Numerical Methods for Fluid Dynamics: TD8

Emmanuel DORMY (dormy@dma.ens.fr)

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Forewords

Download TD8.tar, and then type :

```
> tar xvf TD8.tar
> cd TD8
> ipython --pylab
```

1 The Von Karman street

We want to solve for the Navier-Stokes equations around an immersed obstacle. We will use and modify the code written during tutorial 7 (`NSjet.py`).

This code is provided as is (i.e. for the problem of tutorial 7), but with two additional functions : `vorticity` for the plotting, and `Force` to define an arraw corresponding to the immersed solid.

A/ Modify the boundary conditions so that the injection at velocity 1 takes place over the full left boundary.
B/ Modify the code so as to introduce the penalisation.
C/ What is the value of the Reynolds number ? Is it relevant ?

2 Moffatt vortices

During tutorial 3, we constructed Moffatt vortices in a recangular cavity. Here we will focus on triangular cavities. We thus want to solve for the Stokes equation using a finite element method. The upper side of the triangle has size 1, it is an isocele triangle, and its hight is 2 . The applied velocity is parallel to the top boundary and is scaled to 1 ($u = 1$; $v = 0$), whereas it should vanish on the two ther boundaries. The non-dimensional Stokes equations takes the form

$$\Delta \boldsymbol{u} = \boldsymbol{\nabla} p \qquad \text{and} \qquad \boldsymbol{\nabla} \cdot \boldsymbol{u} = 0 \, .$$

It can be written in the form of a bi-harmonic equation for the stream function

$$\Delta\Delta\psi = 0\,.$$

First instal FreeFem from `http://www3.freefem.org/`
A/ in a terminal, type

```
> cd Moffatt
> FreeFem++ Moffatt1.edp
```

What happens ? Open the code `Moffatt1.edp` in a text editor. What equation is being solved ?
B/ Modify the boundary conditions so as to resolve the relevant problem. Ensure the vortices are visible.
C/ Now open the `Moffatt2.edp` file in a text editor. This code intends to solve for the Stokes equation in velocity-pressure formulation. Check that this is the case and that the boundary conditions are correct.
D/ in a terminal, type

```
> FreeFem++ Moffatt2.edp
```

What happens ? Modify the code so as to get Moffatt vortices...