Flow transition over surface gaps in 2D incompressible laminar boundary layers

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2 July 2025

Motivation

Figure: Wing of a Boeing 737-800

Setup

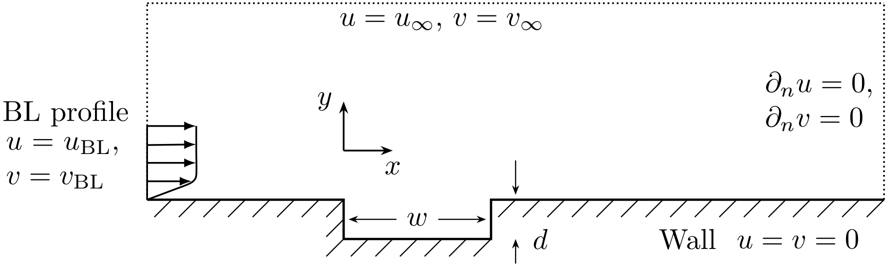

Figure: Domain setup for the steady-state finder

- **Aim:** Study the stability of the system as a function of the depth d and width w of the gap.
- 2D incompressible NS
- $\mathrm{Re}_{\delta*} = 1000 \implies \mathrm{Re}_x = 3.38 \times 10^5$

Stability results

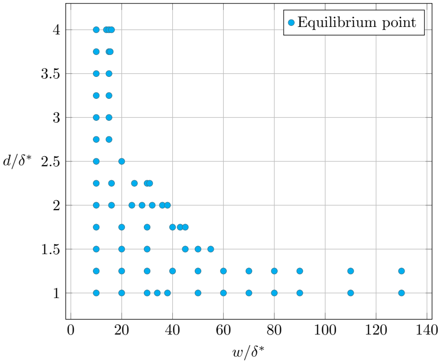

Figure: Classification of the stability of points downstream of the gap.

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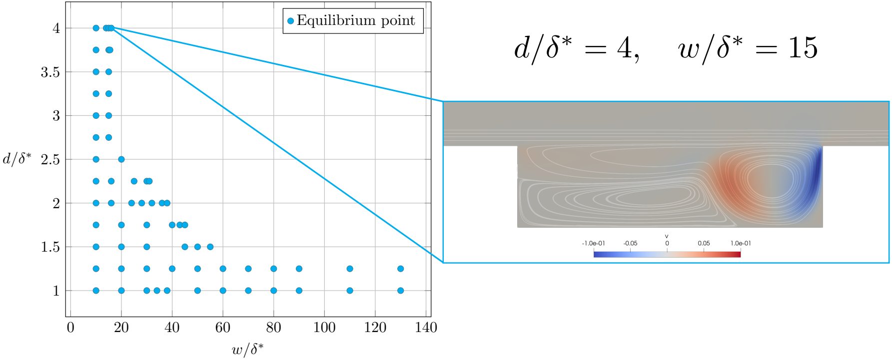

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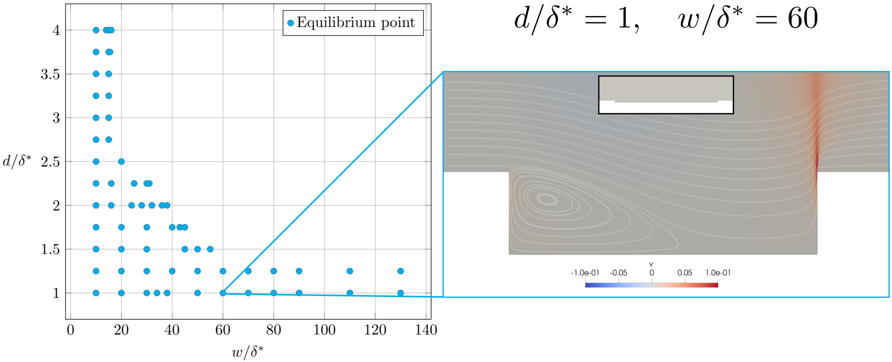

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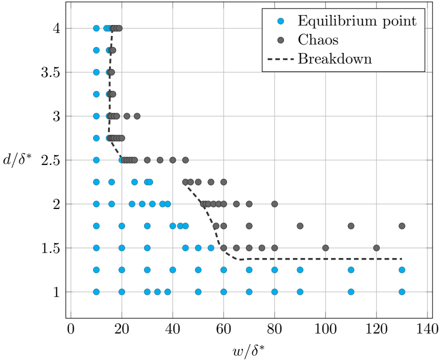

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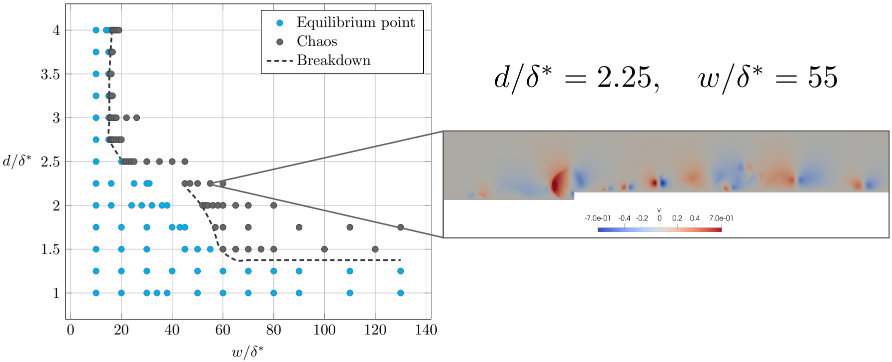

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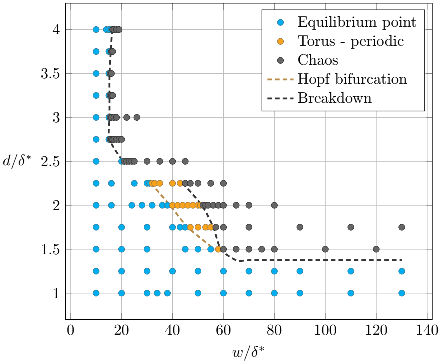

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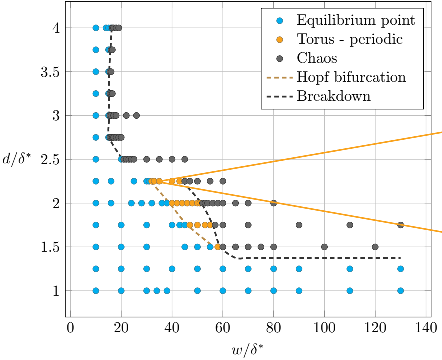
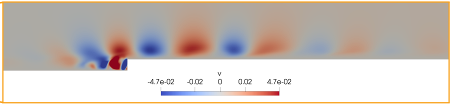


$$d/\delta^* = 2.25, \quad w/\delta^* = 35$$



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Framework for the LST (TS-wave transition)

- We linearize the flow around a steady baseflow:

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- But this is a local representation! To account for streamwise growth in the BL we use the e^N-method. Fixing $\omega \in \mathbb{R}$:

$$n(x, \omega) = -\int_{x_0}^{x} \alpha_i(s, \omega)\,\mathrm{d}s = \log\left(\frac{|\tilde{\mathbf{u}}(x, \omega)|}{|\tilde{\mathbf{u}}_0|}\right)$$
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$\implies$ Disturbances of amplitude A_0 satisfy $A(x) \leq A_0\mathrm{e}^{N(x)}$.

Previous Work

Flow (2022), **2** E8
doi:10.1017/flo.2022.1

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Characterizing surface-gap effects on boundary-layer transition dominated by Tollmien–Schlichting instability

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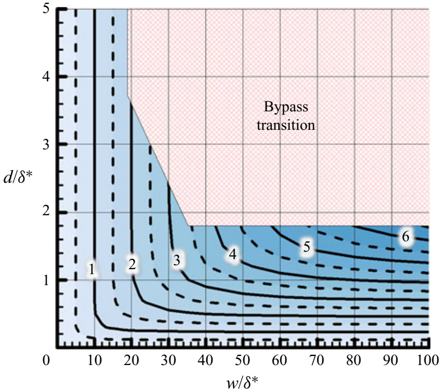

Figure: $\Delta N = N - N_{\text{ref}}$ for different gap dimensions

Crouch JD, Kosorygin VS, Sutanto MI, Miller GD. Characterizing surface-gap effects on boundary-layer transition dominated by Tollmien–Schlichting instability. Flow. 2022;2:E8.

Perturbed system setup

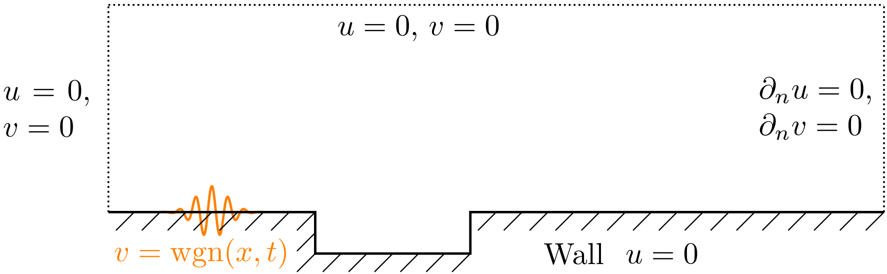

Figure: Domain setup for the perturbed system

e^N-method results

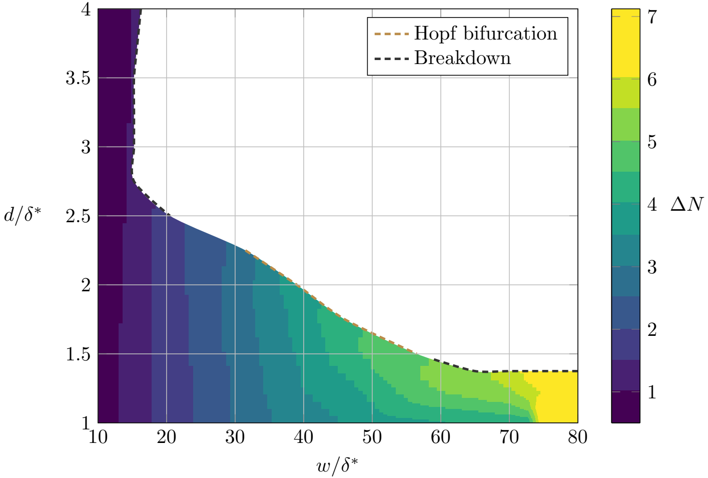

Figure: Interpolated $\Delta N = N - N_{\text{ref}}$ in the globally-stable region.

e^N-method results

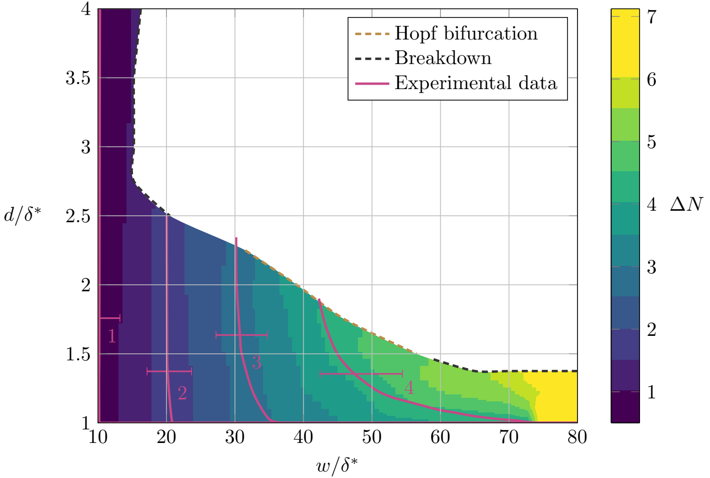

Figure: Interpolated $\Delta N = N - N_{\text{ref}}$ in the equilibra region. Magenta lines indicate the contour levels of the experimental data.

Future Work

- Go to higher Ma (compressible regime).

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- Account for spanwise effects (quasi-3d simulations).