2D Turbulence spreading

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Navier-Stokes equations

Navier-Stokes equations describe the motion of fluids. For an *incompressible fluid* (i.e. volume-preserving), they are given by:

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \boldsymbol{\nabla})\mathbf{u} = -\frac{1}{\rho}\boldsymbol{\nabla} p + \nu \Delta \mathbf{u} + \mathbf{f}$$

$$\boldsymbol{\nabla} \cdot \mathbf{u} = 0$$

where $\mathbf{u}(\mathbf{x}, t)$ is the velocity field, $p(\mathbf{x}, t)$ is the pressure, $\rho(\mathbf{x}, t)$ is the density, ν is the kinematic viscosity, and $\mathbf{f}(\mathbf{x}, t)$ is an external force.

These equations can be simplified by adimensionalizing the variables:

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \boldsymbol{\nabla})\mathbf{u} = -\boldsymbol{\nabla} p + \frac{1}{\mathrm{Re}} \Delta \mathbf{u} + \mathbf{f}$$

$$\boldsymbol{\nabla} \cdot \mathbf{u} = 0$$

where $\mathrm{Re} = \frac{UL}{\nu}$ is the Reynolds number, U is a characteristic velocity and L is a characteristic length.

Figure: Model of an airplane in a wind tunnel

What is turbulence?

There is no universally accepted definition of turbulence, but people agree on:

- In a turbulent flow, the velocity field is chaotic (in the sense of dynamical systems).
- There is wide range of scales involved in the flow.

The onset of turbulence is usually characterized by the Reynolds number (Re), which measures the ratio of inertial forces to viscous forces.

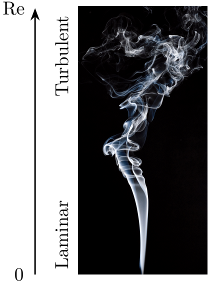

Figure: Smoke from a cigarette.

2D turbulence vs 3D turbulence

They are very different! In a nutshell:

- In 2D, the energy injected at intermediate scales transfers to large scales.
- In 3D, the energy injected at large scales transfers to small scales.

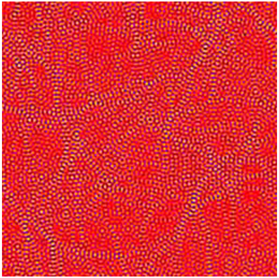

Figure: 2D turbulence without forcing

Our problem

We consider a periodic box in which we constantly inject energy randomly in the middle. We want to study how this energy spreads in the box.

- Will the energy spread to the whole box?
- If so, how long will it take? Which distribution (as a function of the distance to the center) will it follow?

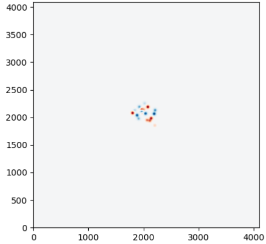

Figure: Vorticity ($\nabla \times \mathbf{u}$) forcing

State-of-the-art and progress

- In some 3D cases, the energy dissipates before reaching the boundaries of the box [Ale23].
- In 2D... seems to spread to the whole box!!

We would like to see that even decreasing the perturbation region, the vortices fill the whole box as $t \to \infty$ and $\mathrm{Re} \to \infty$.

Figure: 2D turbulence

Post-motivation

The main goal is to understand how flows in the atmosphere work.

- In this case, the energy is injected by different local sources, such as storms.
- The atmosphere is a very thin 3D domain.

In these circumstances, will the energy spread to the whole atmosphere? In other words, could a cyclone in Florida affect significantly the weather in Europe?

Bibliography

[Ale23] Alexandros Alexakis. "How far does turbulence spread?" In: *Journal of Fluid Mechanics* 977 (2023), R1. DOI: 10.1017/jfm.2023.951.