Week1 答疑

Wednesday, June 17, 2020 7:47 PM

①

$$\vec{w} = \text{Proj}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\vec{v} = \frac{1\times1+3\times1+(-2)\times(-1)}{1+1+1}\langle 1, 1, -1\rangle$$

$$= \frac{1+3+2}{3}\langle 1, 1, -1\rangle$$

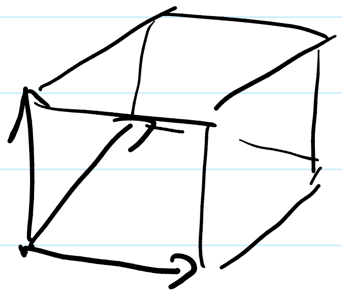
$$= \langle 2, 2, -2\rangle$$

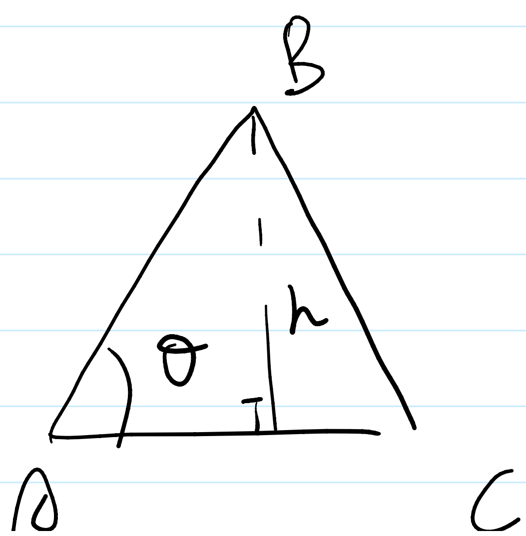
$$\vec{u} - \vec{w} = \langle 1-2,\ 3-2,\ -2-(-2)\rangle$$

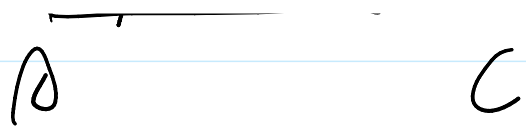
$$= \langle -1,\ 1,\ 0\rangle$$

$$\therefore (\vec{u}-\vec{w})\cdot\vec{v} = -1\times1 + 1\times1 = 0.$$

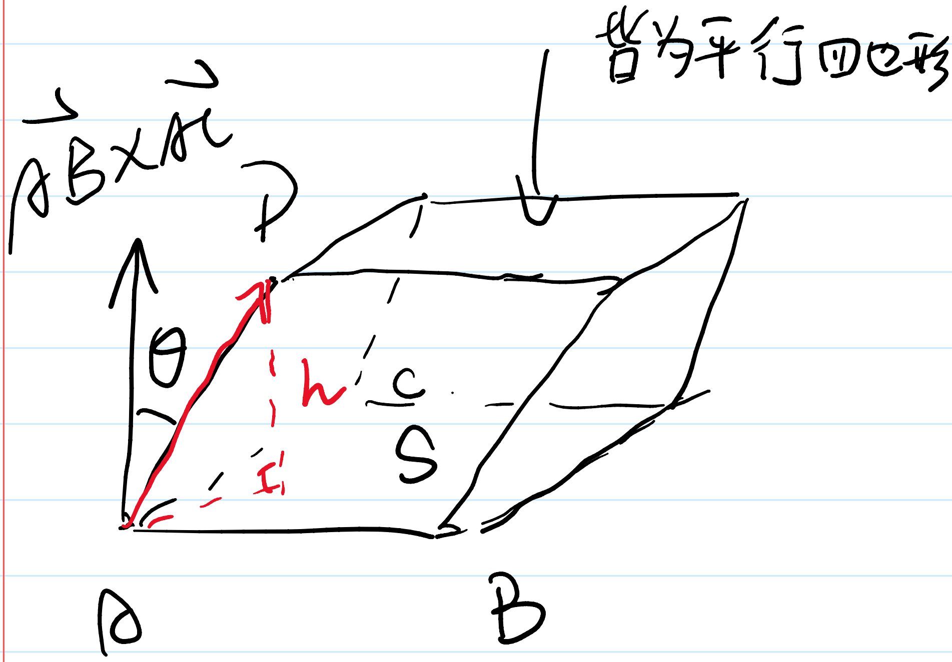
②







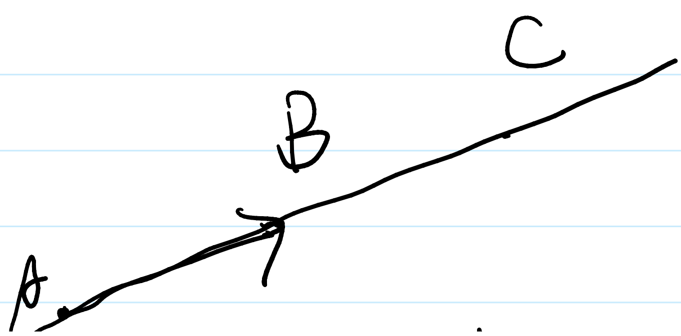
$$S = \frac{1}{2}\underbrace{AC}_{底}\cdot\underbrace{AB\sin\theta}_{高}$$

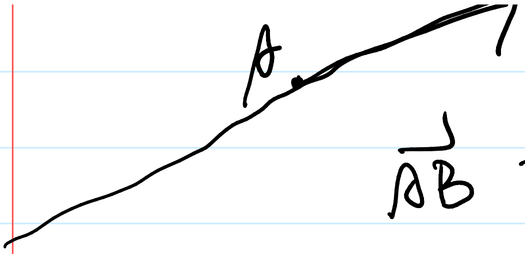


$$\left|(\vec{AB}\times\vec{AC})\cdot\vec{AD}\right| = \left|S_{底}\underbrace{|\vec{AD}|\cos\theta}_{h}\right|$$
$$= S_{底}h = V$$

③ Vector eqn:

(1, 2, -1) (3, 2, -1)





$$\vec{AB} = k\vec{AC}$$

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$\vec{r}(t) = \langle 2-3t, 3+t, -1 \rangle$

direction v of $\vec{r}(t) = \langle -3, 1, 0 \rangle$

$\therefore -3x + 1y + 0z = d$

contains $(1, 2, -3)$

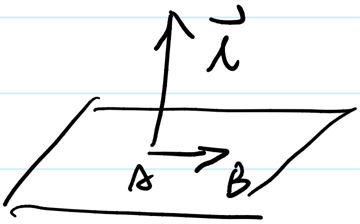
$\therefore -3\cdot 1 + 1\cdot 2 + -3\cdot 0 = d$

$\therefore d = -3+2 = -1$

$\therefore -3x + y = -1$

$3x - y = 1$

plane $ax+by+cz=d$.



suppose

$\vec{n} = \langle a, b, c \rangle$

A (x_1, y_1, z_1)

B (x_2, y_2, z_2)

$\therefore$ For $\forall\ x, y, z$

$\vec{n} \cdot \vec{AB} = \langle a, b, c \rangle \cdot \langle (x_1 - x_2), (y_1 - y_2), z_1 - z_2 \rangle$

$= a(x_1 - x_2) + b(y_1 - y_2) + c(z_1 - z_2)$

$= ax_1 + by_1 + cz_1 - (ax_2 + by_2 + cz_2)$

$= a(x_1 - x_2) + b(y_1 - y_2) + c(z_1 - z_2)$

$= ax_1 + by_1 + cz_1 - (ax_2 + by_2 + cz_2)$

$\because$ Both A, B in plane $ax + by + cz = d$

$\therefore \vec{l} \cdot \vec{AB} = 0$ for any $\vec{AB}$ in plane

$\therefore l \perp$ plane.

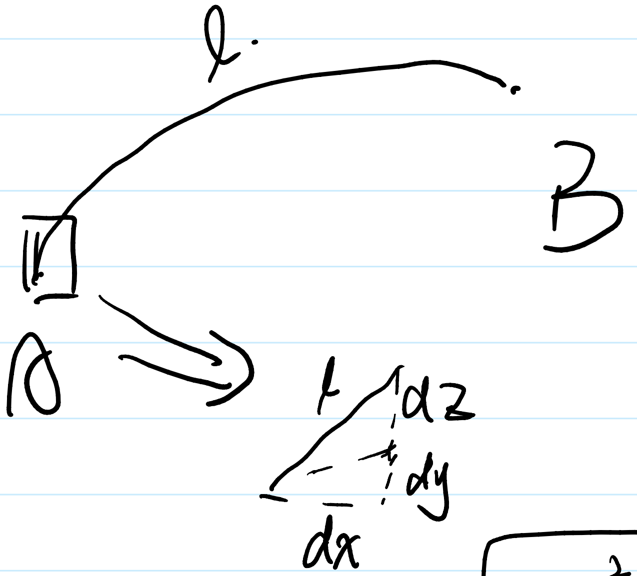
⑤

$$\int -2\cos t\,dt = -2\int \cos t\,dt = -2\sin t + C_1$$

$$\int 2\sin t\,dt = 2\int \sin t\,dt = -2\cos t + C_2$$

$$\int 0\,dt = 0 + C_3$$

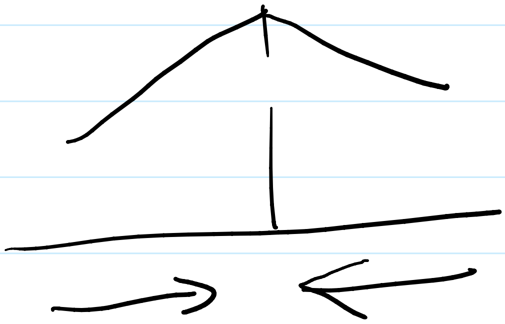
⑥

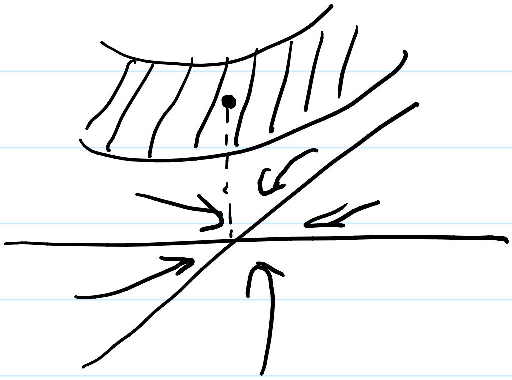


$$dl = \sqrt{(dx^2 + dy^2 + dz^2)}$$

$$L = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,dt$$

⑦





along $x=y$

$$\lim_{y\to 0} f(x,y) = \frac{y^3}{y^2+y^4} = \frac{1}{y+\frac{1}{y}} = 0$$

along $x=2y$

$$\lim_{x,y\to 0} f(x,y) = \frac{2y^3}{4y^2+y^4} = \frac{2}{y+\frac{4}{y}} = 0$$

不行

along $x=y^2$

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{y\to 0} \frac{y^2\cdot y^2}{(y^2)^2+y^4} = \lim_{y\to 0} \frac{y^4}{2y^4}$$

$$= \lim_{y\to 0} \frac{1}{2} = \frac{1}{2}$$

along $x=2y^2$

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{y\to 0} \frac{2y^2\cdot y^2}{(2y^2)^2+y^4} = \lim_{y\to 0} \frac{2y^4}{5y^4}$$

$$= \lim_{y\to 0} \frac{2}{5} = \frac{2}{5} \neq \frac{1}{2}$$

⑨ $\vec{r}'(t) = \langle -10\sin t, 5\cos t \rangle$

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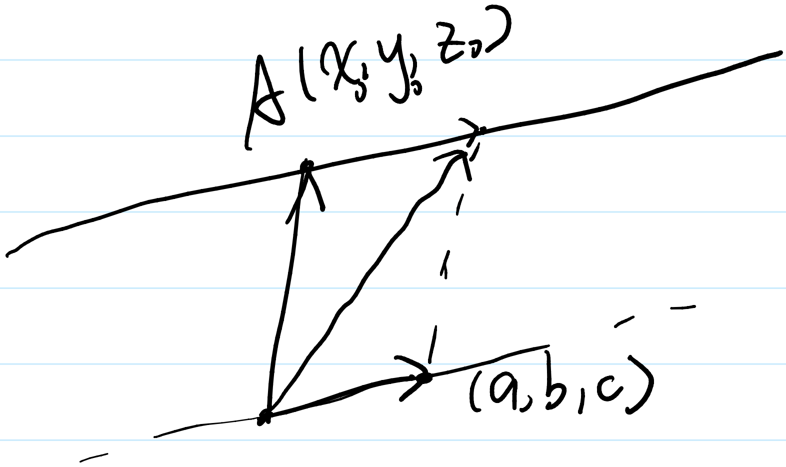
$$\vec{r}'(t) = \langle -10\sin t, 5\cos t \rangle$$

$$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \quad \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$t = \frac{\pi}{4} \quad \vec{r}(t) = \langle 5\sqrt{2}, \frac{5\sqrt{2}}{2} \rangle$$

$$\vec{r}'(t) = \langle -5\sqrt{2}, \frac{5\sqrt{2}}{2} \rangle$$



$$\vec{a} \times \vec{a} = \vec{0}$$

$$\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$$

自己尝试证明

$$\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$$