Chapter 7 Cox-Snell Residuals

The ***Cox-Snell*** residual method can be applied to any parametric model. The ***Cox-Snell*** residual, r_i, for the i^{th} subject with survival time t_i, is

$$r_i = -\ln[\hat{S}(t_i)]$$

Where $\hat{S}(t_i)$ is the estimated survival based on maximum likelihood estimation. Recall that the cumulative hazard function $H(t_i) = -\ln[\hat{S}(t_i)]$. The important aspect of this result is that if the selected family of distributions is indeed the correct one, then the $r_i, i = 1, \cdots, n$ should follow an exponential distribution. Recall earlier that we said, the CDF evaluated at a random observation follows a uniform distribution and the CDF evaluated at the order statistics should appear as the order statistics from a uniform distribution. If one computes the distribution of $-\ln(u_i) \sim \exp(1)$.

Collect the ***Cox-Snell*** residuals from the uncensored observations, where the residuals were obtained based on the MLE's. This approach streamlines the process by using PROC LIFEREG in SAS on the CRES option which directly outputs the Cox-Snell residuals. Rerun the PROC LIFEREG in SAS using CRES instead of the actual times.

```
data lymphoma;
PROBLEM 1: /* The following data are the survival times in years of 21 6MP
patients whom we studied earlier in the semester. The data are given in LW
and in the lecture notes on page 112. Their survival times are given in
years. Obtain the MLE of the parameters and median survival times assuming
a) one parameter exponential and
b) a Weibull distribution and
c) compare the two fit plots. */
input time @@;
if time lt 0.0 then censor=1; else censor=0;
time=abs(time);
cards;
 0.150  0.150  0.150  0.175  0.250  0.325  0.400  0.550  0.575
-0.150 -0.225 -0.250 -0.275 -0.425 -0.475 -0.500 -0.625 -0.800
-0.800 -0.850 -0.875
;
proc lifereg data=lymphoma covout outest=estim plots=probplot;
model time*censor(1) = / covb d=weibull;
output out=outcsr CRES=CoxSnell;
run;
proc print data=estim;
run;
proc lifereg data=outcsr outest=estimb plots=probplot;
  model CoxSnell*censor(1) = / covb distribution=weibull;
  output out=exp cdf=f;
run;
```

```
data tumorfree;
PROBLEM 2: /* Consider the tumor free days in ten animals. Assume the
data follow a log-logistic distribution. Their times are given in days.
Obtain the MLE of the parameters and median survival times assuming
a) a Weibull distribution and
b) a Log-logistic distribution and
c) compare the two fit plots. */
input time @@;
time=abs(time);
cards;
 2.0 3.5 5.0 7.0 9.0 10.0 15.0 20.0 30.0 40.0
;
proc print;
proc lifereg data=tumorfree covout outest=estim plots=probplot;
model time = / covb d=llogistic;
output out=outcsr CRES=CoxSnell;
run;

proc lifereg covout outest=estimb plots=probplot;
model CoxSnell = / covb d=weibull;
output out=outexp cdf=f;
run;
```