Problem 5

The dimension of ∇f_1 is 1×2

$$\nabla f_1 = \begin{pmatrix} \cos(x_1)\cos(x_2) & -\sin(x_1)\sin(x_2) \end{pmatrix}$$

The dimension of ∇f_2 is $1 \times n$

$$\nabla f_2 = y^T$$

The dimension of ∇f_3 is $n \times n \times n$

$$(\nabla f_3)_{ijk} = \frac{\partial(x_i x_j)}{\partial x_k} = x_i \delta_{jk} + x_j \delta_{jk}$$

Problem 6

$$\frac{\partial f}{\partial t_i} = \cos(\log(t^T t))\frac{\partial}{\partial t_i}\log(t_j t_j) = \frac{\cos(\log(t^T t))}{t^T t}\frac{\partial}{\partial t_i}(t_j t_j) = 2\frac{\cos(\log(t^T t))}{t^T t}t_i$$

$$\therefore \frac{df}{dt} = 2\frac{\cos(\log(t^T t))}{t^T t}t^T$$

$$\frac{\partial g}{\partial x_{ij}} = \frac{\partial}{\partial x_{ij}}(A_{kl}X_{lm}B_{mk}) = A_{ki}B_{jk} = (BA)_{ji} = (BA)^T_{ij}$$

$$\therefore \frac{dg}{dx} = (BA)^T$$

Problem 8

$$\frac{\partial f}{\partial x_i} = \frac{\partial f}{\partial z}\frac{\partial z}{\partial y_j}\frac{\partial y_j}{\partial x_i} = -\frac{1}{2}e^{-z/2}(S^{-1}_{ij}y_j + y_j S^{-1}_{ji}) = -\frac{1}{2}e^{-z/2}(S^{-1}y + y^T S^{-1})_i$$

$$\therefore \frac{df}{dx} = -\frac{1}{2}e^{-z/2}(S^{-1}y + y^T S^{-1})$$

$$\frac{\partial f}{\partial x_i} = \frac{\partial}{\partial x_i}(x_i x_i + D\sigma^2) = 2x_i$$

$$\therefore \frac{df}{dx} = 2x^T$$

$$\frac{\partial f_i}{\partial x_j} = \frac{\partial f_i}{\partial z_k}\frac{\partial z_k}{\partial x_j} = \operatorname{sech}^2(z_i)\delta_{ik}A_{kj} = \operatorname{sech}^2(z_i)A_{ij}$$