IMPERIAL

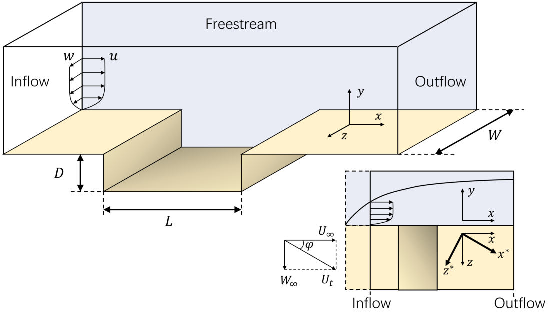
Linear instabilities with the Incompressible solver

Víctor Ballester
January 9, 2025

Domain

Data

- $\mathsf{L}/\mathsf{D} = 4$
- $\mathsf{W}/\mathsf{D} = 2$
- $\varphi_{\mathsf{sweep}} = 30°$
- Reynolds studied (based on the depth of the gap): $\mathsf{Re}_\mathsf{D} = 1500, 7500$



From Ganlin's thesis

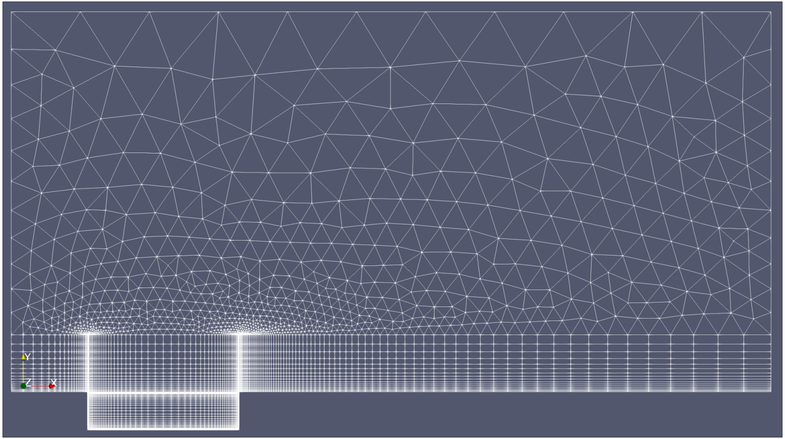
Mesh (quasi-3D simulations)

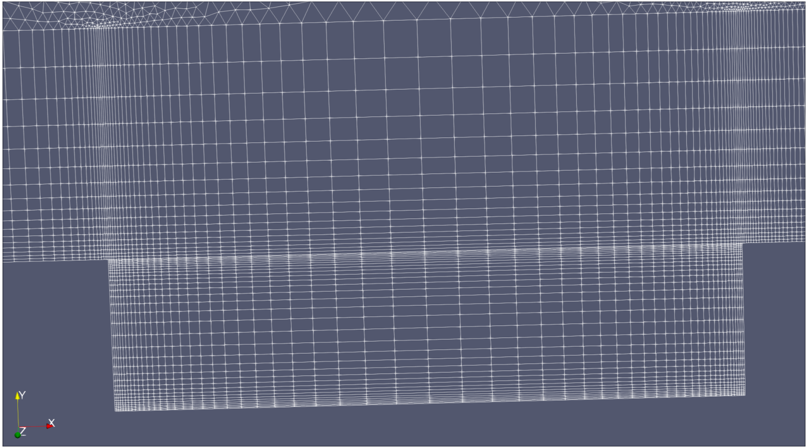
- Element-based mesh in the plane $\mathsf{x} - \mathsf{y}$.
- Fourier expansion in z direction.
- 5th-order polynomial expansion for the velocity field and 4th-order for the pressure, in the hp/Spectral formulation.

Domain

Boundary conditions

- Inlet & outlet flows.
- Free stream at the top and no-slip at the bottom.
- Periodic in z direction.




Y
X
Z

Baseflow

- Quasi-3D simulations with 16 Fourier modes in the z direction at $\mathrm{Re_D} = 1500$ lead to a 2D stable baseflow (see pictures below).
- Quasi-3D simulations with only the constant mode in the z direction at $\mathrm{Re_D} = 7500$ lead to an unstable baseflow, which after using a feedback control in order to stabilize the problem, we get a baseflow similar to one the below.

u component

v component

w component

Preliminary LSA results ($\mathsf{Re_D} = 7500$)

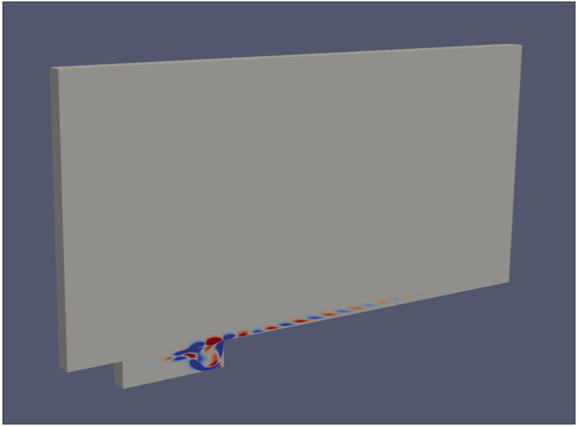
Single mode simulations by modifying the length of the domain in the z direction. In each case we assume that the perturbation is of the form:

$$\tilde{u}(x, y, z, t) = q(x, y) \exp(i\beta_n z) \exp(\lambda t) + c.c.$$

where $\beta_n = 2\pi n / L_z$ is the mode. We ran the linear Navier-Stokes for $n = 1, \ldots, 8$ in order to see which eigenpairs (λ, q) are unstable.

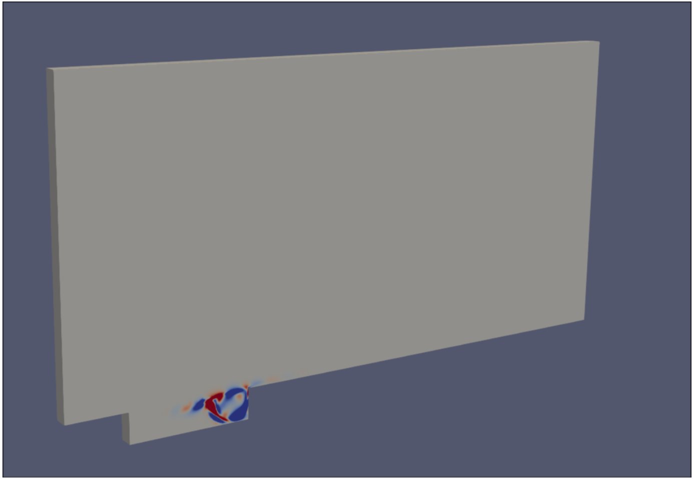
Modes 1, 2, 5, 6, 7 and 8 are stable.

The leading eigenvalues of modes 3 and 4 have positive real part leading also to an instability.

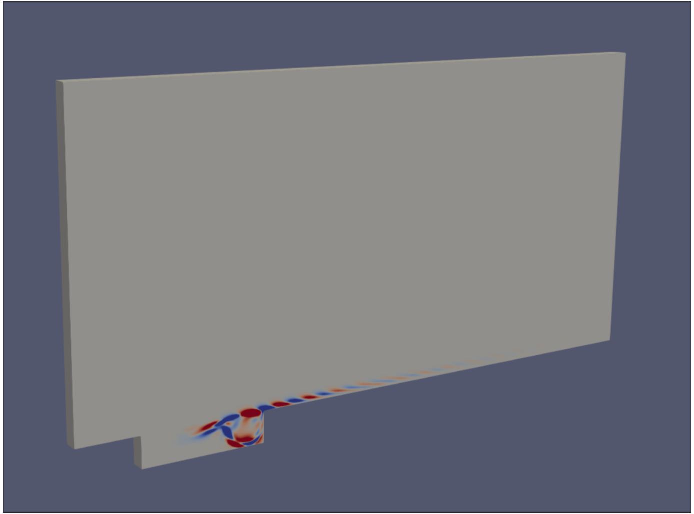


u component of the velocity of an unstable mode with β_3

All the unstable modes that we plot so far look similar. Also w component is similar to u component.



v component of the velocity of the same unstable mode of above, with β_3



u component of the velocity of an unstable mode with β_4

Next steps, questions

- How can we distinguish between convective instabilities within the gap and absolute instabilities? Because the domain is periodic in the z direction, so a priori the latter ones may camouflage the former ones.
- Is the domain appropriate for Boeing's problem? Specific data is needed. Should we change the length ratios (1st slide)? In particular, the sweep angle may influence the appearance of certain unstable modes or not.
- Are we interested in the onset of the instabilities as a function of the Reynolds number?
- Should we start doing DNS simulations with the compressible solver?

IMPERIAL

Domain sensitivity analysis

Víctor Ballester
January 30, 2025

Summary

Our goal here is to determine the appropriate domain for which there is no dependence on the final solution in the length scales of the domain, say the x distance before and after the gap and the height of the domain.

Data

- $\mathrm{Re}_{\delta^*} = 1000$
- $\mathrm{D}/\delta^* = 4$
- $\mathrm{W}/\delta^* = 15$

(for not forgetting it in the future) for Blasius profile $\delta = 2.85\delta^*$ (if the other text books say slightly different values, it is because they are using approximations. I computed the **exact** value.)

Domains considered

Coding system: ioh, where i, o, h $\in \{1, 2, 3\}$ and i is the inflow, o is the outflow and h is the height of the domain, and the numbers mean from small size (1) to large size (3).

133 (small inflow)

233 (medium inflow)

313 (small outflow)

323 (medium outflow)

333 (large everything)

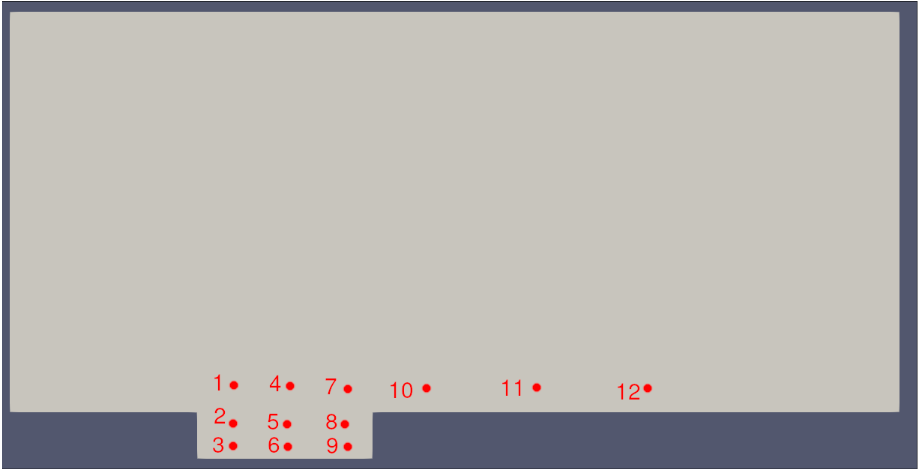
331 (small height)

332 (medium height)

original domain

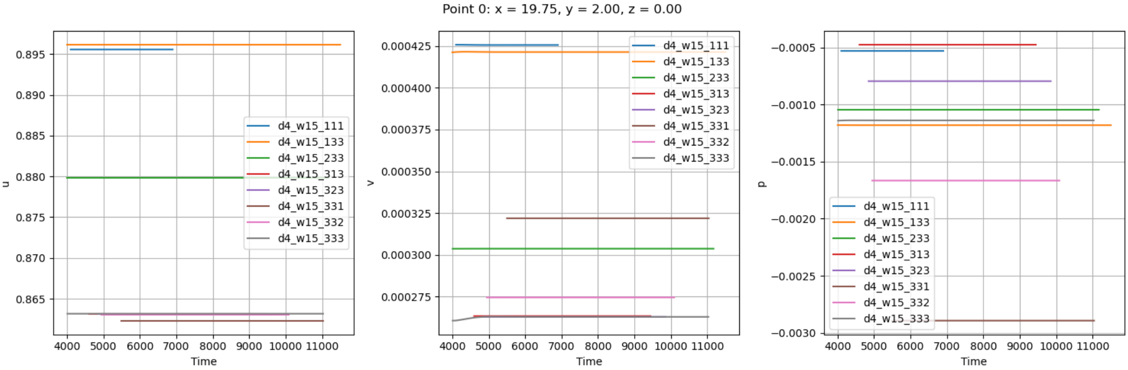
Points of interest

We fix some points in the domain to study their time evolution:



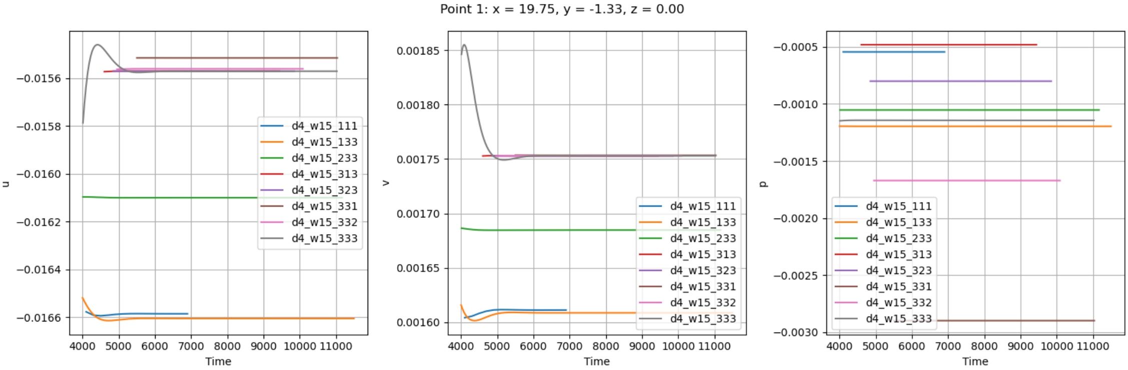
Results

Point 1:



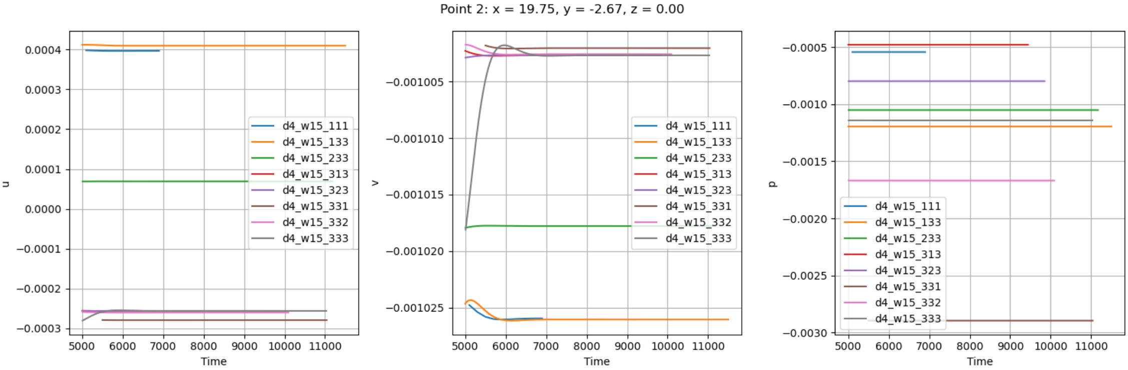
Results

Point 2:



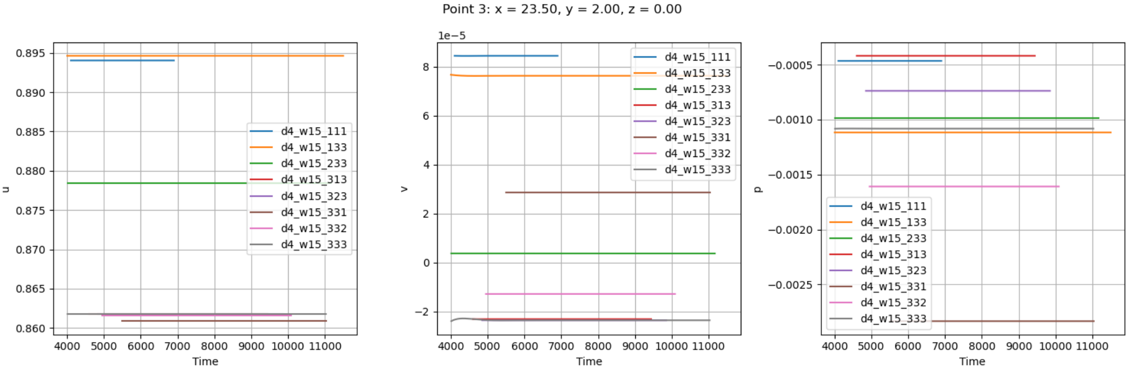
Results

Point 3:



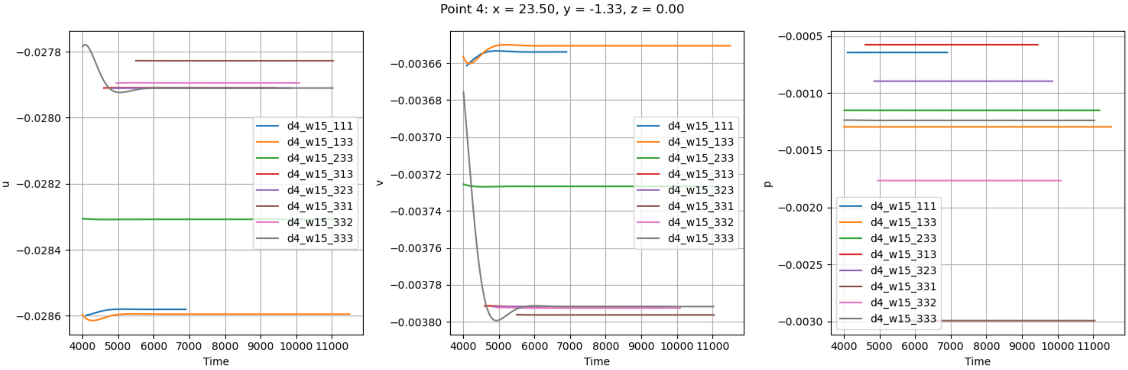
Results

Point 4:



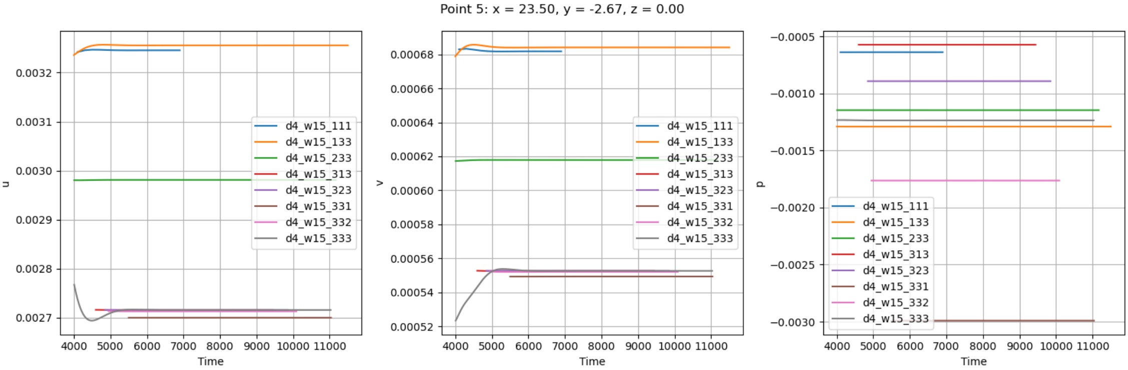
Results

Point 5:



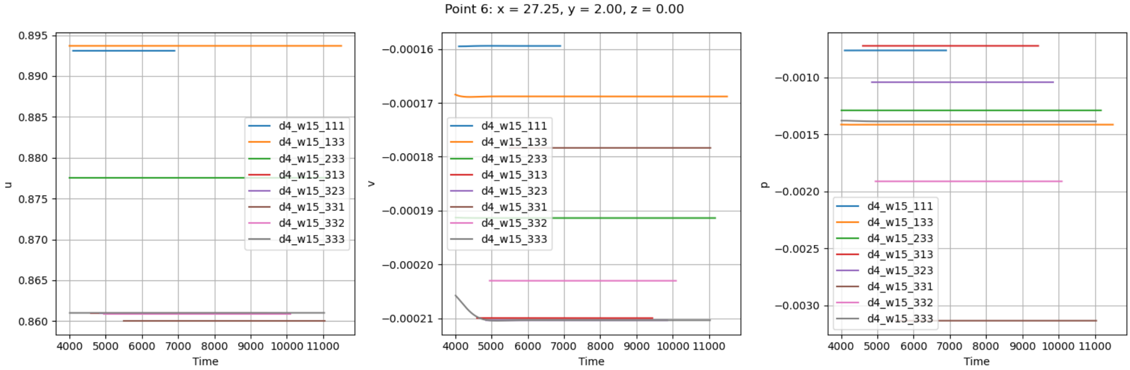
Results

Point 6:



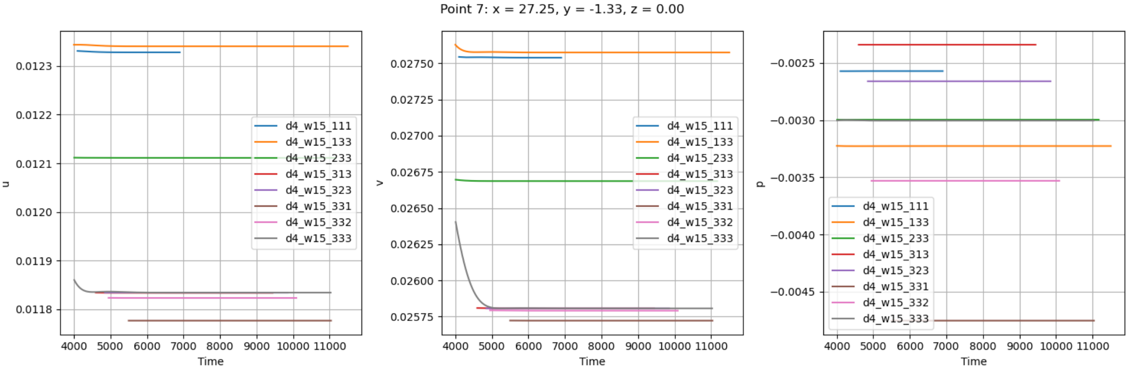
Results

Point 7:



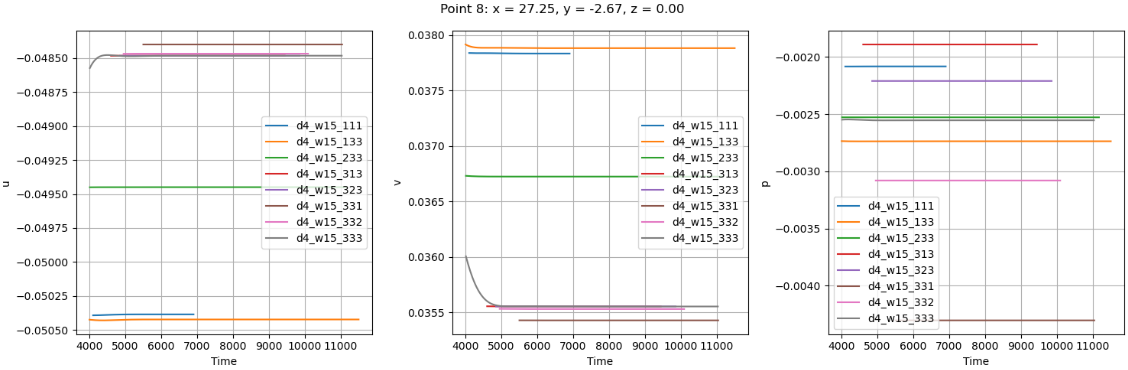
Results

Point 8:



Results

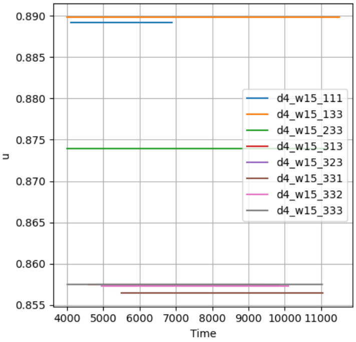
Point 9:

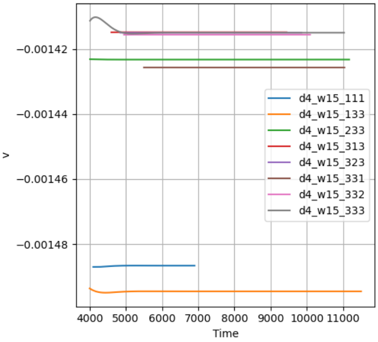


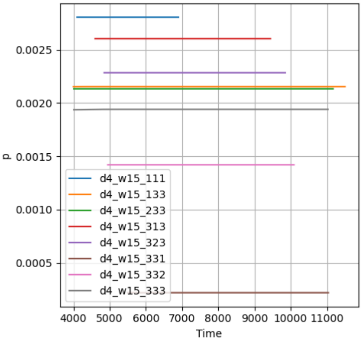
Results

Point 10:

Point 9: x = 31.00, y = 2.00, z = 0.00

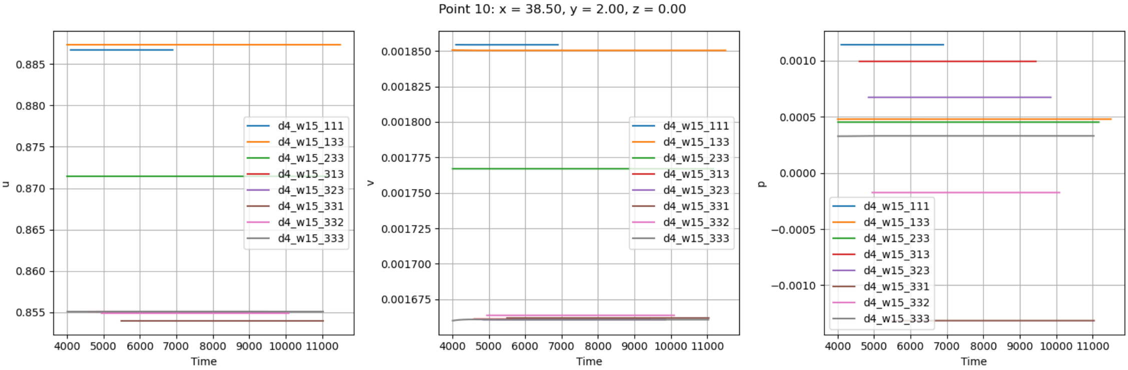






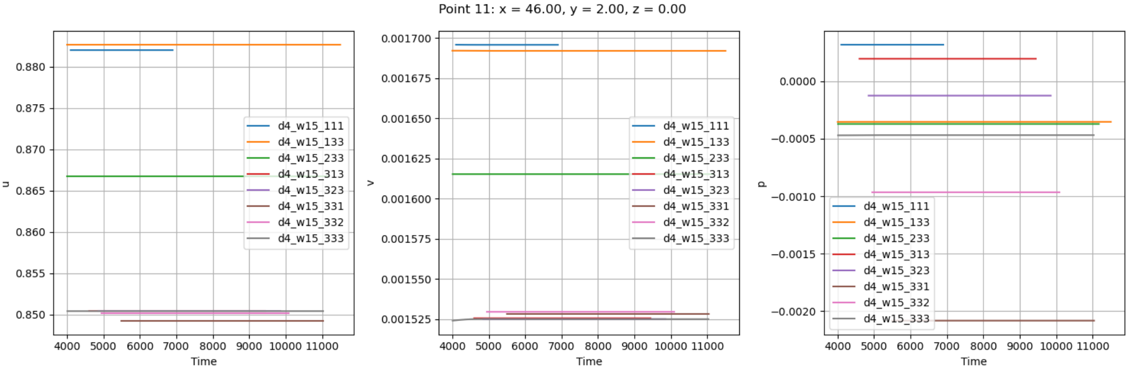
Results

Point 11:



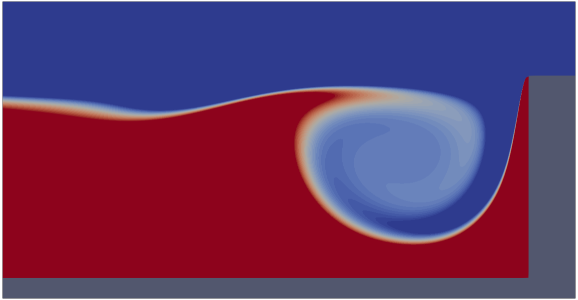
Results

Point 12:



Conclusions

- The distance after the gap is not that important for u and v.
- The pressure is the most sensitive to the domain size.



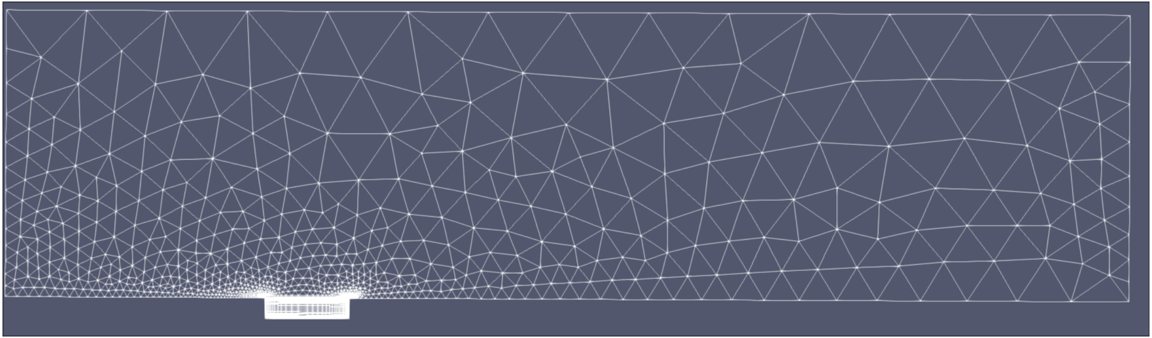
The resolution of the vortex was decent

IMPERIAL

Sparser-mesh computations

Víctor Ballester
February 13, 2025

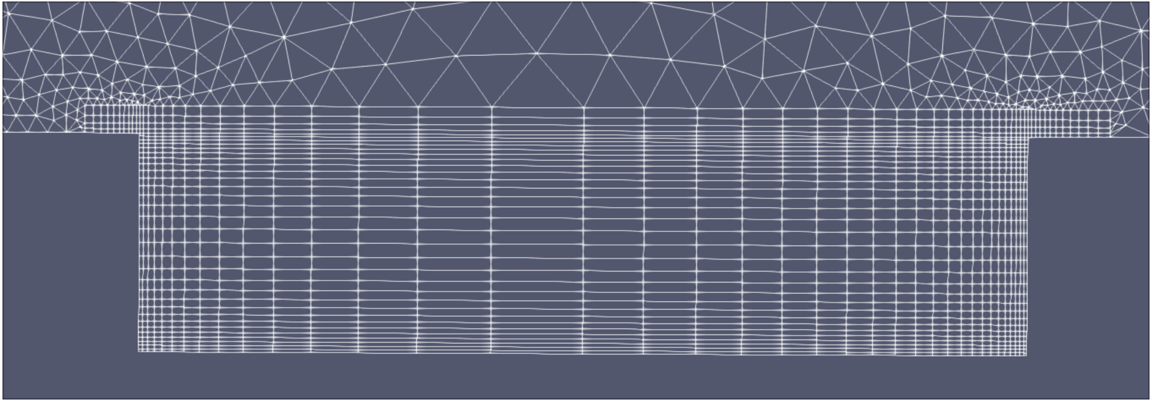
New mesh



Num elements: From ~ 24000 (with the previous mesh) to ~ 3000! One problem is that the downstream region is usually not well resolved.

Polynomial order for ($\mathbf{u}$,p): (7,6) initially and then (9,8) to converge quicker to the Steady State solution.

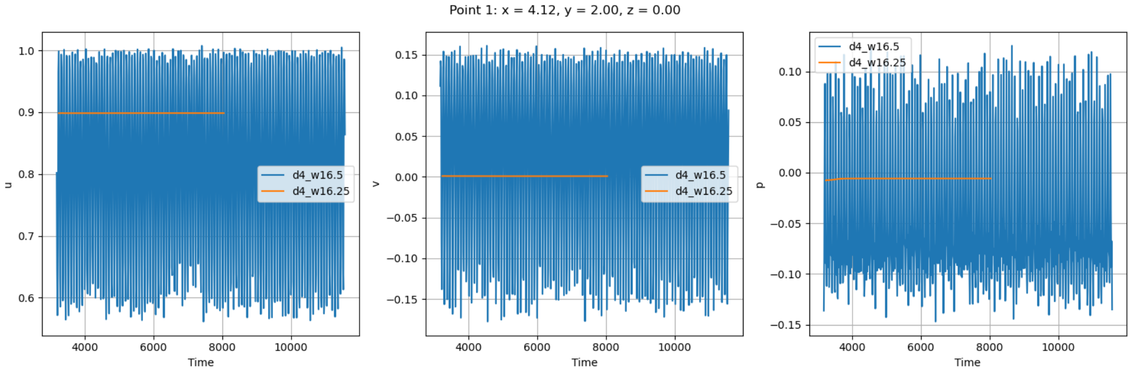
New mesh



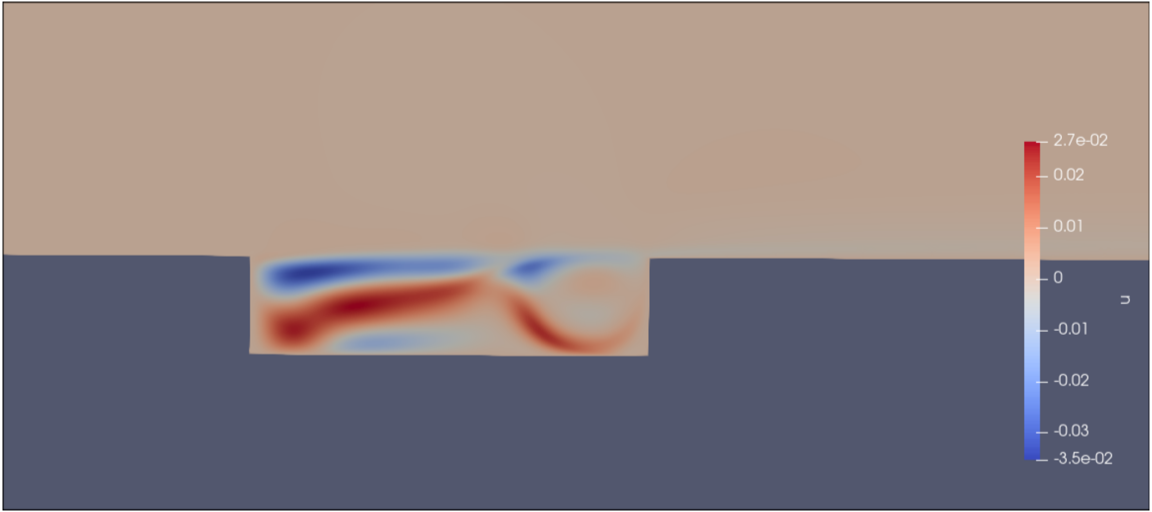
Stability results

Everything at $\mathrm{Re}_{\delta^*} = 1000$.

Using D $= 4\delta^*$, the system with w $= 16.25\delta^*$ is stable, but the system with w $= 16.5\delta^*$ didn't stabilize (I run it till t $= 11574$).

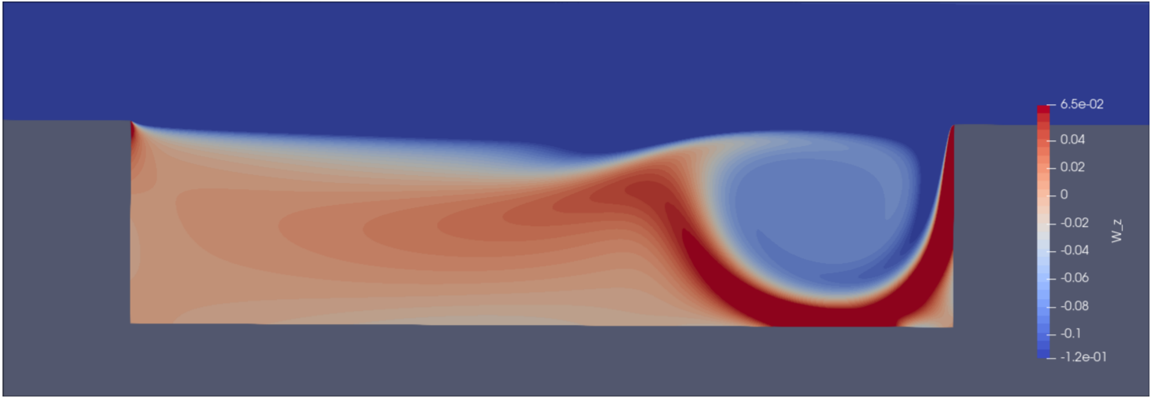


Stable node $w = 16.25\delta^*$



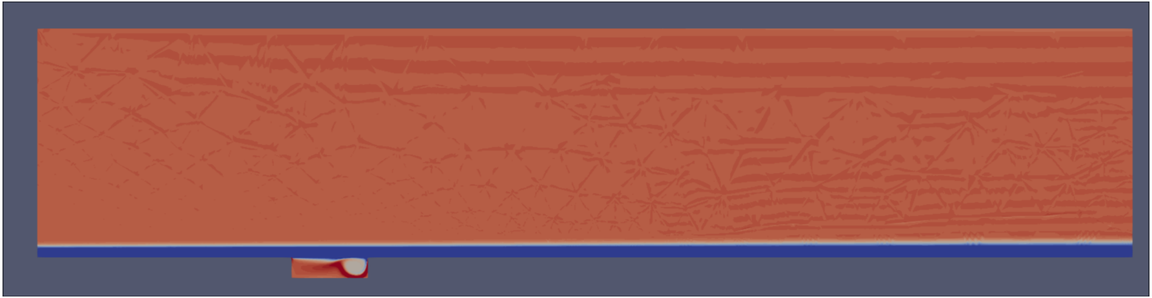
Baseflow w $= 16.25\delta^*$

Vorticity field.



Baseflow w $= 16.25\delta^*$

Vorticity field.



Stable node w $= 16.25\delta^*$

Does it look ok (the downstream region)?

Concerns about convergence

- I stopped the simulation in the steady state solution when the variation in the u component is less than 10^{-7}. In the linear NavierStokes I stop the computation of the eigenvectors when their residual is of the order of 10^{-5}.
- My concern is that the dependence in the domain aspect ratios is much higher than that. And probably the resolution of the smallest element as well (e.g. order of the polynomials). I want to check this. Any suggestions?

IMPERIAL

Flat domain

Víctor Ballester
February 19, 2025

Domain

- Flat domain. I ran the baseflow till numerical steady state (I could set large dt).
- At the end I am using a full (small in height) BL of quads for the mesh.
- Previous length (upstream + gap + downstream): L $= (50 + 16.25 + 150)\delta^* = 216.25\delta^*$.
- New length: $650\delta^*$.

Most unstable mode

First two eigenvalues are real.

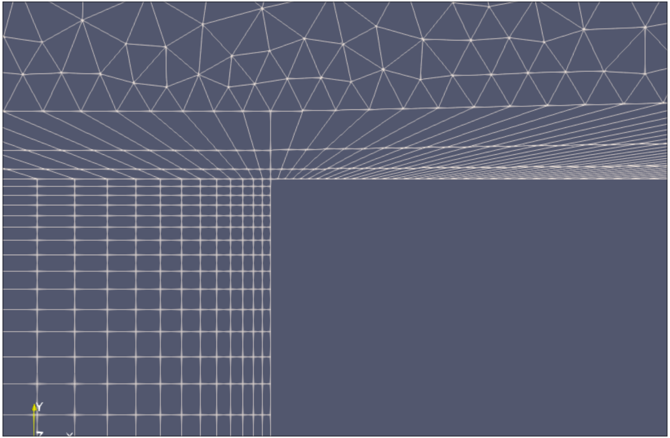
There's an accumulation of energy at the outflow boudnary. I tried to increase the length of the domain to avoid this but it didn't work. Is it physical the mode?

A modification of the mesh

Víctor Ballester
February 27, 2025

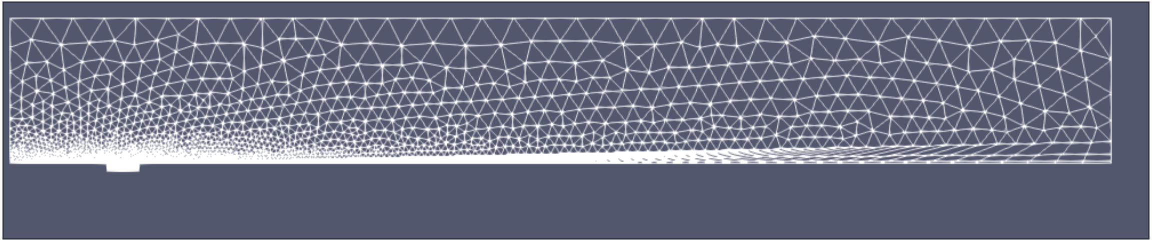
New domain

- Points on the quads edges at different geometric progressions.
- **Aim:** “homogenize” the CFL condition through the domain, in this case by increasing the size of the first triangles.
- So far: $\mathrm{dt}_{\mathrm{old}} \sim 0.0012 \implies \mathrm{dt}_{\mathrm{new}} \sim 0.003$



New domain

- Boundary layer of quads also changed. Now its height increases with x.
- **Aim 1:** take advantage of the efficiency of quads integration and mirror the boundary layer growth
- **Aim 2:** have similar aspect ratios sizes at the outflow to match triangles sizes (in order to avoid jumps). Because the mesh at the outflow is very sparse.



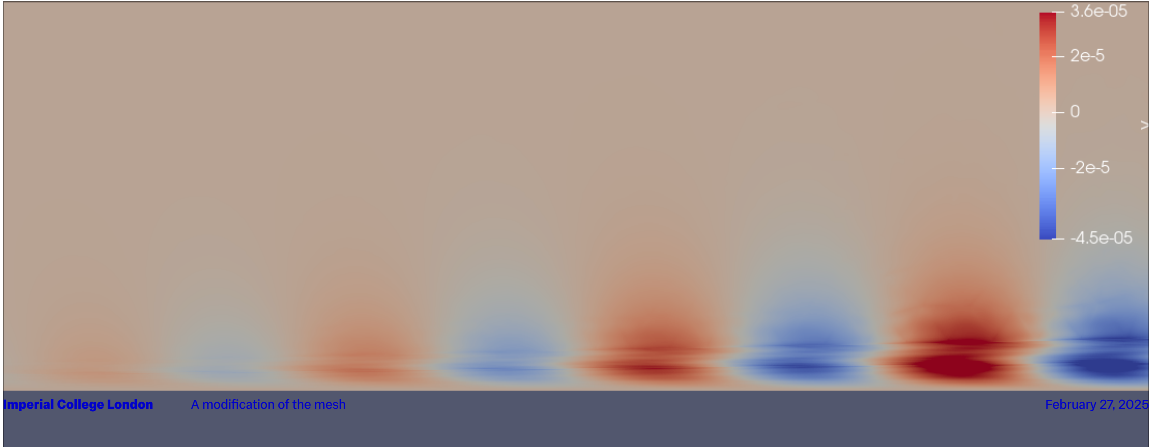
TS waves!!!

- New width considered, w $= 16.35\delta^*$, which doesn't have global instability. (Reminder: we already know that w $= 16.5\delta^*$ has an global instability).

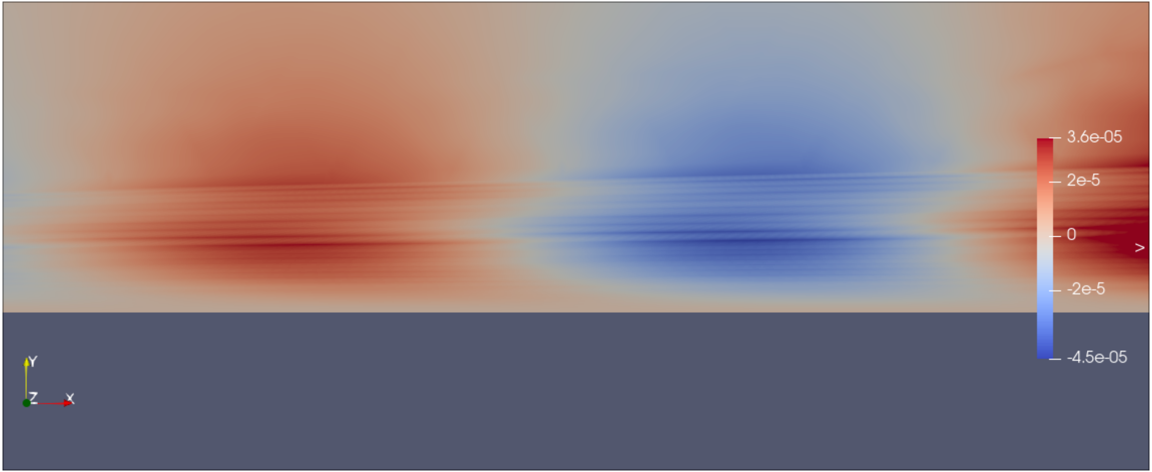
v component of the most unstable mode.

Some problems

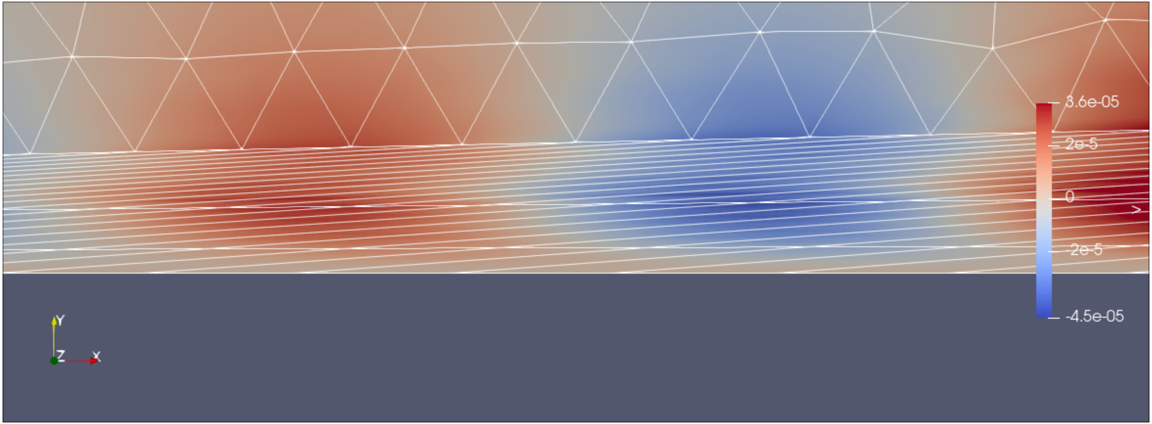
- Sheared boundary layer. Probably because I am keeping the same polynomial order of approximation on the quads, but one of the dimensions is changing (stretching) **a lot** (see figures below).
- Post-processing improves something, but we need to modify the integration as well.



Some problems



Some problems



IMPERIAL

TS mode!

Víctor Ballester
March 13, 2025

Summary

- I ran the case with w $= 16.5\delta^*$ (remember that w $= 16.35\delta^*$ is naturally stable as t $\to \infty$).
- So I needed to use SFD to get a good baseflow (accurate enough, up to residuals of 10^{-6}).
- Then, I ran the linearized solver to get the global modes. Interestingly, changing the number of steps for the Arnoldi iteration I got two different results.
  - With 'few" steps I got the TS mode with growth rate of -0.00543859 and frequency of ± 0.0260322.
  - With more steps I got a greater-in-growth-rate mode, with growth rate of -0.00258415 and frequency of ± 0.00276843. And I 'showed" this is the greatest growth rate.

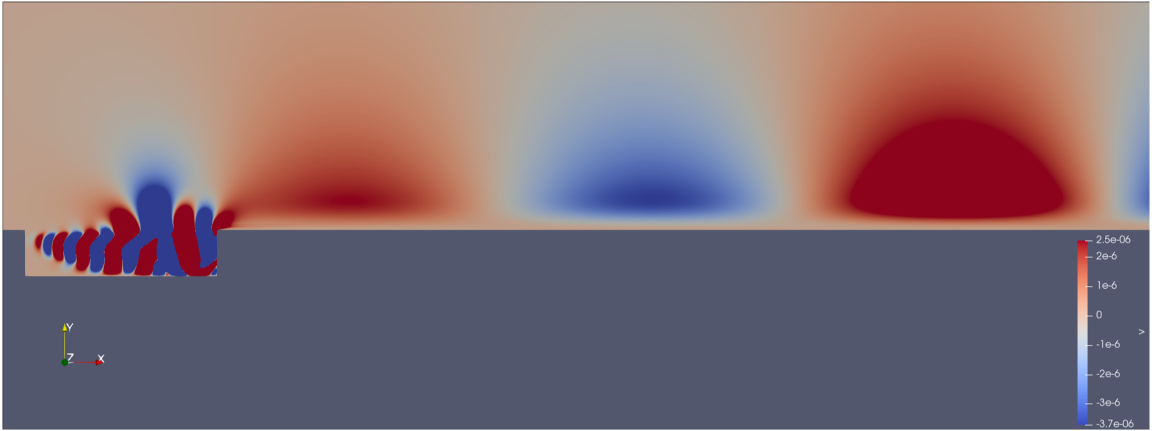
TS mode

u component

v component

TS mode

v component



Huge Mode

u component

Questions

- What is destabilizing my system if every global mode is stable?

IMPERIAL

Consistency between Nonlinear and linear solver

Víctor Ballester

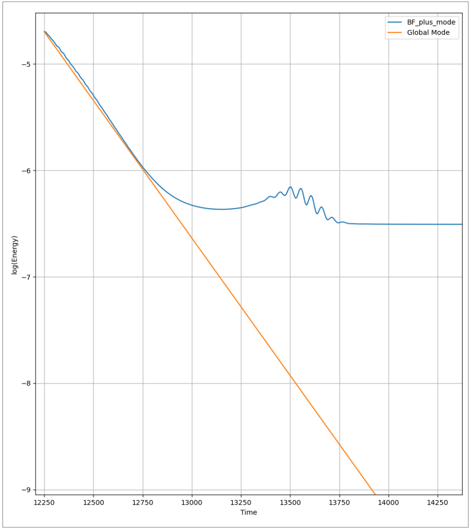
March 20, 2025

1st test

- Run case w $= 16.35\delta^*$ (initially stable with smoothing techniques (SVV)) without SVV and without spectral HP dealiasing (talk with Spencer) as well.
- Result: still naturally stable steady state.
- I got a lot of problems though with the outflow BCs.

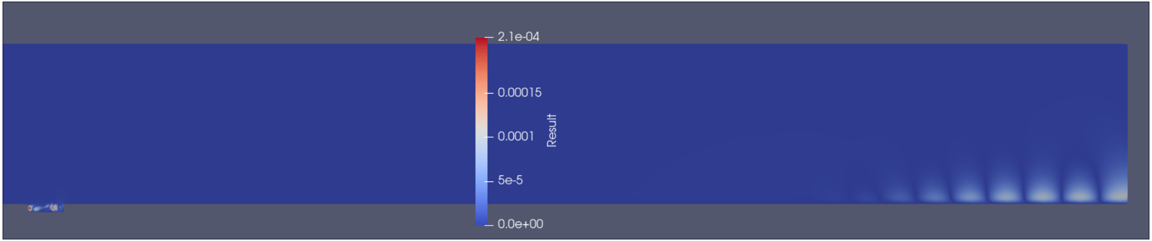
2st test

- With the runs already done (e.g. baseflow and eigenmodes with SVV and spectral HP dealiasing), we wanted to observed the decay in energy of the most unstable mode using the nonlinear solver (with initial conditions the baseflow plus the most unstable mode).



v-component L2 norm comparison

weird wiggles



3rd test

- Compute growth rate of global modes for a case with wider gap.
- I chose w $= 18\delta^*$ but I **forgot** to disable the SVV.
- Still running the EV solver, but it looks again negative growth rate.
- Could it be that SVV is damping too much the modes?

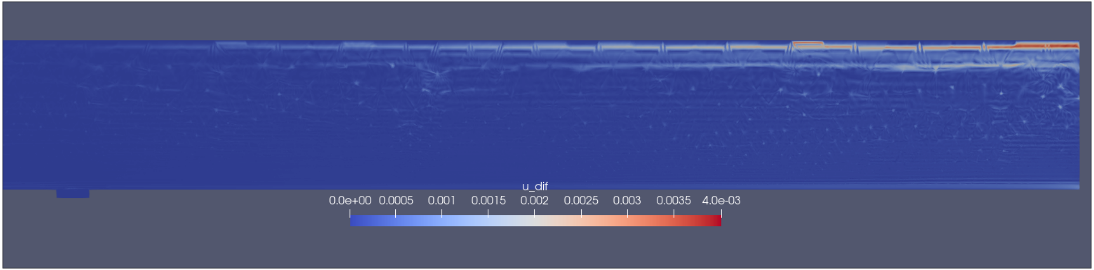
IMPERIAL

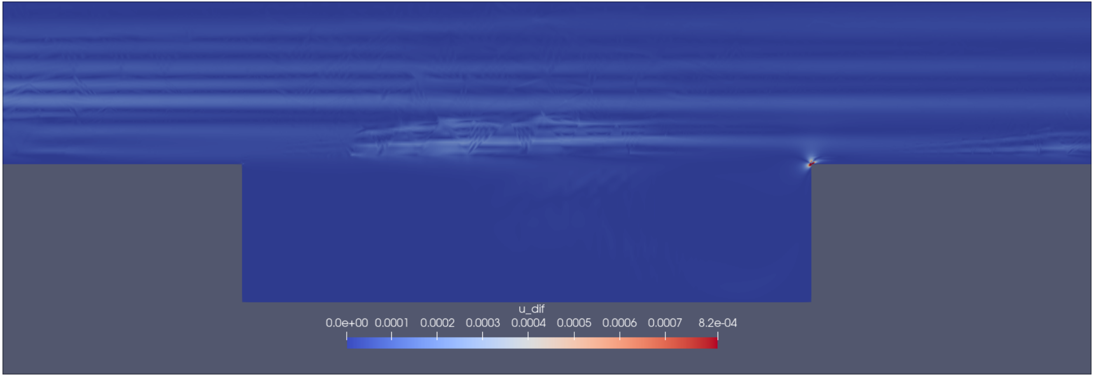
Roughnesses in the domain

Víctor Ballester
March 26, 2025

SVV vs no SVV (in SFD)

- $\mathrm{w} = 16.5\delta^*$
- Plotted we show $|\mathrm{u}_{\mathrm{SVV}} - \mathrm{u}_{\mathrm{noSVV}}|$.



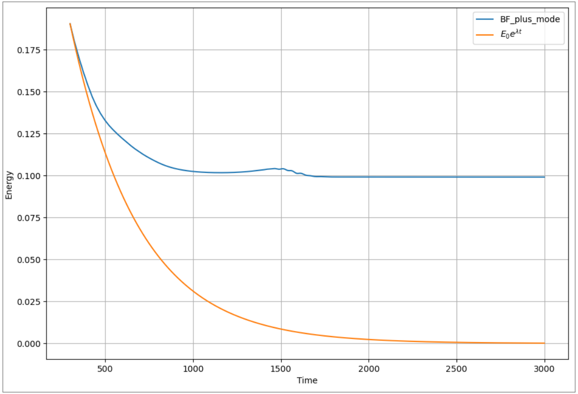

u_dif
0.0e+00 0.0001 0.0002 0.0003 0.0004 0.0005 0.0006 0.0007 8.2e-04

About SFD parameters

- I changed the filter width Δ from 2 to 4, but it is taken so long to converge.

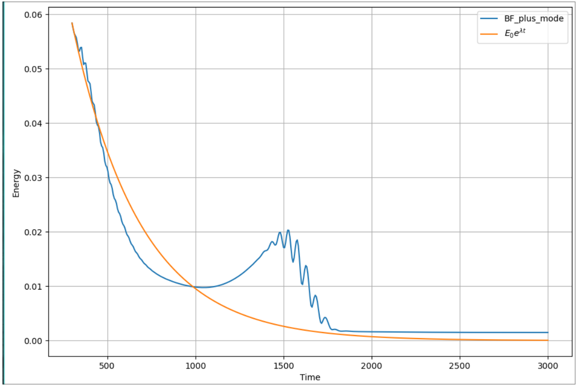
$$\dot{\mathsf{q}} = \mathsf{f}(\mathsf{q}) - \chi(\mathsf{q} - \bar{\mathsf{q}})$$
$$\dot{\bar{\mathsf{q}}} = \frac{\mathsf{q} - \bar{\mathsf{q}}}{\Delta}$$

Baseflow + eigenmode



L2 norm of the u-component (nonlinear vs linear theory).

Baseflow + eigenmode



L2 norm of the v-component (nonlinear vs linear theory).

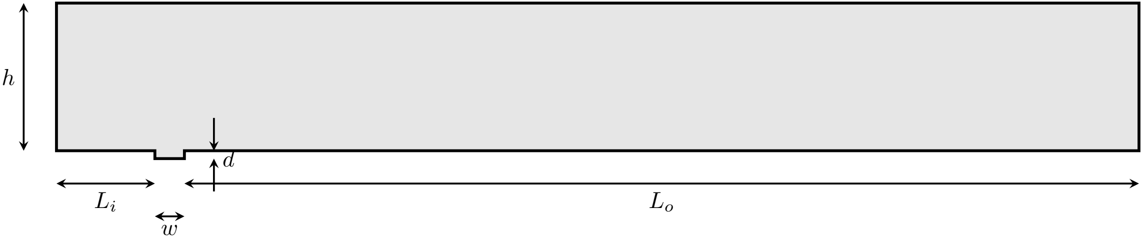
IMPERIAL

Summary 1st term 2025

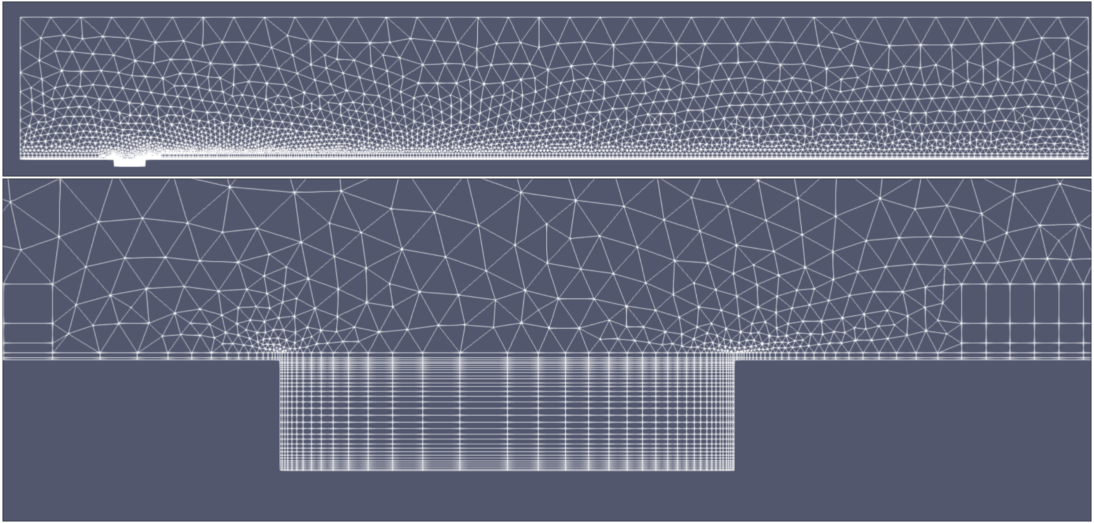
Víctor Ballester
April 2, 2025

Domain

- δ^*: measured at the upstream edge of the gap in a surface free of discontinuities.
- $\mathrm{Re}_{\delta^*} = 1000$
- $\mathrm{L_i} = 50\delta^*$
- $\mathrm{L_o} = 500\delta^*$
- $\mathrm{h} = 75\delta^*$
- $\mathrm{d} = 4\delta^*$
- $\mathrm{w} \in \mathrm{A}\delta^*$, $\mathrm{A} = \{10, \ldots, 30\}$
- Inflow BC: Blasius profile
- Top BC: Far-field BC
- Wall BC: No-slip
- Outflow BC: Convective BC (Robin), or normal Neumann BC.



Current mesh

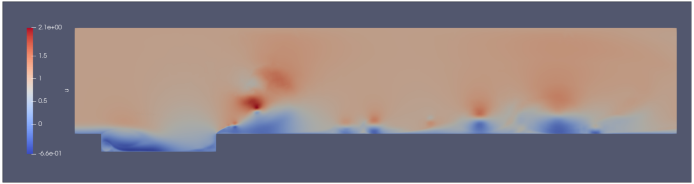


Study - range of w

- For w $\leq 16.4\delta^*$, we observe a steady state from nonlinear simulations ("global" attractor).
- As we increase w $\geq 16.5\delta^*$, we observe a divergent behavior. Thus, to do LSA with those cases we have to make use of Selective Frequency Damping (SFD) (e.g. control theory) to get a locally stable baseflow.
- **Conjecture**: the critical width at d $= 4\delta^*$ lies in the interval $(16.4\delta^*, 16.5\delta^*)$ (up to numerical sensitivity).

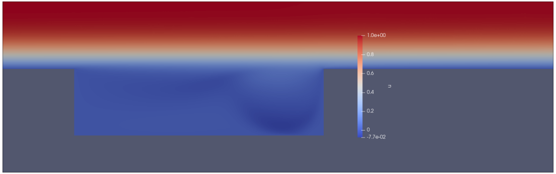
Unstable baseflows

u-component of dns simulations with w $= 16.5\delta^*$ at time t $\sim 11\,500$.

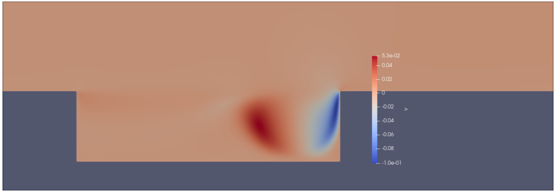


u-component of dns simulations with w $= 26\delta^*$ at time t ~ 1500 (old run with a short domain).

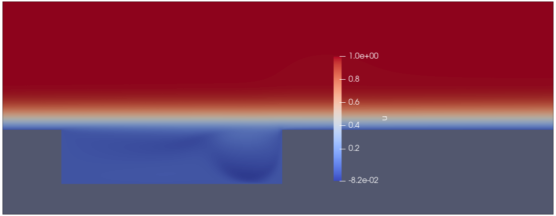
Stable baseflows



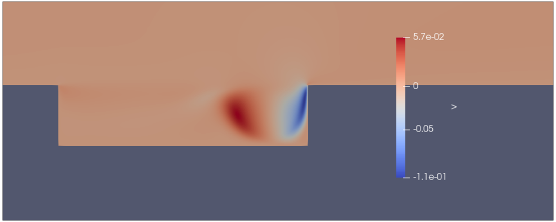
u-component natural baseflow with w $= 15\delta^*$



v-component natural baseflow with w $= 15\delta^*$

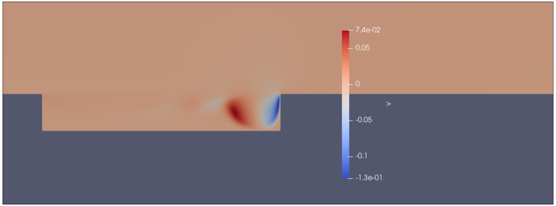


u-component SFD baseflow with w $= 16.5\delta^*$

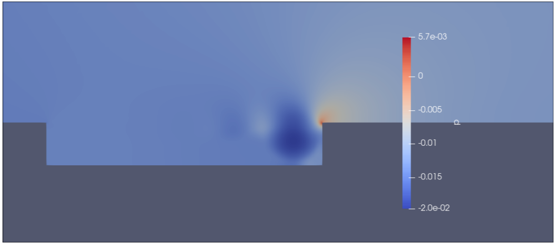


v-component SFD baseflow with w $= 16.5\delta^*$

u-component SFD baseflow with w $= 26\delta^*$



v-component SFD baseflow with w $= 26\delta^*$



pressure SFD baseflow with w $= 26\delta^*$

Linear stability analysis

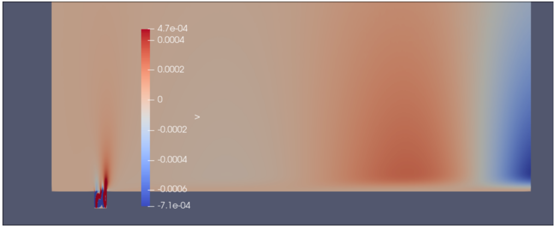
Case w $= 15\delta^*$. Most unstable mode:

- Growth rate: -0.00279
- Frequency: ± 0.00304

Comments:

- This is not a TS mode. We observe a huge mode in the BL.
- The magnitude of the fields is much higher inside the gap than on the BL (around 2-10 orders, depending on the x-position in the BL)

u-component of the most unstable eigenmode with w $= 15\delta^*$ (domain scaled in the x-dir by 0.1)



v-component of the most unstable eigenmode with w $= 15\delta^*$ (domain scaled in the x-dir by 0.1)

Linear stability analysis

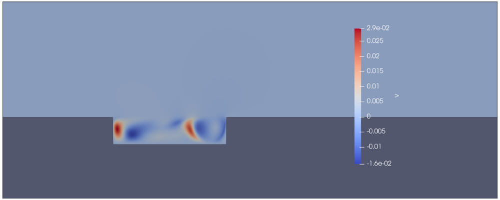
Case w $= 16.5\delta^*$ (very similar to w $= 15\delta^*$).
Most unstable mode:

- Growth rate: -0.00258
- Frequency: ± 0.00276

Comments:

- We do **not** observe a mode with positive growth rate, which suggests (up to numerical sensitivity and other possible errors) that the nature of the instability would be nonlinear instead of linear.

u-component of the most unstable eigenmode with w $= 16.5\delta^*$ (emphasizing colors in the gap)



v-component of the most unstable eigenmode with w $= 16.5\delta^*$ (emphasizing colors in the gap)

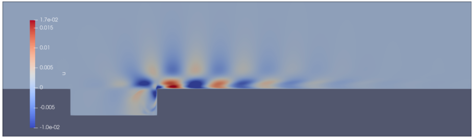
Linear stability analysis

Case w $= 26\delta^*$. Most unstable mode:

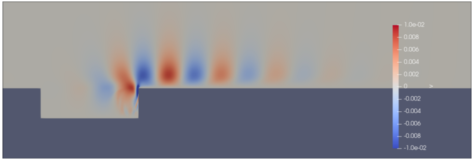
- Growth rate: 0.00859
- Frequency: ± 0.00887

Comments:

- This mode looks like the result of an absolute instability in the gap because the amplitude of the waves decreases as they move downstream.



u-component of the most unstable eigenmode with w $= 26\delta^*$ (domain scaled in the x-dir by 0.5)



v-component of the most unstable eigenmode with w $= 26\delta^*$ (domain scaled in the x-dir by 0.5)

Nature of the instability (work in progress)

- Runs doing global stability analysis from a baseflow just above the critical width where non-steadiness shows up on dns (e.g $\mathrm{w_{critical}}\big|_{\mathrm{d}=4\delta^*} \in (16.4\delta^*, 16.5\delta^*)$ show eigenmodes with a **negative** growth rate.
- We did check that adding those modes to the baseflow as initial conditions in the nonlinear solver leads to a steady solution (we ran that for $\mathrm{w} = 16.5\delta^*$ and $\mathrm{w} = 18\delta^*$). This does **not** happen though for $\mathrm{w} = 26\delta^*$.

Coherence between DNS and LSA

Let

$$\varphi(t;(u_0,v_0)) = (\varphi_u(t;(u_0,v_0)), \varphi_v(t;(u_0,v_0)))$$

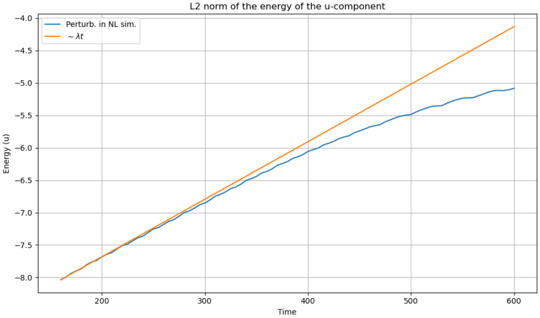
the flow of NS eqs at time t with initial conditions (u_0, v_0). Let (U, V) be the baseflow of our system and $(\tilde{u}, \tilde{v})$ the most unstable eigenmode with λ the respective growth rate. We plot the following quantities

$$t \mapsto \frac{\|\varphi_u(t;(U+\tilde{u}, V+\tilde{v})) - U\|_{L^2}}{\|U\|_{L^2}}$$

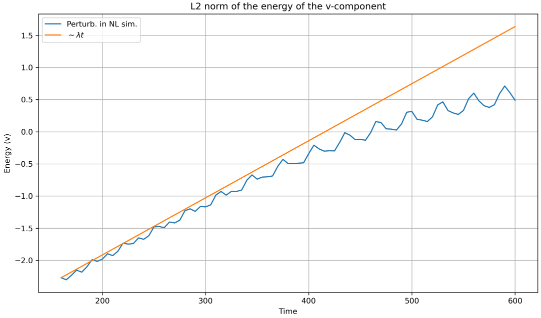
$$t \mapsto \frac{\|\varphi_v(t;(U+\tilde{u}, V+\tilde{v})) - V\|_{L^2}}{\|V\|_{L^2}}$$

for different w. This should tell us how the energy of the perturbation evolves in time. We compare this with the theoretical evolution of the energy based on LSA, which goes as $\|u_0\|_{L^2} e^{\lambda t}$ and $\|v_0\|_{L^2} e^{\lambda t}$, respectively.

Case w $= 26\delta^*$



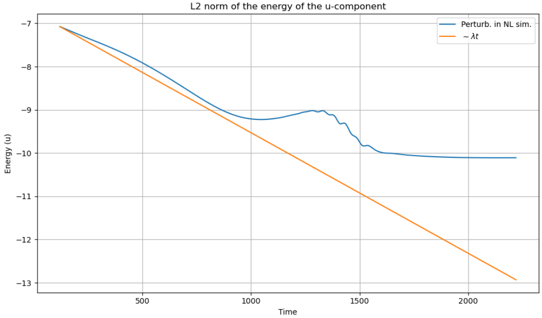
u-component



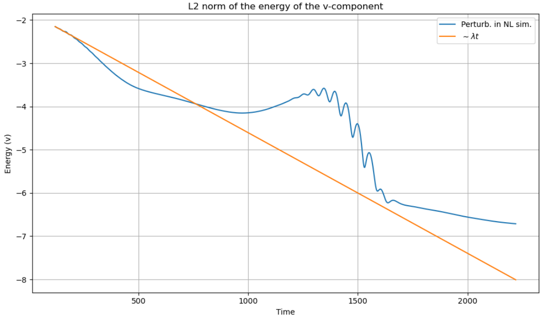
v-component

- We are plotting the log of the energy.
- Initially the nonlinear results fits well the linear prediction after which (t ~ 450) the nonlinear effects start to contribute significantly.

Case w $= 15\delta^*$ (similar to w $= 16.5\delta^*$)



u-component

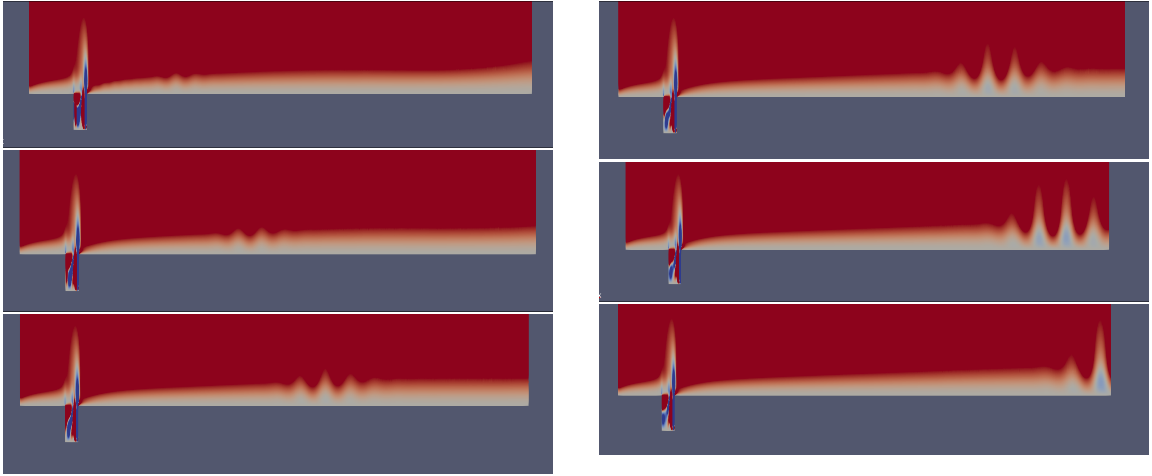


v-component

- We are plotting the log of the energy.
- The fit is less nicer, and we observe a weird bump appearing on the middle of the time interval considered.

The bump

u-component the perturbed system at different times (domain scaled in the x-dir)



The bump

- Looks like the modes may be providing the system with enough energy inside the gap to trigger the convective instability nature of the system. **Why?** The stable modes computed for w $= 15\delta^*$ and w $= 16.5\delta^*$ showed bigger magnitudes inside the gap than on the BL.

Questions

- What range of d should we consider in the future?
- Should we first move to 3D case or to 2D compressible case?

IMPERIAL

Flat domain

Víctor Ballester
April 9, 2025

Domain

- I am getting a real mode, even enlarging the domain.

Inconsistency between numerical reynolds number and theoretical one

- Since ν and U_{inf} are fixed, the only way to change the Reynolds number is by changing δ^*. So we look for a formula to calculate $\delta^*(x_1)$ based on $\delta^*(x_0)$.

We have $\delta^*(x) = C\frac{x}{\sqrt{Re_x}} = C\frac{\sqrt{x}}{\sqrt{U/\nu}}$, $C \simeq 1.72$. Thus, $\frac{\delta^*(x_1)}{\delta^*(x_0)} = \frac{\sqrt{x_1}}{\sqrt{x_0}}$. Now, we have:

If $x_1 = x_0 + \ell\delta^*(x_0)$, we have:

$$\delta^*(x_1) = \delta^*(x_0)\sqrt{1 - \ell C^2/Re_{\delta^*(x_0)}}$$

IMPERIAL

Blowing Suction in flat plate

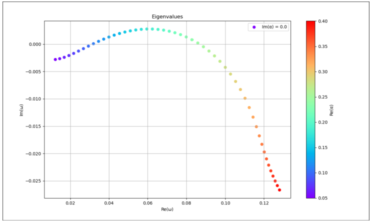
Víctor Ballester
April 24, 2025

Blowing and suction

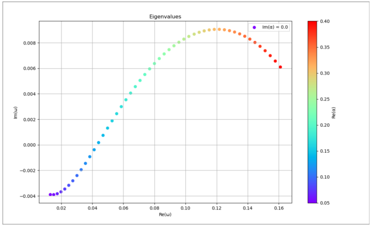
- I modify the BC of v on the wall with:

$$\mathrm{v}(\mathrm{x},\mathrm{t}) = \mathrm{A}\sin\theta(1-\cos\theta)\sin(\omega \mathrm{t})\mathbf{1}_{\mathrm{x}\in(\mathrm{x}_0,\mathrm{x}_0+\ell)}$$

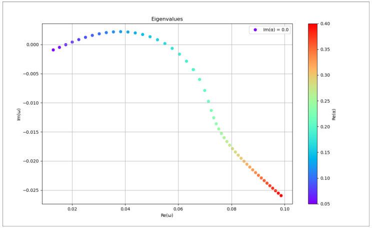
  where $\mathrm{A} = 0.003$, $\mathrm{x}_0 = 0$ (for the flat plate!), $\ell = 2\pi/\alpha_\mathrm{r}$, $\omega = \omega_\mathrm{r}$ and $\theta = \alpha_\mathrm{r}(\mathrm{x}-\mathrm{x}_0)$. The pair $(\alpha_\mathrm{r}, \omega_\mathrm{r})$ is taken from Orr-Sommerfeld eq. Probably I will increase a little bit ω because ω_r is very small.
- I found the function $\sin\theta(1-\cos\theta)$ in a paper. I assume they use it to make a $\mathcal{C}^2$ contact as well as keep the integral of the curve big, in order to “maximize” the forcing.
- Orr-Sommerfeld analysis for $\mathrm{w} = 15$:



$\mathrm{x} = 15\delta^*$ (downstream edge)



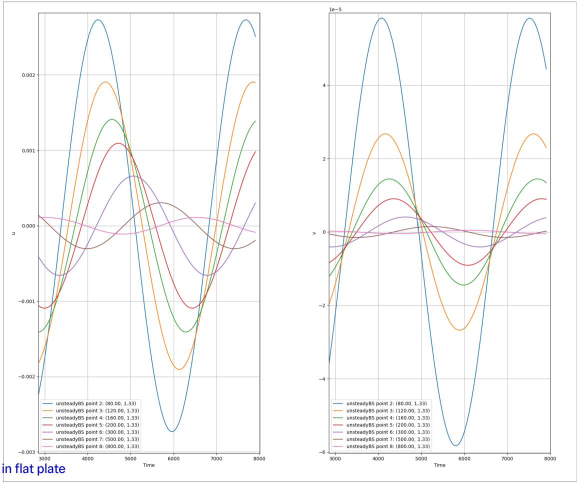
$\mathrm{x} = 300\delta^*$ (downstream edge)



$\mathrm{x} = 900\delta^*$ (downstream edge)

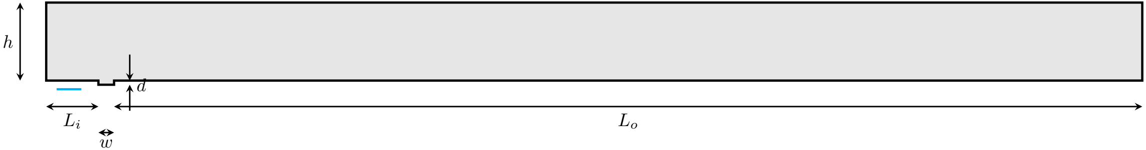
Flat DNS TS amplitudes

- Evolution of different points in the domain of the u (left) and v (right) fields (linear solver with blowing and suction).



General comments

- Should I increase the upstream region for blowing and suction in the gap version?



- I was not able to make the Coupled direct solver work, but I haven't tried too much.
- I have all the ingredients to compute the $n(x)$ curves. I am in the postprocessing part now.

$$n(x) = \max_{\omega} \ln \left(\frac{A(x, \omega)}{A_0(\omega)} \right) = \max_{\omega} \int_{x_0}^{x} -\alpha_i(x, \omega) dx$$

for a perturbation of the form $\tilde{v}(x, y, t) = v(y) \exp\{i(\alpha x - \omega t)\}$ and $\alpha_i = \text{Im}(\alpha)$.

IMPERIAL

Exploring more depths

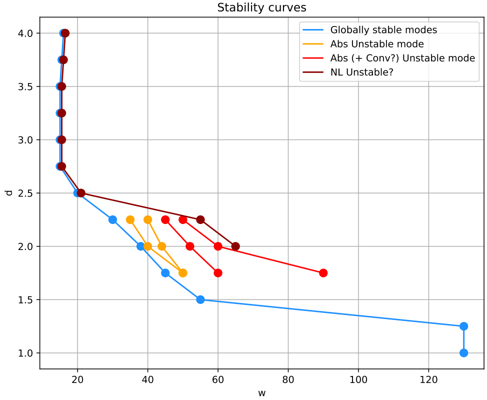
Víctor Ballester
April 30, 2025

General Comments

- I confirm, we may still have a subcritical bifurcation in the vicinity of d $= 4\delta^*$, w $= 16\delta^*$. I ran a perturbed system in an unstable configuration and I got a stable solution.
- Modified Arnoldi still doesn't output the TS mode for the stable case, now that the BC are (more) correct. We still get that huge mode. Should we go for blowing and suction?
- I ran a lot of runs varying d$/\delta^* \in [1, 4]$ and w$/\delta^* \in [15, 130]$.

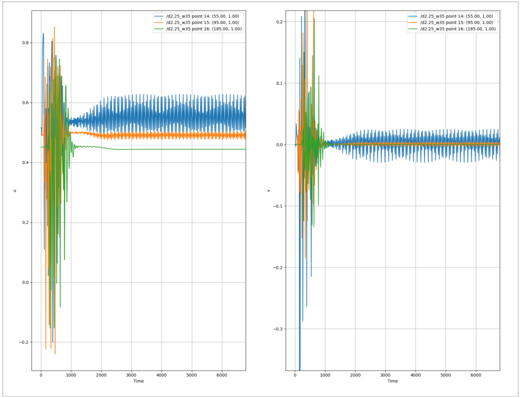
Results

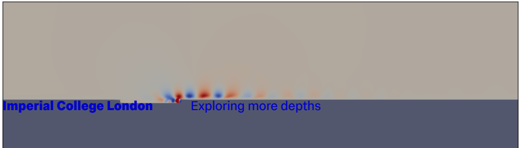
We know there's always a convectively unstable TS mode, even in the blue curve. My hypothesis (taken with caution) for the region $\{\mathrm{d}/\mathrm{delta}^* \leq 2.25\}$ is that if at some fixed d we get an absolute unstable mode localized in the downstream edge of the gap, then as we increase w, the wavelength associated to the absolute mode becomes convectively unstable.



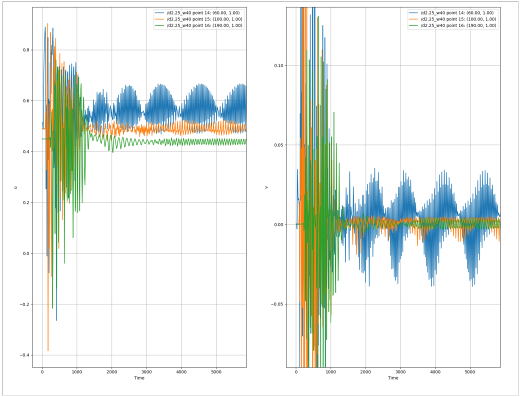
$\mathsf{d} = 2.25\delta^*$

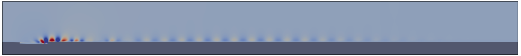
$\mathsf{w} = 35\delta^*$





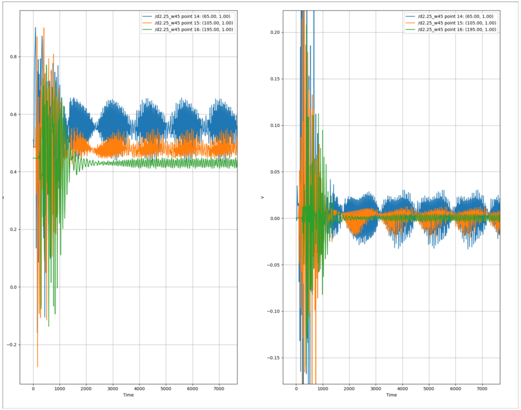
$\mathsf{w} = 40\delta^*$

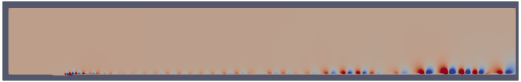




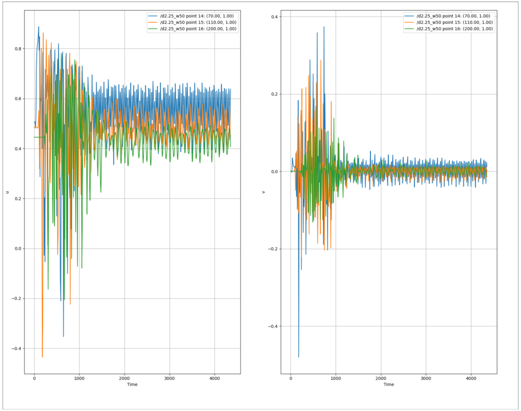
d $= 2.25\delta^*$

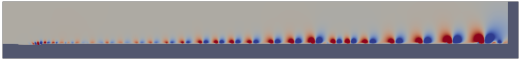
w $= 45\delta^*$





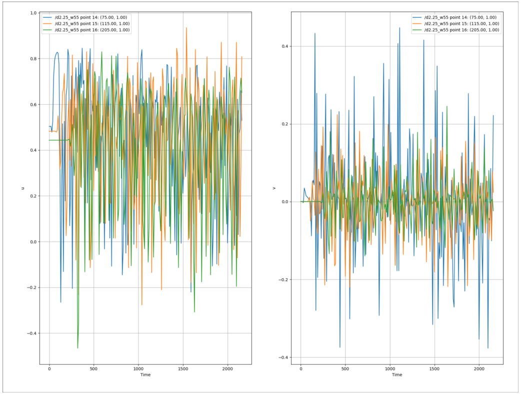
w $= 50\delta^*$

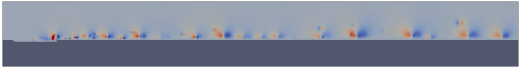




$\mathsf{d} = 2.25\delta^*$

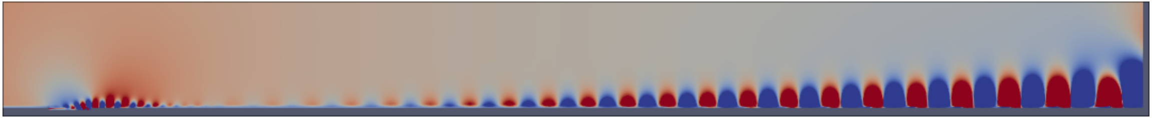
$\mathsf{w} = 55\delta^*$





Absolutely unstable mode exciting TS waves

$\mathrm{d} = 2\delta^*, \mathrm{w} = 40\delta^*$

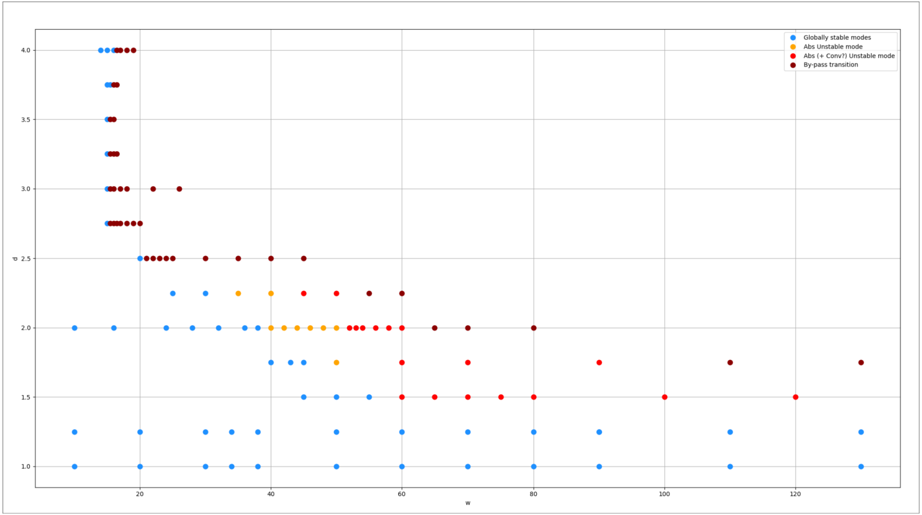


IMPERIAL

n factor

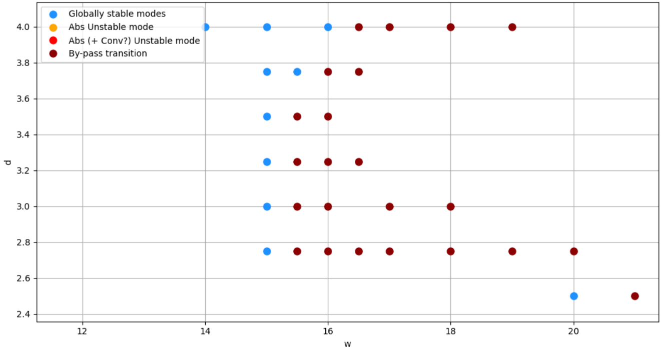
Víctor Ballester
May 6, 2025

Recap



Globally stable modes
Abs Unstable mode
Abs (+ Conv?) Unstable mode
By-pass transition
d
w
4.0
3.5
3.0
2.5
2.0
1.5
1.0
20
40
60
80
100
120

Backward behaviour in w in the stability line



$n(x)$ factor

- Blowing-suction upstream as:

$$v(x,t) = A\sin(\alpha_r(x - x_0))^3 \sin(\omega_r t) 1_{x_0 \le x \le x_0 + \ell}$$

where $\ell = \pi/\alpha_r$, $A = 0.003$, $x_0 = -70\delta^*$, $\alpha_r = 0.1428, \omega_r = 0.04618$ (both from Orr-Sommerfeld eq).

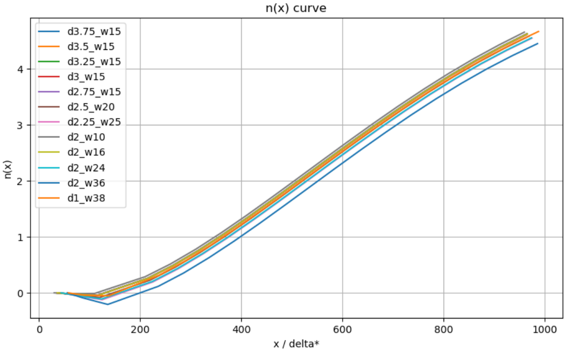
$$n(x, \omega_r, \alpha_r) = \log\left(\frac{A^{TS}(x, \omega_r, \alpha_r)}{A_0^{TS}(\omega_r, \alpha_r)}\right)$$

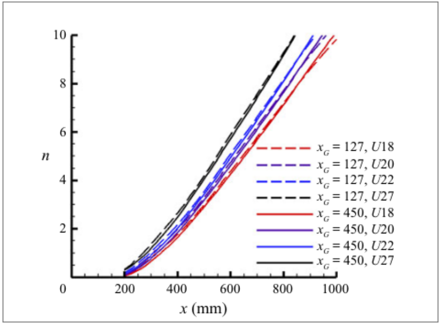
where $A^{TS}(x, \omega_r, \alpha_r)$ is the amplitude of the TS mode at x and $A_0^{TS}(\omega_r, \alpha_r)$ is the amplitude of the TS mode at $x = x_0$ (how to choose x_0?).

And then

$$N(x) = \max_{\omega, \alpha} n(x, \omega, \alpha)$$

It would be nice to have our $n(x, \omega_r, \alpha_r)$ as closest as possible to the $N(x)$. Having observed the huge importance of the v component of the baseflow, do we need to try weakly non-parallel Orr-Sommerfeld?


n(x) curve
d3.75_w15
d3.5_w15
d3.25_w15
d3_w15
d2.75_w15
d2.5_w20
d2.25_w25
d2_w10
d2_w16
d2_w24
d2_w36
d1_w38
n(x)
x / delta*
0
1
2
3
4
0
200
400
600
800
1000


x_G = 127, U18
x_G = 127, U20
x_G = 127, U22
x_G = 127, U27
x_G = 450, U18
x_G = 450, U20
x_G = 450, U22
x_G = 450, U27
n
x (mm)
0
200
400
600
800
1000
2
4
6
8
10

jeff ΔN

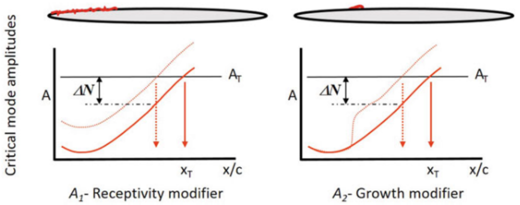


Fig. 5 Schematic showing linkage between a linear amplitude method and the variable N-factor approach

waves [8] and crossflow vortices [36, 37]. Using the envelope n of the physical-mode growth curves m, the amplitude threshold can be expressed as

$$n(x) = max_{\omega} max_{\beta} [m(x; \omega, \beta)] \geq N, \quad (2)$$

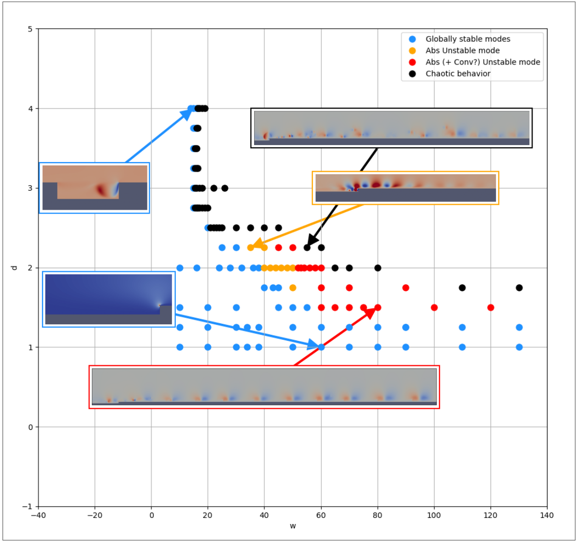
where $N \propto ln(A_T/A_0)$. The value of N can be linked to a reference condition with $N = N_{ref} - \Delta N$, where ΔN captures changes to A_0 or to $\Delta m_{crit}(x_T)$. Figure 5 illustrates the linkage between ΔN and the linear amplitude for both paths A_1 and A_2. The solid line in the figures corresponds to a critical mode causing transition for a given reference condition. The dashed line corresponds to a critical mode arising from enhanced receptivity or enhanced growth. In both cases, the forward movement of transition can be linked to a change in the transition N-factor ΔN.

The variable N-factor approach provides a method for a-priori predictions to account for varying surface-induced flow distortions. This method can also be used

Tries and errors

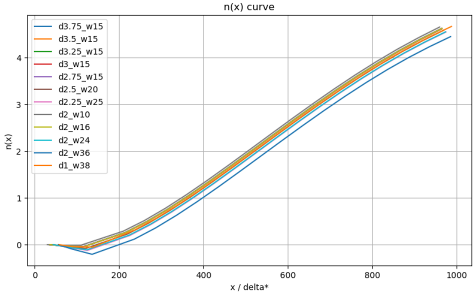
Víctor Ballester
May 14, 2025

Stability plot with images



Comments

- I realized I input the wrong frequency when blowing and suction, up to a factor of 1.72. Now I am redoing the computations. Could that explain that all the curves where clustered together?
- I also try global stability analysis with the coupled approach, which worked (so far with low polynomial order), but it only does kdim iterations and kdim is bounded by 500 in the code. Why?



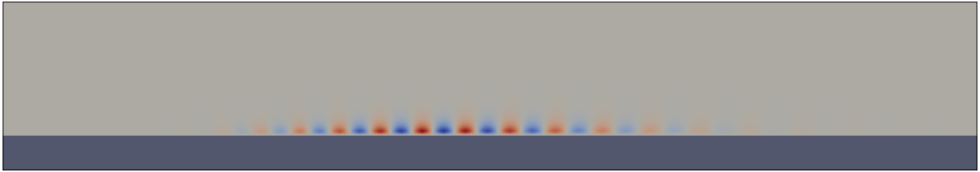
IMPERIAL

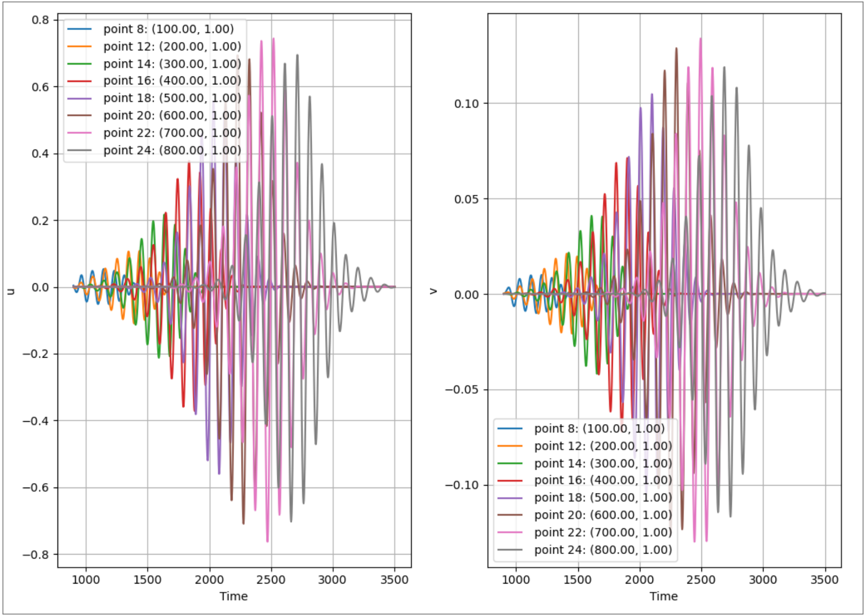
Transient growth in flat plate

Víctor Ballester
May 21, 2025

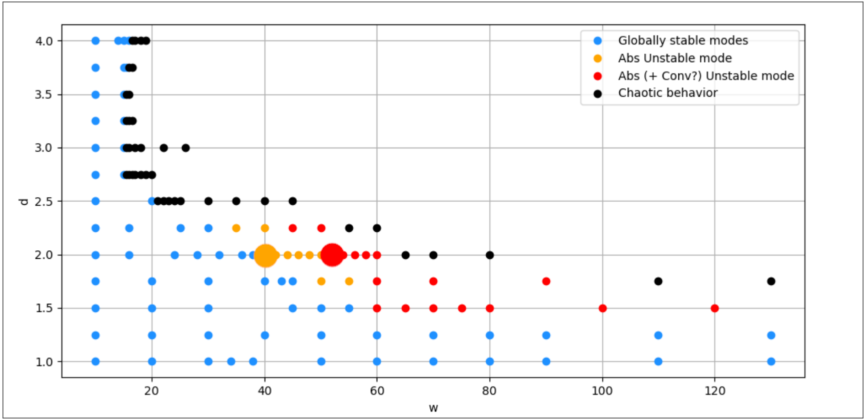
Transient growth in flat plate

- I tried transient growth in the flat plate.
- The length of the domain is $1000(\text{downstream}) - (-100\text{upstream}) = 1100$, as if it was a $0-$width gap domain. So I tried transient growth with $\tau = 450, 900$, τ being the time the simulation is evolved forwards and then backwards. It turned out that the phase speed of the TS modes is not 1 (of course, I didn't thought about that), it is much less (I estimated it to be around 0.35), so I get and amplification just until half of the domain more or less. Even though, **should I increase the length of the domain?**
- Now I am running the case for larger τ.




u
v
Time
Time
point 8: (100.00, 1.00)
point 12: (200.00, 1.00)
point 14: (300.00, 1.00)
point 16: (400.00, 1.00)
point 18: (500.00, 1.00)
point 20: (600.00, 1.00)
point 22: (700.00, 1.00)
point 24: (800.00, 1.00)

Harmonic analysis of the orange-to-red change



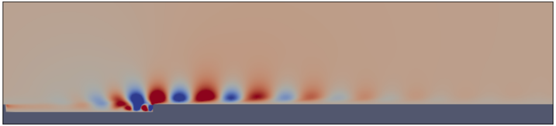
$\mathsf{d} = 2, \mathsf{w} = 40$

Freq with highest amplitude for u

$\omega : 0.00000000$, Amplitude : 226.23665390
$\omega : 0.16174536$, Amplitude : 0.33523849
$\omega : 0.23173134$, Amplitude : 0.25998462
$\omega : 0.23328658$, Amplitude : 5.02840019
$\omega : 0.23484183$, Amplitude : 0.28651745

Freq with highest amplitude for v

$\omega : 0.00000000$, Amplitude : 1.77348566
$\omega : 0.16174536$, Amplitude : 0.40399285
$\omega : 0.23173134$, Amplitude : 0.33448704
$\omega : 0.23328658$, Amplitude : 6.55379325
$\omega : 0.23484183$, Amplitude : 0.37806377



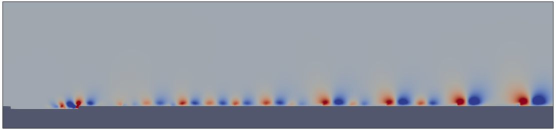
$\mathsf{d} = 2, \mathsf{w} = 52$

Freq with highest amplitude for u

$\omega : 0.00000000$, Amplitude : 48.38968907
$\omega : 0.02559816$, Amplitude : 3.03884201
$\omega : 0.06050475$, Amplitude : 2.12776977
$\omega : 0.08610291$, Amplitude : 2.39670993
$\omega : 0.08843002$, Amplitude : 1.72555528

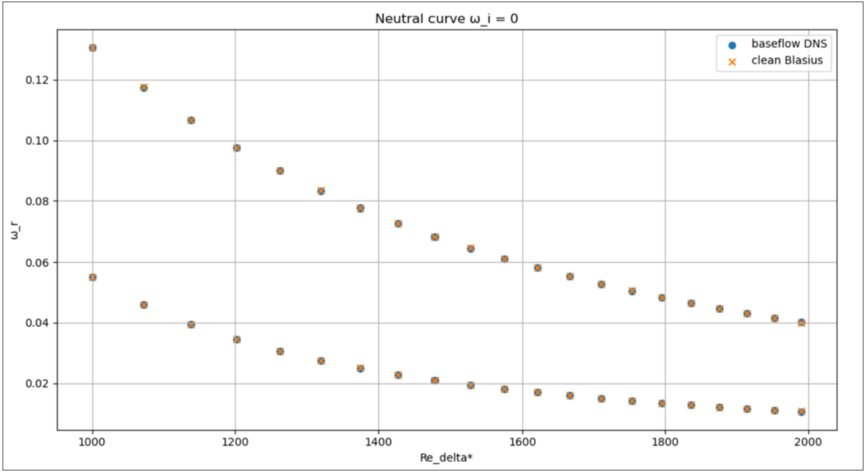
Freq with highest amplitude for v

$\omega : 0.02559816$, Amplitude : 3.28989416
$\omega : 0.06050475$, Amplitude : 1.56887596
$\omega : 0.08610291$, Amplitude : 2.97408531
$\omega : 0.08843002$, Amplitude : 2.26994456



Neutral curve blasius

- Inside the curve, convectively unstable; outside, convectively stable.



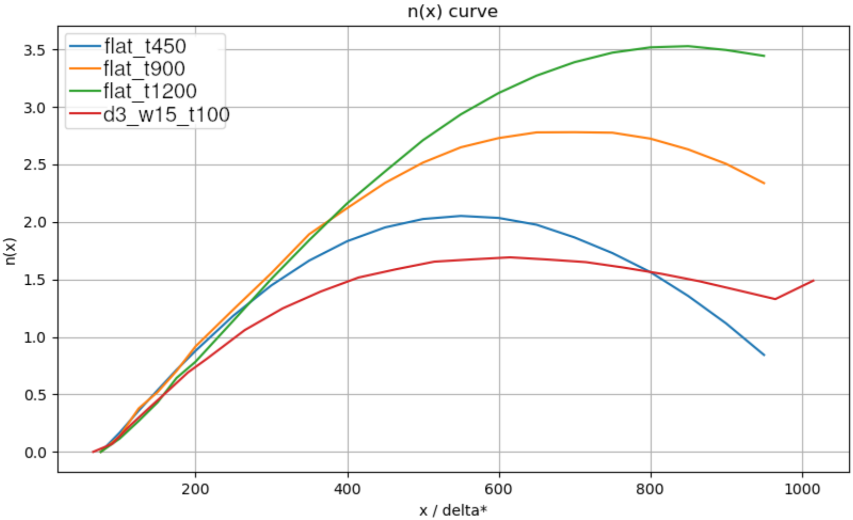
IMPERIAL

Bad transient growth for gap-domains

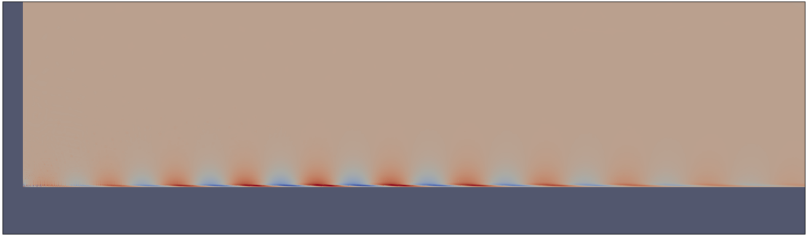
Víctor Ballester

May 28, 2025

Transient growth comparison



Comparison TG solutions



Eigenvector solution from TG in flat-plate

Eigenvector solution from TG in gap-domain d=3, w=15 (same scale as the one above)

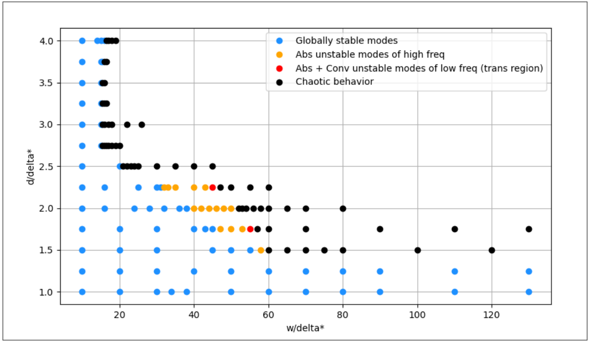
IMPERIAL

Jeff update

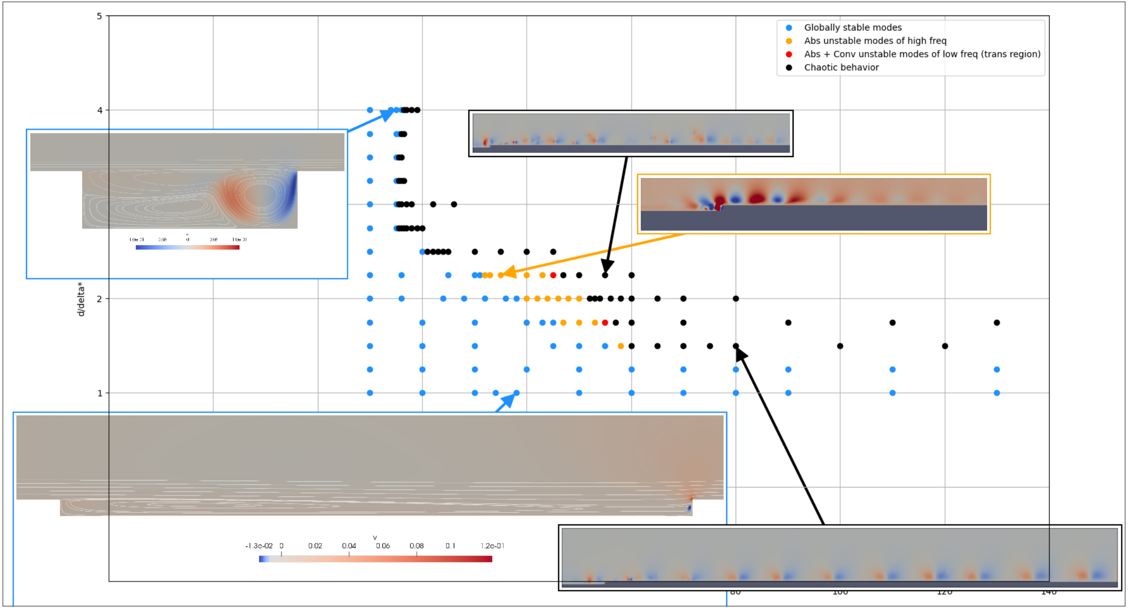
Víctor Ballester
May 26, 2025

Stability analysis diagram

- So far everything is 2d + incompressible.
- Orange means there is an absolute instability in the downstream edge of the gap.
- Usually that absolute instability has high temporal frequency which lies much above the upper limit of the neural curve (of convective instability) for the blasius profile (see figure below). We still need to check this more rigorously with SPOD.
- The red is the transition region, where some small enough frequencies are excited near the absolute instability region, which then grow downstream due to the convective instability. May not be 100% accurate right now, but roughly.

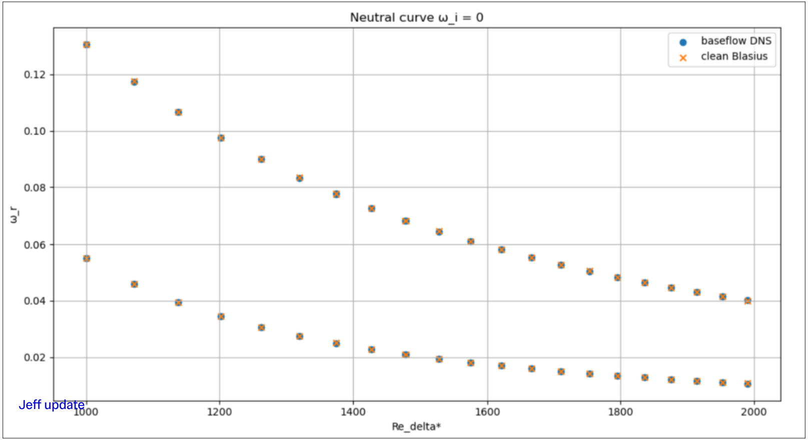


Check next slide to see figures (all representing the v component of the velocity) in some cases.


Globally stable modes
Abs unstable modes of high freq
Abs + Conv unstable modes of low freq (trans region)
Chaotic behavior
d/delta*
5
4
2
1
80
100
120
140
v
-1.3e-02
0
0.02
0.04
0.06
0.08
0.1
1.2e-01

Neutral curve for Blasius profile

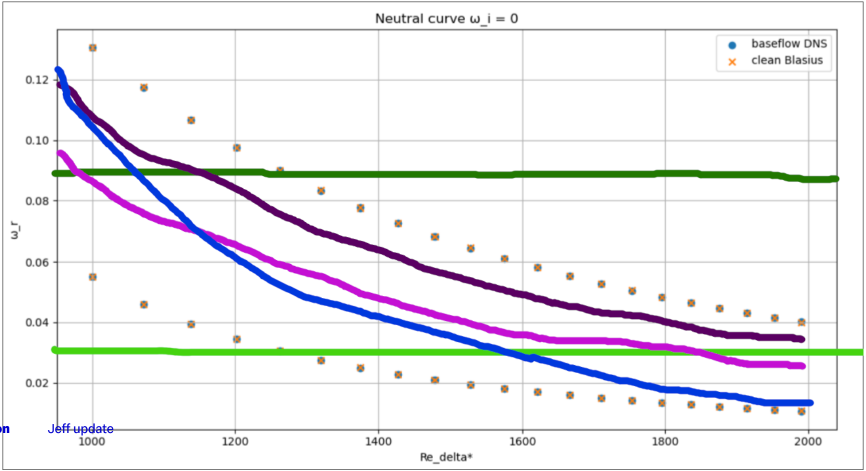
- This is orr-sommerfeld analysis for the Blasius profile and for different profiles (varying x location) of the flat plate baseflow from DNS. They match pretty well.
- Increasing Reynolds = moving downstream in the domain.
- Neutral curve is for the growth rate of the TS mode, y-axis is the temporal frequency of the waves.



Ways to compute the n-factor

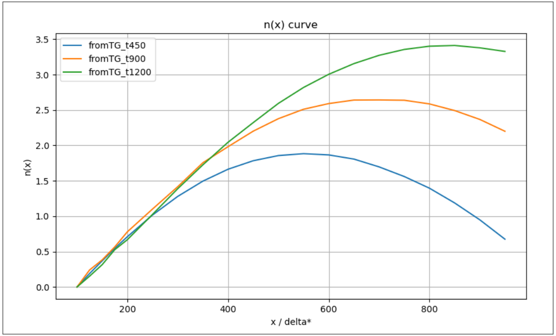
- As we commented last time, we were unable to obtain the TS mode using our ‘default” EV finder (which founds the EVs with greatest real part). We tried another one, which should find the eigenvectors with eigenvalues closes to the origin, but it didn’t work either (in this case, due to software problems, not physical ones).
- Then, we tried blowing and suction, which partially worked. By partially I mean that we were able to excite the TS mode, but they become damped at some point further downstream (see picture next slide).
- Now we started trying with transient growth in the flat-plate case and the results are promising.

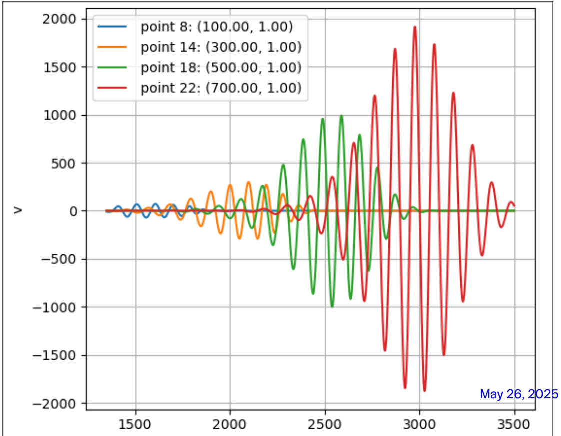
- When blowing an suctioning at the same frequency (green) there are times when the TS mode are damped (bad). The ideal case would be to follow any of the purple curves (even more ideal, the centered one, corresponding to the most unstable frequency at any position), that is, getting the TS modes for a frequency inside the neutral curve and follow them downstream NATURALLY, as opposed to the blue curve.



N-factor

- The N-factor for several transient growth (TG) analysis.
- The result of TG is an the optimal initial condition giving the maximum energy at time τ. Plot below shows TG for $\tau = 450, 900, 1200$.
- Transient growth give us a wave packet that grows as it moves downstream (figure on the right).

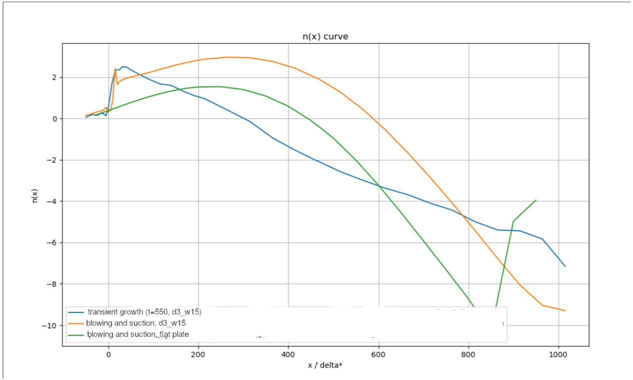




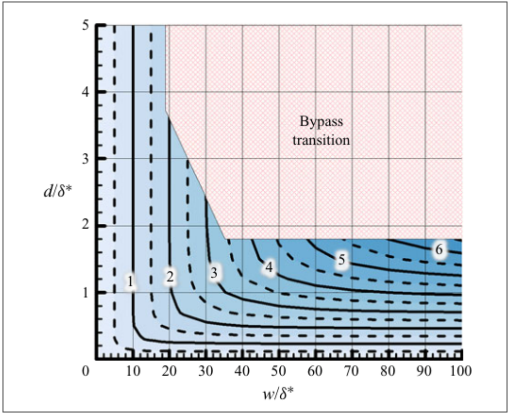
IMPERIAL

Comparison between n-factor computations

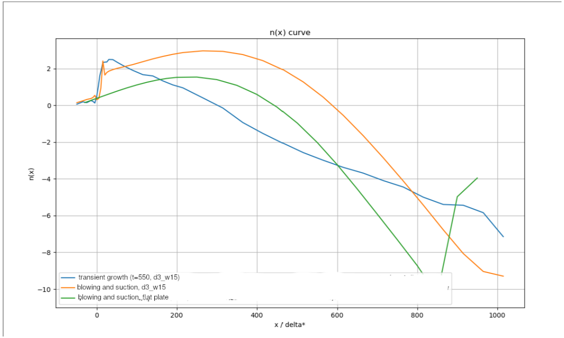
Víctor Ballester
June 4, 2025



Comparison of the n factor with different methods



As we can see the ΔN factor that Jeff defined (as the difference between the n-factor with the gap and the n-factor without the gap) is more or less of the same order of magnitude as the experimental data.



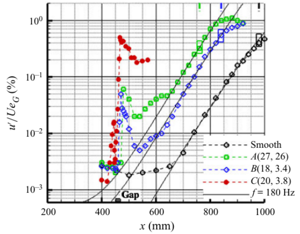
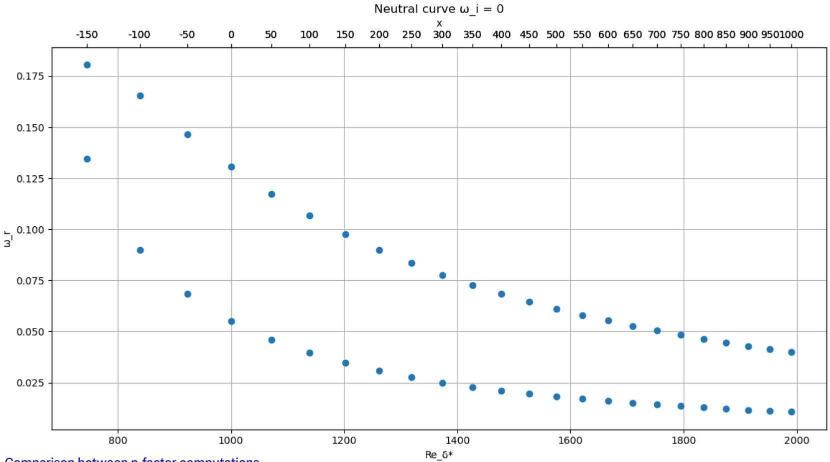


Figure 6. *Streamwise variation of the disturbance amplitude for four different conditions with* x_G = 450 mm*: smooth plate at U18,* f = 178 Hz*; case A*($w/\delta^* = 27$, $d/\delta^* = 2.60$) *at U18,* f = 175 Hz*; case B*($w/\delta^* = 18$, $d/\delta^* = 3.45$) *at U18,* f = 183 Hz*; case C*($w/\delta^* = 20$, $d/\delta^* = 3.83$) *at U22,* f = 830 Hz*. Symbols are measurements and thin lines are linear theory for U18. Rectangles show the measured transition locations.*

We also observe the small peak that we get in the n-factor with blowing and suction in the experimental data.

Neutral curve blasius profile

- I tried Blowing and Suction at $\omega = 0.09$.



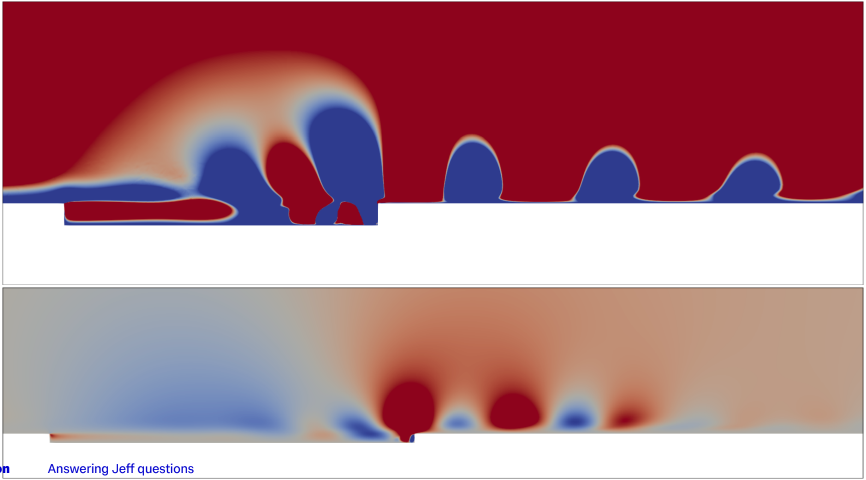
IMPERIAL

Answering Jeff questions

Víctor Ballester
June 16, 2025

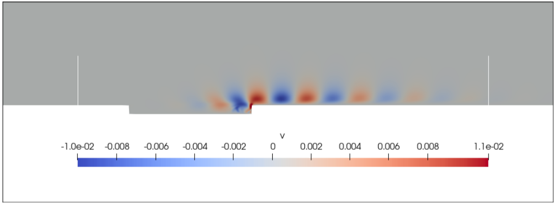
Absolute instability going up to the upstream edge of the gap?

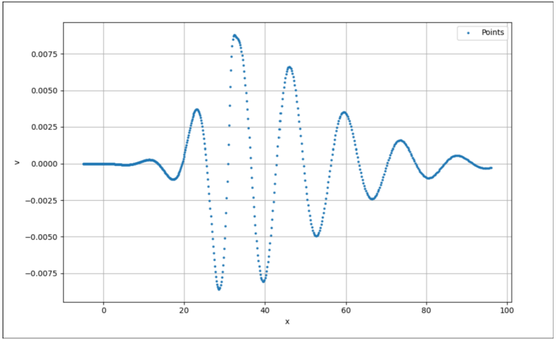
- No. The colors are stressed, but they show that the absolute instability is localized near the downstream edge of the gap.



Aboslute instability in the gap

SFD for d $= 2.25$, w $= 32$ Most unstable mode (with positive growth rate):





Frequency comparison in a chaotic regime

- We study the case $\mathsf{d} = 3.83$, $\mathsf{w} = 20$ (see the figure below)
- Jeff claims that the dominant nondimensional frequency before transitioning is $\mathsf{F} = 10^6 \omega \nu / \mathsf{U}^2 = 168$
- In our case, using that formula we have that $\mathsf{F} = 10^3 \omega$ (right? because $\mathsf{U} = 1$ and $\nu = \frac{1 \times 1}{1000}$).

$\mathsf{x} = \mathsf{w} + 5\delta^*$

Dominant freq for **u**

$\omega : 66.428, \widehat{\mathsf{f}}(\omega) : 784.87$

$\omega : 67.377, \widehat{\mathsf{f}}(\omega) : 539.07$

$\omega : 65.479, \widehat{\mathsf{f}}(\omega) : 224.81$

$\omega : 200.234, \widehat{\mathsf{f}}(\omega) : 204.94$

$\omega : 68.326, \widehat{\mathsf{f}}(\omega) : 202.40$

Dominant freq for **v**

$\omega : 66.428, \widehat{\mathsf{f}}(\omega) : 100.81$

$\omega : 133.806, \widehat{\mathsf{f}}(\omega) : 73.76$

$\omega : 67.377, \widehat{\mathsf{f}}(\omega) : 68.54$

$\omega : 668.080, \widehat{\mathsf{f}}(\omega) : 62.39$

$\omega : 801.886, \widehat{\mathsf{f}}(\omega) : 56.22$

$\mathsf{x} = \mathsf{w} + 25\delta^*$

Dominant freq for **u**

$\omega : 66.856, \widehat{\mathsf{f}}(\omega) : 578.61$

$\omega : 133.712, \widehat{\mathsf{f}}(\omega) : 390.67$

$\omega : 200.567, \widehat{\mathsf{f}}(\omega) : 239.15$

$\omega : 401.135, \widehat{\mathsf{f}}(\omega) : 152.63$

$\omega : 267.423, \widehat{\mathsf{f}}(\omega) : 147.29$

Dominant freq for **v**

$\omega : 401.135, \widehat{\mathsf{f}}(\omega) : 94.62$

$\omega : 467.990, \widehat{\mathsf{f}}(\omega) : 80.02$

$\omega : 534.846, \widehat{\mathsf{f}}(\omega) : 77.23$

$\omega : 133.712, \widehat{\mathsf{f}}(\omega) : 63.12$

$\omega : 334.279, \widehat{\mathsf{f}}(\omega) : 62.55$

$\mathsf{x} = \mathsf{w} + 75\delta^*$

Dominant freq for **u**

$\omega : 133.712, \widehat{\mathsf{f}}(\omega) : 413.21$

$\omega : 66.856, \widehat{\mathsf{f}}(\omega) : 364.18$

$\omega : 267.423, \widehat{\mathsf{f}}(\omega) : 351.61$

$\omega : 200.567, \widehat{\mathsf{f}}(\omega) : 284.69$

$\omega : 334.279, \widehat{\mathsf{f}}(\omega) : 154.04$

Dominant freq for **v**

$\omega : 267.423, \widehat{\mathsf{f}}(\omega) : 152.65$

$\omega : 133.712, \widehat{\mathsf{f}}(\omega) : 74.76$

$\omega : 334.279, \widehat{\mathsf{f}}(\omega) : 71.96$

$\omega : 200.567, \widehat{\mathsf{f}}(\omega) : 63.06$

$\omega : 467.990, \widehat{\mathsf{f}}(\omega) : 49.21$

$\mathsf{x} = \mathsf{w} + 175\delta^*$

Dominant freq for **u**

$\omega : 133.917, \widehat{\mathsf{f}}(\omega) : 279.11$

$\omega : 66.490, \widehat{\mathsf{f}}(\omega) : 195.30$

$\omega : 71.173, \widehat{\mathsf{f}}(\omega) : 158.22$

$\omega : 132.981, \widehat{\mathsf{f}}(\omega) : 149.71$

$\omega : 67.427, \widehat{\mathsf{f}}(\omega) : 148.11$

Dominant freq for **v**

$\omega : 133.917, \widehat{\mathsf{f}}(\omega) : 45.03$

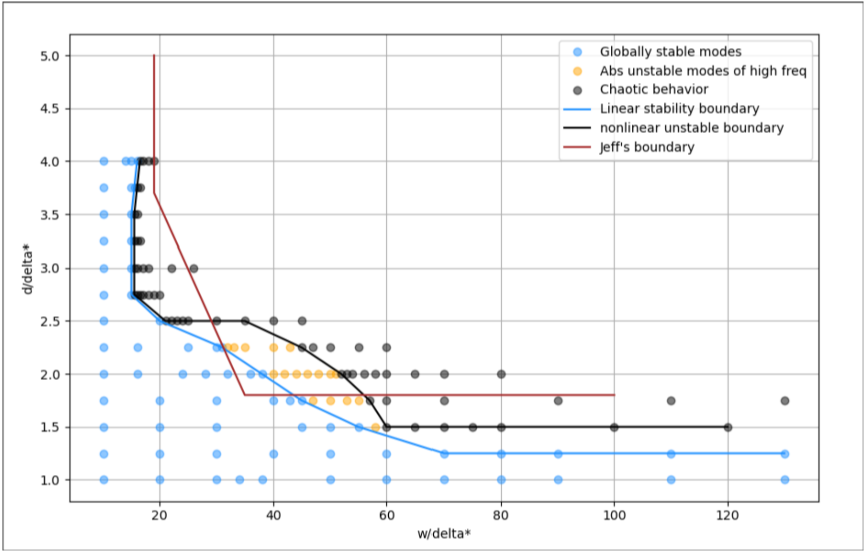
$\omega : 200.408, \widehat{\mathsf{f}}(\omega) : 42.09$

$\omega : 271.581, \widehat{\mathsf{f}}(\omega) : 32.03$

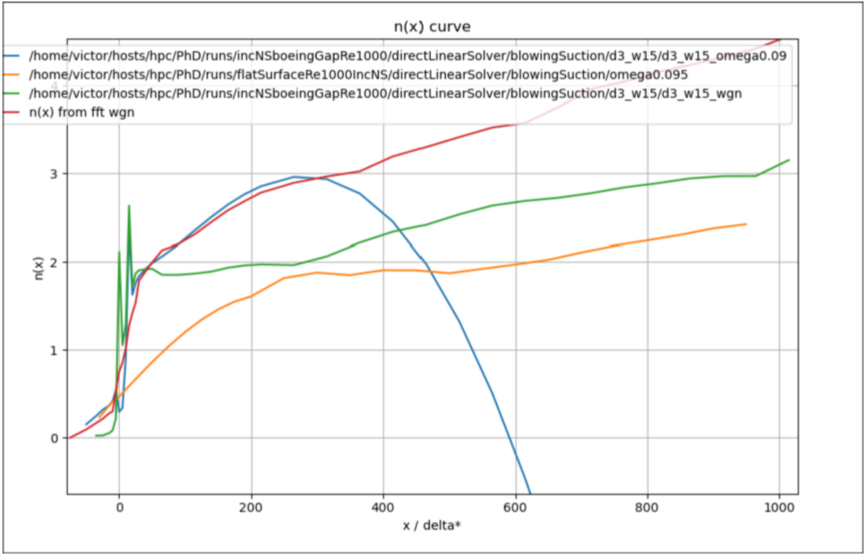
$\omega : 195.725, \widehat{\mathsf{f}}(\omega) : 27.78$

$\omega : 138.600, \widehat{\mathsf{f}}(\omega) : 26.89$

Stability curve



n-factor computations



n-factor computations