Numerical study of hydrodynamic stability of Blasius boundary layer flow using Spectral method

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***Abstract*— Our purpose in this project is to obtain the critical wave number and the critical Reynolds number that cause the instability of Blasius Boundary Layer flow. Therefore, in the first step Blasius boundary layer equation was solved numerically by shooting method and the obtained values were validated. Then in order to study the instability ,a small, two-dimensional perturbation was applied to the boundary layer and then by applying the normal mode analysis method, Orr-Sommerfeld equation was obtained and rewritten dimensionless . The discretization of this equation was performed by spectral method and then it was solved numerically. Finally, the overall result of the project was presented as a neutral Instability curve and the critical Reynolds number and the critical wave number that cause this instability were obtained. These values were also validated after obtaining.**

Keywords—Stability, Instability, Blasius equation, Shooting method, spectral method, Critical reynolds number,Critical wave number.

1. INTRODUCTION

Like any fuid-flow situation, boundary layers may become unstable. Usually, instabilities in boundary layers manifest themselves in turbulence.That is, a laminar boundary layer that becomes unstable usually becomes a turbulent boundary layer.The properties of laminar and turbulent boundary layers are quite different. For example, the angle to the location of the separation point on a circular cylinder, measured from the front stagnation point, is about 82 for a laminar boundary layer and about 108 for a turbulent boundary layer. This signi¢cant change in location of the separation points results in an appreciable drop in the drag coefficient. It is therefore of some interest to investigate the stability of boundary layers. The basis of our stability calculation will be to introduce a small disturbance into the boundary-layer variables and determine whether this disturbance grows or decays with time. If the disturbance grows with time, the boundary layer will be classi¢ed as unstable, and if the disturbance decays with time, the boundary layer will be classified as stable [1].

In this project, the stability of the Blasius flat-plate boundary layer flow is studied, using expansion in Chebyshev polynomials to approximate the solutions of the Orr-Sommerfeld equation. Chebyshev approximations require considerably less computer time and storage to achieve reasonably accurate flow simulations than that required by finite difference approximations. Also, Chebyshev approximations permit simulations of very high accuracy with little extra computation [2]. In 1908, Blasius solved the equation by using a series expansions method. In 1914, Toepfer solved the Blasius equation numerically using Runge-Kutta method. In 1938, Howarth solved the Blasius equation more accurately by numerical methods and tabulated the result [3].

2. BOUNDARY LAYER THEORY

A very satisfactory explanation of the physical process in the boundary layer between a fluid and a solid body could be obtained by the hypothesis of an adhesion of the fluid to the walls, that is, by the hypothesis of a zero relative velocity between fluid and wall. If the viscosity was very small and the fluid path along the wall not too long, the fluid velocity ought to resume its normal value at a very short distance from the wall. In the thin transition layer however, the sharp changes of velocity, even with small coefficient of friction, produce marked results.(Ludwig Prandtl, 1904) [4].

The above quotation is taken from an historic paper given by Ludwig Prandtl at the third Congress of Mathematicians at Heidelberg, Germany, in 1904. In this paper, the concept of the boundary layer was first introduced—a concept which eventually revolutionized the analysis of viscous flows in the twentieth century and which allowed the practical calculation of drag and flow separation over aerodynamic bodies. Before Prandtl’s 1904 paper, the Navier-Stokes were well known, but fluid dynamicists were frustrated in their attempts to solve these equations for practical engineering problems. After 1904, the picture changed completely. Using Prandtl’s concept of a boundary layer adjacent to an aerodynamic surface, the Navier-Stokes equations can be reduced to a more tractable form called the boundary-layer equations. In turn, these boundary-layer equations can be solved to obtain the distributions of shear stress and aerodynamic heat transfer to the surface. Prandtl’s boundary-layer concept was an advancement in the science of fluid mechanics of the caliber of a Nobel prize, although he never received that honor. The purpose of this chapter is to present the general concept of the boundary layer.

What is a boundary layer? The boundary layer is the thin region of flow adjacent to a surface, where the flow is retarded by the influence of friction between a solid surface and the fluid.

Note how thin the boundary layer is in comparison with the size of the body; however, although the boundary layer occupies geometrically only a small portion of the flow field, its influence on the drag and heat transfer to the body is immense—in Prandtl's own words as quoted above, it produces "marked results.

Consider the viscous flow over a flat plate as sketched in Figure 1. The viscous effects are contained within a thin layer adjacent to the surface; the thickness is exaggerated in Figure 1 for clarity. Immediately at the surface, the flow velocity is zero; this is the "no-slip" condition. In addition, the temperature of the fluid immediately at the surface is equal to the temperature of the surface; this is called the wall temperature Tw, as shown in Figure 1. Above the surface, the flow velocity increases in the y direction until, for all practical purposes, it equals the freestream velocity. This will occur at a height above the wall equal to δ, as shown in Figure 1. More precisely, δ is defined as that distance above the wall where $u = 0.99u_e$; here, ue is the velocity at the outer edge of the boundary layer. In Figure 1, which illustrates the flow over a flat plate, the velocity at the edge of the boundary layer will be V∞; that is, u_e= V∞. For a body of general shape, ue is the velocity obtained from an inviscid flow solution evaluated at the body surface. The quantity δ is called the velocity boundary-layer thickness. At any given x station, the variation of u between y = 0 and y = δ, that is, u = u(y), is defined as the velocity profile within the boundary layer, as sketched in Figure 1. This profile is different for different x stations. Similarly, the flow temperature will change above the wall, ranging from $T = T_W$ at y = 0 to $T = 0.99T_e$ at y = δT . Here, δT is defined as the thermal boundary-layer thickness. At any given x station, the variation of T between y = 0 and y = δT , that is, T = T (y), is called the temperature profile within the boundary layer, as sketched in Figure 1. Hence, two boundary layers can be defined: a velocity boundary layer with thickness δ and a temperature boundary layer with thickness δT[4].

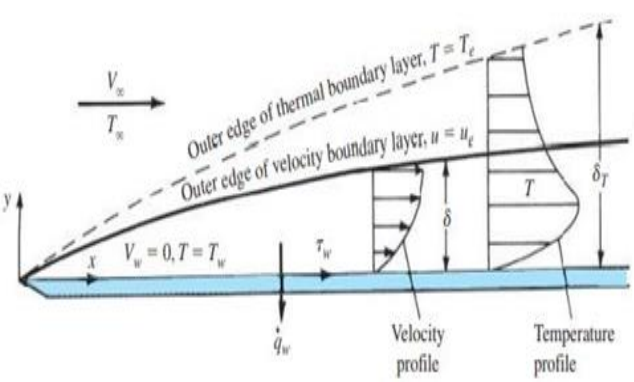

Figure 1. Boundary layer properties[4]

2.1. Prandtl Boundary Layer Equations

Solving boundary layer equations and obtaining the corresponding parameters have always been under great attention. Various experimental, analytical, and numerical methods have been used to calculate these parameters by different people. In this section, boundary layer equations are calculated, first and the governing assumptions are discussed in detail. Then, the numerical solution of the Blasius boundary layer flow equation which is the equation to be discussed in this project will be presented.

Continuity and Navier Stokes equation are proposed for the beginning of the boundary layer theory. In this theory, the fluid is assumed to be stable and incompressible. In this theory, it is also assumed that the flow is two-dimensional in the (x,y) plane and the last assumption corresponds to the negligence of gravity's effect. This is because gravity has a minor influence on the flow inside the boundary layer. These assumptions will change the continuity and Navier Stokes equations [5].

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

Before continuing, we will discuss another important assumption. In this assumption, the thickness of the boundary layer is thin in comparison with the body length. According to figure 2, we have:

$$\frac{\delta}{c} \ll 1$$

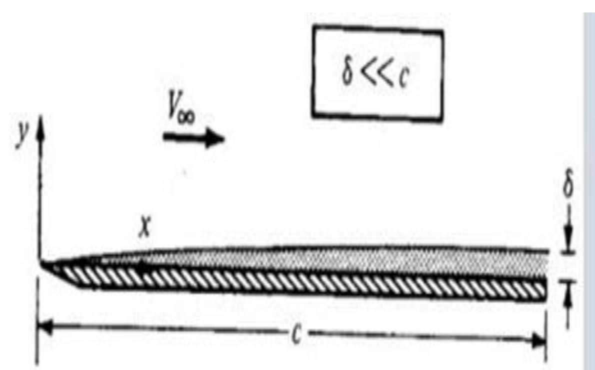

Figure 2. schematic representation of a flat plate with a thin boundary layer [5]

By applying this assumption which is one of the most important assumptions in boundary layer equations extraction, equations of motion and continuity could be estimated using magnitude order method as well as simplification of Navier Stokes equations and elimination of a few terms as below [5]:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \qquad (4)$$

$$\frac{\partial p}{\partial y}\approx 0 \qquad (5)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\approx -\frac{1}{\rho}\frac{dp}{dx}+\nu\left(\frac{\partial^2 u}{\partial y^2}\right) \qquad (6)$$

In this equation, pressure is not among the unknowns, and due to the thinness of the boundary layer, the inside boundary layer pressure could be estimated to the outside of the boundary layer pressure by writing the Bernoulli equation between infinity and outside the boundary layer flow [5].

$$U(x)\frac{dU(x)}{dx}=-\frac{1}{\rho}\frac{dp}{dx} \qquad (7)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\nu\left(\frac{\partial^2 u}{\partial y^2}\right) \qquad (8)$$

2.2. Blasius Flow

The first problem which was solved by boundary layer estimation was the flow over a static flat plate. Pressure gradient in this special geometry is zero.

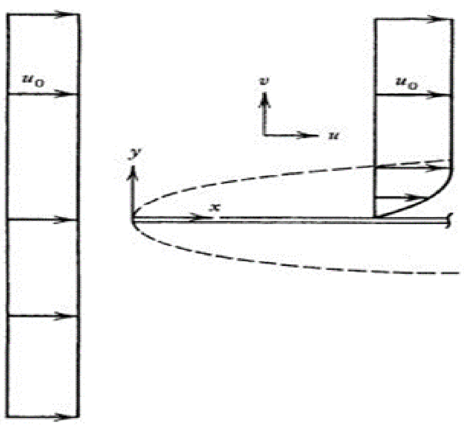

Figure 3. A flat plate exposed in a uniform flow[6]

The boundary conditions are as below:

$$@y=0\,,\quad u=v=0$$
$$@\,y\to\infty,\qquad u\to U_\infty$$

Blasius reckoned that the velocity profiles are the same:

$$\frac{u(x,y)}{U}=g\left(\frac{y}{\delta(x)}\right)=f'(\eta)\,, u=Uf' \qquad (9)$$

Also, from the continuity equation we have:

$$v(x,y)=\sqrt{\frac{\nu x}{4x}}(\eta f'-f) \qquad (10)$$

In case of $\delta(x)$ Blasius used the idea that inertia and viscosity terms are of the same order:

$$\delta(x)\sim\sqrt{\frac{\nu x}{U_\infty}} \qquad (11)$$

$\frac{y}{\zeta}$ could be used rather than $\frac{y}{\delta}$ as the similarity variable. By doing this, the similarity of velocity profiles is maintained.

$$\zeta=\sqrt{\frac{\nu x}{U_\infty}}\Rightarrow\delta=k\zeta \qquad (12)$$

After substituting (9) and (10) equations into (8), and supposing the pressure gradient equal to zero$\frac{dU}{dx}=0$, the following ordinary differential equation is resulted [6]:

$$2f'''+ff'=0 \qquad (13)$$
$$BC: f(0)=f'(0)=0\;,\qquad f'(\infty)=1$$

This equation will be solved numerically in the result section.

3. INSTABILITY THEORY

3.1. Instability of parallel or almost parallel flows

Instability of parallel or almost parallel flows constitute a large group of instability problems. In this case, we can refer to the following flows as an example: Instability of plate Couette flow, Instability of plate Poiseuille flow, Instability of the Poiseuille flow in the tube, Kelvin-Helmholtz flow instability (shear layer), Blasius flow instability (almost parallel flow) and Jet and trail instability. The analysis of these instability problems is performed using the Orr-Sommerfeld equation.

3.2. Orr-Sommerfeld Equation for Blasius Flow

Assume that in a parallel and one-dimensional flow, the velocity and pressure profiles of the base solution are given, which are basically derived from the constant flow analysis: V(y) و $p_0(x)$. We apply a small two-dimensional time function perturbation to this basic solution as follows, and the two-dimensionality of this perturbation is important because, according to Squire, two-dimensional perturbations are more dangerous than three-dimensional perturbations, because in three-dimensional perturbations the third dimension damps some turbulence. Therefore, two-dimensional perturbation analysis can be more important.

$$u(x,y,t) = V(y) + u'(x,y,t)$$
$$v(x,y,t) = 0 + v'(x,y,t)$$
$$p(x,y,t) = p_0(x) + p'(x,y,t)$$

At first, we assume than perturbations are small which means:

$$\left|\frac{u'}{V}\right|, \left|\frac{v'}{V}\right|, \left|\frac{p'}{p_0}\right| \ll 1$$

Obviously, the turbulent velocity and pressure fields must also apply to the Navier-Stokes and continuity equations:

$$x:\ \frac{\partial u'}{\partial t} + (u'+V)\frac{\partial u'}{\partial x} + v'\left(\frac{\partial u'}{\partial y} + \frac{dV}{dy}\right) = -\frac{1}{\rho}\left(\frac{\partial p'}{\partial x} + \frac{dp_0}{dx}\right) + \nu\left(\frac{\partial^2 u'}{\partial x^2} + \frac{\partial^2 u'}{\partial y^2} + \frac{d^2 V}{dy^2}\right) \quad (14)$$

$$y:\ \frac{\partial v'}{\partial t} + (u'+V)\frac{\partial v'}{\partial x} + v'\frac{\partial v'}{\partial y} = -\frac{1}{\rho}\frac{\partial p'}{\partial y} + \nu\left(\frac{\partial^2 v'}{\partial x^2} + \frac{\partial^2 v'}{\partial y^2}\right) \quad (15)$$

Continuity: $\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial x} = 0 \quad (16)$

In the case of no perturbation, we have the momentum equation in the x direction: $-\frac{1}{\rho}\frac{dp_0}{dx} + \nu\frac{d^2V}{dy^2} = 0 \quad (17)$

And also, by neglecting the turbulent terms we will have:

$$x:\ \frac{\partial u'}{\partial t} + V\frac{\partial u'}{\partial x} + v'\frac{dV}{dy} = -\frac{1}{\rho}\frac{\partial p'}{\partial x} + \nu\left(\frac{\partial^2 u'}{\partial x^2} + \frac{\partial^2 u'}{\partial y^2}\right) \quad (18)$$

$$y:\ \frac{\partial v'}{\partial t} + V\frac{\partial v'}{\partial x} = -\frac{1}{\rho}\frac{\partial p'}{\partial y} + \nu\left(\frac{\partial^2 v'}{\partial x^2} + \frac{\partial^2 v'}{\partial y^2}\right) \quad (19)$$

Continuity: $\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial x} = 0 \quad (20)$

Also, due to the two-dimensionality of perturbation, the concept of flow function can be used for it:

$$u' = \frac{\partial \psi'}{\partial y} \quad , \quad v' = -\frac{\partial \psi'}{\partial x} \quad (21)$$

By placing these equations (21) in (18, 19, 20) and removing the pressure between the two equations, we arrive at a linear equation of order 4:

$$\left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial x}\right)\left(\frac{\partial^2 \psi'}{\partial y^2} + \frac{\partial^2 \psi'}{\partial x^2}\right) - \frac{d^2V}{dy^2}\frac{\partial \psi'}{\partial x} = \nu\left(\frac{\partial^4 \psi'}{\partial y^4} + 2\frac{\partial^4 \psi'}{\partial x^2 \partial y^2} + \frac{\partial^4 \psi'}{\partial x^4}\right) \quad (22)$$

If we solve this equation numerically, we get the perturbation flow function. By determining the perturbation flow function, we can reach the perturbation velocity field to see for what parameters the velocity profile makes the flow unstable. But in the following, we will deal with a simpler method called normal modes analysis.

We consider the perturbation flow function as a combination of infinitely normal modes (of the moving wave type). In other words, we use the Fourier integral to introduce the perturbation flow function:

$$\psi'(x,y,t) = \int_0^\infty \tilde{v}\, e^{i\alpha(x-ct)} d\alpha \quad (23)$$

$\tilde{v}$: Perturbation domain
c: wave velocity
$\alpha = \frac{2\pi}{\lambda}$: Wave number

By replacing this equation (23) in the equation (22), we will have:

$$\int_0^\infty [(-i\alpha c + i\alpha V)(\tilde{v}'' - \alpha^2 \tilde{v}) - i\alpha\, \tilde{v}\, V''] e^{i\alpha(x-ct)} d\alpha = \int_0^\infty [\nu(\tilde{v}'''' - 2\alpha^2 \tilde{v}'' + \alpha^4 \tilde{v})] e^{i\alpha(x-ct)} d\alpha \quad (24)$$

This equation must hold for every perturbation and every wavelength, so the values below the two integrals must be equal:

$$(V-c)(\tilde{v}'' - \alpha^2 \tilde{v}) - V''\tilde{v} = \frac{\nu}{i\alpha}(\tilde{v}'''' - 2\alpha^2 \tilde{v}'' + \alpha^4 \tilde{v}) \quad (25)$$

This equation (25) is called the Orr-Sommerfeld equation.
Boundary conditions are defined in such a way that the perturbation velocities at infinity and near the wall are equal to zero [1]:

$$u'(x,0,t) = v'(x,0,t) = 0$$
$$u'(x,0,t) = v'(x,0,t) \ \rightarrow\ 0 \qquad y \rightarrow \infty$$

Which will turn to the following equations:

$$\tilde{v}(0) = \tilde{v}'(0) = 0 \ ,\ \tilde{v}(0) = \tilde{v}'(0) \rightarrow 0 \ ,\ y \rightarrow \infty$$

3.3. Making the Orr-Sommerfeld Equation Dimensionless

The Orr-Sommerfeld equation can be dimensionless as follows:

$$\hat{v} = \frac{\tilde{v}}{LV_\infty} \ ,\ \hat{U}(y) = \frac{V(y)}{V_\infty}, \hat{c} = \frac{c}{V_\infty} \ ,\ \hat{x} = \frac{x}{L}, \hat{\alpha} = \alpha L \ ,\ Re = \frac{V_\infty L}{\nu}$$

In this case, after insertion, the Orr-Sommerfeld equation can be written in the following dimensionless form:

$$v'''' - 2\alpha^2 v'' + \alpha^4 v - i\alpha Re[(U-c)(v'' - \alpha^2 v) - U''v] \quad 26$$

The equation can be written as $F(Re,\alpha,c) = 0$. Given that α and c in general can be complex numbers, by equating the imaginary and real parts of this equation (26) to zero, we actually get two equations and five unknown variables, which are: $Re, c_r, c_i, \alpha_r, \alpha_i$. As a result, we have to choose three parameters from these five parameters so that we can solve the problem in the form of two equations and two unknowns. There are two ways for doing this:

1. Temporal instability analysis 2. Spatial instability analysis

Both analyzes give a critical answer to Reynolds. But because of the simplicity of this project, we use time analysis.

3.3.1. Temporal instability analysis

In this analysis, out of the five parameters of the imaginary part, we consider the wave number to be zero $\alpha_i = 0$. By the way, we assume Re and α as known, so we just need to get c_i , c_r . What is important here and in the occurrence of instability is the term c_i , which appears in the exponential expression as $e^{-i\alpha(c_r+ic_i)t}$ If c_i> 0, the flow becomes unstable, and if c_i<0, the perturbations gradually damp, and in the case of c_i= 0, we reach a state of neutral instability.

4. DISCRETIZATION OF OSE USING SPECTRAL METHOD

After extracting the OSE equation as mentioned in the previous section, we will now solve this equation numerically using spectral method. Now we will rewrite the OSE equation as follows:

$$\left(\frac{\mathrm{df}}{\mathrm{d\eta}} - \mathrm{c}\right)(\mathrm{D}^2 - \alpha^2)\mathrm{v} - \mathrm{v}\mathrm{D}^2\left(\frac{\mathrm{df}}{\mathrm{d\eta}}\right) = \frac{1}{\mathrm{i\alpha Re}}(\mathrm{D}^4 + \alpha^4 - 2\alpha^2\mathrm{D}^2)\mathrm{v} \quad (27)$$

$$\mathrm{D}^{(\mathrm{n})} = \frac{\mathrm{d}^{\mathrm{n}}}{\mathrm{d\eta}^{\mathrm{n}}} \quad (28)$$

BC:

$$\mathrm{v}|_{\eta=0} = \frac{\mathrm{dv}}{\mathrm{d\eta}}|_{\eta=0} = 0 \quad (29)$$

$$\lim_{\eta\to\infty} v = \lim_{\eta\to\infty}\frac{\mathrm{dv}}{\mathrm{d\eta}} = 0 \quad (30)$$

In this numerical method, we first write the main variable of Equation (v) in terms of a series. We use Chebyshev Polynomial Functions due to its high stability. Which is defined as follows:

$$v(\zeta) = \sum_{m=0}^{N+1} a_m T_m(\zeta) \quad (31)$$

$$T_m(\zeta) = cos\left(mcos^{-1}(\zeta)\right) \quad (32)$$

And as it turns out the scope of this function is:

$$-1 < \zeta < 1$$

So if we want to use this series, we have to take the problem variable that is in the η domain to the ζ domain. For this purpose, we use the following conversion function:

$$\zeta = \frac{2\eta - \eta_\infty}{\eta_\infty} \quad (33)$$

Also, according to the derivatives appearing in the equation, the following equation can be used to transfer them to the ζ domain:

$$\frac{\mathrm{d}^{\mathrm{k}}}{\mathrm{d\eta}^{\mathrm{k}}} = \left(\frac{2}{\eta_\infty}\right)^{\boldsymbol{k}} \frac{\mathrm{d}^{\mathrm{k}}}{\mathrm{d\zeta}^{\mathrm{k}}} \quad k = 0,1,2 \ldots \quad (34)$$

Suppose we divide the interval $0 < \eta < \eta_\infty$ into n points such that n = 1 represents the lower bound and n = N indicates the upper bound. Now considering that the main equation has two conditions at the lower bound and two conditions at the upper bound and $2 < n < N - 1$ represent interior points, for interior points it is enough to satisfy the differential equation in the ζ domain and for boundary points the boundary conditions in the ζ domain must be satisfied. So we have N + 2 equations. Now notice the other points called in the Chebyshev function. To call these points, we must obtain them, which are defined as follows [7]:

$$\zeta = cos\left(\frac{J-1}{N-1}\pi\right) \quad J = 1,2,3 \ldots N \quad (35)$$

BC:

$$\sum_{m=0}^{N+1} a_m T_m(\zeta)|_{\zeta=1} = 0 \quad (36)$$

$$\sum_{m=0}^{N+1} a_m \frac{dT_m}{d\zeta}|_{\zeta=1} = 0 \quad (37)$$

$$\sum_{m=0}^{N+1} a_m T_m(\zeta)|_{\zeta=-1} = 0 \quad (38)$$

$$\sum_{m=0}^{N+1} a_m \frac{dT_m}{d\zeta}|_{\zeta=-1} = 0 \quad (39)$$

Also according to the derivatives appearing for $T_m(\zeta)$ the following equations can be used:

$$\frac{dT_m}{d\xi} = \frac{m}{(1-\xi^2)^{\frac{1}{2}}} \sin mcos^{-1}(\xi) \quad (40)$$

$$\frac{d^2T_m}{d\xi^2} = \frac{m\xi}{(1-\xi^2)^{\frac{3}{2}}} \sin mcos^{-1}(\xi) - \frac{m^2}{1-\xi^2} mcos^{-1}(\xi) \quad (41)$$

$$\frac{d^3T_m}{d\xi^3} = m\left[\frac{(1-m^2)(1-\xi^2)+3\xi^2}{(1-\xi^2)^{\frac{5}{2}}}\right] sin\, mcos^{-1}(\xi) - 3m^2\left[\frac{\zeta}{(1-\xi^2)^2}\right] cosmcos^{-1}(\xi) \quad (42)$$

$$\frac{d^4T_m}{d\xi^4} = m\left[\frac{m(m^2(\xi^2-1)+11\xi^2+4)cosmcos^{-1}(\xi)}{(\xi^2-1)^3} + \frac{3\xi(2m^2(\xi^2-1)+2\xi^2+3)sinmcos^{-1}(\xi)}{(\xi^2-1)^{\frac{7}{2}}}\right] \quad (43)$$

Finally, after transferring the differential equation to the ζ domain and satisfying the boundary conditions, we arrive at the following equation: $AX = cBX \ \rightarrow (A - cB)X = 0$ (44) Which are c Eigen Values and X Eigen Functions. By obtaining Eigen Values (c) as described in the previous section, we will examine the instability of this flow.

5. RESULT AND DISCUSSION

5.1. Numerical result for 3rd order Blasius differential Equation

In this section, we will solve the 3rd order Blasius differential equation. It is solved using MATLAB and the Shooting method (the MATLAB code is attached to this file). The solution results are indicated below:

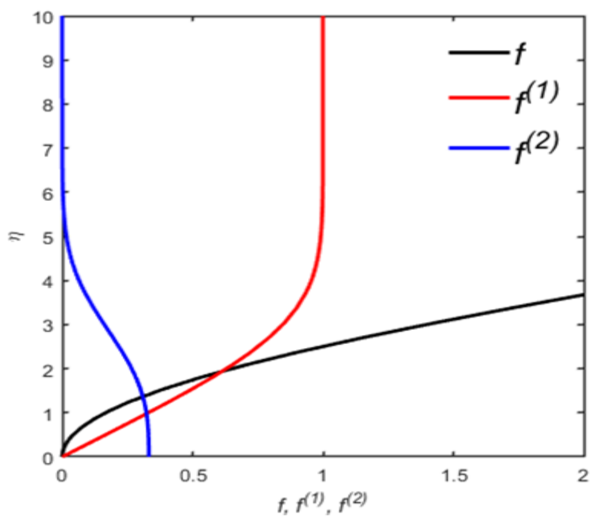

Figure 4. Numerical solution results of Blasius equation using Shooting method

Drawing the non-dimensional velocity profile (f `) with respect to the similarity variable of η indicates that in η=5, the difference between boundary layer velocity and free steam velocity is about 1 percent and it could be claimed that we are at the edge of the boundary layer so according to [1] our result is correct about this numerical solotion.

$$\eta = \frac{y}{\zeta} = \frac{\delta}{\zeta} = 5 \Rightarrow \delta(x) = 5\sqrt{\frac{\nu x}{U_\infty}} \quad (14)$$

Therefore, the proportion coefficient in Equation (12) is equal to 5.

5.2. Numerical result for Orr-Sommerfeld Equation

In the part of the results obtained from the numerical solution of the Orr-Sommerfeld equation, the critical Reynolds value and the critical wave number are extracted by plotting the neutral stability curve. The following are the results of the required curves in this project.

5.2.1. Neutral istability curves:

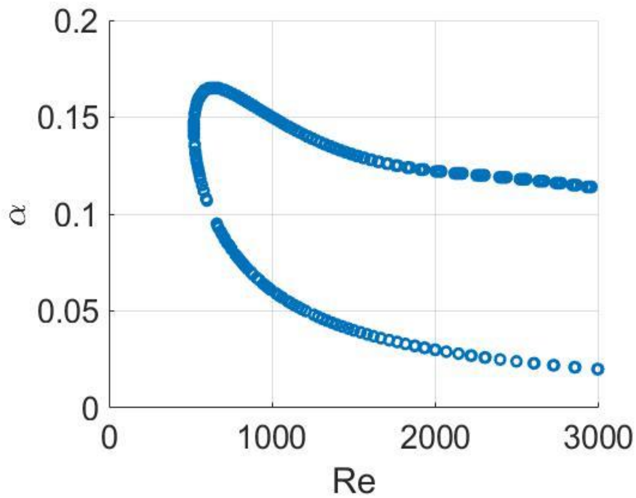

Figure 5. Neutral instability curves

Number of problem solving points, Reynolds interval and wave number:

$0.01 < \alpha < 0.5$: Step=0.001
$100 < Re < 3000$:Step=5
Number of nodes=N=20

5.2.2 critical Reynolds and the critical wave number:

As can be seen, the previous curve (Figure 5) predicts a critical Reynolds number of about 520 and a critical wave number of about 0.145, which is in good agreement with [2] and indicates an acceptable numerical solution in this project.

5.2.3. Range of eigen values at the critical point

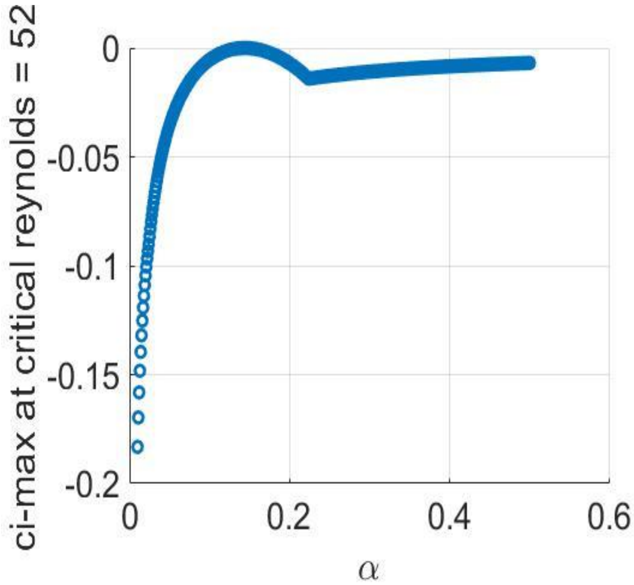

Figure 6. Neutral stability curves

As can be seen in the figure, all eigenvalues in this Reynolds are negative so according to the contents in the section 3.3.1 disturbances are damped.

5.2.4. Range of eigen values at Re=2200

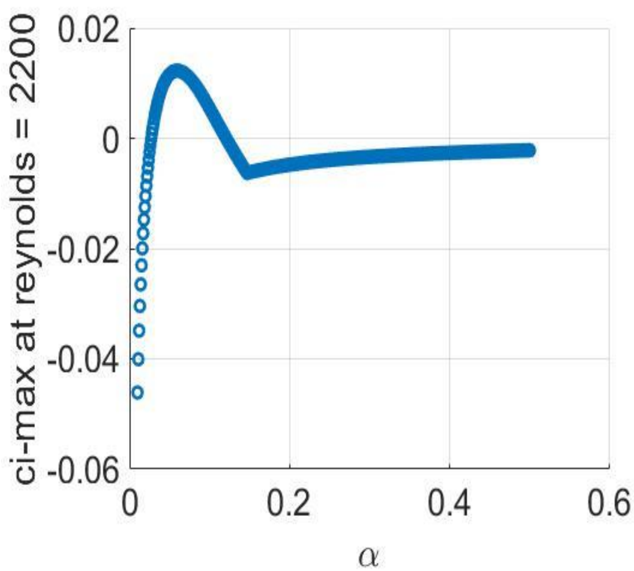

Figure 7. Range of eigen values at Re=2200

As shown in the figures, these curves have two roots, which actually means that in this Reynolds two wave numbers have zero eigenvalues, which represent the boundary between stability and instability.

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