Examples of exercises you should be able to solve

Ideal Gas Law

a) Isothermic expansion of an ideal gas - work done and heat gained

Consider a piston chamber which is undergoing an expansion at constant temperature. The fluid within the chamber is helium, which behaves as ideal gas, and has molar mass of 4 kg/kmol and R = 2.07 kJ/kgK. The temperature is 100 °C, the initial pressure 30 bar, and the final pressure 1 bar. Calculate:

Helium has the ideal gas law equation of state

$$pV = Nk_bT$$

Where N is the number of Helium atoms, and k_b is boltzmann's constant.

We can rewrite this in terms of the number of moles of the gas n:

$$pV = nN_Ak_bT$$

Where n is the number of moles, and N_A is avogadro's constant.

We can also write this in terms of the mass of gas m by substituting $n = \frac{m}{M}$, where M is the molar mass (mass of one mole of Helium)

$$pV = \left(\frac{m}{M}\right)N_Ak_bT$$

$$pV = m\left(\frac{N_Ak_b}{M}\right)T$$

$$p = mR_s\frac{T}{V}$$

where R_s is the *specific gas constant*, in our case $2.07\ \text{kJ}\ \text{kg}^{-1}K^{-1}$

The expansion work p-V is:

$$dW = \int_{V_1}^{V_2} pdV$$

$$dW = \int_{V_1}^{V_2} \frac{mR_sT}{V}dV$$

$$\Delta W = mR_sT\ln\left(\frac{V_2}{V_1}\right)$$

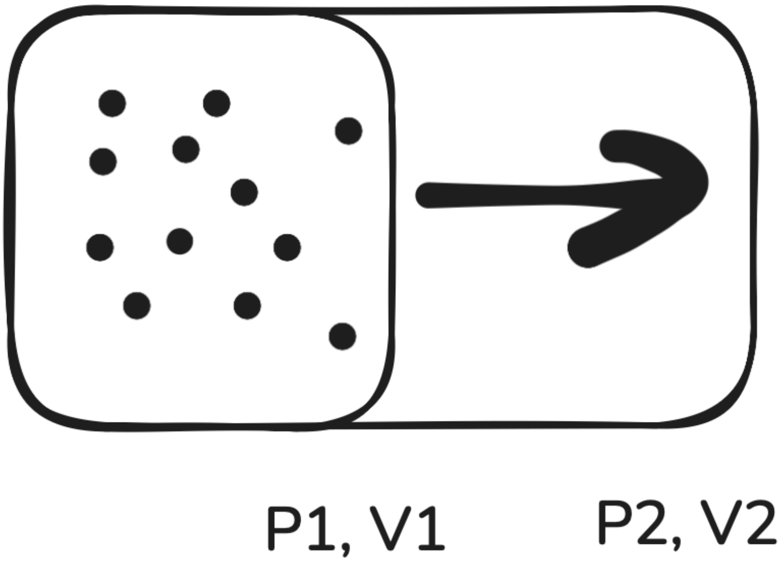

Figure 1: Diagram to show initial and final pressure, volume

The specific expansion work is

$$\frac{\Delta W}{m} = R_s T \ln\left(\frac{V_2}{V_1}\right)$$

For constant temperature, $\frac{V_2}{V_1} = \frac{p_1}{p_2}$

$$\frac{\Delta W}{m} = R_s T \ln\left(\frac{p_1}{p_2}\right)$$

Where $R_s = 2.07 \text{ kJ kg}^{-1}\text{K}^{-1}$, and $T = 373K$, $\frac{p_1}{p_2} = 30$

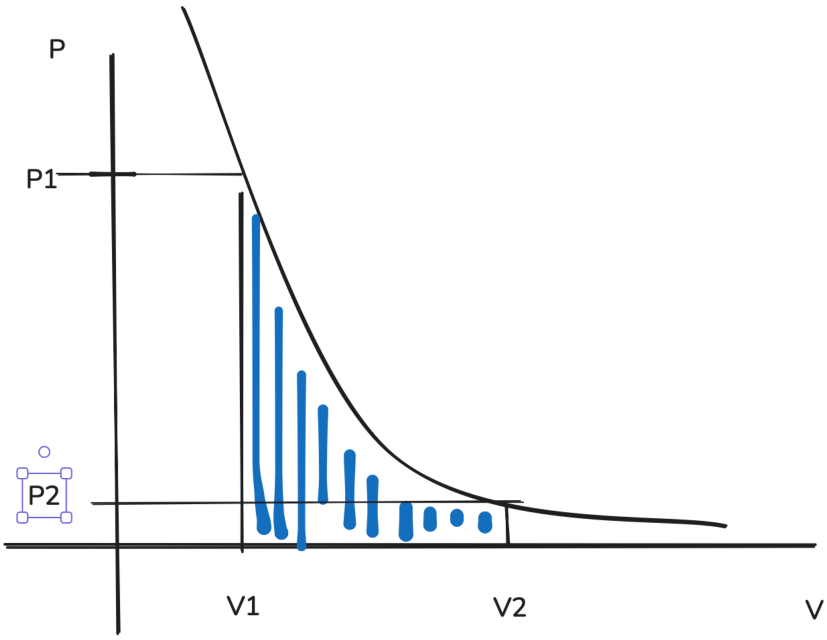

Figure 2: Blue area is the work done by the gas

We can do this calculation in the Python REPL:

```
>>> import math
>>> (2.07) * 373 * math.log(30)
2626.0985103551666
```

hence:

$$\Delta W = 2626 \text{ kJ kg}^{-1}$$

For an *isothermal* process of an ideal gas:

$\Delta T = 0 \rightarrow \Delta U = 0$

By the first law of thermodynamics, $\Delta U = Q - W$

Q is the energy added to the system as heat W is the work done by the system on its surroundings

The heat supplied to the gas equals the work done by it, since the internal energy isn't changing.

Hence, 2627 kJ kg^{-1} has to be supplied

b) entropy change of isothermically expanding gas

From thermodynamics:

$$dU = TdS - pdV$$

For an isothermic, ideal gas

$$dU = 0$$

$$TdS = pdV$$

$$dS = \frac{p}{T}dV$$

where

$$p = mR\frac{T}{V}$$

so

$$dS = m\frac{R}{V}dV$$

$$\Delta S = mR\ln\left(\frac{V_2}{V_1}\right)$$

$$\Delta S = mR\ln\left(\frac{p_1}{p_2}\right)$$

$$\frac{\Delta S}{m} = \Delta s = R\ln\left(\frac{p_1}{p_2}\right) = 7.04 \text{ kJ kg}^{-1}K^{-1}$$

The cylinder has a piston. You can lock the piston in place with a pin. You can add or remove masses from the piston. You can place the entire cylinder in a hot or cool liquid.

a) Can you decrease the volume without changing the pressure?

Yes:

$$pV = mRT$$

You can reduce the V by reducing T. This can be done by placing the cylinder in a cold liquid.

b) Can you decrease the volume without changing the temperature?

$$V = mR\frac{T}{p}$$

The only way to do this would be to increase the pressure by adding some masses on to the piston

c) Can you decrease the pressure without changing the temperature?

$$p = mR\frac{T}{V}$$

Yeah, we can increase the volume by removing masses and thus decrease the pressure

d) Can you decrease the pressure without changing the volume? If so, how?

$$p = mR\frac{T}{V}$$

Keep it locked with a pin to keep V the same. Dunk in cold liquid.

b) Mass and energy balances

The energy entering the chamber is due to the internal energy of the bit of mass δm_w and due to the pV wor it does $pvdm_w$

Mass balance equation:

$$\dot{m}_w$$

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>>> ((293.07-42) * 150 + 190) / (2769 - 293.07)
15.28738696166693
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