Flow transition over surface gaps in 2D incompressible laminar boundary layers

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Motivation

Figure: Wing of a Boeing 737-800

Framework

- 2D Incompressible Navier-Stokes

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- We linearize the flow around a steady baseflow:

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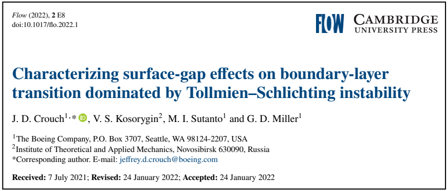
$\implies$ Disturbances of amplitude A_0 satisfy $A(x) \leq A_0\mathrm{e}^{N(x)}$.

Previous Work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Characterizing surface-gap effects on boundary-layer transition dominated by Tollmien–Schlichting instability

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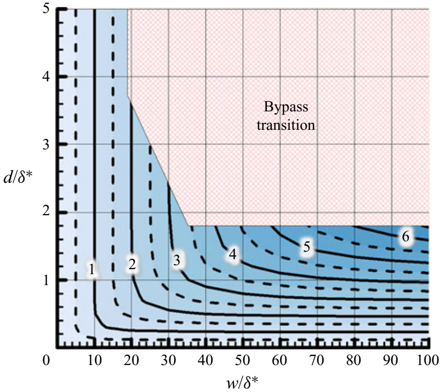


Figure: $\Delta N = N - N_{\text{ref}}$ for different gap dimensions

Crouch JD, Kosorygin VS, Sutanto MI, Miller GD. Characterizing surface-gap effects on boundary-layer transition dominated by Tollmien–Schlichting instability. Flow. 2022;2:E8.

Setup

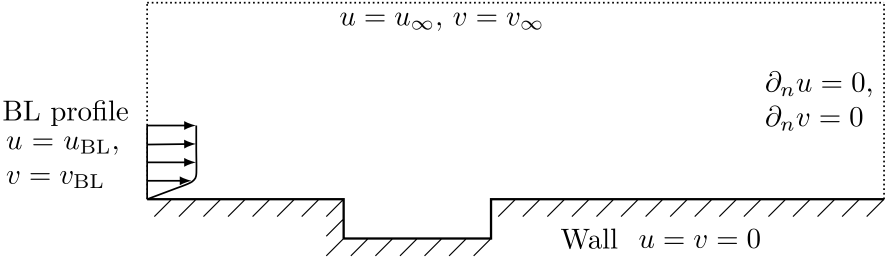


Figure: Domain setup for the steady-state finder

- $\mathrm{Re}_{\delta^*} = 1000 \implies \mathrm{Re}_x = 3.38 \times 10^5$
- δ^* measured at the upstream edge of the gap in the smooth flat plate.

Computational cost

- Mesh: $\sim 10\,000$ elements
- Polynomial order: $(7,6)$
- Time step: from 5×10^{-4} (on transient conditions)
  to 5×10^{-3} (on steady conditions)
- HPC: 1 node with 256 CPUs

Stability results

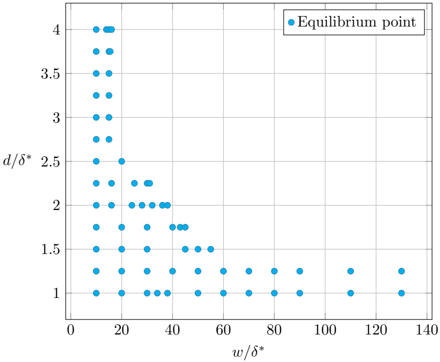
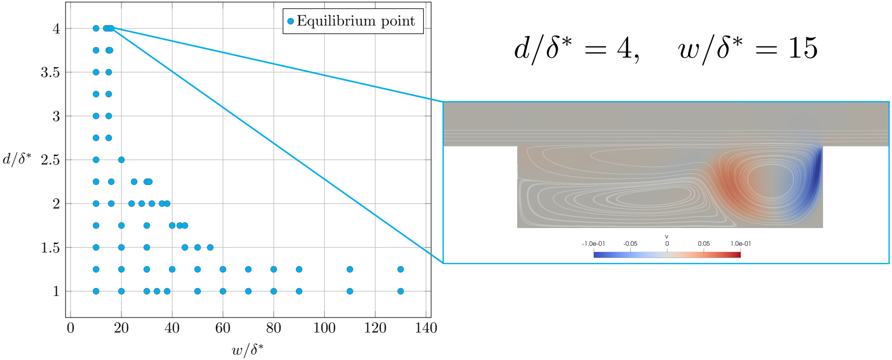


Figure: Classification of the topological behavior of points downstream of the gap.

Stability results



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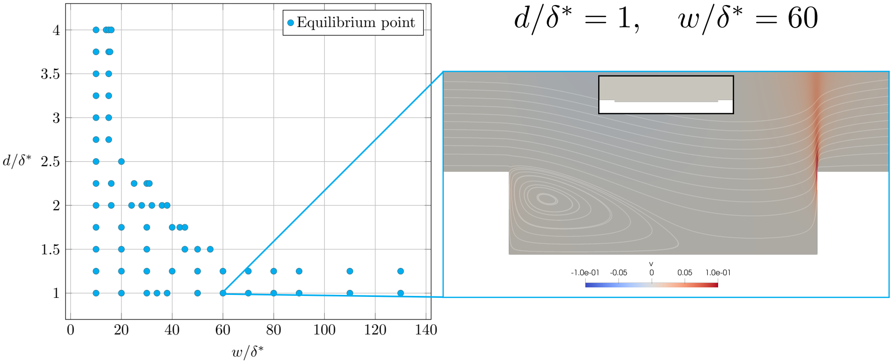

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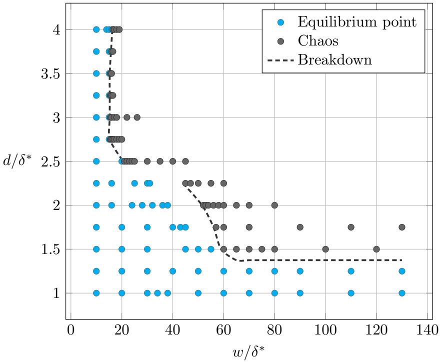

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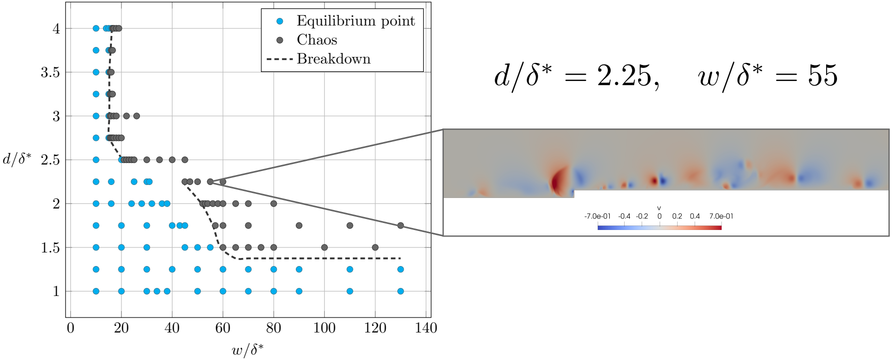

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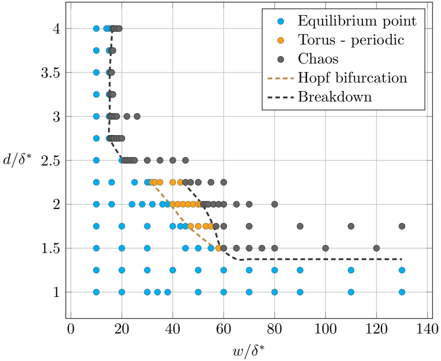

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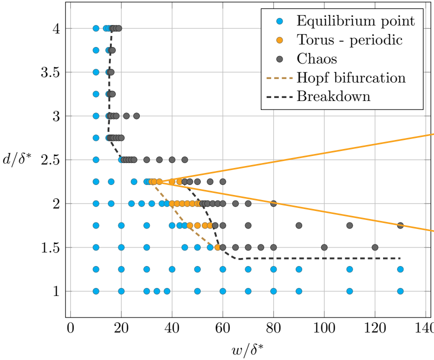
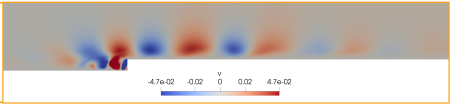


$$d/\delta^* = 2.25, \quad w/\delta^* = 35$$



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Perturbed system setup

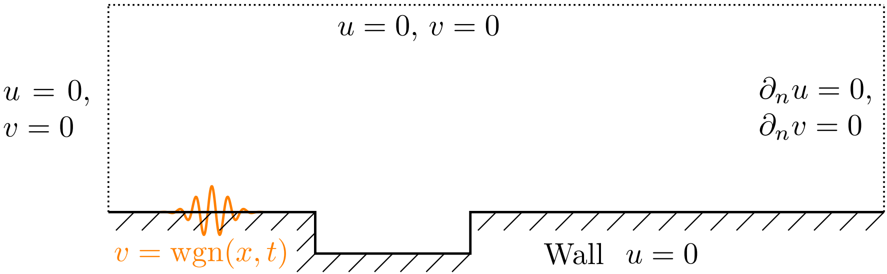

Figure: Domain setup for the perturbed system

e^N-method results

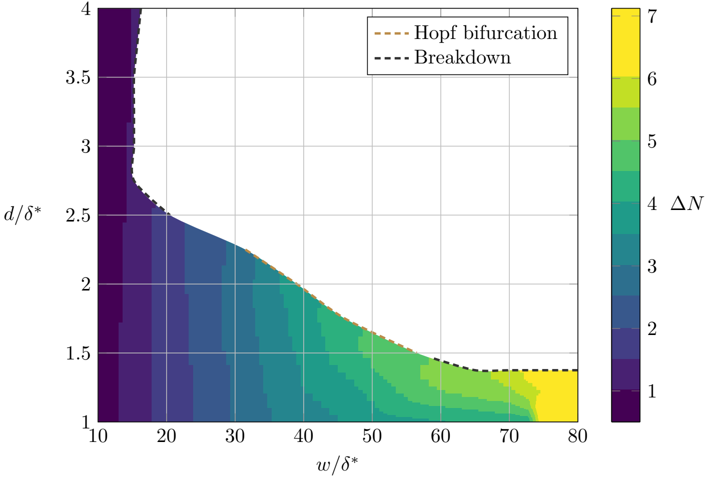

Figure: Interpolated $\Delta N = N - N_{\text{ref}}$ in the equilibra region.

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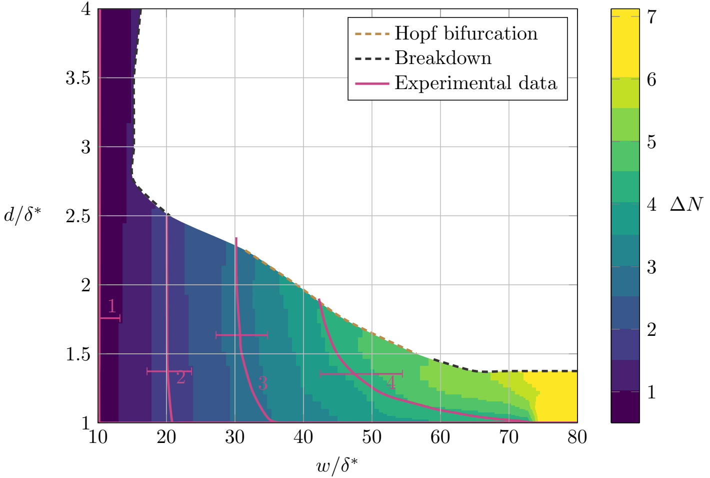

Figure: Interpolated $\Delta N = N - N_{\text{ref}}$ in the equilibra region. Magenta lines indicate the contour levels of the experimental data.

Future Work

- Go to higher Ma number (compressible regime)

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- Account for spanwise effects (quasi-3d simulations)