IMPERIAL

Summary 1st term 2025

Víctor Ballester
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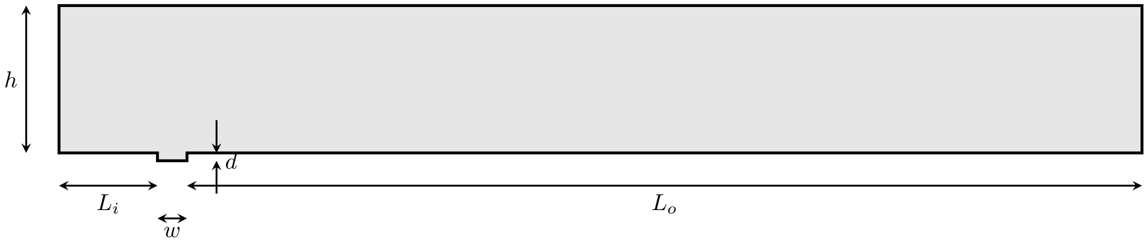
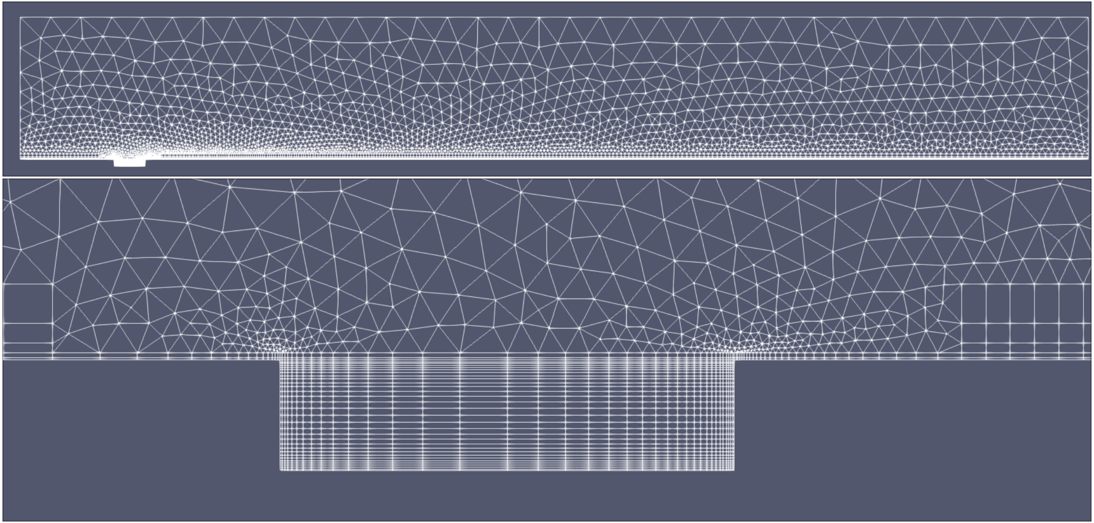
Domain

- δ^*: measured at the upstream edge of the gap in a surface free of discontinuities.
- $\mathrm{Re}_{\delta^*} = 1000$
- $\mathrm{L_i} = 50\delta^*$
- $\mathrm{L_o} = 500\delta^*$
- $\mathrm{h} = 75\delta^*$
- $\mathrm{d} = 4\delta^*$
- $\mathrm{w} \in \mathrm{A}\delta^*$, $\mathrm{A} = \{10, \ldots, 30\}$
- Inflow BC: Blasius profile
- Top BC: Far-field BC
- Wall BC: No-slip
- Outflow BC: Convective BC (Robin), or normal Neumann BC.



Current mesh

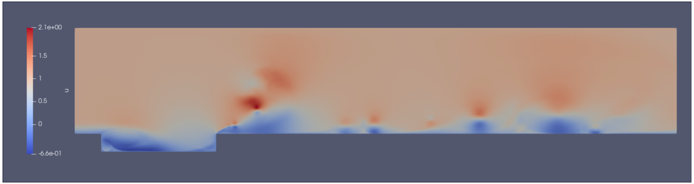


Study - range of w

- For w $\leq 16.4\delta^*$, we observe a steady state from nonlinear simulations (“global” attractor).
- As we increase w $\geq 16.5\delta^*$, we observe a divergent behavior. Thus, to do LSA with those cases we have to make use of Selective Frequency Damping (SFD) (e.g. control theory) to get a locally stable baseflow.
- **Conjecture**: the critical width at d $= 4\delta^*$ lies in the interval $(16.4\delta^*, 16.5\delta^*)$ (up to numerical sensitivity).

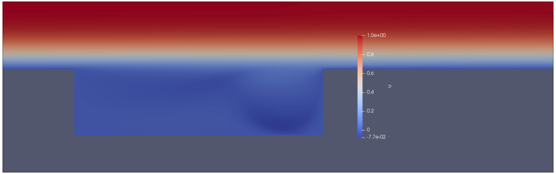
Unstable baseflows

u-component of dns simulations with w $= 16.5\delta^*$ at time t $\sim 11\,500$.

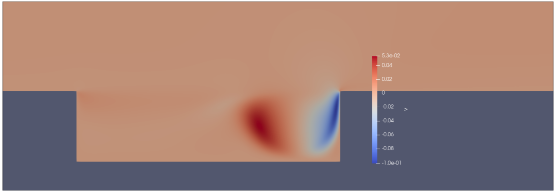


u-component of dns simulations with w $= 26\delta^*$ at time t ~ 1500 (old run with a short domain).

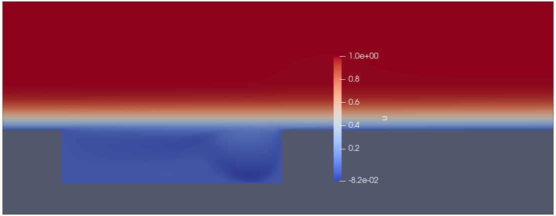
Stable baseflows



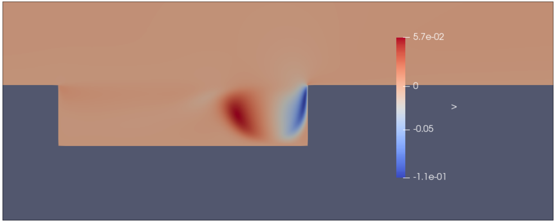
u-component natural baseflow with w $= 15\delta^*$



v-component natural baseflow with w $= 15\delta^*$

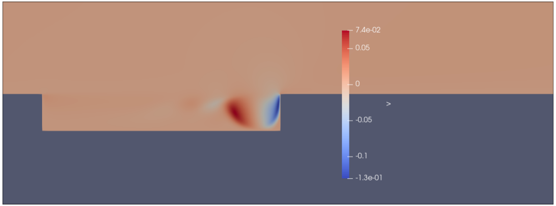


u-component SFD baseflow with w $= 16.5\delta^*$

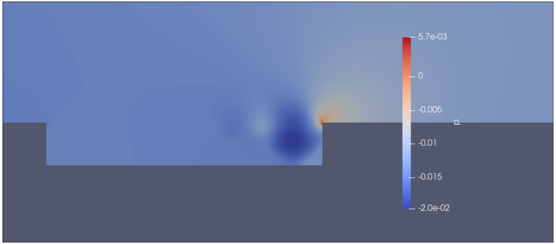


v-component SFD baseflow with w $= 16.5\delta^*$

u-component SFD baseflow with w $= 26\delta^*$



v-component SFD baseflow with w $= 26\delta^*$



pressure SFD baseflow with w $= 26\delta^*$

Linear stability analysis

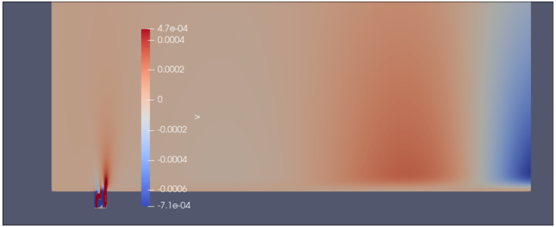
Case w $= 15\delta^*$. Most unstable mode:

- Growth rate: -0.00279
- Frequency: ± 0.00304

Comments:

- This is not a TS mode. We observe a huge mode in the BL.
- The magintude of the fields is much higher inside the gap than on the BL (around 2-10 orders, depending on the x-position in the BL)

u-component of the most unstable eigenmode with w $= 15\delta^*$ (domain scaled in the x-dir by 0.1)



v-component of the most unstable eigenmode with w $= 15\delta^*$ (domain scaled in the x-dir by 0.1)

Linear stability analysis

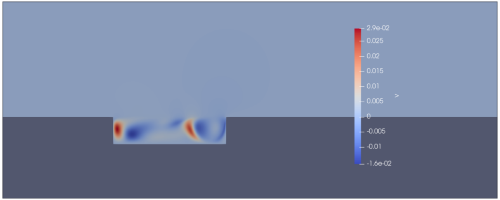
Case w $= 16.5\delta^*$ (very similar to w $= 15\delta^*$).
Most unstable mode:

- Growth rate: -0.00258
- Frequency: ± 0.00276

Comments:

- We do **not** observe a mode with positive growth rate, which suggests (up to numerical sensitivity and other possible errors) that the nature of the instability would be nonlinear instead of linear.

u-component of the most unstable eigenmode with w $= 16.5\delta^*$ (emphasizing colors in the gap)



v-component of the most unstable eigenmode with w $= 16.5\delta^*$ (emphasizing colors in the gap)

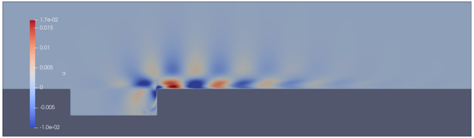
Linear stability analysis

Case w $= 26\delta^*$. Most unstable mode:

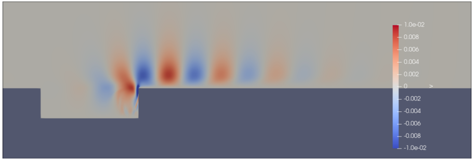
- Growth rate: 0.00859
- Frequency: ± 0.00887

Comments:

- This mode looks like the result of an absolute instability in the gap because the amplitude of the waves decreases as they move downstream.



u-component of the most unstable eigenmode with w $= 26\delta^*$ (domain scaled in the x-dir by 0.5)



v-component of the most unstable eigenmode with w $= 26\delta^*$ (domain scaled in the x-dir by 0.5)

Nature of the instability (work in progress)

- Runs doing global stability analysis from a baseflow just above the critical width where non-steadiness shows up on dns (e.g $\mathrm{w_{critical}}|_{\mathrm{d}=4\delta^*} \in (16.4\delta^*, 16.5\delta^*)$ show eigenmodes with a **negative** growth rate.
- We did check that adding those modes to the baseflow as initial conditions in the nonlinear solver leads to a steady solution (we ran that for $\mathrm{w} = 16.5\delta^*$ and $\mathrm{w} = 18\delta^*$). This does **not** happen though for $\mathrm{w} = 26\delta^*$.

Coherence between DNS and LSA

Let

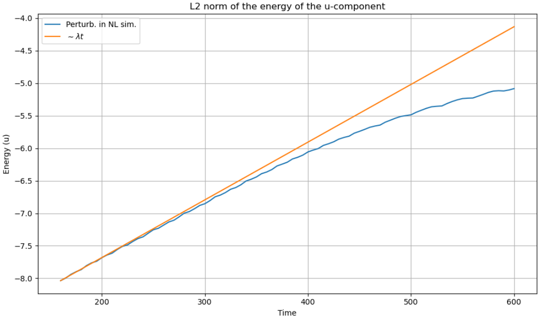
$$\varphi(t;(u_0,v_0)) = (\varphi_u(t;(u_0,v_0)),\varphi_v(t;(u_0,v_0)))$$

the flow of NS eqs at time t with initial conditions (u_0, v_0). Let (U, V) be the baseflow of our system and $(\tilde{u}, \tilde{v})$ the most unstable eigenmode with λ the respective growth rate. We plot the following quantities

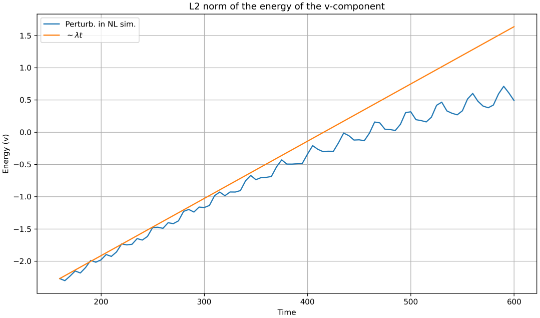
$$t \mapsto \frac{\|\varphi_u(t;(U+\tilde{u},V+\tilde{v}))-U\|_{L^2}}{\|U\|_{L^2}}$$
$$t \mapsto \frac{\|\varphi_v(t;(U+\tilde{u},V+\tilde{v}))-V\|_{L^2}}{\|V\|_{L^2}}$$

for different w. This should tell us how the energy of the perturbation evolves in time. We compare this with the theoretical evolution of the energy based on LSA, which goes as $\|u_0\|_{L^2}e^{\lambda t}$ and $\|v_0\|_{L^2}e^{\lambda t}$, respectively.

Case w $= 26\delta^*$



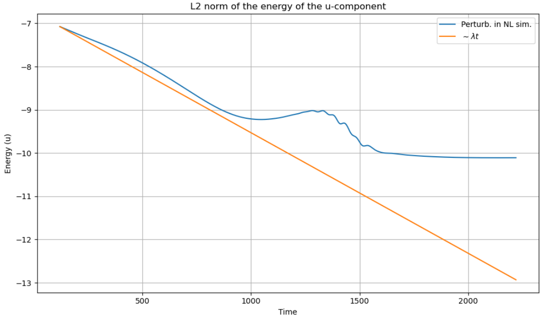
u-component



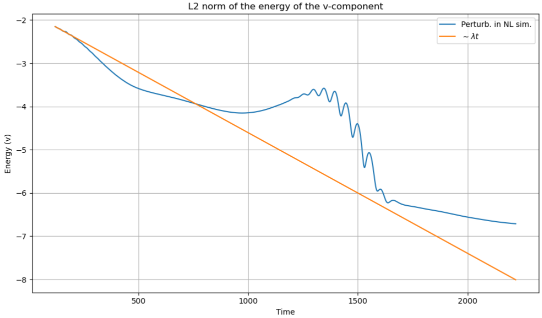
v-component

- We are plotting the log of the energy.
- Initially the nonlinear results fits well the linear prediction after which (t ~ 450) the nonlinear effects start to contribute significantly.

Case w $= 15\delta^*$ (similar to w $= 16.5\delta^*$)



u-component

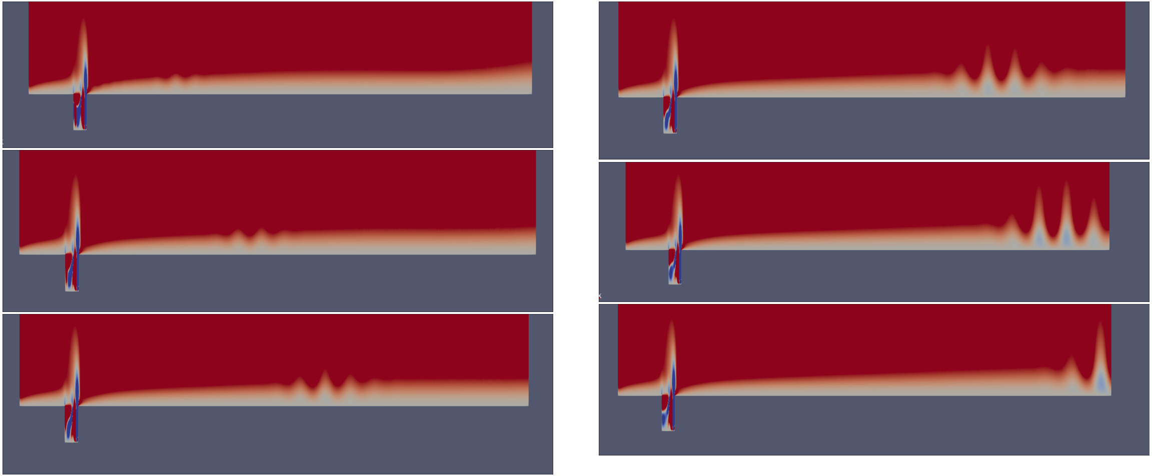


v-component

- We are plotting the log of the energy.
- The fit is less nicer, and we observe a weird bump appearing on the middle of the time interval considered.

The bump

u-component the perturbed system at different times (domain scaled in the x-dir)



The bump

- Looks like the modes may be providing the system with enough energy inside the gap to trigger the convective instability nature of the system. **Why?** The stable modes computed for w $= 15\delta^*$ and w $= 16.5\delta^*$ showed bigger magnitudes inside the gap than on the BL.

Questions

- What range of d should we consider in the future?
- Should we first move to 3D case or to 2D compressible case?