Nr. de condiționare

$$\begin{matrix} x \\ x^* \end{matrix} \longrightarrow \boxed{\;f\;} \longrightarrow \begin{matrix} y \\ y^* \end{matrix}$$

Er. abs.: $|x - x^*| = |\Delta x|$

$|y - y^*| = |\Delta y|$

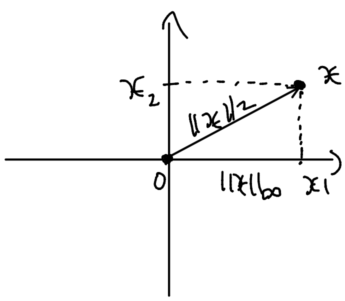
Er. rel.: $|\delta x| = \dfrac{|\Delta x|}{|x|}$

$|\delta y| = \dfrac{|\Delta y|}{|y|}$

$$\frac{|\delta y|}{|\delta x|} \underset{\simeq}{\leq} \operatorname{cond} f(x)$$

$f : \mathbb{R}^m \to \mathbb{R}^n$

$x = [x_1 \ldots x_m]^T$



Ex. norme (p-norme)

- Euclidiană: $\|x\|_2 = \sqrt{x_1^2 + \ldots + x_m^2}$
- Cebîșev/supremum: $\|x\|_\infty = \max\{|x_1|, \ldots, |x_m|\}$
- Taxi/Manhattan: $\|x\|_1 = |x_1| + \ldots + |x_m|$

$A \in \mathcal{M}_{m,n}(\mathbb{R})$, $\displaystyle \|A\|_p = \max_{\substack{x \in \mathbb{R}^m \\ x \neq 0_m}} \frac{\|A \cdot x\|_p}{\|x\|_p}$ $\to$ normă matriceală

2 def. ale nr. cond. pt. $f = (f_1, \ldots, f_n)$

1) $\Gamma(x) = \left(\dfrac{x_j \cdot \frac{\partial f_i}{\partial x_j}(x)}{f_i(x)} \right)_{\substack{i=\overline{1,n} \\ j=\overline{1,m}}}$ $\operatorname{cond}_p f(x) = \|\Gamma(x)\|_p$

2) $\dfrac{\partial f}{\partial x}(x) = \left(\dfrac{\partial f_i}{\partial x_j}(x) \right)_{\substack{i=\overline{1,n} \\ j=\overline{1,m}}}$ $\operatorname{cond}_p f(x) = \dfrac{\|x\|_p \cdot \left\| \frac{\partial f}{\partial x}(x) \right\|_p}{\|f(x)\|_p}$.

Ex.: • $f : \mathbb{R} \to \mathbb{R}$, $x \in \mathbb{R}^*$, $f(x) \neq 0 \Rightarrow \operatorname{cond} f(x) = \dfrac{|x| \cdot |f'(x)|}{|f(x)|}$

• $\left.\begin{matrix} A \cdot y = b \text{ (sist.)} \\ A \in \mathcal{M}_{n,m}(\mathbb{R}), \det A \neq 0 \end{matrix}\right\} \Rightarrow y = A^{-1} \cdot b$

$f(b) = A^{-1} \cdot b$

$A \in \mathcal{M}_{n,n}(\mathbb{R}), \det A \neq 0$

$$b \longrightarrow \boxed{f} \longrightarrow y \qquad y = f(b) = A^{-1} \cdot b$$

$$\text{cond}_p f(b) = \frac{\|b\|_p \cdot \|A^{-1}\|_p}{\|A^{-1} b\|_p}$$

$\downarrow$ Def. 2)

$$\underline{\text{cond}_p A} := \max_{\substack{b \in \mathbb{R}^n \\ b \neq 0_n}} \text{cond}_p f(b) = \underline{\|A\|_p \cdot \|A^{-1}\|_p}.$$

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>> cond(A,p)
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A Vandermonde matrix has the form:

   c(1)^(n-1)  ... c(1)^2  c(1)   1
   c(2)^(n-1)  ... c(2)^2  c(2)   1
        .       .     .      .    .
        .         .   .      .    .
        .           . .      .    .
   c(n)^(n-1)  ... c(n)^2  c(n)   1
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<u>Temă</u>: ec. alg.: $p(x) = 0$, unde $p(x) = x^n + a_1 x^{n-1} + \dots + a_{n-1} x + a_n$

are o răd. nenulă şi simplă $\xi(a)$

$$(\xi(a) \neq 0, \; p(\xi(a)) = 0, \; p'(\xi(a)) \neq 0)$$

<u>ex</u>.: $p(x) = x^3 - x^2 = x^2(x-1) \Rightarrow \xi = 1$ răd. simplă

$$(p(1) = 0; \; p'(x) = 3x^2 - 2x \; ; \; p'(1) = 1)$$

Completaţi condpol.m a.î. să returneze

$$\text{cond}_1 \xi(a) = \frac{\sum_{j=1}^{n} \left| a_j \cdot \xi(a)^{n-j} \right|}{\left| \xi(a) \cdot p'(\xi(a)) \right|}$$

$\downarrow$ Def. 1)

$a = [a_1 \dots a_n]$

$$a \longrightarrow \boxed{\xi} \longrightarrow \xi(a)$$