

2D Turbulence spreading

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Navier-Stokes equations

Navier-Stokes equations describe the motion of fluids. For an *incompressible fluid* (i.e. volume-preserving), they are given by:

$$\begin{aligned}\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} &= -\frac{1}{\rho} \nabla p + \nu \Delta \mathbf{u} + \mathbf{f} \\ \nabla \cdot \mathbf{u} &= 0\end{aligned}$$

where $\mathbf{u}(\mathbf{x}, t)$ is the velocity field, $p(\mathbf{x}, t)$ is the pressure, $\rho(\mathbf{x}, t)$ is the density, ν is the kinematic viscosity, and $\mathbf{f}(\mathbf{x}, t)$ is an external force. These equations can be simplified by adimensionalizing the variables and they read:

$$\begin{aligned}\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} &= -\nabla p + \frac{1}{\text{Re}} \Delta \mathbf{u} + \mathbf{f} \\ \nabla \cdot \mathbf{u} &= 0\end{aligned}$$

where $\text{Re} = \frac{UL}{\nu}$ is the Reynolds number, U is a characteristic velocity and L is a characteristic length.

What is turbulence?

There is no universally accepted definition of turbulence, but people agree on:

- In a turbulent flow, the velocity field is chaotic (in the sense of dynamical systems).
- There is wide range of scales involved in the flow.

The onset of turbulence is usually characterized by the **Reynolds number** (Re), which is a dimensionless quantity that measures the ratio of inertial forces to viscous forces.

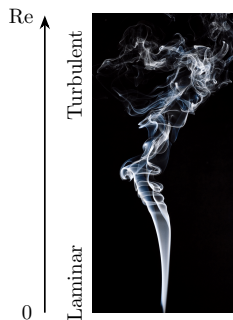


Figure: Smoke from a cigarette.

2D turbulence vs 3D turbulence

They are very different! In a nutshell:

- In 2D, the energy injected at intermediate scales transfers to large scales.
- In 3D, the energy injected at large scales transfers to small scales.



Figure: 2D turbulence

Our problem

We consider a periodic box in which we constantly inject energy randomly in the middle. We want to study how this energy spreads in the box.

- Will the energy spread to the whole box?
- If so, how long will it take? Which distribution (as a function of the distance to the center) will it follow?

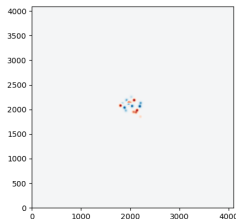


Figure: Vorticity ($\nabla \times \mathbf{u}$) forcing

State-of-the-art and advances

- In some 3D cases, the energy dissipates before reaching the boundaries of the box [Ale23].
- In 2D... seems to spread to the whole box!!

We would like to see that even decreasing the perturbation region, the vortices fill the whole box as $t \rightarrow \infty$ and $Re \rightarrow \infty$.



Figure: 2D turbulence

Post-motivation

The main goal is to understand how flows in the atmosphere work.

- In this case, the energy is injected by different local sources, such as storms.
- The atmosphere is a very thin 3D domain.

In these circumstances, will the energy spread to the whole atmosphere? That is, could a cyclone in Florida affect significantly the weather in Europe?

Bibliography

- [Ale23] Alexandros Alexakis. “How far does turbulence spread?”
In: *Journal of Fluid Mechanics* 977 (2023), R1. DOI: [↗](#).