Solutions for MATH2901 - 2011 paper.

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Let some random univariate (**r.v.**) $X \sim \mathcal{P}(\lambda)$ be *Poisson* distributed with parameter λ. Then X has density,

$$f_X(x;\lambda) = \frac{\lambda^x e^{-\lambda}}{x!} : x = 0, 1, 2, \ldots$$

2.1 a.)

2.1.1 i.)

Suppose $\{X_i\}_{i=1}^n$ denotes an **i.i.d** random sample and $\mathcal{L}_i(\lambda|x) = f_{X_i}(x;\lambda)$ denotes the likelihood function of the **r.v.** X with any $x \in \{X_i\}_{i=1}^n$. Then the loglikelihood $\ell(\lambda;x)$ defined on the random sample is given by,

$$\begin{aligned}
\ell(\lambda;x) &= \log_2 \mathcal{L}(\lambda|x) \\
&= log_2 \prod_{i=1}^n \mathcal{L}_i(\lambda|x) && \text{(by indepenance of the sample)} \\
&= \log_2 \prod_{i=1}^n f_{X_i}(x;\lambda) \\
&= \sum_{i=1}^n \log_2 f_{X_i}(x;\lambda) \\
&= \sum_{i=1}^n \log_2 \left\{ \frac{\lambda^{x_i} e^{-\lambda}}{x_i!} \right\} \\
&= \sum_{i=1}^n \left\{ \log_2 e^{-\lambda} + \log_2 \frac{\lambda^{x_i}}{x_i!} \right\} \\
&= \sum_{i=1}^n \{ x_i \log_2 \lambda - \log_2 x_i! - \lambda \} \\
\Rightarrow \ell(\lambda;x) &= -n\lambda + \sum_{i=1}^n \{ x_i \log_2 \lambda - \log_2 x_i! \} .
\end{aligned}$$

In particular, we may ignore the constant term that does not depend on λ,

$$\Rightarrow \ell(\lambda; x) = n\bar{X}\log_2(\lambda) - n\lambda.$$