

Octave problems

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Integració numèrica d'equacions en derivades parcials
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Exercice 1. For values of $x \in [-1, 3]$ and $t \in [0, 2.4]$, compare the following numerical schemes for the one-dimensional wave equation

$$u_t + u_x = 0$$

with initial condition

$$u(0, x) = \begin{cases} \cos(\pi x)^2 & |x| \leq \frac{1}{2} \\ 0 & \text{en cas otherwise} \end{cases}$$

and boundary condition $u(t, -1) = 0$. Use spatial time steps of $h = 1/10$, $h = 1/20$, and $h = 1/40$.

- Forward-time, backward-space (FTBS) with $\lambda = 0.8$.
- Forward-time, centered-space (FTCS) with $\lambda = 0.8$.
- Lax-Friedrichs (LF) with $\lambda = 0.8$ and $\lambda = 1.6$.
- Leapfrog (L) with $\lambda = 0.8$ and using the forward-time, centered-space scheme for the first step.

For schemes **b**, **c**, and **d**, use the numerical boundary condition $v_M^{n+1} = v_{M-1}^{n+1}$.

Resolution. In the next table we expose the error in L^∞ norm of all the experiments that we have done. From

Scheme	$h = 1/10$	$h = 1/20$	$h = 1/40$
FTBS ($\lambda = 0.8$)	0.309408	0.188814	0.105457
FTCS ($\lambda = 0.8$)	30.065494	4532.489959	2117202272.413460
LF ($\lambda = 0.8$)	0.475755	0.331514	0.206424
LF ($\lambda = 1.6$)	37.449901	3672.928504	771466658.304601
L ($\lambda = 0.8$)	0.179728	0.076990	0.055387

Taula 1: Error in L^∞ norm for the different schemes.

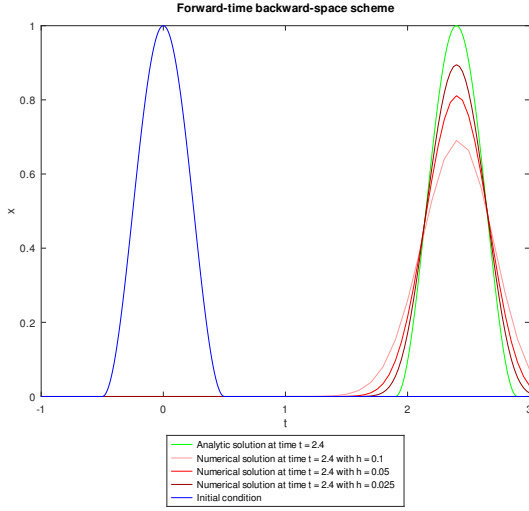
here we can conclude that the useful schemes are the FTBS with $\lambda = 0.8$, the Lax-Friedrichs with $\lambda = 0.8$ and the Leapfrog with $\lambda = 0.8$. And the other ones are useless as the error seems to approach to infinity as we decrease the step size h . The next figures shows the solutions of the three convergent methods with the three spatial steps mentioned above.

Exercice 2. Solve $u_t + u_x = 0$, $x \in [-1, 1]$, $t \in [0, 2.1]$, with initial condition $u(0, x) = \sin(2\pi x)$ and periodicity $u(t, -1) = u(t, 1)$. Use the following schemes:

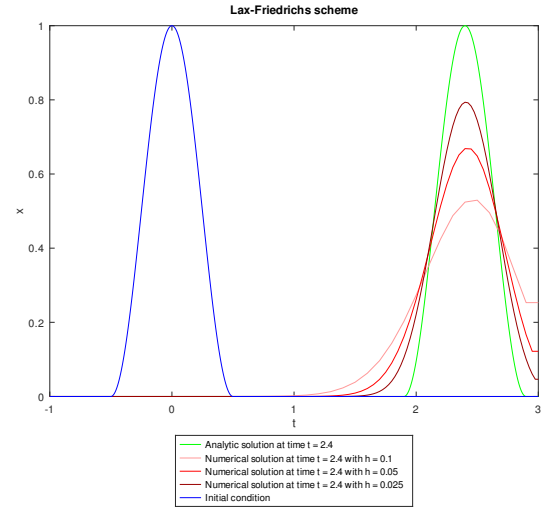
- Forward-time, backward-space (FTBS) with $\lambda = 0.8$.
- Lax-Wendroff (LW) with $\lambda = 0.8$.

Demonstrate the first-order accuracy of the FTBS scheme and the second-order accuracy of the LW scheme using time steps of $h = 1/10$, $h = 1/20$, $h = 1/40$ and $h = 1/80$. Do it in both the norm L^2 and the norm L^∞ .

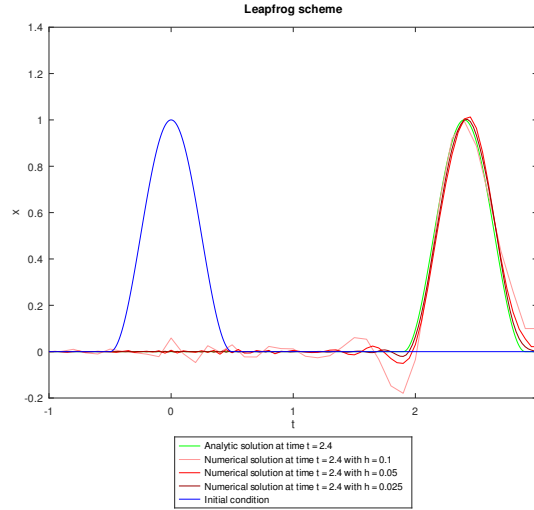
Resolution. The next table summarizes all the experiments that we have done.



(a) FTBS with $\lambda = 0.8$.



(b) Lax-Friedrichs with $\lambda = 0.8$.

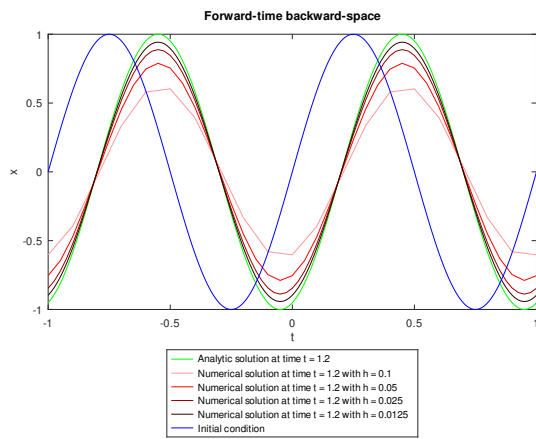


(c) Leapfrog with $\lambda = 0.8$.

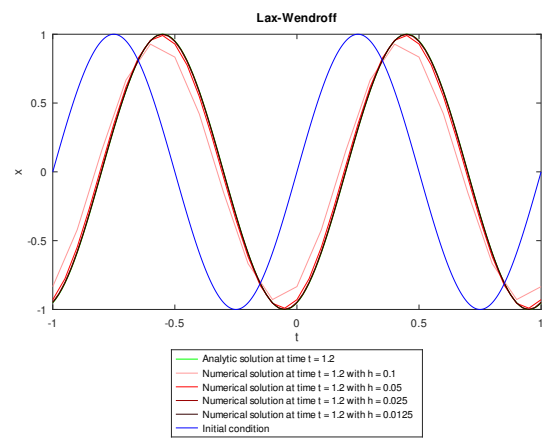
Figure 1: Plot of the analytical and numerical solutions of the convergent schemes

h	FTBS				LW			
	L^2		L^∞		L^2		L^∞	
	Error	Rate	Error	Rate	Error	Rate	Error	Rate
1/10	0.379872	-	0.371158	-	0.169042	-	0.166403	-
1/20	0.211259	1.7981	0.210930	1.7596	0.044196	3.8248	0.043954	3.7858
1/40	0.111737	1.8907	0.111688	1.8886	0.011139	3.9677	0.011121	3.9523
1/80	0.057504	1.9431	0.057498	1.9425	0.002789	3.9939	0.002788	3.9889

Taulla 2: Error in L^∞ norm for the different schemes.



(a) FTBS with $\lambda = 0.8$.



(b) Lax-Wendroff with $\lambda = 0.8$.

Figure 2: Plot of the analytical and numerical solutions of the convergent schemes