1. Description of the membership functions that you have designed for the output variable and the rule base defined. Argue why you have chosen these membership and rules.

The range proposed in the problem for the thrust [-25 25]. The range for the error is defined in the range [-80 80] and the range for the error derivative in the range [-5 5].

$$E := \Delta\theta = \theta_{ref} - \theta_t \qquad \dot{E} := \frac{\partial E}{\partial t} \approx \frac{\Delta E}{\Delta t} = E_t - E_{t-1}$$

The first equation shows the definition for the Error as the difference between the reference angle and the current angle. The second and third equations are two alternate but equivalent definitions of the error derivative in the discrete domain as a discrete difference of errors or angles. Considering the previous, we defined 3 possible states for each of the two variables that the fuzzy system is dependant on:

- $E < 0$: Case when Θ_t is greater than Θ_{ref}, thus the bar is located to the right of the expected position.
  - $\dot{E} < 0 \rightarrow (|E_t| > |E_{t-1}|) \cap (E_t < E_{t-1}) \cap (\Theta_t > \Theta_{t-1} > \Theta_{ref}) \rightarrow T < 0$
  - $\dot{E} = 0 \rightarrow (E_t = E_{t-1}) \cap (\Theta_t = \Theta_{t-1} > \Theta_{ref}) \rightarrow T < 0$
  - $\dot{E} > 0 \rightarrow (|E_t| < |E_{t-1}|) \cap (E_t > E_{t-1}) \cap (\Theta_{t-1} > \Theta_t > \Theta_{ref}) \rightarrow T = 0$

- $E = 0$: Case where Θ_t is equal to Θ_{ref}. The target angle has been achieved.
  - $\dot{E} < 0 \rightarrow (E_t < E_{t-1}) \cap (E_{t-1} > 0) \cap (\Theta_t = \Theta_{ref} > \Theta_{t-1}) \rightarrow T < 0$
  - $\dot{E} = 0 \rightarrow (E_t = E_{t-1}) \cap (\Theta_t = \Theta_{t-1} = \Theta_{ref}) \rightarrow T = 0$
  - $\dot{E} > 0 \rightarrow (E_t > E_{t-1}) \cap (E_{t-1} < 0) \cap (\Theta_t = \Theta_{ref} < \Theta_{t-1}) \rightarrow T > 0$

- $E > 0$: Case where Θ_t is smaller than Θ_{ref}, thus the bar is located to the left of the expected position.
  - $\dot{E} < 0 \rightarrow (|E_t| < |E_{t-1}|) \cap (E_t < E_{t-1}) \cap (\Theta_{t-1} < \Theta_t < \Theta_{ref}) \rightarrow T = 0$
  - $\dot{E} = 0 \rightarrow (E_t = E_{t-1}) \cap (\Theta_{t-1} = \Theta_t < \Theta_{ref}) \rightarrow T > 0$
  - $\dot{E} > 0 \rightarrow (|E_t| > |E_{t-1}|) \cap (E_t > E_{t-1}) \cap (\Theta_t < \Theta_{t-1} < \Theta_{ref}) \rightarrow T > 0$

Once these rules were established, we defined the Fuzzy System and tweaked the membership functions and values using as baseline parameters $\Theta_{ref} = -10$ and Ideal Thrust = -0.062 while analyzing the system performance on θ Error and Thrust distance to the ideal value.

Table 1 represents the experiments performed in order to find the best approximation to the ideal memberships functions and its values. Two triangular membership functions experiments were developed of which the second was found to be better with the triangular functions centered at -15 0 and 15. However, the experiment done with *gauss2mf* membership functions was found to be the best overall, with an improvement of over 2 orders of magnitude over the first experiment in terms of distance to the ideal Thrust value.

We can conclude that the *gauss2mf* Membership Function with variance 3 and centers: -17.3 -12.7 -2.295 2.295 12.7 17.3 yields the best results with respect to the optimal theoretical result.

2. **Plots of the results that you get for the following values with a simulation stop time of 80:**

2.1) θ_{ref} = 20º and Thrust = 0.123

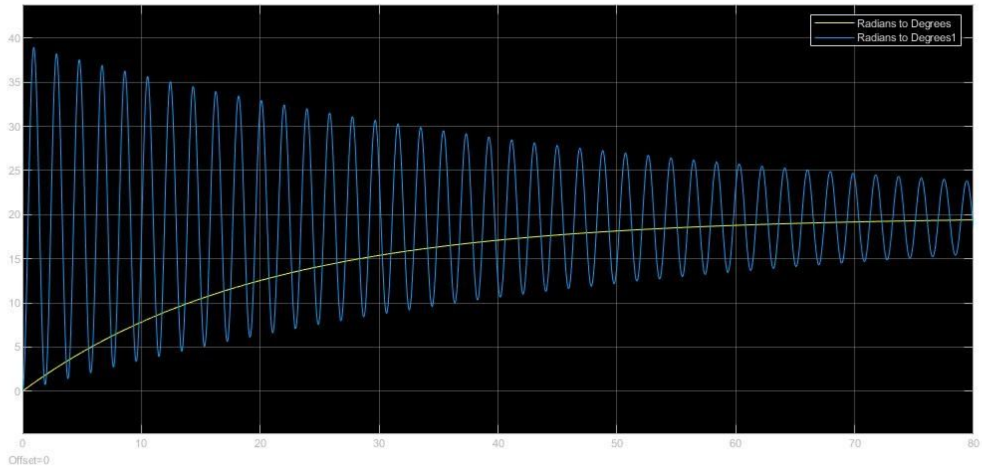

Figure 1.The figure shows the response of the system under fuzzy control (yellow) compared with the response of the open loop or no-control system (blue) with a constant thrust of 0.123 for the first 80 seconds. Although the system or the pendulum itself is able to stabilize in both cases at 20º (Θ_{ref}), the fuzzy-controlled one does it much faster with no overshoot.

2.2) θ_{ref} = -10º and Thrust = -0.062

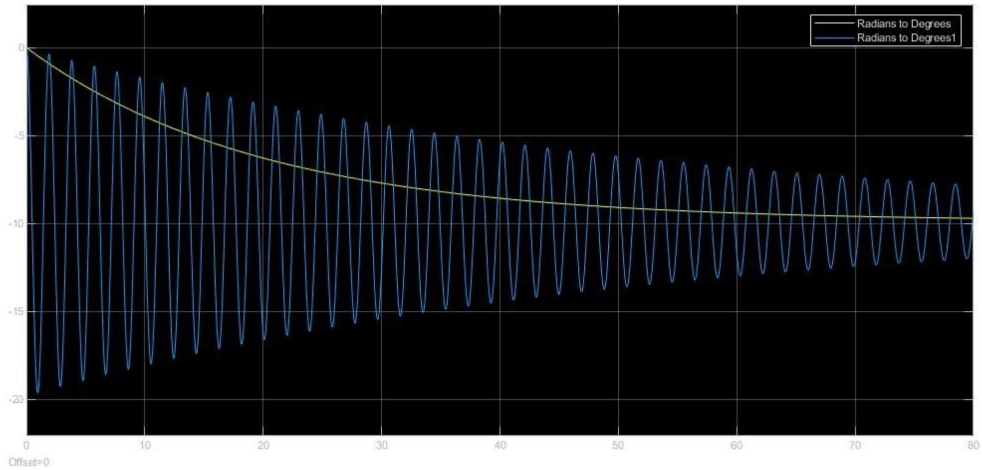

Figure 2.The figure shows the response of the system under fuzzy control (yellow) compared with the response of the open loop or no-control system (blue) with a constant thrust of -0.062 for the first 80 seconds. Although the system or the pendulum itself is able to stabilize in both cases at -10º (Θ_{ref}), the fuzzy-controlled one does it much faster with no overshoot.

Write your own comments about the results that you get.

In both plots we can observe a comparison between the system under fuzzy control and the open loop system with constant torque value that in both cases produces similar results, the fuzzy control system is able to generalize well given their inputs, output, membership functions and rule sets to provide a much quicker and stable result.

Although the fuzzy control system exhibits a correct behaviour that is good enough to provide a solution to the problem, we still feel like the stabilization time -- or the time required for the pendulum to stop at the expected angle -- could be optimized so the system reaches its goal much quicker. In order to do so, we reckon we'll require more membership functions in the output, specially convenient for two cases:

- $E < 0 \cap \dot{E} > 0$: In this case, as $\Theta_{t-1} > \Theta_t > \Theta_{ref}$, the pendulum is located to the right of the expected position (Θ_{ref}) but still moving down (to the left) towards it. Therefore, we assumed that given just 3 MF, the Thrust should be 0 as the pendulum is already moving to the desired location and does not need extra help. With another MF, we might add a small force to help the pendulum out.

- $E > 0 \cap \dot{E} < 0$: In this case, as $\Theta_{t-1} < \Theta_t < \Theta_{ref}$, the pendulum is located to the left of the expected position (Θ_{ref}) but still moving down (to the right) towards it. Therefore, we assumed that given just 3 MF, the Thrust should be 0 as the pendulum is already moving to the desired location and does not need extra help. With another MF, we might add a small force to help the pendulum out.

- $E = 0 \cap \dot{E}! = 0$: In this case, as $\Theta_t = \Theta_{ref}$, the pendulum is located at the expected position (Θ_{ref}) but still moving in any direction away from it. Therefore, we assumed that given just 3 MF, the Thrust should not be 0 as the pendulum is still moving away from the desired location. With another MF, we might reduce this amount of Thrust to a smaller and more appropriate (considering that the pendulum is currently at the correct location) value.

3. What happens if you increase the number of membership functions of the output?

We tested the θ_{ref} = -10º and ideal thrust = -0.062 scenario (the same described as in section 2.2) with two systems. The first was the best case scenario from section 1 (3 gauss2mf). The second one was a FIS block with 5 MFs of gauss2mf type. Each gauss2mf function has 4 parameters; std center for the first and the second gaussian functions. To distribute the range [-25 25] uniformly between the 10 gaussian distributions, the 12 equidistant points between those values were computed (10 gaussians and the 2 boundaries). These values are: -25.0000, -20.4545, -15.9091, -11.3636, -6.8182, -2.2727, 2.2727, 6.8182, 11.3636, 15.9091, 20.454 and 25.0000. All gaussians were defined with 3 as the standard deviation. The extra 2 MFs were used to define intermediate ranges in the thrust range.

The rules are also to be changed to take advantage of the extra MFs that have been defined. For that we redefined the rules where all the rules that resulted in $T > 0$ or $T < 0$ were redefined as $T >> 0$ and $T << 0$ respectively. The rules were changed in that way:

	$E > 0$	$E = 0$	$E < 0$
$\dot{E} > 0$	$T >> 0$	$T >> 0$	$T > 0$
$\dot{E} = 0$	$T > 0$	$T = 0$	$T < 0$
$\dot{E} < 0$	$T < 0$	$T << 0$	$T << 0$

The comparison previously defined yielded the results presented in Table 2. In the experiments we can observe clear improvements to the system by adding the two intermediate ranges and their MFs. The results show an improvement of 0.1 degrees compared to the best result of exercise 1 even though the values of the distance to the ideal thrust are worse. However as shown in Figures 3 and 4, the main advantage of this modification is the speed of convergence of the system for the two scenarios of the problem.

Annex

Fis file name	MF Type	Lower MF Range	Medium MF Range	Higher MF Range	Final θ Error	Distance to ideal Thrust
workingFis1	trimf	[-50 -25 -5]	[-20 0 20]	[5 25 50]	0.5325	3.412e-6
workingFis2	trimf	[-25 -15 0]	[-15 0 15]	[0 15 25]	0.3549	6.402e-7
workingFis3	gaussmf	[6.5 -15]	[6.5 0]	[6.5 15]	0.4649	1.741e-6
workingFis4	gaussmf	[5 -15]	[5 0]	[5 15]	0.3206	3.105e-8
workingFis5	**gauss2mf**	**[3 -17.3 3 -12.7]**	**[3 -2.295 3 2.295]**	**[3 12.7 3 17.3]**	**0.2941**	**1.769e-8**
workingFis6	2 sigmf - dsigmf	[0.5 -15]	[0.5 -10 0.5 10]	[0.5 15]	0.4967	9.728e-7

Table 1

Fis file name	Low MF Range	Mid-Low MF Range	Middle MF Range	Mid-High MF Range	Higher MF Range	Final θ Error	Distance to ideal Thrust
workingFis5	[3 -17.3 3 -12.7]	NA	[3 -2.295 3 2.295]	NA	[3 12.7 3 17.3]	0.2941	1.769e-8
output5mfFis	**[3 -20.45 3 -15.9]**	**[3 -11.36 3 -6.81]**	**[3 -2.27 3 2.27]**	**[3 6.81 3 11.36]**	**[3 15.9 3 20.45]**	**0.1904**	**5.191e-7**

Table 2

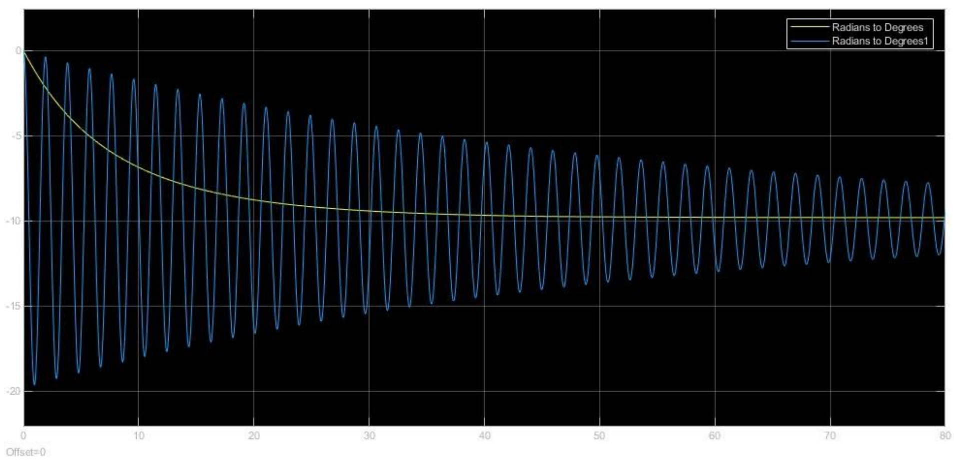

Figure 3. $\theta_{ref} = 20^o$, Thrust = 0.123 - 5 MFs Fuzzy System

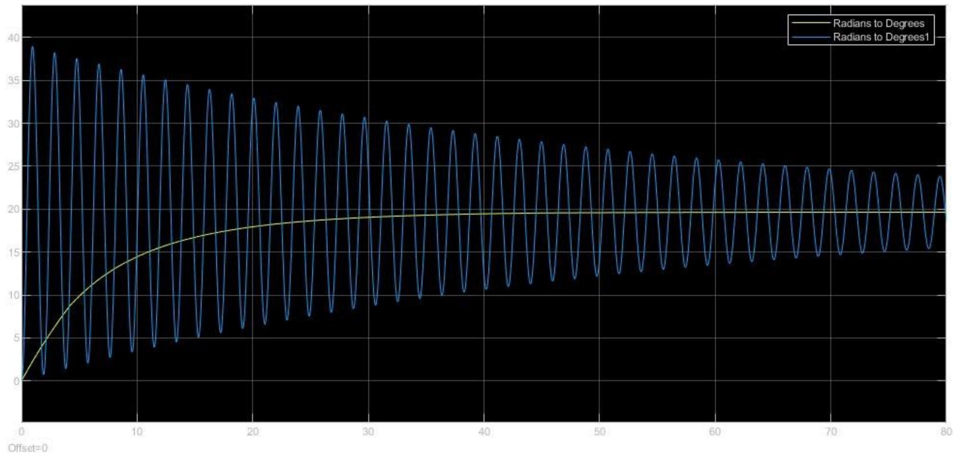

Figure 4. θ_{ref} = -10º , Thrust = -0.062 - 5 MFs Fuzzy System

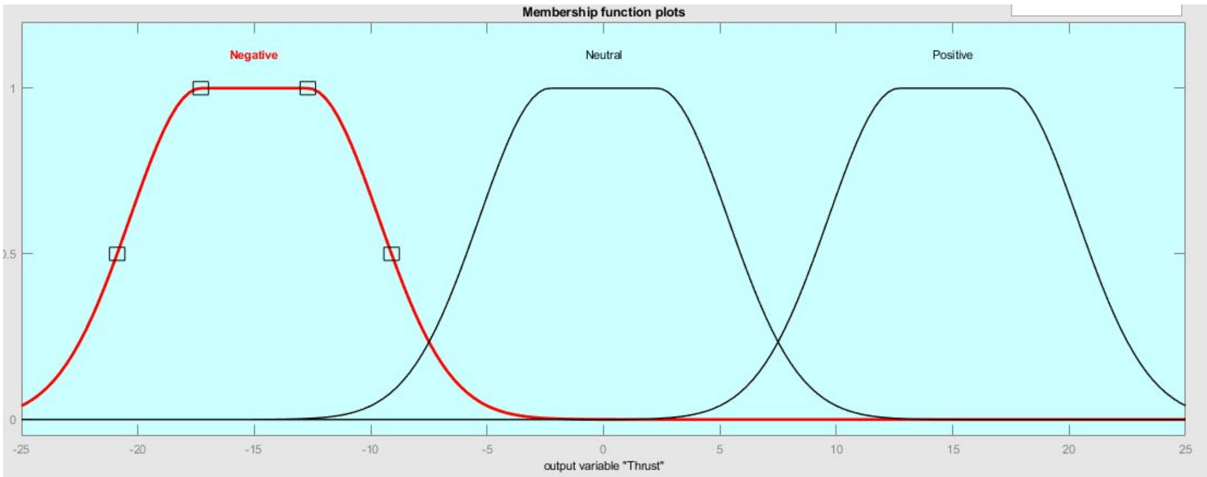

Figure 5. Fuzzy System's 3 Output MFs that yielded the best results.

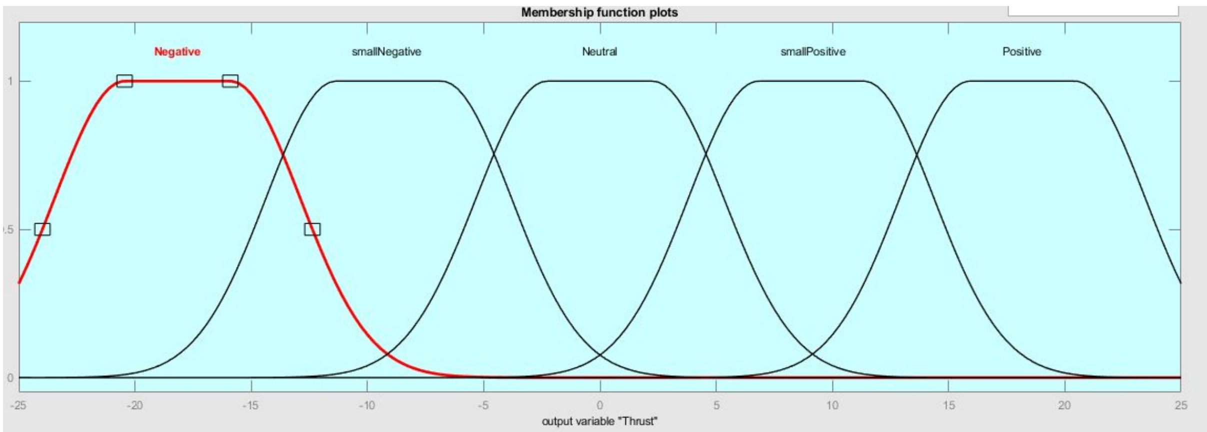

Figure 6. Fuzzy System's 5 Output MFs that yielded the best results.