Numerical study of the 2D Kuramoto-Sivashinsky equation

Víctor Ballester

Instabilities and nonlinear phenomena
M2 - Applied and Theoretical Mathematics
Université Paris-Dauphine & PSL
December 14, 2023

Abstract

This is an abstract.

Contents

1 Introduction

2d KS equation:

$$\begin{cases} u_t = -\frac{1}{2}\left|\boldsymbol{\nabla} u\right|^2 - \Delta u - \Delta^2 u & \text{in } (0,\infty)\times[0,L_x)\times[0,L_y) \\ u(t,x,y) = u(t,x+L_x,y) & \text{in } [0,\infty)\times\mathbb{R}\times[0,L_y) \\ u(t,x,y) = u(t,x,y+L_y) & \text{in } [0,\infty)\times[0,L_x)\times\mathbb{R} \\ u(0,x,y) = u_0(x,y) & \text{for all } x\in[0,L_x), y\in[0,L_y) \end{cases} \tag{1}$$

If we rescale the spatial domain to $[0,2\pi)^2$ and the time domain using the transformations

$$x_{\text{new}} = \frac{2\pi}{L_x}x \qquad y_{\text{new}} = \frac{2\pi}{L_y}y \qquad t_{\text{new}} = \left(\frac{L_x}{2\pi}\right)^2 t \tag{2}$$

we get the following equation

$$u_t = -\frac{1}{2}\left|\boldsymbol{\nabla}_\nu u\right|^2 - \Delta_\nu u - \nu_1 {\Delta_\nu}^2 u \tag{3}$$

where we used a slightly-modified notation from the one in [KKP15]

$$\boldsymbol{\nabla}_\nu = \left(\partial_x, \sqrt{\frac{\nu_2}{\nu_1}}\partial_y\right) \qquad \textbf{div}_\nu = \partial_x + \sqrt{\frac{\nu_1}{\nu_2}}\partial_y \tag{4}$$

$$\Delta_\nu = \textbf{div}_\nu(\boldsymbol{\nabla}_\nu) = \partial_{xx} + \frac{\nu_2}{\nu_1}\partial_{yy} \qquad {\Delta_\nu}^2 = \Delta_\nu(\Delta_\nu) = \partial_{xxxx} + 2\frac{\nu_2}{\nu_1}\partial_{xxyy} + \frac{\nu_2^2}{\nu_1^2}\partial_{yyyy} \tag{5}$$

and $\nu_1 = \left(\frac{L_x}{2\pi}\right)^2$, $\nu_2 = \left(\frac{L_y}{2\pi}\right)^2$ and we have dropped the subindices *new* for simplicity.

Note that the equation is invariant under the transformation $(t,x,y,\nu_1,\nu_2) \mapsto \left(\frac{\nu_2}{\nu_1}t, y, x, \nu_2, \nu_1\right)$. That is, if $u(t,x,y)$ is a solution of the equation with parameters (ν_1,ν_2), then $u\left(\frac{\nu_2}{\nu_1}t, y, x\right)$ is the solution of the equation for the parameters (ν_2,ν_1).

References

[KKP15] A. Kalogirou, E. E. Keaveny, and D. T. Papageorgiou. "An in-depth numerical study of the two-dimensional Kuramoto-Sivashinsky equation." In: *Proc. R. Soc. A* 471.20140932 (2015). DOI: .