Stepping Motors

Unit: 4/EE/A0 Mechatronics

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Number of Lectures: 10

Recommended text book:
P. P. Acarnley:
Stepping Motors: A Guide to Modern Theory and Practice
Peter Peregrinus (for IEE)
ISBN 0-86341-027-8

Syllabus

Types of stepping motor: variable-reluctance, permanent-magnet, hybrid, single-phase.

Stepping motor drivers, H-bridge, resistor ballasting, chopper drives, drive sequences

Microprocessor control of stepping motors

Static torque characteristic, dynamic response, resonance, pull-in and pull-out characteristics, micro stepping

Stepping motor model, high-speed operation, simulation, velocity-error plane diagrams

Closed-loop control of stepping motors

Stepping Motors

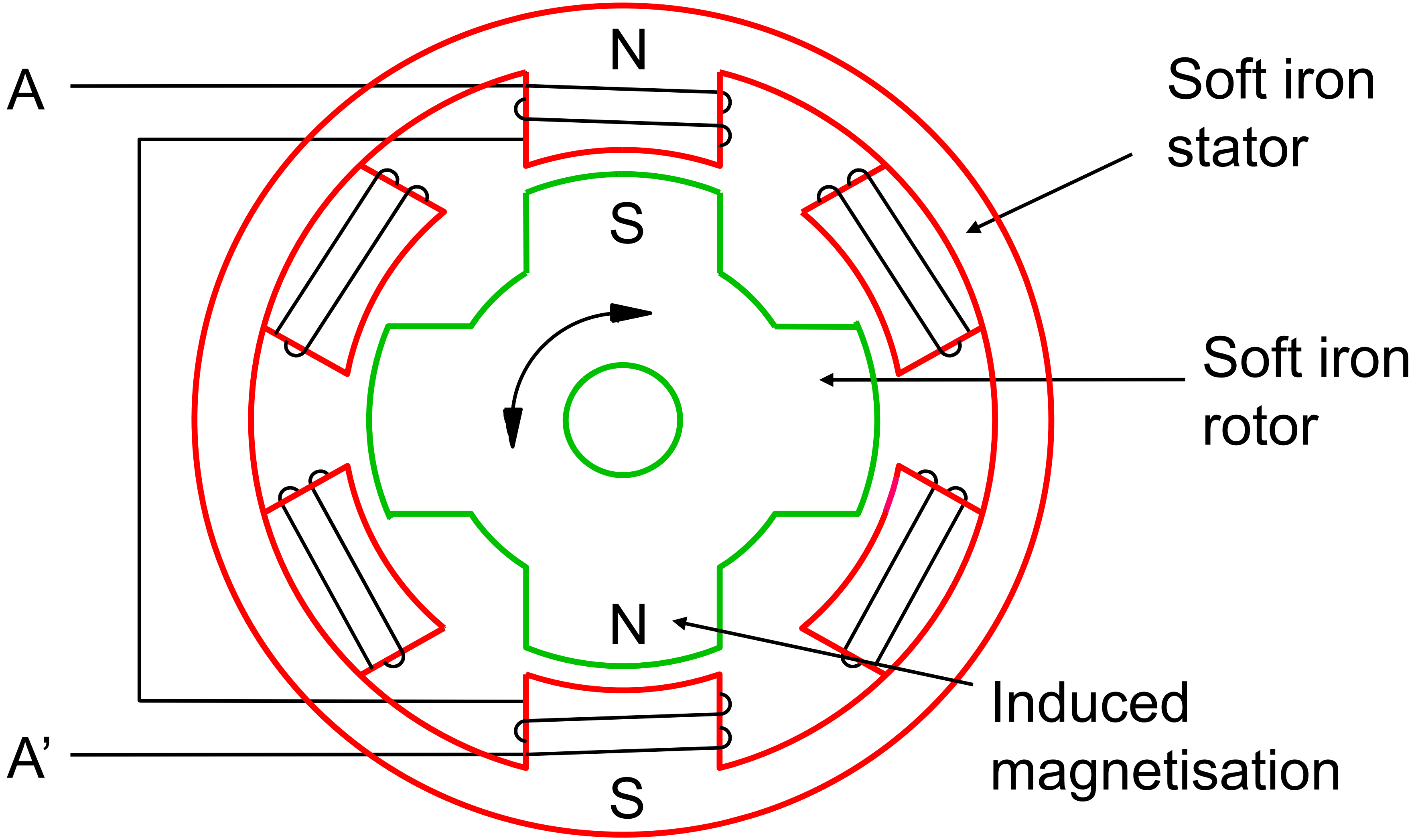
Stepping motors convert switched excitation changes to precise increments of rotation

This property allows stepping motors to be used in positioning systems without the need for feedback

Rotor positioning is achieved by magnetic alignment of rotor and stator poles

There are 3 classes of stepping motor: variable-reluctance motors, permanent-magnet motors and hybrid motors.

Variable-Reluctance Stepping Motor



Variable-Reluctance Stepping Motor

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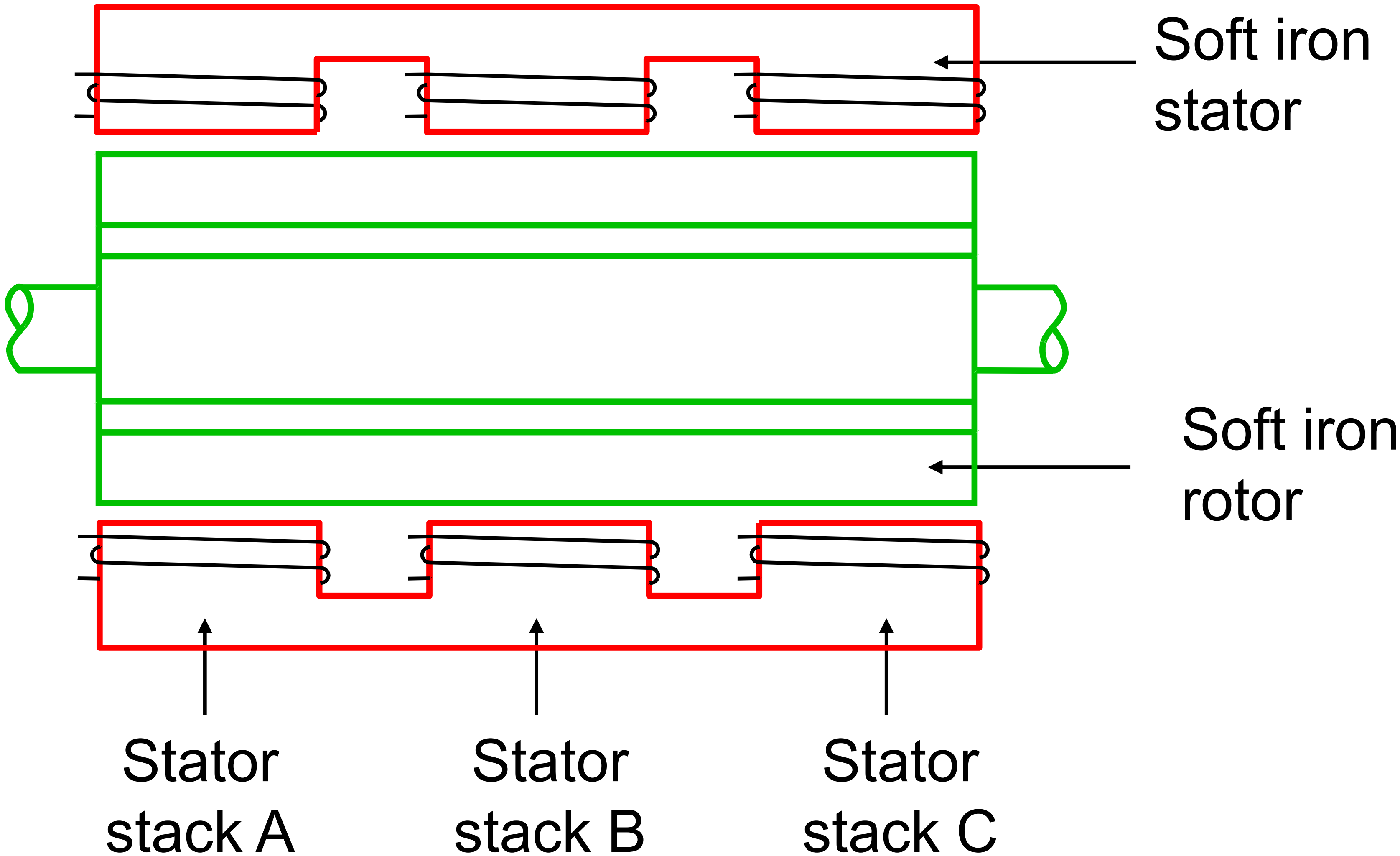
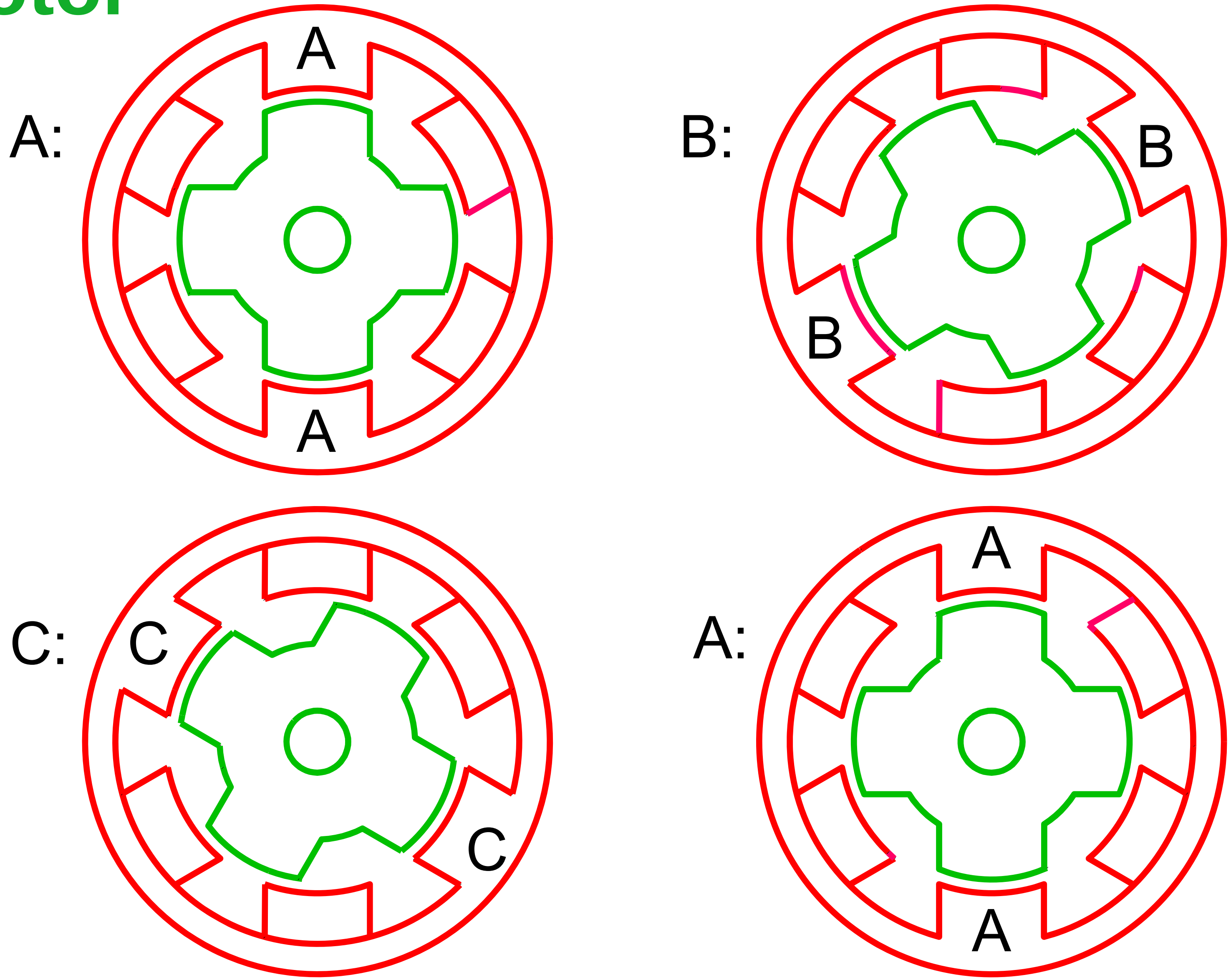


Variable-Reluctance Stepping Motor



Variable-Reluctance Stepping Motor

Anti-clockwise rotation can be produced by exciting the stator windings in the sequence:

A B C A B C A ..

and clockwise rotation can be produced by the sequence:

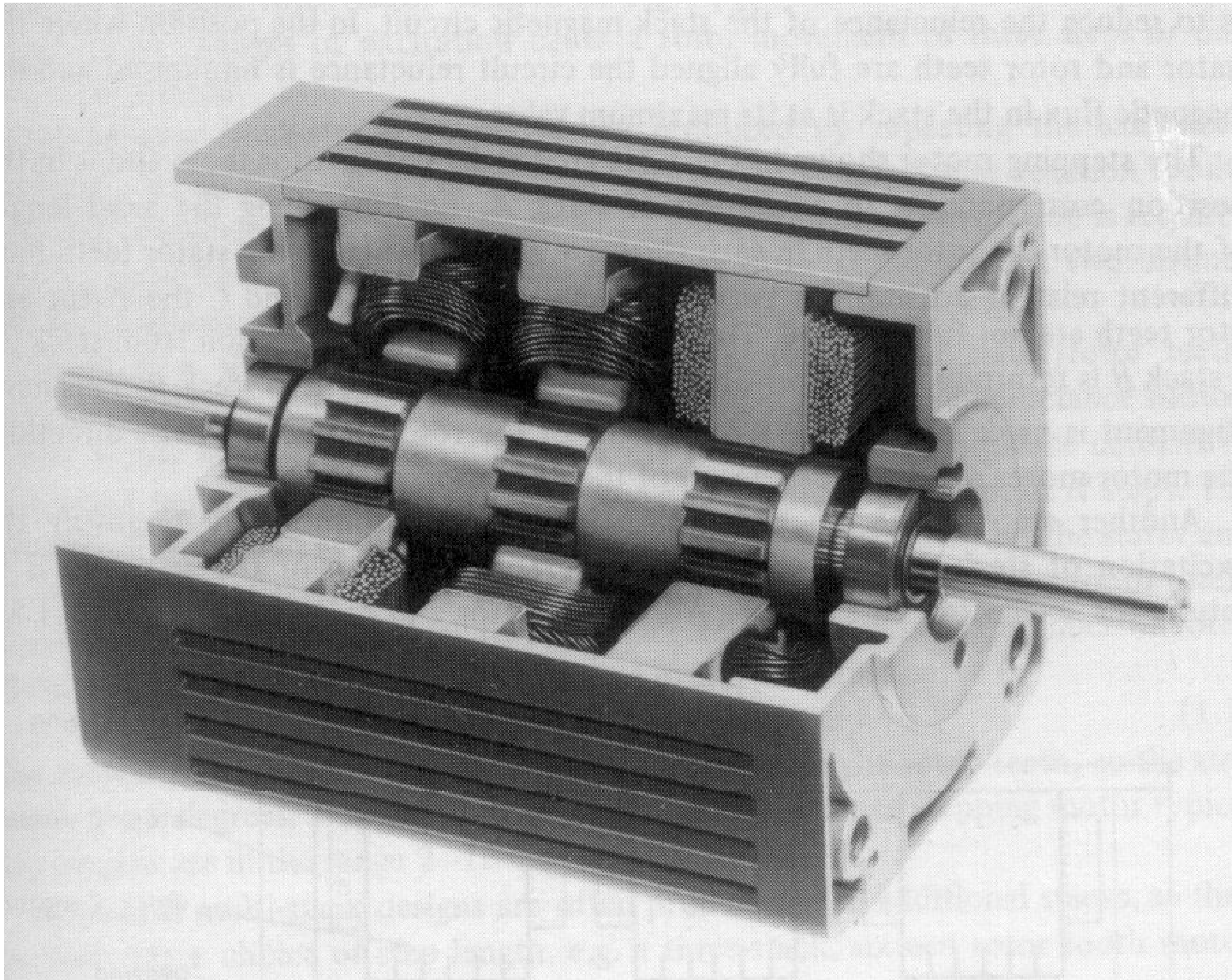
A C B A C B A ..

If the windings A B C A are excited in turn, the rotor moves by one rotor tooth pitch. Thus if p is the number of rotor teeth then the step angle α_s is given by:

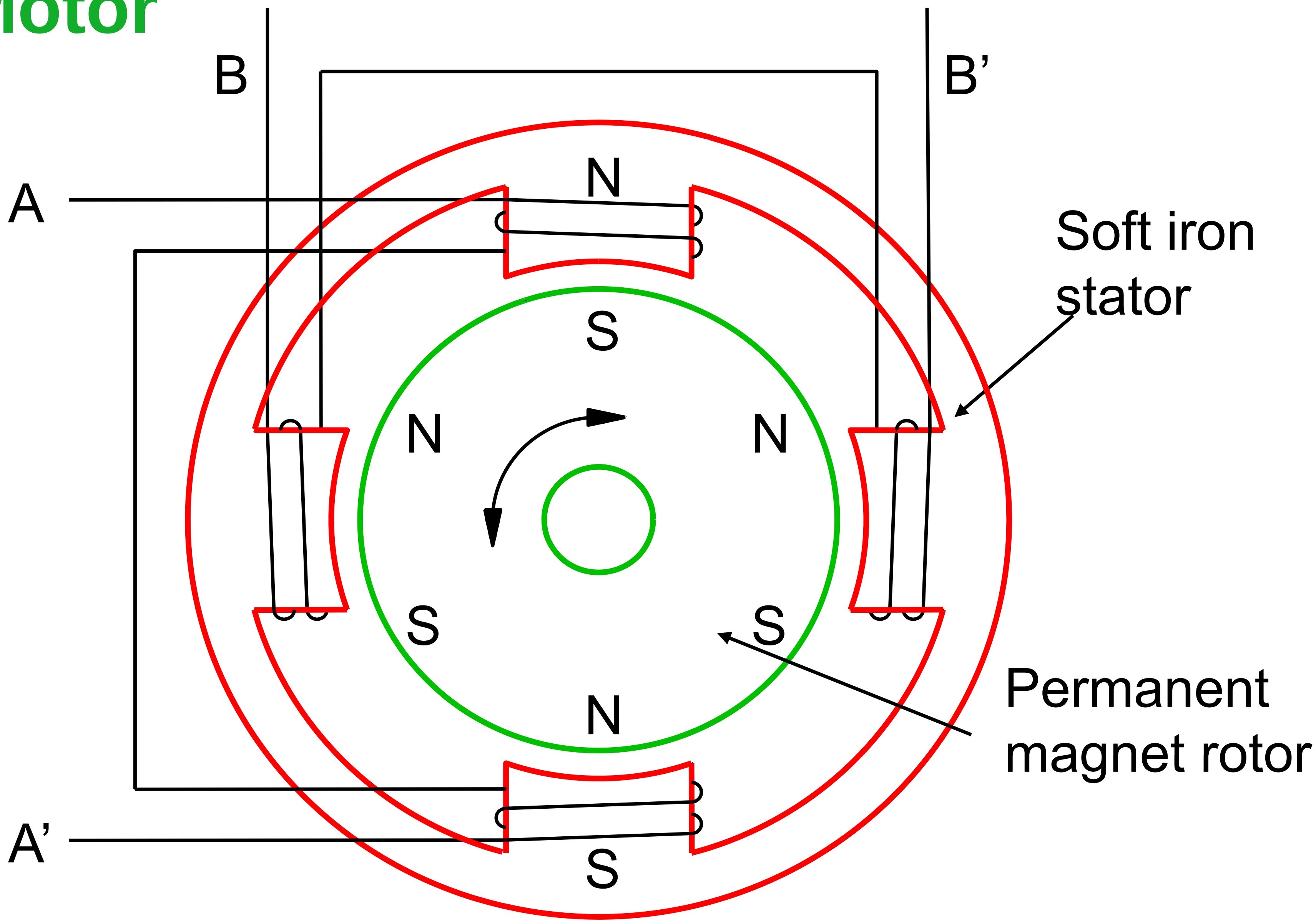
$$\alpha_s = 360/3p = 120/p$$

A typical variable-reluctance stepping motor has 8 rotor teeth giving a stepping angle of 15º

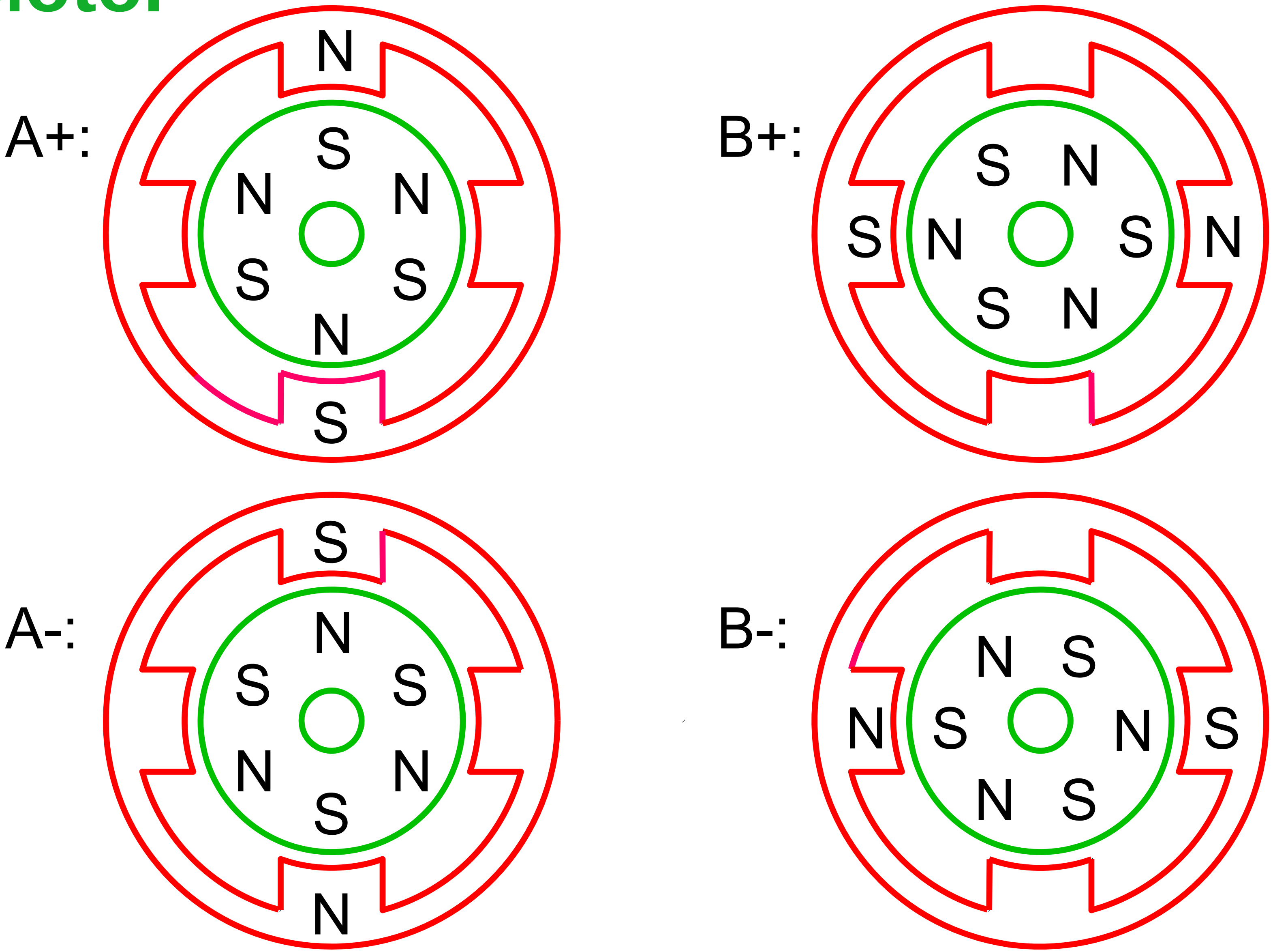
Variable-Reluctance Stepping Motor



Permanent-Magnet Stepping Motor



Permanent-Magnet Stepping Motor



Permanent-Magnet Stepping Motor

Clockwise rotation can be produced by exciting the stator windings in the sequence:

A+ B+ A- B- A+ B+ A- ..

and anti-clockwise rotation can be produced by the sequence:

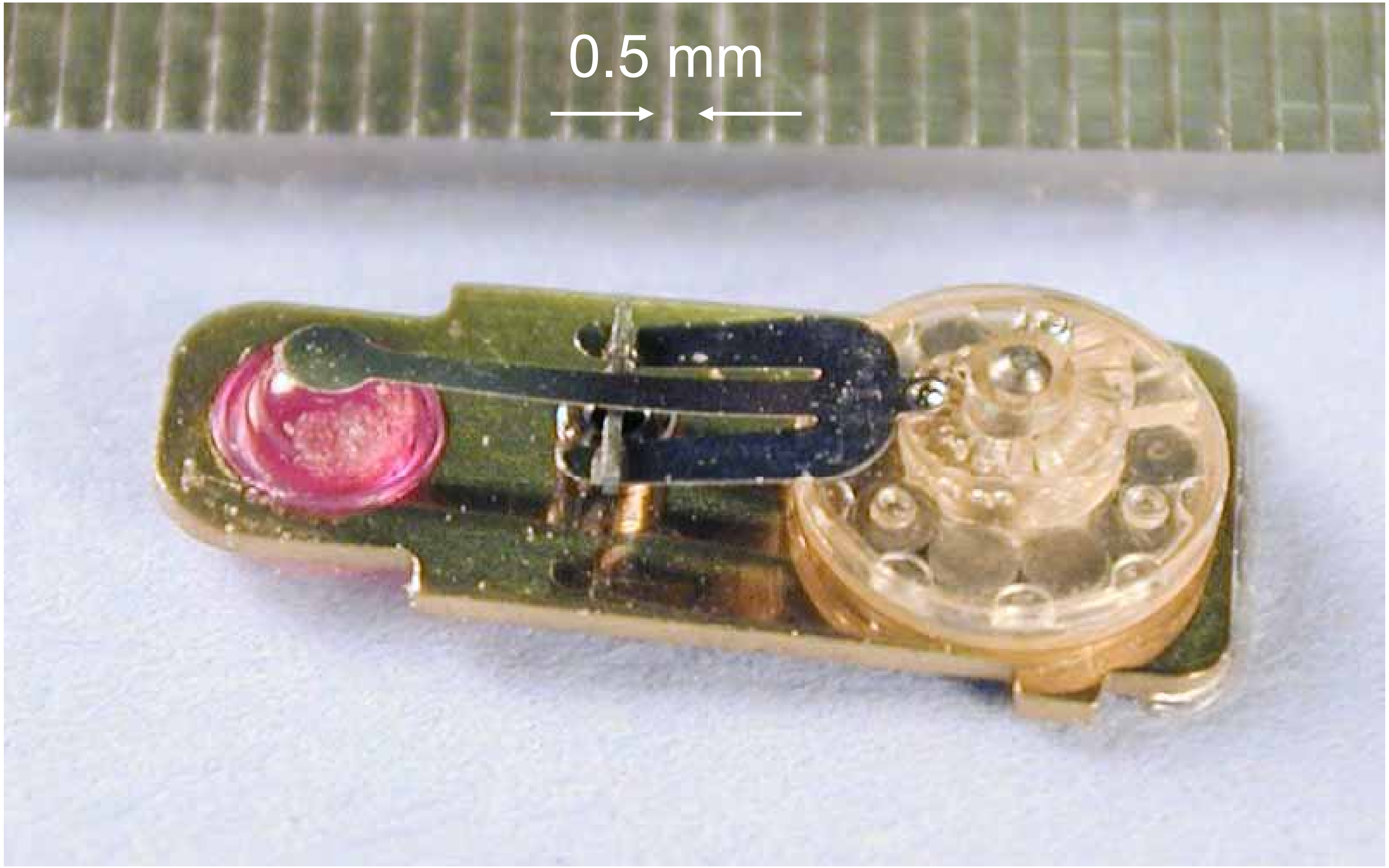
A+ B- A- B+ A+ B- A- ..

If the windings A+ B+ A- B- A+ are excited in turn, the rotor moves by one rotor N pole pitch. Thus if p is the number of rotor N poles then the step angle α_s is given by:

$$\alpha_s = 360/4p = 90/p$$

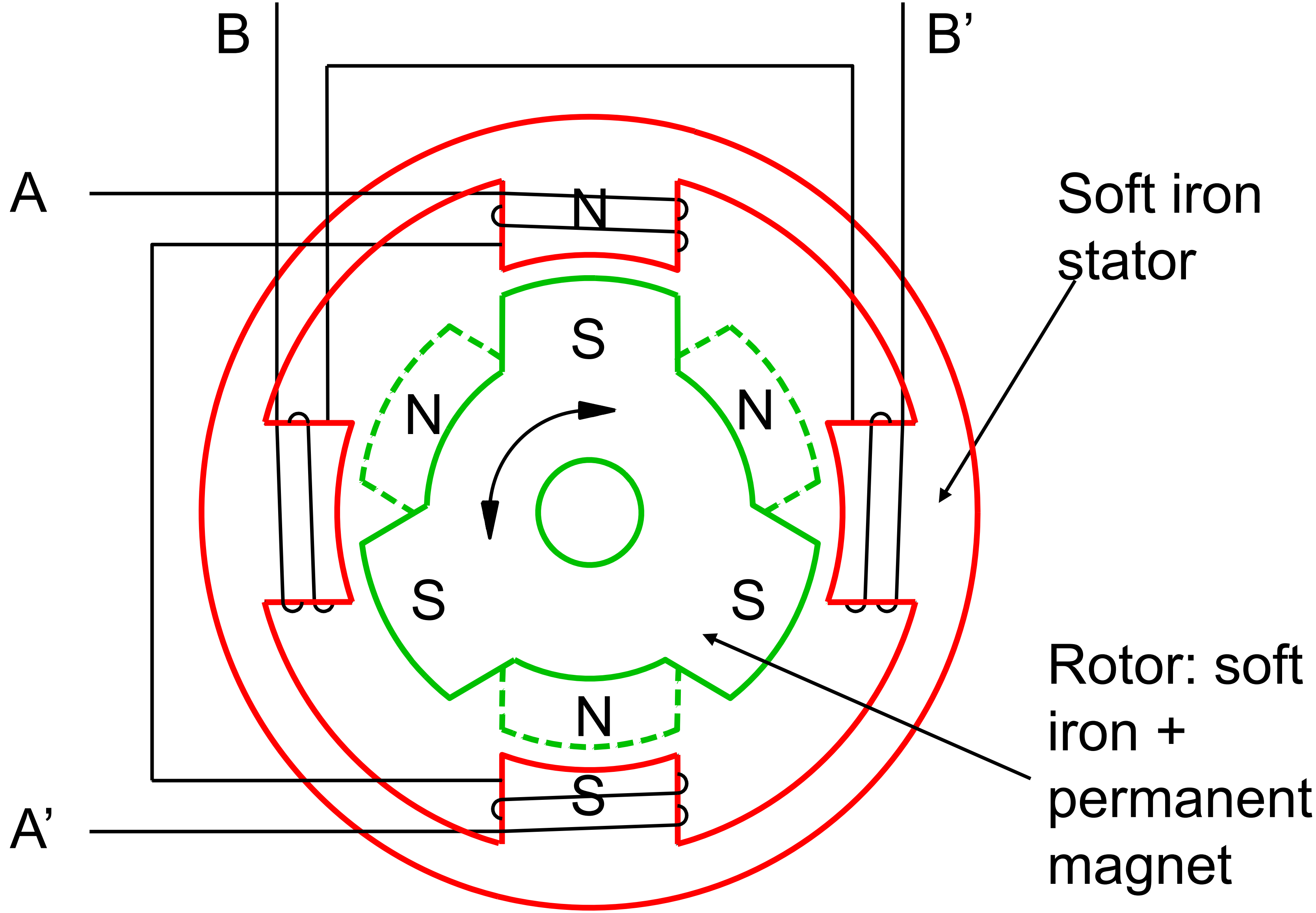
A typical permanent magnet stepping motor has 4 N poles giving a stepping angle of 22.5º

Permanent-Magnet Stepping Motor

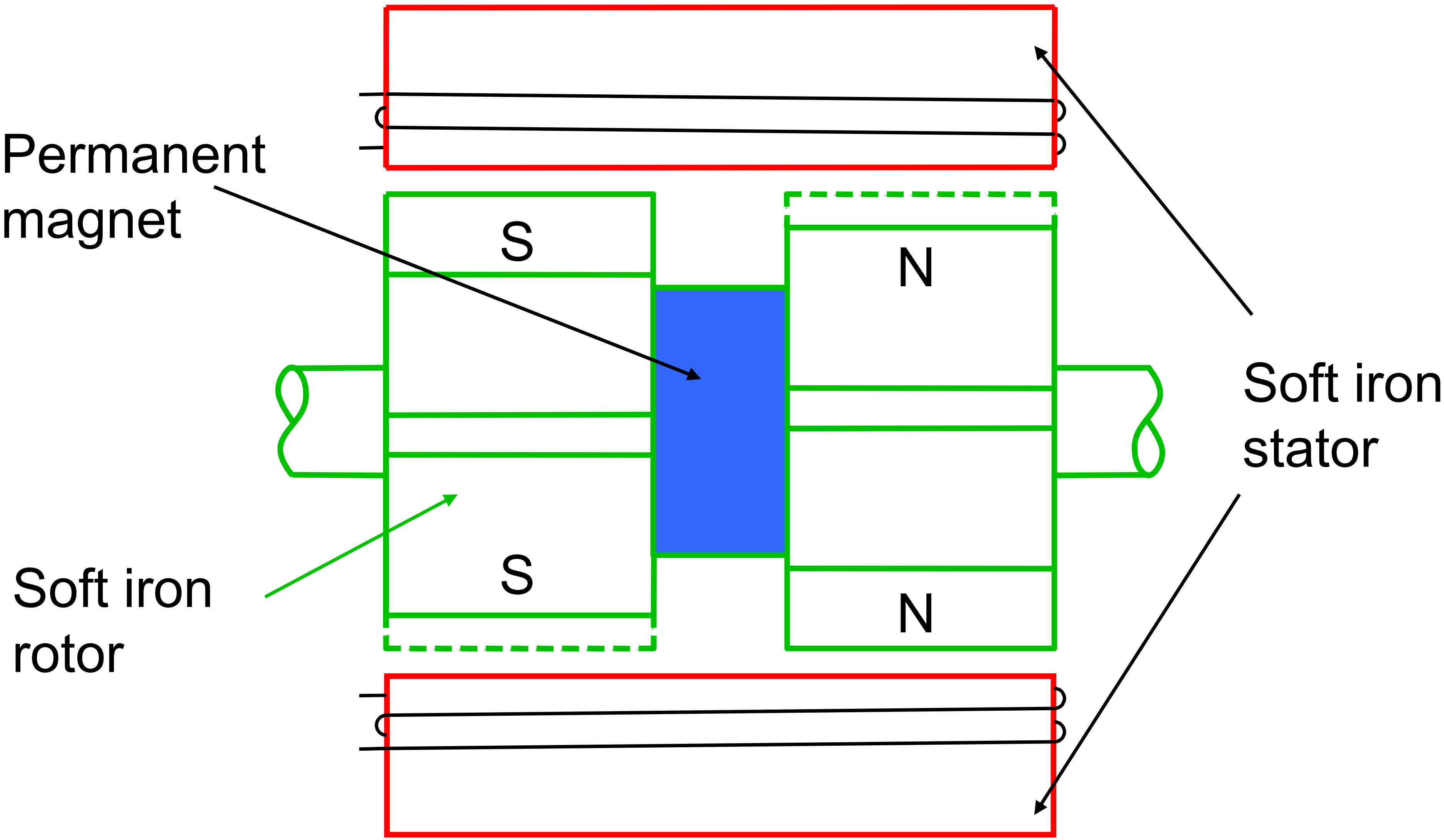


Implantable pressure release valve

Hybrid Stepping Motor



Hybrid Stepping Motor



Hybrid Stepping Motor

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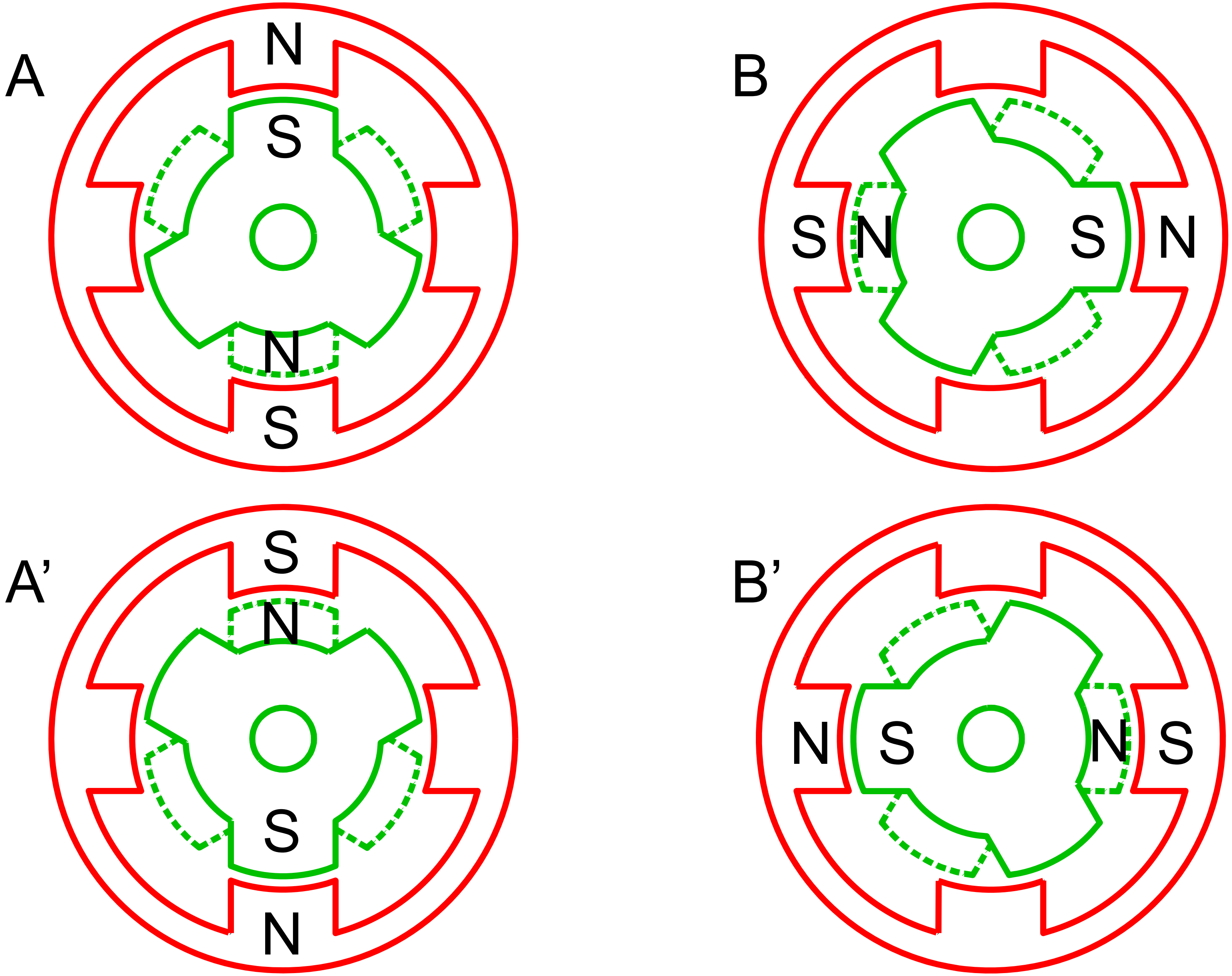


Hybrid Stepping Motor

Anti-clockwise rotation can be produced by exciting the stator windings in the sequence:

A+ B+ A- B- A+ B+ A- ..

and clockwise rotation can be produced by the sequence:

A+ B- A- B+ A+ B- A- ..

If the windings A+ B+ A- B- A+ are excited in turn, the rotor moves by one rotor tooth pitch. Thus if p is the number of rotor teeth then the step angle α_s is given by:

$$\alpha_s = 360/4p = 90/p$$

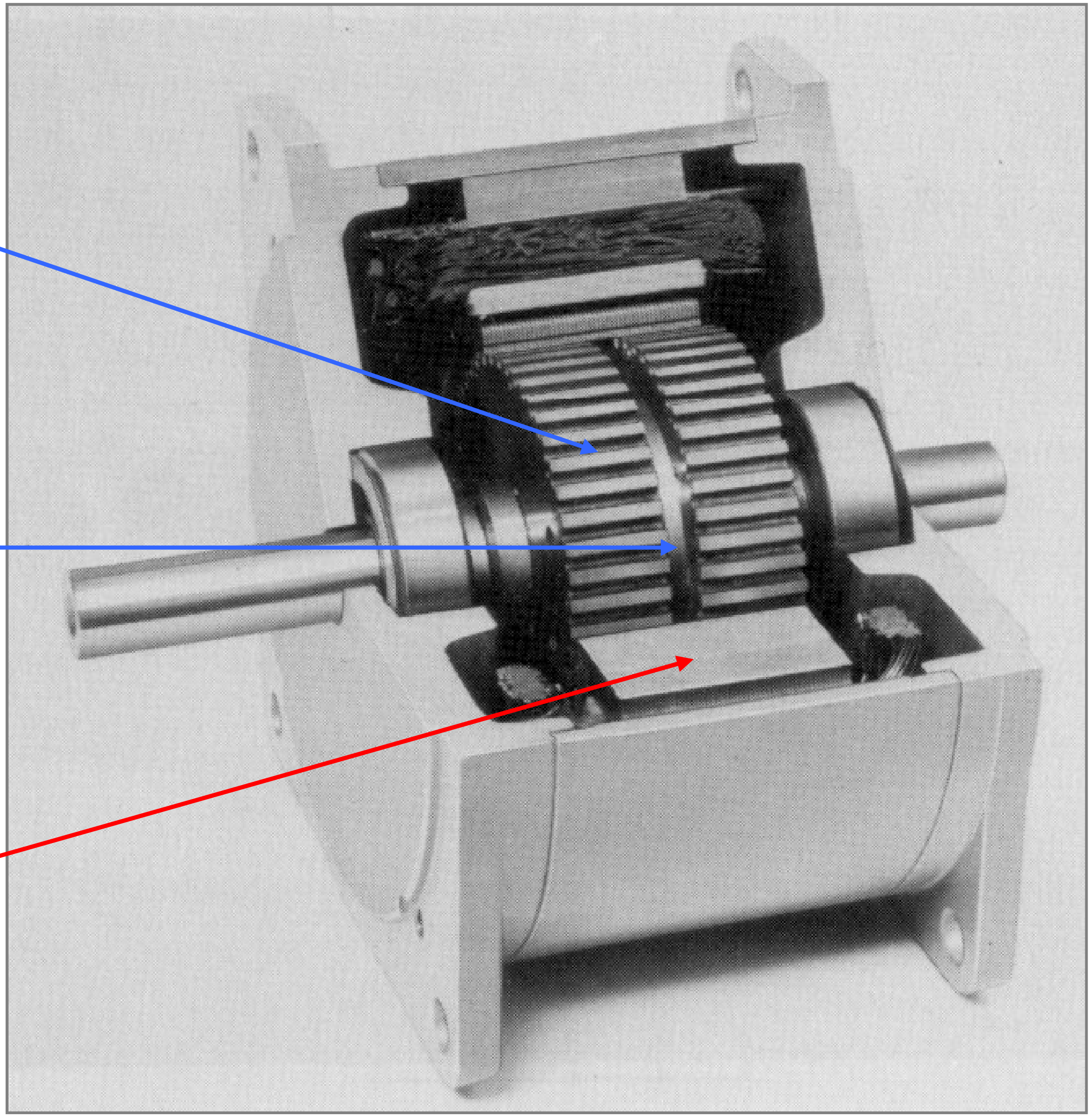
A typical hybrid stepping motor has 50 rotor teeth giving a stepping angle of 1.8º

Hybrid Stepping Motor

Soft iron rotor

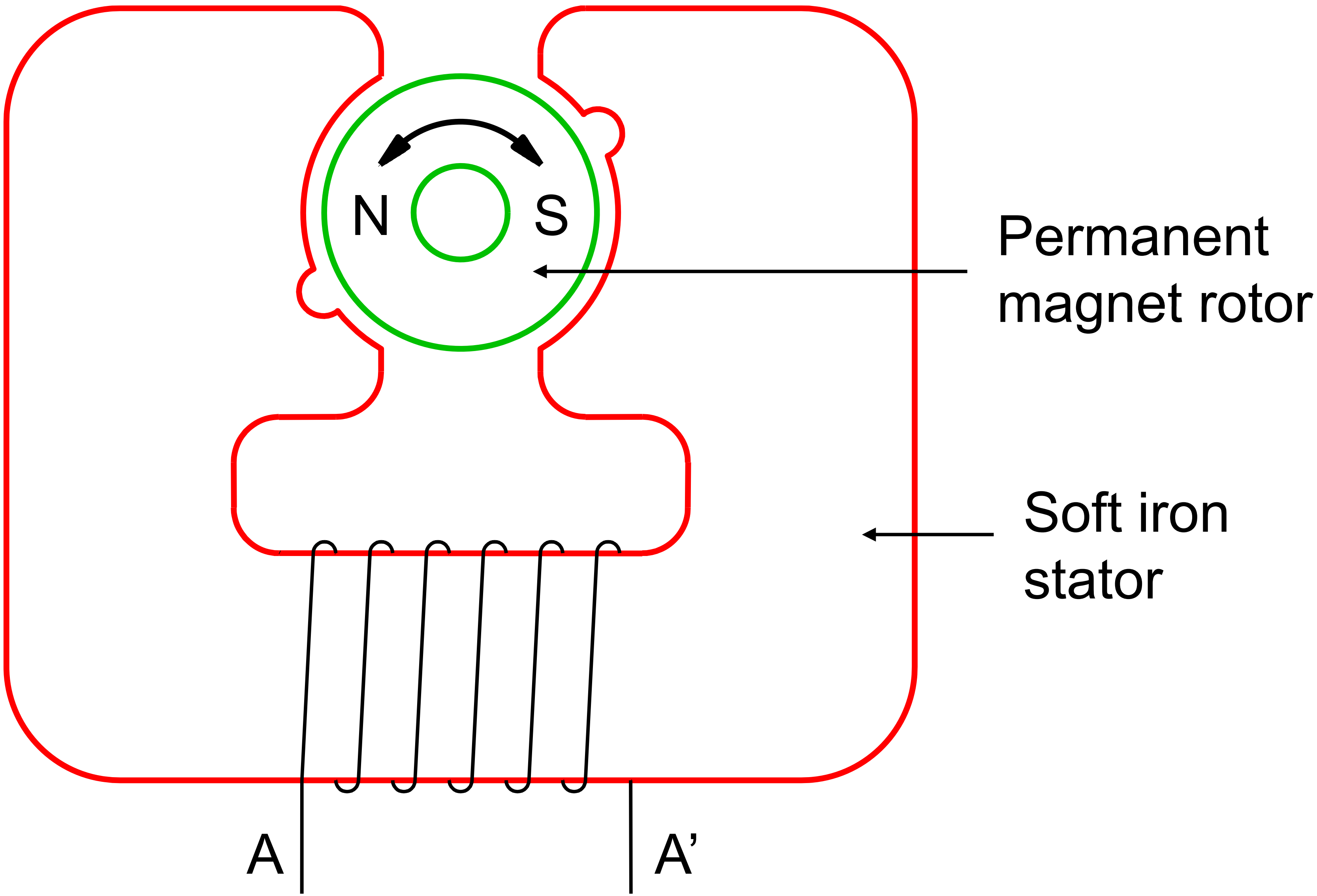
Permanent magnet

Soft iron stator



Hybrid Stepping Motor

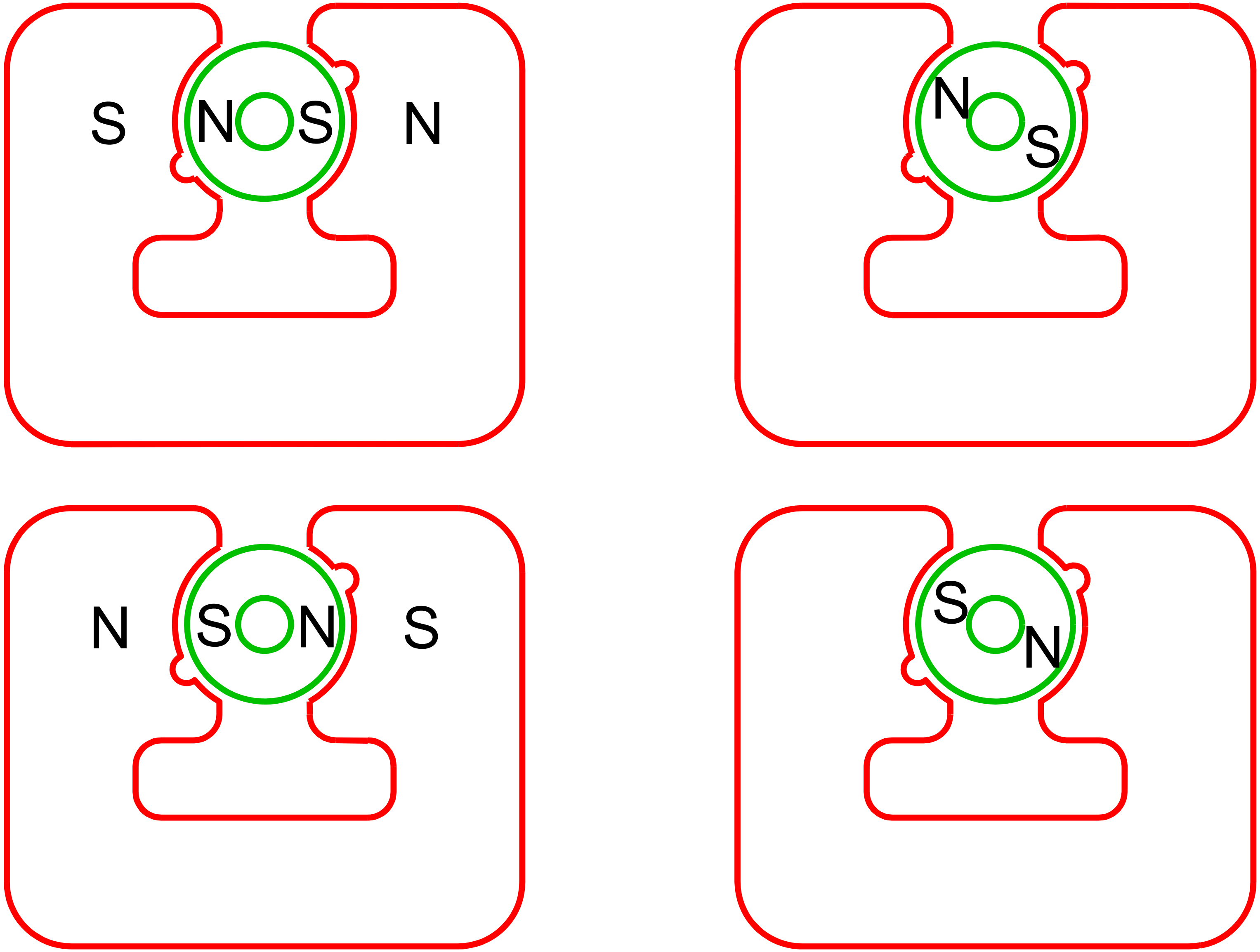
Single-Phase Stepping Motor



Single-Phase Stepping Motor

A+:

A-:



Comparison of Motor Types

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Permanent-magnet stepping motors are inferior in performance to hybrid motors, and are only used in specialised applications

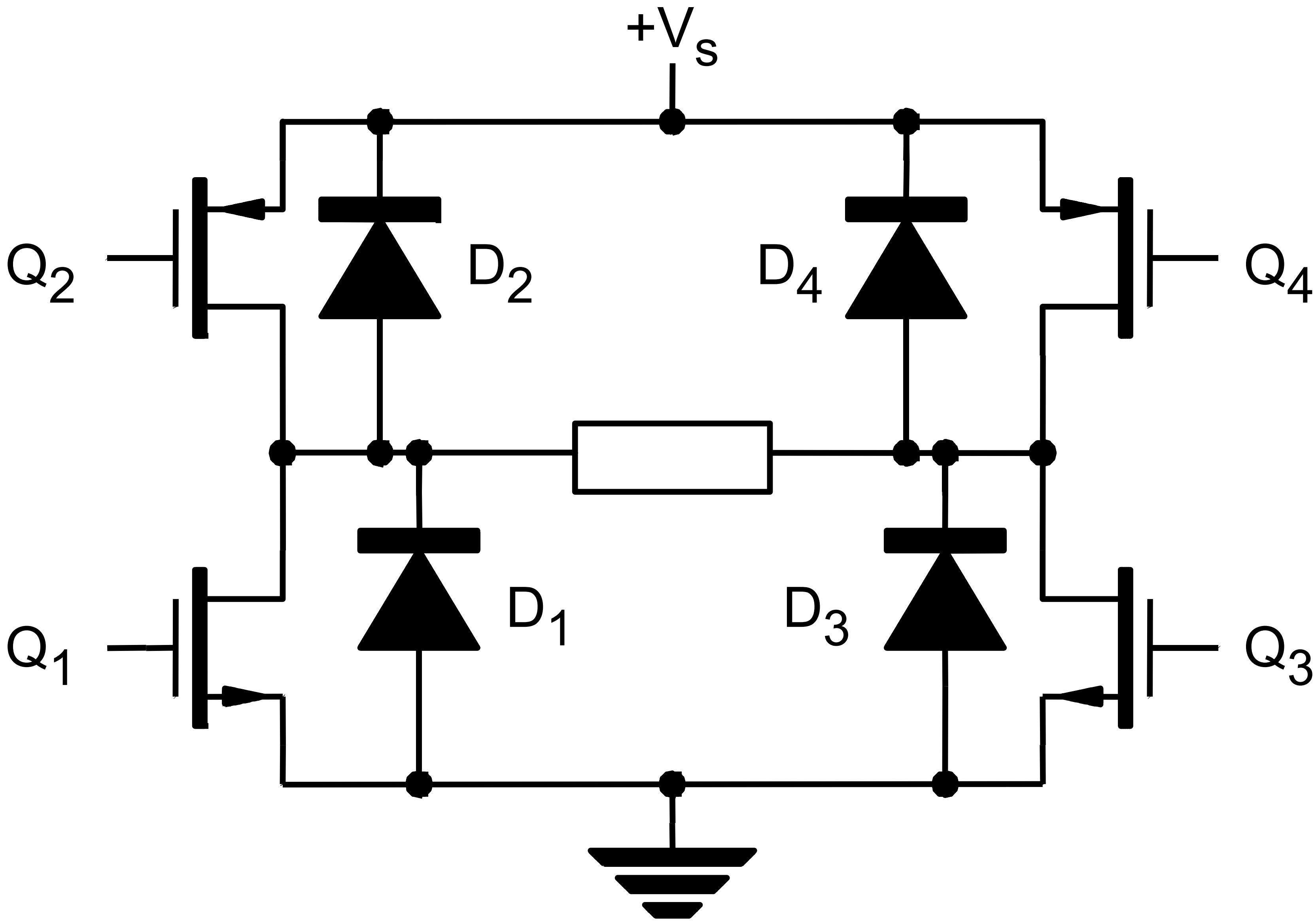
Hybrid motors have a smaller step size and a higher torque than a similar VR motor

Hybrid motors also have a detent torque

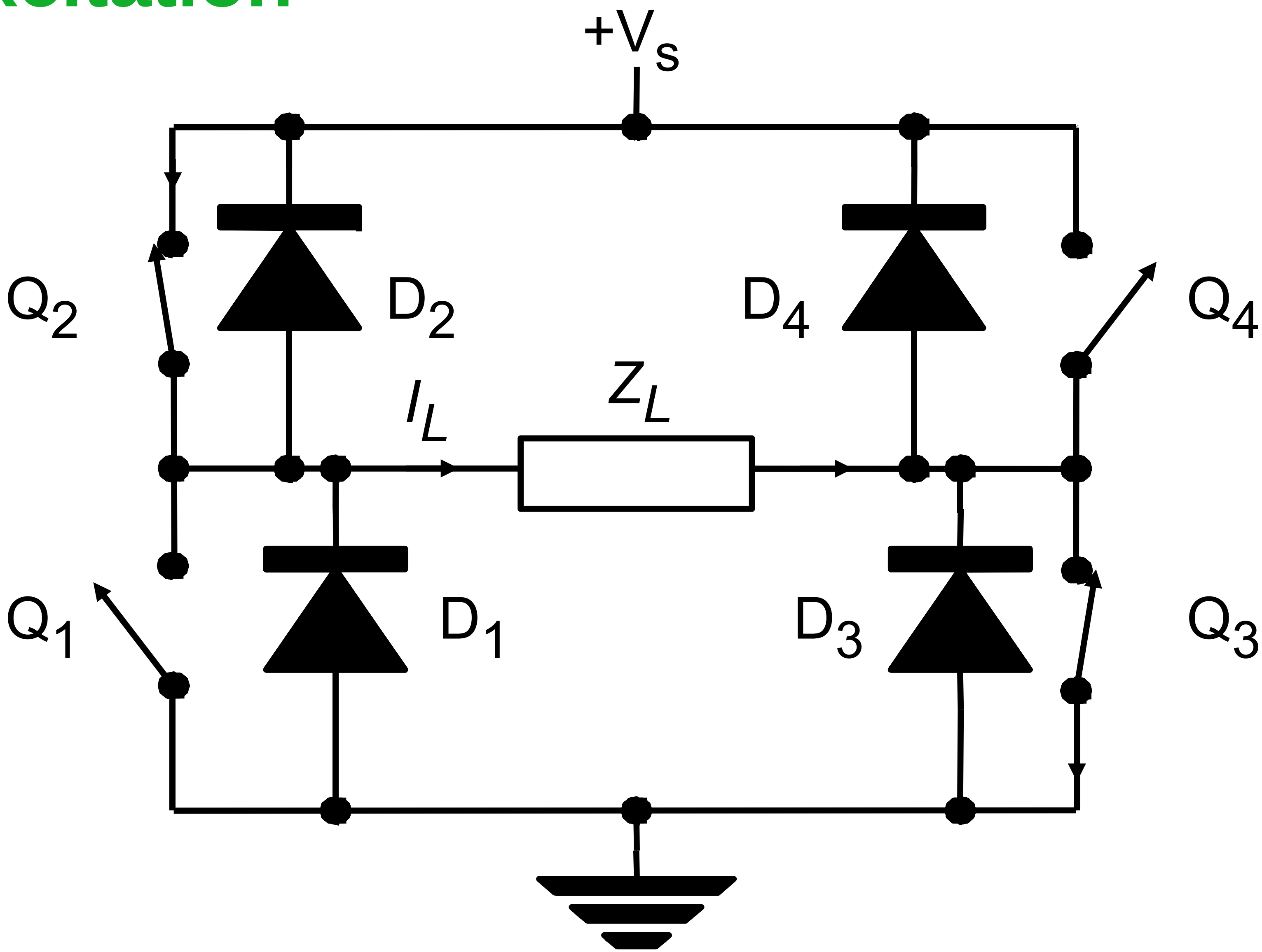
Hybrid motors have 2, rather than 3, windings

VR motors have a lower rotor inertia than hybrid motors

H-Bridge: Voltage Drive

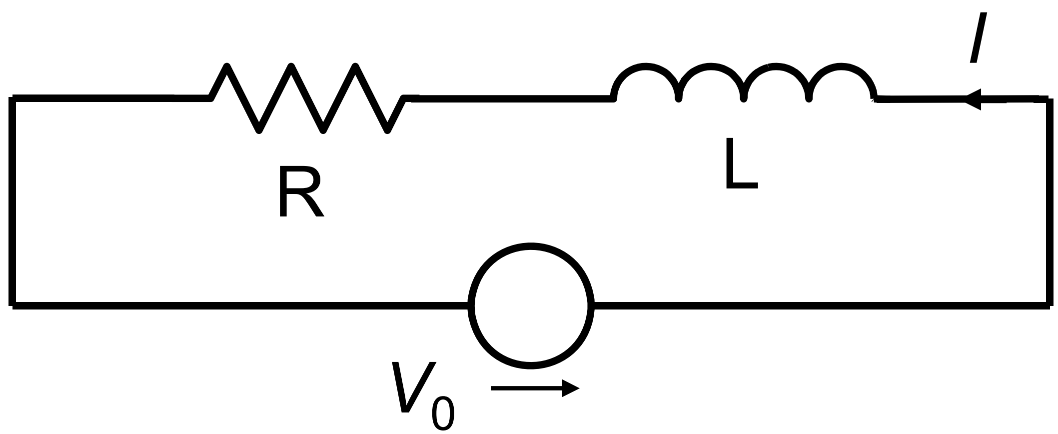


H-Bridge: Voltage Drive, Positive Excitation



H-Bridge: Voltage Drive

Stator winding excitation:



$$R.I + L.\frac{\mathrm{d}I}{\mathrm{d}t} = V_0$$

$$I = I_0.\left\{1 - \exp\frac{-t}{T_0}\right\} \quad \text{where:} \quad I_0 = \frac{V_0}{R} \quad T_0 = \frac{L}{R}$$

H-Bridge: Voltage Drive

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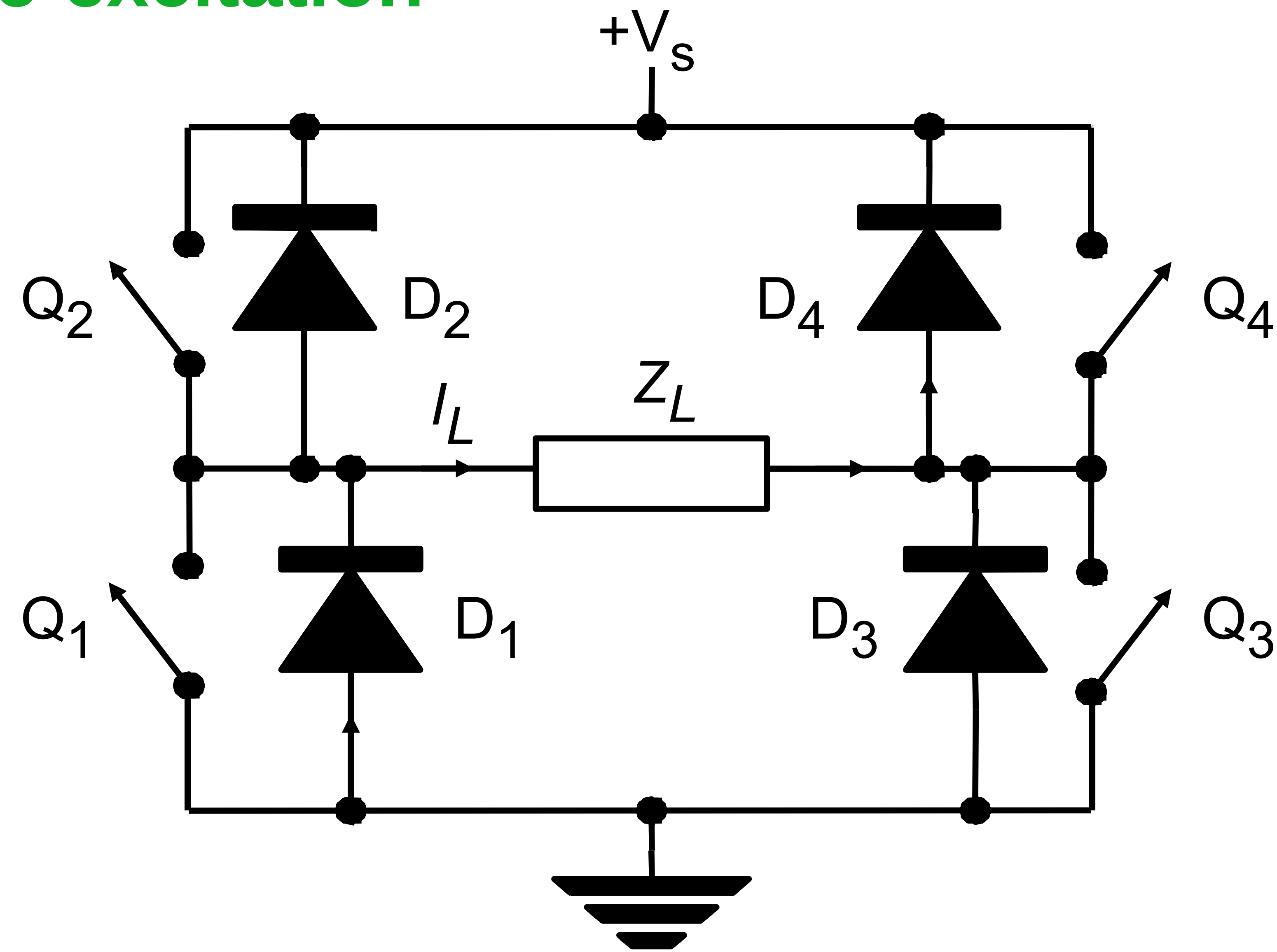
Motor parameters: (type ID31 motor)

Number of rotor poles:	$N_r = 50$
Rotor inertia:	$J_r = 1.16\times10^{-5}$ kg m^2
Coupling coeff:	$K_c = 0.121$ V rad^{-1}s
Viscous damping:	$D_r = 0.0006$ Nmrad^{-1}s
Coulomb friction:	$F_r = 0.000$ Nm
Stator winding resistance:	$R_w = 0.66\ \Omega$
Stator winding inductance:	$L_w = 1.52\times10^{-3}$ H
Nominal stator current:	$I_w = 2.0$ A

Thus: $V_0 = R_w \times I_w = 1.32$ V

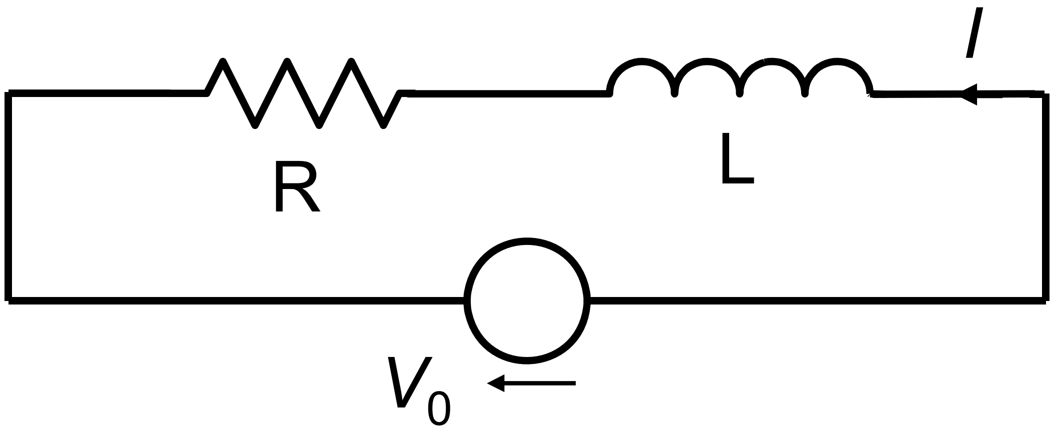
and: $T_0 = L_w / R_w = 2.3$ ms

H-Bridge: Voltage Drive, De-excitation



H-Bridge: Voltage Drive

Stator winding de-excitation:

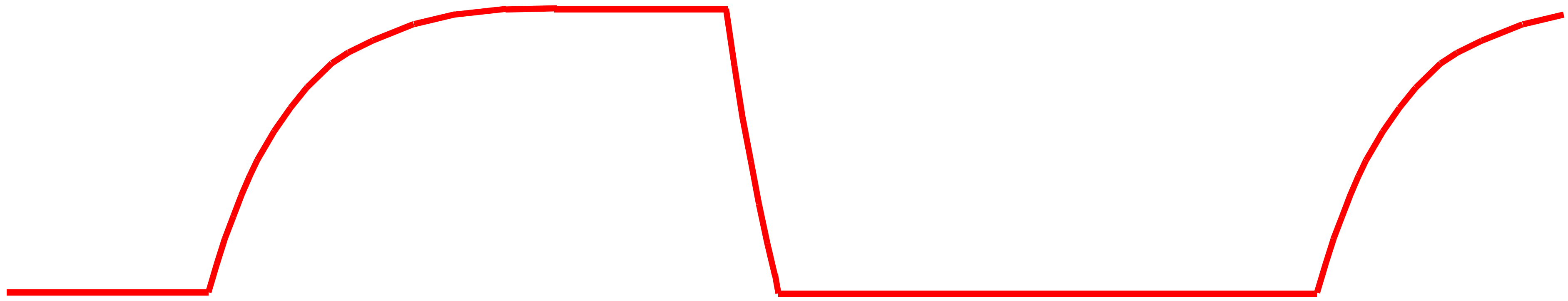


$$R.I + L.\frac{\mathrm{d}I}{\mathrm{d}t} = -V_0$$

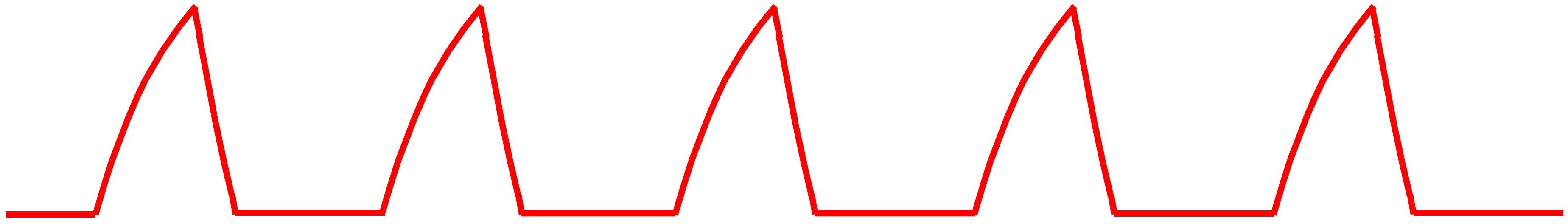
$$I = I_0.\left\{2\exp\frac{-t}{T_0} - 1\right\} \quad \text{where:} \quad I_0 = \frac{V_0}{R} \quad T_0 = \frac{L}{R}$$

H-Bridge: Voltage Drive

Low stepping rate:



High stepping rate:



H-Bridge: Resistor Ballast Drive

For the ID31 motor:

$$T_0 = \frac{L}{R} = \frac{1.52 \times 10^{-3}}{0.66} = 2.3\,\mathrm{ms}$$

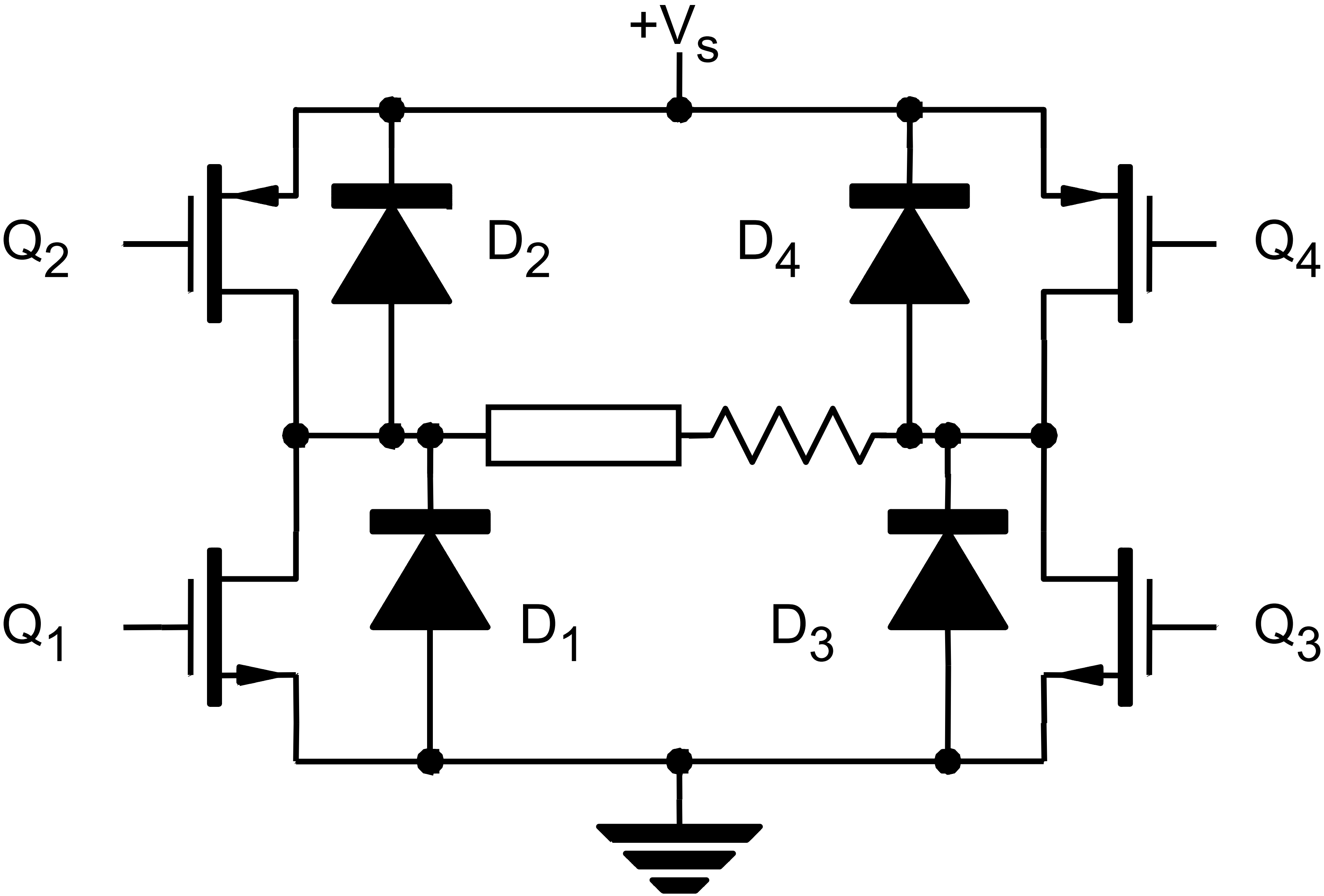
Stepping rate should be less than 400 step/s

To increase the stepping rate it is necessary to reduce T_0

Since it is not possible to reduce L the only alternative is to increase R: an external ballast resistance is placed in series with the winding

This reduces T_0 at the expense of efficiency

H-Bridge: Resistor Ballast Drive



H-Bridge: Resistor Ballast Drive

Use a series ballast resistance of 11.34 Ω:

$$T_0 = \frac{L}{R} = \frac{1.52 \times 10^{-3}}{11.34 + 0.66} = 0.13\,\text{ms}$$

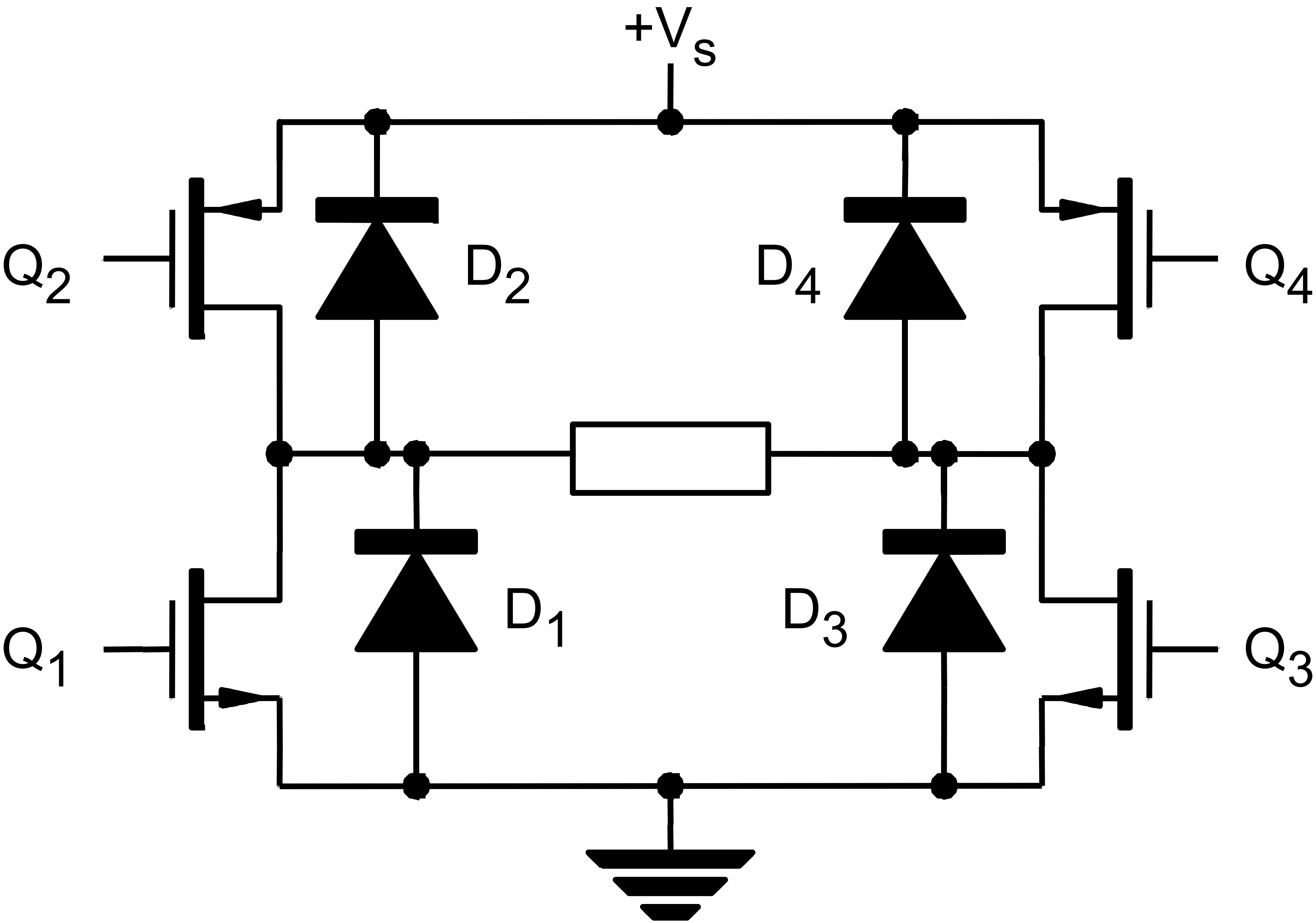
Stepping rate is increased to 8000 step/s

To maintain a stator current of 2 A requires a supply voltage of 24 V

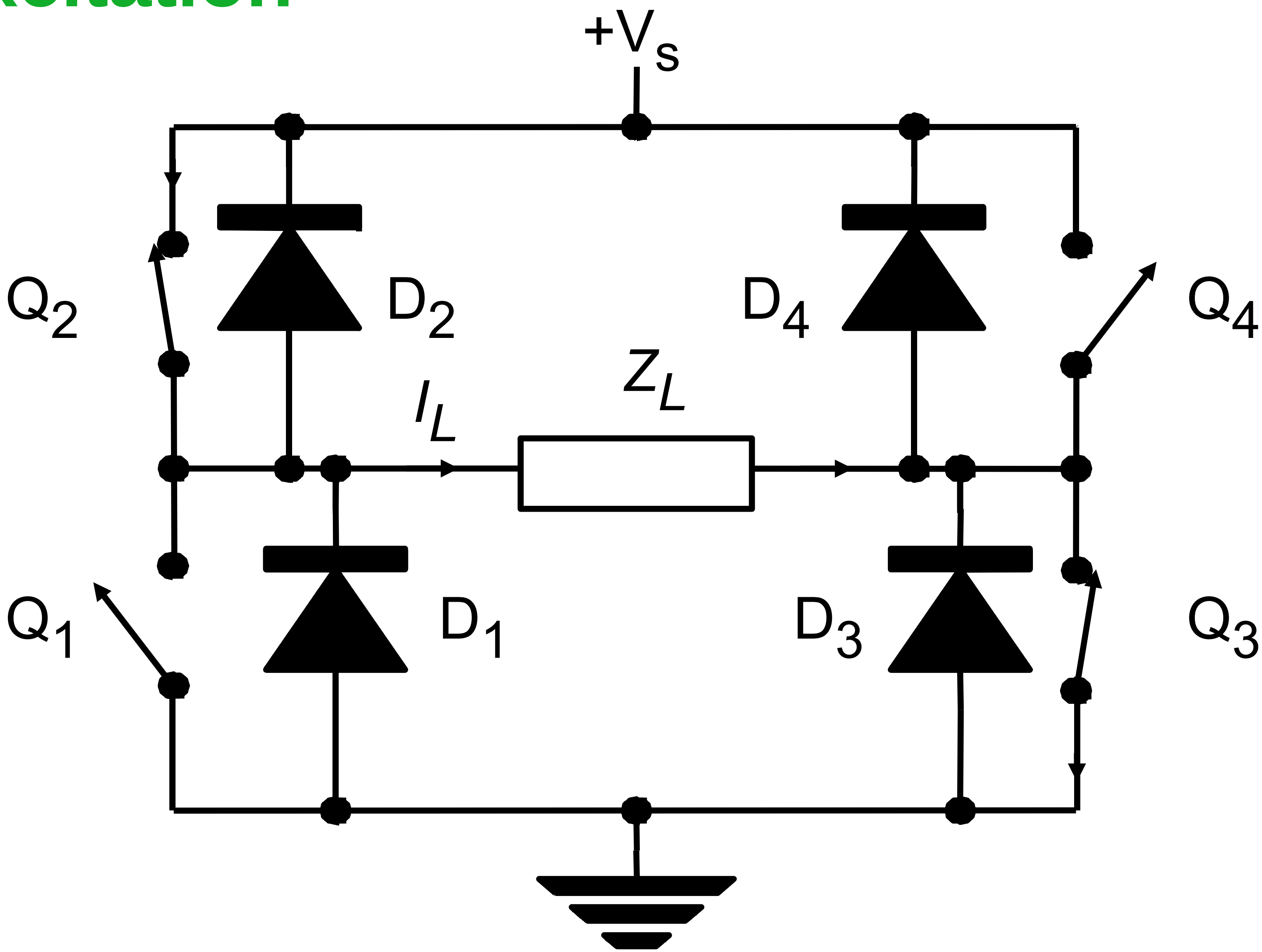
Static power dissipation has increased from 2.64 W to 48 W

The poor efficiency of resistor ballast drive limits application to low power motors

H-Bridge: Chopper Drive



H-Bridge: Chopper Drive, Initial Excitation



H-Bridge: Chopper Drive

Stator winding initial excitation:

$$R.I + L.\frac{\mathrm{d}I}{\mathrm{d}t} = V_0 \quad \text{where}: V_0 >> R.I$$

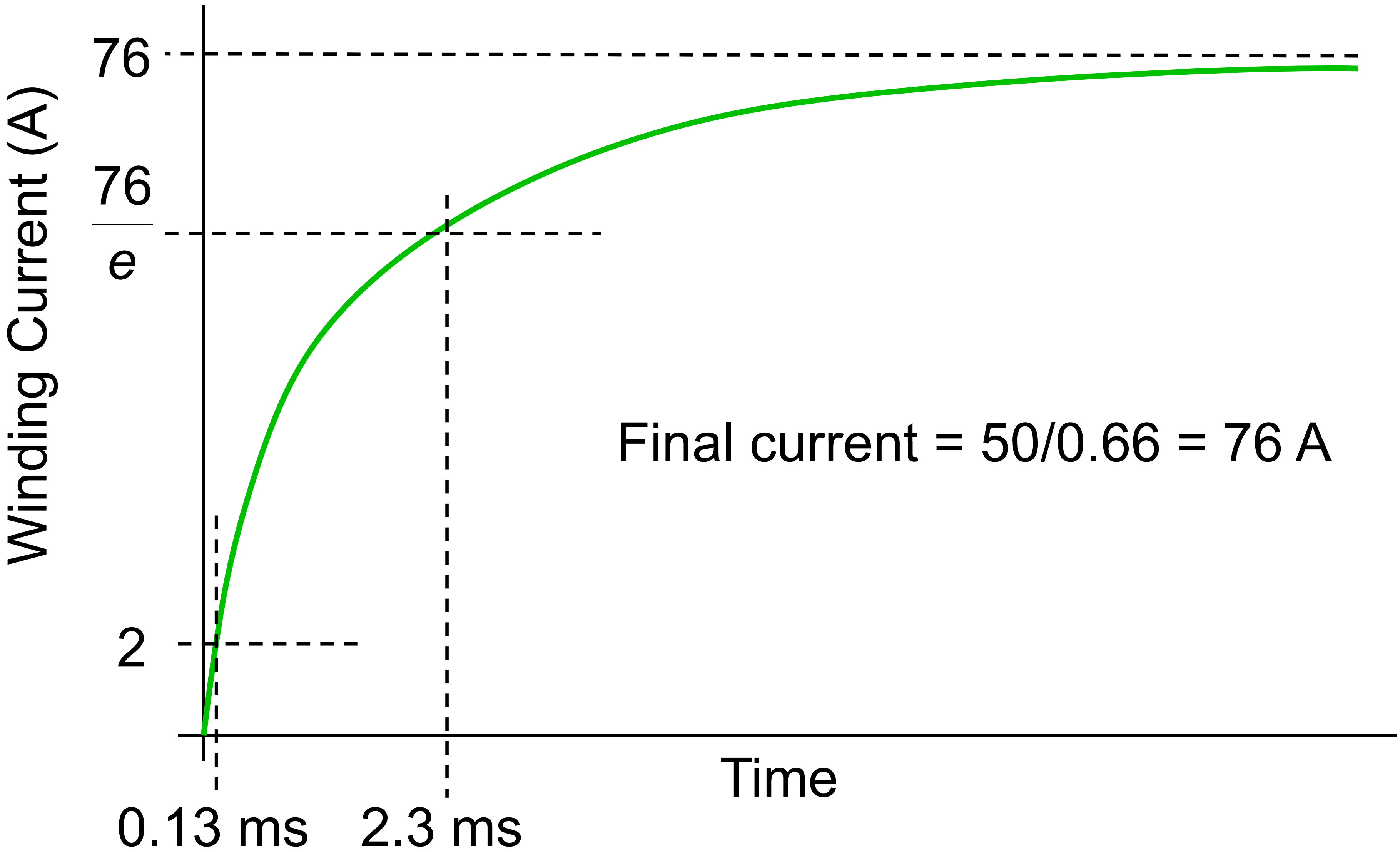
$$I \approx \frac{V_0}{L}.t \quad \text{or}: T_0 \approx \frac{I_0.L}{V_0}$$

T_0 is the time for the current to reach the nominal stator current I_0

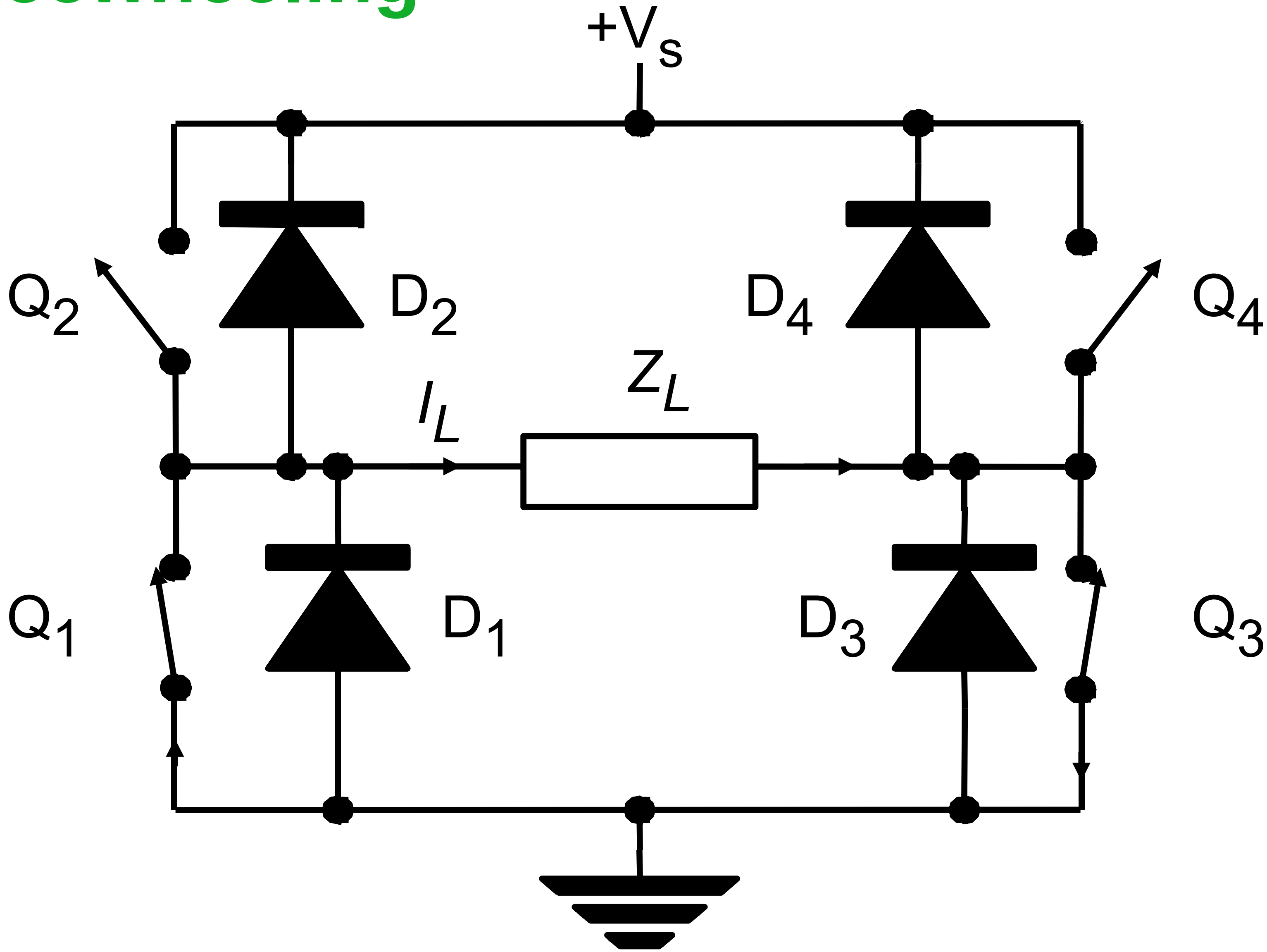
$$T_0 \approx \frac{I_0.L}{V_0} = \frac{2 \times 1.52 \times 10^{-3}}{24} = 0.13\,\mathrm{ms}$$

When stator current reaches nominal current the chopper drives goes into freewheel mode

H-Bridge: Chopper Drive



H-Bridge: Chopper Drive, Freewheeling



H-Bridge: Chopper Drive

Stator winding freewheeling:

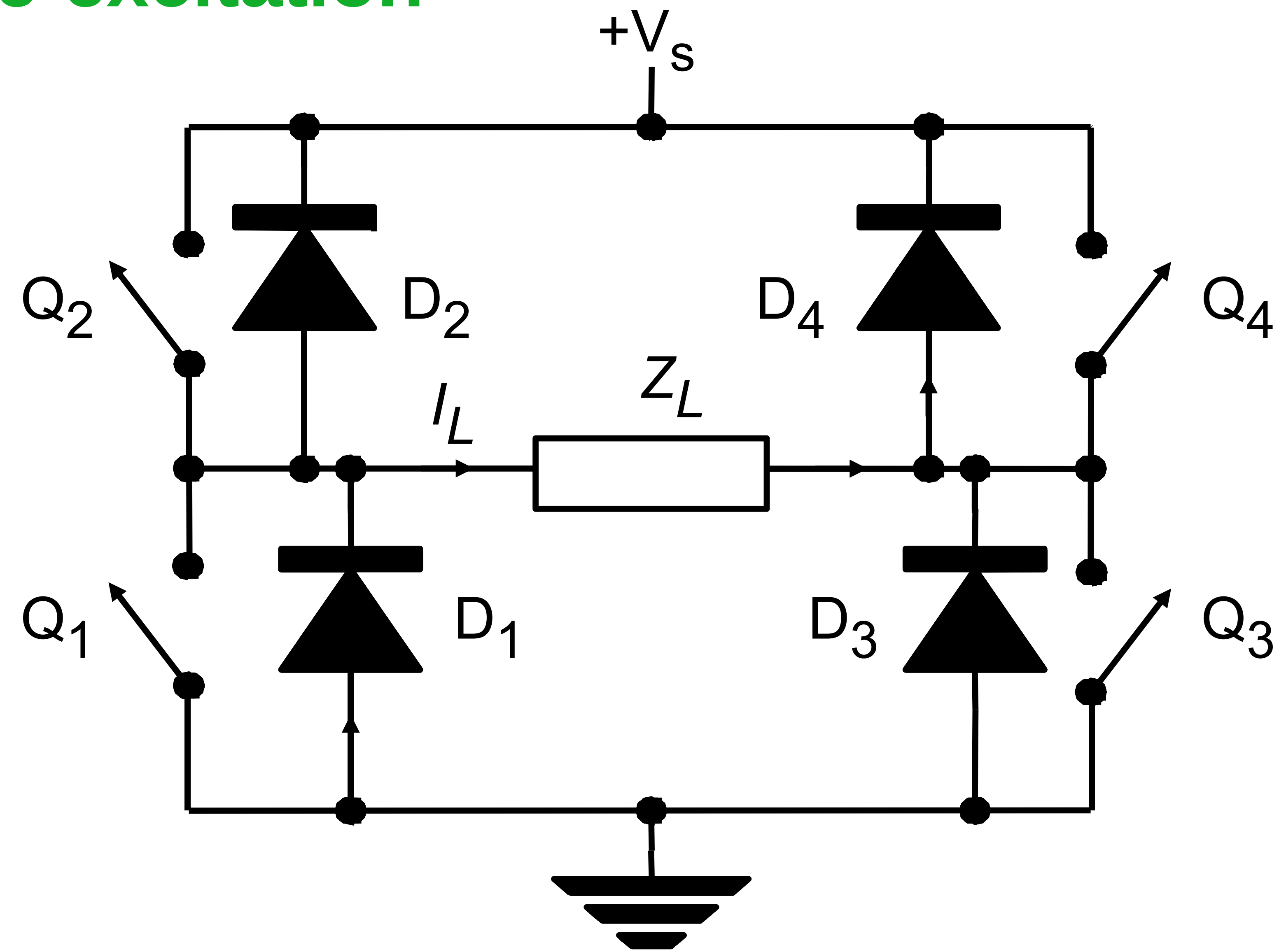
$$R.I + L.\frac{\mathrm{d}I}{\mathrm{d}t} = V_0 = 0$$

$$I = I_0.\exp\frac{-t}{T_0} \quad \text{where :} \quad T_0 = \frac{L}{R}$$

Stator current decays with time constant T_0 towards zero

By alternately applying the full supply voltage, and the freewheeling, the stator current is maintained close to the nominal current.

H-Bridge: Chopper Drive, De-excitation



H-Bridge: Chopper Drive

Stator winding de-excitation:

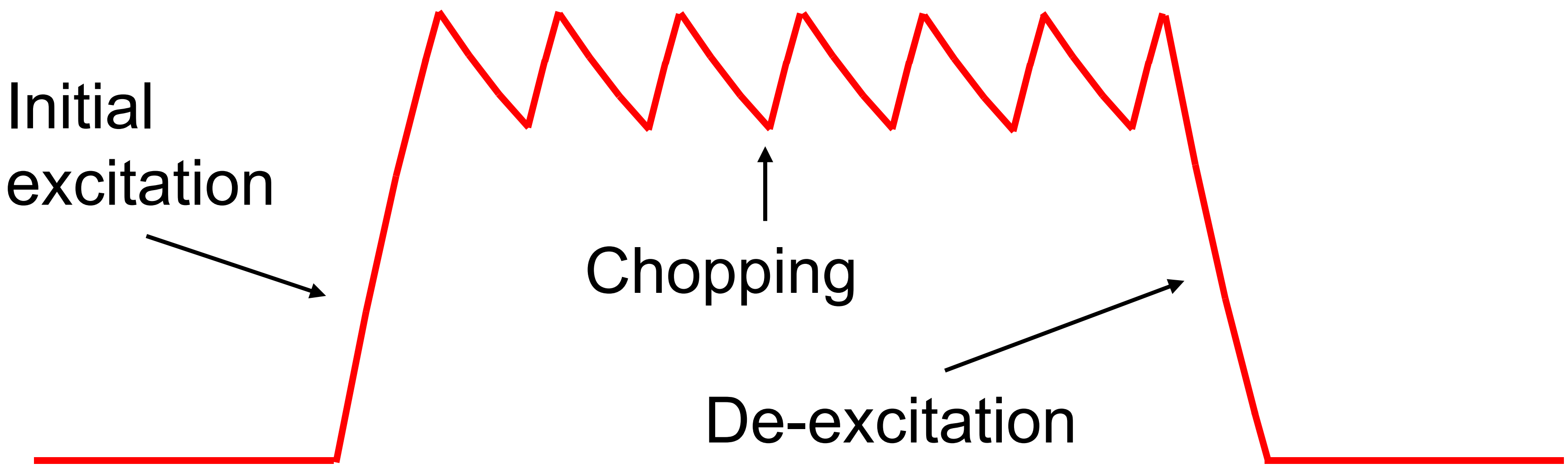
$$R.I + L.\frac{dI}{dt} = -V_0 \quad \text{where}: |V_0| >> R.I$$

$$I = I_0 - \frac{V_0}{L}.t \qquad \text{or}: T_0 = \frac{I_0.L}{V_0}$$

T_0 is the time for the current to fall to zero, and is the same as the excitation time

The current in the stator winding is normally sensed by placing a small resistor in series with the source terminals of Q1 and Q3

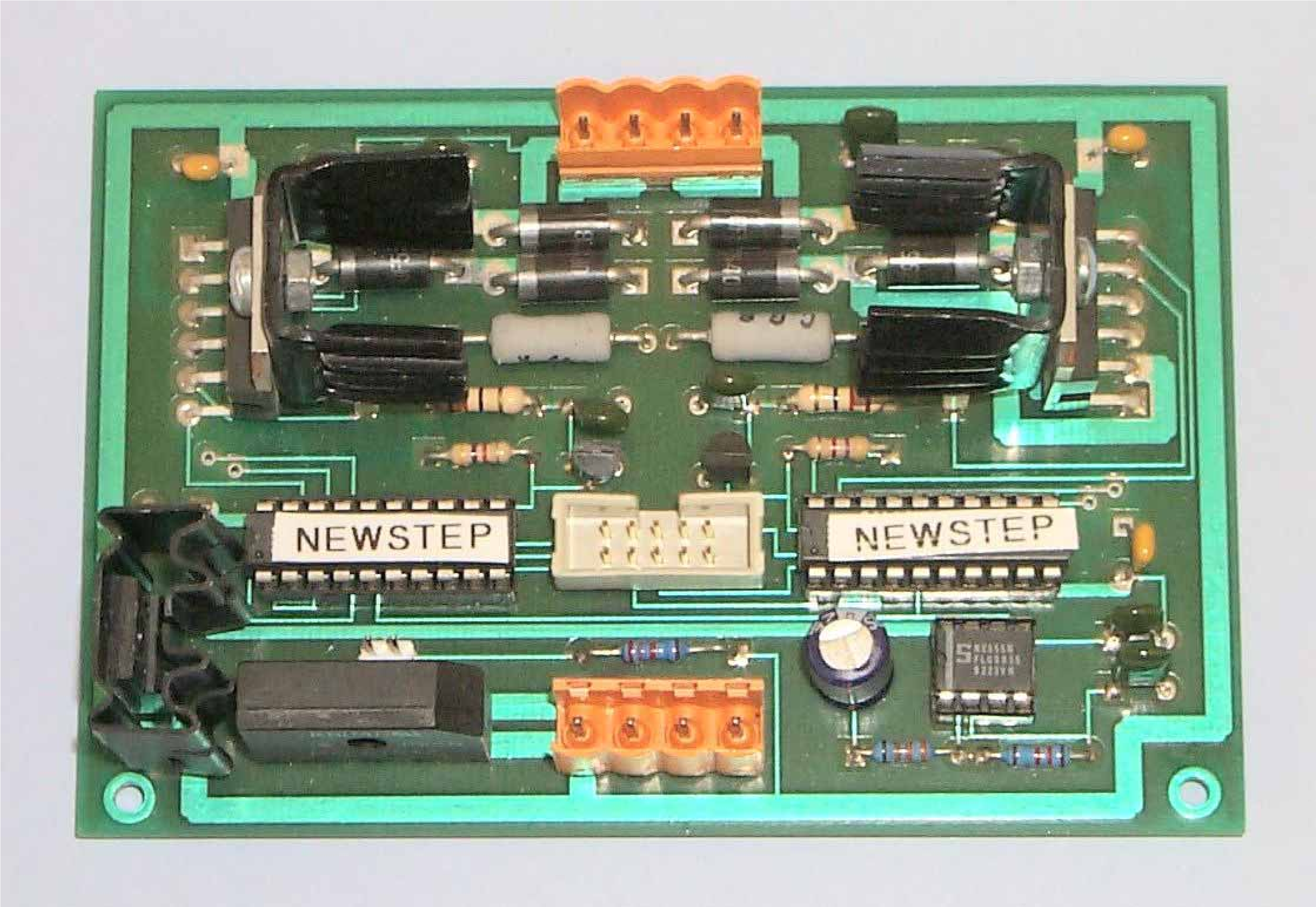
H-Bridge: Chopper Drive



Because of magnetostriction the motor will generate noise at the chopping frequency

To prevent this causing annoyance the chopping frequency is normally chosen to be greater than 25 kHz

H-Bridge: Chopper Drive



Dual 2 A chopper drive

H-Bridge: Chopper Drive

Commercial 4A chopper drive (Mclennan Servo Supplies Ltd)

Stepping Motor Drive Sequences

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The simplest drive sequence is the *one-winding-on* sequence:

A+	B+	A–	B–
1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

This sequence repeats after 4 steps

It is rarely used because a better performance can be obtained from the *two-windings-on* or *wave* sequence

Stepping Motor Drive Sequences

The *two-windings-on* or *wave* sequence:

A+	B+	A–	B–
1	1	0	0
0	1	1	0
0	0	1	1
1	0	0	1

This sequence also repeats after 4 steps

It provides $\sqrt{2}=1.4$ times the torque of the *one-winding-on* sequence, a the expense of twice the static power consumption

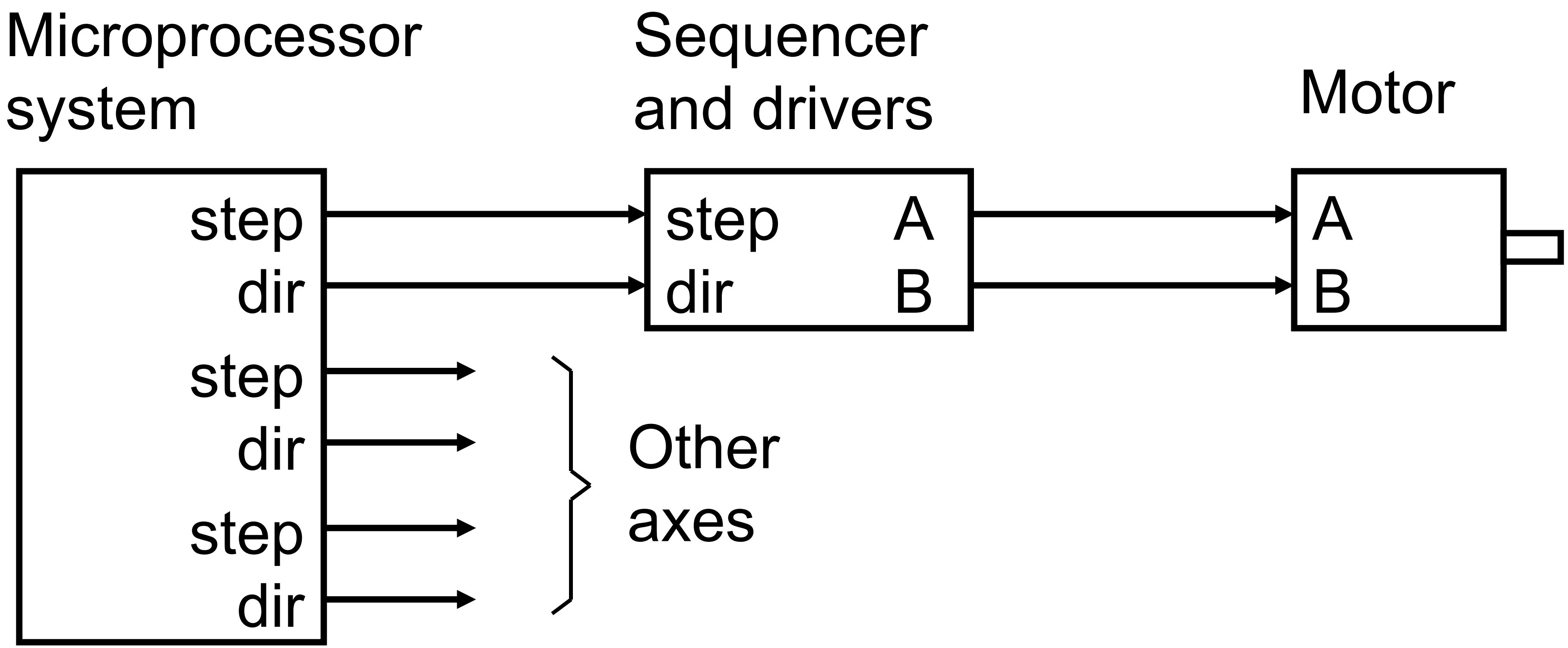
Stepping Motor Drive Sequences

The *half-step* sequence:

A+	B+	A–	B–
1	0	0	0
1	1	0	0
0	1	0	0
0	1	1	0
0	0	1	0
0	0	1	1
0	0	0	1
1	0	0	1

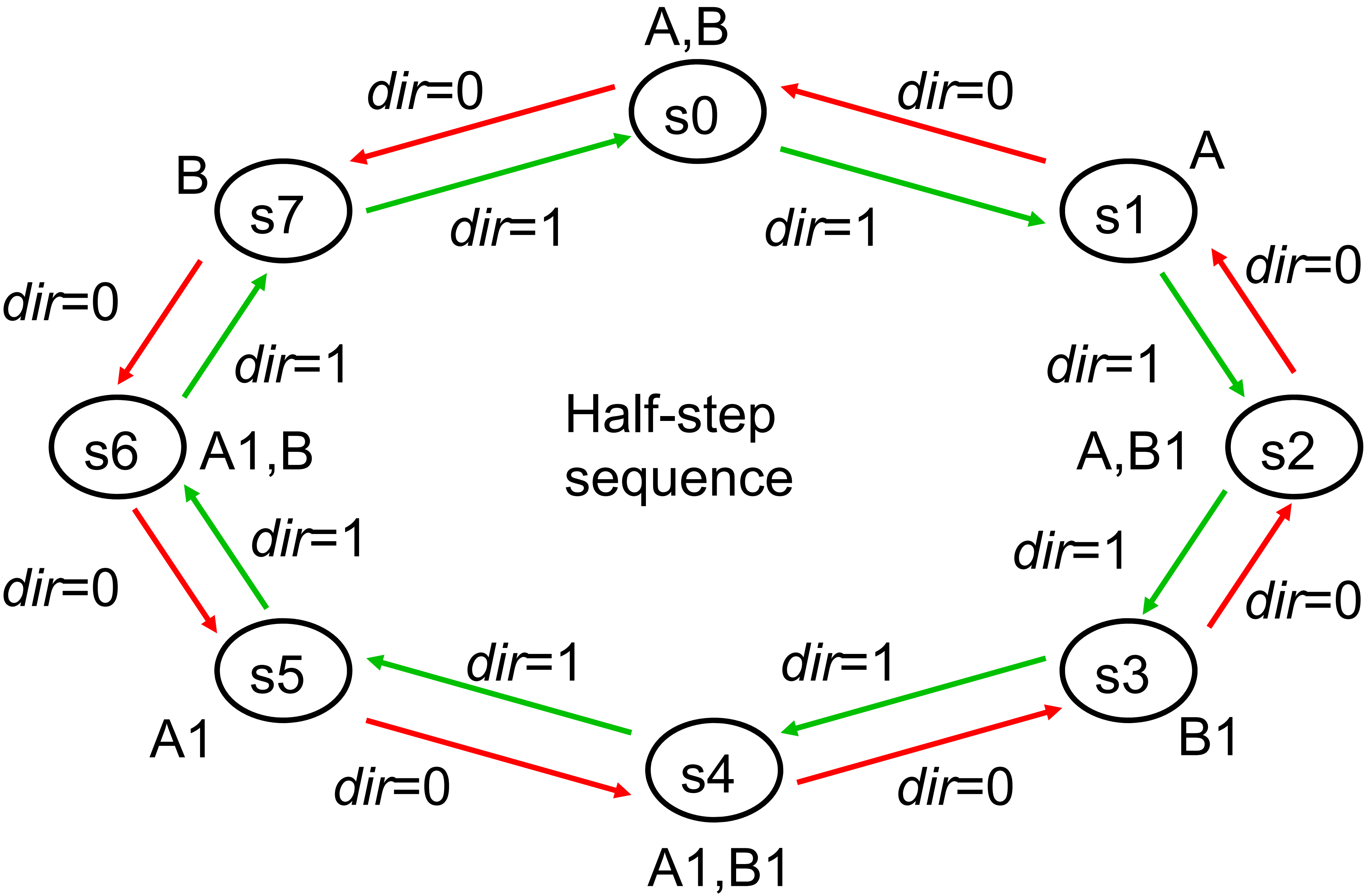
This sequence repeats after 8 steps and provides twice the precision of other sequences

Microprocessor Control



The most common interface between a microprocessor system and a sequencer/driver is by *step* and *dir* signals

Sequencer State Machine



Sequencer State Machine

```
PIN 1  = clock;            /* inputs */
PIN 2  = dir;

PIN 16 = A;                /* outputs */
PIN 17 = A1;
PIN 18 = B;
PIN 19 = B1;

PIN [12..14] = [y2..0]; /* state vars */

FIELD stepper = [y2..0];
$DEFINE s0 'b'000          /* states */
$DEFINE s1 'b'001
$DEFINE s2 'b'011
$DEFINE s3 'b'010
$DEFINE s4 'b'110
$DEFINE s5 'b'111
$DEFINE s6 'b'101
$DEFINE s7 'b'100
```

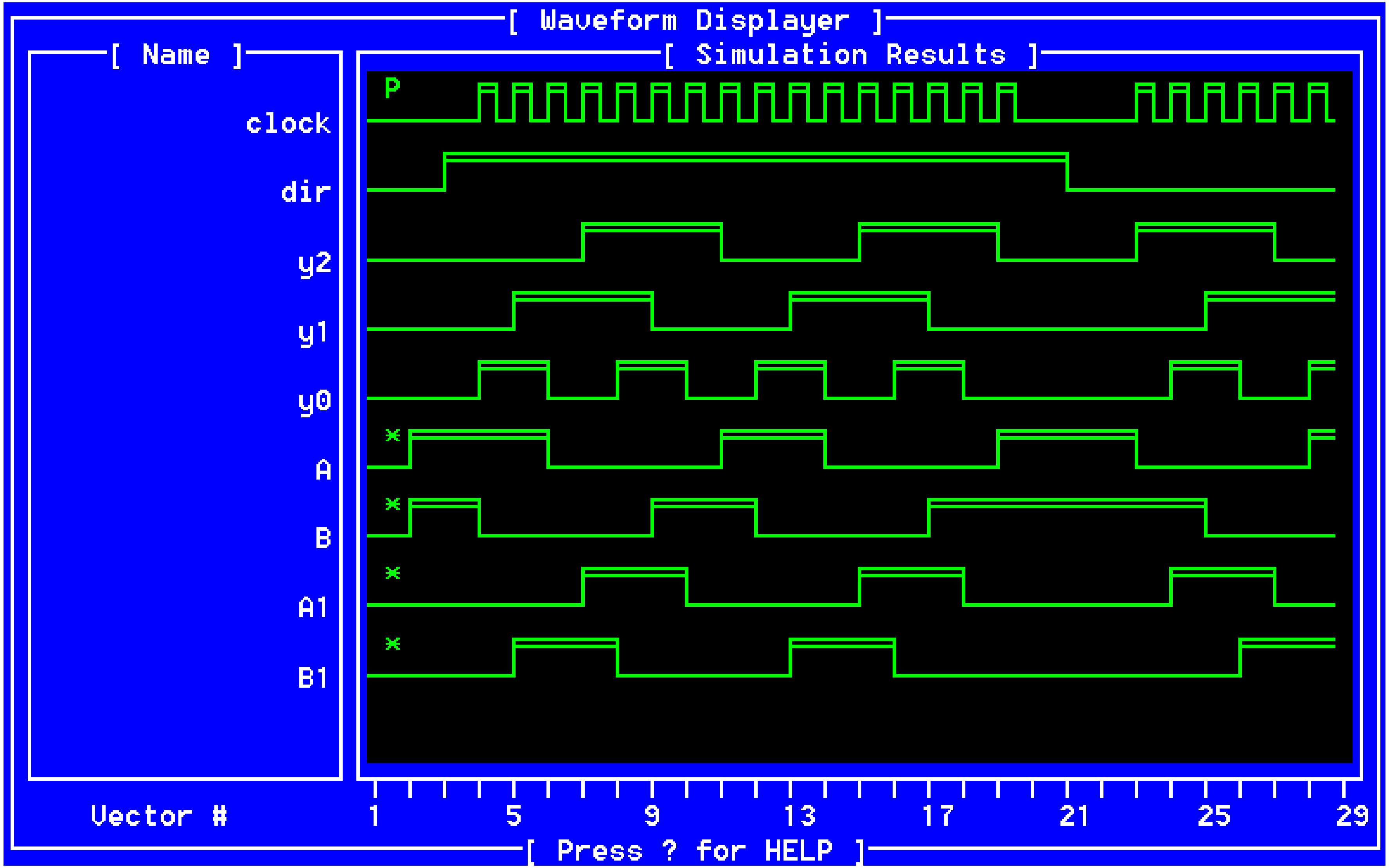
Sequencer State Machine

```
sequence stepper {
    present s0
        if ( dir ) next s1;
        if (!dir ) next s7;
        out A,B;
    present s1
        if ( dir) next s2;
        if (!dir) next s0;
        out A;
    present s2
        if ( dir ) next s3;

    ......................

present s7
        if ( dir) next s0;
        if (!dir) next s6;
        out B;
}
```

Sequencer State Machine



Sequencer state machine simulation

Sequencer State Machine

```
               _________________
              |    Stepping     |
  clock x---|1              20|---x Vcc
    dir x---|2              19|---x B1
        x---|3              18|---x B
        x---|4              17|---x A1
        x---|5              16|---x A
        x---|6              15|---x
        x---|7              14|---x y0
        x---|8              13|---x y1
        x---|9              12|---x y2
    GND x---|10             11|---x
              |_________________|
```

GAL16V8: connection details

Microprocessor Coordinator

Step class (base class) definition:

```
class step {
public:
    step() {}
    // step class constructor
    ~step() {}
    // step class destructor
    void up();
    // move one step clockwise
    void dn();
    // move one step anti-clockwise
};
```

Microprocessor Coordinator

Step class (base class) implementation:

```
#include "step.h"
#define p ((volatile unsigned char *) ...
#define dir_bit 0x01
#define step_bit 0x02

void step::up() {
    *p = dir_bit;
    *p = dir_bit | step_bit;
    *p = dir_bit;
}

void step::dn() {
    *p = 0;
    *p = step_bit;
    *p = 0;
}
```

Microprocessor Coordinator

Move class (derived class) definition:

```
#include "step.h"

class move: private step {
private:
    long int pos;
public:
    move() { pos = 0; }
    // move class constructor
    ~move() {}
    // move class constructor
    void go(long int x, long int s);
    // move to position x at speed s
};
```

Microprocessor Coordinator

Move class (derived class) implementation:

```
#include "move.h"

void move::go(long int x, long int s)
{
    const long int q = 1000000 / s;
    long int k;
    while (pos != x) {
        for (k = q; k > 0; --k);
        if (pos < x) {
            up(); pos++;
        }
        else {
            dn(); pos--;
        }
    }
}
```

Microprocessor Coordinator

Delay loop:

```
; D7 <- k
; D5 <- q
; for (k = q; k > 0; --k);
000078 2E05                 MOVE.L D5,D7
00007A 6002                 BRA _44
00007C 5387          _43    SUBQ.L #--1,D7
00007E 4A87          _44    TST.L D7
000080 6EFA                 BGT _43
```

There are 3 instructions in main delay loop: at 250ns per instruction this gives Δt=750ns

20,000 steps/s $\rightarrow$ 50 μs per step
50.750 μs per step $\rightarrow$ 19704 steps/s

Microprocessor Coordinator

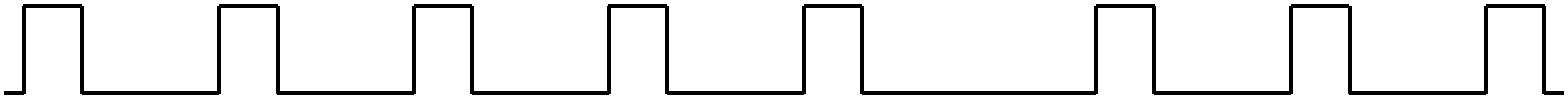
Improved move class (derived class) implementation:

```
#include "move.h"
void move::go(long int x, long int s)
{
    unsigned long int k = 0;
    while (pos != x) {
        k += s;
        if (k > 1000000) {
            k -= 1000000;
            if (x > pos) {
                up(); pos++;
            }
            else {
                dn(); pos--;
            }
        }
    }
}
```

Microprocessor Coordinator

```
k += s;
if (k > 1000000)
    k -= 1000000;
```

s=456789:



k: 400000 856789 313578 770367 227156 683945 140734 597523 054312 511101 967890 424697 881468 338257 795046 251835

s=123456:

k: 400000 523456 646912 770368 893824 017280 140736 264193 387648 511104 634560 758016 881472 004928 128384 251840

Microprocessor Coordinator

Each iteration of the **while** loop involves around 24 machine instructions taking Δt=6μs.

The proportion of iterations generating a step is *s*/1,000,000 and varies from 0 to 1 (as *s* varies from 0 to 1,000,000)

The maximum step frequency is therefore 160,000 steps/s and the frequency increment is 160,000/1,000,000 = 0.16 steps/s

There will be a jitter in the step pulses of Δt=6μs

Microprocessor Coordinator

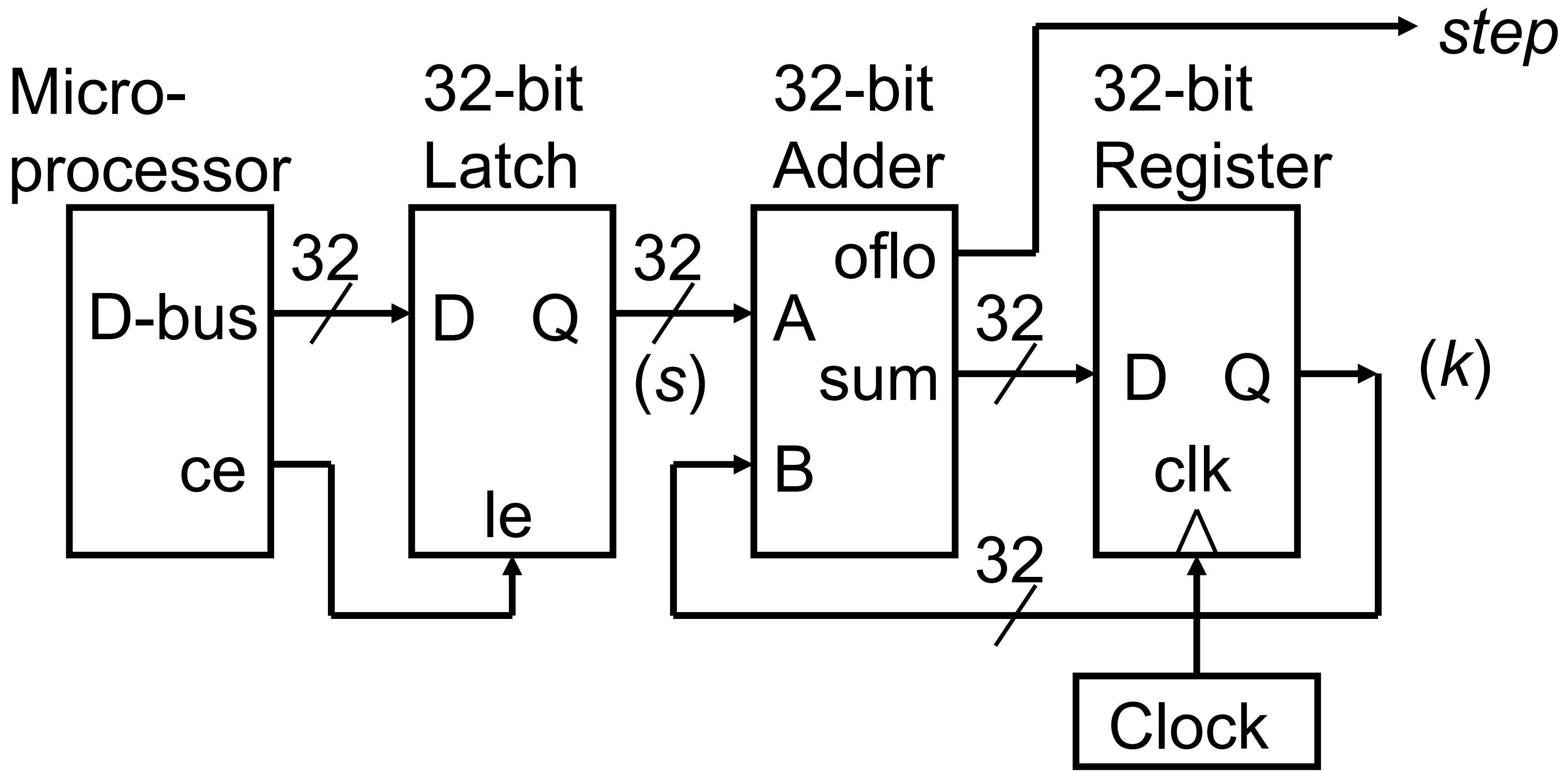
Main loop:

```
#include "move.h"

int main()
{
    move s;

    for (;;) {
        delay(100000);
        s.go(800, 800);
        delay(100000);
        s.go(0, 1200);
    }
    return 0;
}
```

Hardware Implementation



At each clock edge: oflo = (k + s) > 2^{32}

k = (k + s) mod 2^{32}

Microprocessor Coordinator

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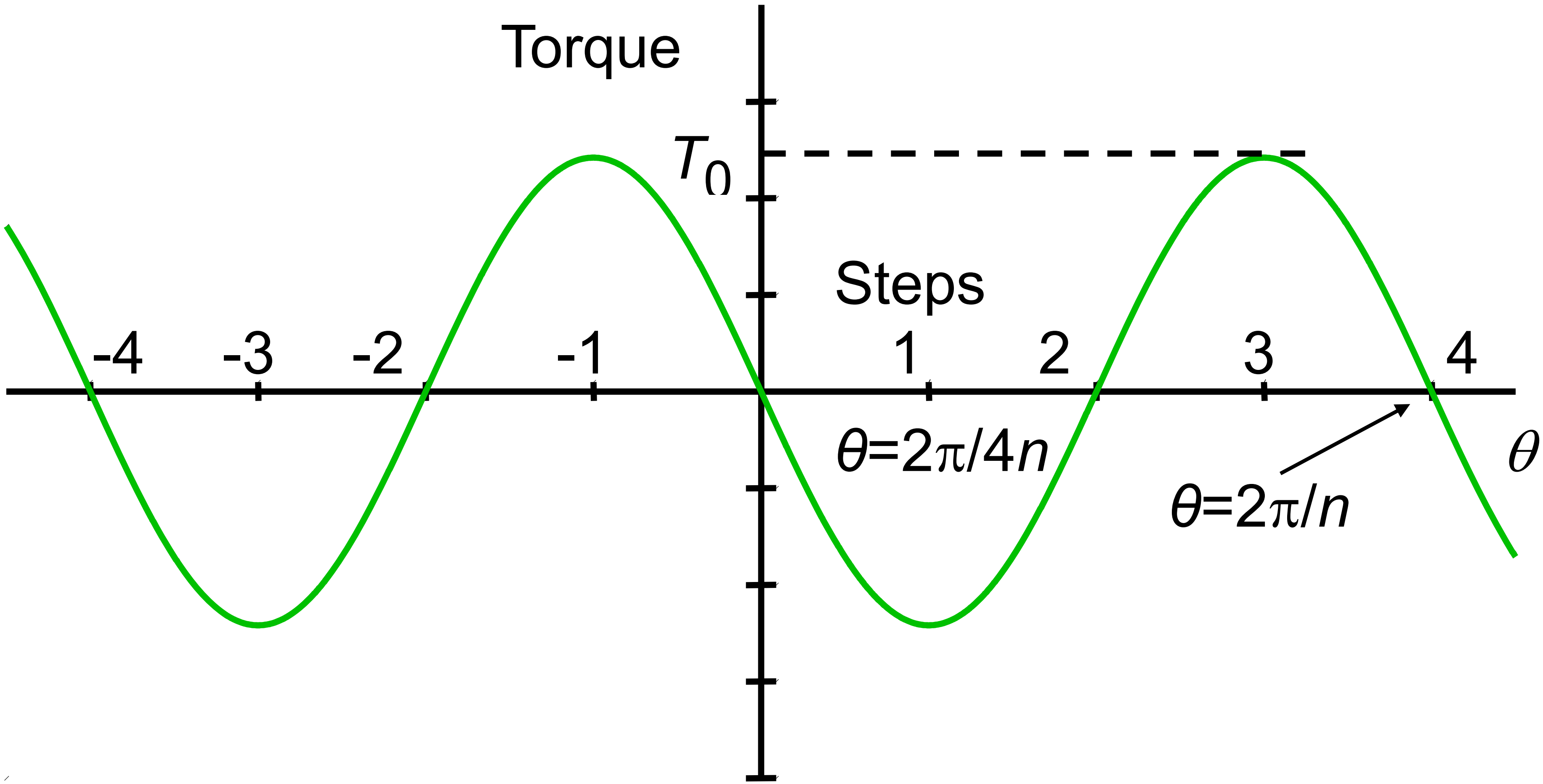
Commercial microprocessor-based coordinator
(Mclennan Servo Supplies Ltd)

Static Torque Characteristic



$$T = -T_0 \sin n\theta$$

where n is the number of rotor teeth and θ is the rotor angle

Static Torque Characteristic

The motor torque is given by:

$$T = -T_0 \sin n\theta$$

If a load torque T_L is applied to the motor then the rotor will be displaced to an angle θ_e where:

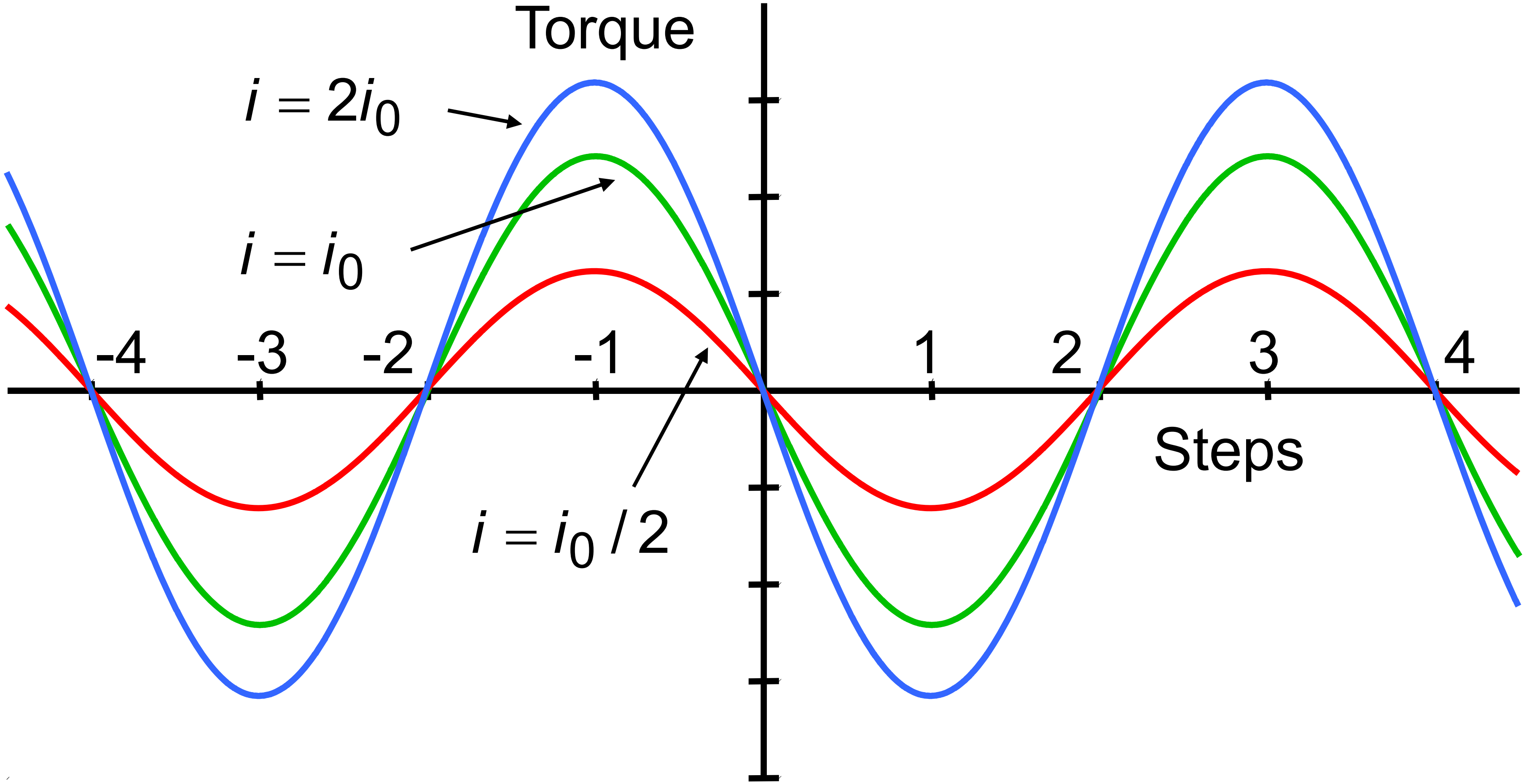
$$T_L = -T_0 \sin n\theta$$

$$\theta_e = \frac{\sin^{-1}(-T_L / T_0)}{n}$$

This formula is true for $|T_L| < T_0$

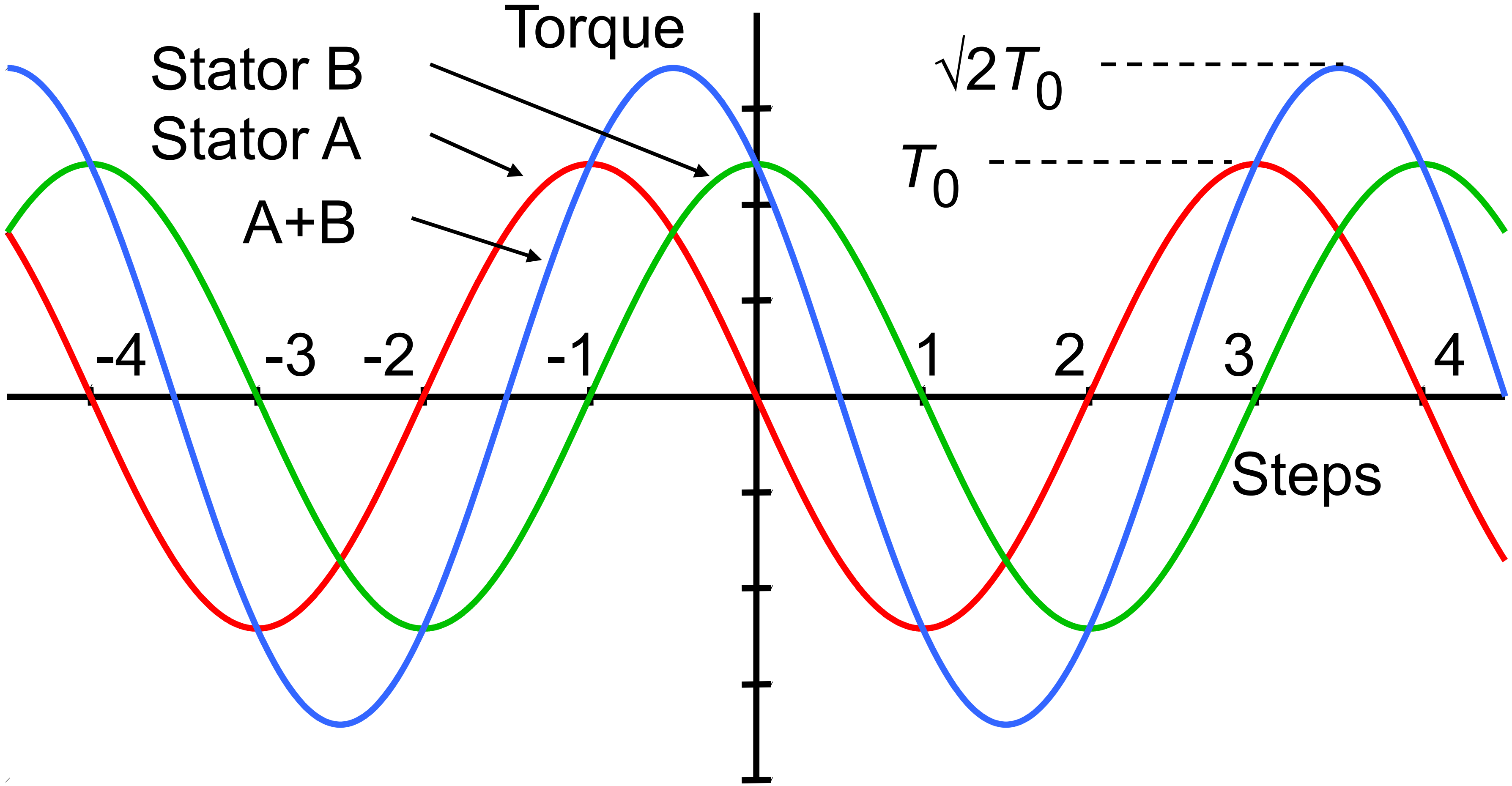
If $|T_L|$ exceeds T_0 then synchronisation is lost

Static Torque Characteristic



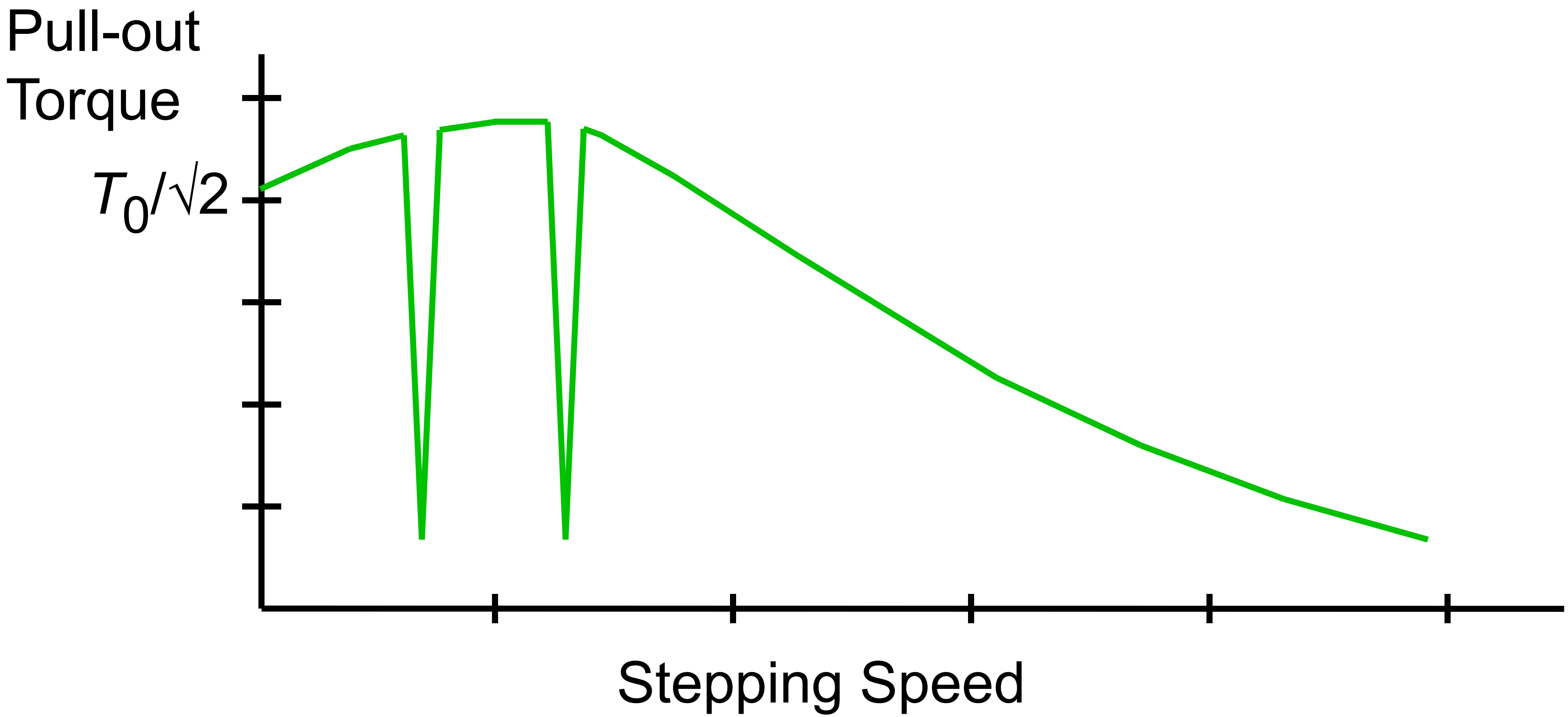
Stator is nearly saturated at nominal current i_0, so doubling current does not double torque

Static Torque Characteristic



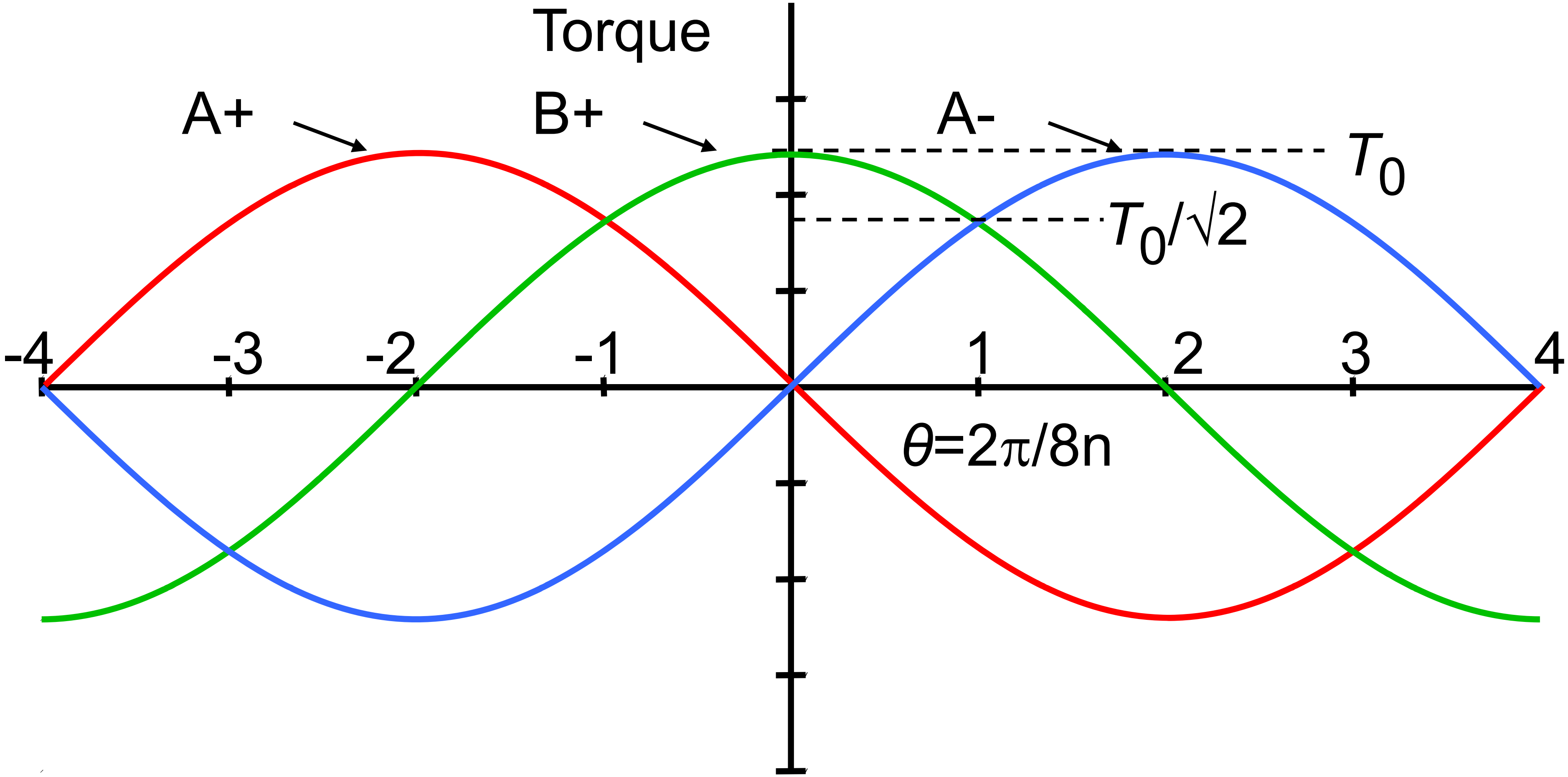
By exciting 2 stator windings, the torque is increased from T_0 to $\sqrt{2}T_0$

Torque/Speed Characteristic



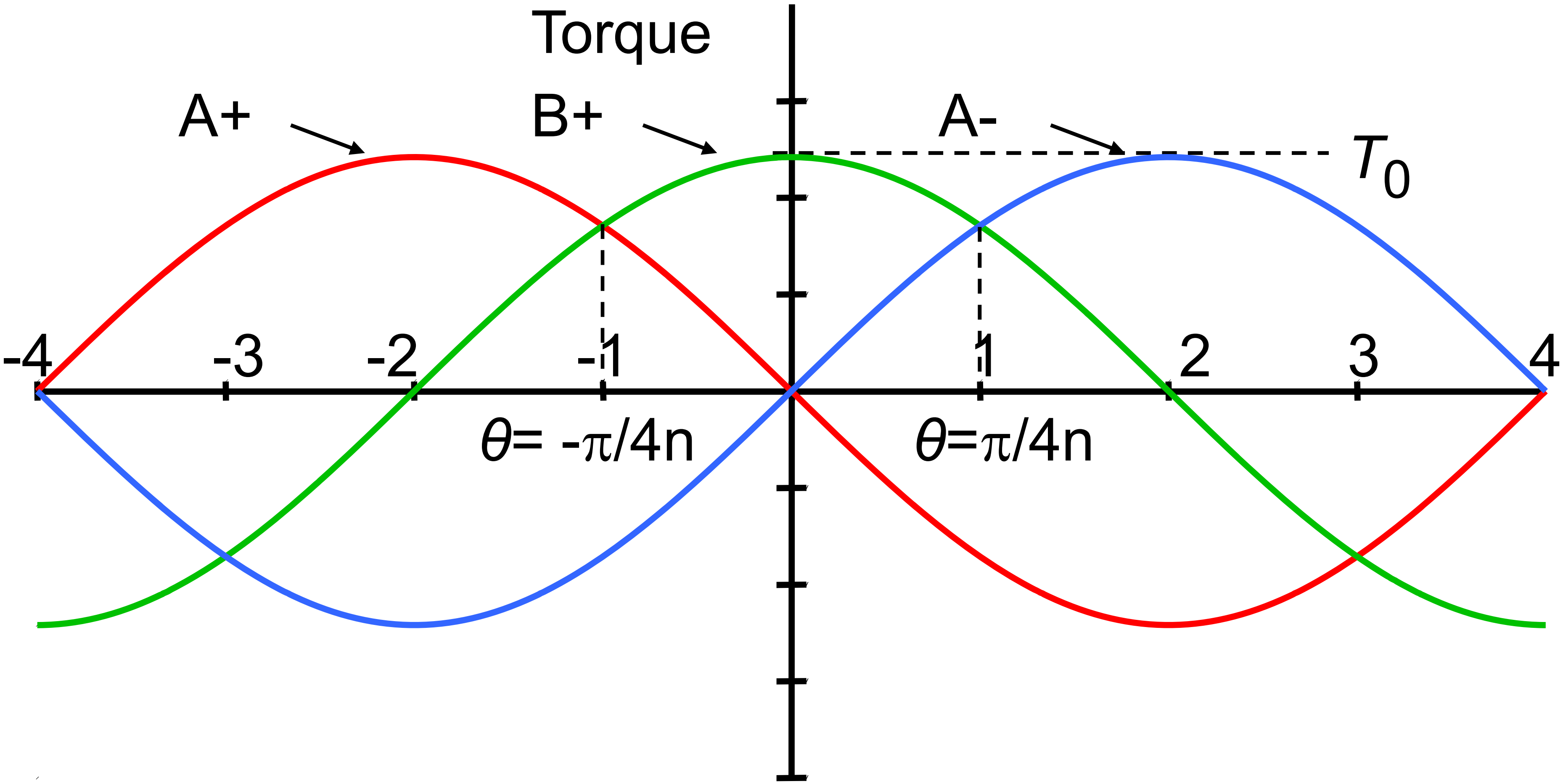
Pull-out torque is the torque that the motor can generate at a given stepping rate: if the load torque exceeds this then the motor loses synchronisation

Static Torque Characteristic



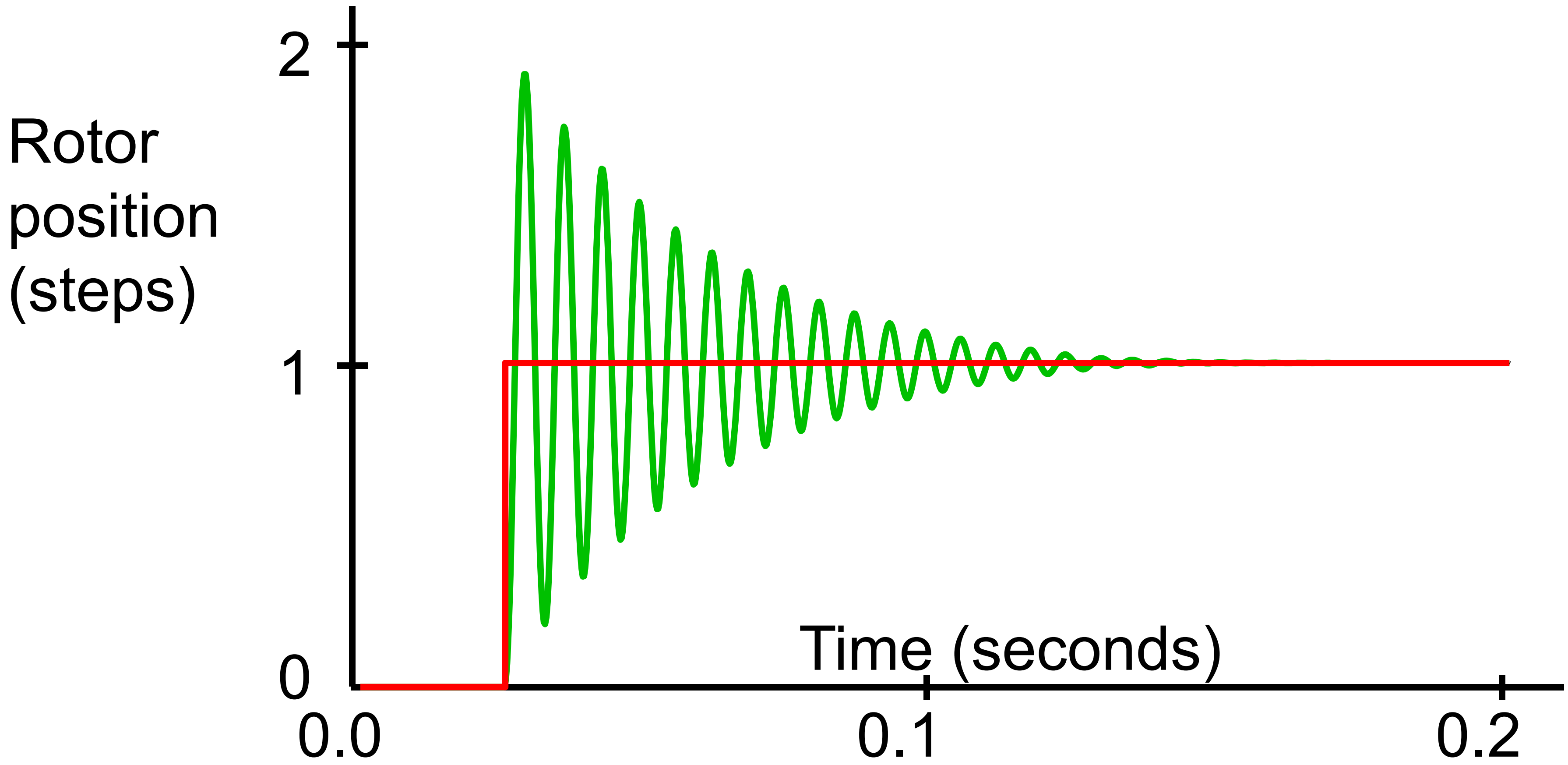
$$T_{\min} = T_0 \cos n.\frac{2\pi}{8n} = T_0 \cos\frac{\pi}{4} = \frac{T_0}{\sqrt{2}}$$

Static Torque Characteristic



$$T_{avg} = \frac{2n}{\pi} \int_{-\pi/4n}^{\pi/4n} T_0 \cos n\theta \, \mathrm{d}\theta = \frac{2T_0}{\pi} \int_{-\pi/4}^{\pi/4} \cos\phi \, \mathrm{d}\phi = \frac{2\sqrt{2}T_0}{\pi}$$

Single Step: Dynamic Response



This shows the rotor position following a change of excitation from A+ to B+ (1 step, wave sequence)

Single Step: Dynamic Response

In the absence of load torque:

$$J_r \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + D_r \frac{\mathrm{d}\theta}{\mathrm{d}t} = -T_0 \sin n\theta$$

Where J_r is the rotor inertia, and D_r is the viscous damping coefficient

For small displacements from equilibrium:

$$J_r \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + D_r \frac{\mathrm{d}\theta}{\mathrm{d}t} \cong -T_0 n\theta$$

This is the equation for damped simple harmonic motion with resonance frequency:

$$f_0 = \frac{1}{2\pi}\sqrt{nT_0 / J_r}$$

Single Step: Dynamic Response

For the ID31 Motor:
Number of rotor poles: $N_r = 50$
Rotor inertia: $J_r = 1.16 \times 10^{-5}$ kg m^2
Peak torque: $T_0 = 0.242$ N m

$$f_0 = 1/2\pi \sqrt{nT_0 / J_r}$$
$$= 162\,\text{Hz}$$

Any attempt to step the motor at the resonance rate will lead to loss of synchronisation

The transient response from one step will reinforce the response from a previous step, and eventually the amplitude will exceed two steps.

Multiple Step: Dynamic Response

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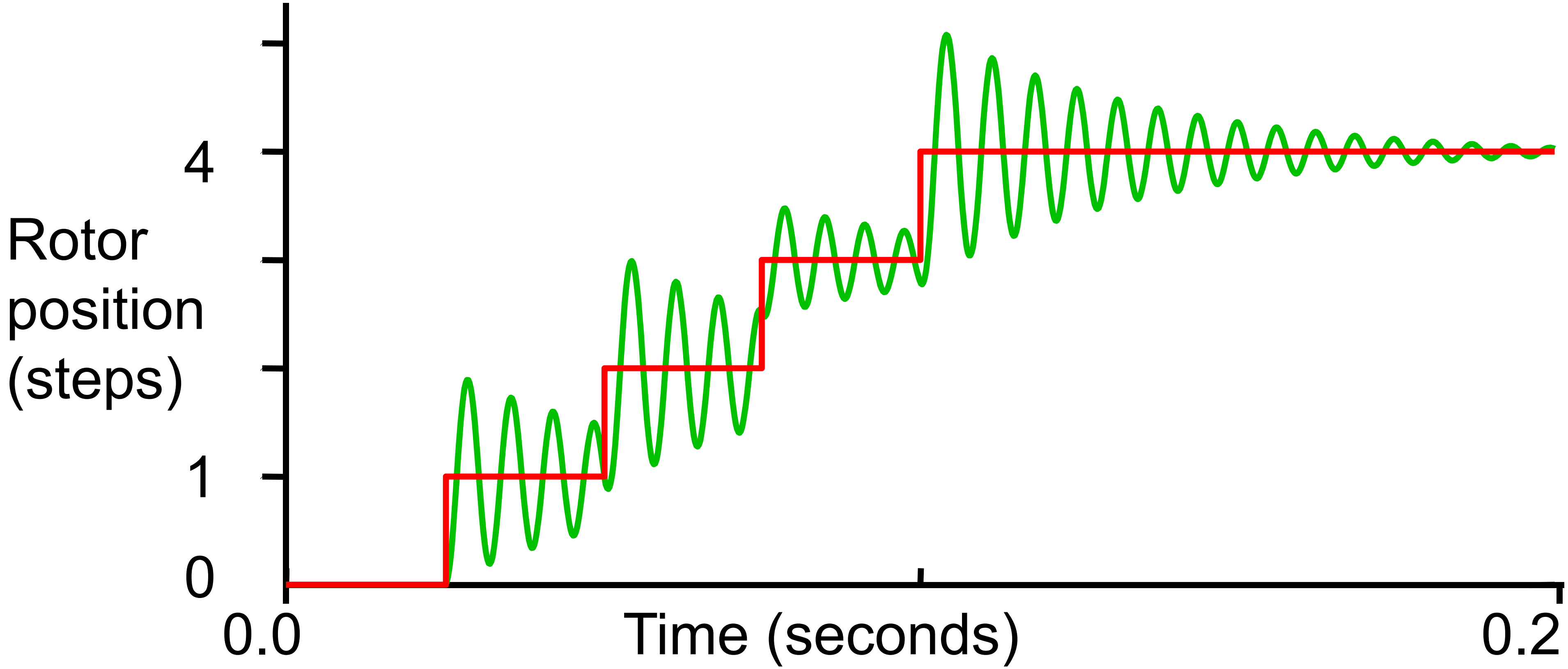
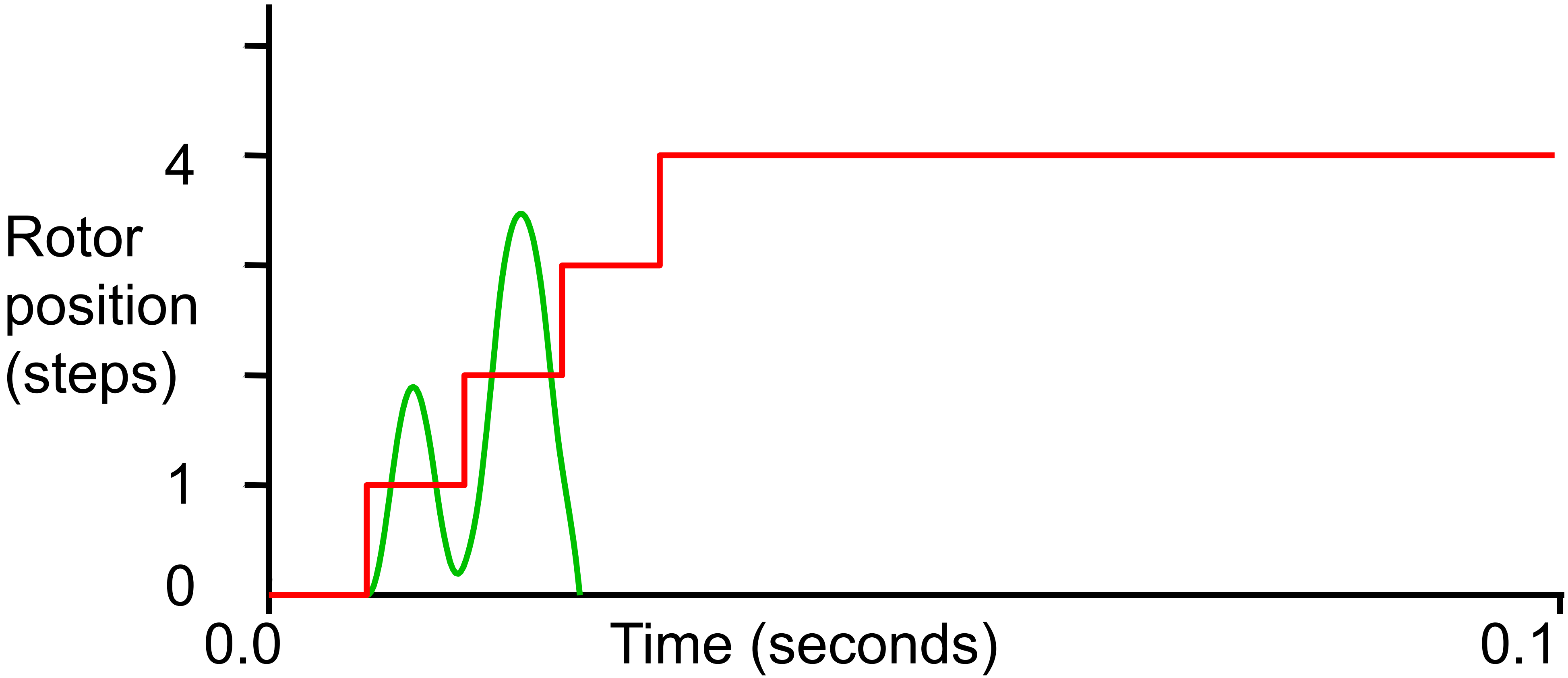


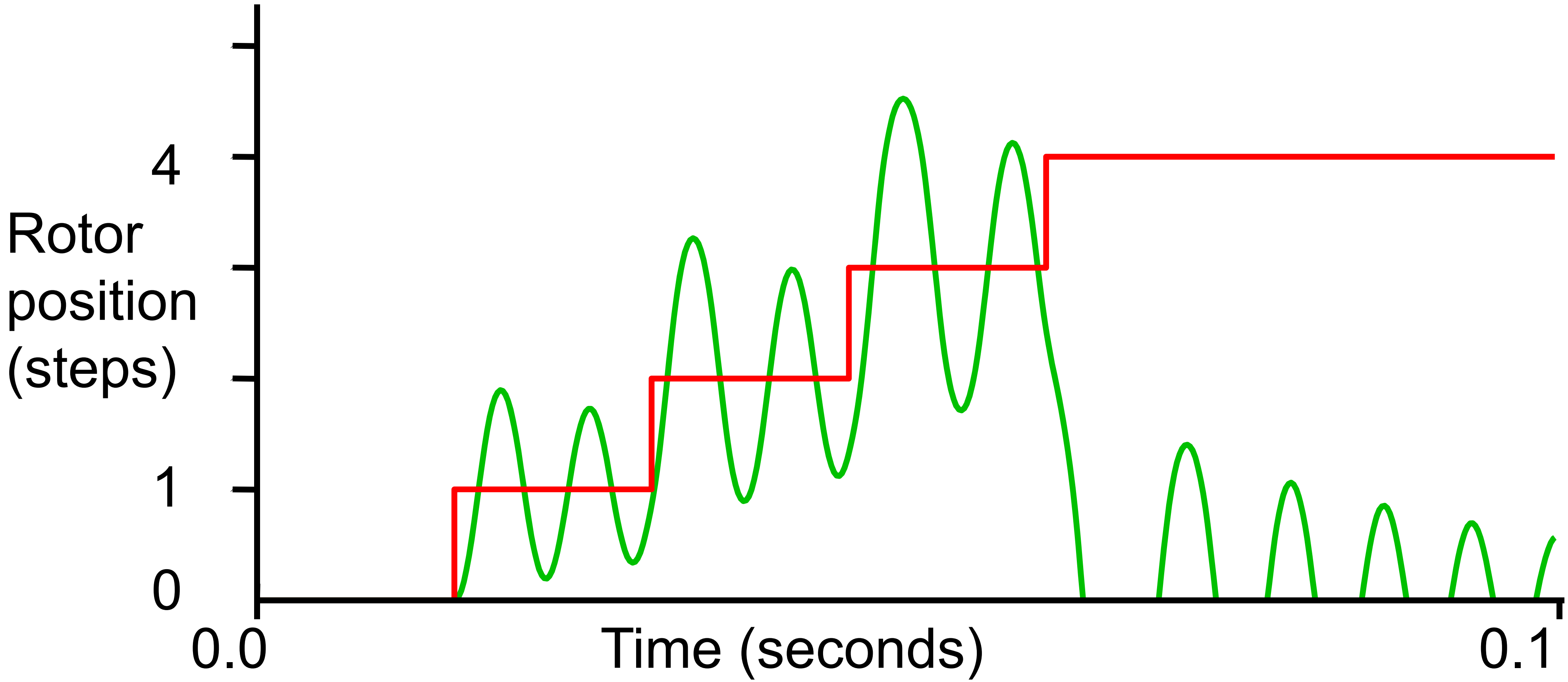
This shows 4 steps at a stepping rate of 40 steps/s

Multiple Step: Dynamic Response



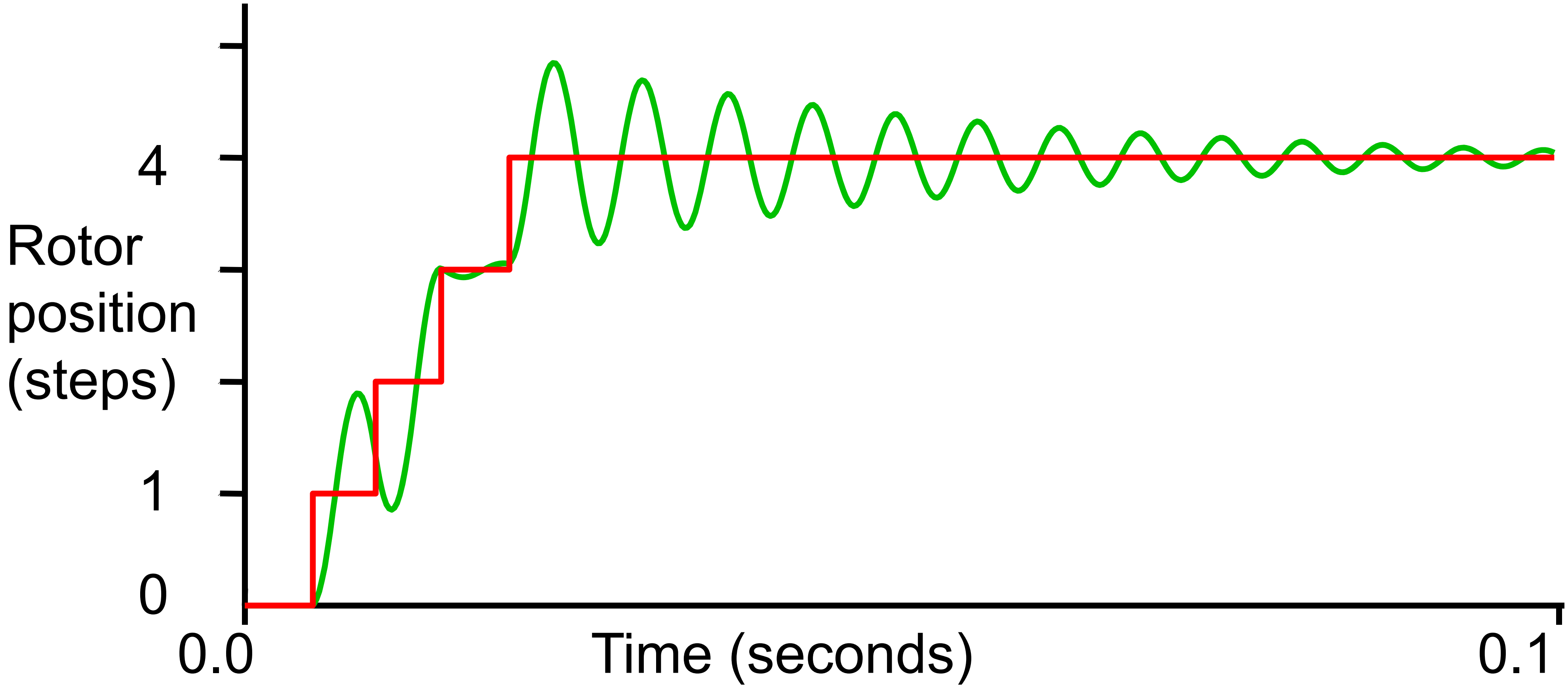
This shows loss of synchronisation as a result of 4 steps at a stepping speed of 132 steps/s (the resonance speed)

Multiple Step: Dynamic Response



This shows loss of synchronisation as a result of 4 steps at a stepping speed of 66 steps/s (half the resonance speed)

Multiple Step: Dynamic Response



This shows 4 steps at a stepping speed of 200 steps/s (well above the resonance speed)

The Problem of Resonance

Smaller step size of half-step sequence reduces amplitude of oscillation

Resonance can be eliminated by micro-stepping

Resonance can be reduced by applying extra damping

Simple viscous or Coulomb damping reduces the perfomance (maximum stepping speed)

1. Apply electrical damping to non-excited stator winding

2. Apply mechanical damping such as Viscously-Coupled Inertial Damping (VCID)

Electrical Damping of Resonance

When one stator winding is excited the rotor move to a position where the torque, and the rate-of-change of magnetic flux, are zero

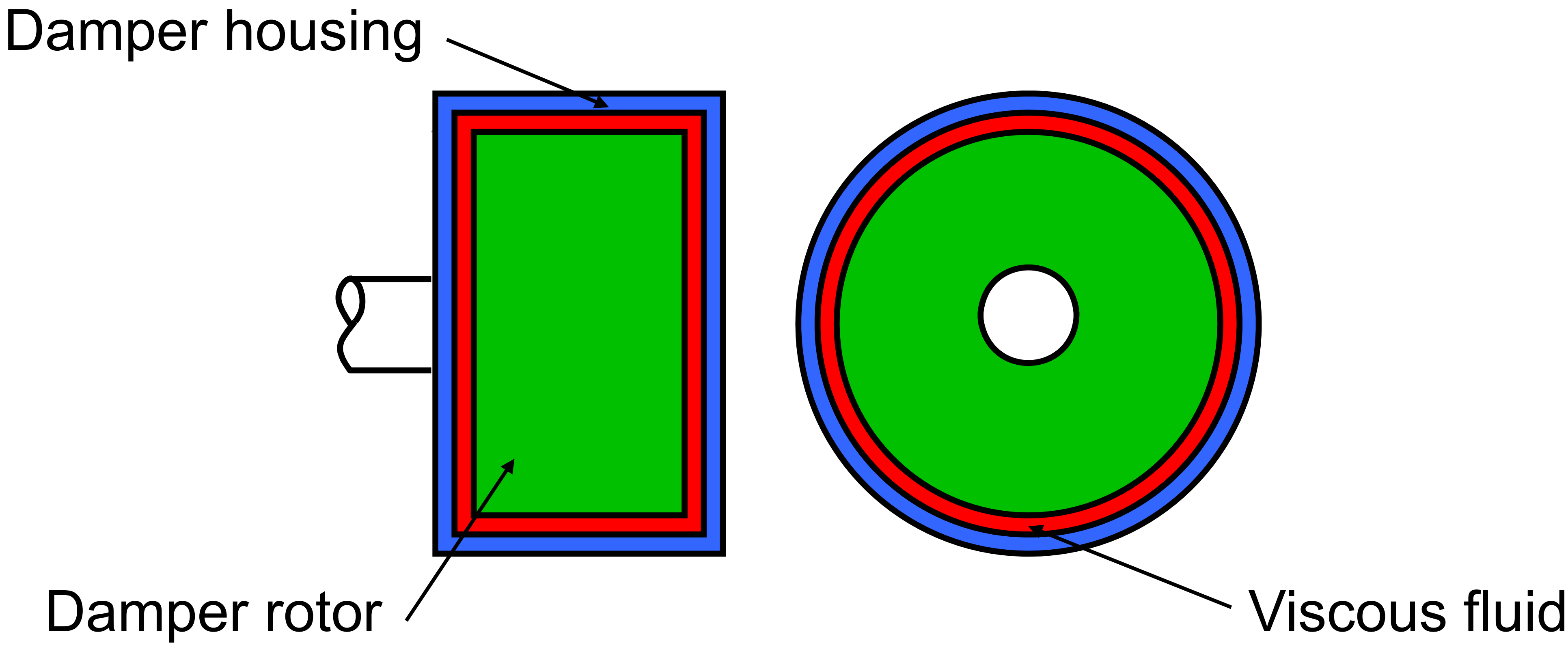
At this point the rate-of-change of magnetic flux in the orthogonal winding is a maximum

Any change in rotor position induces a voltage in the orthogonal winding

A resistor connected across the orthogonal winding will extract energy and damp the oscillation

Mechanical Damping of Resonance

Viscously-Coupled Inertial Damper (VCID), aka Lanchester damper:



There is no power loss during constant rotation

Micro-Stepping

Micro-stepping involves interpolating between full or half-step positions

This is achieved by linear control of the stator winding drive currents

Micro-stepping provides greater precision and smoother operation at low speeds, and eliminates resonance

Micro-stepping requires complex linear drives together with DACs to set the winding currents

Micro-Stepping

In sine-cosine micro-stepping the currents in the A and B stator windings are given by:

$$i_a = i_0 \sin\alpha$$

$$i_b = i_0 \cos\alpha$$

where varying α from 0 to $\pi/2$ moves the rotor position by one full step

In principle there is no limit to the number to the number of micro-step precision

In practice there is little point in using more than 256 micro-steps between full steps

Micro-Stepping

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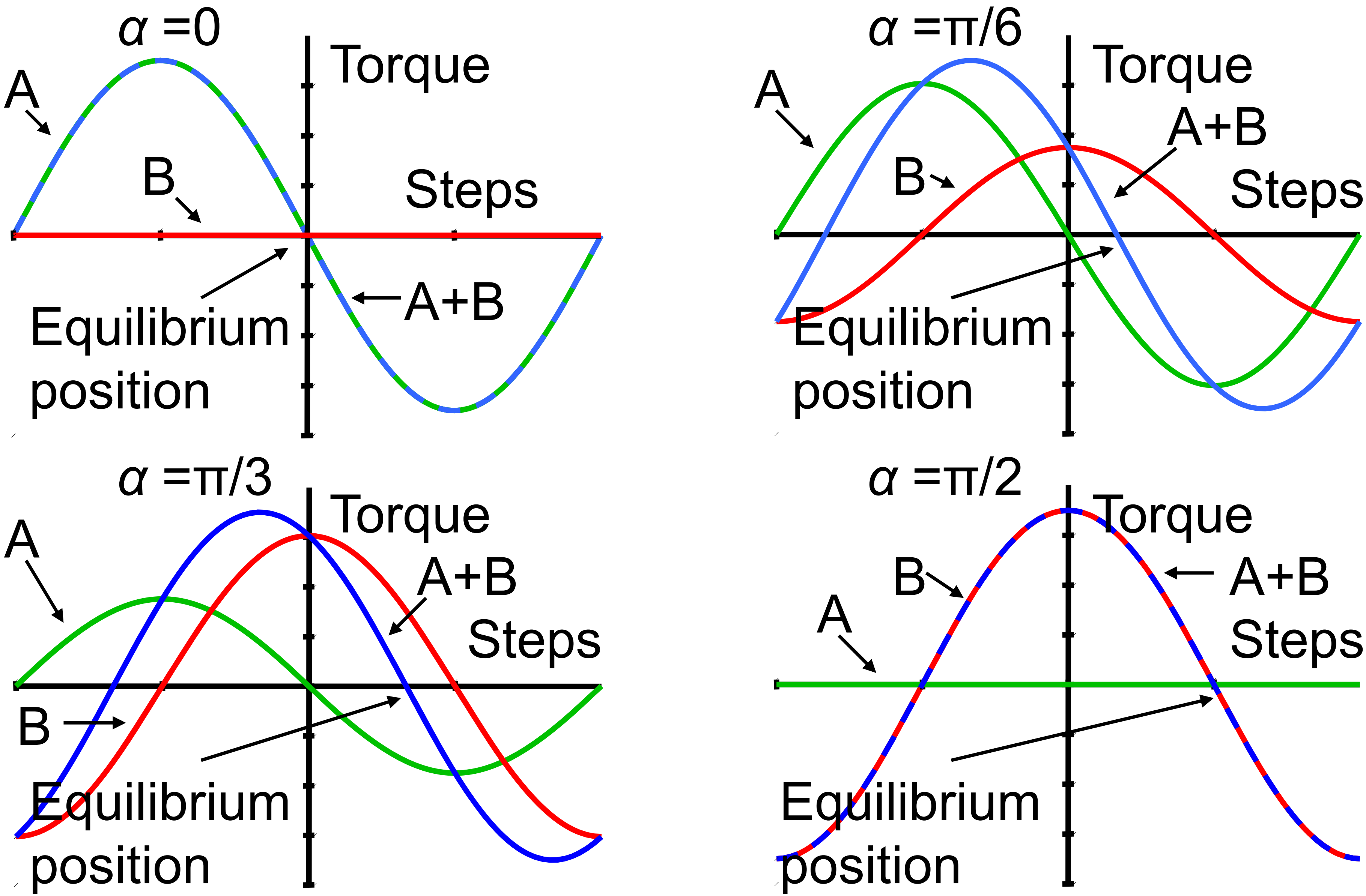


Micro-Stepping

The apparent superior precision of micro-stepping is only realised in practice in the absence of significant coulomb friction and load torque

The actual shape of the static torque curve is not exactly sinusoidal; this results in the micro steps being non-uniformly spaced

DAC quantisation will also result in non-uniformly spaced micro steps

Very high step rates are necessary to achieve normal rotation speeds

Start/Stop Operation

The pull-out torque-speed characteristic may extend to 10,000 or 20,000 steps/s

However, a motor cannot be started or stopped at these speeds

To move a large number of steps quickly a motor must be started at low speed and then accelerated to high speed

The range of load torques and speeds for which the motor will start and stop without loss of synchronisation is known as the pull-in characteristic

Start/Stop Operation

Consider a motor initially at rest

The first step of a sequence will cause the motor to accelerate and it must move far enough, and attain sufficient speed, for synchronisation to be maintained

If the motor moves 1/2 step during the period of the first step then a high torque will continue to be available during the next step

The maximum pull-in torque occurs at zero speed, and is the same as the zero-speed pull-torque

Start/Stop Operation

The maximum pull-in speed occurs when the load torque is zero and all the motor torque is available for acceleration:

$$J_r \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = T_{avg} = \frac{2T_0\sqrt{2}}{\pi}$$

Integrating:

$$\theta = \frac{1}{2}\frac{2T_0\sqrt{2}}{\pi J_r}t^2 \geq \frac{1}{2}.\frac{2\pi}{4n}$$

$$t \geq \sqrt{\frac{\pi^2 J_r}{4nT_0\sqrt{2}}}$$

Maximum pull-in speed:

$$f_{\max} = \frac{2}{\pi}\sqrt{\frac{nT_0\sqrt{2}}{J_r}}$$

Start/Stop Operation

For the ID31 Motor:

Number of rotor poles: $N_r = 50$

Rotor inertia: $J_r = 1.16\times10^{-5}$ kg m^2

Peak torque: $T_0 = 0.242$ N m

Maximum pull-in speed:

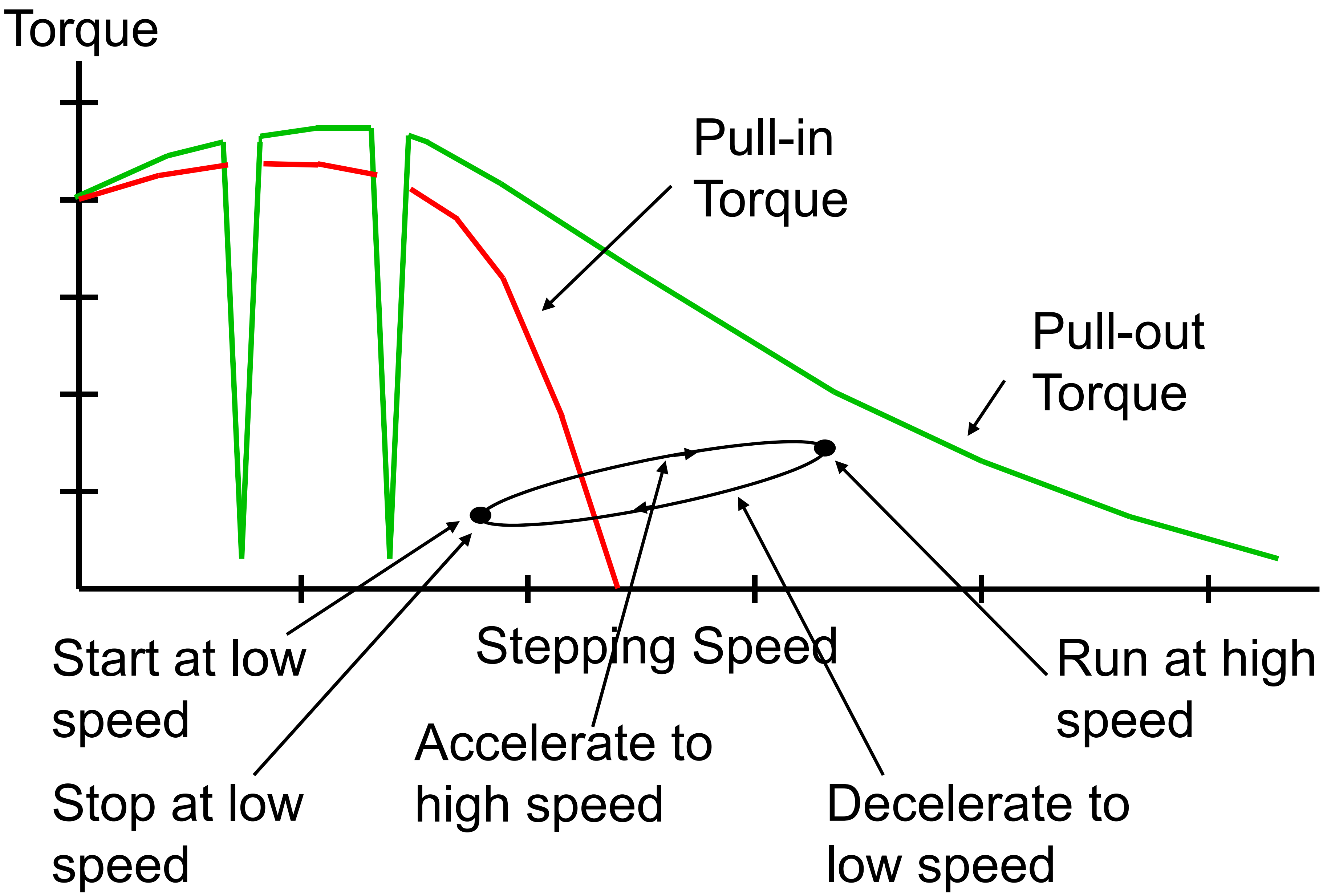
$$f_{\max} = \frac{2}{\pi}\sqrt{\frac{nT_0\sqrt{2}}{J_r}}$$

$$= 773 \text{ step/s}$$

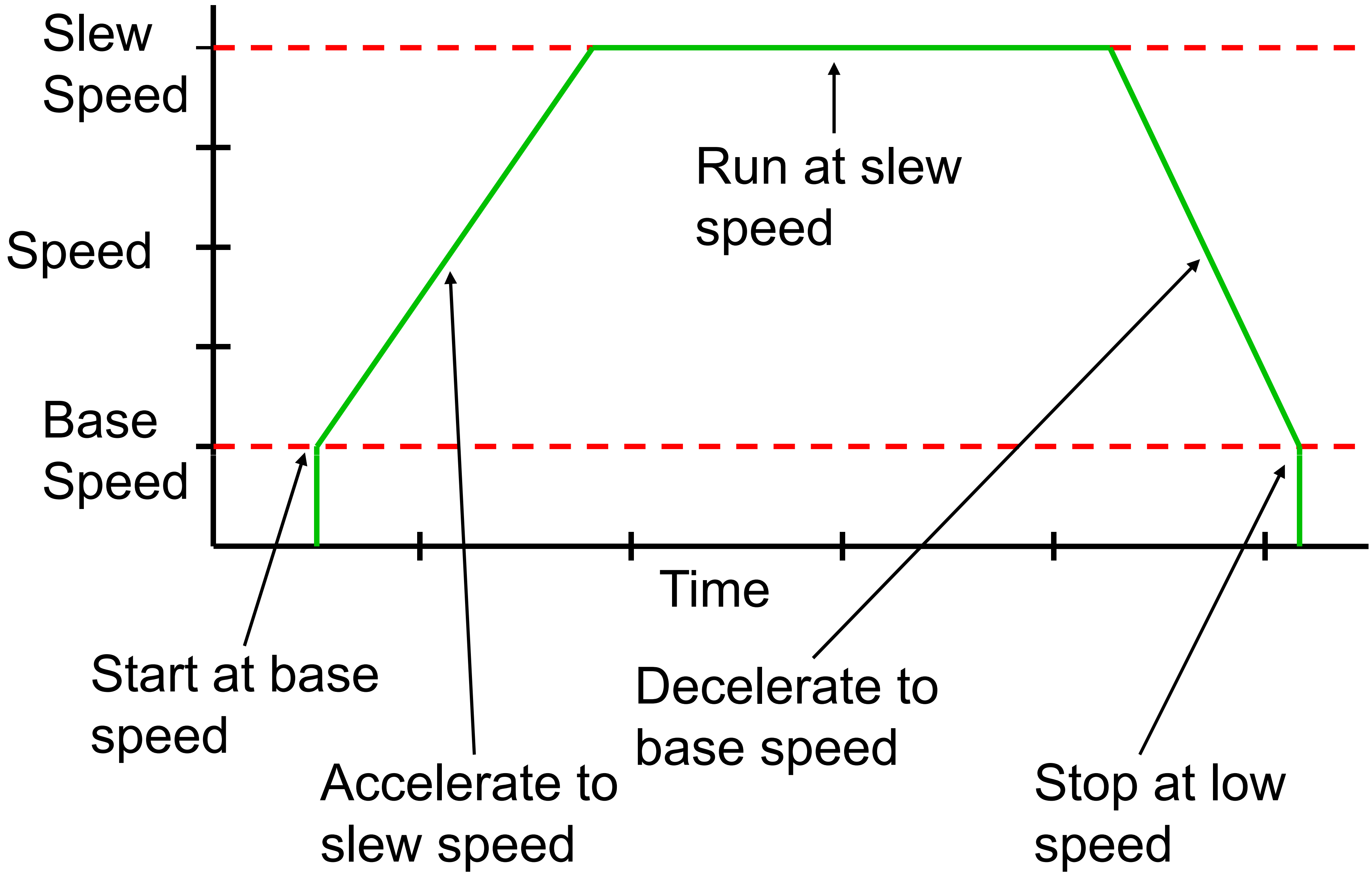
In general $f_{\max}$ is related to the resonant rate f_0:

$$f_{\max} = \frac{2}{\pi}\sqrt{\frac{nT_0\sqrt{2}}{J_r}} = 4\sqrt[4]{2}f_0 = 4.8f_0$$

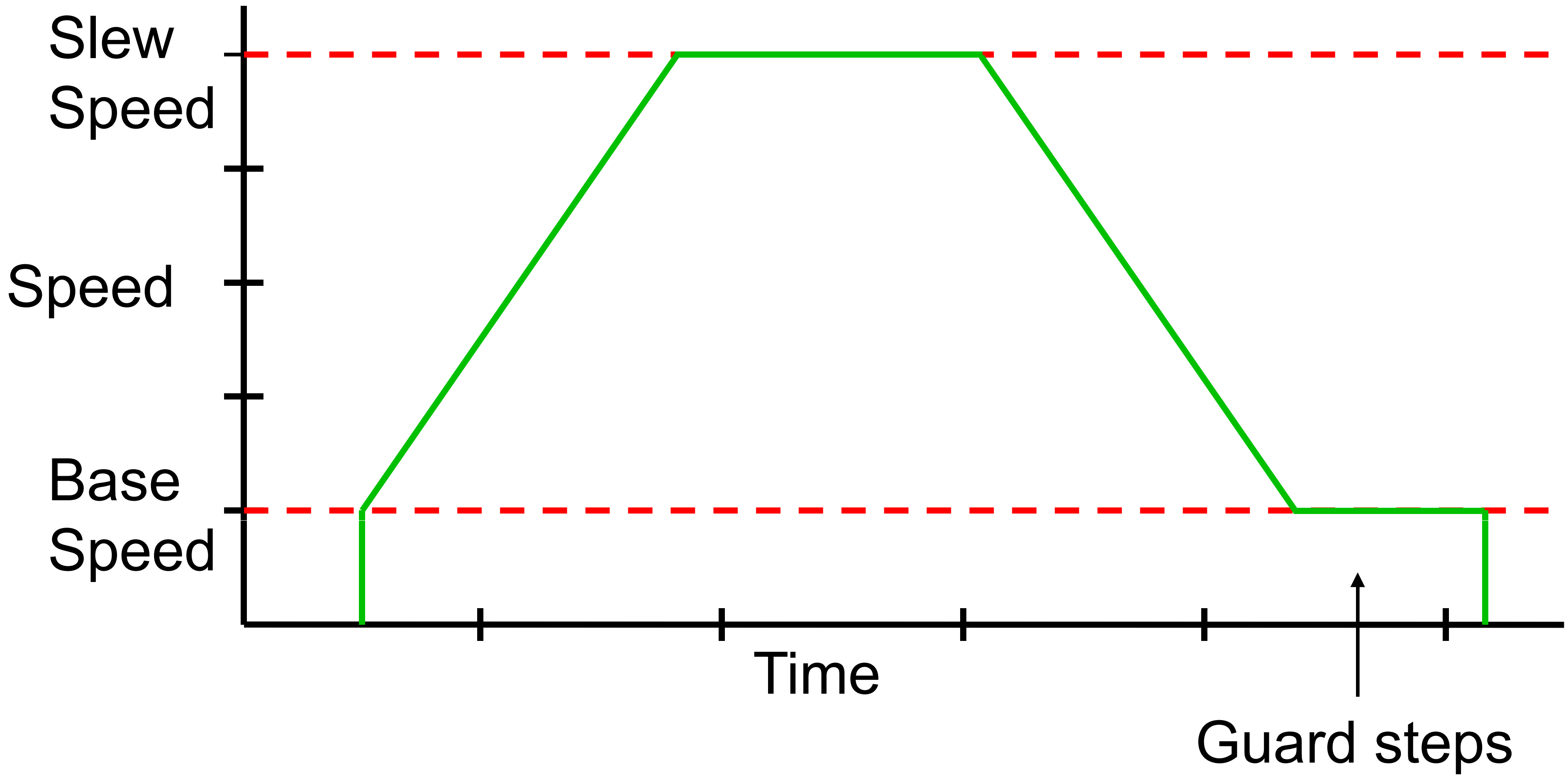
Start/Stop Operation



Start/Stop Operation

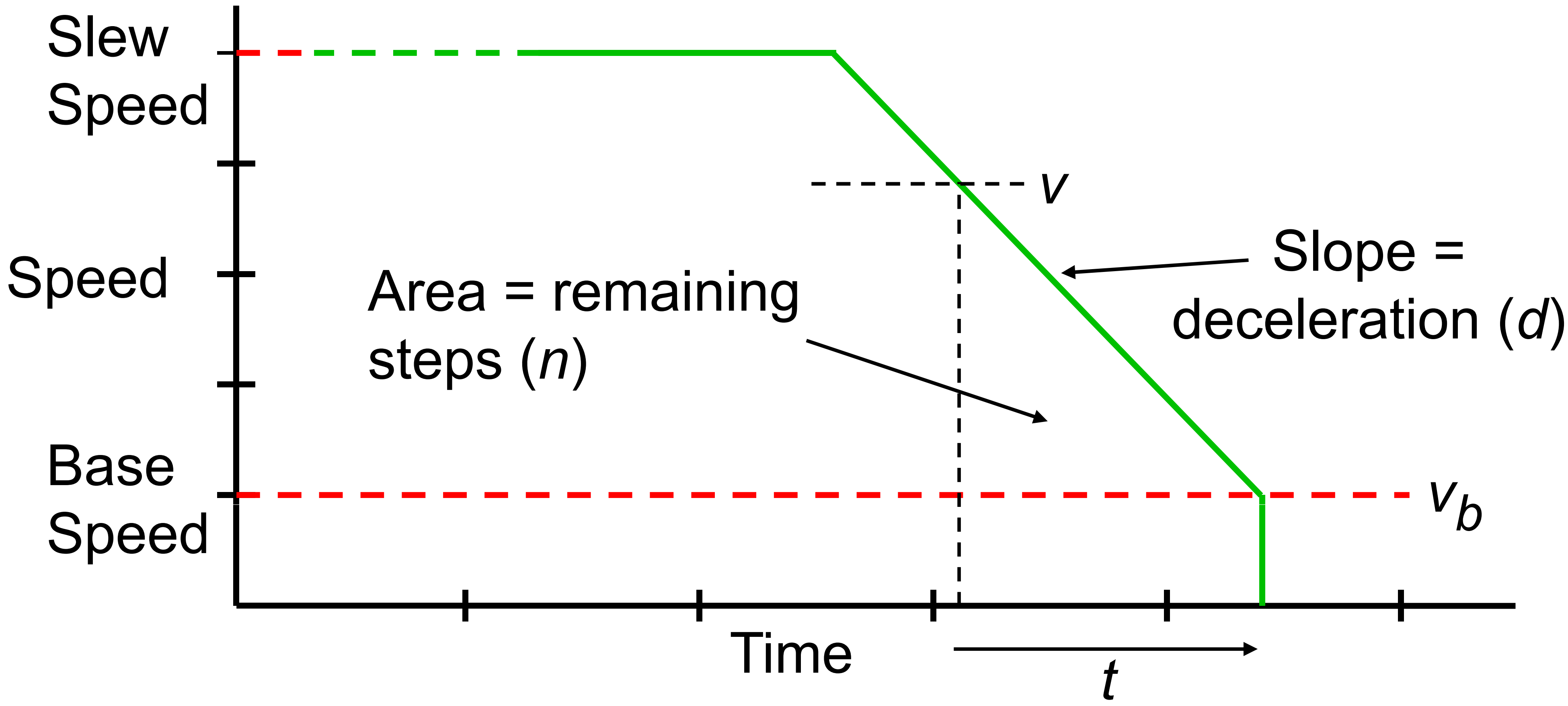


Start/Stop Operation



Count steps in acceleration phase: assume that the same number of steps will be required for deceleration

Start/Stop Operation

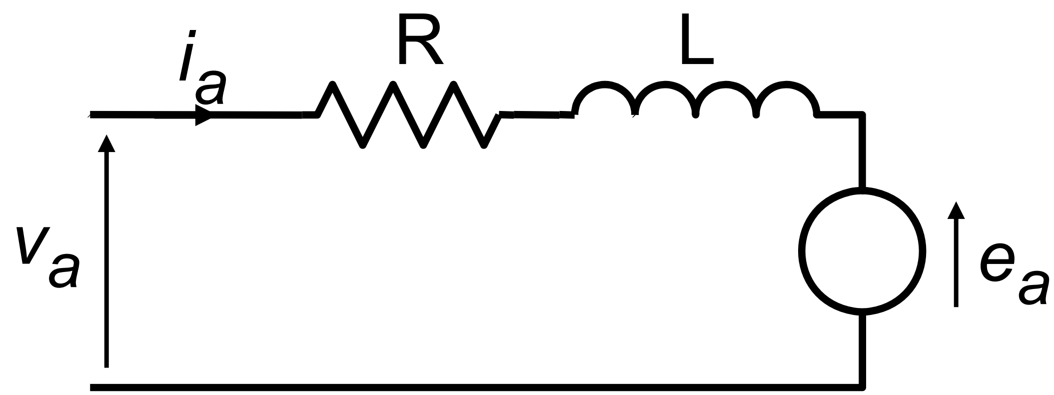


$$n = \frac{1}{2}t(v + v_b) \qquad v - v_b = d \times t$$

$$v = \sqrt{v_b^2 + 2nd}$$

Stepping Motor Model

Stator winding A:



v_a is the voltage applied to the winding
i_a is the winding current
e_a is the flux-induced voltage in the winding

$$v_a = Ri_a + L\frac{\mathrm{d}i_a}{\mathrm{d}t} + e_a$$

and for stator winding B:

$$v_b = Ri_b + L\frac{\mathrm{d}i_b}{\mathrm{d}t} + e_b$$

Stepping Motor Model

ψ_a and ψ_b are the magnetic flux in stator windings A and B, where:

$$\Psi_a = \Psi_m \cos n\theta$$

$$\Psi_b = \Psi_m \sin n\theta$$

and ψ_m is the maximum stator flux

The voltages e_a and e_b that are induced in the stator windings are given by:

$$e_a = m\frac{\mathrm{d}\Psi_a}{\mathrm{d}t} = -mn\Psi_m \sin n\theta.\frac{\mathrm{d}\theta}{\mathrm{d}t} = -K_c\omega \sin n\theta$$

$$e_b = m\frac{\mathrm{d}\Psi_b}{\mathrm{d}t} = -mn\Psi_m \cos n\theta.\frac{\mathrm{d}\theta}{\mathrm{d}t} = K_c\omega \cos n\theta$$

where m is the number of turns on the stator winding

Stepping Motor Model

Conservation of energy: mechanical power out = electrical power in:

$$\omega T_a = i_a e_a \quad \text{and:} \quad \omega T_b = i_b e_b$$

so:

$$T_a = -i_a K_c \sin n\theta \quad \text{and:} \quad T_b = i_b K_c \cos n\theta$$

Complete model:

$$J_r \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + D_r \frac{\mathrm{d}\theta}{\mathrm{d}t} = T + T_a + T_b$$

$$= T - i_a K_c \sin n\theta + i_b K_c \cos n\theta$$

$$v_a = Ri_a + L\frac{\mathrm{d}i_a}{\mathrm{d}t} - \omega K_c \sin n\theta$$

$$v_b = Ri_b + L\frac{\mathrm{d}i_b}{\mathrm{d}t} + \omega K_c \cos n\theta$$

Stepping Motor Model

State variable form:

$$\frac{d\theta}{dt} = \omega$$

$$\frac{d\omega}{dt} = \frac{T}{J_r} - \frac{D_r}{J_r}\omega - i_a \frac{K_c}{J_r}\sin n\theta + i_b \frac{K_c}{J_r}\cos n\theta$$

$$\frac{di_a}{dt} = \frac{1}{L}v_a - \frac{R}{L}i_a + \omega\frac{K_c}{L}\sin n\theta$$

$$\frac{di_b}{dt} = \frac{1}{L}v_b - \frac{R}{L}i_b - \omega\frac{K_c}{L}\cos n\theta$$

These equations can be solved numerically using, for example, the Runge-Kutta method

High Speed Operation

To determine the performance of a stepping motor with chopper drive it is necessary to resort to computer simulation

However, if some simplifying assumptions are made an expression describing the torque-speed characteristic can be derived

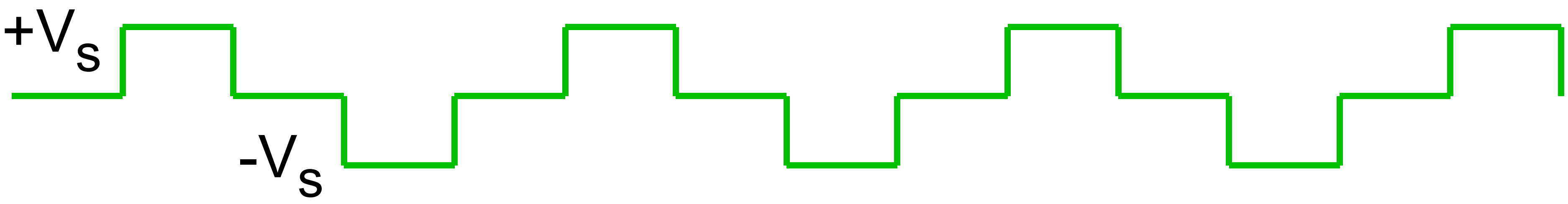
If resistor-ballast drive is used then the drive voltage waveforms are simple square waves

Because of the low-pass filtering action of the winding inductance the current waveforms are likely to be close to sinusoidal

High Speed Operation

To estimate the high-speed motor characteristic the winding currents are assumed to be sinusoidal

One winding on:



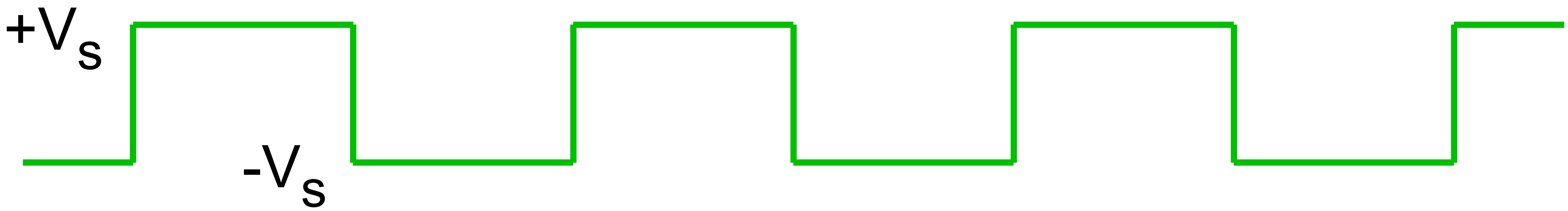
Fundamental sinusoidal component:

$$a_1 = \frac{1}{\pi}\int_0^{2\pi} f(\theta)\cos\theta\,\mathrm{d}\theta$$

$$= \frac{2V_s}{\pi}\int_{-\pi/4}^{\pi/4}\cos\theta\,\mathrm{d}\theta = \frac{4V_s}{\pi}\sin\frac{\pi}{4} = \frac{2\sqrt{2}V_s}{\pi} = 0.9V_s$$

High Speed Operation

Two windings on:



$$a_1 = \frac{2V_s}{\pi}\int_{-\pi/2}^{\pi/2}\cos\theta\,\mathrm{d}\theta = \frac{4V_s}{\pi}\sin\frac{\pi}{2} = \frac{4V_s}{\pi} = 1.3V_s$$

Half step:

$$a_1 = \frac{2V_s}{\pi}\int_{-3\pi/8}^{3\pi/8}\cos\theta\,\mathrm{d}\theta = \frac{4V_s}{\pi}\sin\frac{3\pi}{8} = 1.2V_s$$

High Speed Operation

Torque from stator winding A:

$$T_a = -i_a K_c \sin n\theta = -i_a K_c \sin n\omega t$$

Current in stator winding A:

$$i_a = i_0 \sin(n\omega t + \beta)$$

So stator torque is given by:

$$\begin{aligned}
T_a &= -i_0 \sin(n\omega t + \beta).K_c \sin n\omega t \\
&= -i_0 K_c.\{\sin n\omega t.\cos\beta + \cos n\omega t.\sin\beta\}.\sin n\omega t \\
&= -i_0 K_c.\{\sin^2 n\omega t.\cos\beta + \sin n\omega t.\cos n\omega t.\sin\beta\} \\
&= -\frac{i_0 K_c}{2}.\{1 - \cos 2n\omega t\}.\cos\beta - \frac{i_0 K_c}{2}\sin 2n\omega t.\sin\beta
\end{aligned}$$

High Speed Operation

Average stator torque:

$$\overline{T_a} = -\frac{i_0 K_c}{2}\cos\beta \qquad \overline{T} = \overline{T_a} + \overline{T_b} = -i_0 K_c \cos\beta$$

Voltage applied to stator winding A:

$$v_a = v_0 \sin(n\omega t + \alpha) = Ri_a + L\frac{\mathrm{d}i_a}{\mathrm{d}t} + e_a$$

$$= Ri_0 \sin(n\omega t + \beta) + Ln\omega i_0 \cos(n\omega t + \beta) - K_c\omega \sin n\omega t$$

Expanding trigonometrical functions:

$$v_0 \sin n\omega t.\cos\alpha + v_0 \cos n\omega t.\sin\alpha$$

$$= Ri_0 \sin n\omega t.\cos\beta + Ri_0 \cos n\omega t.\sin\beta$$

$$+ Ln\omega i_0 \cos n\omega t.\cos\beta - Ln\omega i_0 \sin n\omega t.\sin\beta - K_c\omega \sin n\omega t$$

High Speed Operation

Comparing coefficients of sin$n\omega t$ and cos$n\omega t$:

$$v_0 \cos\alpha = Ri_0 \cos\beta - Ln\omega i_0 \sin\beta - K_c\omega$$

$$v_0 \sin\alpha = Ri_0 \sin\beta + Ln\omega i_0 \cos\beta$$

Substitute for $i_0$sin β:

$$R^2 i_0 \cos\beta - Ln\omega\{v_0 \sin\alpha - Ln\omega i_0 \cos\beta\} = RK_c\omega + Rv_0 \cos\alpha$$

or:

$$\{R^2 + L^2n^2\omega^2\} i_0 \cos\beta = v_0\{Ln\omega \sin\alpha + R\cos\alpha\} + RK_c\omega$$

$$= v_0\sqrt{R^2 + L^2n^2\omega^2}\sin(\alpha + \gamma) + RK_c\omega$$

High Speed Operation

So:

$$i_0 \cos\beta = \frac{v_0 \sin(\alpha+\gamma)}{\sqrt{R^2 + L^2 n^2 \omega^2}} + \frac{R K_c \omega}{R^2 + L^2 n^2 \omega^2}$$

Total motor torque:

$$\overline{T} = -\frac{K_c v_0 \sin(\alpha+\gamma)}{\sqrt{R^2 + L^2 n^2 \omega^2}} - \frac{R K_c^2 \omega}{R^2 + L^2 n^2 \omega^2}$$

Maximum motor torque:

$$\overline{T} = \frac{K_c v_0}{\sqrt{R^2 + L^2 n^2 \omega^2}} - \frac{R K_c^2 \omega}{R^2 + L^2 n^2 \omega^2}$$

High Speed Operation

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Maximum motor torque:

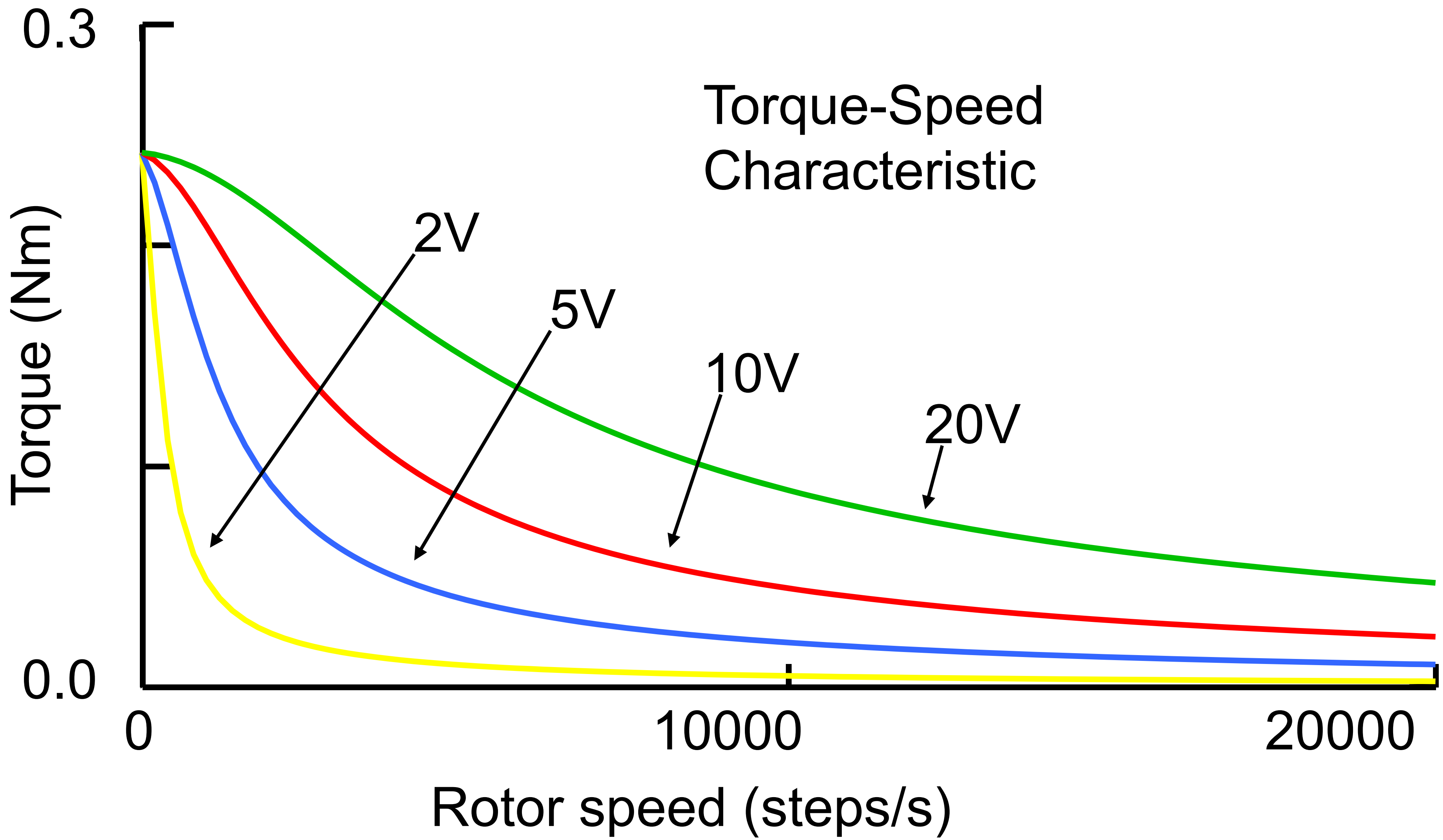
$$\bar{T} = \frac{K_c v_0}{\sqrt{R^2 + L^2 n^2 \omega^2}} - \frac{R K_c^2 \omega}{R^2 + L^2 n^2 \omega^2}$$

Firstly the drive voltage is chosen, using the appropriate sinusoidal component of the actual voltage drive waveform

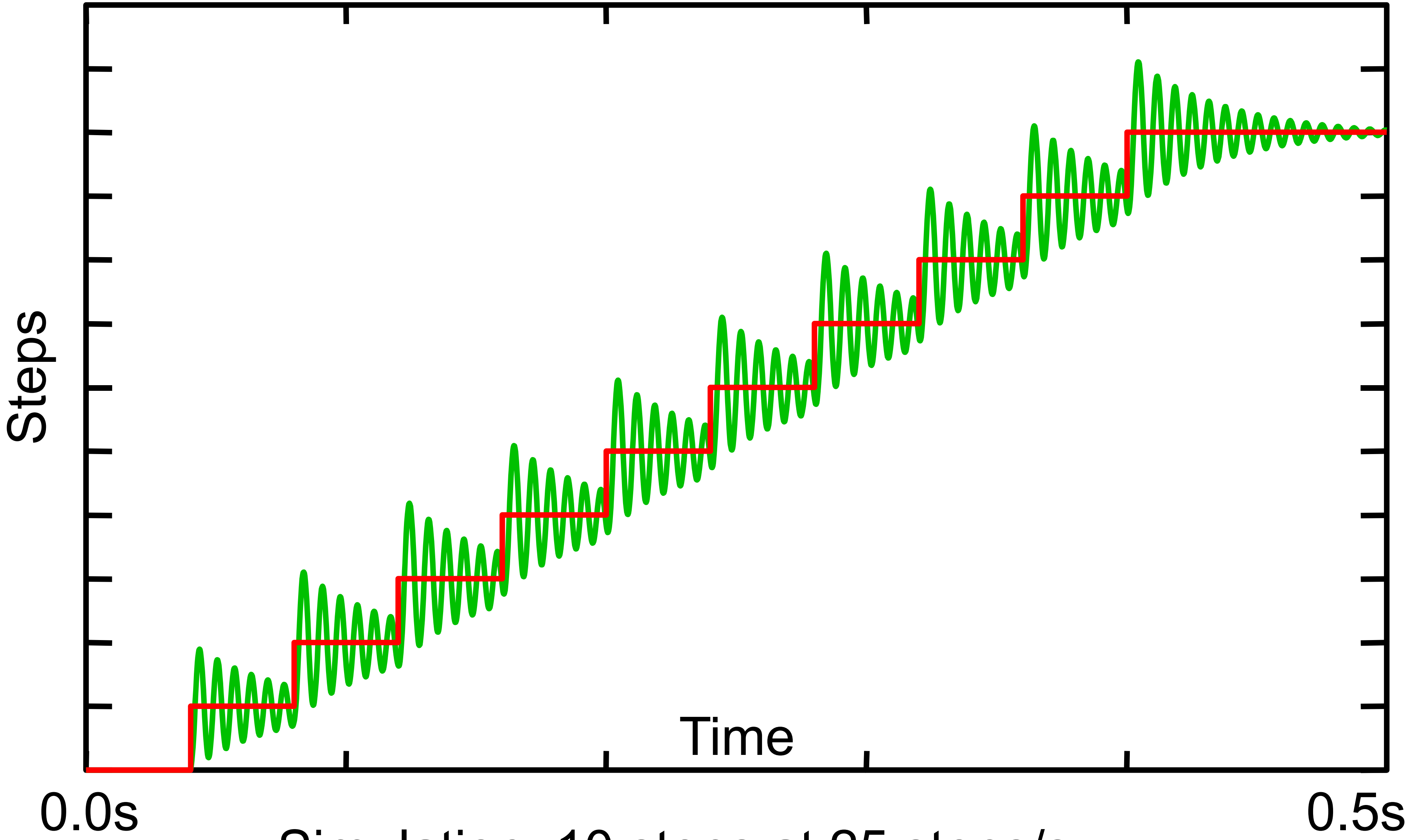
The resistance *R* comprises the motor winding resistance plus the ballast resistance (chosen to give the correct winding current)

Surprisingly a significant torque is available at speeds above $\omega K_c = v_0$

High Speed Operation

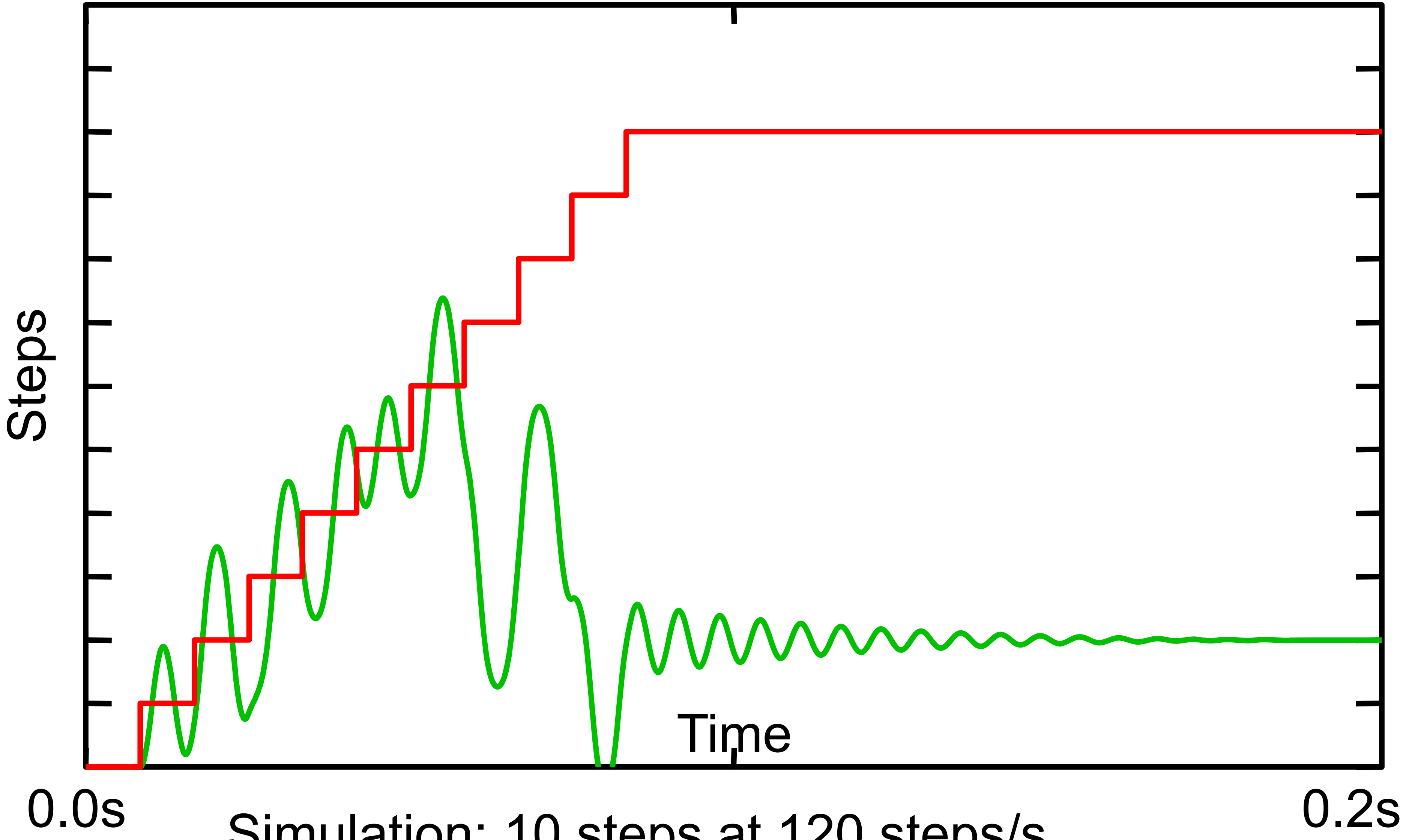


Stepping Motor Simulation



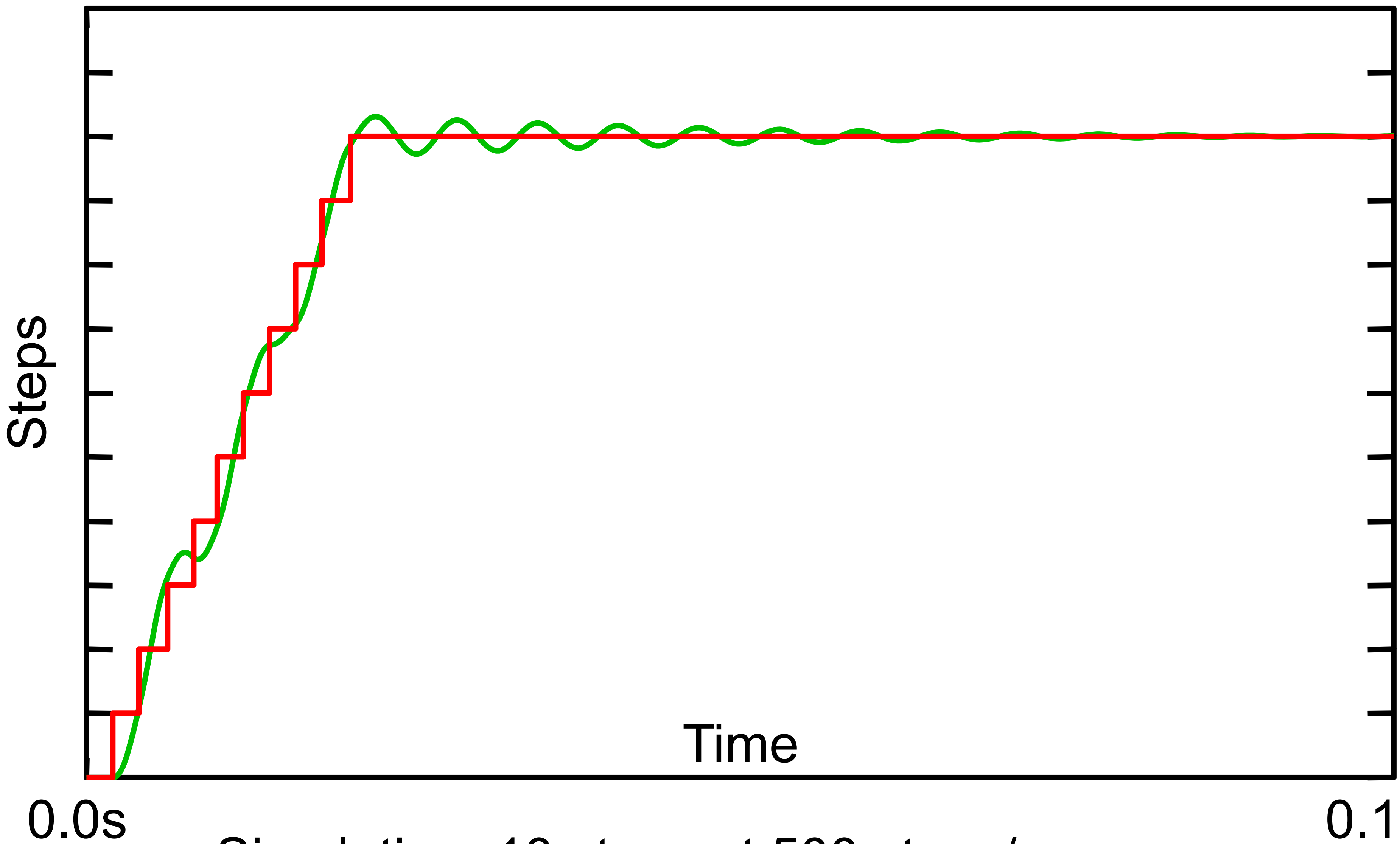
Simulation: 10 steps at 25 steps/s

Stepping Motor Simulation



Simulation: 10 steps at 120 steps/s

Stepping Motor Simulation



Simulation: 10 steps at 500 steps/s

Stepping Motor Simulation

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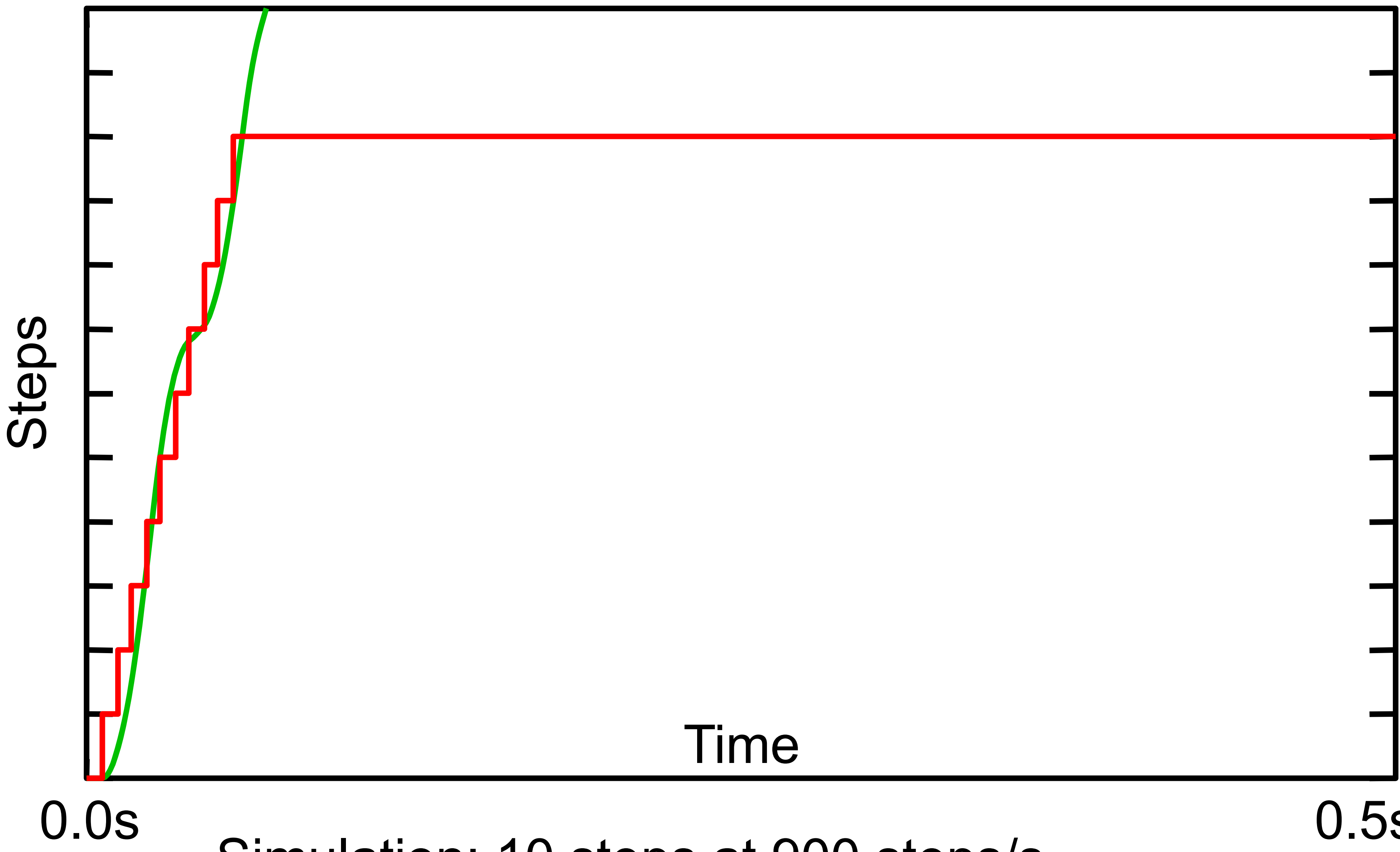


Simulation: 10 steps at 900 steps/s

Velocity-Error Plane

It is convenient to display the performance of a stepping motor on the velocity-error plane

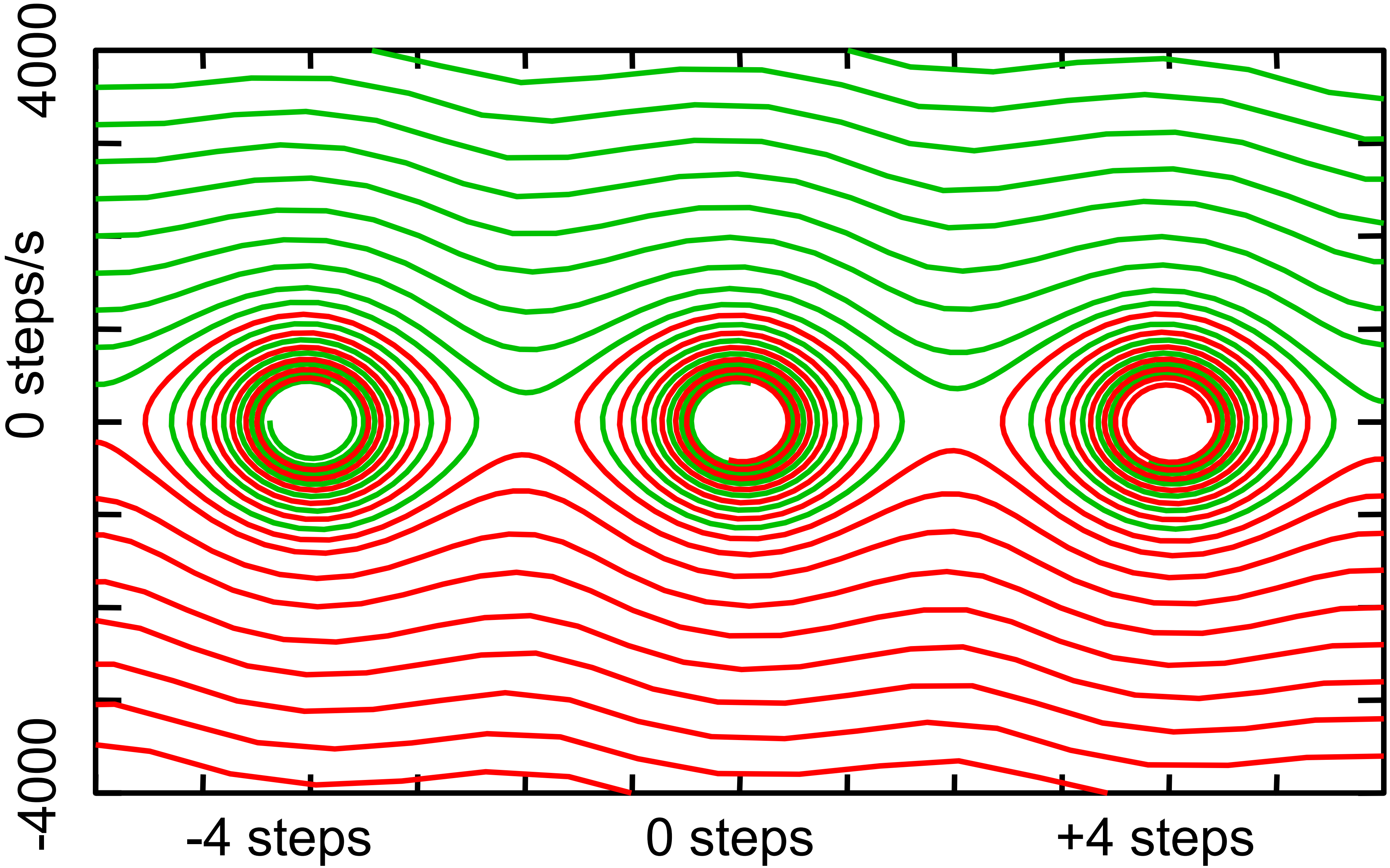
This plane has positional error as the horizontal axis, and velocity as the vertical axis

A velocity-error plane trajectory always finishes at the zero-error, zero-velocity point if the motor remains synchronised

It is therefore obvious from a simulation whether or not synchronisation has been lost

The velocity-error plane is sometimes called the “phase plane”

Velocity-Error Plane Trajectories



Low damping coefficient: D_r=0.0006

Velocity-Error Plane Trajectories

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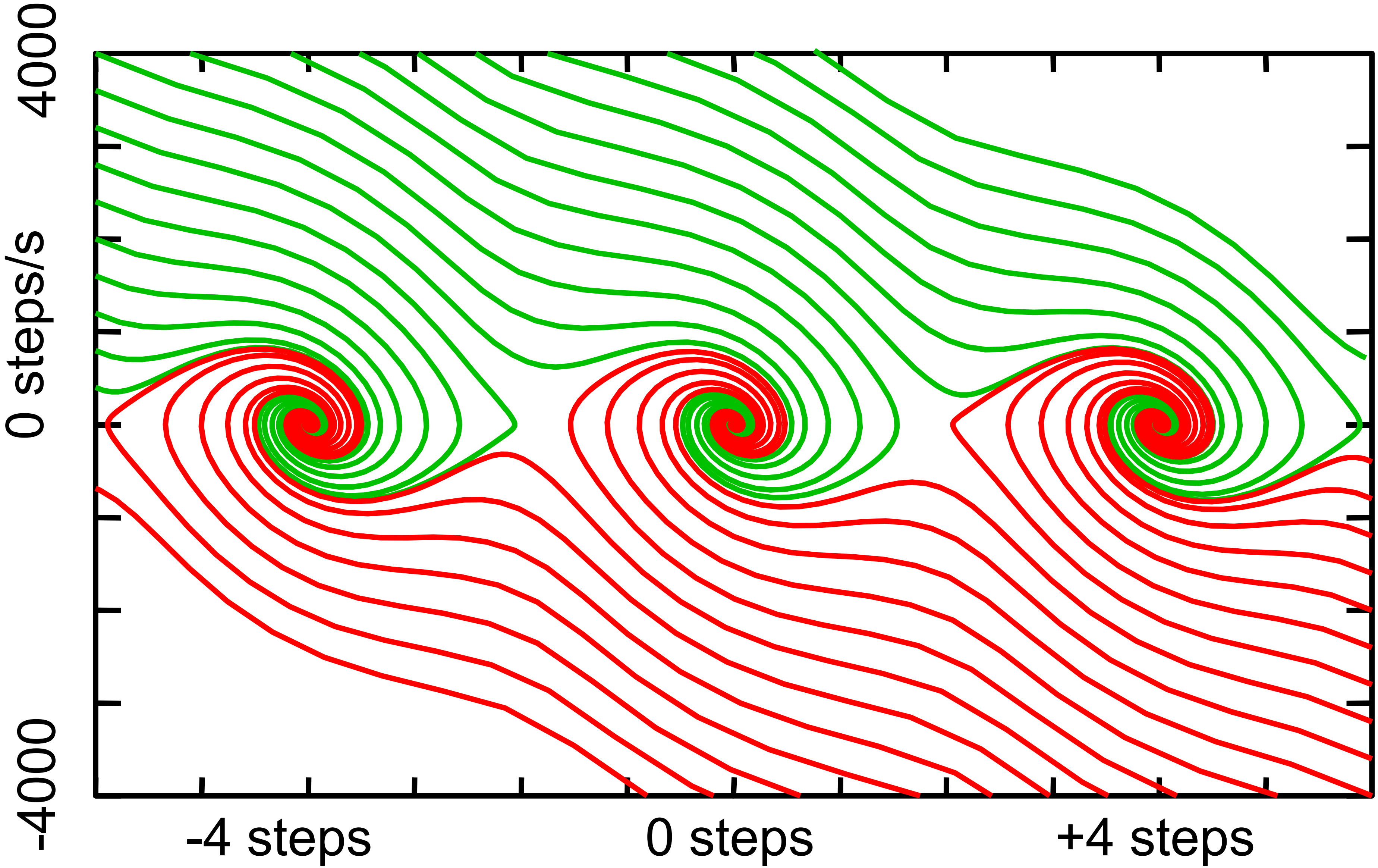
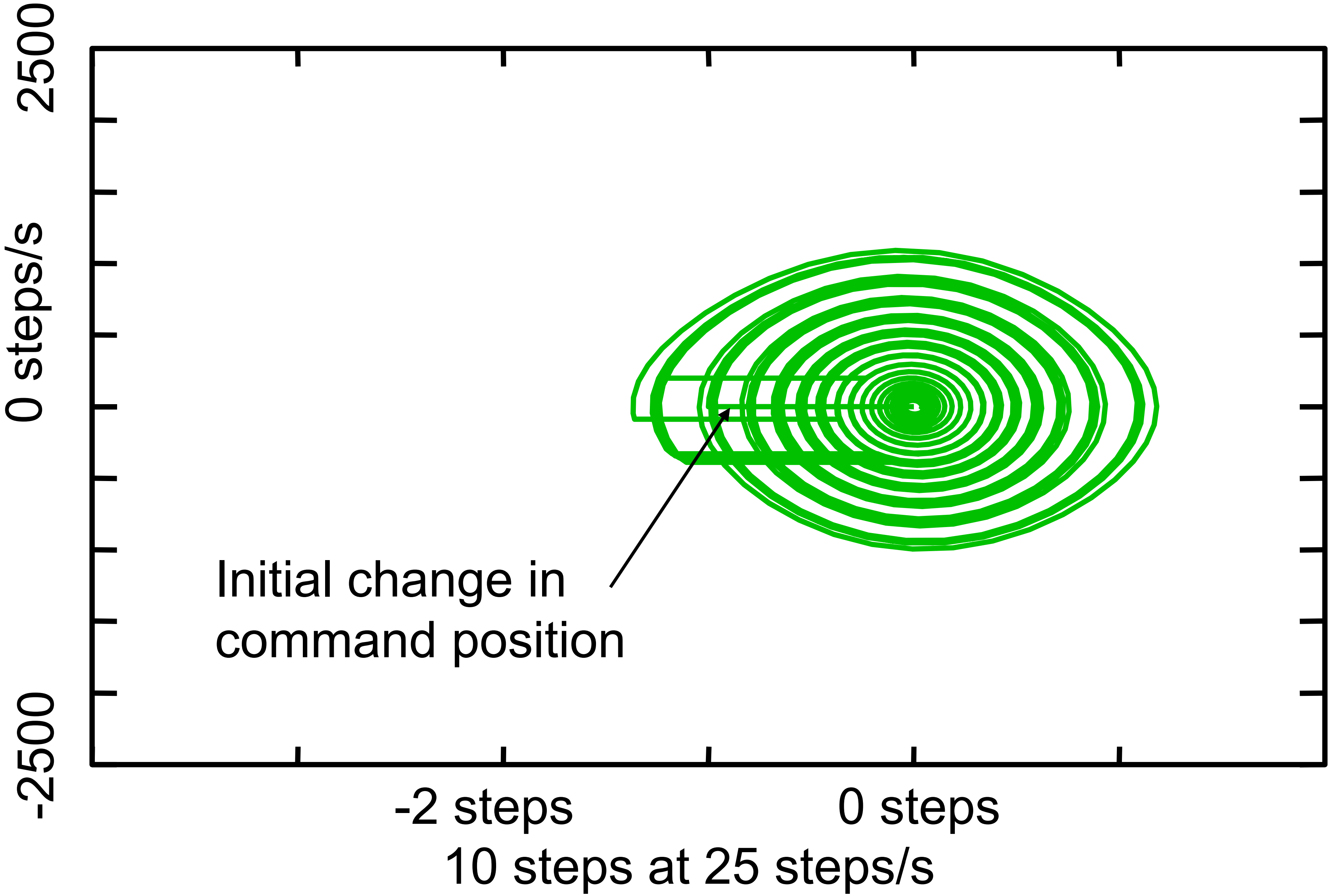
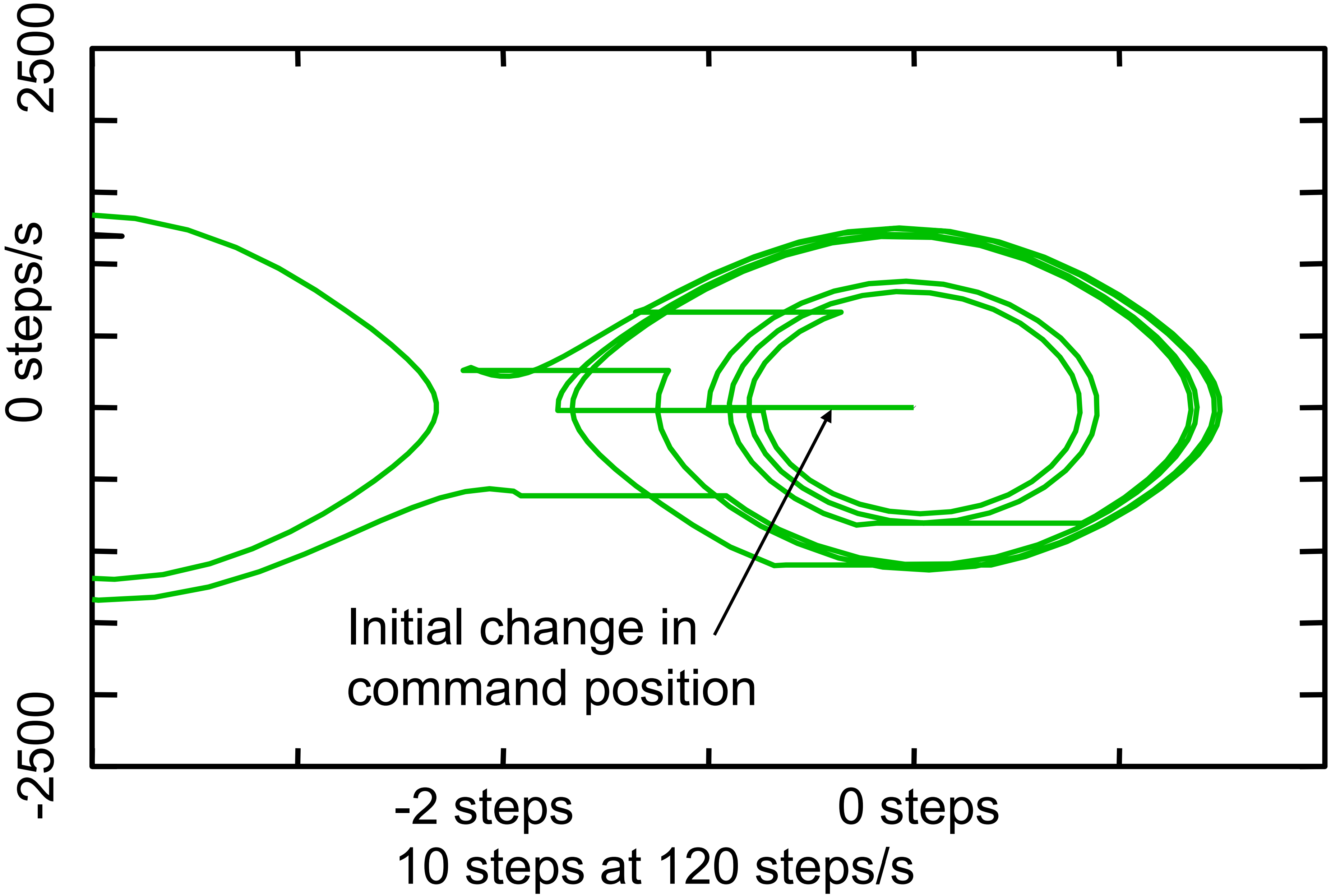


High damping coefficient: D_r=0.006

Velocity-Error Plane Trajectories



Velocity-Error Plane Trajectories



Velocity-Error Plane Trajectories

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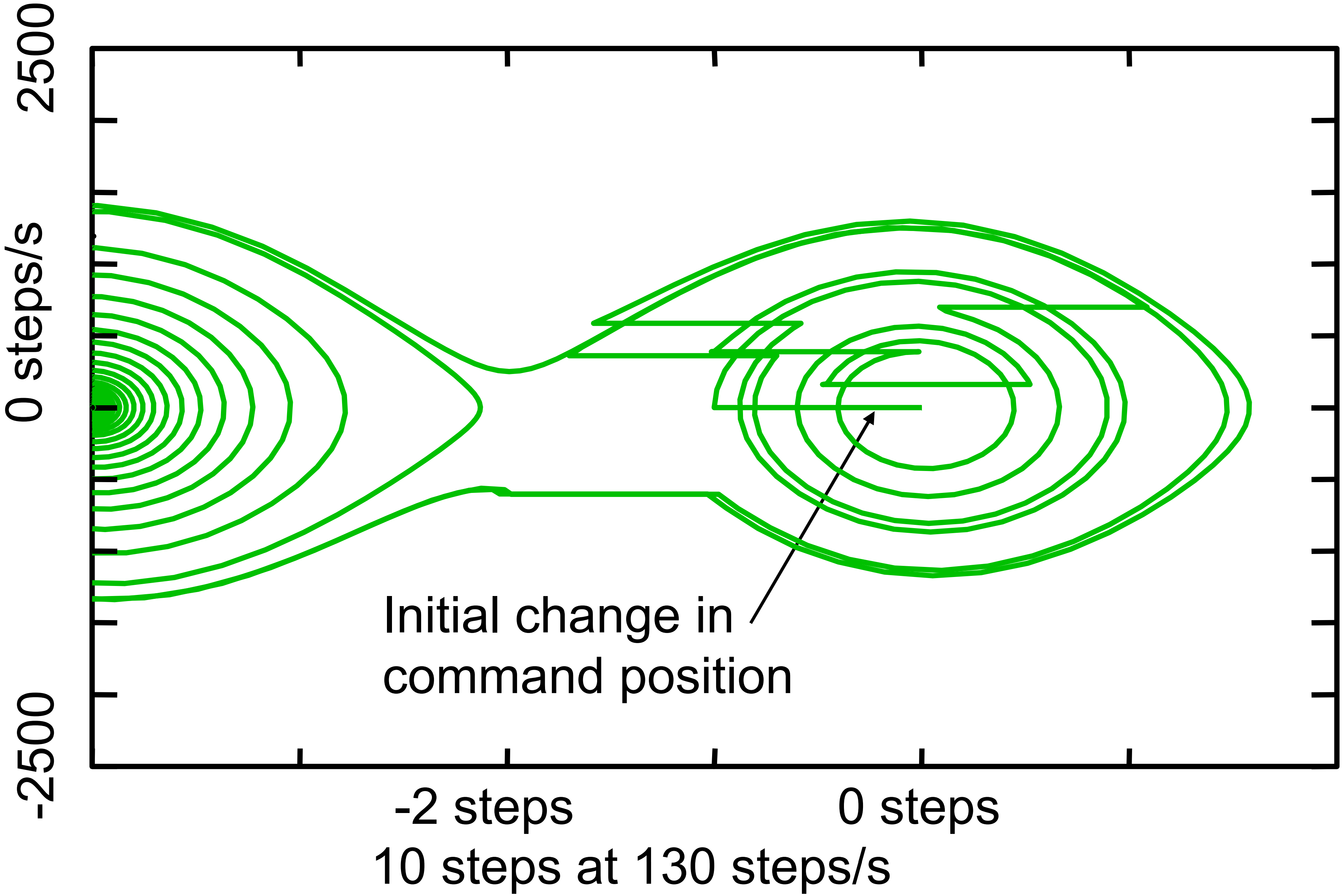
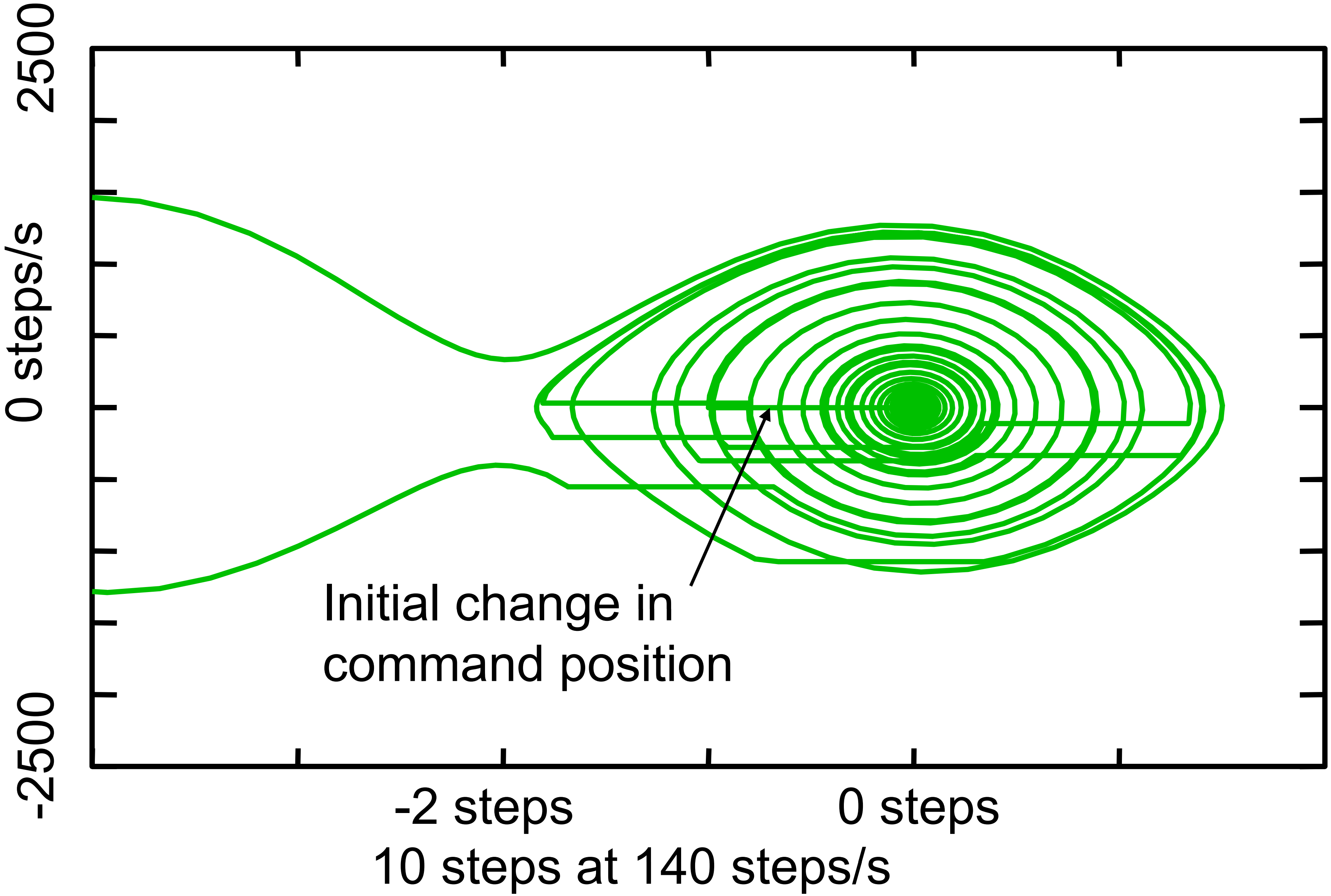
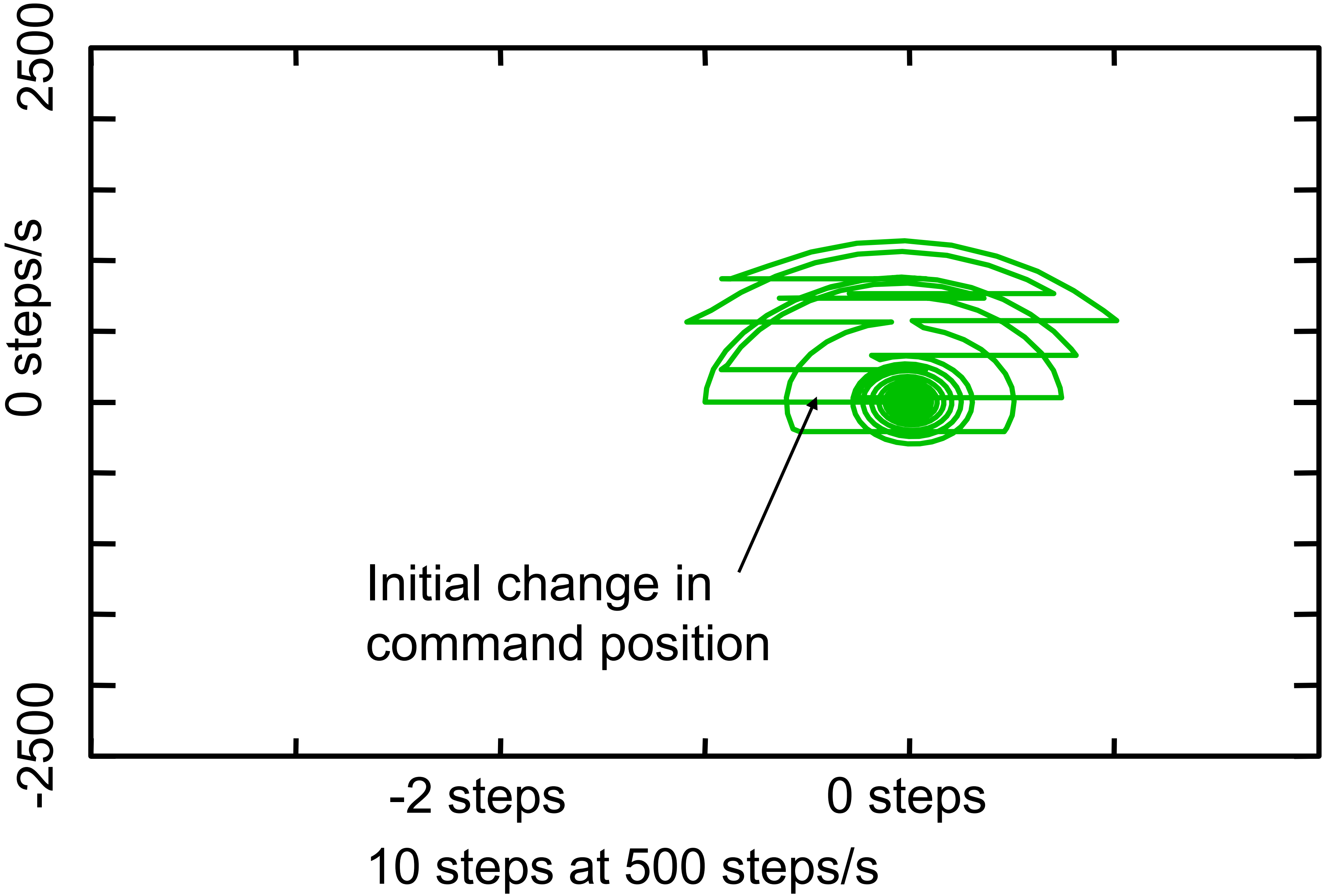


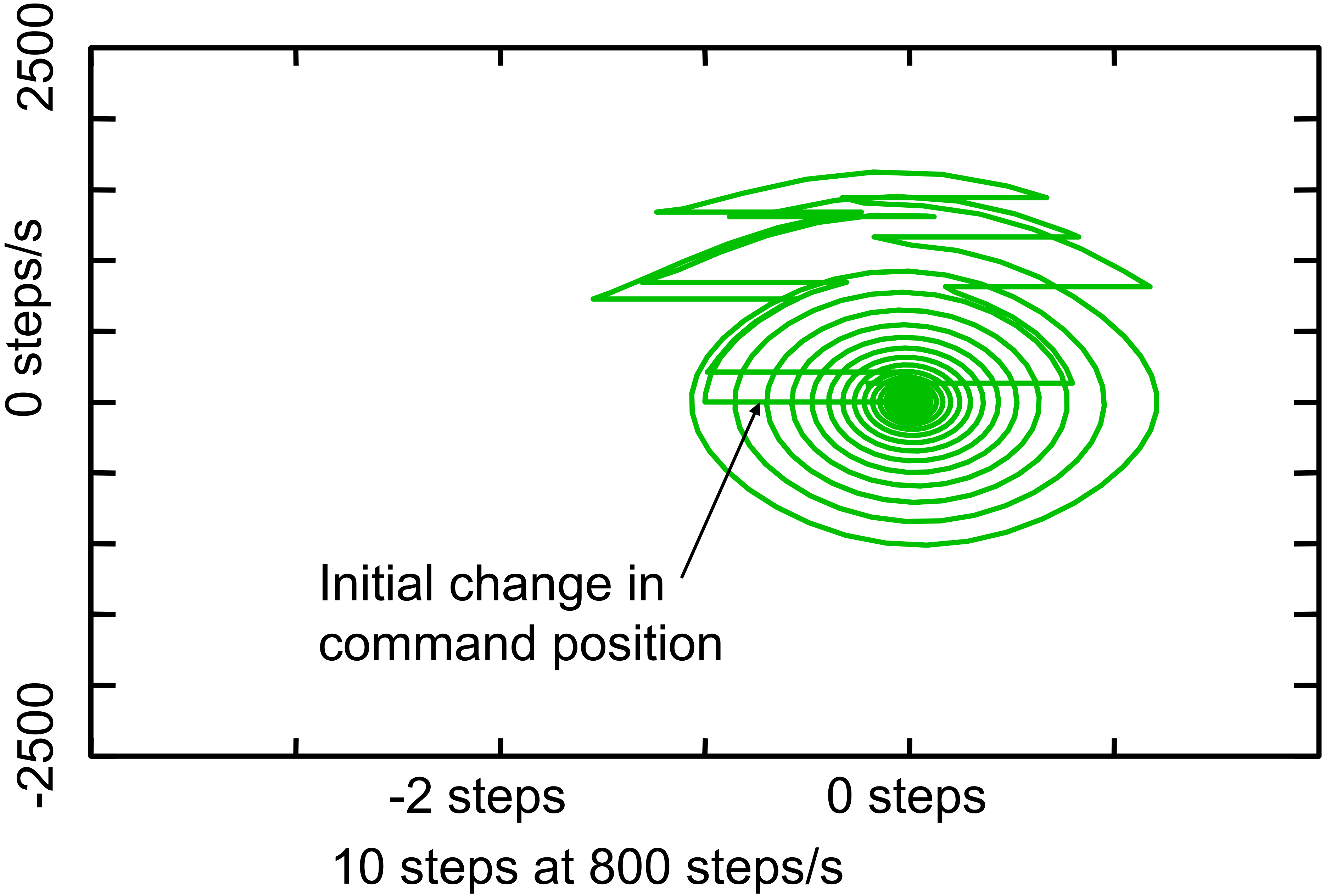
Velocity-Error Plane Trajectories



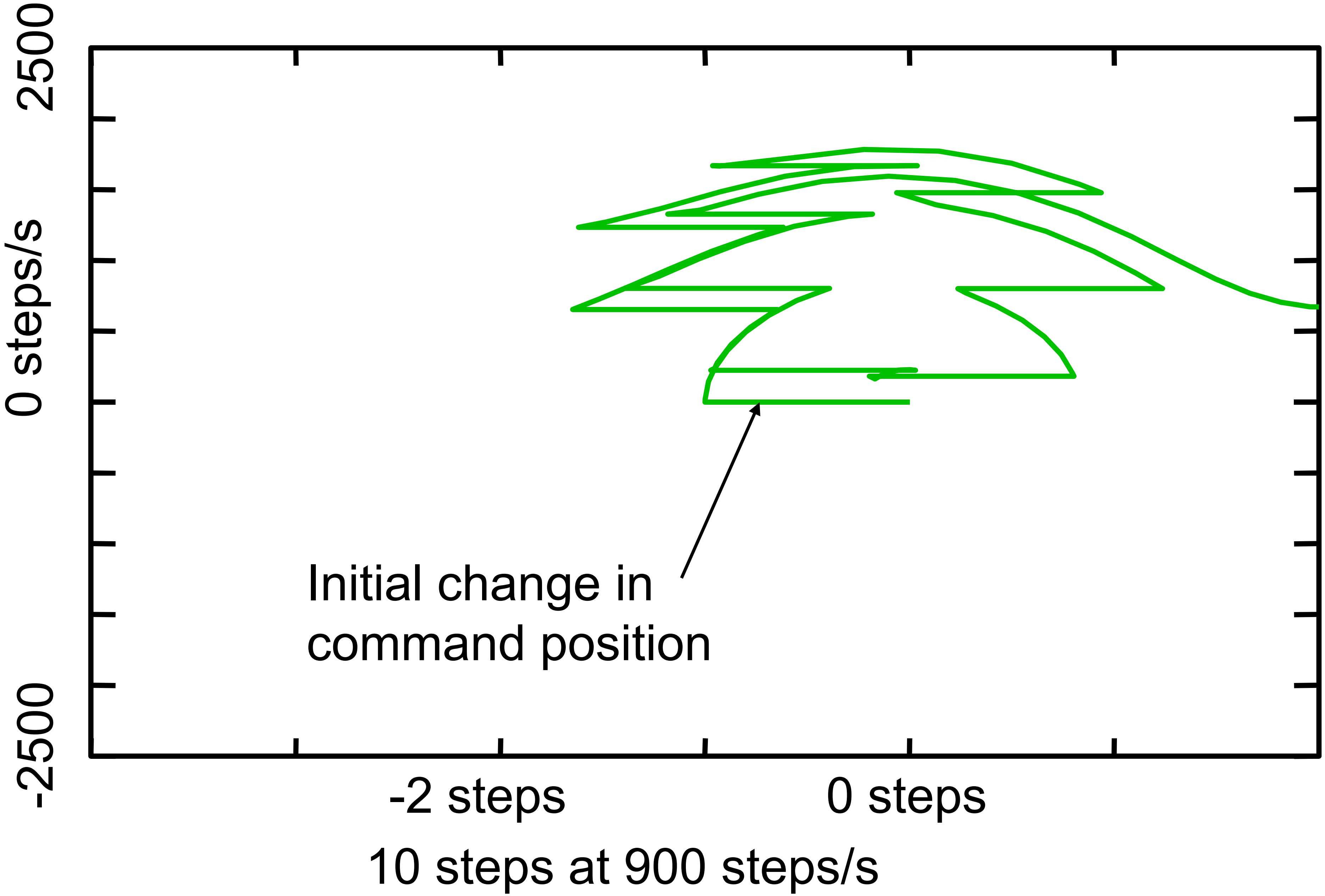
Velocity-Error Plane Trajectories



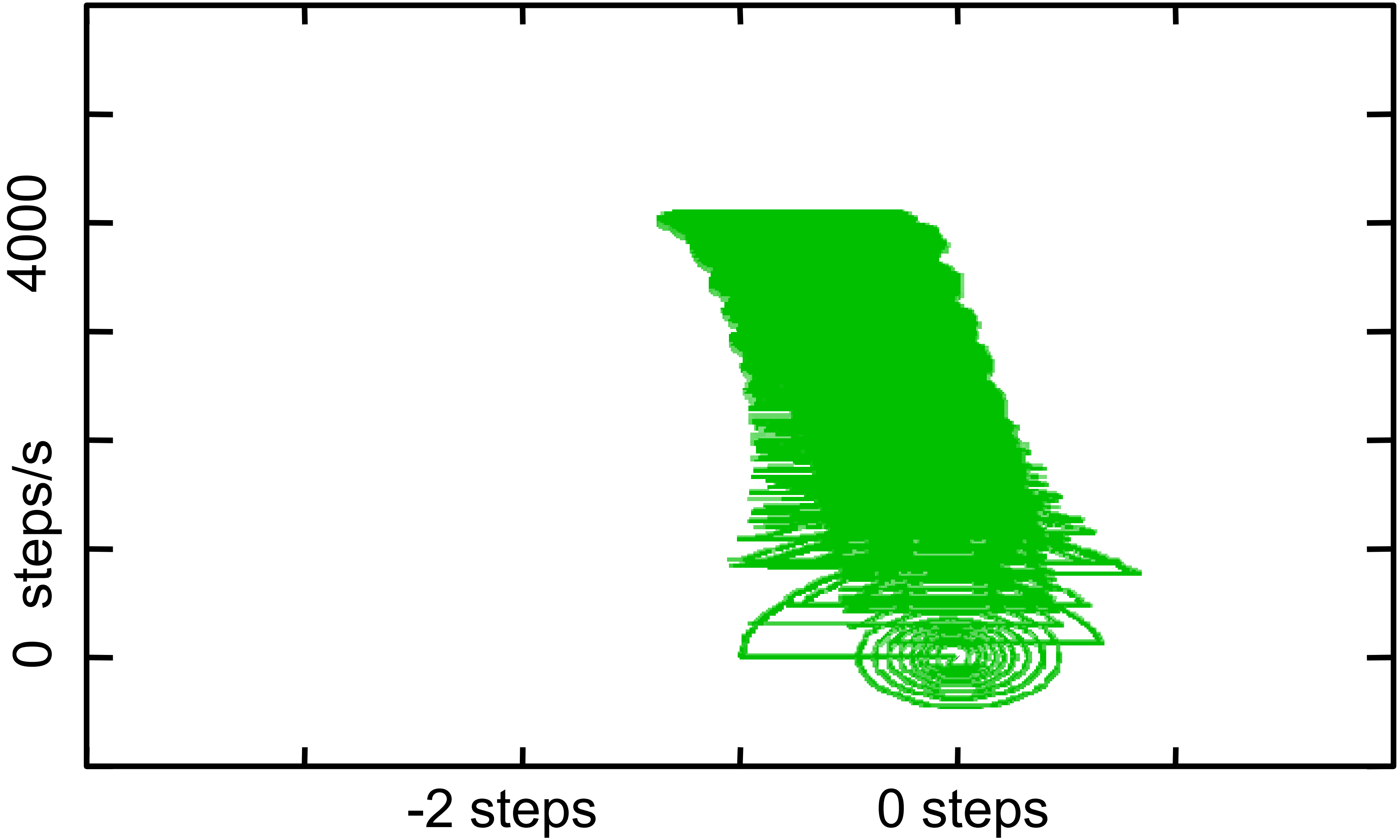
Velocity-Error Plane Trajectories



Velocity-Error Plane Trajectories

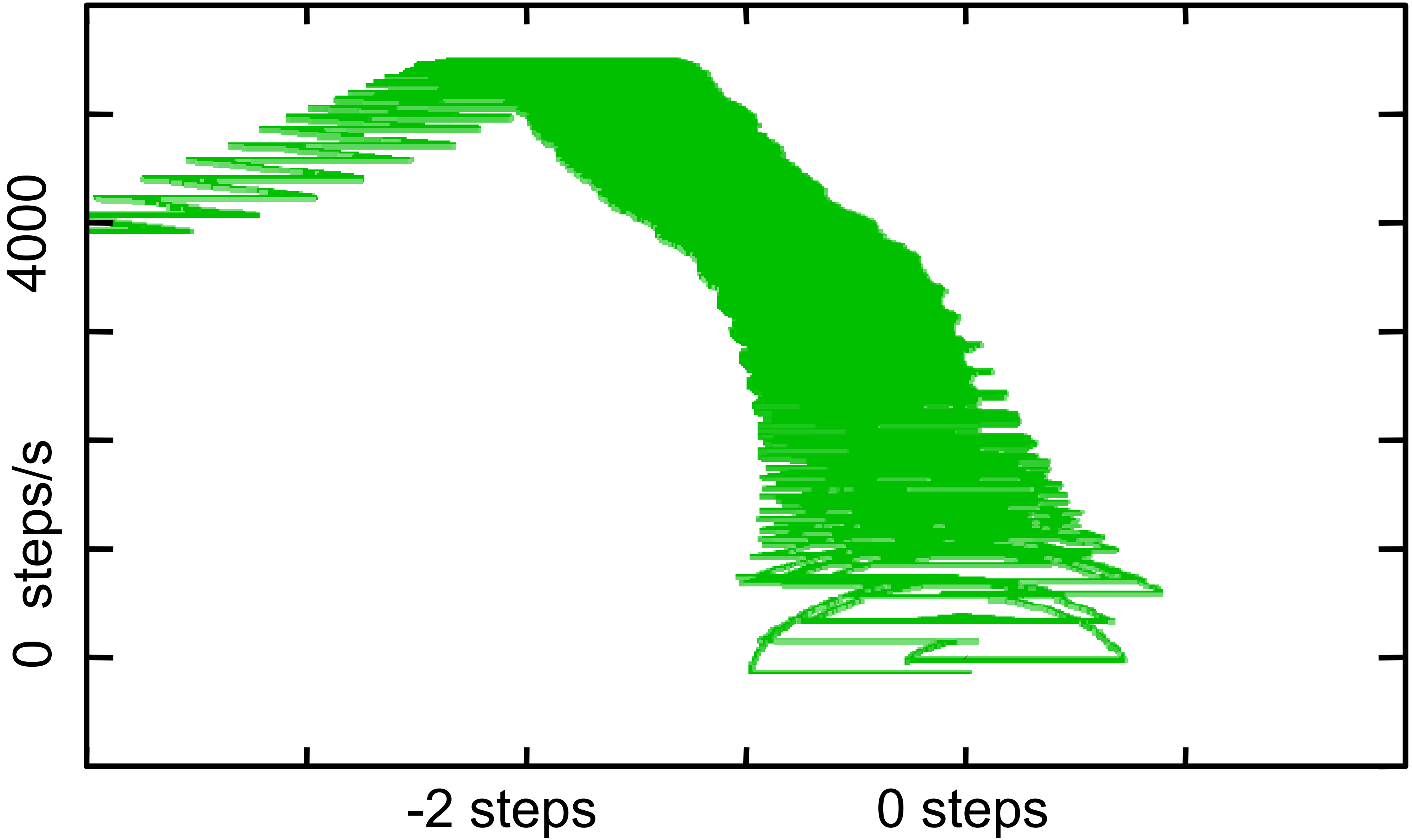


Velocity-Error Plane Trajectories



Base=400, slew=4000, accel=32000, decel=48000

Velocity-Error Plane Trajectories



Base=400, slew=6000, accel=32000, decel=48000

Closed-Loop Control

Stepping motor systems are simple, robust and very reliable

Open-loop stepping motors lose synchronisation as a result of:

excessive load torque
operation at the resonant step rate
too rapid acceleration or deceleration

In open-loop systems loss of synchronisation can be neither detected nor corrected

This problem can be overcome by the use of positional feedback

Closed-Loop Control

In closed-loop stepping motor systems an incremental shaft encoder is used to sense rotor position

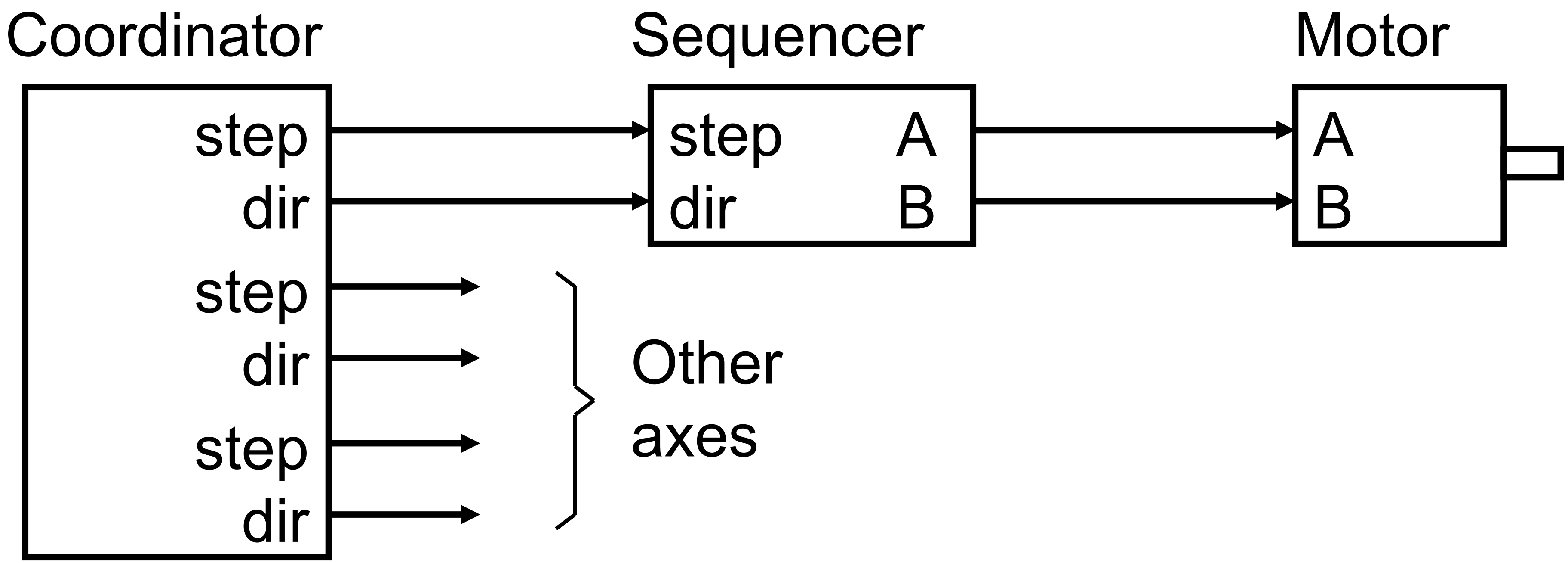
Alternatively the current waveforms in the stator windings can be used to sense position

The information gained can be used to detect and correct loss of synchronisation

The excess torque capability that must be built into open-loop systems is no longer required

Closed-loop stepping motor systems share the reliability and robustness of open-loop systems

Open-Loop Control



The coordinator generates *step* and *dir* signals for several axes

The sequencer generates the drive sequences for the stator windings

Closed-Loop Control

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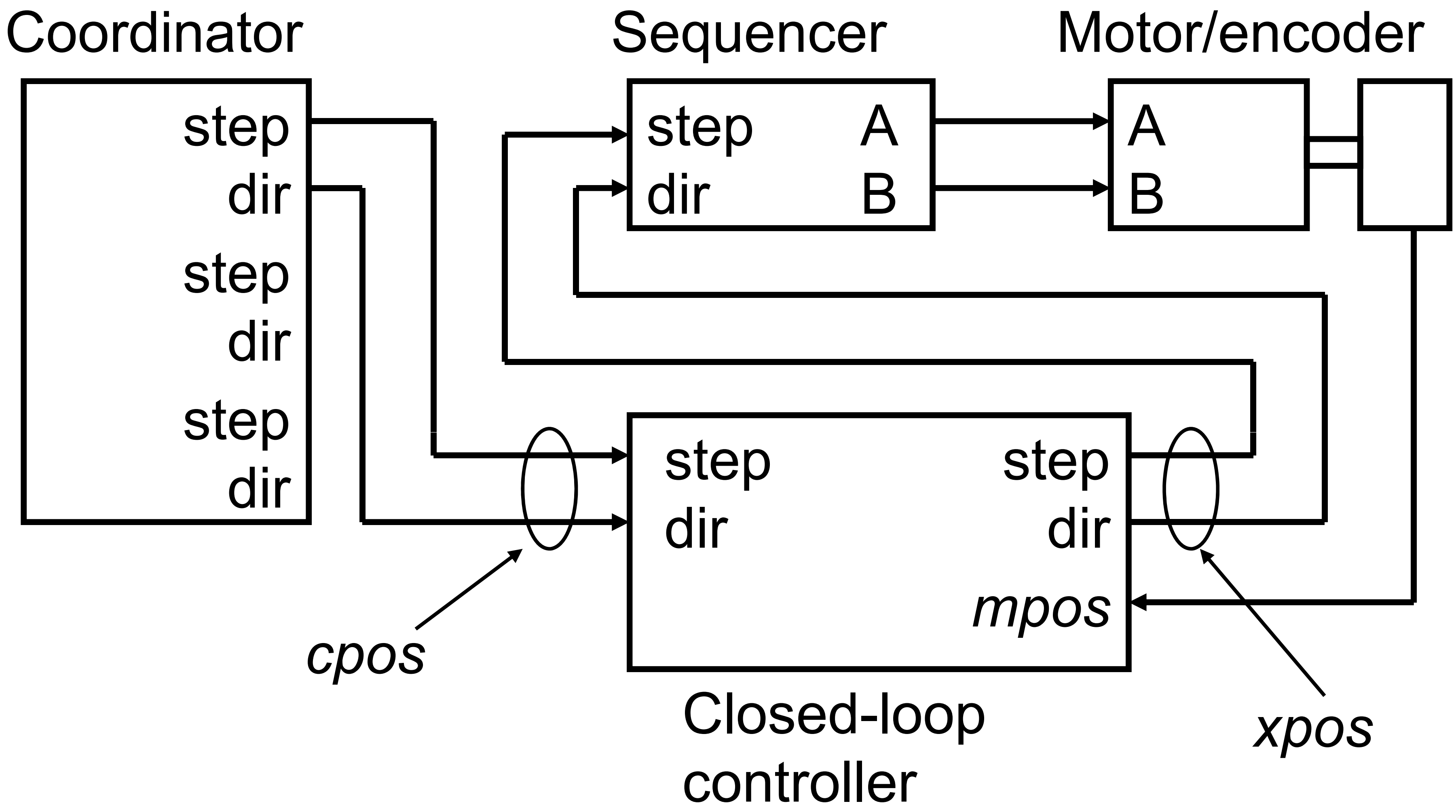
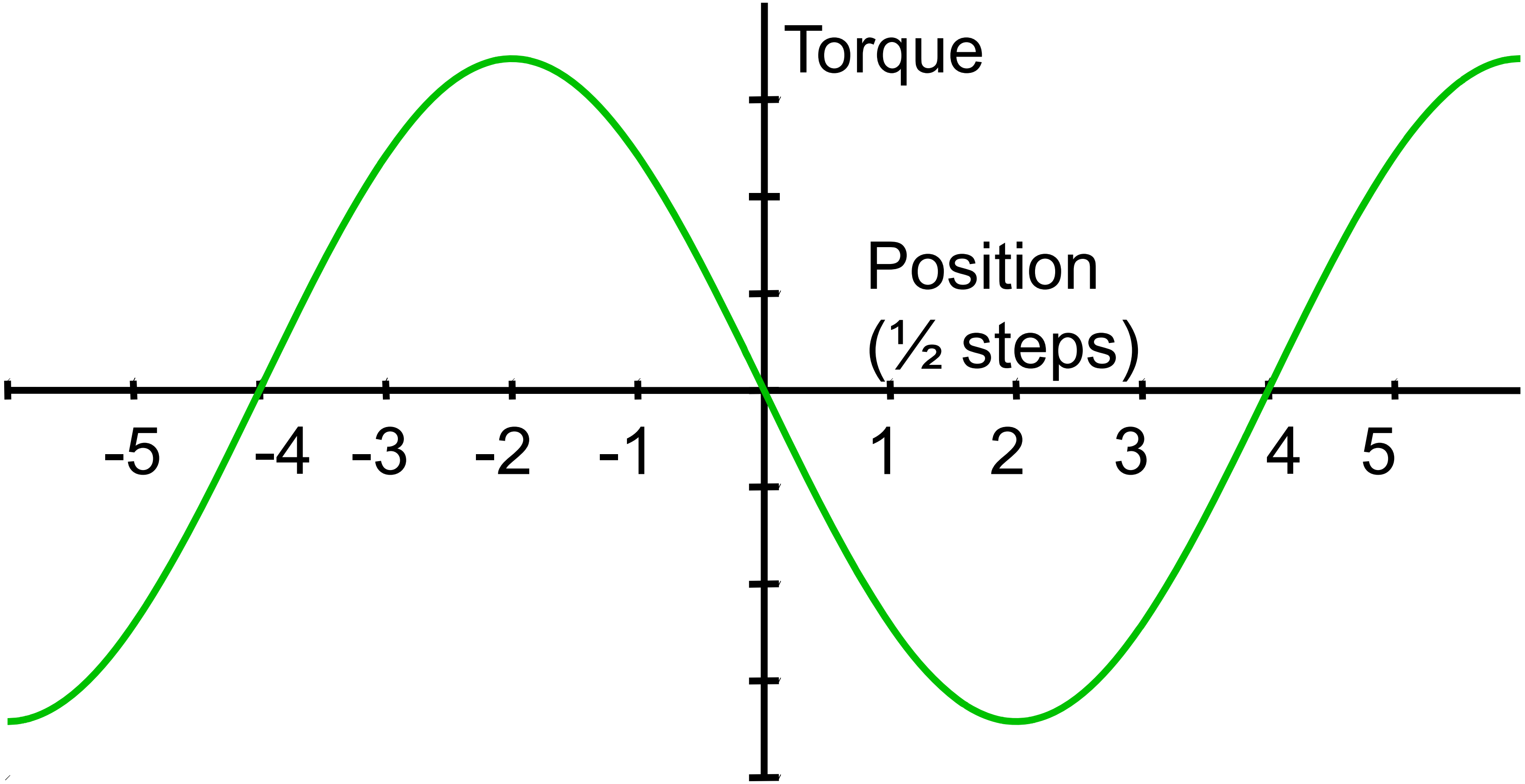
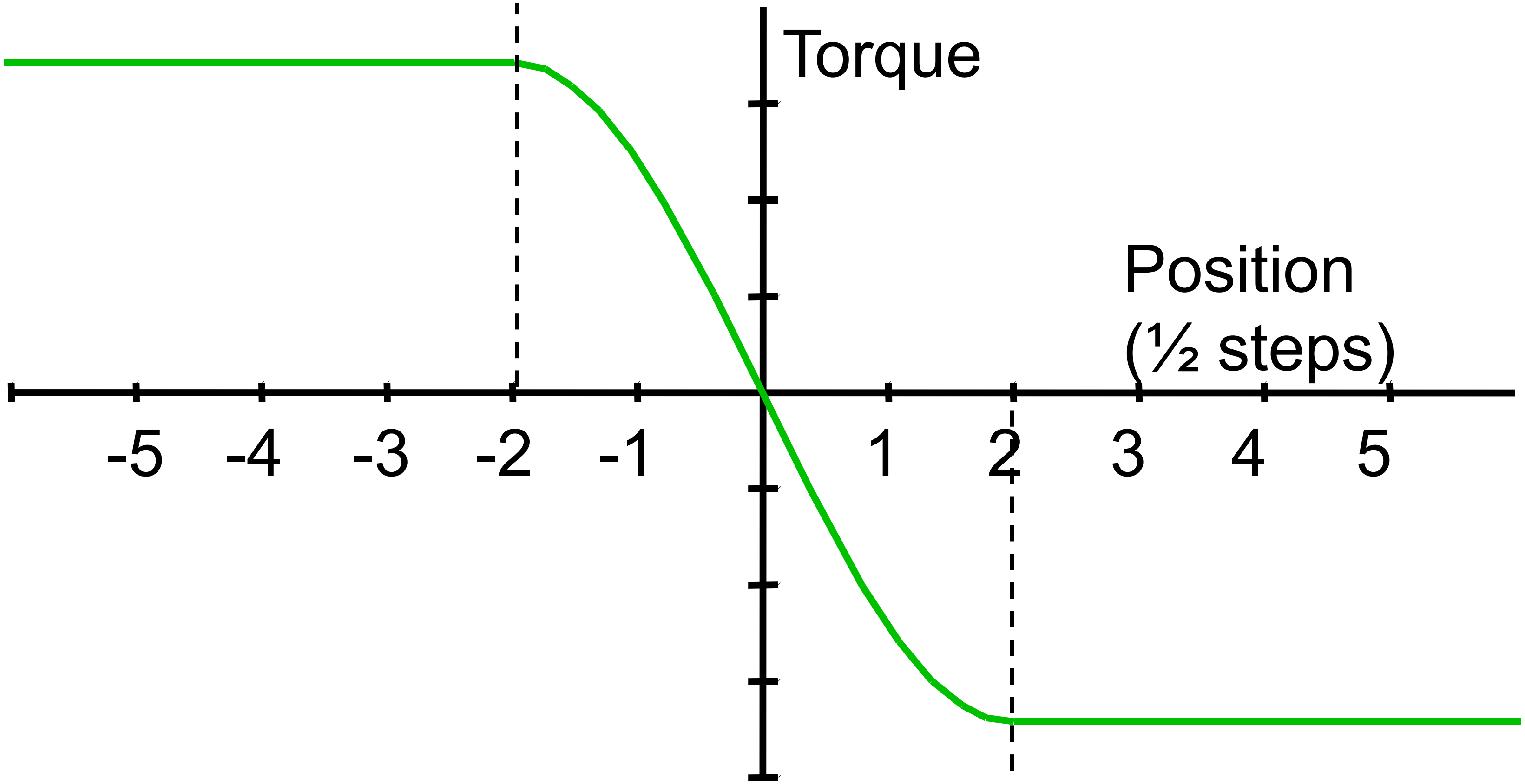


Open-Loop Control



Static torque characteristic: 2 windings on

Closed-Loop Control



Ideal closed-loop static torque characteristic to maintain a position of 0

Closed-Loop Control Algorithm

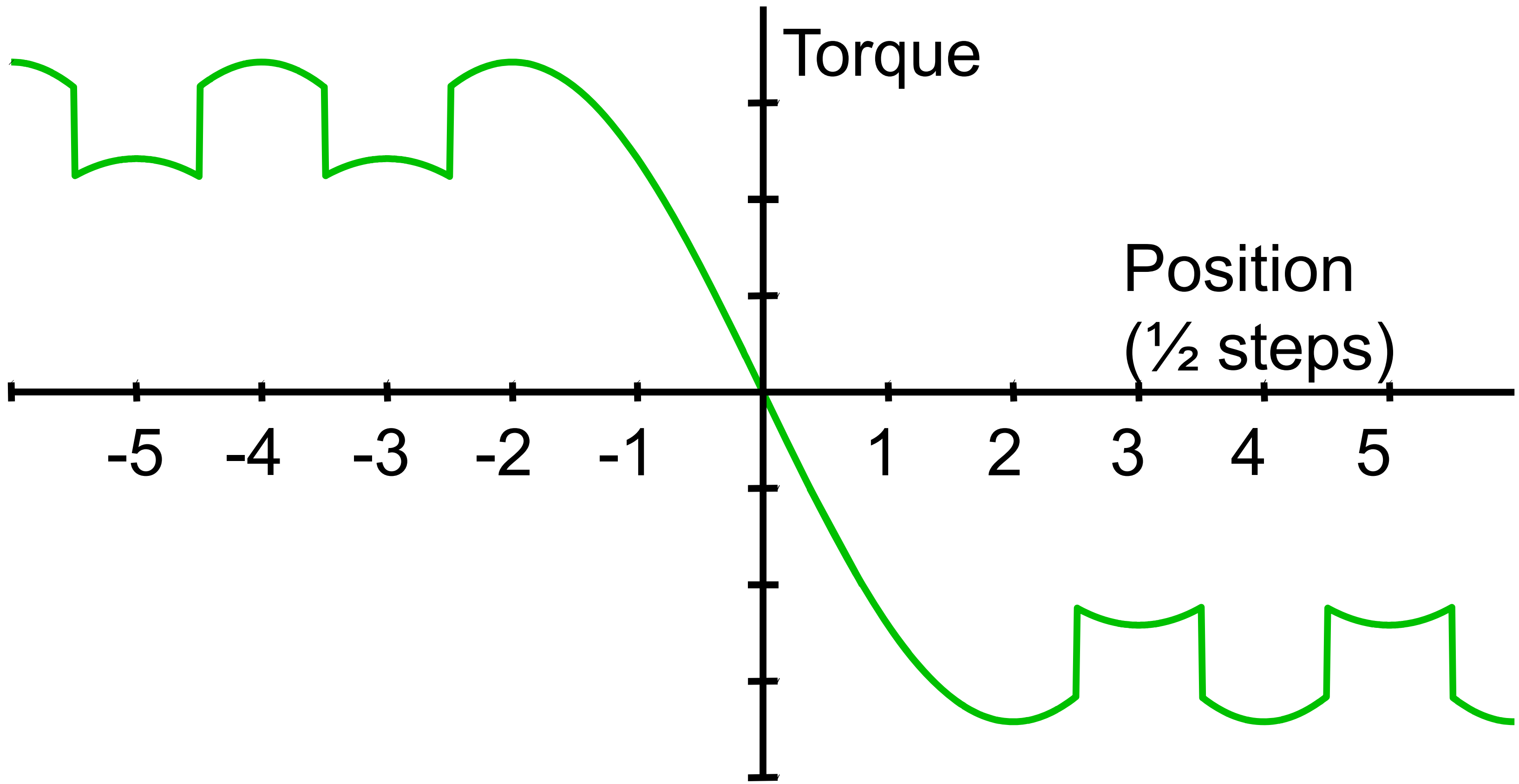
Closed-loop control algorithm to maintain a position of 0:

```
if (mpos < -2)
        xpos = mpos + 2;
else if (mpos > 2)
        xpos = mpos - 2;
else
        xpos = 0;
```

Provided that the motor position remains within 2 steps the excitation remains constant

If motor position exceeds 2 steps the excitation changes to provide maximum restoring torque

Closed-Loop Control



Closed-loop static torque characteristic - phase excitations change to maintain torque

Closed-Loop Control Algorithm

Closed-loop control algorithm to maintain a position of *cpos*:

```
if (mpos - cpos < -2)
        xpos = mpos + 2;
else if (mpos - cpos > 2)
        xpos = mpos - 2;
else
        xpos = cpos;
```

Provided that the error does not exceed 2 steps the behaviour is identical to open-loop

If error exceeds 2 steps the excitation changes to provide maximum correcting torque

Closed-Loop Control Algorithm

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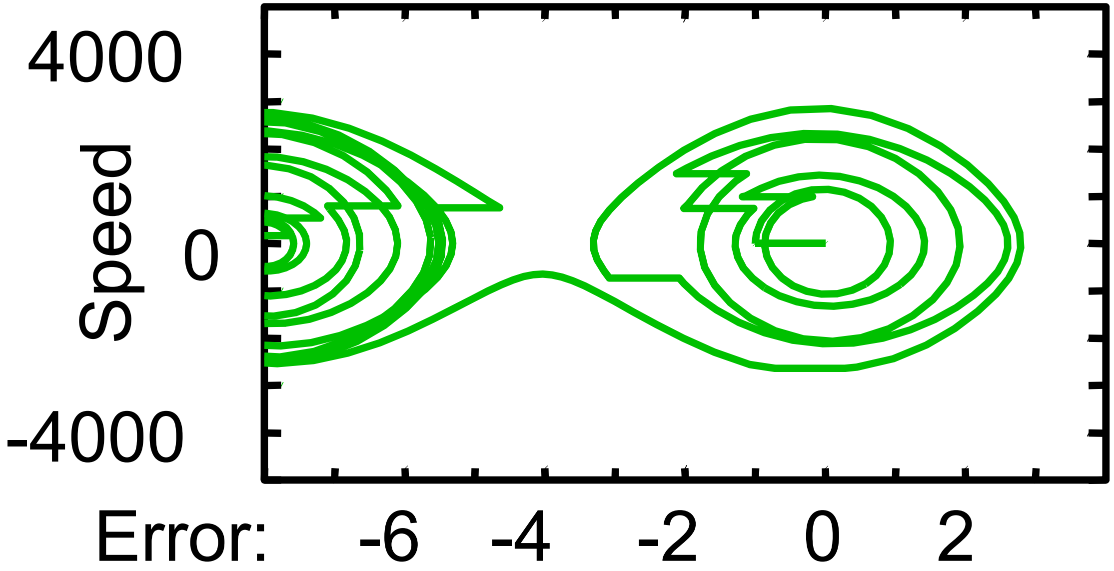
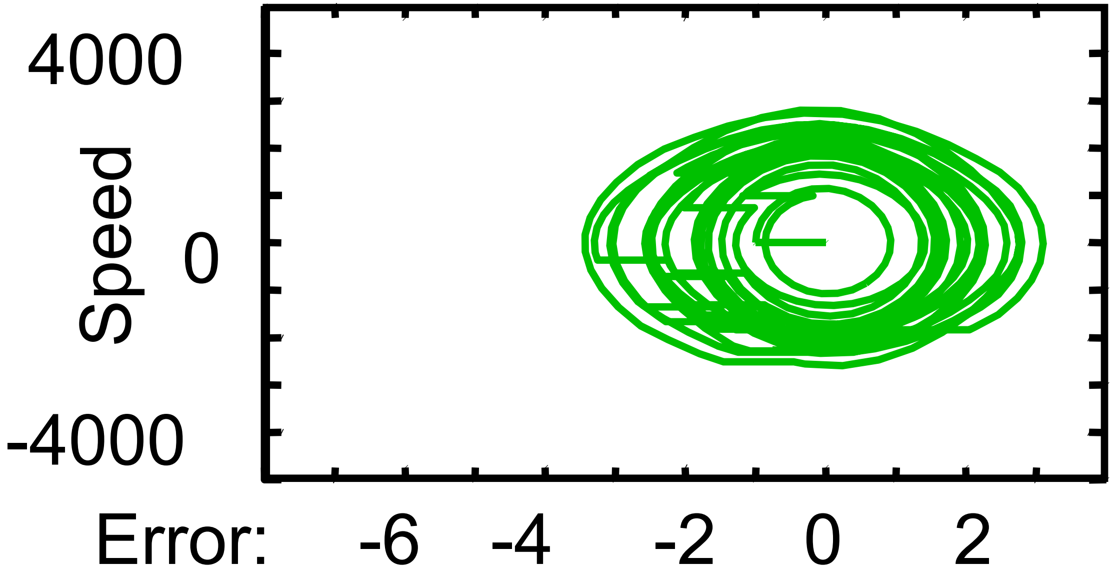
Open loop:

Operation at the resonant step rate (20steps at 153steps/s)



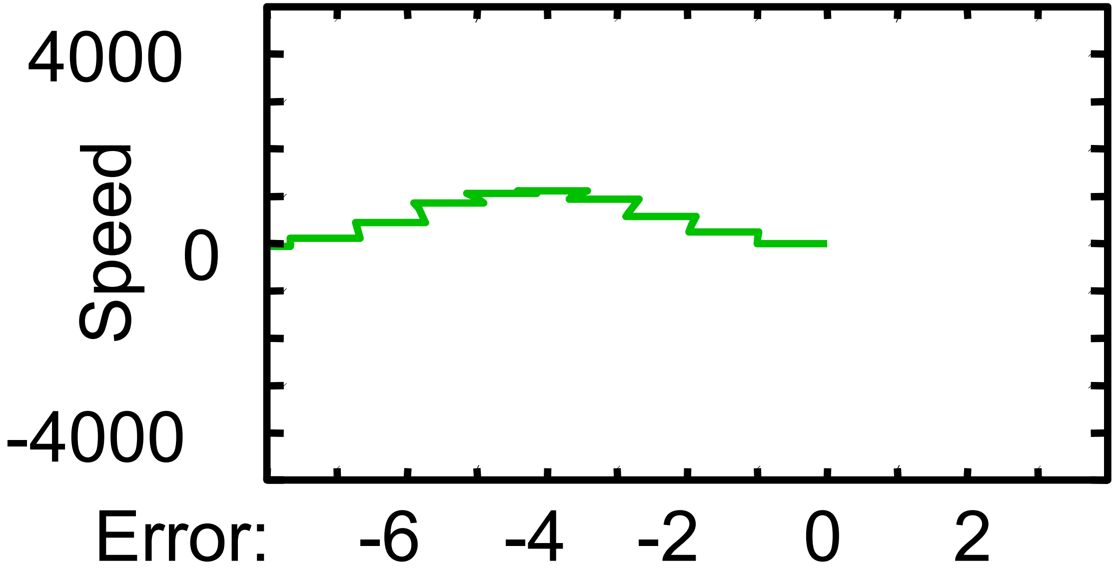
Closed loop:



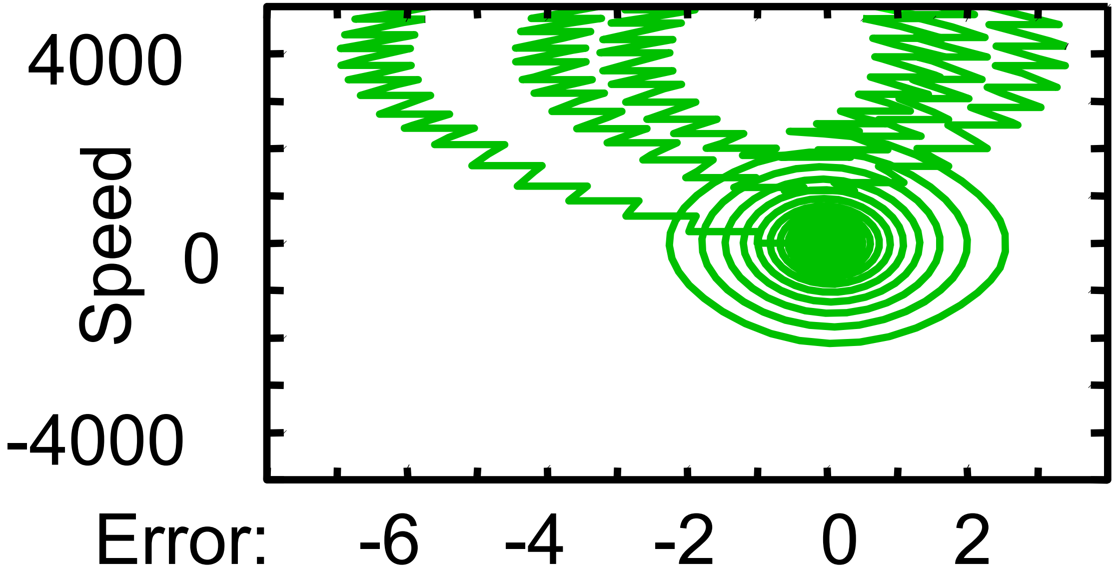
Closed-Loop Control Algorithm

Open loop:

Start-stop operation (100steps at 4000steps/s)



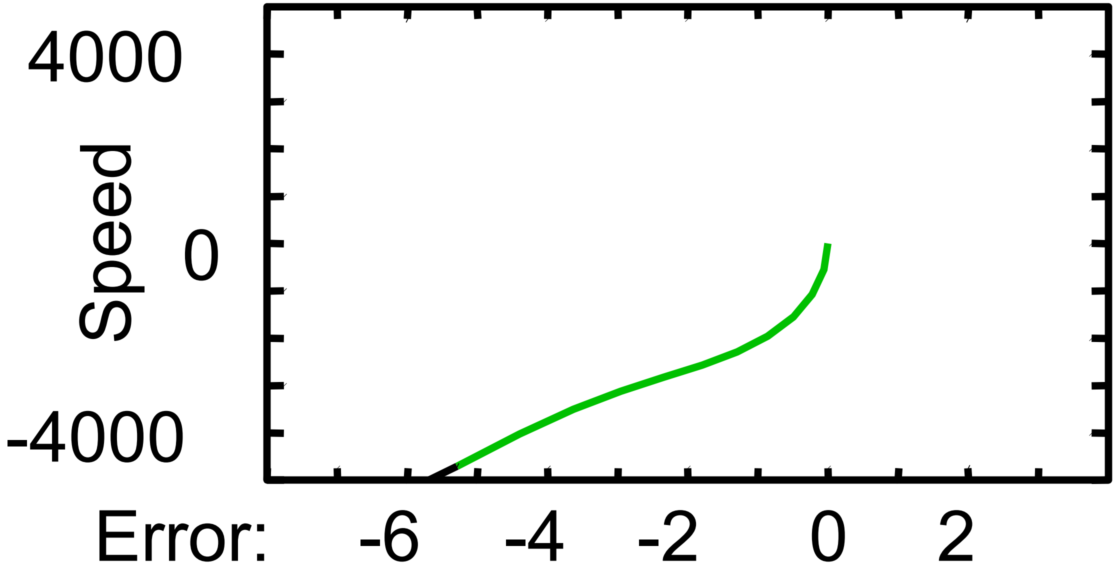
Closed loop:



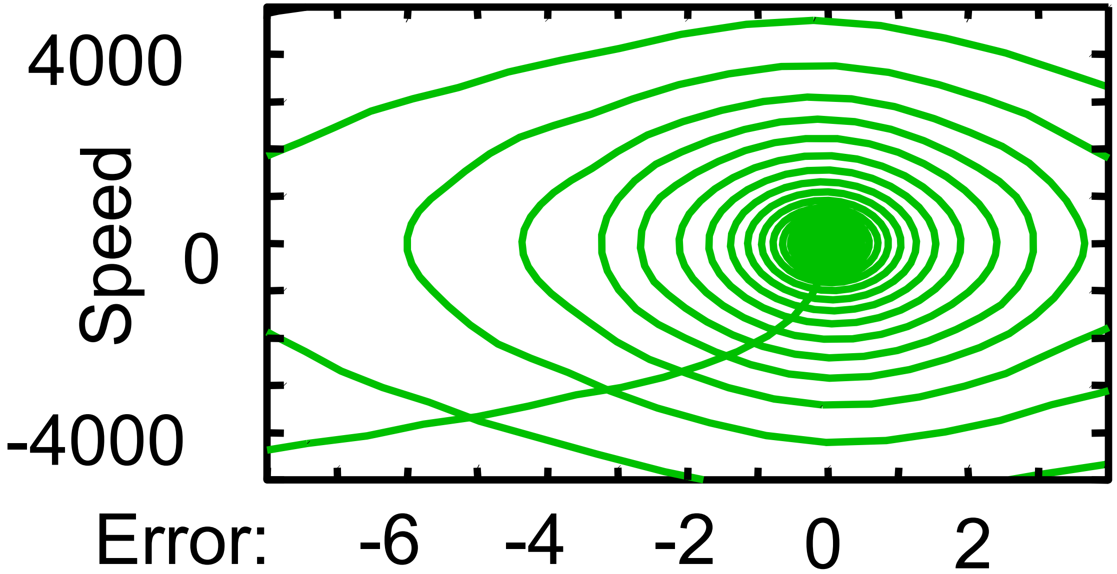
Closed-Loop Control Algorithm

Open loop:

Disturbance response (0.5Nm for 10ms)



Closed loop:



Stepping Motors