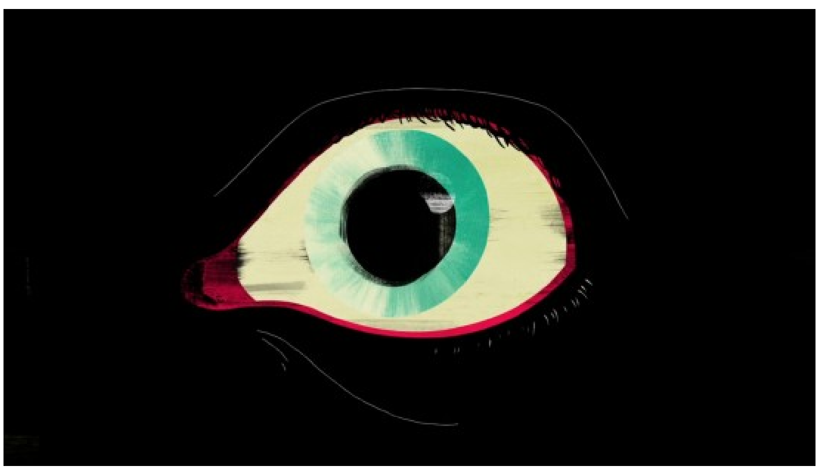
Foundations of Type

Programming in Mathematics

Edward O’Callaghan



"Foundations of Type"

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This book is freely available at http://victoredwardocallaghan.github.io/book/

Acknowledgment

Thanks to github...

Preface

Book development

The following people participated in the development of this text.

Edward O'Callaghan

About this book

This is a textbook that I am writing on a number of topics that have a deep unexpected connection. In particular, this book introduces programming and the foundations of mathematics from scratch by way of *homotopy type* and *category theory* and explores aspects of these topics to lead to a new era in high assurance software design and implementations.

Foundations of Type
Edward O'Callaghan
Australia, Jun 2013

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1.1 Groupoids

The extension from *group* to *groupoid* is motivated with the desire to describe reversible mappings which may traverse a number of states. Intuitively, in *group theory* we study mappings of the form $\tau : \mathcal{G} \times \mathcal{G} \to \mathcal{G}$ that are total functions. Whereas in *groupoid theory* one considers a set of compositable and invertible mappings of the form $\phi \circ \psi : \mathcal{G} \times \mathcal{G} \to \mathcal{G}$ where ϕ and ψ need only be partial functions on $\mathcal{G}$.

Definition 1.1.1 (Groupoid)**.** A **groupoid** is a small category in which every morphism is an isomorphism.

Thus, as a result one may view groups as a groupoid with one object. More precisely, a groupoid X with one object x is uniquely determined by its automorphism group $\mathcal{G} = \mathrm{Aut}(x)$. Consequently, any group may be regarded as a groupoid with one object.

Notice that can naturally view morphisms between *groupoids* are *functors* between categories.

Appendix

Index of symbols

From the Introduction:

We present *Homotopy type theory* as the corner stone to understanding both mathematics and programming.

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