Octave problems

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Integració numèrica d'equacions en derivades parcials
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Febrer de 2023

Exercice 1. *For values of $x \in [-1, 3]$ and $t \in [0, 2.4]$, compare the following numerical schemes for the one-dimensional wave equation*

$$u_t + u_x = 0$$

with initial condition

$$u(0, x) = \begin{cases} \cos(\pi x)^2 & |x| \leq \frac{1}{2} \\ 0 & \textit{en cas otherwise} \end{cases}$$

and boundary condition $u(t, -1) = 0$. Use spatial time steps of $h = 1/10$, $h = 1/20$, and $h = 1/40$.

a. *Forward-time, backward-space (FTBS) with $\lambda = 0.8$.*

b. *Forward-time, centered-space (FTCS) with $\lambda = 0.8$.*

c. *Lax-Friedrichs (LF) with $\lambda = 0.8$ and $\lambda = 1.6$.*

d. *Leapfrog (L) with $\lambda = 0.8$ and using the forward-time, centered-space scheme for the first step.*

For schemes b, c, and d, use the numerical boundary condition $v_M^{n+1} = v_{M-1}^{n+1}$.

Resolution. In the next table we expose the error in L^∞ norm of all the experiments that we have done. From

Scheme	$h = 1/10$	$h = 1/20$	$h = 1/40$
FTBS ($\lambda = 0.8$)	0.309408	0.188814	0.105457
FTCS ($\lambda = 0.8$)	30.065494	4532.489959	2117202272.413460
LF ($\lambda = 0.8$)	0.475755	0.331514	0.206424
LF ($\lambda = 1.6$)	37.449901	3672.928504	771466658.304601
L ($\lambda = 0.8$)	0.179728	0.076990	0.055387

Taula 1: Error in L^∞ norm for the different schemes.

here we can conclude that the useful schemes are the FTBS with $\lambda = 0.8$, the Lax-Friedrichs with $\lambda = 0.8$ and the Leapfrog with $\lambda = 0.8$. And the other ones are useless as the error seams to approach to infinity as we decrease the step size h. The next figures shows the solutions of the three convergent methods with the three spatial steps mentioned above.

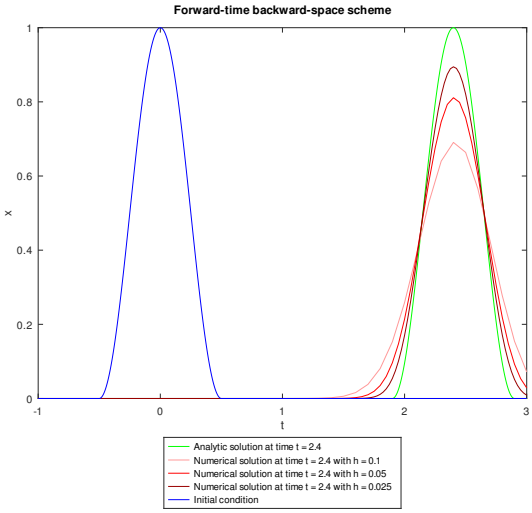
Exercice 2. *Solve $u_t + u_x = 0$, $x \in [-1, 1]$, $t \in [0, 2.1]$, with initial condition $u(0, x) = \sin(2\pi x)$ and periodicity $u(t, -1) = u(t, 1)$. Use the following schemes:*

a. *Forward-time, backward-space (FTBS) with $\lambda = 0.8$.*

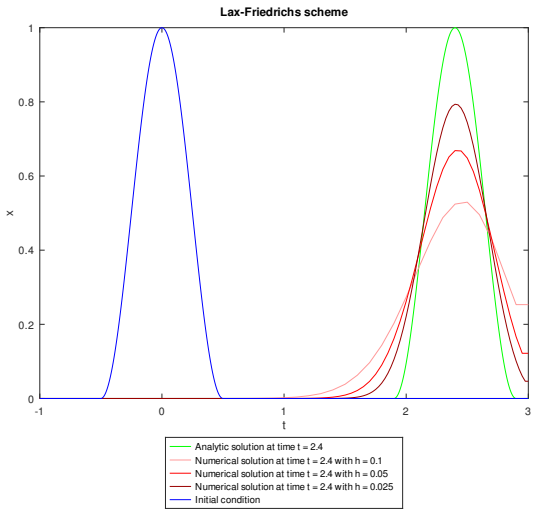
b. *Lax-Wendroff (LW) with $\lambda = 0.8$.*

Demonstrate the first-order accuracy of the FTBS scheme and the second-order accuracy of the LW scheme using time steps of $h = 1/10$, $h = 1/20$, $h = 1/40$ and $h = 1/80$. Do it in both the norm L^2 and the norm L^∞.

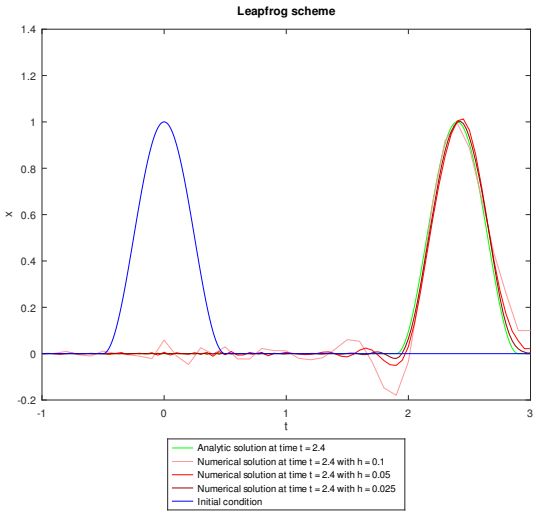
Resolution. The next table summarizes all the experiments that we have done.



(a) FTBS with $\lambda = 0.8$.



(b) Lax-Friedrichs with $\lambda = 0.8$.

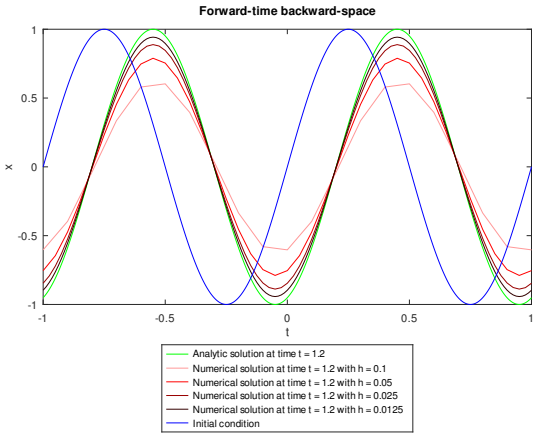


(c) Leapfrog with $\lambda = 0.8$.

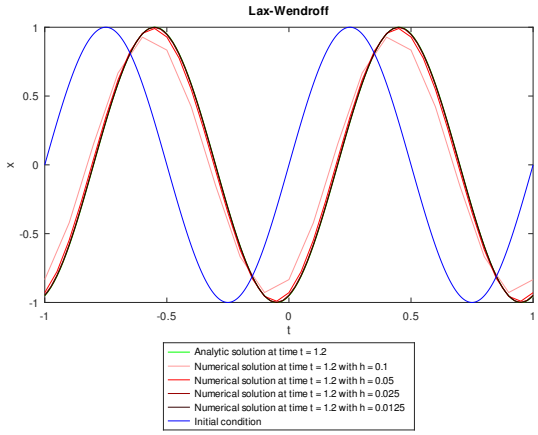
Figura 1: Plot of the analytical and numerical solutions of the convergent schemes

	FTBS				LW			
	L^2		L^∞		L^2		L^∞	
h	Error	Rate	Error	Rate	Error	Rate	Error	Rate
1/10	0.379872	-	0.371158	-	0.169042	-	0.166403	-
1/20	0.211259	1.7981	0.210930	1.7596	0.044196	3.8248	0.043954	3.7858
1/40	0.111737	1.8907	0.111688	1.8886	0.011139	3.9677	0.011121	3.9523
1/80	0.057504	1.9431	0.057498	1.9425	0.002789	3.9939	0.002788	3.9889

Taula 2: Error in L^∞ norm for the different schemes.



(a) FTBS with $\lambda = 0.8$.



(b) Lax-Friedrichs with $\lambda = 0.8$.

Figura 2: Plot of the analytical and numerical solutions of the convergent schemes