Numerical Methods for Fluid Dynamics: TD5

Emmanuel DORMY (dormy@dma.ens.fr)

15 February 2024

Forewords

Download TD5.tar, and then type :

```
> tar xvf TD5.tar
> cd TD4
> ipython --pylab
```

1 Chebychev polynomia discrete scheme

We would like to compare the accuracy of a compact finite difference scheme and a Chebychev discretisation. To do this we consider the function $f(x) = \sin(\pi x)$ on $-1 \leq x \leq 1$. For a given number of points we want to compare the discrepancy between the numerically computed derivative and the exact one. A *Python* code is provided in the `Cheb` directory.

A/ Execute it and understand it.

B/ Modify this code to compare the accuracy of both schemes on the second derivative.

2 The vorticity equation

We consider a two-dimensional periodic flow with a few initial vortices. We want to resolve the 2D Navier-Stokes equation formulated in terms of vorticity.
For a velocity noted

$$\mathbf{u} = u\,\mathbf{e}_x + v\,\mathbf{e}_y,$$

the vorticity and the streamfunction are defined as

$$\omega\,\mathbf{e}_z = \boldsymbol{\nabla} \wedge \mathbf{u} \quad \text{et} \quad u = \partial_y \psi\,,\; v = -\partial_x \psi\,. \tag{1}$$

The Navier-Stokes then become

$$\frac{\mathrm{D}}{\mathrm{D}t}\omega = \frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla} \omega = \frac{1}{Re} \Delta \omega \,, \tag{2a}$$

$$\Delta \psi = -\omega \,. \tag{2b}$$

A/ Edit `Periodic_NS.py` in the directory `Vorticity`. What is *w_hat*?

B/ Let us start by considering the linearised Navier-Stokes equations

$$\frac{\partial \omega}{\partial t} = \frac{1}{Re} \Delta \omega \,, \tag{3a}$$

$$\Delta \psi = -\omega \,. \tag{3b}$$

Modify the code `Periodic_NS.py` to resolve the evolution of $\hat{\omega}$, Fourier transform of the vorticity. Starting with equation 3a, we will use a Crank-Nicholson scheme. What do you observe ?

We will now implement the non-linear term, to solve equation 2a. To achieve this we need to know the velocity field, which we must deduce from the stream function.

C/ Compute the Fourier transform $\hat{\psi}$ of the stream function from $\hat{\omega}$ at the timestep n.

D/ Use the function `spec_phys` (provided) to compute the velocity field and the vorticity gradient at the timestep n.

E/ Now compute the advection term and use the function `phys_spec` (provided) to compute its transform.

F/ Introduce this advection term in the time stepping, what do you observe ?