Numerical Methods for Fluid Dynamics TD1

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Forewords

Download the TD1.tar archive from the course website. Move it to a directory dedicated to the course. Then type

```
\$ tar xvf TD1.tar
\$ cd TD1
\$ ipython --pylab
```

1 2D Poisson problem

We are interested in the following elliptic problem

$$\Delta u = f\,, \qquad \mathbf{x} \in \Omega \equiv [0, L_x] \times [0, L_y]\,,$$

$$u = 0\,, \qquad \mathbf{x} \in \partial\Omega\,.$$

1. Edit the *Poisson.py* file.
   What do the DXX, DYY and LAP matrices correspond to ?
2. What is the right-hand-side f being considered here ?
3. Let us now consider the case

   $$f = \left(\left(\frac{\pi}{L_x}\right)^2 + \left(\frac{\pi}{L_y}\right)^2\right) \sin\left(\frac{\pi x}{L_x}\right) \sin\left(\frac{\pi y}{L_y}\right)\,.$$

   Compute the exact analytical solution to that problem.
4. Complete the end of the code using the analytic expression you computed.
5. Now run the full code and conclude on the convergence of the scheme.

Homework 1 : 1D Poisson problem

Write your own Python code to solve the following 1D elliptic problem

$$\partial_{xx} u = f\,, \qquad \mathbf{x} \in [0,\, L_x]\,,$$

$$u = 0\,, \qquad \mathbf{x} \in \partial\Omega\,.$$

considering

$$f = \left(\frac{\pi}{L_x}\right)^2 \sin\left(\frac{\pi x}{L_x}\right)\,.$$

Perform a convergence study as in the previous exercise using the classical centered 3 points finite difference scheme (2nd order). Then perform the same study using a 3 point compact finite difference scheme (4th order).
Possible extension : write a compact scheme for exercise 1.