Numerical Methods for Fluid Dynamics

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CFD

Maths students (Dauphine):
Homework today
(OK until tomorrow by email)

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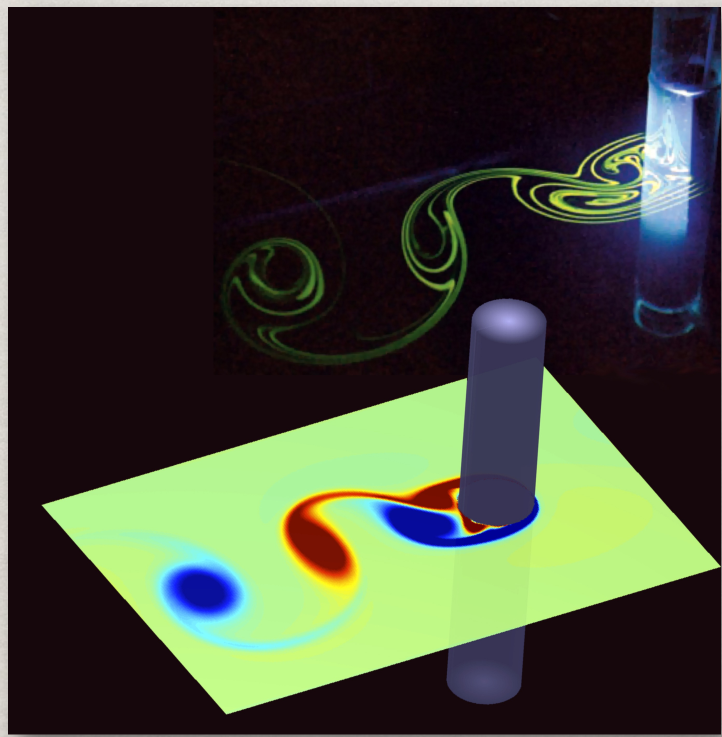

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11. Prospects

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How to choose:

- Physical problem

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How to choose:

- Physical problem
- <u>Type of PDE</u>

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Stiff problems:

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}f}{\mathrm{d}t} - \varepsilon^{-1}(y - f)$$

$$y(t = 0) = y_0$$

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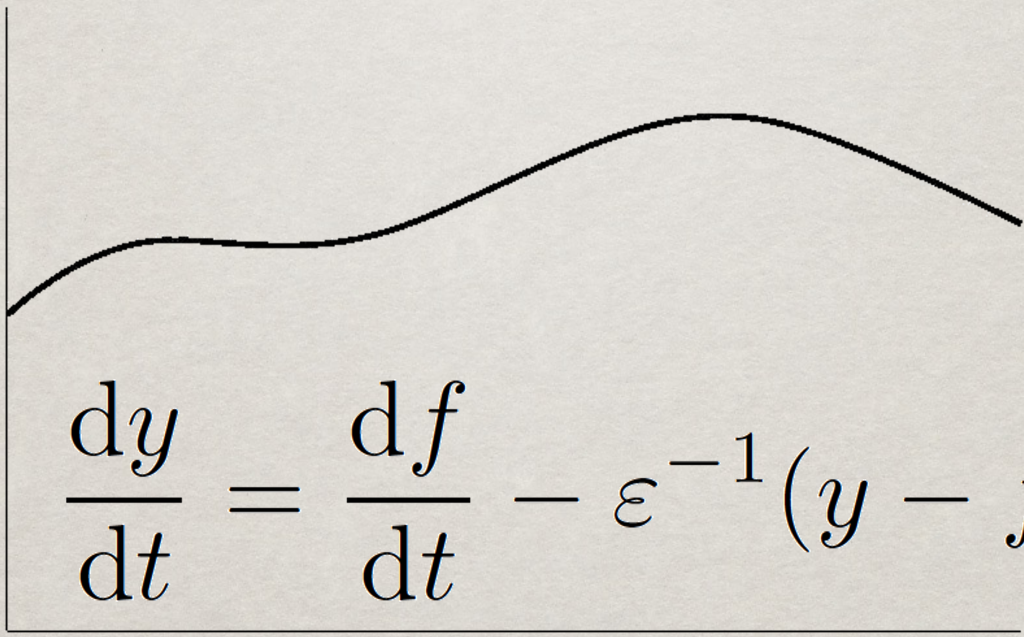
Stiff problems:



$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}f}{\mathrm{d}t} - \varepsilon^{-1}(y - f)$$

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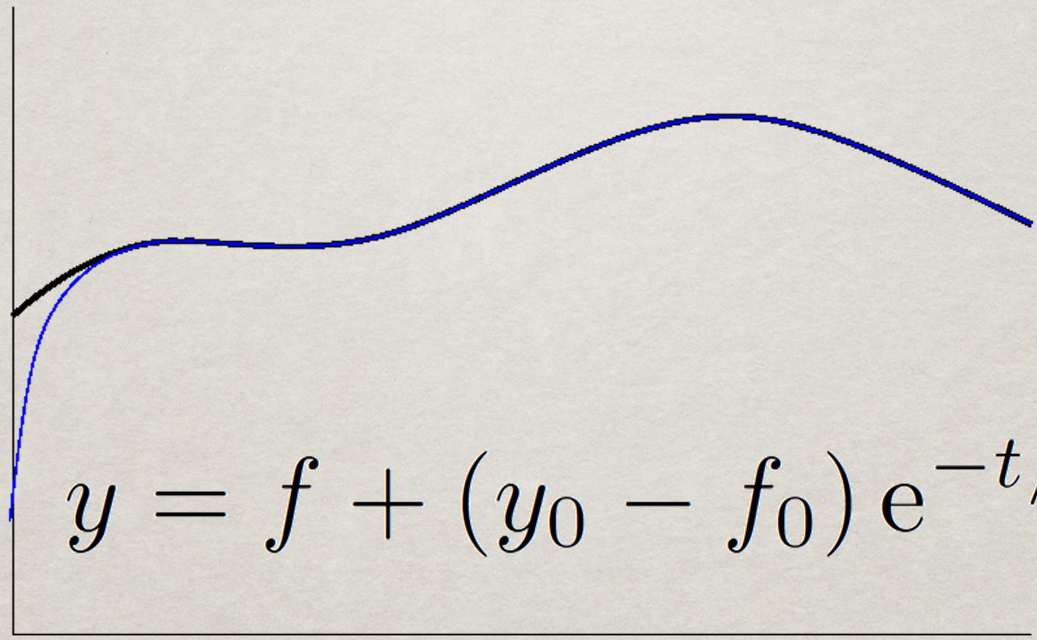
Stiff problems:



$$y = f + (y_0 - f_0)\, \mathrm{e}^{-t/\varepsilon}$$

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How to choose:

- Physical problem
- <u>Type of PDE</u>
- Shape of the domain

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How to choose:

- Physical problem
- Type of PDE
- Shape of the domain
- Anticipated measurements

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How to choose:

- Physical problem
- Type of PDE
- Shape of the domain
- Anticipated measurements
- Type of computer(!)

The importance of debugging and validation…

Ariane 5

(june 1996)

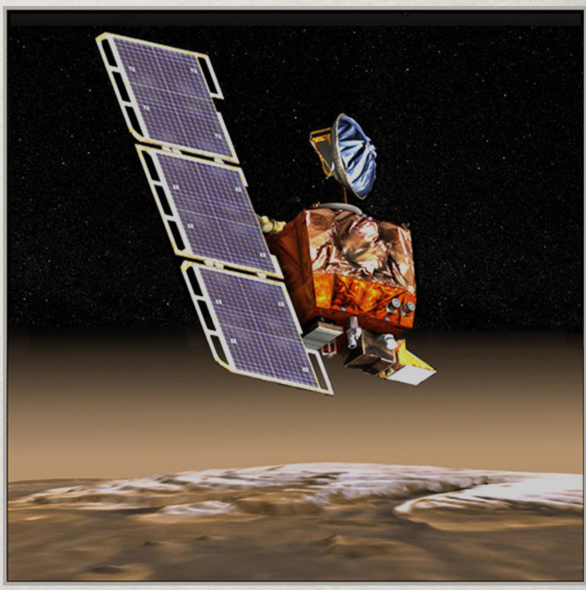
Ariane 5

(june 1996)

10 years development time, $7 billion
- June 5, 1996: Ariane 5 rocket lifts off from Kourou, French Guyana.
- And it explodes 40 seconds after lift-off
- Inertial guidance system uses gyroscopes and accelerometers to guide its course.
- Conversion of 64-bit floating-point numbers to 16-bit causes an overflow error and shuts down the system.
- On-board computer steers off-course based on erroneous input.

Mars Climate Orbiter

(september 1999)



Study the weather, climate and CO_2 budget of Mars.
$330 million
- Intended to orbit Mars 140-150 km above the surface, but it reached as low as 57 km.
- The spacecraft was destroyed by atmospheric stresses and friction.
- The navigation error arose because the contractors used US units, while the spacecraft expected SI units.

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Storm Lothar

(december 1999)

- 26 December, 1999: a very strong front passes France, Germany and Switzerland. 29 people died.

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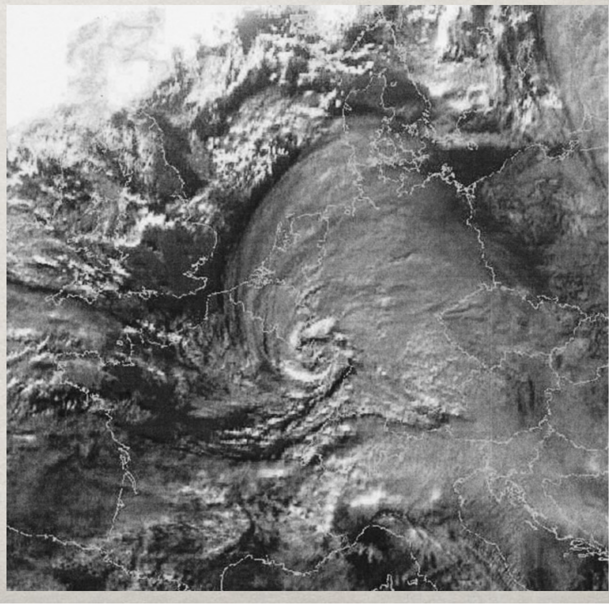
Storm Lothar

(december 1999)

- DWD interpreted the observations of an exceptional pressure drop as an error.
- It warned for strong winds, but not a devastating storm.



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Computational Mythology…

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Myth

Meteorology, four models:

- ARPEGE (Meteo-France)
- ECMWF (European model)
- GFS (US model)
- UKMO (UK model)

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Hurricane Matthew

(October 2016)

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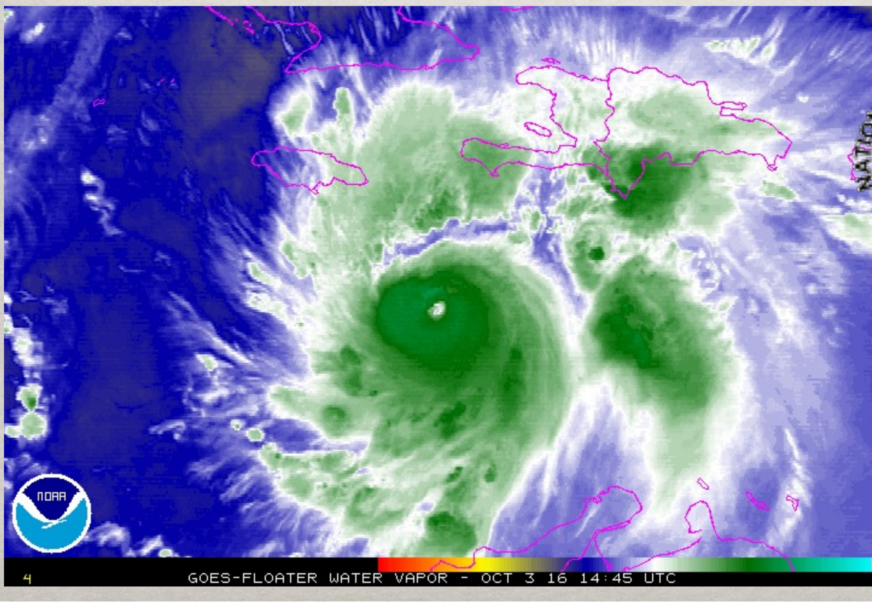
Hurricane Matthew

(October 2016)

NAM: North American Model



NATIONAL CENTERS FOR ENVIRONMENTAL PREDICTION
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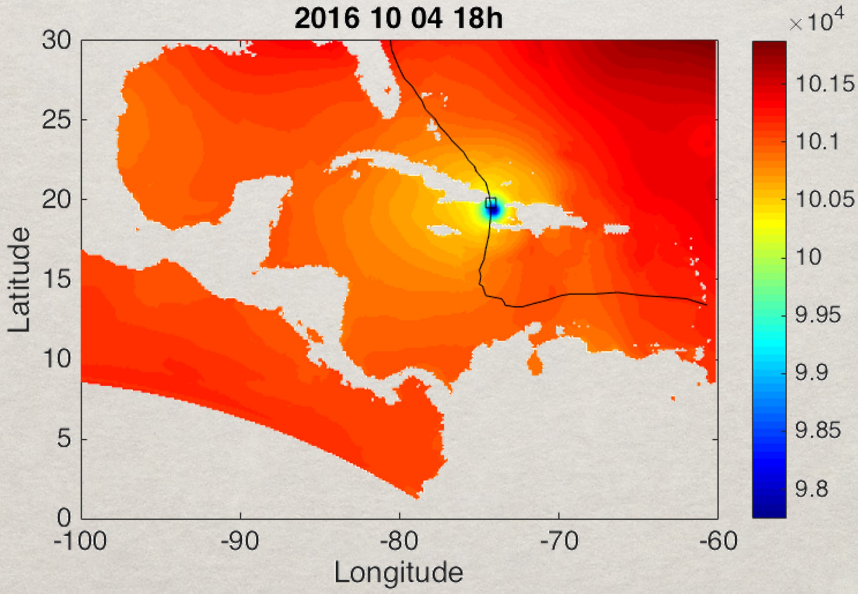
Hurricane Matthew

(October 2016)



Caraïbe 12km

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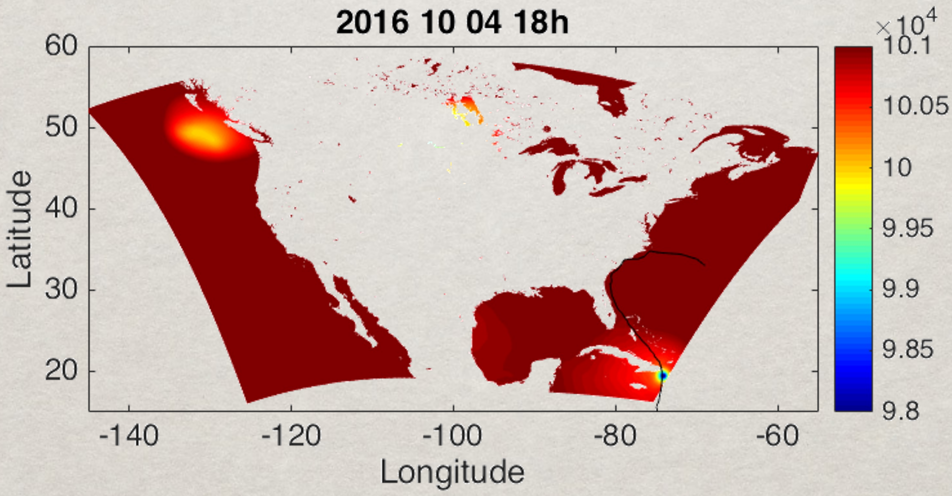
Hurricane Matthew

(October 2016)



Conus 4-5 km

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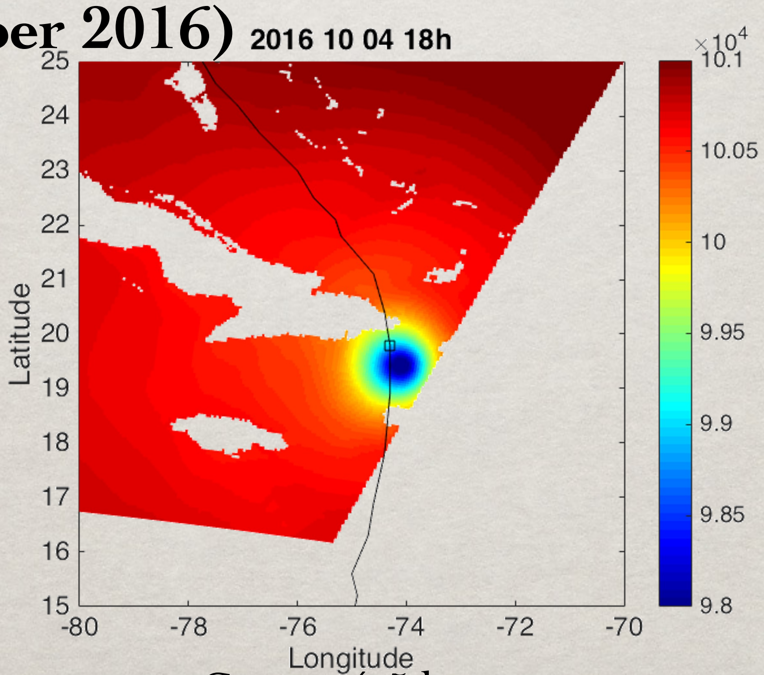
Hurricane Matthew

(October 2016)



Conus 4-5 km

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Hurricane Matthew

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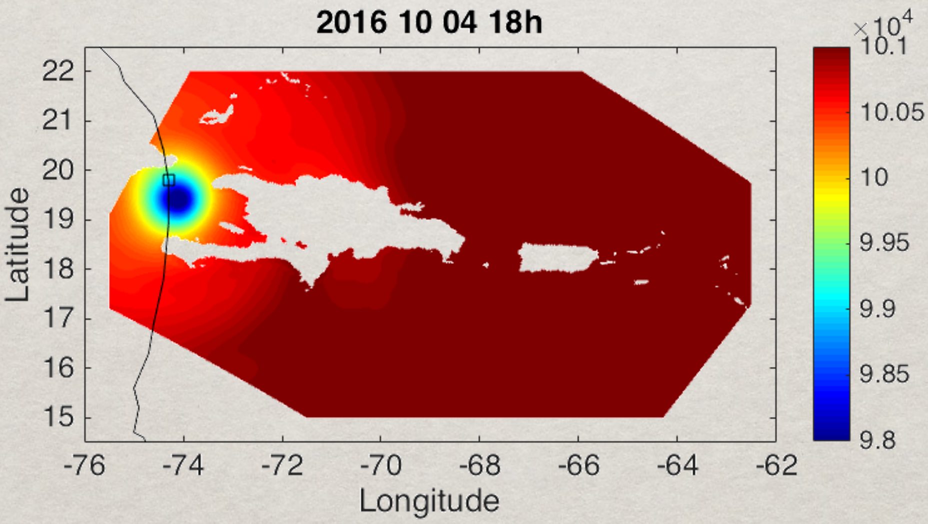

Puerto Rico 3km

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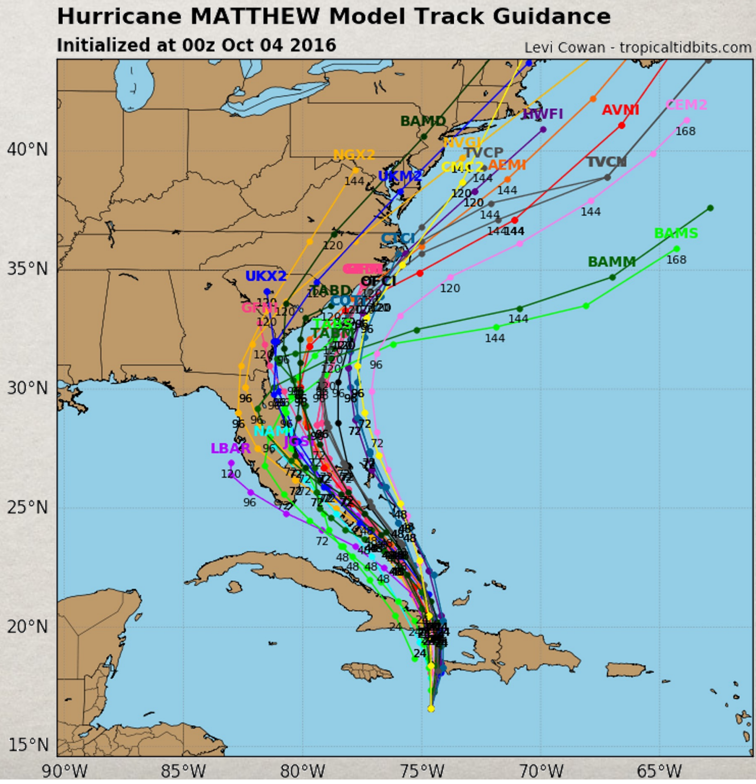
Hurricane Matthew

(October 2016)



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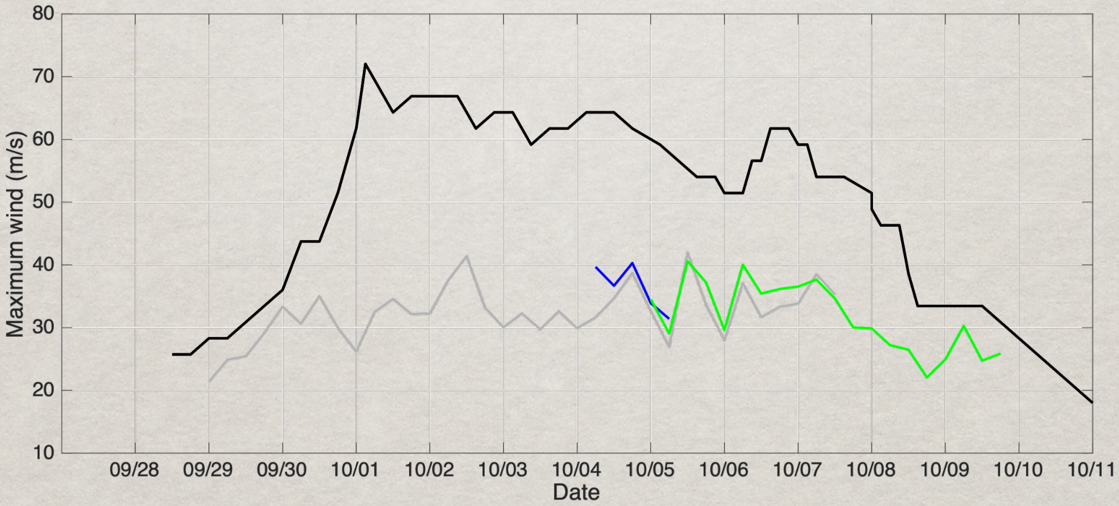
Hurricane Matthew

(October 2016)



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Hurricane Matthew

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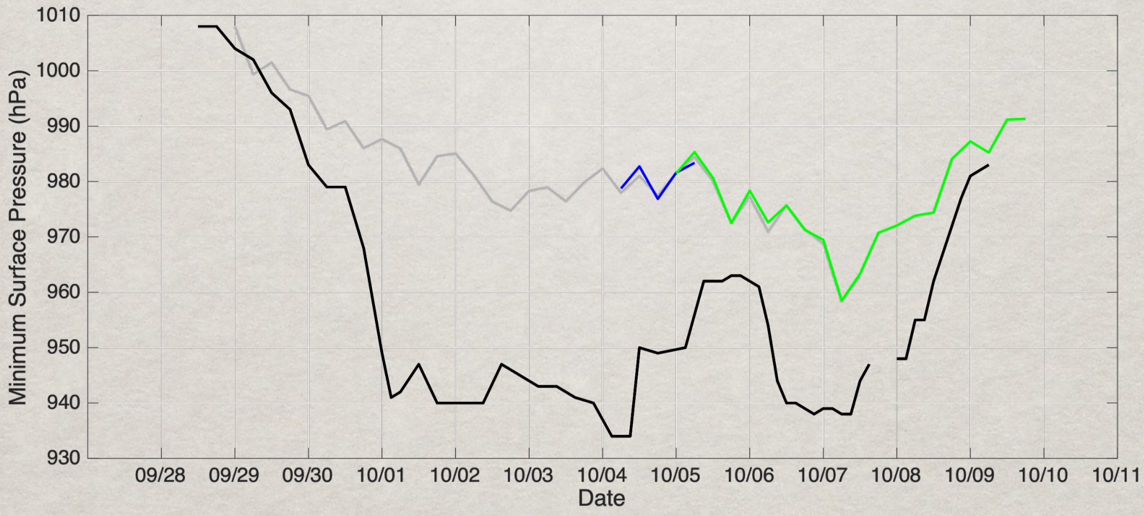

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Myth

Dassault Falcon F7X

« Aucun prototype ni maquette n'ont été réalisés.
Les formes et l'architecture de l'avion ont
été conçues uniquement sur maquette numérique. »

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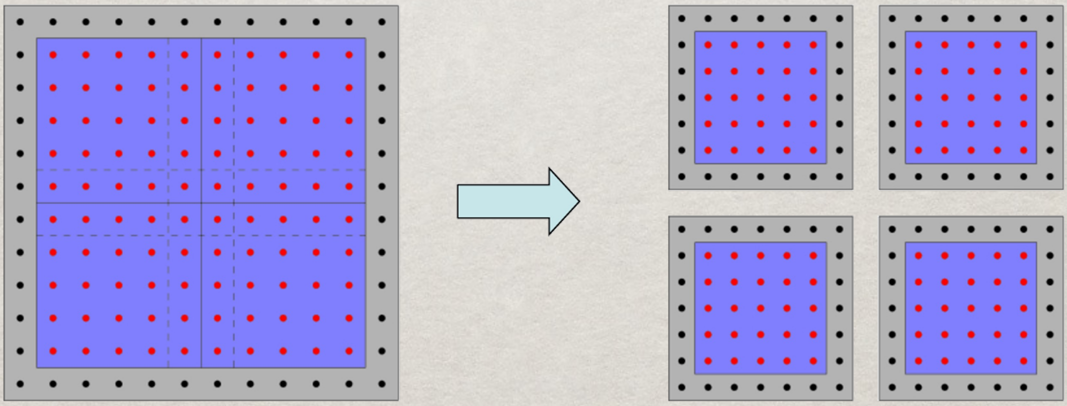
Parallel Computing

Domain decomposition

General idea: patching of simple domains to build a complex domain; solution transfer across patches



simple for hyperbolic problems   difficult for elliptic problems

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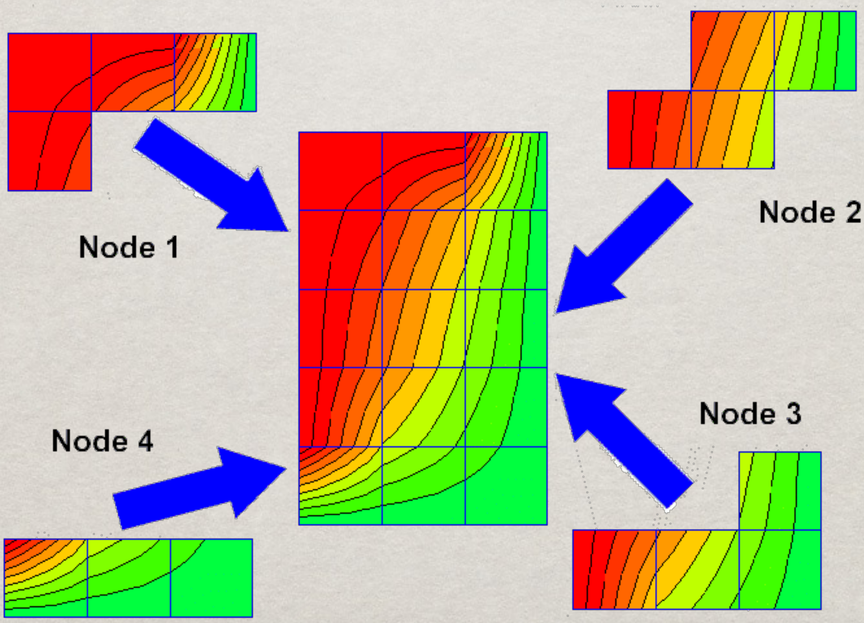
Domain decomposition



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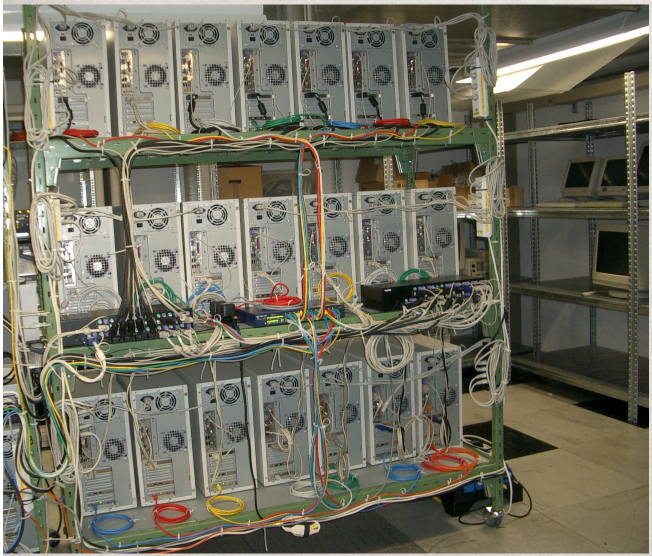
Domain decomposition



Clusters

Massively Parallel Computers

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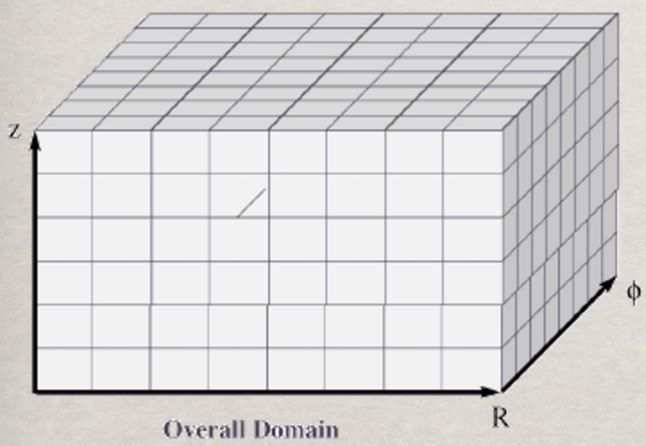
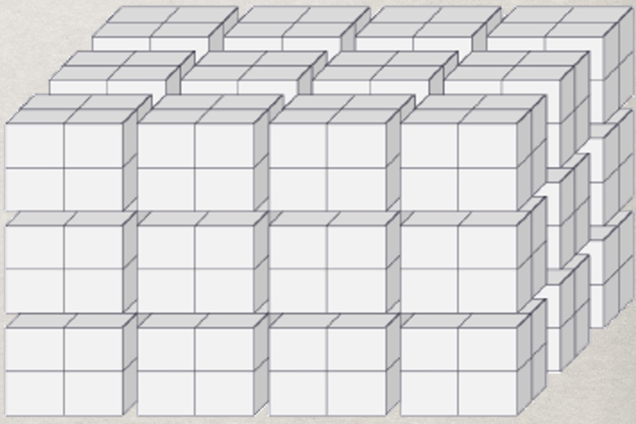
Domain decomposition





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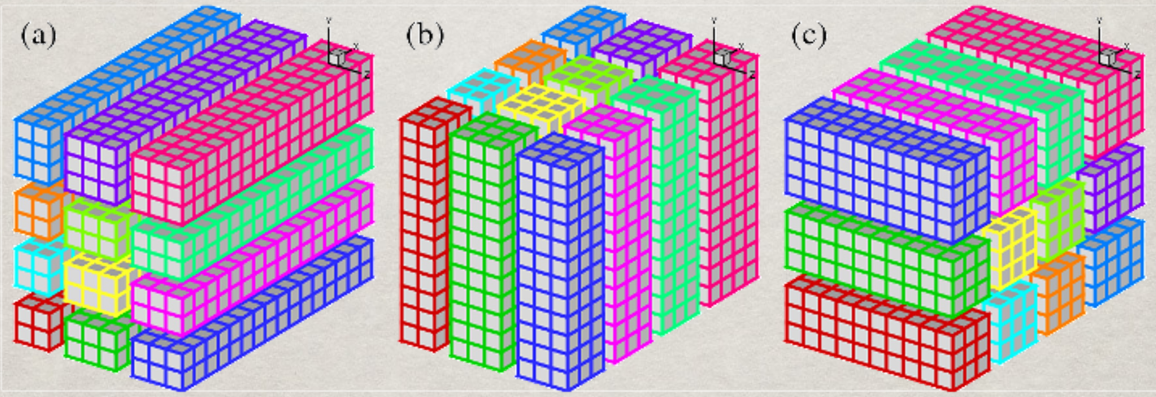
Domain decomposition



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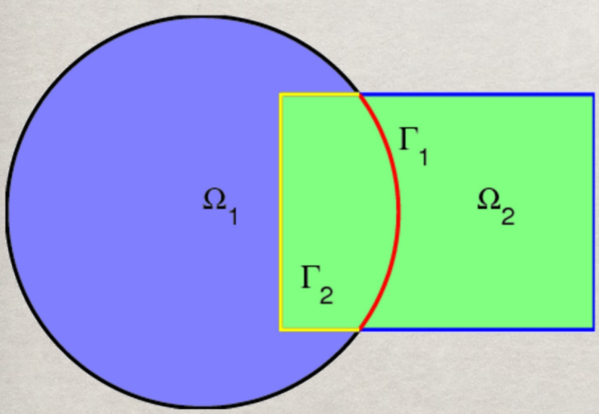
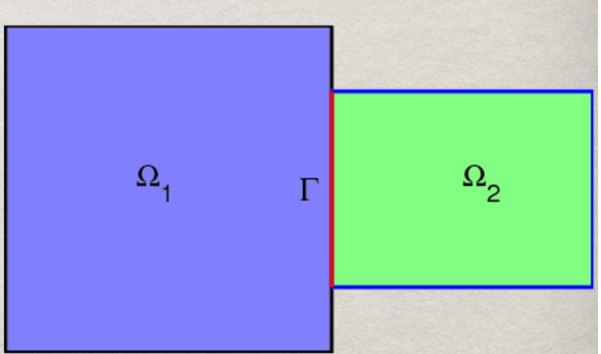
Domain decomposition

Two concepts of domain decomposition: **overlapping** and **non-overlapping**





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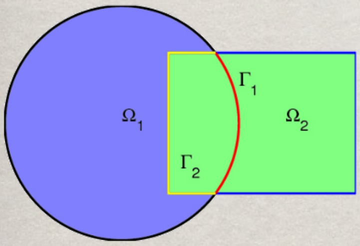
Domain decomposition



$$
\begin{aligned}
\mathbf{A}_1\mathbf{u}_1^{(n)} &= \mathbf{f}_1 && \text{in} \quad \Omega_1 \\
\mathbf{u}_1^{(n)} &= \mathbf{g} && \text{on} \quad \partial\Omega_1 \backslash \Gamma_1 \\
\mathbf{u}_1^{(n)} &= \mathbf{u}_2^{(n-1)}|_{\Gamma_1} && \text{on} \quad \Gamma_1
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{A}_2\mathbf{u}_2^{(n)} &= \mathbf{f}_2 && \text{in} \quad \Omega_2 \\
\mathbf{u}_2^{(n)} &= \mathbf{g} && \text{on} \quad \partial\Omega_2 \backslash \Gamma_2 \\
\mathbf{u}_2^{(n)} &= \mathbf{u}_1^{(n)}|_{\Gamma_2} && \text{on} \quad \Gamma_2
\end{aligned}
$$

alternating Schwarz method

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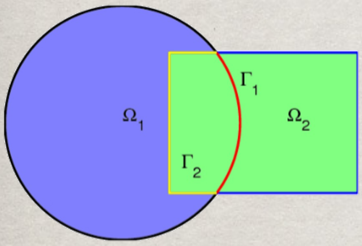
Domain decomposition



stop the iteration when

$$\|\mathbf{u}^{(n+1)}|_{\Gamma_1} - \mathbf{u}^{(n)}|_{\Gamma_1}\| \leq \epsilon$$

$$\|\mathbf{u}^{(n+1)}|_{\Gamma_2} - \mathbf{u}^{(n)}|_{\Gamma_2}\| \leq \epsilon$$

alternating Schwarz method

iterates

$$\mathbf{u}^{(n+1)}(x,y) = \begin{cases} \mathbf{u}_2^{(n+1)}(x,y) & \text{if} \quad (x,y) \in \Omega_2 \\ \mathbf{u}_1^{(n+1)}(x,y) & \text{if} \quad (x,y) \in \Omega \backslash \Omega_2 \end{cases}$$

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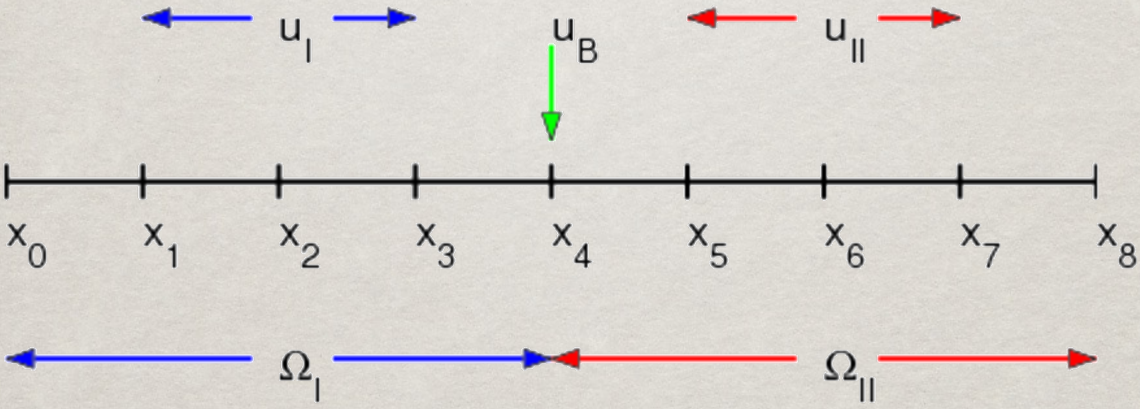
Domain decomposition

Example: one-dimensional Poisson equation



$$u'' = f \qquad u(0) = u(1) = 0$$

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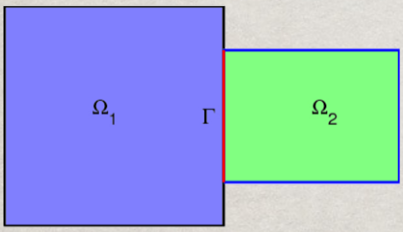
Schwartz algorithm



$$\nabla^2 u_i = f_i \qquad \text{in} \quad \Omega_i$$

$$u_i = 0 \qquad \text{in} \quad \partial\Omega \cap \partial\Omega_i$$

transmission conditions

$$u_1 = u_2 \qquad \text{on} \quad \Gamma$$

$$\frac{\partial u_1}{\partial n} = \frac{\partial u_2}{\partial n} \qquad \text{on} \quad \Gamma$$

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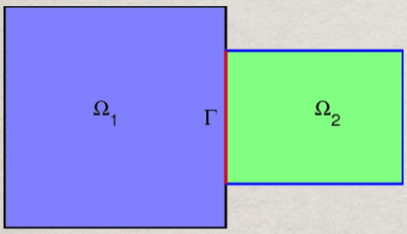
Schwartz algorithm



General idea: solve equations on each subdomain subject to mixed boundary conditions on the common interface; after each solution exchange the boundary data on the interface

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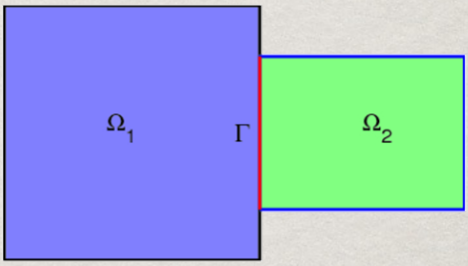
Schwartz algorithm



$$\Delta u_i^{(n+1)} = f_i \quad \text{in} \quad \Omega_i\,,$$

$$u_i^{(n+1)} = 0 \quad \text{on} \quad \partial\Omega \cap \partial\Omega_i\,,$$

$$\frac{\partial u_i^{(n+1)}}{\partial n_{ij}} + u_i^{(n+1)} = \frac{\partial u_j^{(n)}}{\partial n_{ij}} + u_j^{(n)} \quad \text{on} \quad \partial\Omega_i \cap \partial\Omega_j \quad (i \neq j)\,.$$

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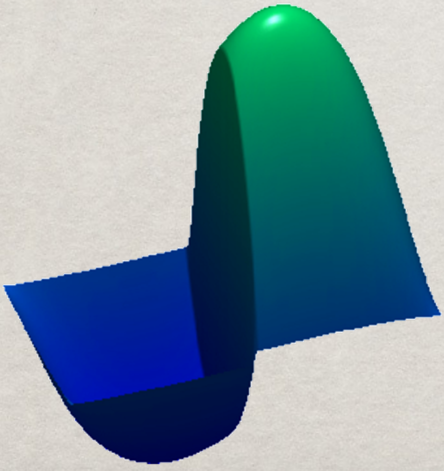
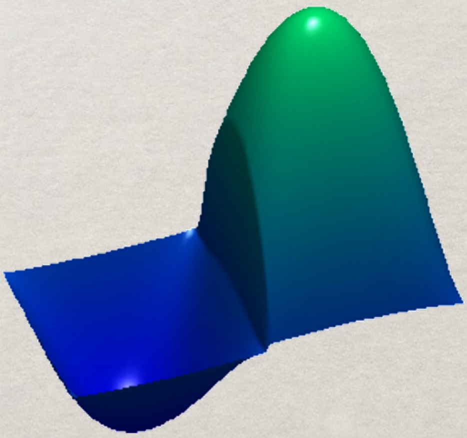
Domain decomposition

apply to two-dimensional Poisson equation





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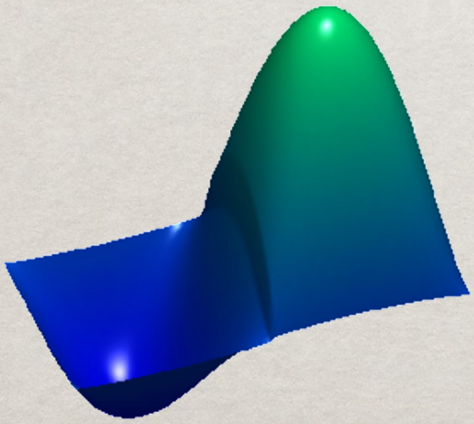
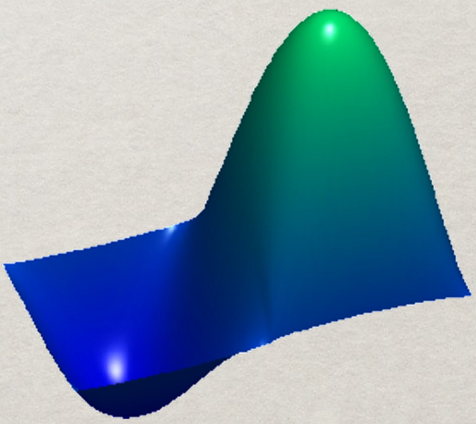
Domain decomposition

apply to two-dimensional Poisson equation





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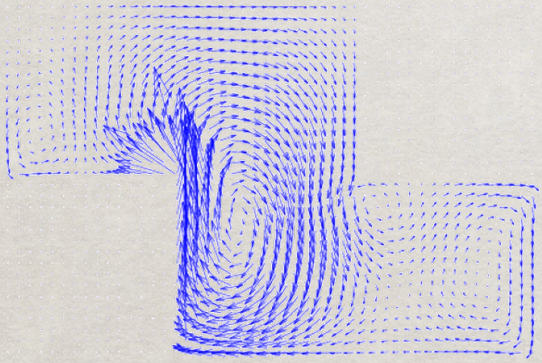
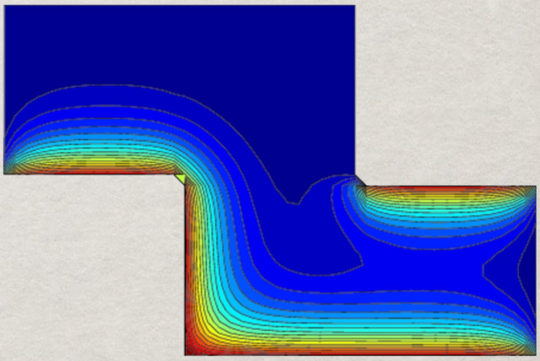
Domain decomposition

apply to buoyancy-driven flow





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Combining these methods:

- In space
- In time

Combining these methods:

- In space
- In time

Alternative methods...

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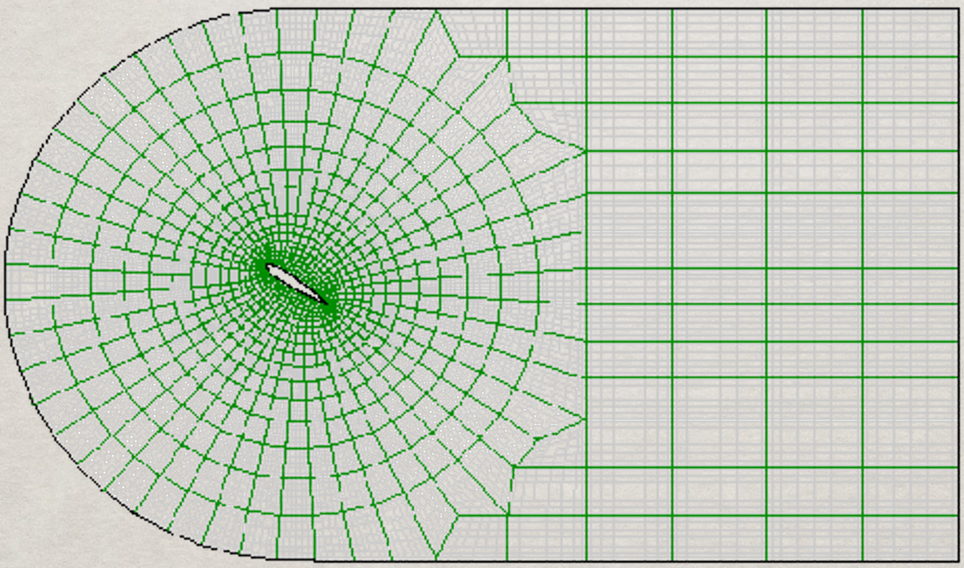
Spectral Elements



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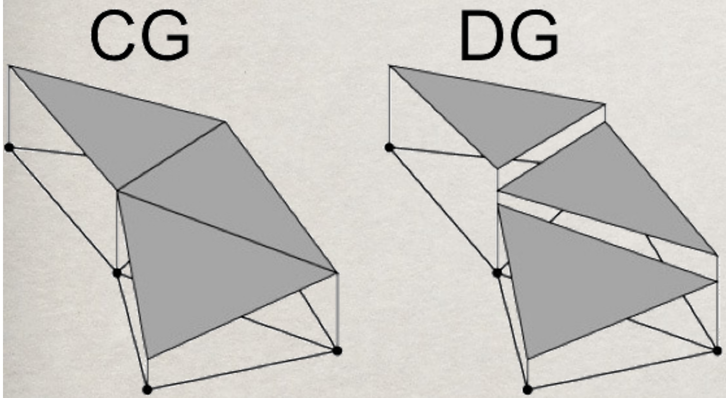
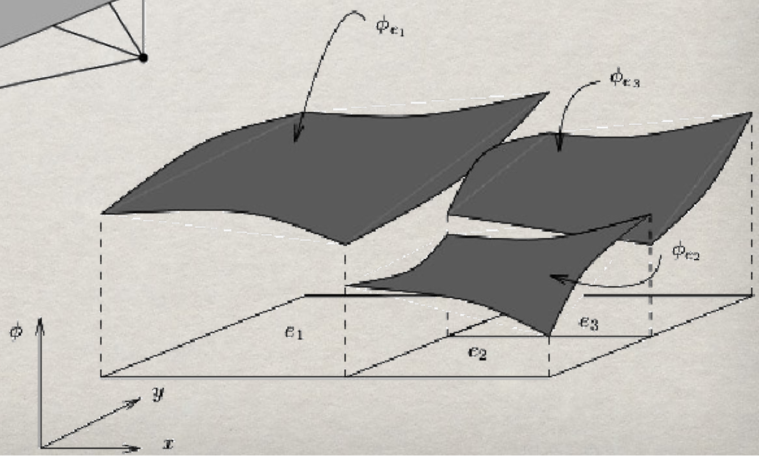
Discontinuous Galerkin





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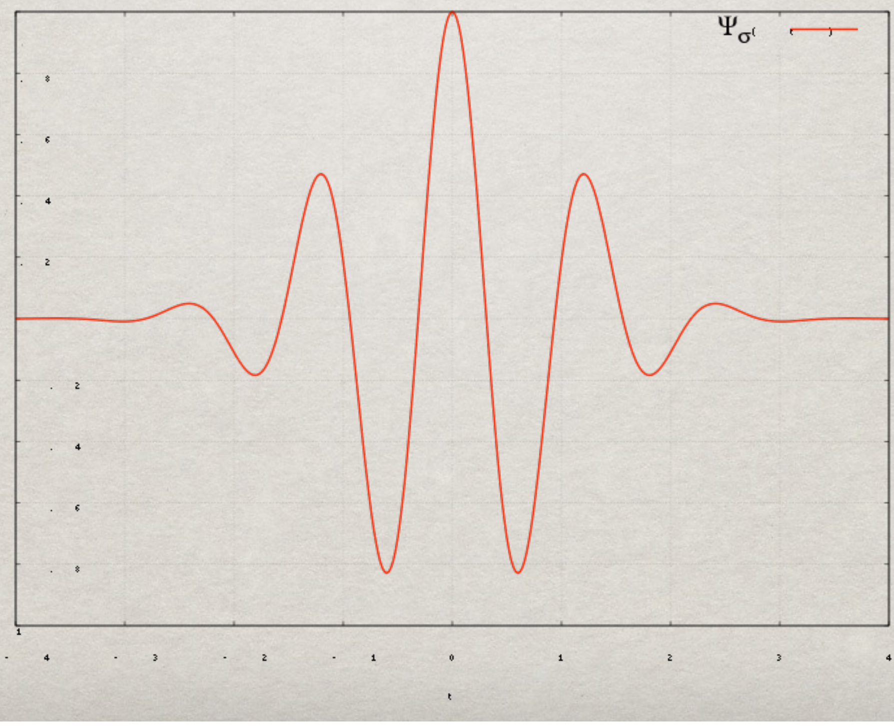
Wavelets methods



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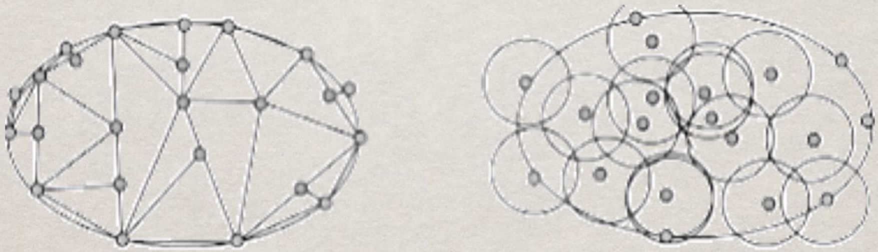
Meshless methods



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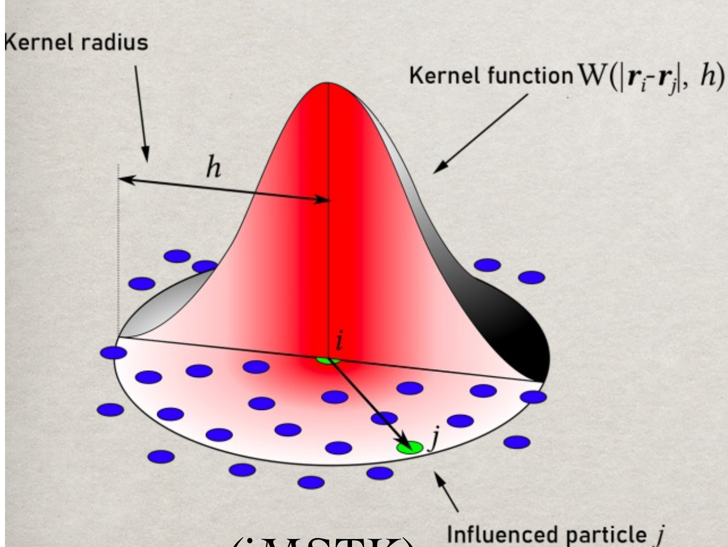
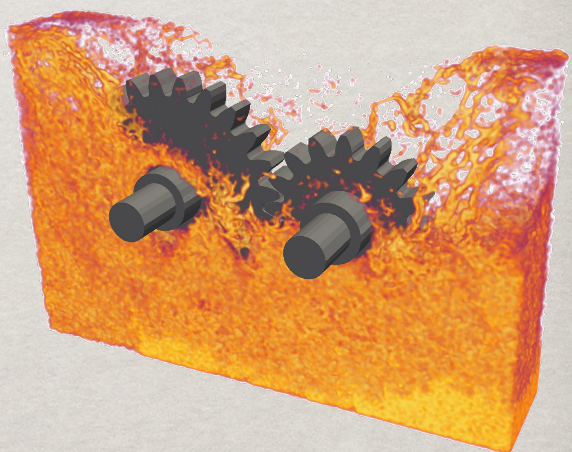
SPH methods



(iMSTK)



(Altair)

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The vortex method

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u} = -\boldsymbol{\nabla}\pi\,,$$

$$\boldsymbol{\nabla}\cdot\mathbf{u} = 0\,.$$

CFD

The vortex method

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u} = -\boldsymbol{\nabla}\pi\,,$$

$$\boldsymbol{\nabla}\cdot\mathbf{u} = 0\,.$$

$$\Rightarrow \quad \frac{D\omega}{Dt} = 0$$

The vortex method

velocity to vorticity $\omega = \nabla \times u$

vorticity to velocity $\nabla^2 \Psi = -\omega$

$$u = \int \nabla_x \times G(x, x')\omega(x')\, dx'$$

$G(x, x')$ Green's function

The vortex method

$$u = \int \nabla_x \times G(x, x')\omega(x')\, dx'$$

solve for the Biot-Savart kernel using complex variables

$$u - iv = \frac{1}{2\pi i z} \qquad z = x + iy$$

this is the velocity field induced by a point vortex

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The vortex method

$$\frac{d\bar{z}}{dt}(\Gamma, t) = \frac{1}{2\pi i} \int \frac{d\tilde{\Gamma}}{z(\Gamma, t) - z(\tilde{\Gamma}, t)}$$

Cauchy principal value integral

The vortex line is now parameterized by its strength

$$z = z(\Gamma, t)$$

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Vortex-blob:

$$K(z) = \frac{1}{2\pi i z}$$

$$K_\delta(z) = \frac{1}{2\pi i z} \frac{|z|^2}{|z|^2 + \delta^2}$$

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Vortex-blob:

$$\frac{\mathrm{d}\bar{z}_j}{\mathrm{d}t} = \int K_\delta(z\Gamma, t - z(\tilde{\Gamma}, t))\,\mathrm{d}\tilde{\Gamma}$$

$$\frac{\mathrm{d}\bar{z}_j}{\mathrm{d}t} = \sum_{\substack{k=1\\k\neq j}}^{N} K_\delta(z_j - z_k)\,\Gamma_k$$

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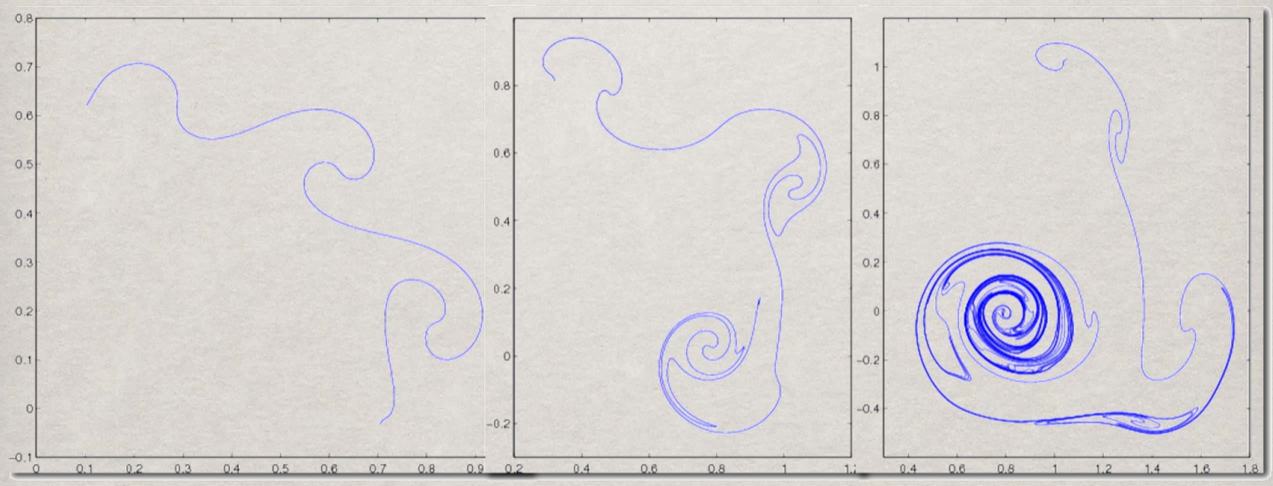
The vortex method

Illustrations



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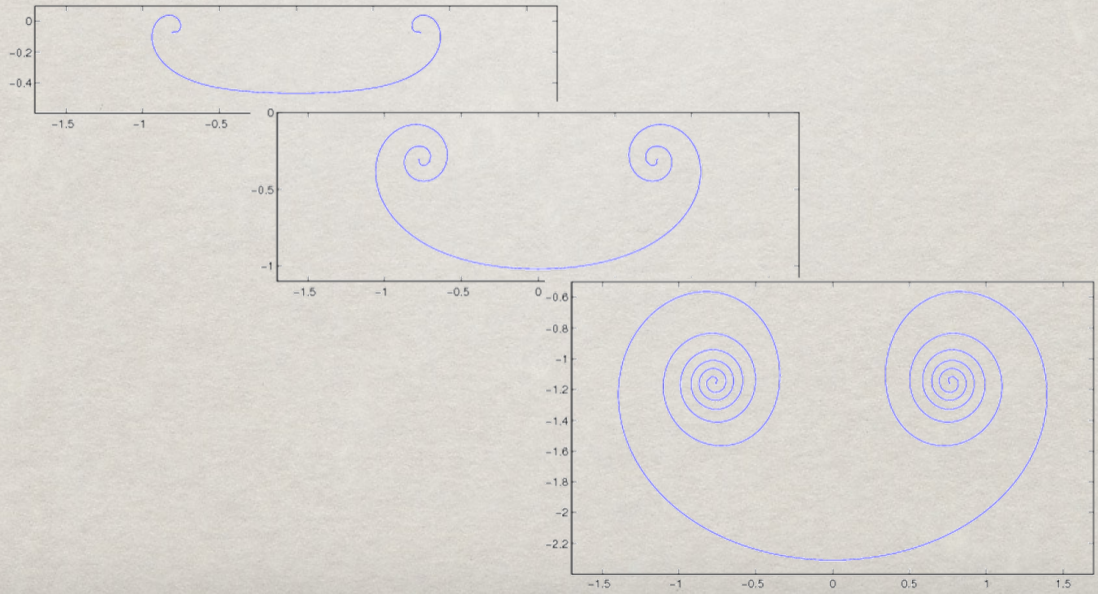
The vortex method

Illustrations



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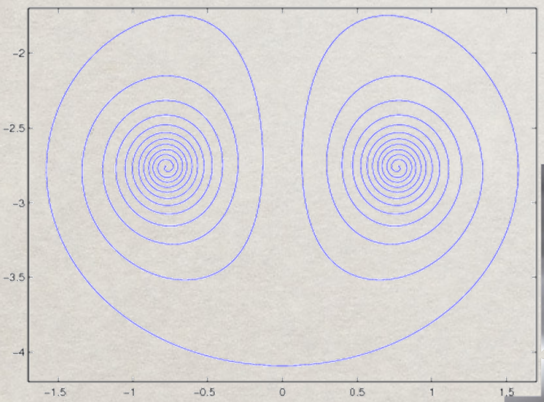
The vortex method

Illustrations



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Illustrations

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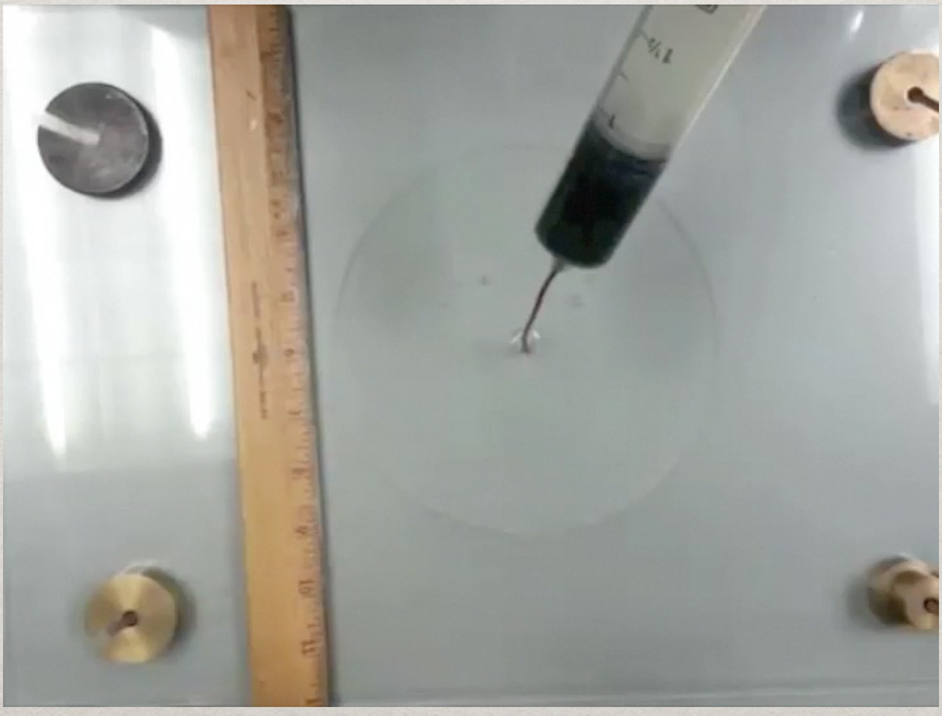
Hele-Shaw cell



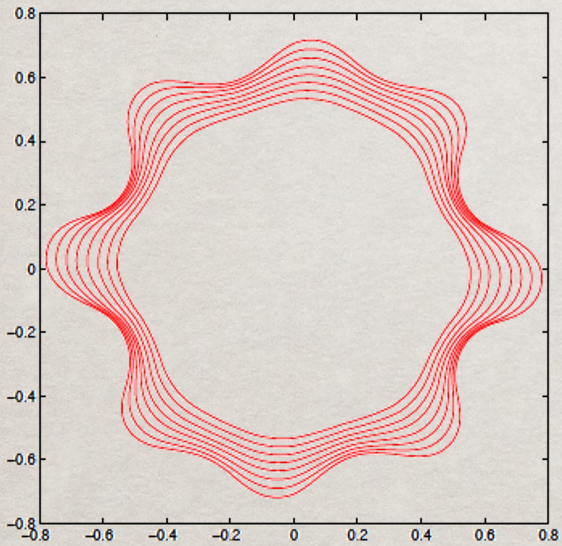
The vortex method

Hele-Shaw cell

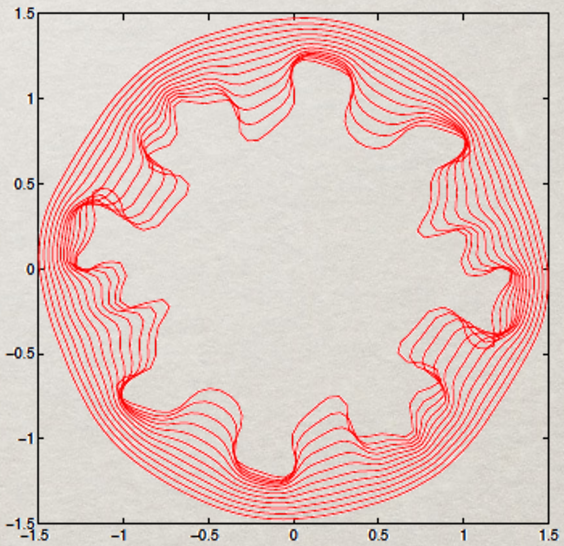
Two immiscible fluids between two narrow parallel plates.

Interface motion (vortex blob, with equation for the sheet strength).

Saffman-Taylor



Injection



Suction

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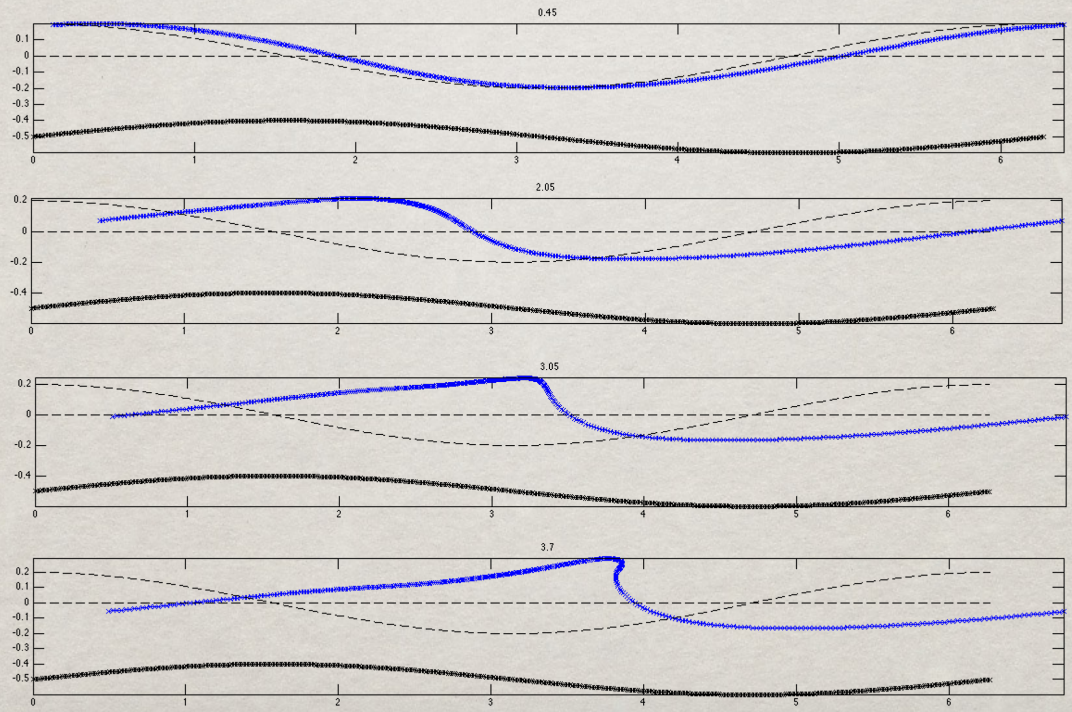
Wave breaking



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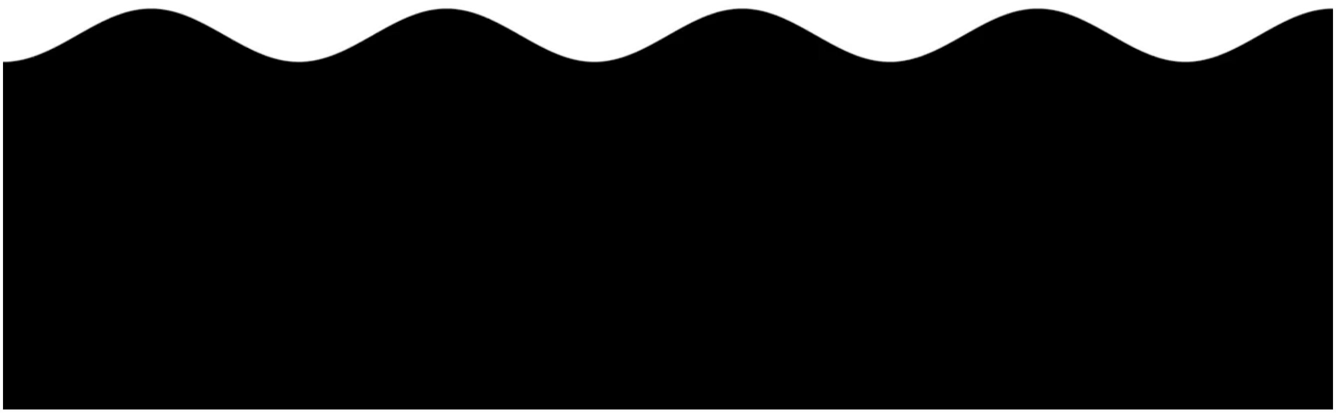
Rayleigh-Taylor

(Movie Mark Stock)



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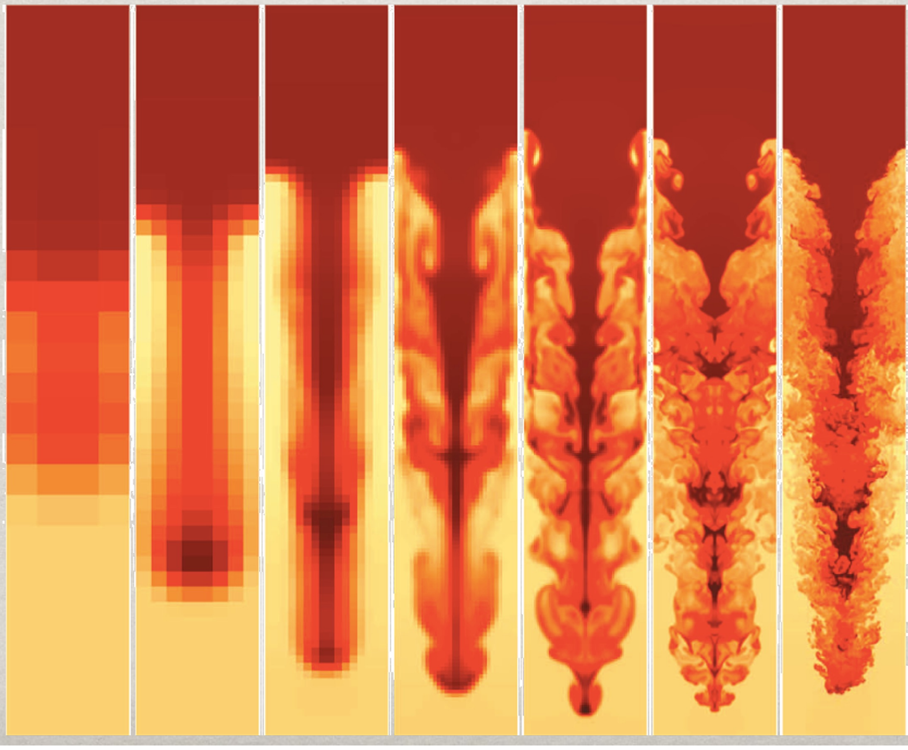
Rayleigh-Taylor (L. Kadanoff, 2004)



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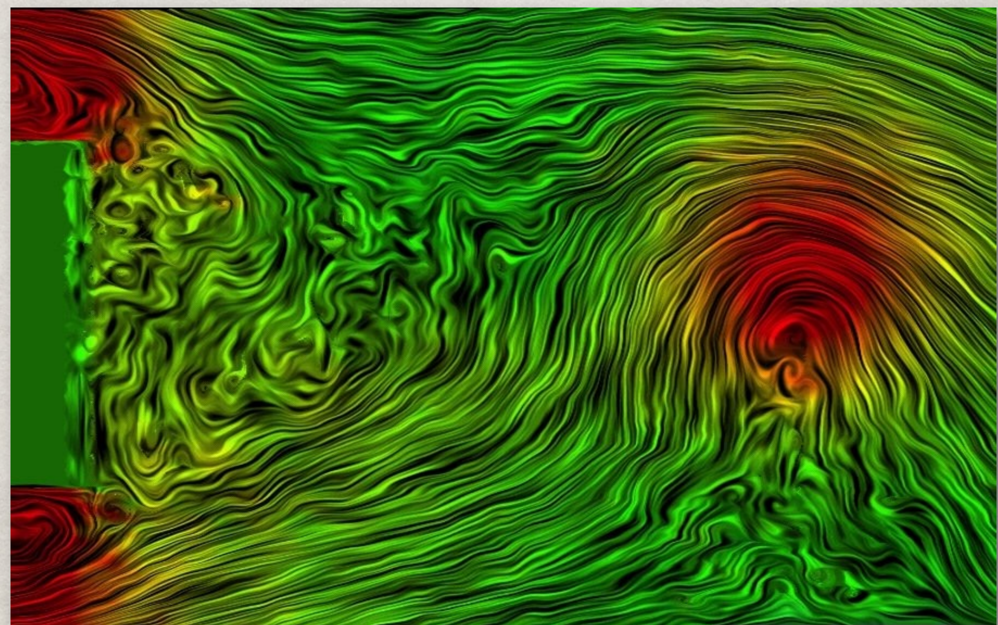
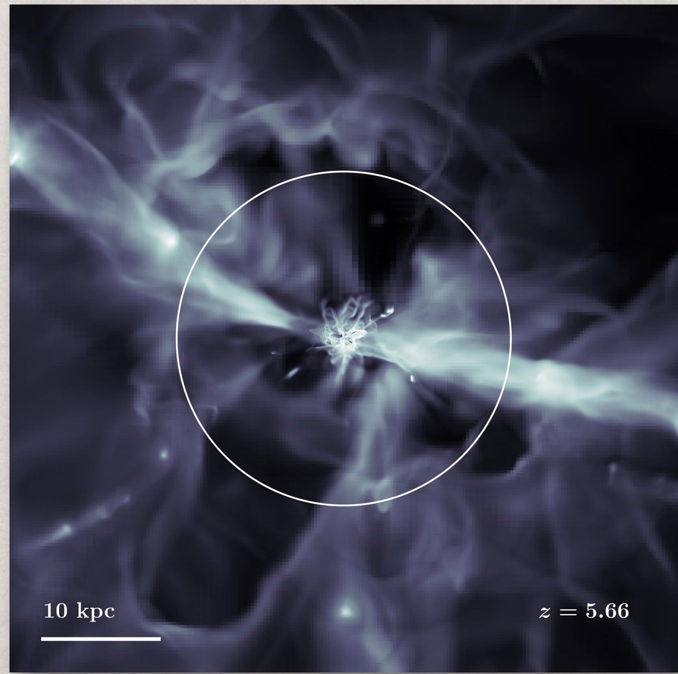
Scientific visualisation



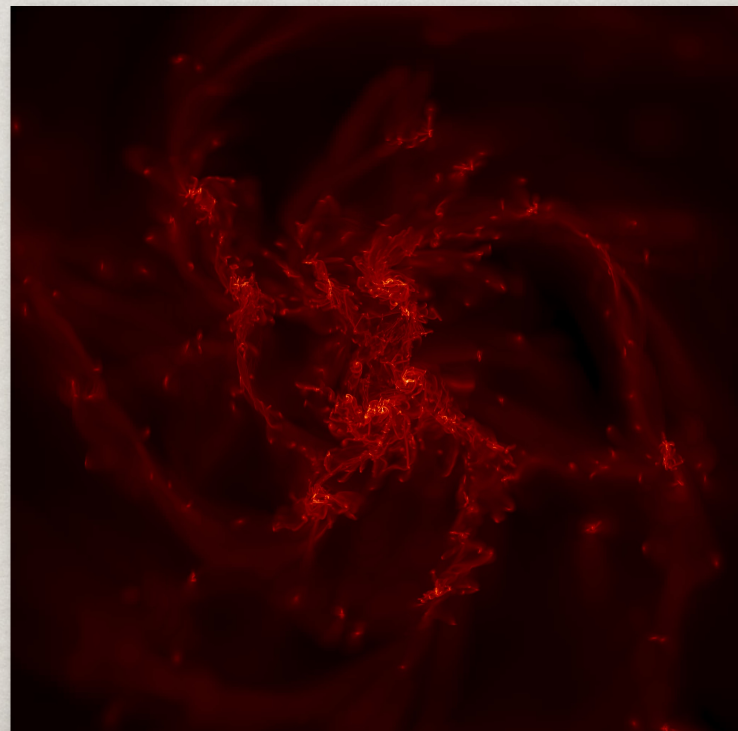
CFD

Scientific visualisation



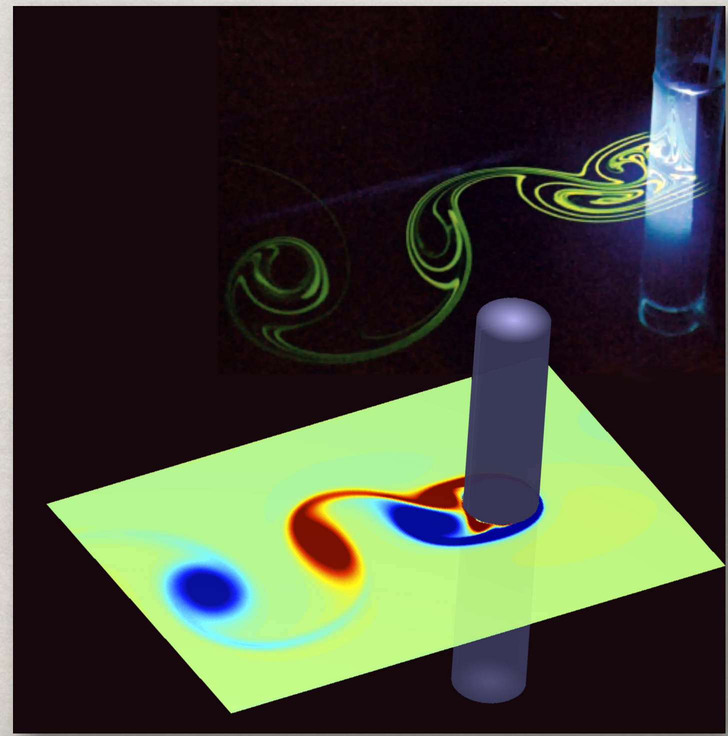
CFD

Scientific visualisation



CFD

Scientific visualisation



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Final Projects:

Defense (20 minutes presentation + questions)

On April 4 (Thursday), reports+codes by April 2

<u>Share the presentation time</u>

Final Projects:

Send me an email TODAY at

dormy@dma.ens.fr

including the students names and the subject

(+ which Master !)

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Final Projects:

Today's tutorial devoted to the projects.
Binomes work together