IMPERIAL

n factor

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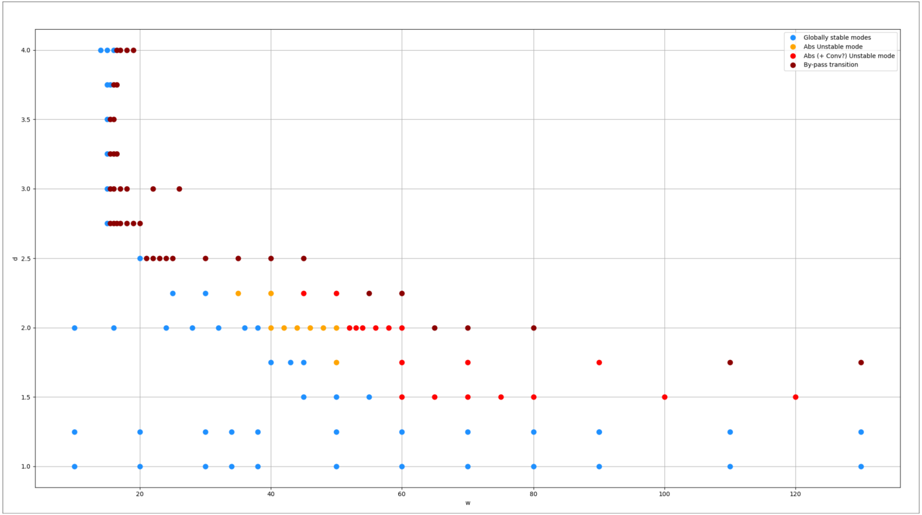
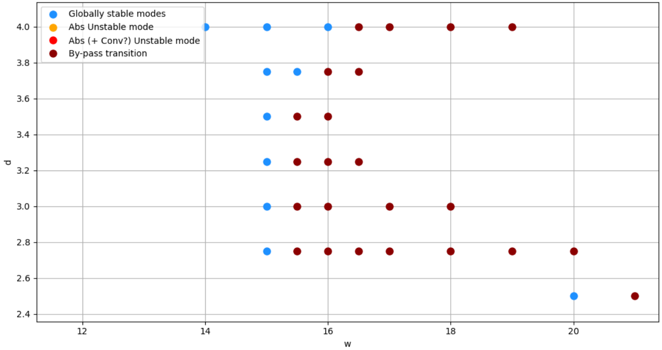
Recap


Globally stable modes
Abs Unstable mode
Abs (+ Conv?) Unstable mode
By-pass transition
d
w
4.0
3.5
3.0
2.5
2.0
1.5
1.0
20
40
60
80
100
120

Backward behaviour in w in the stability line



$\mathsf{n}(\mathsf{x})$ factor

- Blowing-suction upstream as:

$$\mathsf{v}(\mathsf{x},\mathsf{t}) = \mathsf{A}\sin(\alpha_\mathsf{r}(\mathsf{x}-\mathsf{x}_0))^3\sin(\omega_\mathsf{r}\mathsf{t})1_{\mathsf{x}_0\le\mathsf{x}\le\mathsf{x}_0+\ell}$$

where $\ell = \pi/\alpha_\mathsf{r}$, $\mathsf{A} = 0.003$, $\mathsf{x}_0 = -70\delta^*$, $\alpha_\mathsf{r} = 0.1428, \omega_\mathsf{r} = 0.04618$ (both from Orr-Sommerfeld eq).

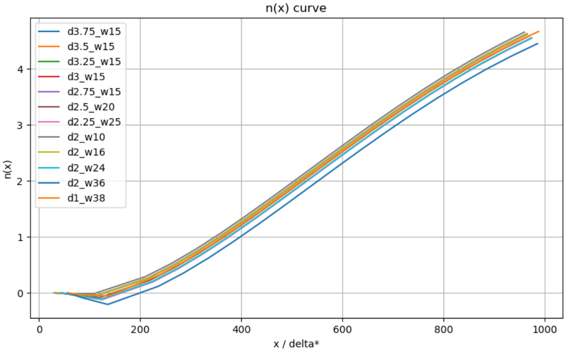
$$\mathsf{n}(\mathsf{x},\omega_\mathsf{r},\alpha_\mathsf{r}) = \log\left(\frac{\mathsf{A}^{\mathsf{TS}}(\mathsf{x},\omega_\mathsf{r},\alpha_\mathsf{r})}{\mathsf{A}_0^{\mathsf{TS}}(\omega_\mathsf{r},\alpha_\mathsf{r})}\right)$$

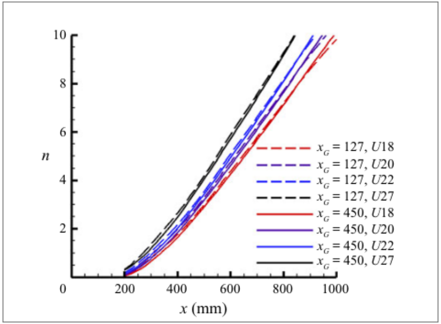
where $\mathsf{A}^{\mathsf{TS}}(\mathsf{x},\omega_\mathsf{r},\alpha_\mathsf{r})$ is the amplitude of the TS mode at x and $\mathsf{A}_0^{\mathsf{TS}}(\omega_\mathsf{r},\alpha_\mathsf{r})$ is the amplitude of the TS mode at $\mathsf{x} = \mathsf{x}_0$ (how to choose x_0?).

And then

$$\mathsf{N}(\mathsf{x}) = \max_{\omega,\alpha}\mathsf{n}(\mathsf{x},\omega,\alpha)$$

It would be nice to have our $\mathsf{n}(\mathsf{x},\omega_\mathsf{r},\alpha_\mathsf{r})$ as closest as possible to the $\mathsf{N}(\mathsf{x})$. Having observed the huge importance of the v component of the baseflow, do we need to try weakly non-parallel Orr-Sommerfeld?


n(x) curve
d3.75_w15
d3.5_w15
d3.25_w15
d3_w15
d2.75_w15
d2.5_w20
d2.25_w25
d2_w10
d2_w16
d2_w24
d2_w36
d1_w38
n(x)
x / delta*
0
1
2
3
4
0
200
400
600
800
1000


x_G = 127, U18
x_G = 127, U20
x_G = 127, U22
x_G = 127, U27
x_G = 450, U18
x_G = 450, U20
x_G = 450, U22
x_G = 450, U27
n
x (mm)
0
2
4
6
8
10
200
400
600
800
1000

jeff ΔN

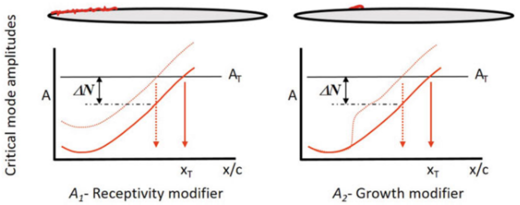


Fig. 5 Schematic showing linkage between a linear amplitude method and the variable N-factor approach

waves [8] and crossflow vortices [36, 37]. Using the envelope n of the physical-mode growth curves m, the amplitude threshold can be expressed as

$$n(x) = max_{\omega} max_{\beta} [m(x; \omega, \beta)] \geq N, \tag{2}$$

where $N \propto ln(A_T/A_0)$. The value of N can be linked to a reference condition with $N = N_{ref} - \Delta N$, where ΔN captures changes to A_0 or to $\Delta m_{crit}(x_T)$. Figure 5 illustrates the linkage between ΔN and the linear amplitude for both paths A_1 and A_2. The solid line in the figures corresponds to a critical mode causing transition for a given reference condition. The dashed line corresponds to a critical mode arising from enhanced receptivity or enhanced growth. In both cases, the forward movement of transition can be linked to a change in the transition N-factor ΔN.

The variable N-factor approach provides a method for a-priori predictions to account for varying surface-induced flow distortions. This method can also be used