to select geat based on engine load, driving style, and road conditions.
Copy machine	Canon	Using for adjusting drum voltage based on picture density, humidity, and temperature.
Cruise control	Nissan, Isuzu, Mitsubishi	Use it to adjusts throttle setting to set car speed and acceleration
Dishwasher	Matsushita	Use for adjusting the cleaning cycle, rinse and wash strategies based depend upon the number of dishes and the amount of food served on the dishes.
Elevator control	Fujitec, Mitsubishi Electric, Toshiba	Use it to reduce waiting for time-based on passenger traffic
Golf diagnostic system	Maruman Golf	Selects golf club based on golfer's swing and physique.
Fitness management	Omron	Fuzzy rules implied by them to check the fitness of their employees.
Kiln control	Nippon Steel	Mixes cement
Microwave oven	Mitsubishi Chemical	Sets lunes power and cooking strategy
Palmtop computer	Hitachi, Sharp, Sanyo, Toshiba	Recognizes handwritten Kanji characters
Plasma etching	Mitsubishi Electric	Sets etch time and strategy

Example: Dinner for two

Rule Editor

Example: Dinner for two (Mamdani)
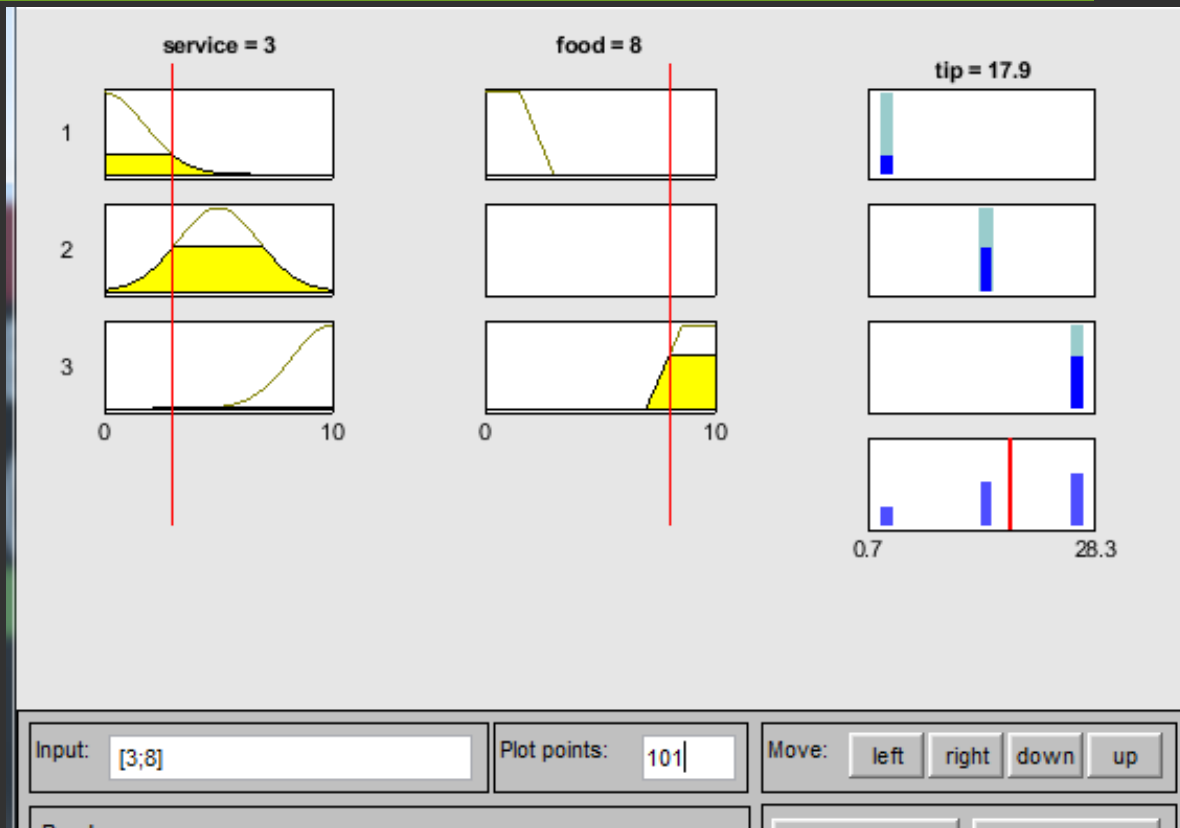
Rule Viewer
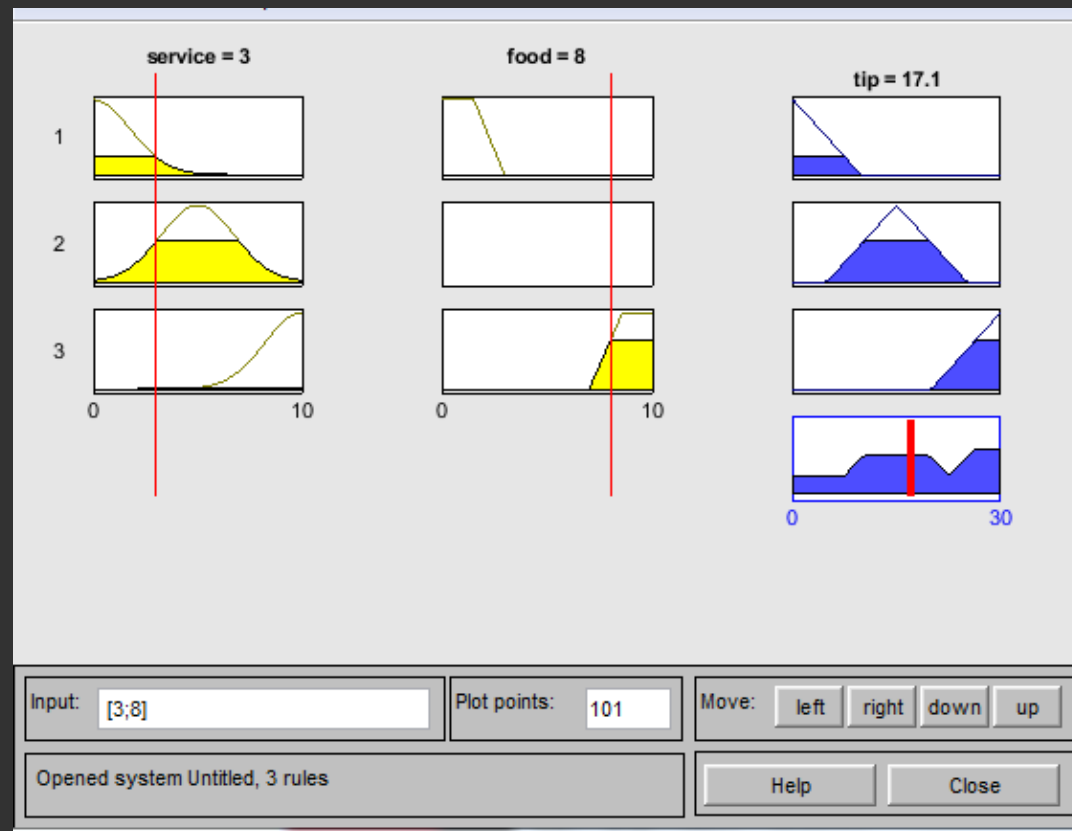
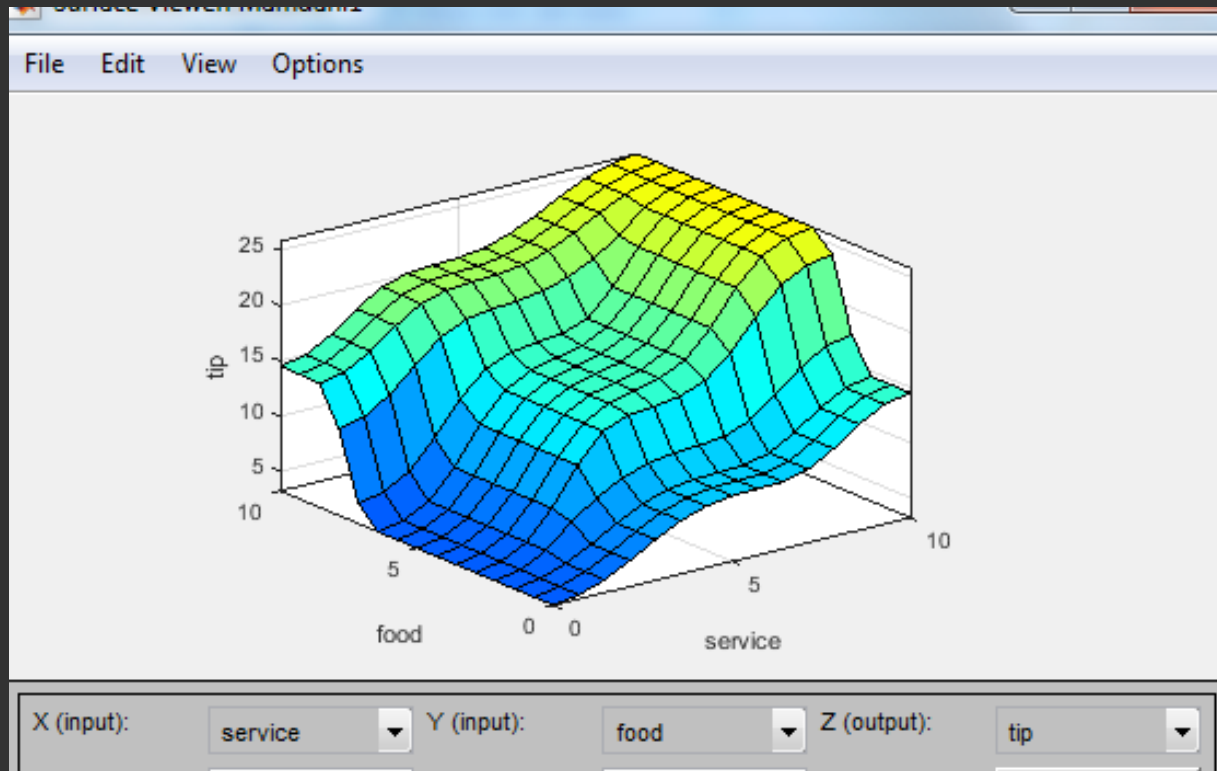# Example: Dinner for two (Mamdani)
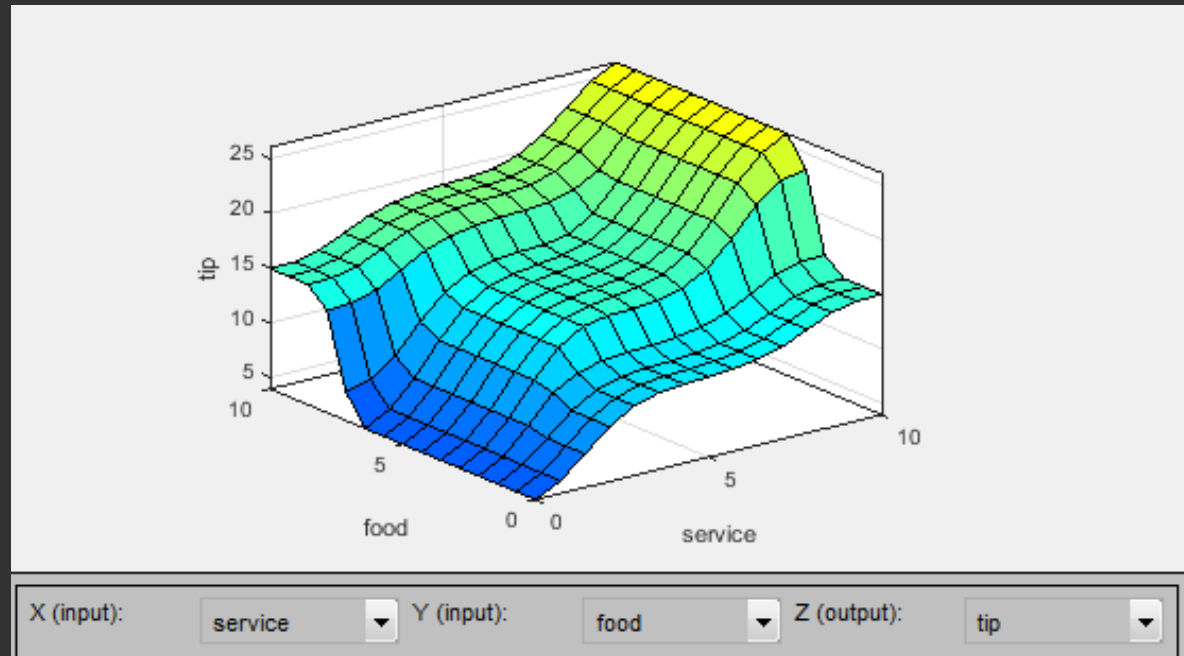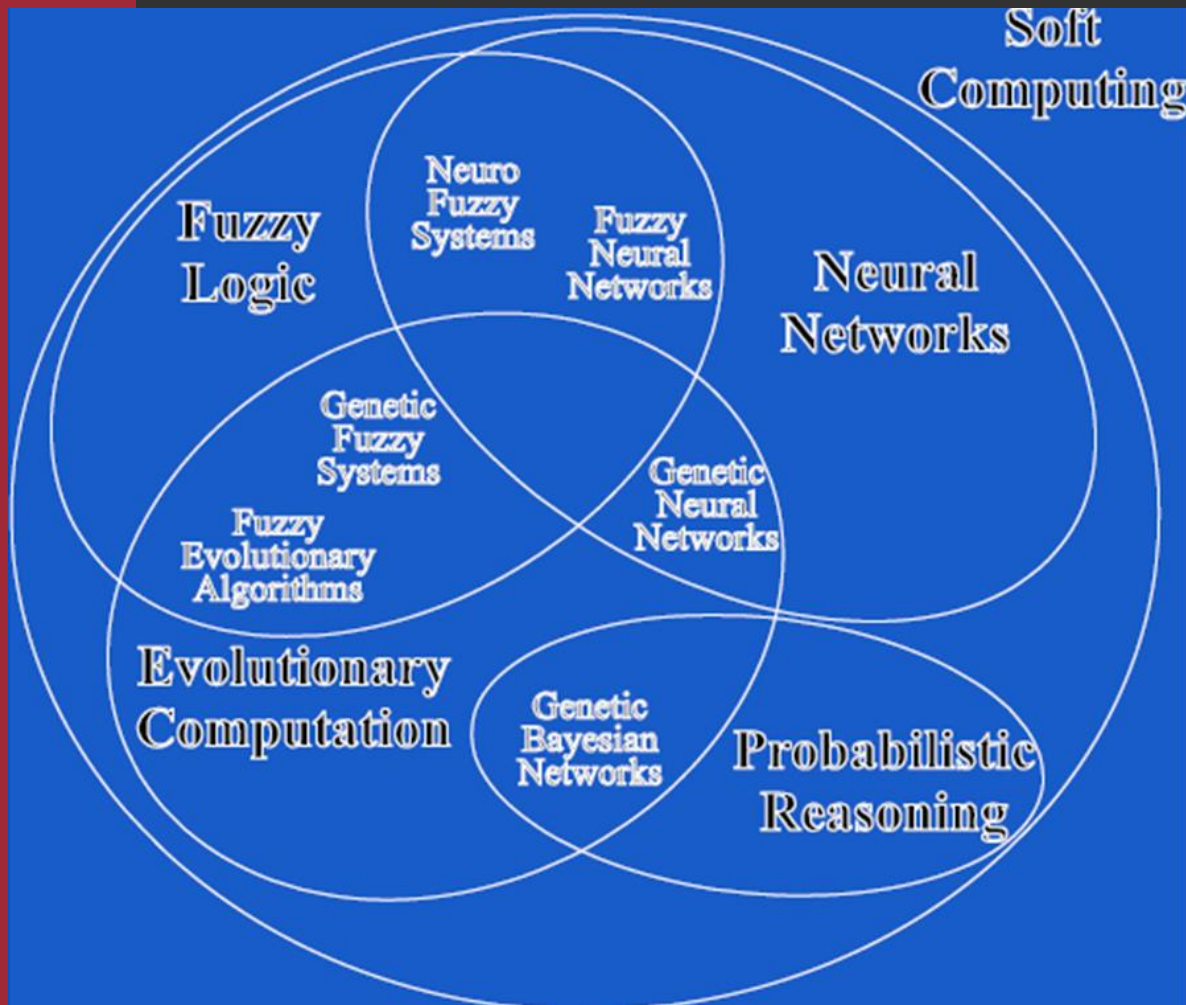
Example: Dinner for two (Sugeno)

Rule Viewer

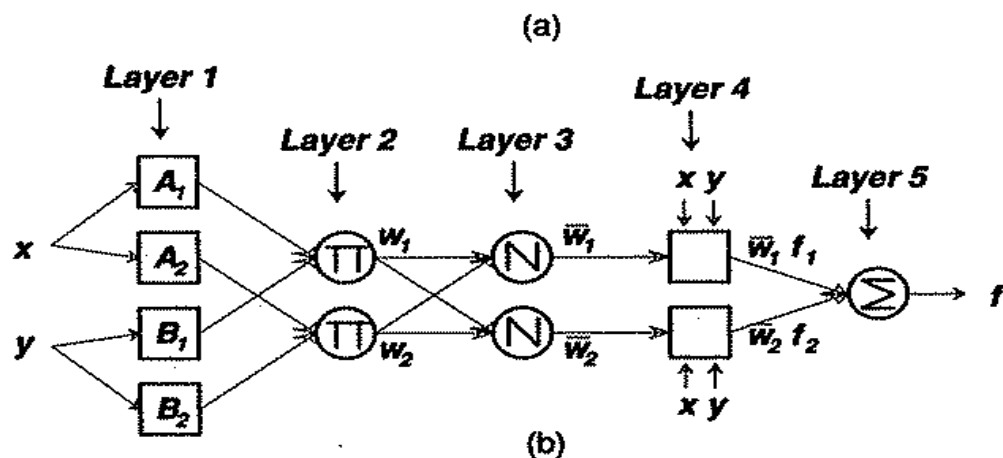# Example: Dinner for two (Sugeno)
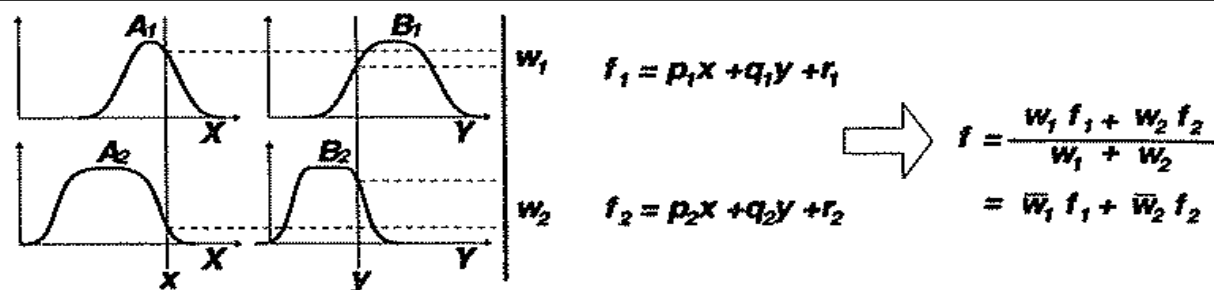
Hybridization

- FSs are neither capable of learning, adaptation or parallel computation, whereas these characteristics are clearly attributed to NNs.

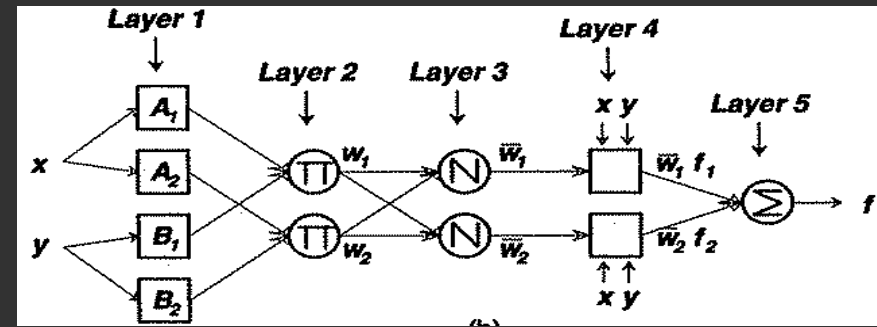- NNs lack flexibility, human interaction or knowledge representation, which lies at the core of FL.

ANFIS: Adaptive Neuro-Fuzzy Inference System

Sugeno

Rule 1: If x is A_1 and y is B_1, then $f_1 = p_1 x + q_1 y + r_1$,
Rule 2: If x is A_2 and y is B_2, then $f_2 = p_2 x + q_2 y + r_2$.

$f_1 = p_1 x + q_1 y + r_1$

$f_2 = p_2 x + q_2 y + r_2$

$$f = \frac{w_1 f_1 + w_2 f_2}{w_1 + w_2} = \bar{w}_1 f_1 + \bar{w}_2 f_2$$

(a)

(b)

ANFIS: Architecture

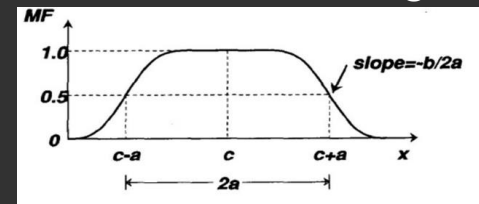**Layer 1:** Every node *i* is an adaptive node with a node function:

$$O_{1,i} = \mu_{A_i}(x), \qquad \text{for } i = 1, 2, \text{ or}$$
$$O_{1,i} = \mu_{B_{i-2}}(y), \qquad \text{for } i = 3, 4,$$

where *x* (or *y*) is the input to node *i* and A_i (or B_{i-2}) is a linguistic label ("small", "large",…) associated with this node

The membership function for *A* or B (=A_1, A_2, B_1 or B_2) can be any appropriate parameterized membership function such as the generalized bell function:

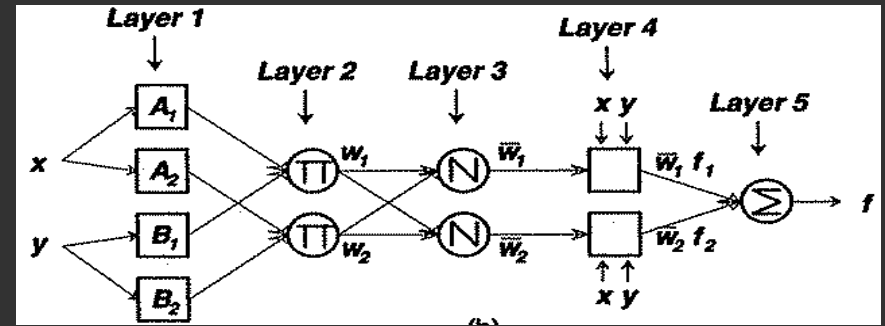$$\mu_A(x) = \frac{1}{1 + \left| \frac{x - c_i}{a_i} \right|^{2b}},$$

where {a_i, b_i, c_i} is the parameter set (**premise parameters**)

Layer 2: Every node in this layer is a fixed node labeled Π, whose output is the result of applying any other T-norm operator that performs fuzzy AND

$$O_{2,i} = w_i = \mu_{A_i}(x)\mu_{B_i}(y), \ i = 1, 2.$$

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ANFIS: Architecture

Layer 3: Every node in this layer is a fixed node labeled N. The ith node calculates the ratio of the ith rule's firing strength to the sum of all rules' firing strengths

$$O_{3,i} = \overline{w}_i = \frac{w_i}{w_1 + w_2}, \quad i = 1, 2.$$

Outputs of this layer are called **normalized firing strengths**

**Layer 4:** Every node in this layer is an adaptive node with a node function

$$O_{4,i} = \overline{w}_i f_i = \overline{w}_i(p_i x + q_i y + r_i),$$

where $\overline{w}_i$ is a normalized firing strength from layer 3 and $\{p_i, q_i, r_i\}$ is the parameter set of this node (**consequent parameters**)

Layer 5: The single node in this layer is a fixed node labeled Σ, which computes the overall output as the summation of all incoming signals:
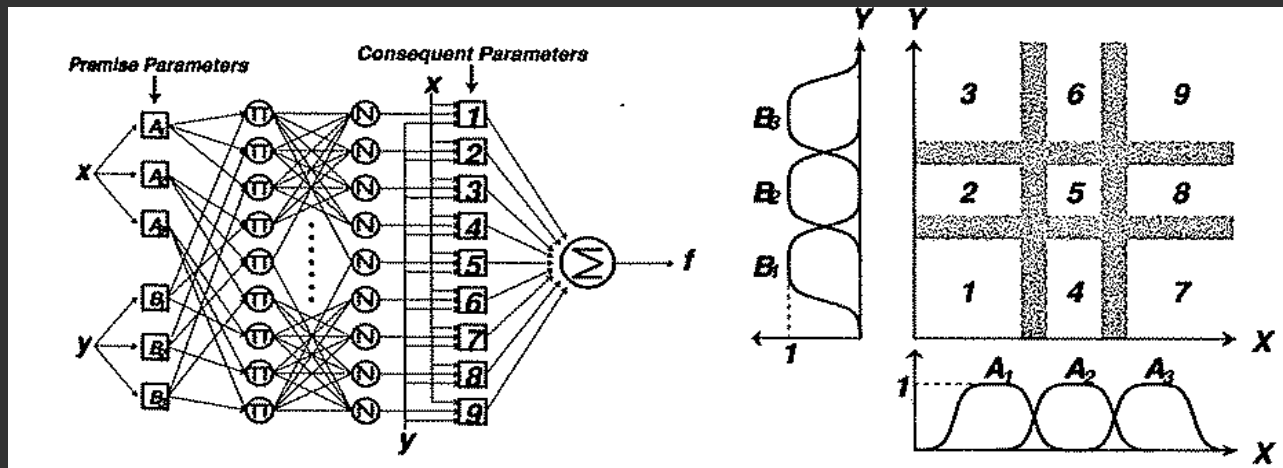
$$O_{5,1} = \sum_i \overline{w}_i f_i = \frac{\sum_i w_i f_i}{\sum_i w_i}$$

ANFIS: Another Architecture
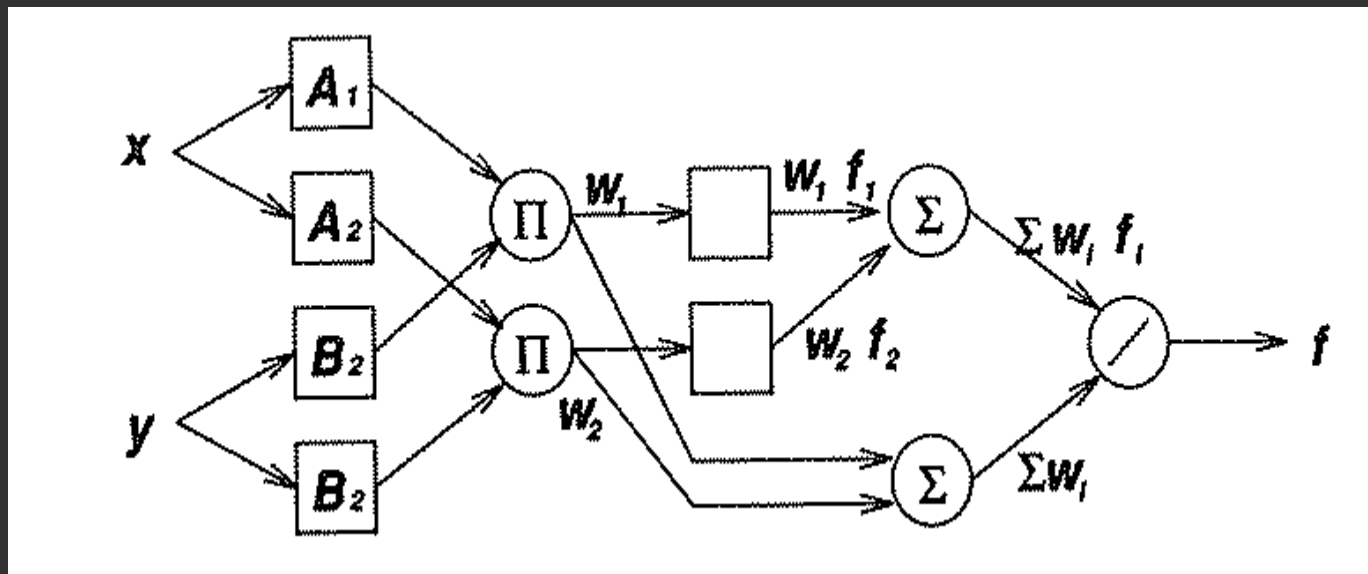
Weight normalization is performed at the very last layer

ANFIS: Hybrid Learning

The premise part of a rule defines a fuzzy region, while the consequent part specifies the output within the region

	Forward pass	Backward pass
Premise parameters	Fixed	Gradient descent
Consequent parameters	Least-squares estimator	Fixed

S1= Set of premise (nonlinear) parameters
S2= Set of consequent (linear) parameters
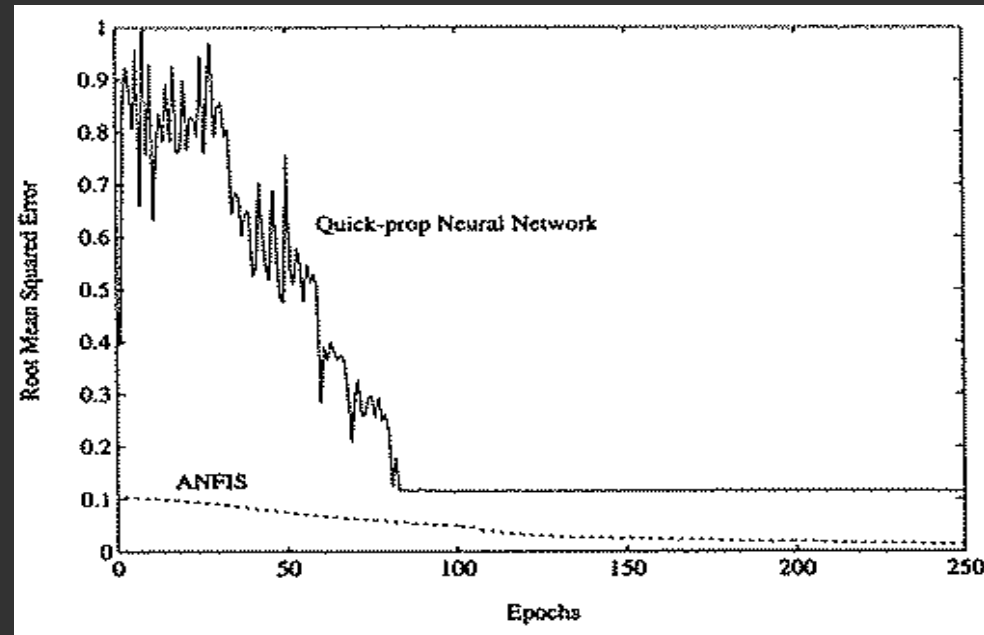
Example 1: Two-Input Sinc Function

$$z = \text{sinc}(x, y) = \frac{\sin(x)\sin(y)}{xy}.$$

ANFIS
- 121 training data pairs
- 16 rules with 4 membership functions assigned to each input
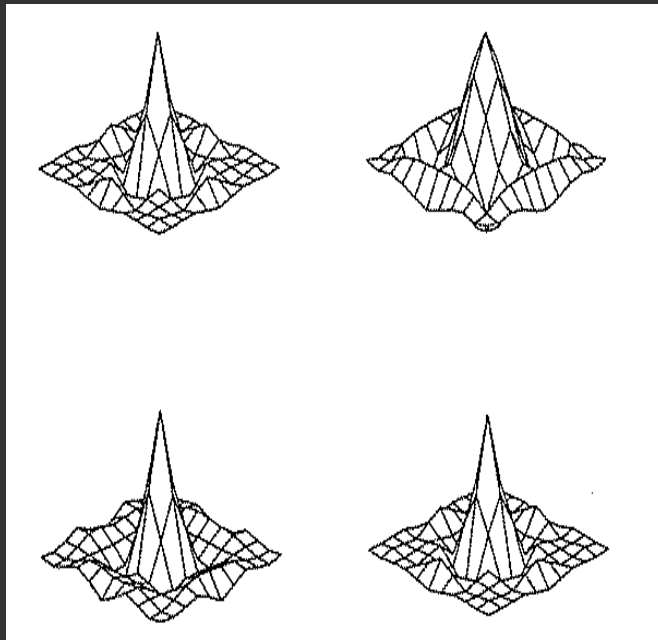- 72 total fitting parameters (24 premise and 48 consequent)

2-18-1 Backpropagation MLP
- 121 training data pairs
- Trained with quick propagation
- 73 total fitting parameters (connection weights and thresholds)
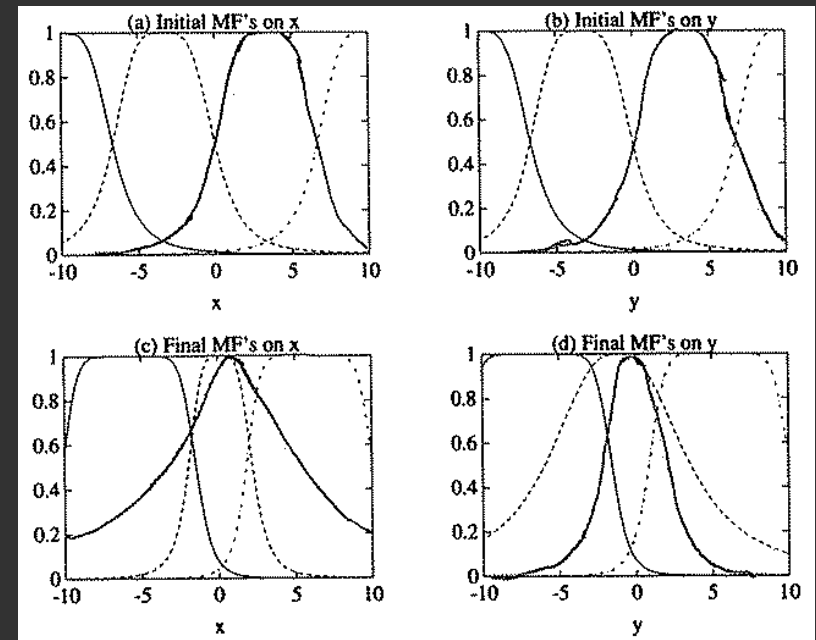

ANFIS approximates the highly nonlinear surface more effectively than the MLP

Example 1: Two-Input Sinc Function

- Training data and reconstructed surfaces at different epochs: 0.5, 99.5 and 249.5

- Initial and final membership functions

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Example 2: Predicting Mackey-Glass Chaotic Time Series

- Desired and predicted values for both training and test data are essentially the same

NDEI: root mean square error divided by the standard deviation of the target series; Prediction: 500 values test data set

(a) Mackey-Glass Time Series

(b) Prediction Errors
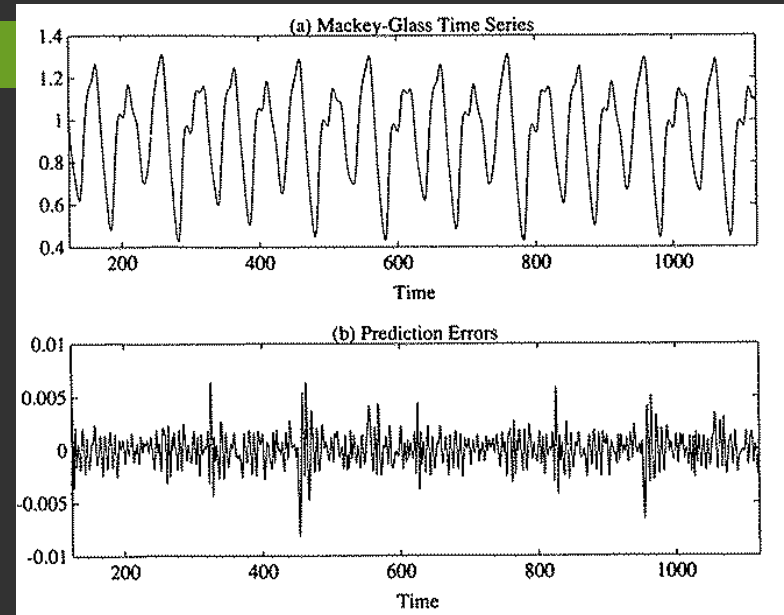
Method	Training cases	Non-dimensional error index	PARAM
ANFIS	500	0.007	(104)
AR model	500	0.19	
Cascaded-correlation NN	500	0.06	(693)
Backpropagation MLP	500	0.02	(540)
6th-order polynomial	500	0.04	
Linear predictive method	2000	0.55	

Advantages of ANFIS

The remarkable generalization capability of ANFIS is due to:

1. ANFIS can achieve a highly nonlinear mapping. Therefore, it is superior to common linear methods in reproducing nonlinear time series.

2. The initial parameters are intuitively reasonable and all the input space is covered properly (fast convergence).

3. ANFIS consists of fuzzy rules that are local mappings instead of global ones, then facilitate the minimal disturbance principle (the adaptation should not only reduce the output error for the current training pattern but also minimize disturbance to response already learned).

CART: Classification and Regression Trees

- ANFIS only deals with **parameter identification**, we still need methods for **structure identification** to determine an initial ANFIS architecture.

- CART generates a tree partitioning of the input space, which relieves the "curse of dimensionality" problem (num. of rules increasing exponentially with num. of inputs)
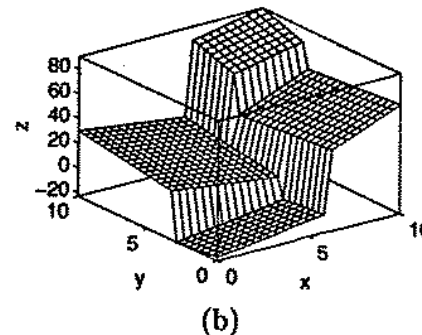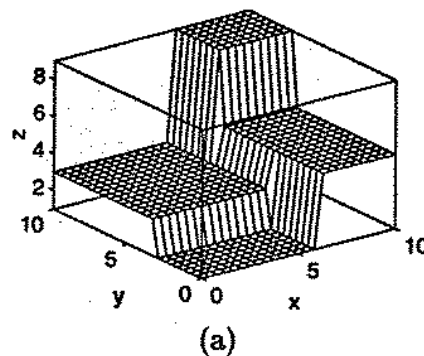
CART: Classification and Regression Trees

Binary regression tree with two inputs and one output

Internal node

Terminal node

Constants or equations
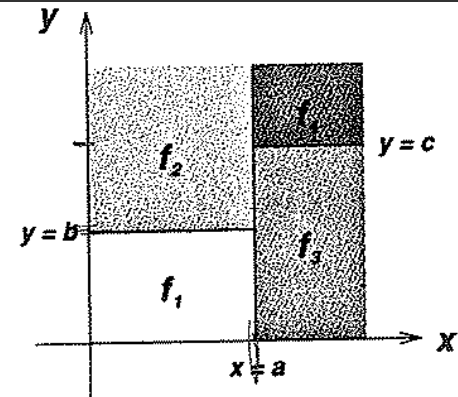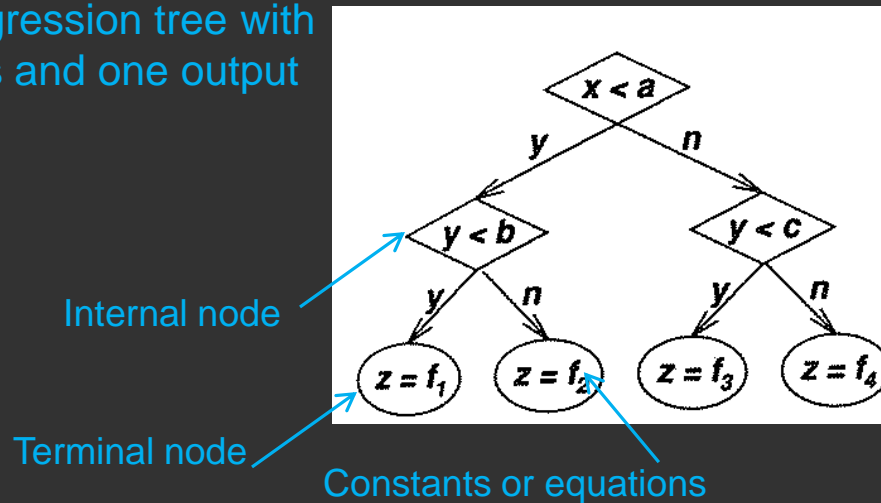
The decision tree partitions the input space into 4 regions

Constants: a=6, b=3, c=7
f1=1, f2=3, f3=5, f4=9

Linear functions:
f1= 2x-y-20,
f2= -2x+2y+10,
f3=6x-y+5,
f4=3x+4y+20

CART: Classification and Regression Trees

CART first **grows** the tree extensively based on a training data set, and then **prunes** the tree back based on a minimum cost-complexity principle

Tree Growing

▪ CART grows a decision tree by determining a succession of splits (decision boundaries) that partition the training data into disjoint subsets

▪ Starting from the root node (contains all the training data), an exhaustive search is performed to find the split that best reduces an error measure (cost function)

▪ The data set is partitioned into two subsets and the same splitting method is applied to both child nodes

▪ This recursive procedure terminates either when the <u>error measure associated with a node falls below a certain tolerance level</u>, or when the <u>error reduction does not exceed a certain threshold value</u>

CART: Classification and Regression Trees

For a regression tree, the error measure that quantifies the performance of a node t in separating data into disjoint subsets is usually taken as the square error (or residual) of a local model employed to fit the data set of the node:

$$E(t) = \min_{\boldsymbol{\theta}} \sum_{i=1}^{N(t)} (y_i - d_t(\mathbf{x}_i, \boldsymbol{\theta}))^2,$$

where $\{x_i, y_i\}$ is a data point, $d_t(\mathbf{x}_i, \boldsymbol{\theta})$ is a local model for node t (with modifiable parameter θ) and E(t) is the mean-square error of fitting the local model to the data set in the node

Constant $(d(x, \theta) = \theta)$: the minimizing of $E(t)$ is the average value of the desired output

$$\theta^* = \frac{1}{N(t)} \sum_{i=1}^{N(t)} y_i.$$

CART: Classification and Regression Trees

Linear model (d(x, θ) = linear model with linear parameters): least-squares methods to identify the minimizing θ*

For any split s of node t into t_l and t_r, the change in the error measure is expressed as:
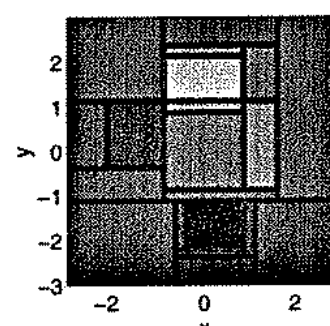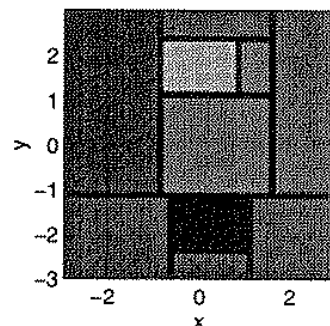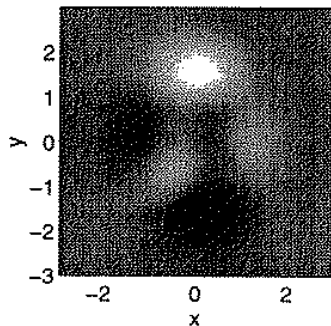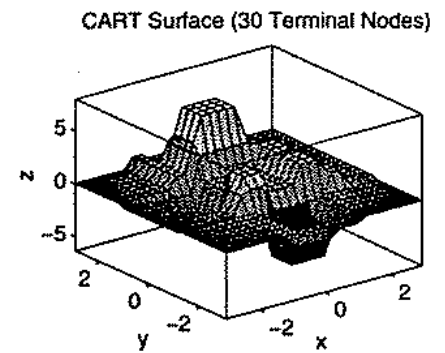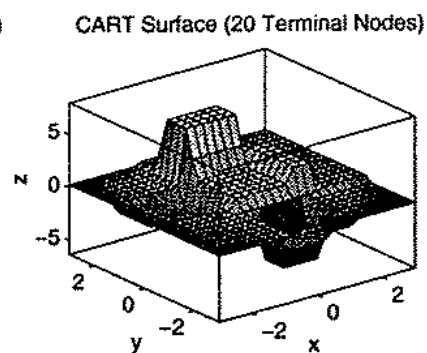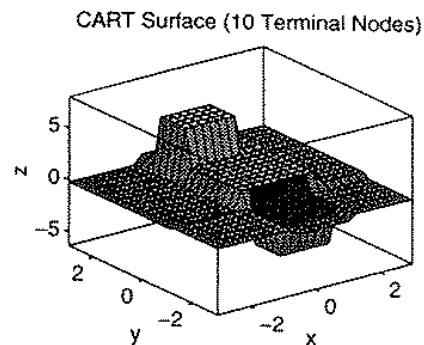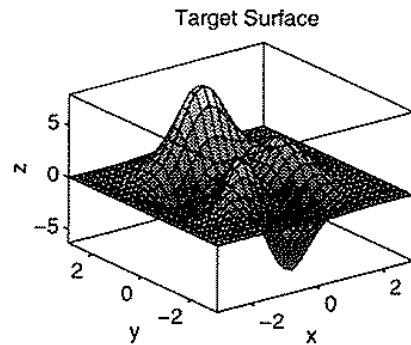
$$\Delta E(s,t) = E(t) - E(t_l) - E(t_r)$$

the best split s^* is the one that maximizes the decrease in the error measure:

$$\Delta E(s^*,t) = \max_{s \in S} \Delta E(s,t).$$

The strategy for growing a regression tree is to split nodes (or data set) iteratively and thus maximize the decrease in $E(T) = \sum_{t \in \tilde{T}} E(t)$, the overall error measure (or cost) of the tree.

CART: Classification and Regression Trees

| Target Surface | CART Surface (10 Terminal Nodes) | CART Surface (20 Terminal Nodes) | CART Surface (30 Terminal Nodes) |
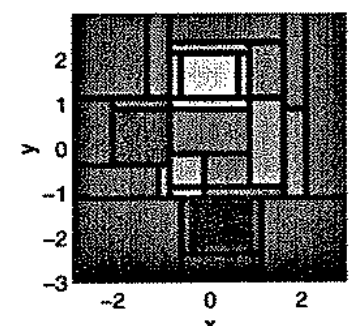
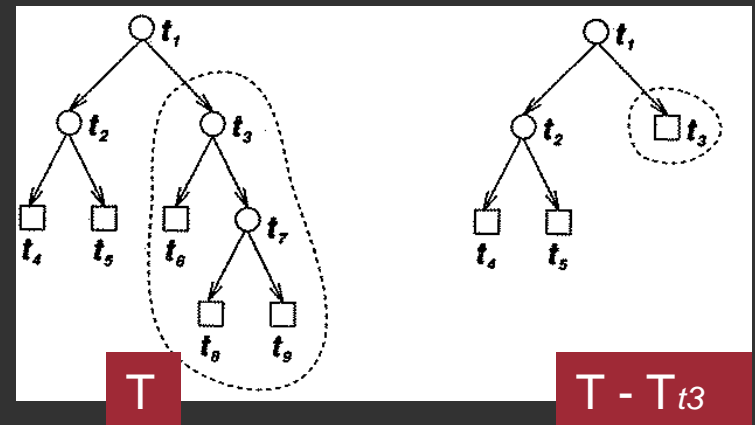# CART: Classification and Regression Trees

<u>Tree Pruning</u>

▪ The tree that the growing procedure yields is often too large and it is biased towards the training data set (overfitting)

Principle of minimum cost-complexity: prune the tree finding the weakest subtree in it, i.e. considering both the <u>training error measure</u> and the <u>number of terminal nodes</u> (measure of tree's complexity)

The internal node with the smallest α_t is chosen as the target node for shrinking

$$\alpha_t = \frac{E(T) - E(T_t)}{|\tilde{T}_t| - 1}.$$

number of terminal nodes in t

T

T - T$_{t3}$

CART: Classification and Regression Trees

Therefore, a tree-pruning cycle consists of the following tasks:
1. Calculate α_t for each internal node t in Ti
2. Find the minimal α_t and choose $T - T_t$ as the next minimizing tree

Next step: select one of these candidate trees as the optimum-size tree:

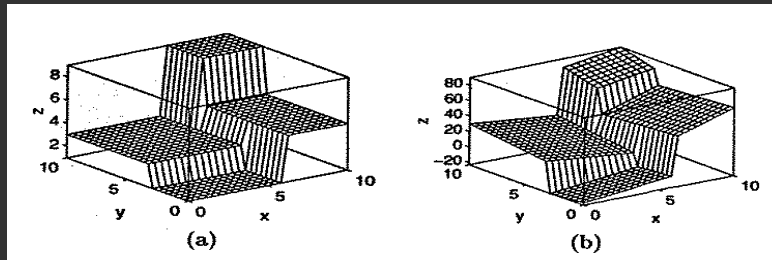• use an independent test (checking)
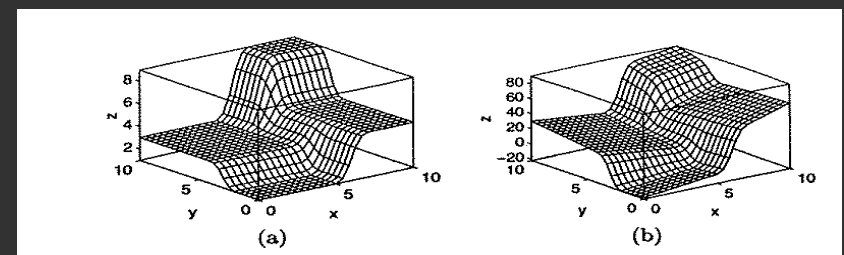• cross-validation
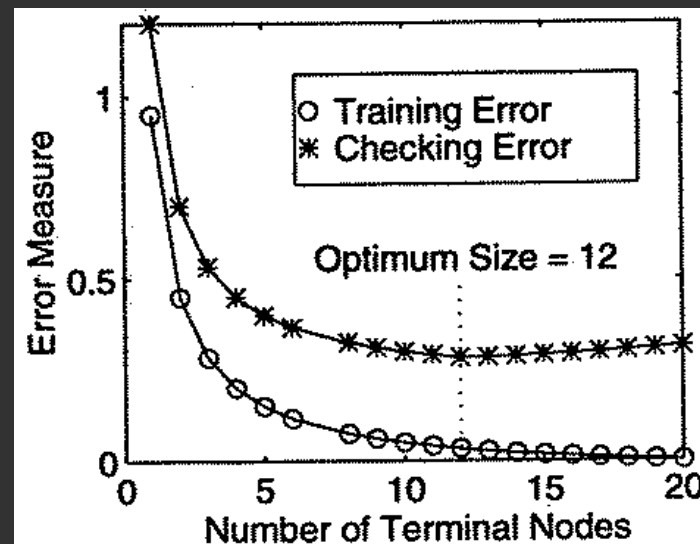
CART - ANFIS

CART is used to find the number of ANFIS rules and the initial locations of membership functions before training

$$\begin{cases} \text{If } x < a \text{ and } y < b, \text{ then } z = f_1. \\ \text{If } x < a \text{ and } y \geq b, \text{ then } z = f_2. \\ \text{If } x \geq a \text{ and } y < c, \text{ then } z = f_3. \\ \text{If } x \geq a \text{ and } y \geq c, \text{ then } z = f_4. \end{cases}$$
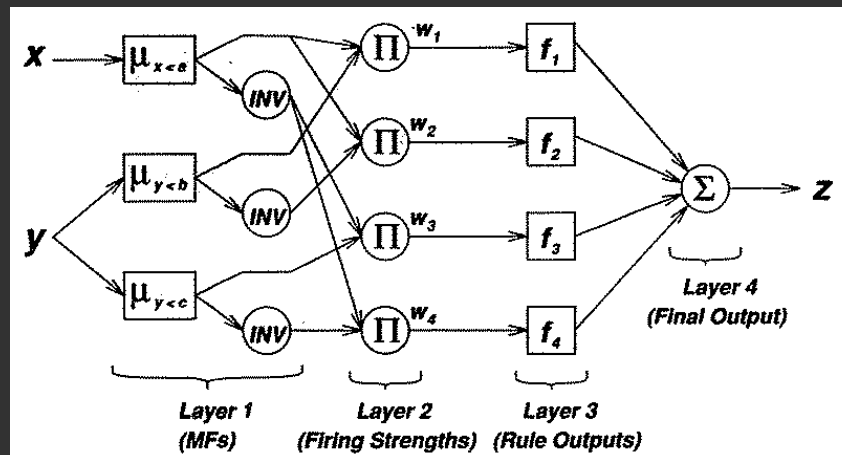
a=6; b=3; c=7

Crisp: discontinuous boundaries


Fuzzy: continuous boundaries

Extracting Fuzzy Models from Data

In the context of intelligent data analysis it is of great interest how such fuzzy models can automatically be derived from example data. Since, besides prediction, understandability is of prime concern, the resulting fuzzy model should offer insights into the underlying system. To achieve this, different approaches exist that construct grid-based rule sets defining a global granulation of the input space, as well as fuzzy graph based structures.

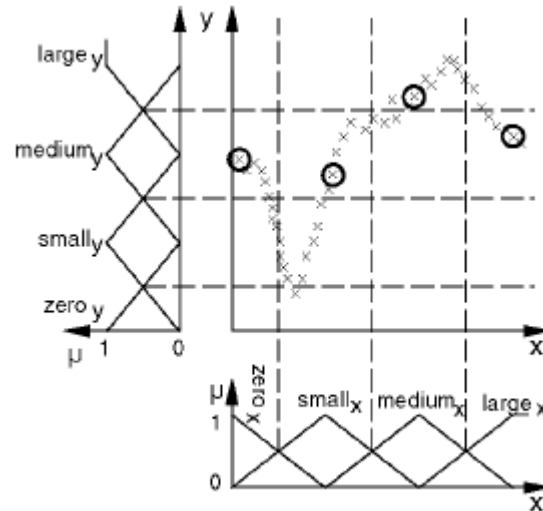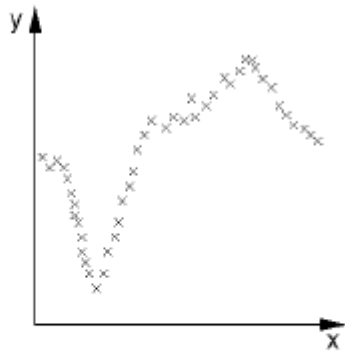# Extracting Grid-Based Fuzzy Models from Data

Grid-based rule sets model each input variable through a usually small set of linguistic values. The resulting rule base uses all or a subset of all possible combinations of these linguistic values for each variable, resulting in a global granulation of the feature space into "tiles":
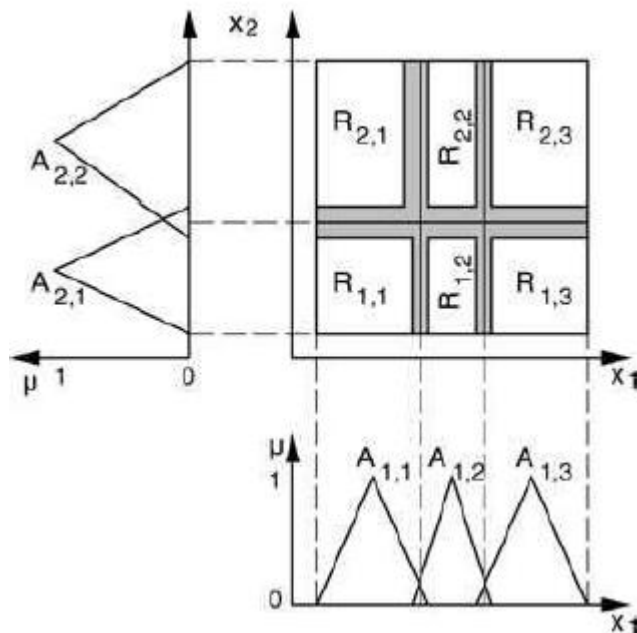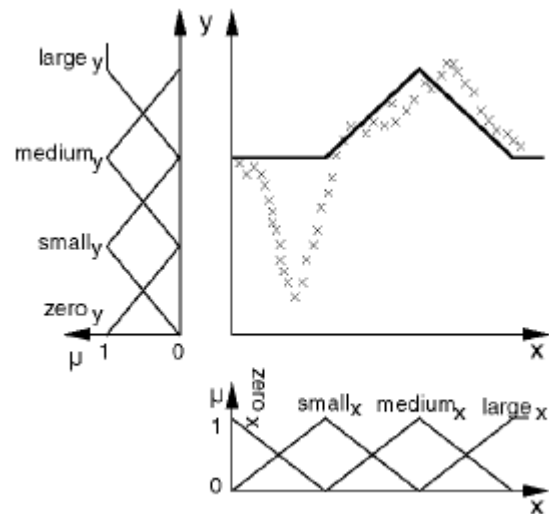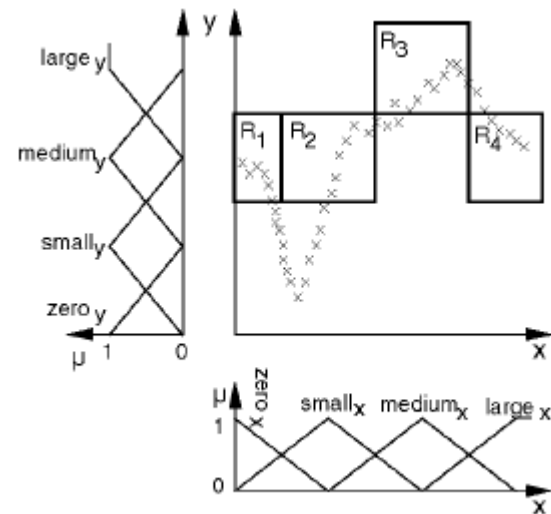
Extracting Grid-Based Fuzzy Models from Data: Wang&Mendel

Wang&Mendel [531] presented a straightforward approach to learning fixed-grid Mamdani models. After predefinition of the granulation of all input variables and also the output variable, one sweep through the entire dataset determines the closest example to the geometrical center of each rule, assigning the closest output fuzzy value to the corresponding rule:

$R_1 :$ IF x IS $zero_x$ THEN y IS $medium_y$
$R_2 :$ IF x IS $small_x$ THEN y IS $medium_y$
$R_3 :$ IF x IS $medium_x$ THEN y IS $large_y$
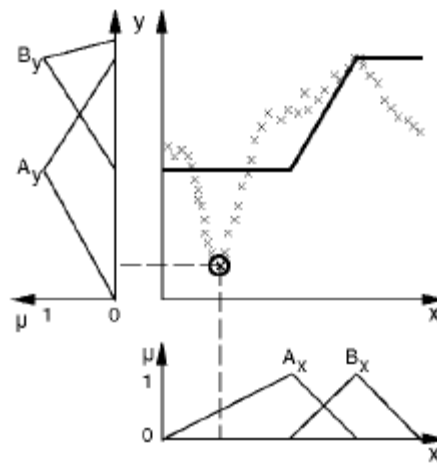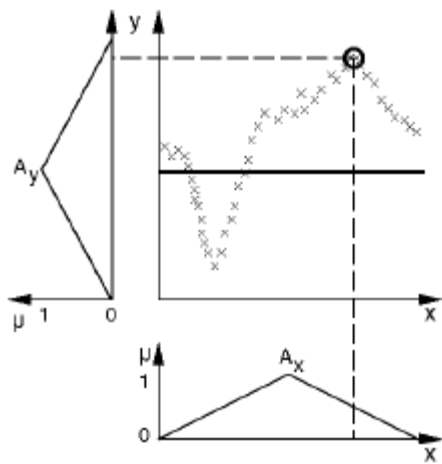$R_4 :$ IF x IS $large_x$ THEN y IS $medium_y$

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Extracting Grid-Based Fuzzy Models from Data: Higgins&Goodman

For practical applications it is obvious, however, that using such a predefined, fixed grid results in a fuzzy model that will either not fit the underlying functions very well or consist of a large number of rules. This is why approaches are of more interest that fine-tune or even automatically determine the granulations of both input and output variables.

Higgins&Goodman: Initially only one membership function is used to model each of the input variables as well as the output variable, resulting in one large rule covering the entire feature space. Subsequently new membership functions are introduced at points of maximum error. This is done until a maximum number of divisions is reached or the approximation error remains below a certain threshold.

IF x IS A_x THEN y IS A_y

IF x IS B_x THEN y IS B_y
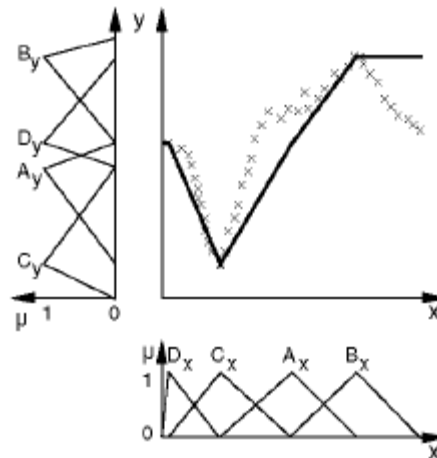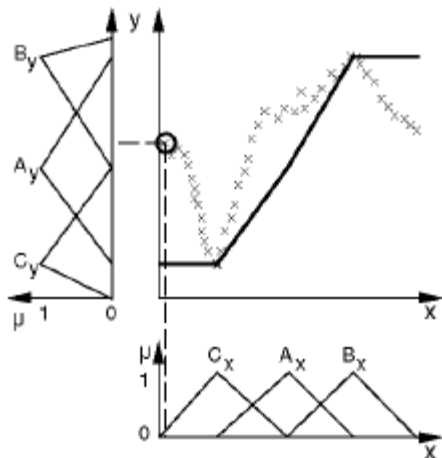
IF x IS C_x THEN y IS C_y

IF x IS A_x THEN y IS D_y
IF x IS B_x THEN y IS B_y
IF x IS C_x THEN y IS C_y
IF x IS D_x THEN y IS D_y

This approach is able to model extrema much better than Wang&Mendel algorithm, but has a strong tendency to concentrate on outliers

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