Numerical propagation of errors of Earth-orbiting spacecraft

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Motivation

- Around 7000 satellites (active and inactive) orbiting the Earth.
- Various unintentional collisions in the past.
- Point Earth model is not enough. We need a more accurate model.

Goal:

- Develop a geopotential model of the Earth.
- Propagate the position of a satellite adding different perturbations.
- Estimate the error of the trajectory.

Equation for V

Let $\Omega \subseteq \mathbb{R}^3$ be the region that occupies the Earth.

$$\mathbf{g} = -\int\limits_{\Omega} G \frac{\mathbf{r}-\mathbf{s}}{\|\mathbf{r}-\mathbf{s}\|^3} \rho(\mathbf{s}) \mathrm{d}^3\mathbf{s} = \boldsymbol{\nabla} V$$

$$V = \int\limits_{\Omega} G \frac{\rho(\mathbf{s})}{\|\mathbf{r}-\mathbf{s}\|} \mathrm{d}^3\mathbf{s}$$

Theorem

V satisfies the following exterior boundary problem:

$$\begin{cases} \Delta V = 0 & \textit{in } \Omega^c \\ V = f & \textit{on } \partial\Omega \\ \lim\limits_{\|\mathbf{r}\| \to \infty} V = 0 \end{cases}$$

where $f : \partial\Omega \to \mathbb{R}$ is the gravitational potential on the surface of the Earth.

Separation of variables: $V = R(r)\Theta(\theta)\Phi(\phi)$

$$\begin{cases} \dfrac{(r^2R')'}{R} = n(n+1) \\ \dfrac{1}{\Theta}\Theta'' = -m^2 \\ \dfrac{\sin\phi}{\Phi}(\sin\phi\Phi')' + n(n+1)(\sin\phi)^2 = m^2 \end{cases} \qquad n, m \in \mathbb{N} \cup \{0\}, m \leq n$$

Imposing boundary conditions:

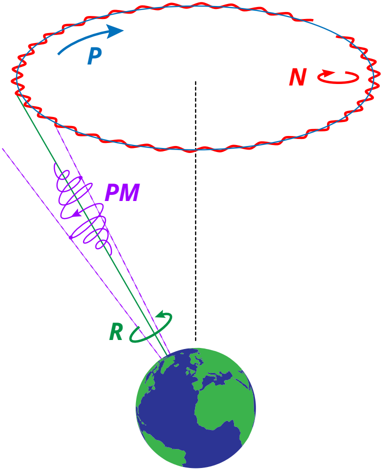
$$V = \frac{GM_\oplus}{R_\oplus} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left(\frac{R_\oplus}{r} \right)^{n+1} (\bar{C}_{n,m} Y^{\mathrm{c}}_{n,m}(\theta, \phi) + \bar{S}_{n,m} Y^{\mathrm{s}}_{n,m}(\theta, \phi))$$

where

$$\begin{aligned} Y^{\mathrm{c}}_{n,m}(\theta, \phi) &= N_{n,m} P_{n,m}(\cos\theta) \cos(m\phi) \\ Y^{\mathrm{s}}_{n,m}(\theta, \phi) &= N_{n,m} P_{n,m}(\cos\theta) \sin(m\phi) \end{aligned}$$

are the **spherical harmonics**.

Deviations of the Earth's rotation axis



- Precession
- Nutation
- Rotation
- Polar motion

Inertial and non-inertial reference frames

Quasi-inertial:

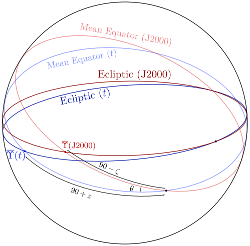
- x-axis: pointing towards the $\overline{\Upsilon}$ of the J2000 date
- z-axis: perpendicular to the equator of the J2000 date

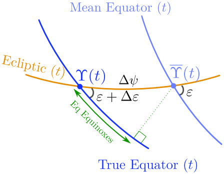
Non-inertial (Earth fixed):

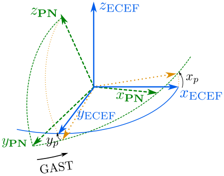
- x-axis: pointing towards the zero meridian
- z-axis: perpendicular to the equator

In both systems y-axis is chosen in order to complete a right-handed system.

Transformation between systems







Other perturbations

$\mathbf{r}$ = satellite position with respect to Earth's center of mass

- Third body perturbations (Moon and Sun):

$$\frac{GM}{\|\mathbf{s}-\mathbf{r}\|^2}(\mathbf{s}-\mathbf{r}) - \frac{GM}{\|\mathbf{s}\|^2}\mathbf{s}$$

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- Atmospheric drag:

$$-\frac{1}{2}C_{\mathrm{D}}\frac{A}{m}\rho v_{\mathrm{rel}}\mathbf{v}_{\mathrm{rel}}$$

where $\mathbf{v}_{\mathrm{rel}} = \dot{\mathbf{r}} - \boldsymbol{\omega}_{\oplus} \times \mathbf{r}$.

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- Solar radiation pressure:

$$-P_{\odot} C_{\mathrm{R}} \frac{A_{\odot}}{m} \frac{\mathbf{s}_{\odot} - \mathbf{r}}{\|\mathbf{s}_{\odot} - \mathbf{r}\|}$$

Differential system

Using the Runge-Kutta-Fehlberg method of order 7(8) we will integrate the differential system:

$$\begin{cases} \dot{\mathbf{r}} = \mathbf{v} \\ \dot{\mathbf{v}} = \mathbf{a}_{\mathrm{GP}} + \delta_{\mathrm{D}}\mathbf{a}_{\mathrm{D}} + \delta_{\mathrm{R}}\mathbf{a}_{\mathrm{R}} + \delta_{\mathrm{sun}}\mathbf{a}_{\mathrm{sun}} + \delta_{\mathrm{moon}}\mathbf{a}_{\mathrm{moon}} \end{cases}$$

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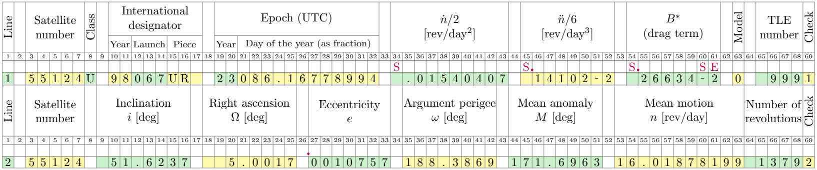
Initial conditions from **TLEs** (Two Line Elements sets).



Line		Satellite number					Class		International designator									Epoch (UTC)															$\dot{n}/2$ [rev/day^2]											$\ddot{n}/6$ [rev/day^3]									B^* (drag term)									Model		TLE number				Check
									Year		Launch			Piece				Year		Day of the year (as fraction)																																																
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69
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1		5	5	1	2	4	U		9	8	0	6	7	U	R			2	3	0	8	6	.	1	6	7	7	8	9	9	4			.	0	1	5	4	0	4	0	7			1	4	1	0	2	-	2			2	6	6	3	4	-	2		0			9	9	9	1

Line		Satellite number						Inclination i [deg]									Right ascension Ω [deg]									Eccentricity e								Argument perigee ω [deg]									Mean anomaly M [deg]									Mean motion n [rev/day]											Number of revolutions					Check
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69
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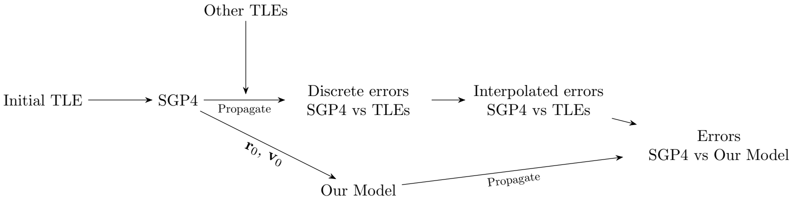
Results

Scheme of our simulation:



Other TLEs

Initial TLE

SGP4

Propagate

Discrete errors
SGP4 vs TLEs

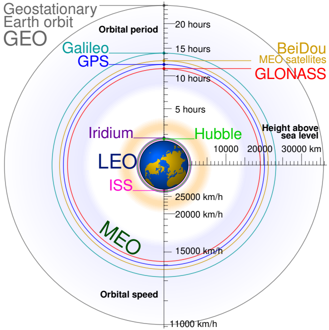
Interpolated errors
SGP4 vs TLEs

Errors
SGP4 vs Our Model

$\mathbf{r}_0,\ \mathbf{v}_0$

Our Model

Propagate

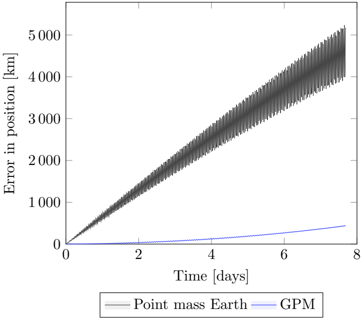


Zones that we will explore:

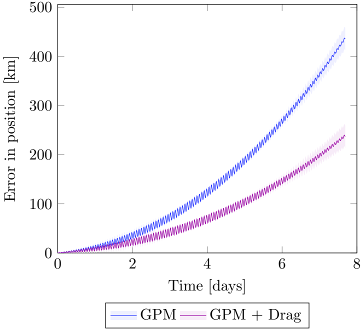
- **L**ow **E**arth **O**rbit satellites
- **M**edium **E**arth **O**rbit satellites
- **G**eostationary **E**arth **O**rbit satellites

Results - LEO

- ISS makes $\sim$ 16 orbits per day.
- LEO satellites interact with the atmosphere.
- The atmospheric drag is difficult to predict.

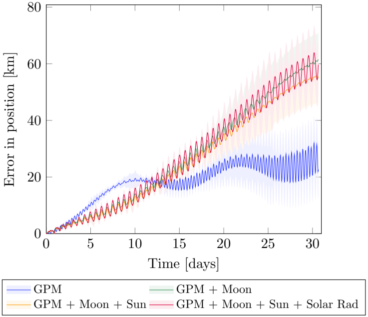


ISS

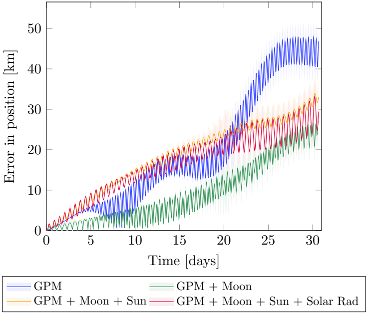


ISS

Results - MEO



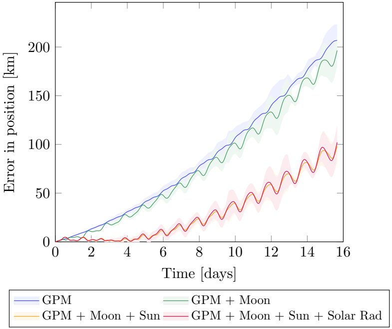
Sirius-3



Galileo-20

- GPM is very oscillatory.
- Sun and Moon reduce the oscillations.
- Solar radiation increases the oscillations.

Results - GEO



TDRS-3

- Adding the Sun and the Moon the errors are reduced.
- Solar radiation again increases the oscillations.
- Maneuver at around the 13th day.

Conclusions

- The point mass model is not enough to predict the orbit of a satellite.
- Adding both together the Sun and the Moon the variability of the errors is reduced.
- Radiation pressure increases the oscillations.

Improvements:

- Atmospheric drag and solar radiation bad modelled.
- Study the influence of the inclination and eccentricity on the errors.