Master Thesis

2D Turbulence spreading

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Abstract

This will be an abstract.

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prova[1] Alexakis 2023.

2 Dynamics of the problem

As said in the intruduction, the primary problem considered in this work is the integration of the incompressible Navier-Stokes equations with a random forcing term:

$$\partial_t \boldsymbol{u} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u} = -\boldsymbol{\nabla} p + \nu\Delta\boldsymbol{u} + \boldsymbol{f} \tag{1}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{u} = 0 \tag{2}$$

where $\boldsymbol{u}$ is the velocity field, p is the pressure, ν is the kinematic viscosity, and $\boldsymbol{f}$ is a random forcing term. The forcing term is chosen to be gaussian for the streamfunction formulation, which is detailed below.

2.1 Streamfunction formulation

In Batchelor 2000, a new variable is introduced in order to simplify the integration of the 2D incompressible Navier-Stokes equations. This quantity, called *streamfunction* and denoted by ψ, is defined as:

The forcing term is assumed to be Gaussian, in particular of the form:

3 Simulation

3.1 Introduction

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References

Alexakis, Alexandros (2023). "How far does turbulence spread?" In: *Journal of Fluid Mechanics* 977, R1. DOI: 10.1017/jfm.2023.951.

[1] This is a footnote.

Batchelor, G. K. (2000). *An Introduction to Fluid Dynamics*. Cambridge Mathematical Library. Cambridge University Press.