Tutorial 1: Fundamentals

Answers

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1 Thermodynamic properties and tables

Objectives of this exercise:

1. Get familiar with steam/water tables.
2. Get familiar with the enthalpy of reaction.

Problem 1: Steam expansion

A turbine expands steam to a pressure of 0.01 MPa and a quality of 0.92. Determine the temperature and specific volume at this state.

Solution:
See also Figure 1.
$T = T_{\text{sat}}$ @ $P_{\text{sat}} = 318.6$ K
$v_{\text{f}} = 0.001$ m^3/kg
$v_{\text{g}} = 14.6$ m^3/kg
$v_{0.92} = (1 - x)v_{\text{f}} + xv_{\text{g}} = 13.496$ m^3/kg

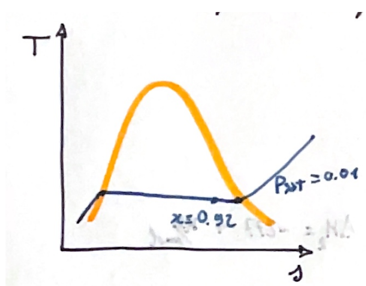

Figure 1: Saturated steam expansion.

Problem 2: Isentropical steam expansion

Superheated steam at 700 K and 10 MPa is expanded at 1 MPa isentropically. Determine the values of v, h, T, and x at the final state.

Solution:
See also Figure 2.
$s_1 = s(700\text{K}, 10\text{MPa}) = 6.33\text{kJ/kgK}$
$s_2 = s_1$
$x_2 = \frac{s_2 - s_\text{f}}{s_\text{g} - s_\text{f}}$
$h = (1 - x_2)h_\text{f} + xh_\text{g}$
$v = (1 - x_2)v_\text{f} + xv_\text{g}$
$T = T_\text{sat}@\ P_\text{sat}$

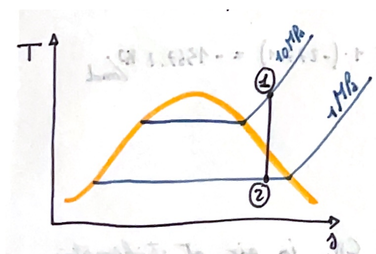

Figure 2: Isentropical expansion of superheated steam.

Problem 3: Enthalpy of reaction

Enthalpy of reaction ΔH_R: $v_A A + v_B B \rightleftharpoons v_c C + v_D D$ @ standard reference temperature

$$\Delta H_R = \sum_{\text{products}} v_p \Delta H_p^{\text{formation}} - \sum_{\text{reactants}} v_r \Delta H_r^{\text{formation}} \tag{1}$$

if $T \neq T_{\text{ref}}$

$$\Delta H_R(T) = \underbrace{\Delta H_R(T_{\text{ref}})}_{\text{reaction at } T_{\text{ref}}} + \underbrace{\Delta H(T)}_{\text{sensible heat}} \tag{2}$$

Now, compute the standard enthalpy of reaction for

$$C_2H_5OH + 3\,O_2 \rightleftharpoons 2\,CO_2 + 3\,H_2O \tag{3}$$

Solution: Ethanol: $2\,C + 3\,H_2 +$ 0.5 $O_2 \rightarrow C_2H_5OH$ $\Delta H_R =$ -277.1 kJ/mol
O_2:
CO_2: $C + O_2 \rightleftharpoons CO_2$ $\Delta H_R =$ -393.4 kJ/mol
H_2O: $H_2 + 0.5O_2 \rightleftharpoons H_2O$ $\Delta H_R =$ -285.8 kJ/mol
$\Delta H_R(T) = 2 \cdot (-393.4) + 3 \cdot (-285.8) - (-277.1) =$ -1367.7 kJ/mol

Problem 4: Adiabatic flame temperature

At T_{ad}, $H_{\text{prod}}(T_{\text{ad}}) = H_{\text{reac}}(T_{\text{in}})$.
Calculate the adiabatic flame temperature of CH_4 in air, at stoichiometric conditions. $T_{\text{in}} =$ 298 K. Assume constant specific heat.

Solution:
$CH_4 + 2(O_2 + 3.7N_2) \rightleftharpoons CO_2 + 2\,H_2O + 2(3.7N_2)$
$H_{\text{prod}}(T_{\text{ad}}) = H_{\text{react}}(T_{\text{in}})$
$H_{\text{react}}(298\text{K}) = 1 \cdot \Delta H_{CH_4} + \cancel{2 \cdot \Delta H_{O_2}} + \cancel{7.52 \cdot \Delta H_{N_2}} = 1 \cdot \Delta H_{CH_4}$ = -74.83 kJ
$H_{\text{prod}}(T_{\text{ad}}) = 1 \cdot (\Delta H_{CO_2} + C_p^{CO_2}(T_{\text{ad}} - 298)) + 2 \cdot (\Delta H_{H_2O} + C_p^{H_2O}(T_{\text{ad}} - 298)) + C_p^{N_2}(T_{\text{ad}} - 298)$
$T_{\text{ad}} = 2318$ K

2 Equilibrium: Dissociation of carbon dioxide

Problem 5: Dissociation of CO_2

Let's assume the dissociation of CO_2:

$$CO_2 \rightleftharpoons CO + \frac{1}{2}O_2 \tag{4}$$

We want to use equilibrium to find the composition of the mixture starting from pure CO_2 at 2500 K and ambient pressure.

- Known: CO_2 in the reactants, p,T.
- Unknown: x_i where $x_i = \frac{p_i}{p_{\mathrm{tot}}}$.
- Assumptions:
  1. High temperature and low pressure (ideal gas).
  2. No other species are present in the system.

Solution:
$\kappa_p = \exp\left(-\frac{\Delta G_T^\circ}{RT}\right)$
* $\kappa_p = \Pi x_i^{\nu_i} = \frac{x_{\mathrm{CO}} \cdot x_{\mathrm{O_2}}^{1/2}}{x_{\mathrm{CO_2}}}$
$\Delta G_T^\circ = [\cancel{0.5 G_{\mathrm{O_2}}^\circ} + 1 \cdot G_{\mathrm{CO}}^\circ - 1 \cdot \Delta G_{\mathrm{CO_2}}^\circ]_{@T=2500\mathrm{K}} = -327.245 - (-396.152) = 68.907$ kJ/mol
$\kappa_p = 0.03635$
* $\frac{\#\mathrm{C}}{\#\mathrm{O}} = \frac{1}{2} \Rightarrow \frac{x_{\mathrm{CO}} + x_{\mathrm{CO_2}}}{x_{\mathrm{CO}} + 2x_{\mathrm{CO_2}} + 2x_{\mathrm{O_2}}} = \frac{1}{2}$
* $x_{\mathrm{CO}} + x_{\mathrm{CO_2}} + x_{\mathrm{O_2}} = 1$
Solving * gives $x_{\mathrm{CO}} = 0.121$, $x_{\mathrm{CO_2}} = 0.8185$, $x_{\mathrm{O_2}} = 0.0605$

3 Berrnoulli equation

Problem 6: Application of Bernoulli equation

Water flows through a pipe that reduces in diameter from 200 mm to 100 mm. The pressure at the inlet diameter is 300 kPa, and the velocity is 2 m/s. The pipe is made of commercial steel and the flow is fully developed. The length of the pipe section is 20 m. The outlet discharged at ambient pressure (101.3 kPa).
Calculate the maximum elevation charge between the two points. You need to account for friction losses. The temperature of water is 20 °C.

Solution:
Also see Figure 3.
Using Bernoulli equation (1 as inlet, 2 as outlet)

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2 + h_f \tag{5}$$

Rearranging, we get $z_2 - z_1 = \frac{p_1 - p_2}{\rho g} + \frac{1}{2g}(v_1^2 - v_2^2) - h_f$
Now, we acquire v_2 from continuity, and h_f from Moody's diagram with Re, ϵ, ν.

$$A_1 v_1 = A_2 v_2 \tag{6}$$

$$v_2 = \frac{A_1 v_1}{A_2} = 8\text{m/s} \tag{7}$$

$$Re_1 = \frac{v_1 D_1}{\nu} = 398800 > 4000 \tag{8}$$

$$Re_2 = \frac{v_2 D_2}{\nu} = 797608 > 4000 \tag{9}$$

The flow is turbulent. The friction loss is then almost independent of Re. The roughness of steel $\epsilon = 0.046$ mm. Using either diameter of the inlet or outlet will give similar f from Moody's diagram. Let us use $f = 0.018$.

$$\Delta z = \frac{p_1 - p_2}{\rho g} + \frac{v_1^2 - v_2^2}{2g} - h_f = 17\text{m} \tag{10}$$

This means the flow can rise up to 17m while still discharging at atmospheric pressure.

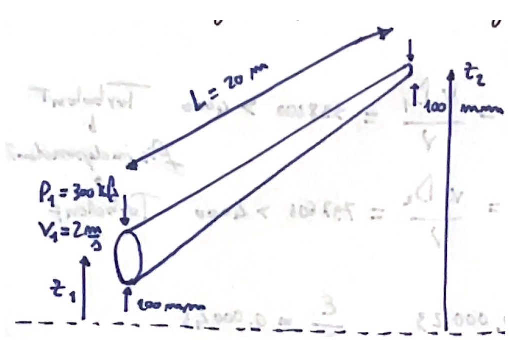

Figure 3: Pipe flow with a reducing diameter.

4 Velocity triangles

Problem 7: Velocity triangle calculation

Let's consider an axial steam turbine working in the following conditions:

- Inlet steam velocity: $v_1 = 250$ m/s.
- Inlet steam flow angle: $\alpha_1 = 20$ °.
- Outlet steam velocity: 220 m/s.
- Blade speed $U = 180$ m/s.

Calculate the turbine specific work for the considered stage. Start with optimal conditions and continue by increasing the outlet flow angle α_2 to 45 °.

Solution:
See also Figure 4.
Let's start by drawing the velocity triangle for optimal conditions

$$W = U_1(v_1, t) - U_2(v_2, t) = U(v_{1,t} - v_{2,t}) \tag{11}$$

$$U_1 = U_2 \tag{12}$$

Moreover, for optimal conditions $v_{2,t} = 0 \Rightarrow \alpha_2 = 90°$.

$$v_{1,t} = v_1 \cos \alpha_1 \tag{13}$$

$$v_{2,t} = v_2 \cos \alpha_2 \tag{14}$$

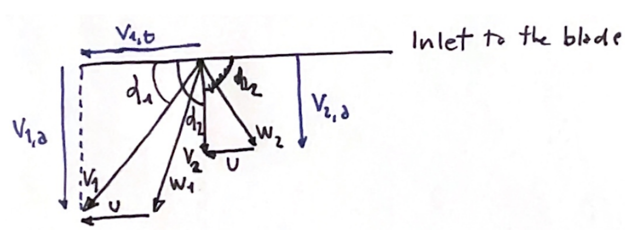

Figure 4: Velocity triangle of a turbine blade.

5 Heat transfer

Problem 8: Radiation loss

First, familiarize yourself with problem and solution of worked example SR3.1 (next page). Then, calculate the heat lost per hour by radiation from the pot in this example. Check that it indeed is less than the heat lost by convection.

Solution:
Heat is lost by radiation from the top and sides of the pot (assumed to be at the temperature of boiling water T1 = 373K) to a completely surrounding space at ambient temperature T2 = 293K. (Note: the heat loss is insensitive to small variations in T2 .)
Radiative heat flow between grey surfaces (T&W, Chapter Review 3. Table C.5)

$$P_{12} = \epsilon_1 \sigma A_1 (T_1^4 - T_2^4) \tag{15}$$

Here, $A_1 = \pi(0.11\text{m})^2 + 2\pi(0.11\text{m})(0.11\text{m}) = 0.114\text{m}^2$. Emittance of aluminium pot is $\epsilon_1 = 0.095$ (from Table B.4 – shiny) or 0.18, when you assumed it to be unpolished. Try to provide some argumentation for the value you assume, and realize the correct application of the theory is most important (note that ϵ_1 should not be 1.0, as this is only true for black bodies!). σ is the Stefan-Boltzman constant.

$$P_{12} = (0.095)(5.67\cdot 10^{-8} W m^{-2} K^{-4})(0.114 m^2)\cdot[(373K)^4 - (293K)^4] = 7.4\text{W} = 26\text{kJ/h} \tag{16}$$

which is much less than the 700 kJ/h lost by convection (last line of example SR3.1).

Problem 9: Heat through glass windows

A room has two glass windows each 1.5 m high, 0.8 m wide and 5.0 mm thick. The temperature of the air and wall surface inside the room is 20 °C. The temperature of the outside air is 0 °C. There is no wind. See Figure 5.

1. Calculate the heat loss through the glass assuming (falsely) that the only resistance to heat flow is from conduction through the material of the glass.

2. Calculate the heat loss through the glass assuming allowing (correctly) for the thermal resistance of the air boundary layers against the glass, as in the figure. Hint: Assume as a first approximation that $T_2 \approx T_3 \approx (T_1 + T_4)/2$. Justify this assumption afterwards.

Solution:
(1)
Total area of the two windows considered together is $A = 2 \times 1.5\text{m} \times 0.8\text{m} = 2.4\text{m}^2$.
Conduction alone:
Using Eqn. R3.10 from book T&W and data from Table B.3,

$$R_{g1} = \frac{\Delta x}{kA} = \frac{(5.0 \cdot 10^{-3}\text{m})}{(0.96\text{W/mK})(1.2\text{m}^2)} = 4.34 \cdot 10^{-3}\text{KW}^{-1} \tag{17}$$

The identical windows are in parallel between the inside and outside, so the total resistance is half the single resistance,

$$\frac{1}{R_{14}} = \frac{1}{R_{g1}} + \frac{1}{R_{g1}} \Rightarrow R_{14} = \frac{1}{2}R_{g1} \tag{18}$$

The heat loss is

$$P_{14} = \frac{T_1 - T_4}{R_{14}} = \frac{20\text{K}}{2.17 \cdot 10^{-3}\text{KW}^{-1}} = 9.2 \cdot 10^3\text{W} = 9.2\text{kW} \tag{19}$$

(2)
The convective resistance of unit area on each side of the window is (R3.16)

$$r_{12} = \frac{X}{Nk} \tag{20}$$

the characteristic dimension X is the vertical length of the windows, so $X = 1.5\text{m}$. From Table B1, the thermal conductivity of air at about 10 °C is k =2.49x10-2 $\text{Wm}^{-1}\text{K}^{-1}$ (interpolated between values for 0 °C and 20°C) To find the Nusselt number N we first calculate the Rayleigh number A from (R3.21).

$$A = \frac{g\beta}{\kappa\nu}X^3\Delta T \tag{21}$$

Using Table B.1,

$$\frac{A}{X^3\Delta T} = 1.25 \cdot 10^8\text{m}^3\text{K}^{-1} \Rightarrow A = 4.219 \cdot 10^9 \tag{22}$$

The flow is therefore turbulent and we use formula C.6.

$$N = 0.2A^{0.4} = 1.416 \cdot 10^3 \tag{23}$$

Using R3.16

$$r_{12} = r_{34} = \frac{1.5m}{(1.416 \cdot 10^3)(2.49 \cdot 10^{-2}\text{Wm}^{-1}\text{K}^{-1})} = 0.04254\text{m}^2\text{KW}^{-1} \tag{24}$$

So the convective resistance on each side is

$$R = \frac{r_{34}}{A} = \frac{0.04254\text{m}^2\text{KW}^{-1}}{1.2\text{m}^2} = 0.0354\text{KW}^{-1} \tag{25}$$

For heat loss from the room, the two windows are in parallel

$$R_{12} = R_{34} = \frac{1}{2}R = 0.0177\text{KW}^{-1} \tag{26}$$

The total thermal resistance is

$$R_{14} = R_{12} + R_{23} + R_{34} = 0.0177 + 0.0022 + 0.0177 = 0.0376\text{KW}^{-1} \tag{27}$$

The heat flow is

$$P_{14} = \frac{T_1 - T_4}{R_{14}} = \frac{20\text{K}}{0.0376\text{KW}^{-1}} = 532\text{W} \tag{28}$$

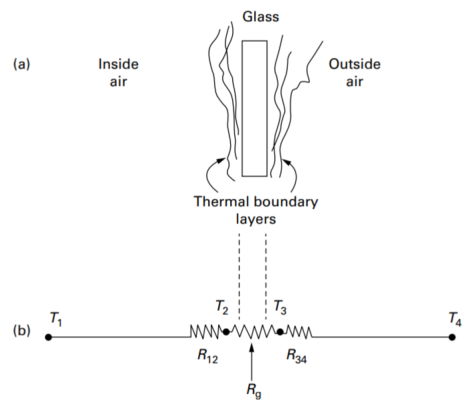

Figure 5: Heat loss through a window; see problem 9.

Worked Example SR3.1 Convective cooling of a cooking pot

A metal cooking pot with a shiny outside surface, of the dimensions shown in Fig. H1 is filled with food and water and placed on a cooking stove. What is the minimum energy required to maintain it at boiling temperature for one hour, (1) if it is sheltered from the wind (2) if it is exposed to a breeze of 3.0 m/s?

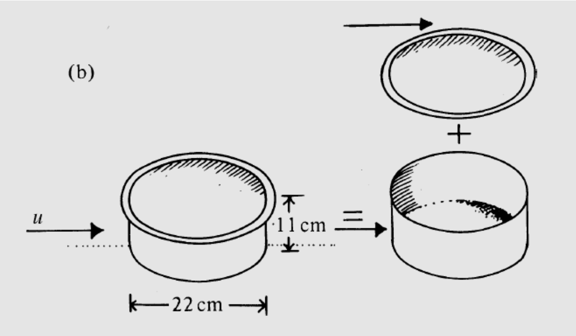

Figure H1: Diagram of a cooking pot with lid.

We assume that the lid is tight, so that there is no heat loss by evaporation. We also neglect heat loss by radiation. Since the conductive resistance of the pot wall is negligible, the problem is then reduced to calculating the convective heat loss from the top and sides of a cylinder with a surface temperature of 100°C. We shall consider the ambient (air and surrounding walls) temperature to be 20°C. Therefore, heat transfer properties of the air are evaluated at the mean temperature T = 60°C.

(1) Free convection alone. For the top: (R3.21)

$$\mathcal{A} = \frac{g\beta X^3 \Delta T}{\kappa \nu}$$

However, easier is using table B.1 – which already gives a value for $\mathcal{A}/X^3\Delta T$. For $T = 60℃$ (mean temperature):

$$\mathcal{A} = (5.8 \cdot 10^7 m^{-3} K^{-1})(0.22m)^3(80K) = 5.0 \cdot 10^7$$

The geometry is that of C.2, therefore:

$$\mathcal{N} = 0.14 \cdot \mathcal{A}^{0.33} = 48.5$$

and (from R3.1 + R3.15)

$$P_{top} = \frac{Ak\mathcal{N}\Delta T}{X} = \frac{(\pi/4)(0.22\text{ m})^2(2.88 \cdot 10^{-2}\text{ Wm}^{-1}\text{K}^{-1})(48.5)(80\text{ K})}{(0.22\text{ m})} = 19.3\text{ W}$$

For the sides, X = 0.11 m:

$$\mathcal{A}_{side} = \mathcal{A}_{top}\left(\frac{0.11\text{ m}}{0.22\text{ m}}\right)^3 = 6.2 \cdot 10^6$$

and from (C.5)

$$\mathcal{N} = 0.56 \cdot \mathcal{A}^{0.25} = 27.9$$

So (R3.1 + R3.15)

$$P_{side} = \frac{\pi(0.22\text{ m})(0.11\text{ m})(2.88 \cdot 10^{-2}\text{ Wm}^{-1}\text{K}^{-1})(27.9)(80\text{ K})}{(0.11\text{ m})} = 44.5\text{ W}$$

Hence

$P_{\text{free}} = P_{\text{top}} + P_{\text{side}} = 63.8\ \text{W} = (0.0638\text{kW})(3600\ \text{s/h}) \approx 0.2\ \text{MJ/h}$

(2) *Forced plus free convection*. Here we calculate the forced convective power losses separately, and add them to those already calculated for free convection, in order to obtain an estimate of the total convective heat loss P_{total}.

For the top (*R3.18):*

$$\mathcal{R} = \frac{(3\ \text{ms}^{-1})(0.22\ \text{m})}{(1.9 \cdot 10^{-5}\ \text{m}^2\text{s}^{-1})} = 3.5 \cdot 10^4$$

$\mathcal{P} = \nu/\kappa = (1.9 \cdot 10^{-5}\ \text{m}^2\text{s}^{-1})/(2.7 \cdot 10^{-5}\ \text{m}^2\text{s}^{-1}) = 7.0 \cdot 10^{-1}$ *(R3.19)*

which suggests the use of (C.8):

$\mathcal{N} = 0.66 \cdot \mathcal{R}^{0.5}\mathcal{P}^{0.33} = 111$

So (from R3.1 + R3.15)

$$P_{top} = \frac{Ak\mathcal{N}\Delta T}{X} = \frac{(\pi/4)(0.22\ \text{m})^2(2.88 \cdot 10^{-2}\ \text{Wm}^{-1}\text{K}^{-1})(111)(80\ \text{K})}{(0.22\ \text{m})} = 44.0\ \text{W}$$

For the sides, as for the top,

$\mathcal{R} = 3.5 \cdot 10^4$

$\mathcal{P} = 7.0 \cdot 10^{-1}$

which suggests the use of (C.11):

$\mathcal{N} = 0.26 \cdot \mathcal{R}^{0.6}\mathcal{P}^{0.3} = 125$

And

$$P_{\text{side}} = \frac{\pi(0.22\text{m})(0.11\text{m})(2.88 \cdot 10^{-2}\text{Wm}^{-1}\text{K}^{-1})(125)(80\text{K})}{(0.22\text{m})} = 99\text{W}$$

Hence

$P_{\text{forced}} = 99 + 44 = 143\text{W}$

The total estimate is

$P_{\text{total}} = P_{\text{forced}} + P_{\text{free}} = 207\text{W} = (0.0207\ \text{kW})(3600\ \text{s/h}) \approx 0.7\text{MJ/h}$

i.e. about 3 times the energy per unit time of the sheltered cooking.