LECTURE 16 – CURRENT MIRRORS AND SIMPLE REFERENCES LECTURE ORGANIZATION

Outline

- MOSFET current mirrors
- Improved current mirrors
- Voltage references with power supply independence
- Current references with power supply independence
- Temperature behavior of voltage and current references

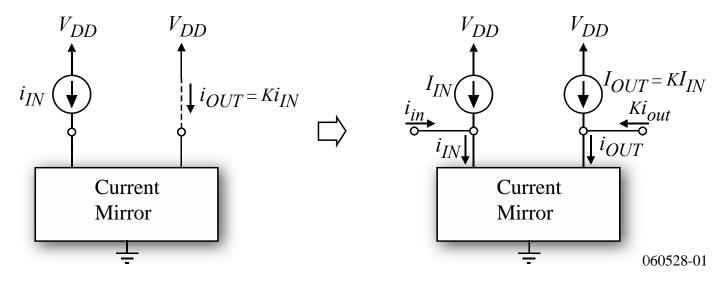
CMOS Analog Circuit Design, 3rd Edition Reference

Pages 138-156

MOSFET CURRENT MIRRORS

What is a Current Mirror?

A current mirror replicates the input current of a current sink or current source as an output current. The output current may be identical to the input current or can be a scaled version of it.



The above current mirrors are referenced with respect to ground. Current mirrors can also be referenced with respect to V_{DD} and can source input and output currents.

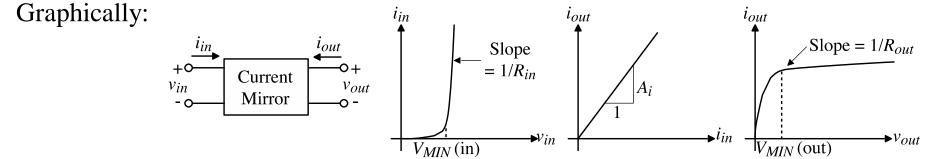
Characterization of Current Mirrors

A current mirror is basically nothing more than a current amplifier. The ideal characteristics of a current amplifier are:

- Output current linearly related to the input current, $i_{out} = A_i i_{in}$
- Input resistance is zero
- Output resistance is infinity

Also, the characteristic V_{MIN} applies not only to the output but also the input.

- V_{MIN} (in) is the range of v_{in} over which the input resistance is not small
- V_{MIN} (out) is the range of v_{out} over which the output resistance is not large



Input Characteristics Transfer Characteristics Output Characteristics Fig. 300-01

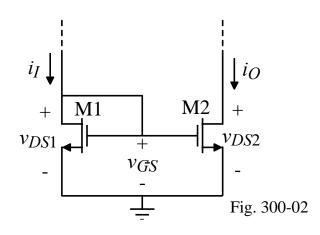
Therefore, R_{out} , R_{in} , V_{MIN} (out), V_{MIN} (in), and A_i will characterize the current mirror.

Simple MOS Current Mirror

Circuit:

Assume that $v_{DS2} > v_{GS} - V_{T2}$, then

$$\frac{i_O}{i_I} = \left(\frac{L_1 W_2}{W_1 L_2}\right) \left(\frac{V_{GS} - V_{T2}}{V_{GS} - V_{T1}}\right) 2 \left[\frac{1 + \lambda v_{DS2}}{1 + \lambda v_{DS1}} \left(\frac{K_2'}{K_1'}\right)\right]$$



If the transistors are matched, then $K_1' = K_2'$ and $V_{T1} = V_{T2}$ to give,

$$\frac{i_O}{i_I} = \left(\frac{L_1 W_2}{W_1 L_2}\right) \left(\frac{1 + \lambda v_{DS2}}{1 + \lambda v_{DS1}}\right)$$

If $v_{DS1} = v_{DS2}$, then

$$\frac{i_O}{i_I} = \left(\frac{L_1 W_2}{W_1 L_2}\right)$$

Therefore the sources of error are:

- 1.) $v_{DS1} \neq v_{DS2}$
- 2.) M1 and M2 are not matched.

Influence of the Channel Modulation Parameter, λ

If the transistors are matched and the W/L ratios are equal, then

$$\frac{i_O}{i_{\rm I}} = \frac{1 + \lambda v_{DS2}}{1 + \lambda v_{DS1}}$$

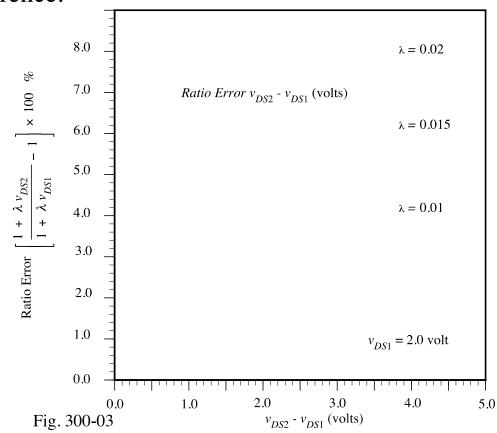
if the channel modulation parameter is the same for both transistors $(L_1 = L_2)$.

Ratio error (%) versus drain voltage difference:

Note that one could use this effect to measure λ .

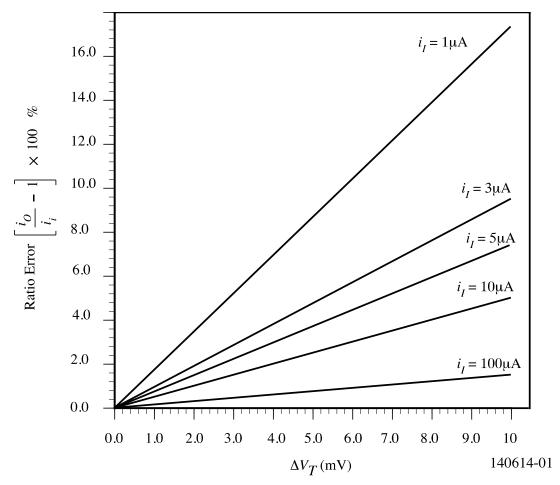
Measure V_{DS1} , V_{DS2} , i_I and i_O and solve the above equation for the channel modulation parameter, λ .

$$\lambda = \frac{\frac{i_O}{i_I} - 1}{v_{DS2} - \frac{i_O}{i_I} \lambda v_{DS1}}$$



<u>Illustration of the Offset Voltage Error Influence</u>

Assume that $V_{T1} = 0.7 \text{V}$ and $K'W/L = 110 \mu \text{A/V}^2$.



Key: Make the part of V_{GS} causing the current to flow, V_{ON} , more significant than V_T .

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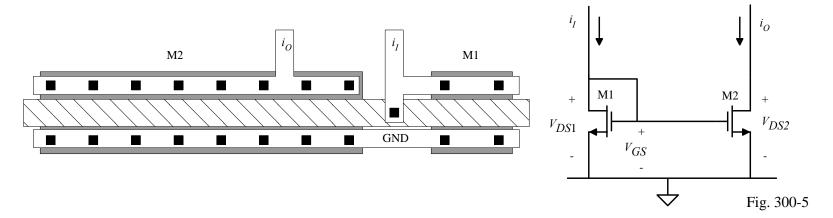
Example 16-1 – Aspect Ratio Errors in Current Mirrors

A layout is shown for a one-to-four current amplifier. Assume that the lengths are identical ($L_1 = L_2$) and find the ratio error if $W_1 = 5 \pm 0.1 \ \mu m$. The actual widths of the two transistors are

$$W_1 = 5 \pm 0.1 \ \mu \text{m}$$
 and $W_2 = 20 \pm 0.1 \ \mu \text{m}$

Solution

We note that the tolerance is not multiplied by the nominal gain factor of 4.



The ratio of W_2 to W_1 and consequently the gain of the current amplifier is

$$\frac{i_O}{i_I} = \frac{W_2}{W_1} = \frac{20 \pm 0.1}{5 \pm 0.1} = 4 \left(\frac{1 \pm (0.1/20)}{1 \pm (0.1/5)} \right) \approx 4 \left(1 \pm \frac{0.1}{20} \right) \left(1 - \frac{\pm 0.1}{5} \right) \approx 4 \left(1 \pm \frac{0.1}{20} - \frac{\pm 0.4}{20} \right) = 4 - (\pm 0.03)$$

where we have assumed that the variations would both have the same sign (correlated). It is seen that this ratio error is 0.75% of the desired current ratio or gain.

Example 16-2 – Reduction of the Aspect Ratio Errors in Current Mirrors

Use the layout technique illustrated below and calculate the ratio error of a current amplifier having the specifications of the previous example.

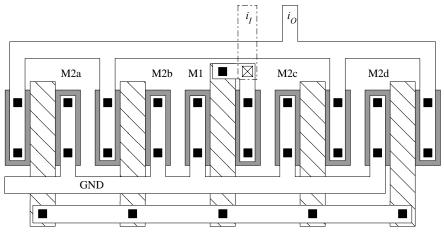
Solutions

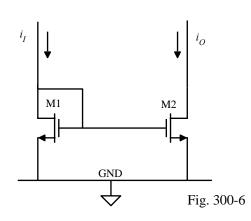
The actual widths of M1 and M2 are

$$W_1 = 5 \pm 0.1 \ \mu \text{m}$$
 and $W_2 = 4(5 \pm 0.1) \ \mu \text{m}$

The ratio of W_2 to W_1 and consequently the current gain is given below and is for all practical purposes independent of layout error.

$$\frac{i_O}{i_I} = \frac{4(5 \pm 0.1)}{5 \pm 0.1} = 4$$

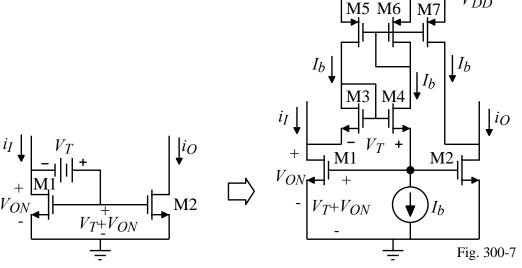




Summary of the Simple MOS Current Mirror/Amplifier

• Minimum input voltage is $V_{MIN}(in) = V_T + V_{ON}$ Okay, but could be reduced to V_{ON} .

Principle:



Will deal with later in low voltage op amps.

- Minimum output voltage is $V_{MIN}(out) = V_{ON}$
- Output resistance is $R_{out} = \frac{1}{\lambda I_D}$
- Input resistance is $R_{in} \approx \frac{1}{g_m}$
- Current gain accuracy is poor because $v_{DS1} \neq v_{DS2}$

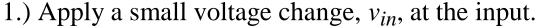
IMPROVED CURRENT MIRRORS

Large Output Swing Cascode Current Mirror

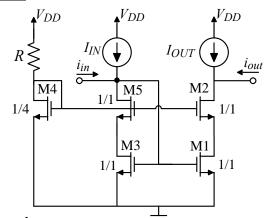
• $R_{out} \approx g_{m2} r_{ds2} r_{ds1}$

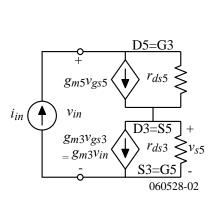
•
$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{r_{ds5} + r_{ds3} + r_{ds3}g_{m5}r_{ds5}}{g_{m3}r_{ds3}(1 + g_{m5}r_{ds5})} \approx \frac{1}{g_{m3}}$$

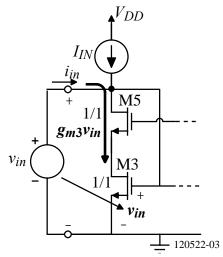
An easier way to find R_{in} :



- 2.) Note that this voltage is equal to v_{gs3} .
- 3.) This small voltage change causes a current change in the drain of M3 of $g_{m3}v_{gs3}$ or $g_{m3}v_{in}$.
- 4.) The current i_{in} is equal to $g_{m3}v_{in}$.
- 5.) Therefore, dividing v_{in} by i_{in} gives $R_{in} = 1/g_{m3}$.
- $V_{MIN}(\text{out}) = 2V_{ON}$
- $V_{MIN}(in) = V_T + V_{ON}$
- Current gain is excellent because $v_{DS1} = v_{DS3}$.







Self-Biased Cascode Current Mirror

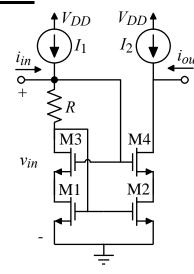
• $R_{in} = ?$

$$v_{in} = i_{in}R + r_{ds3}(i_{in}-g_{m3}v_{gs3}) + r_{ds1}(i_{in}-g_{m1}v_{gs1})$$

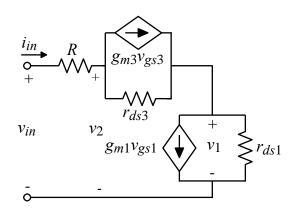
But,

$$v_{gs1} = v_{in} - i_{in}R$$

and



Self-biased, cascode current mirror



Small-signal model to calculate R_{in} . Fig. 310-03

$$v_{gs3} = v_{in} - r_{ds1}(i_{in} - g_{m1}v_{gs1}) = v_{in} - r_{ds1}i_{in} + g_{m1}r_{ds1}(v_{in} - i_{in}R)$$

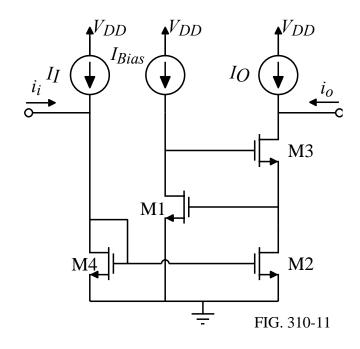
$$v_{in} = i_{in}R + r_{ds}3i_{in} - g_{m3}r_{ds}3[v_{in} - r_{ds}1i_{in} + g_{m1}r_{ds}1(v_{in} - i_{in}R)] + r_{ds}1[i_{in} - g_{m1}(v_{in} + i_{in}R)]$$
$$v_{in}[1 + g_{m3}r_{ds}3 + g_{m1}r_{ds}1g_{m3}r_{ds}3 + g_{m1}r_{ds}1]$$
$$= i_{in}[P + r_{in}1 + r_{in}2 + g_{in}2r_{in}1 + g_{in}1r_{in}R]$$

$$= i_{in}[R + r_{ds1} + r_{ds3} + g_{m3}r_{ds3}r_{ds1} + g_{m1}r_{ds1}g_{m3}r_{ds3}R]$$

$$R_{in} = \frac{R + r_{ds1} + r_{ds3} + g_{m3}r_{ds3}r_{ds1} + g_{m1}r_{ds1}g_{m3}r_{ds3}R}{1 + g_{m3}r_{ds3} + g_{m1}r_{ds1}g_{m3}r_{ds3} + g_{m1}r_{ds1}} \approx \frac{1}{g_{m1}} + R$$

- $R_{out} \approx g_{m4} r_{ds4} r_{ds2}$

MOS Regulated Cascode Current Mirror



- $R_{out} \approx g_m^2 r_{ds}^3$
- $R_{in} \approx \frac{1}{g_{m4}}$
- $V_{MIN}(out) = V_T + 2V_{ON}$ (Can be reduced to $2V_{ON}$)
- $V_{MIN}(in) = V_T + V_{ON}$ (Can be reduced to V_{ON})
- Current gain matching good as long as $v_{DS4} = v_{DS2}$

Summary of MOS Current Mirrors

Current	Accuracy	Output	Input	Minimum	Minimum
Mirror		Resistance	Resistance	Output	Input
				Voltage	Voltage
Simple	Poor	r_{ds}	1	V_{ON}	$V_T + V_{ON}$
			g_m		
Wide Output	Excellent	$g_m r_{ds}^2$	1_	$2V_{ON}$	$V_T + V_{ON}$
Swing			g_m		
Cascode					
Self-biased	Excellent	$g_m r_{ds}^2$	$\mathbf{p} + \frac{1}{\mathbf{p}}$	$2V_{ON}$	V_T +2 V_{ON}
Cascode			$R+\frac{1}{g_m}$		
Regulated	Good-	gm^2rds^3	_1_	V_T +2 V_{ON}	V_T + V_{ON}
Cascode	Excellent		g_m	(Can be	(Can be
				2 <i>V</i> _{ON})	$\approx V_{ON}$)

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VOLTAGE REFERENCES WITH POWER SUPPLY INDEPENDENCE

Power Supply Independence

How do you characterize power supply independence?

Use the concept of:

$$S_{V_{DD}}^{V_{REF}} = \frac{V_{REF}/V_{REF}}{V_{DD}/V_{DD}} = \frac{V_{DD}}{V_{REF}} \left(\frac{V_{REF}}{V_{DD}} \right)$$

Application of sensitivity to determining power supply dependence:

$$\frac{V_{REF}}{V_{REF}} = \left(S_{V_{DD}}^{V_{REF}}\right) \frac{V_{DD}}{V_{DD}}$$

Thus, the fractional change in the reference voltage is equal to the sensitivity times the fractional change in the power supply voltage.

For example, if the sensitivity is 1, then a 10% change in V_{DD} will cause a 10% change in V_{REF} .

Ideally, we want $S_{V\!D\!D}^{V\!REF}$ to be zero for power supply independence.

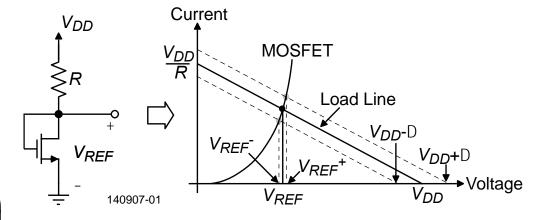
MOSFET-Resistance Voltage References

Simple MOS-R Voltage Reference

$$V_{REF} = V_{GS} = V_{T} + \sqrt{\frac{2(V_{DD}-V_{REF})}{\beta R}}$$

or

$$V_{REF} = V_T - \frac{1}{\beta R} + \sqrt{\frac{2(V_{DD} - V_T)}{\beta R} + \frac{1}{(\beta R)^2}}$$
$$S_{V_{DD}}^{V_{REF}} = \left(\frac{1}{\sqrt{1 + 2\beta(V_{DD} - V_T)R}}\right)\left(\frac{V_{DD}}{V_{REF}}\right)$$

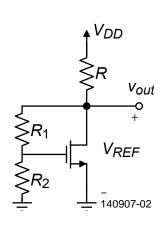


Assume V_{DD} =5V, W/L =100 and R=100k Ω , thus $V_{REF} \approx 0.7875$ V and $S_{V_{DD}}^{V_{REF}} = 0.0653$

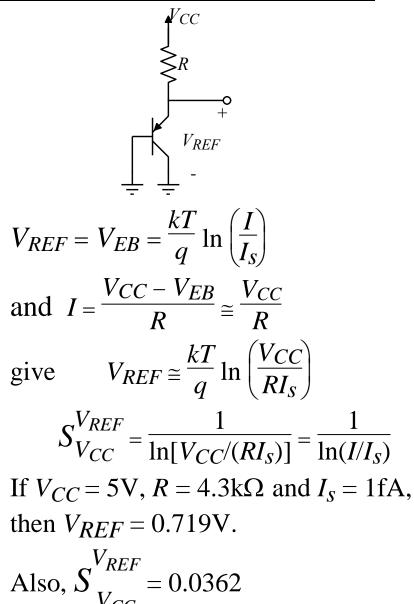
Higher Voltage Simple MOS-R Voltage Reference

This circuit allows V_{REF} to be larger. If the current in R_1 (and R_2) is small compared to the current flowing through the transistor, then

$$V_{REF} \approx \left(\frac{R_1 + R_2}{R_2}\right) V_{GS}$$



Bipolar-Resistance Voltage References



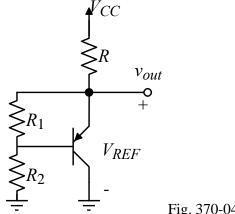


Fig. 370-04

If the current in R_1 (and R_2) is small compared to the current flowing through the transistor, then

$$V_{REF} \approx \left(\frac{R_1 + R_2}{R_1}\right) V_{EB}$$

Can use diodes in place of the BJTs.

CURRENT REFERENCES WITH POWER SUPPLY INDEPENDENCE

Power Supply Independence

Again, we want

$$S_{VDD}^{I_{REF}} = \frac{I_{REF}/I_{REF}}{V_{DD}/V_{DD}} = \frac{V_{DD}}{I_{REF}} \left(\frac{I_{REF}}{V_{DD}} \right)$$

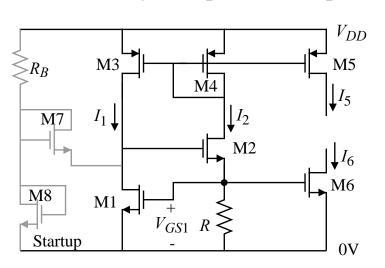
to approach zero.

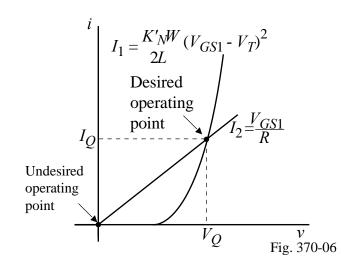
Therefore, as $S_{VDD}^{I_{REF}}$ approaches zero, the change in I_{REF} as a function of a change in V_{DD} approaches zero.

Gate-Source Referenced Current Reference

The circuit below uses both positive and negative feedback to accomplish a current reference that is reasonably independent of power supply.

Circuit:





Principle:

If M3 = M4, then $I_1 \approx I_2$. However, the M1-R loop gives $V_{GS1} = V_{T1} + \sqrt{\frac{2I_1}{K_N'(W_1/L_1)}}$

Solving these two equations gives $I_2 = \frac{V_{GS1}}{R} = \frac{V_{T1}}{R} + \left(\frac{1}{R}\right) \sqrt{\frac{2I_1}{K_N'(W_1/L_1)}}$

The output current, $I_{out} = I_1 = I_2$ can be solved as $I_{out} = \frac{V_{T1}}{R} + \frac{1}{\beta_1 R^2} + \frac{1}{R} \sqrt{\frac{2V_{T1}}{\beta_1 R} + \frac{1}{(\beta_1 R)^2}}$

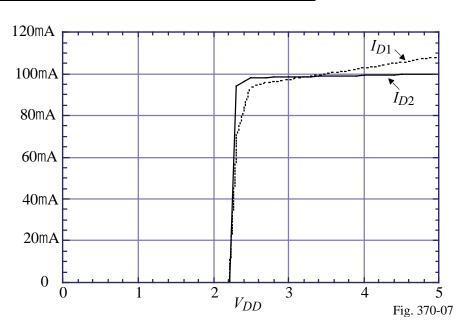
Simulation Results for the Gate-Source Referenced Current Reference

The current I_{D2} appears to be okay, why is I_{D1} increasing?

Apparently, the channel modulation on the current mirror M3-M4 is large.

At
$$V_{DD} = 5V$$
, $V_{SD3} = 2.83V$ and $V_{SD4} = 1.09V$ which gives $I_{D3} = 1.067I_{D4} \approx 107\mu\text{A}$

Need to cascode the upper current mirror. SPICE Input File:



Simple, Bootstrap Current Reference VDD 1 0 DC 5.0

VSS 9 0 DC 0.0

M1 5 7 9 9 N W=20U L=1U

M2 3 5 7 9 N W=20U L=1U

M3 5 3 1 1 P W=25U L=1U M4 3 3 1 1 P W=25U L=1U

M5 9 3 1 1 P W=25U L=1U

D 7 0 10KH OHM

R 7 9 10KILOHM

M8 6 6 9 9 N W=1U L=1U

M7 6 6 5 9 N W=20U L=1U

RB 1 6 100KILOHM

.OP

.DC VDD 0 5 0.1

.MODEL N NMOS VTO=0.7 KP=110U

GAMMA=0.4 +PHI=0.7 LAMBDA=0.04

.MODEL P PMOS VTO=-0.7 KP=50U

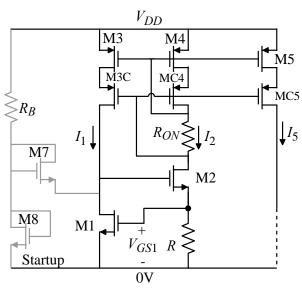
GAMMA=0.57 +PHI=0.8 LAMBDA=0.05

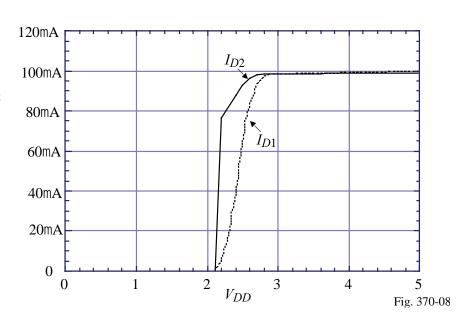
.PRINT DC ID(M1) ID(M2) ID(M5)

.PROBE

.END

Cascoded Gate-Source Referenced Current Reference





SPICE Input File:

Cascode, Bootstrap Current Reference

VDD 1 0 DC 5.0

VSS 9 0 DC 0.0

M1 5 7 9 9 N W=20U L=1U

M2 4 5 7 9 N W=20U L=1U

M3 2 3 1 1 P W=25U L=1U

M4 8 3 1 1 P W=25U L=1U

M3C 5 4 2 1 P W=25U L=1U

MC4 3 4 8 1 P W=25U L=1U

RON 3 4 4KILOHM

M5 9 3 1 1 P W=25U L=1U

R 7 9 10KILOHM

M8 6 6 9 9 N W=1U L=1U

M7 6 6 5 9 N W=20U L=1U

RB 1 6 100KILOHM

.OP

.DC VDD 0 5 0.1

.MODEL N NMOS VTO=0.7 KP=110U

GAMMA=0.4 PHI=0.7 LAMBDA=0.04

.MODEL P PMOS VTO=-0.7 KP=50U

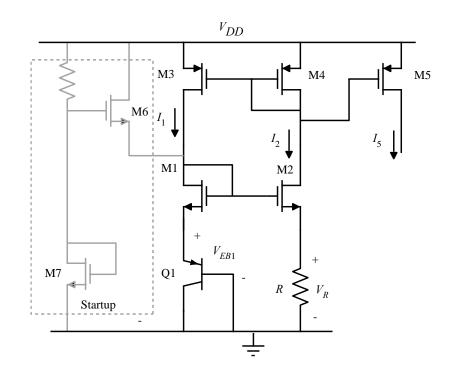
GAMMA=0.57 PHI=0.8 LAMBDA=0.05

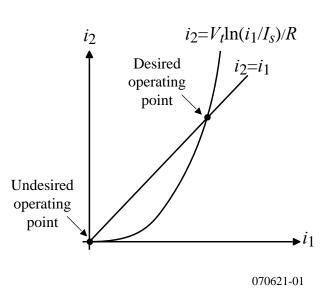
.PRINT DC ID(M1) ID(M2) ID(M5)

.PROBE

.END

Base-Emitter Referenced Circuit





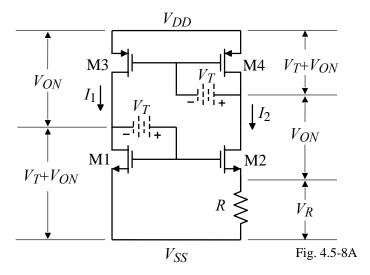
$$I_{out} = I_2 = \frac{V_{EB1}}{R}$$

BJT can be a MOSFET in weak inversion.

Low Voltage Gate-Source Referenced MOS Current Reference

The previous gate-source referenced circuits required at least 2 volts across the power supply before operating.

A low-voltage gate-source referenced circuit:



Without the batteries, V_T , the minimum power supply is $V_T + 2V_{ON} + V_R$. With the batteries, V_T , the minimum power supply is $2V_{ON} + V_R \approx 0.5$ V

Summary of Power-Supply Independent References

- Reasonably good, simple voltage and current references are possible
- Best power supply sensitivity is approximately 0.01 (10% change in power supply causes a 0.1% change in reference)

Type of Reference	V_{REF} I_{REF} S or S V_{PP}	
MOSFET-R	<1	
BJT-R	<<1	
Gate-source Referenced	<<1	
Base-emitter Referenced	<<1	

TEMPERATURE BEHAVIOR OF VOLTAGE AND CURRENT REFERENCES Characterization of Temperature Dependence

The objective is to minimize the fractional temperature coefficient defined as,

$$TC_F = \frac{1}{V_{REF}} \left(\frac{V_{REF}}{T} \right) = \frac{1}{T} S_T^{V_{REF}}$$
 parts per million per °C or ppm/°C

Temperature dependence of PN junctions:

$$i \approx I_{S} \exp\left(\frac{v}{V_{t}}\right)$$

$$I_{S} = KT^{3} \exp\left(\frac{-V_{GO}}{V_{t}}\right)$$

$$\frac{1}{I_{S}}\left(\frac{I_{S}}{T}\right) = \frac{(\ln I_{S})}{T} = \frac{3}{T} + \frac{V_{GO}}{TV_{t}} \approx \frac{V_{GO}}{TV_{t}}$$

$$\frac{dv_{BE}}{dT} \approx \frac{V_{BE} - V_{GO}}{T} = -2\text{mV/}^{\circ}\text{C at room temperature}$$

Temperature dependence of MOSFET in strong inversion:

$$\frac{dv_{GS}}{dT} = \frac{dV_T}{dT} + \sqrt{\frac{2L}{WC_{ox}}} \frac{d}{dT} \left(\sqrt{\frac{i_D}{\mu_o}} \right)$$

$$\mu_o = KT^{-1.5}$$

$$V_T(T) = V_T(T_o) - \alpha(T - T_o)$$

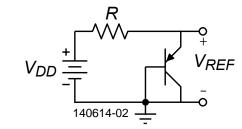
$$\frac{dv_{GS}}{dT} \approx -\alpha \approx -2.3 \frac{\text{mV}}{\text{°C}}$$

Resistors: (1/R)(dR/dT) ppm/°C

Bipolar-Resistance Voltage References

From previous work we know that,

$$V_{REF} = \frac{kT}{q} ln \left(\frac{V_{DD} - V_{REF}}{RI_s} \right)$$



However, not only is V_{REF} a function of T, but R and I_s are also functions of T.

$$\therefore \frac{dV_{REF}}{dT} = \frac{k}{q} ln \left(\frac{V_{DD} - V_{REF}}{RI_s} \right) + \frac{kT}{q} \left(\frac{RI_s}{V_{DD} - V_{REF}} \right) \left[\frac{-1}{RI_s} \frac{dV_{REF}}{dT} - \left(\frac{V_{DD} - V_{REF}}{RI_s} \right) \left(\frac{dR}{RdT} + \frac{dI_s}{I_s dT} \right) \right]$$

$$= \frac{V_{REF}}{T} - \frac{V_t}{V_{DD} - V_{REF}} \frac{dV_{REF}}{dT} - V_t \left(\frac{dR}{RdT} + \frac{dI_s}{I_s dT} \right) = \frac{V_{REF} - V_{GO}}{T} - \frac{V_t}{V_{DD} - V_{REF}} \frac{dV_{REF}}{dT} - \frac{3V_t}{T} - \frac{V_t}{RdT} \frac{dR}{dT}$$

$$\therefore \frac{dV_{REF}}{dT} = \frac{V_{REF} - V_{GO}}{T} - V_t \frac{dR}{RdT} - \frac{3V_t}{T} - V_t \frac{dR}{RdT} - \frac{3V_t}{T}$$

$$TC_F = \frac{1}{V_{REF}} \frac{dV_{REF}}{dT} = \frac{V_{REF} - V_{GO}}{V_{REF}} - \frac{V_t}{V_{REF}} \frac{dR}{RdT} - \frac{3V_t}{V_{REF}} \frac{dR}{RdT} - \frac{3V_t}{V_{REF}} T$$

If $V_{REF} = 0.6$ V, $V_t = 0.026$ V, and the R is polysilicon, then at 27°K the TC_F is

$$TC_F = \frac{0.6 - 1.205}{0.6 \cdot 300} - \frac{0.026 \cdot 0.0015}{0.6} - \frac{3 \cdot 0.026}{0.6 \cdot 300} = 33110^{-6} - 65 \times 10^{-6} - 433 \times 10^{-6} = -3859 \text{ppm}/^{\circ}\text{C}$$

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MOSFET Resistor Voltage Reference

From previous results we know that

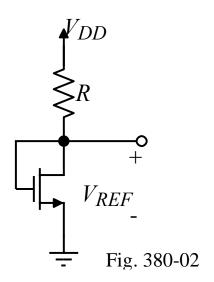
$$V_{REF} = V_{GS} = V_{T} + \sqrt{\frac{2(V_{DD} - V_{REF})}{\beta R}}$$
or
$$V_{REF} = V_{T} - \frac{1}{\beta R} + \sqrt{\frac{2(V_{DD} - V_{T})}{\beta R} + \frac{1}{(\beta R)^{2}}}$$

Note that V_{REF} , V_T , β , and R are all functions of temperature.

It can be shown that the TC_F of this reference is

$$\frac{dV_{\text{REF}}}{dT} = \frac{-\alpha + \sqrt{\frac{V_{DD} - V_{\text{REF}}}{2\beta R}} \left(\frac{1.5}{T} - \frac{1}{R} \frac{dR}{dT}\right)}{1 + \frac{1}{\sqrt{2\beta R} (V_{DD} - V_{\text{REF}})}}$$

$$\therefore TC_F = \frac{-\alpha + \sqrt{\frac{V_{DD} - V_{\text{REF}}}{2\beta R}} \left(\frac{1.5}{T} - \frac{1}{R} \frac{dR}{dT}\right)}{V_{REF}(1 + \frac{1}{\sqrt{2\beta R} (V_{DD} - V_{\text{REF}})})}$$



Example 16-3 - Calculation of MOSFET-Resistor Voltage Reference *TC_F*

Calculate the temperature coefficient of the MOSFET-Resistor voltage reference where W/L=2, V_{DD} =5V, R=100k Ω using the parameters of Table 3.1-2. The resistor, R, is polysilicon and has a temperature coefficient of 1500 ppm/°C.

Solution

First, calculate V_{REF} . Note that $\beta R = 220 \times 10^{-6} \times 10^5 = 22$ and $\frac{dR}{RdT} = 1500 \text{ppm/}^{\circ}\text{C}$

$$\therefore V_{REF} = 0.7 - \frac{1}{22} + \sqrt{\frac{2(5 - 0.7)}{22} + \left(\frac{1}{22}\right)^2} = 1.281V$$

Now,
$$\frac{dV_{REF}}{dT} = \frac{-2.3 \times 10^{-3} + \sqrt{\frac{5 - 1.281}{2(22)}} \left(\frac{1.5}{300} - 1500 \times 10^{-6}\right)}{1 + \frac{1}{\sqrt{2(22)} (5 - 1.281)}} = -1.189 \times 10^{-3} \text{V/°C}$$

The fractional temperature coefficient is given by

$$TCF = -1.189 \times 10^{-3} \left(\frac{1}{1.281} \right) = -928 \text{ ppm/}^{\circ}\text{C}$$

Gate-Source and Base-Emitter Referenced Current Source/Sinks

Gate-source referenced source:

The output current was given as,
$$I_{out} = \frac{V_{T1}}{R} + \frac{1}{\beta_1 R^2} + \frac{1}{R} \sqrt{\frac{2V_{T1}}{\beta_1 R} + \frac{1}{(\beta_1 R)^2}}$$

Although we could grind out the derivative of I_{out} with respect to T, the temperature performance of this circuit is not that good to spend the time to do so. Therefore, let us assume that $V_{GS1} \approx V_{T1}$ which gives

$$I_{out} \approx \frac{V_{T1}}{R}$$
 $\Rightarrow \frac{dI_{out}}{dT} = \frac{1}{R} \frac{dV_{T1}}{dT} - \frac{1}{R^2} \frac{dR}{dT}$

In the resistor is polysilicon, then

$$TC_F = \frac{1}{I_{out}} \frac{dI_{out}}{dT} = \frac{1}{V_{T1}} \frac{dV_{T1}}{dT} - \frac{1}{R} \frac{dR}{dT} = \frac{-\alpha}{V_{T1}} - \frac{1}{R} \frac{dR}{dT} = \frac{-2.3 \times 10^{-3}}{0.7} - 1.5 \times 10^{-3} = -4786 \text{ppm}/^{\circ}\text{C}$$

Base-emitter referenced source:

The output current was given as, $I_{out} = I_2 = \frac{V_{BE1}}{R}$

The
$$TC_F = \frac{1}{V_{BE1}} \frac{dV_{BE1}}{dT} - \frac{1}{R} \frac{dR}{dT}$$

If $V_{BE1} = 0.6$ V and R is poly, then the $TC_F = \frac{1}{0.6} (-2x10^{-3}) - 1.5x10^{-3} = -4833$ ppm/°C.

Low V_{DD} Current Reference

Consider the following circuit with all transistors having a W/L = 10. This is a bootstrapped reference which creates a V_{bias} independent of V_{DD} . The two key equations are:

$$I_3 = I_4 \Rightarrow I_1 = I_2$$

and

$$V_{GS1} = V_{GS2} + I_2R$$

Solving for I_2 gives:

$$I_{2} = \frac{V_{GS1} - V_{GS2}}{R} = \frac{1}{R} \left(\sqrt{\frac{2I_{1}}{\beta_{1}}} - \sqrt{\frac{2I_{2}}{\beta_{2}}} \right) = \frac{\sqrt{2I_{1}}}{R\sqrt{\beta_{1}}} \left(1 - \frac{1}{2} \right)$$

$$\therefore \sqrt{I_2} = \frac{1}{R\sqrt{2\beta_1}} \implies \boxed{I_2 = I_1 = \frac{1}{2\beta_1 R^2}} = \frac{1}{2 \cdot 110 \times 10^{-6} \cdot 10 \cdot 25 \times 10^6} = 18.18 \mu A$$

Now, V_{bias} can be written as

$$V_{bias} = V_{GS1} = \sqrt{\frac{2I_2}{\beta_1}} + V_{TN} = \frac{1}{\beta_1 R} + V_{TN} = \frac{1}{110 \times 10^{-6} \cdot 10 \cdot 5 \times 10^3} + 0.7 = 0.1818 + 0.7 = 0.8818 \text{V}$$

Any transistor with $V_{GS} = V_{bias}$ will have a current flow that is given by $1/2\beta R^2$.

Therefore,
$$g_m = \sqrt{2I\beta} = \sqrt{\frac{2\beta}{2\beta R^2}} = \frac{1}{R} \implies \left[g_m = \frac{1}{R}\right]$$

Summary of Reference Performance

Type of Reference	$S_{V_{DD}}^{V_{REF}}$	TC_F	Comments
MOSFET-R	<1	>1000ppm/°C	
BJT-R	<<1	>1000ppm/°C	
Gate-Source	Good if currents	>1000ppm/°C	Requires start-
Referenced	are matched		up circuit
Base-emitter	Good if currents	>1000ppm/°C	Requires start-
Referenced	are matched		up circuit

- A MOSFET can have zero temperature dependence of i_D for a certain v_{GS}
- If one is careful, very good independence of power supply can be achieved
- None of the above references have really good temperature independence Consider the following example:

A 10 bit ADC has a reference voltage of 1V. The LSB is approximately 0.001V. Therefore, the voltage reference must be stable to within 0.1%. If a 100°C change in temperature is experienced, then the TC_F must be 0.001%/C or multiplying by 10^4 requires a $TC_F = 10$ ppm/°C.