Numerical Methods for Fluid Dynamics: TD6

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Forewords

Download TD6.tar, and then type :

```
> tar xvf TD6.tar
> cd TD6
> ipython --pylab
```

1 Acoustic waves

Let us consider a rectangular cavity of size $0 \leq x \leq L_x = 3$ and $0 \leq y \leq L_y = 1$. The propagation of acoustic waves is modeled by a forced wave equation

$$\partial_t \mathbf{u} = -c_0^2 \, \nabla p \quad \text{and} \quad \partial_t p = -\nabla \cdot \mathbf{u} + s \, .$$

The boundary conditions are periodic in x, no flux at $y = 0$ and a fixed (vanishing) pressure at $y = L_y$.

We consider $c = 1$ and

$$s = a \exp\left(-\frac{\sqrt{(x-x_0)^2 + (y-y_0)^2}}{\delta} \right) \sin\left(\frac{2\pi}{T} \, t \right) \, ,$$

with $a = 20\,,\ T = 0.2\,,\ \delta = 1/50\,,\ x_0 = L_x/4\,,\ y_0 = L_y/2\,.$
The initial conditions are $p(t = 0) = 0$ and $\mathbf{u} = \mathbf{0}\,.$

A/ We will use a staggered-mesh. Choose a positioning of points on the computational grid to be able to easily enforce the boundary conditions. Define in `Waves_staggered` the variables `dx` and `dy` suited for this grid, then construct the positions of grid points by modifying the `linspace` operator.

B/ We will use a two step method : first advance the velocity in time, then the pressure. Implements the velocity evolution equation via a Leapfrog scheme.
Impose the boundary conditions on the velocity.

C/ Now implement the evolution equation for the pressure, using a Leapfrog scheme.
Impose the boundary conditions on the pressure.

D/ Which difference do you notice between the boundary at $y = L_y$ and $y = 0$? (the frequency of forcing can be changed.)

2 Water waves at arbitrarily depth

We now want to solve for the water wave at arbitrarily depth, but small amplitude, via a spectral method.

A/ Analyse the `WaterWaves` code and run it. What do you observe ?

B/ Modify the code to be able to study arbitrary depth.