

# Solutions for MATH2901 - 2011 paper.

Edward O'Callaghan

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## 1 Question 1

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## 2 Question 2

Let some random univariate (**r.v.**)  $X \sim \mathcal{P}(\lambda)$  be *Poisson* distributed with parameter  $\lambda$ . Then  $X$  has density,

$$f_X(x; \lambda) = \frac{\lambda^x e^{-\lambda}}{x!} : x = 0, 1, 2, \dots$$

### 2.1 a.)

#### 2.1.1 i.)

Suppose  $\{X_i\}_{i=1}^n$  denotes an **i.i.d** random sample and  $\mathcal{L}_i(\lambda|x) = f_{X_i}(x; \lambda)$  denotes the likelihood function of the **r.v.**  $X$  with any  $x \in \{X_i\}_{i=1}^n$ . Then the loglikelihood  $\ell(\lambda; x)$  defined on the random sample is given by,

$$\begin{aligned}\ell(\lambda; x) &= \log_2 \mathcal{L}(\lambda|x) \\ &= \log_2 \prod_{i=1}^n \mathcal{L}_i(\lambda|x) && \text{(by indepenance of the sample)} \\ &= \log_2 \prod_{i=1}^n f_{X_i}(x; \lambda) \\ &= \sum_{i=1}^n \log_2 f_{X_i}(x; \lambda) \\ &= \sum_{i=1}^n \log_2 \left\{ \frac{\lambda^{x_i} e^{-\lambda}}{x_i!} \right\} \\ &= \sum_{i=1}^n \left\{ \log_2 e^{-\lambda} + \log_2 \frac{\lambda^{x_i}}{x_i!} \right\} \\ &= \sum_{i=1}^n \{x_i \log_2 \lambda - \log_2 x_i! - \lambda\} \\ \Rightarrow \ell(\lambda; x) &= -n\lambda + \sum_{i=1}^n \{x_i \log_2 \lambda - \log_2 x_i!\}.\end{aligned}$$

In particular, we may ignore the constant term that does not depend on  $\lambda$ ,

$$\Rightarrow \ell(\lambda; x) = n\bar{X} \log_2(\lambda) - n\lambda.$$

### 3 Question 3

#### 3.1 a.)

Suppose that  $X \sim \mathcal{N}(\mu, \sigma^2)$  and  $Y = \frac{1}{|X|}$ .

##### 3.1.1 i.)

We wish to discern if the function  $f(x) = \frac{1}{|x|} : x \neq 0$  has monotonicity.

Clearly we have weak-monotonicity, since if  $0 < x_1 \leq x_2$ , we have that  $f(x_1) \geq f(x_2)$  and if  $x_1 \leq x_2 < 0$  we then have  $f(x_1) \leq f(x_2)$ .

##### 3.1.2 ii.)

Show that  $F_Y(y) = F_X(\frac{1}{y}) - F_X(-\frac{1}{y})$ .