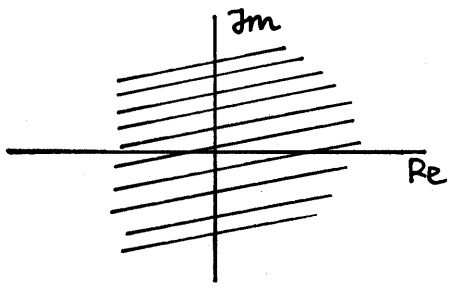
CHAPTER 7

$\boxed{7.1}$

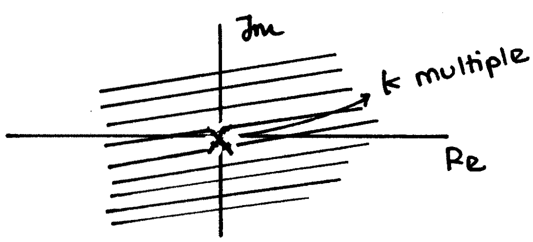
(a) $x[n] = \delta[n]$

$$X(z) = \sum_{n=-\infty}^{\infty} \delta[n] z^{-n} = 1 \quad , \text{ all } z$$



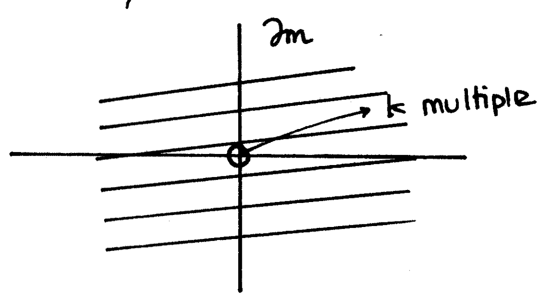
(b) $x[n] = \delta[n-k] \quad , k > 0$

$X(z) = z^{-k} \quad , z \neq 0$



(c) $x[n] = \delta[n-k] \quad , k > 0$

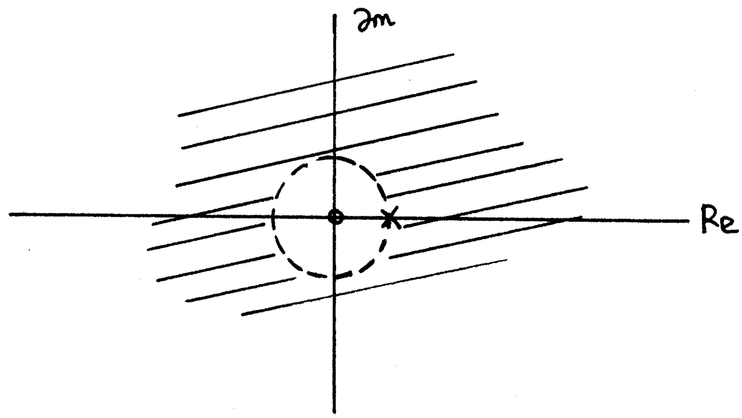
$X(z) = z^{k} \quad , \text{ all } z$



(d) $x[n] = u[n]$

$$X(z) = \sum_{n=0}^{\infty} z^{-n}$$

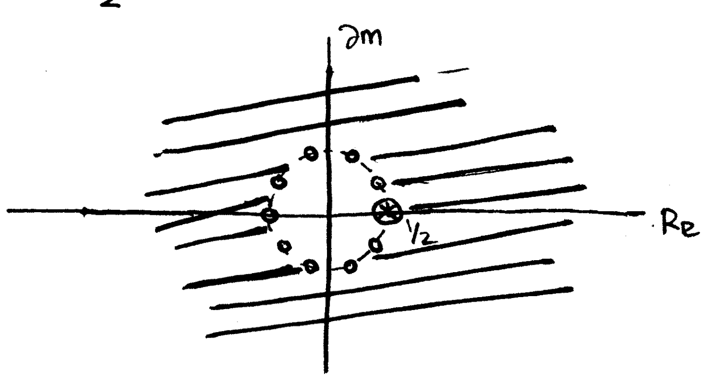
$$= \frac{1}{1 - z^{-1}} \quad , |z| > 1$$



(e) $x[n] = \left(\frac{1}{2}\right)^n \left(u[n] - u[n-10]\right)$

$$X(z) = \sum_{n=0}^{9} \left(\frac{1}{2} z^{-1}\right)^n$$

$$= \frac{1 - \left(\frac{1}{2} z^{-1}\right)^{10}}{1 - \frac{1}{2} z^{-1}} \quad , \text{ all } z$$

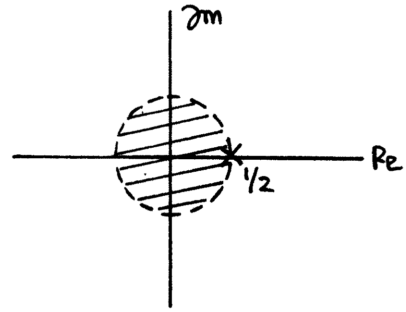


(f) $x[n] = \left(\frac{1}{2}\right)^n u[-n]$

$$X(z) = \sum_{n=-\infty}^{0} \left(\tfrac{1}{2} z^{-1}\right)^n$$

$$= \sum_{n=0}^{\infty} (2z)^n$$

$$= \frac{1}{1-2z} \quad , |z| < \frac{1}{2}$$



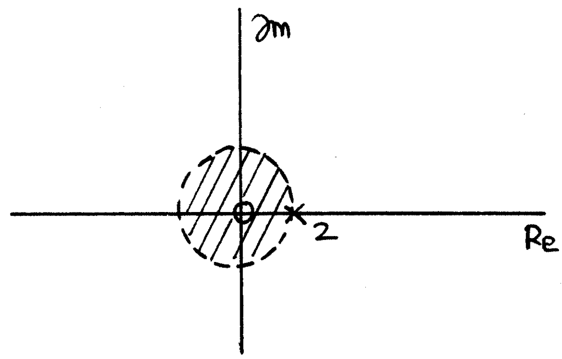
(g) $x[n] = 2^n u[n-1]$

$$X(z) = \sum_{n=-\infty}^{-1} (2z^{-1})^n$$

$$= \sum_{n=1}^{\infty} \left(\tfrac{1}{2} z\right)^n$$

$$= \frac{\tfrac{1}{2} z}{1 - \tfrac{1}{2} z}$$

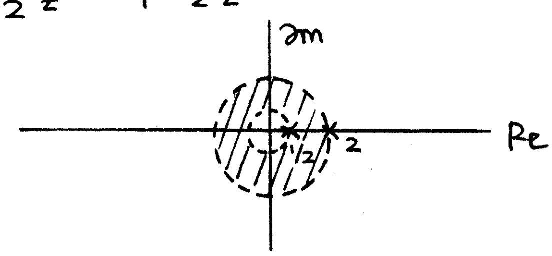
$$= \frac{-1}{1-2z^{-1}} \quad , |z| < 2$$



(h) $x[n] = \left(\tfrac{1}{2}\right)^{|n|}$

$$X(z) = \sum_{n=-\infty}^{-1} (2z^{-1})^n + \sum_{n=0}^{\infty} \left(\tfrac{1}{2} z^{-1}\right)^n$$

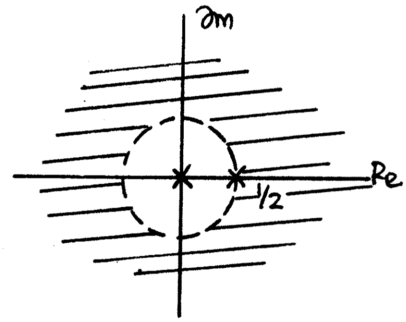
$$= \frac{1}{1-\tfrac{1}{2}z^{-1}} - \frac{1}{1-2z^{-1}} \quad , \qquad \frac{1}{2} < |z| < 2$$



(i) $x[n] = \left(\frac{1}{2}\right)^n u[n-2]$

$$X(z) = \sum_{n=2}^{\infty} \left(\frac{1}{2} z^{-1}\right)^n$$

$$= \frac{\frac{1}{4} z^{-2}}{1 - \frac{1}{2} z^{-1}} \quad , |z| > \frac{1}{2}$$

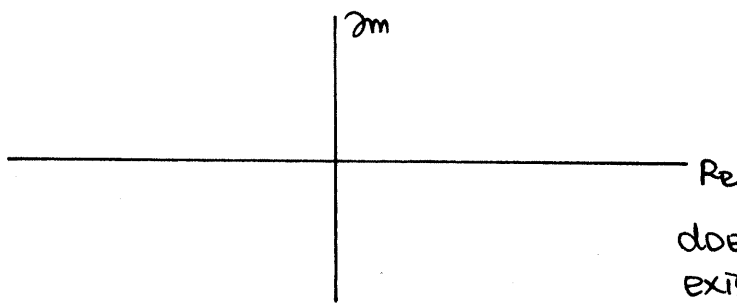


(j) $x[n] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{3}\right)^n u[n-1]$

$$X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{2} z^{-1}\right)^n + \sum_{n=-\infty}^{-1} \left(\frac{1}{3} z^{-1}\right)^n$$

$$= \frac{1}{1 - \frac{1}{2} z^{-1}} - \frac{1}{1 - \frac{1}{3} z^{-1}} \quad , |z| > \frac{1}{2} \cap |z| < \frac{1}{3}$$

doesn't exist



doesn't exist

$\boxed{7.2}$

(a) $$X(z) = \frac{10}{1 + \frac{1}{2} z^{-1}} \quad , |z| > \frac{1}{2}$$

ROC includes $|z| = 1$, DTFT exists

$$X(e^{j\Omega}) = \frac{10}{1 + \frac{1}{2} e^{-j\Omega}}$$

(b) $X(z) = \dfrac{10}{1+\frac{1}{2}z^{-1}}, \quad |z| < \frac{1}{2}$

ROC doesn't include $|z|=1$, DTFT doesn't exist

(c) $X(z) = z^{-5}, \quad |z| > 0$

ROC includes $|z|=1$, DTFT exists

$X(e^{j\Omega}) = e^{-j5\Omega}$

(d) $X(z) = z^{5}, \quad |z| < \infty$

ROC includes $|z|=1$, DTFT exists

$X(e^{j\Omega}) = e^{j5\Omega}$

(e) $X(z) = \dfrac{z^{-1}}{(1-\frac{1}{3}z^{-1})(1+3z^{-1})}, \quad |z| < \frac{1}{3}$

ROC doesn't include $|z|=1$, DTFT doesn't exist

(f) $X(z) = \dfrac{z^{-1}}{(1-\frac{1}{3}z^{-1})(1+3z^{-1})}, \quad \frac{1}{3} < |z| < 3$

ROC includes $|z|=1$, DTFT exists

$$X(e^{j\Omega}) = \frac{e^{-j\Omega}}{(1-\frac{1}{3}e^{-j\Omega})(1+3e^{-j\Omega})}$$

$\boxed{7.3}$

(a) $X(z) = \dfrac{c\,z}{(z+\frac{3}{4})(z-\frac{1}{2})(z-\frac{3}{2})}$

There are 4 possible ROCS :

1) $|z| > \frac{3}{2}$
$x[n]$ is causal and not stable

2) $\frac{3}{4} < |z| < \frac{3}{2}$

$x[n]$ is two sided (not causal) .
The anticausal term is $(-\frac{3}{2})^n$
$x[n]$ is stable (includes $|z| = 1$ circle)

3) $\frac{1}{2} < |z| < \frac{3}{4}$

$x[n]$ is two sided (not causal)
The anticausal term is $(-\frac{3}{2})^n$ and $(\frac{3}{4})^n$
$x[n]$ is unstable

4) $|z| < \frac{1}{2}$
$x[n]$ is anticausal and not stable

(b) $X(z) = \dfrac{c(z^4-1)}{z} = c\,z^3 - c\,z^{-1}$

$|z| > 0$
$x[n]$ is not causal but stable

(c) $X(z) = (z-\frac{1}{2})(z+1)(z^2+\frac{9}{16})\,c$
$|z| < \infty$
$x[n]$ is not causal but stable

$\boxed{7.4}$

(a) Timeshift : $x[n-n_0] \xleftrightarrow{z} z^{-n_0} X(z)$

$$y[n] = x[n-n_0]$$

$$Y(z) = \sum_{n=-\infty}^{\infty} x[n-n_0]\, z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x[n]\, z^{-(n+n_0)}$$

$$= \left(\sum_{n=-\infty}^{\infty} x[n]\, z^{-n} \right) z^{-n_0}$$

$$= z^{-n_0} X(z)$$

(b) Time reversal : $x[n] \xleftrightarrow{z} X\left(\frac{1}{z}\right)$

$$y[n] = x[-n]$$

$$Y(z) = \sum_{n=-\infty}^{\infty} x[-n]\, z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x[n] \left(\frac{1}{z}\right)^{-n}$$

$$= X\left(\frac{1}{z}\right)$$

(c) Multiplication with exponential sequence :

$$\alpha^n x[n] \xleftrightarrow{z} X\left(\frac{z}{\alpha}\right)$$

$$y[n] = \alpha^n x[n]$$

$$Y(z) = \sum_{n=-\infty}^{\infty} \alpha^n x[n]\, z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x[n] \left(\frac{z}{\alpha}\right)^{-n}$$

$$= X\left(\frac{z}{\alpha}\right)$$

(d) Differential in z domain : $n x[n] \xleftrightarrow{z} -z \frac{d}{dz} X(z)$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

differentiate both sides with respect to z

$$\frac{d}{dz} X(z) = \sum_{n=-\infty}^{\infty} -n \, x[n] z^{-n-1}$$

multiple both sides by $-z$

$$-z \frac{d}{dz} X(z) = \sum_{n=-\infty}^{\infty} n \, . x[n] z^{-n}$$

$$\therefore \; n \, x[n] \xleftrightarrow{z} -z \frac{d}{dz} X(z)$$

$\boxed{7.5}$

(a) $x[n] = \left(\frac{1}{2}\right)^n u[n] * \left(\frac{1}{3}\right)^n u[n]$

ROC : $|z| > \frac{1}{2} \cap |z| > \frac{1}{3} = |z| > \frac{1}{2}$ (causal)

$$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \cdot \frac{1}{1-\frac{1}{3}z^{-1}}$$

$$= \frac{3}{1-\frac{1}{2}z^{-1}} + \frac{-2}{1-\frac{1}{3}z^{-1}}$$

(b) $x[n] = n\left(\left(\frac{1}{2}\right)^n u[n] * \left(\frac{1}{2}\right)^n u[n]\right)$

$$X(z) = -z \cdot \frac{d}{dz}\left(\frac{1}{(1-\frac{1}{2}z^{-1})^2}\right)$$

$$= -z \; \frac{(-2)(\frac{1}{2}z^{-2})}{(1-\frac{1}{2}z^{-1})^3}$$

$$X(z) = \frac{z^{-1}}{(1-\frac{1}{2}z^{-1})^3} \qquad |z| > 1/2$$

(c) $x[n] = u[-n]$

$$X(z) = \frac{1}{1-(\frac{1}{z})^{-1}}$$

$$= \frac{1}{1-z} \qquad |z| < 1$$

(d) $x[n] = \sin\left(\frac{\pi}{8}n - \frac{\pi}{4}\right)u[n-2]$

$$= \sin\left(\frac{\pi}{8}(n-2)\right)u[n-2]$$

$$X(z) = \frac{z^{-1}\sin(\frac{\pi}{8})}{1-z^{-1}\,2\cos(\frac{\pi}{8})+z^{-2}}\,(z^{-2}), \quad |z| > 1$$

(e) $x[n] = n.\sin\left(\frac{\pi}{2}n\right)u[-n]$

$$= (-n)\sin\left(-\frac{\pi}{2}n\right)u[n]$$

$$X(z) = -z\frac{d}{dz}\left(\frac{z^{-1}}{1+z^{-2}}\right)\Bigg|_{z=\frac{1}{z}}$$

$$= -z\left[\frac{-z^{-2}}{1+z^{-2}} - \frac{z^{-1}(-2z^{-3})}{(1+z^{-2})^2}\right]\Bigg|_{z=\frac{1}{2}}$$

$$= \frac{z^{-1}}{1+z^{-2}} + \frac{2z^{-3}}{(1+z^{-2})^2} \Bigg|_{z=\frac{1}{2}}$$

$$X(z) = \frac{z}{1+z^2} + \frac{2z^3}{(1+z^2)^2}$$

$\boxed{7.6}$

(a) $y[n] = x[n-4]$

$$Y(z) = \frac{z}{z^2+4} \cdot z^{-4}$$

(b) $y[n] = 2^n x[n]$

$$Y(z) = X\left(\frac{z}{2}\right) = \frac{\frac{z}{2}}{\left(\frac{z}{2}\right)^2+4}$$

(c) $y[n] = x[-n]$

$$Y(z) = X\left(\frac{1}{z}\right) = \frac{\frac{1}{z}}{\frac{1}{z^2}+4} = \frac{z}{1+4z^2}$$

(d) $y[n] = n\,x[n]$

$$Y(z) = -z\frac{d}{dz}X(z)$$

$$= -z\left[\frac{1}{z^2+4} - \frac{z(2z)}{(z^2+4)^2}\right]$$

$$= \frac{2z^3}{(z^2+4)^2} - \frac{z}{(z^2+4)}$$

$$= \frac{z^3-4z}{(z^2+4)^2}$$

(e) $y[n] = x[n+1] + x[n-1]$

$$Y(z) = (z + z^{-1}) \frac{z}{z^2+4}$$

(f) $y[n] = \underbrace{x[n] * x[n] * \ldots\ldots * x[n]}_{m \text{ times}}$

$$Y(z) = \left[\frac{z}{z^2+4}\right]^m$$

(g) $y[n] = (n-3).x[n-2]$
$= (n-2).x[n-2] - x[n-2]$

$$Y(z) = z^{-2}\left(-z\frac{z^3-4z}{(z^2+4)^2} - \frac{z}{z^2+4}\right)$$

$$Y(z) = -z^{-1}\left(\frac{z^3+z^2-4z+4}{(z^2+4)^2}\right)$$

$\boxed{7.7}$

(a) $Y(z) = X(3z) \longleftrightarrow (\frac{1}{3})^n \; x[n]$
$\therefore y[n] = u[n]$

(b) $Y(z) = X(z^{-1}) \longleftrightarrow x[-n]$
$\therefore y[n] = (\frac{1}{3})^n \; u[-n]$

(c) $Y(z) = \frac{d}{dz} X(z) = -z^{-1}\left(-z\frac{d}{dz} X(z)\right)$

$\longleftrightarrow -(n-1)\, x(n-1)$

$\therefore y[n] = -(n-1)\, 3^{n-1} \; u[n-1]$

(d) $Y(z) = \frac{z + z^{-1}}{2} X(z) \longleftrightarrow \frac{1}{2}\left(x[n+1] + x[n-1]\right)$

$\therefore y[n] = \frac{1}{2}\left[3^{n+1} u[n+1] + 3^{n-1} u[n-1]\right]$

(e) $Y(z) = X(z) . X(9z) \longleftrightarrow x[n] * \left(\frac{1}{9}\right)^n x[n]$

$\therefore y[n] = 3^n u[n] * \left(\frac{1}{3}\right)^n u[n]$

$= u[n] \sum_{k=0}^{n} \left(\frac{1}{3}\right)^n 9^k$

$y[n] = \left(\frac{1}{3}\right)^n \frac{9^{n+1} - 1}{8} u[n]$

$\boxed{7.8}$

(a) $X(z) = \frac{\frac{1}{4} z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$, $|z| > \frac{1}{2} \rightarrow$ causal

$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{-1}{1-\frac{1}{4}z^{-1}}$

$x[n] = \left(\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right) u[n]$

(b) same as (a) , $|z| < \frac{1}{4} \rightarrow$ anticausal

$x[n] = \left(\left(\frac{1}{4}\right)^n - \left(\frac{1}{2}\right)^n\right) u[-n-1]$

(c) same as (a) , $\frac{1}{4} < |z| < \frac{1}{2}$

$x[n] = -\left(\frac{1}{4}\right)^n u[n] - \left(\frac{1}{2}\right)^n u[-n-1]$

(d) $X(z) = \frac{z^2 - 3z}{z^2 + \frac{3}{2}z - 1}$, $\frac{1}{2} < |z| < 2$

$$= \frac{1-3z^{-1}}{1+\frac{3}{2}z^{-1}-z^{-2}}$$

$$= \frac{-\frac{14}{5}}{2+z^{-1}} + \frac{-\frac{1}{5}}{-\frac{1}{2}+z^{-1}}$$

$$= \frac{-\frac{7}{5}}{1+\frac{1}{2}z^{-1}} + \frac{\frac{2}{5}}{1-2z^{-1}}$$

$$\underset{\text{causal}}{\uparrow} \qquad\qquad \underset{\text{anticausal}}{\uparrow}$$

$$\therefore\ x[n] = -\frac{7}{5}\left(-\frac{1}{2}\right)^n u[n] - \frac{2}{5}(2)^n u[-n-1]$$

(e) $$X(z) = \frac{12(11z^2-3z)}{12z^2-7z+1}, \quad |z| > \frac{1}{3} \longrightarrow \text{causal}$$

$$X(z) = \frac{12(11-3z^{-1})}{12-7z^{-1}+z^{-2}}$$

$$= \frac{-24}{-3+z^{-1}} + \frac{-12}{-4+z^{-1}}$$

$$= \frac{8}{1-\frac{1}{3}z^{-1}} + \frac{3}{1-\frac{1}{4}z^{-1}}$$

$$\therefore\ x[n] = \left(8\left(\frac{1}{3}\right)^n + 3\left(\frac{1}{4}\right)^n\right) u[n]$$

(f) $$X(z) = \frac{8z^2+4z}{4z^2-4z+1}, \quad |z| > \frac{1}{2} \rightarrow \text{causal}$$

$$X(z) = \frac{8+4z^{-1}}{4-4z^{-1}+z^{-2}} = \frac{4(2-z^{-1})+8z^{-1}}{(2-z^{-1})^2}$$

$$= \frac{4}{2 - z^{-1}} + \frac{8z^{-1}}{(2 - z^{-1})^2}$$

$$= \frac{2}{1 - \frac{1}{2}z^{-1}} + \frac{2z^{-1}}{(1 - \frac{1}{2}z^{-1})^2}$$

$$\therefore x[n] = (2 + 4n)\left(\tfrac{1}{2}\right)^n u[n]$$

(g) $$X(z) = \frac{z^3 + z^2 + \frac{3}{2}z + \frac{1}{2}}{z^3 + \frac{3}{2}z^2 + \frac{1}{2}z}$$

$$= \frac{1 + z^{-1} + \frac{3}{2}z^{-2} + \frac{1}{2}z^{-3}}{1 + \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}}$$

$$= z^{-1} + \frac{2}{1 + z^{-1}} + \frac{-1}{1 + \frac{1}{2}z^{-1}}$$

ROC : $|z| < \frac{1}{2}$ $\longrightarrow$ anticausal

$$\therefore x[n] = \delta[n-1] + \left(\left(-\tfrac{1}{2}\right)^n - 2(-1)^n\right)u[-n-1]$$

(h) $$X(z) = \frac{z^3 + z^2 + \frac{3}{2}z + \frac{1}{2}}{z^2 + \frac{3}{2}z + \frac{1}{2}}, \quad |z| > 1 \rightarrow \text{causal}$$

$$X(z) = \frac{1 + z^{-1} + \frac{3}{2}z^{-2} + \frac{1}{2}z^{-3}}{z^{-1} + \frac{3}{2}z^{-2} + \frac{1}{2}z^{-3}}$$

$$= 1 + \frac{1}{z^{-1}\left(1 + \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}\right)}$$

$$= 1 + \frac{1}{z^{-1}} + \frac{-2}{1 + z^{-1}} + \frac{\frac{1}{2}}{1 + \frac{1}{2}z^{-1}}$$

$$\therefore\ x[n] = \delta[n] + \delta[n+1] - 2u[n] + \tfrac{1}{2}\left(-\tfrac{1}{2}\right)^n u[n]$$

$\boxed{7.9}$

(a) $X(z) = 1 + 2z^{-2} + 4z^{-4}, \quad |z| > 0$

$x[n] = \delta[n] + 2\delta[n-2] + 4\delta[n-4]$

(b) $$X(z) = \sum_{k=5}^{10} \frac{1}{k} z^{-k}, \quad |z| > 0$$

$$x[n] = \sum_{k=5}^{10} \frac{1}{k} \delta[n-k]$$

(c) $X(z) = (1 + z^{-1})^4, \quad |z| > 0$

$$x[n] = (\delta[n] + \delta[n-1]) * (\delta[n] + \delta[n-1]) * (\delta[n] + \delta[n-1]) * (\delta[n] + \delta[n-1])$$

$$x[n] = \{\underset{\uparrow}{1}, 2, 1\} * \{\underset{\uparrow}{1}, 2, 1\}$$

$$= \{\underset{\uparrow}{1}, 4, 6, 4, 1\}$$

$$\therefore\ x[n] = \delta[n] + 4\delta[n-1] + 6\delta[n-2] + 4\delta[n-3] + \delta[n-4]$$

(d) $X(2) = z^4 + 2z^2 + 3 + 2z^{-2} + z^{-4}, \quad |z| > 0$

$x[n] = \delta[n+4] + 2\delta[n+2] + 3\delta[n] + 2\delta[n-2] + \delta[n-4]$

$\boxed{7.10}$

(a) $$H(z) = \frac{3z^{-1}}{(1 - 2z^{-1})^2}$$

(i) Stable, ROC : $|z| < 2$ (includes $|z| = 1$) $\rightarrow$ anticausal

$$h[n] = -\frac{3}{2}(2)^n \, n \, u[-n-1]$$

(ii) Causal, ROC : $|z| > 2$

$$h[n] = \frac{3}{2}(2)^n \, n . u[n]$$

(b) $$H(z) = \frac{12z^2 + 24z}{12z^2 + 13z + 3}$$

$$= \frac{12 + 24z^{-1}}{12 + 13z^{-1} + 3z^{-2}}$$

$$= \frac{-12}{4 + 3z^{-1}} + \frac{12}{3 + z^{-1}}$$

$$H(z) = \frac{-3}{1 + \frac{3}{4}z^{-1}} + \frac{4}{1 + \frac{1}{3}z^{-1}}$$

(i) Stable, ROC : $|z| > \frac{3}{4}$ (includes $|z| = 1$) $\rightarrow$ causal

$$h[n] = \left(-3\left(-\frac{3}{4}\right)^n + 4\left(-\frac{1}{3}\right)^n\right) u[n]$$

(ii) Causal, ROC : $|z| > \frac{3}{4}$

$$h[n] = \left(-3\left(-\frac{3}{4}\right)^n + 4\left(-\frac{1}{3}\right)^n\right) u[n]$$

(c) $$H(z) = \frac{4z}{z^2 - \frac{1}{2}z + \frac{1}{16}}$$

$$= \frac{4z^{-1}}{\left(1 - \frac{1}{4}z^{-1}\right)^2}$$

$$= \frac{16 \cdot \frac{1}{4} z^{-1}}{(1 - \frac{1}{4} z^{-1})^2}$$

(i) Stable , ROC : $|z| > \frac{1}{4}$ (includes $|z| = 1$) $\rightarrow$ causal

$$h[n] = 16\, n \left(\frac{1}{4}\right)^n u[n]$$

(ii) Causal , ROC : $|z| > \frac{1}{4}$

$$h[n] = 16\, n \left(\frac{1}{4}\right)^n u[n]$$

$\boxed{7.11}$

(a) $X(z) = \frac{1}{1 - z^{-2}}$, $|z| > 1$ $\longrightarrow$ causal

$$X(z) = \frac{1}{1 - (z^{-2})}$$

$$X(z) = \sum_{k=0}^{\infty} z^{-2.k}$$

$$x[n] = \sum_{k=0}^{\infty} \delta[n-2k]$$

(b) $X(z) = \frac{1}{1 - z^{-2}}$, $|z| < 1$ $\longrightarrow$ anticausal

$$X(z) = \frac{z^2}{z^2 - 1}$$

$$= -z^2 \frac{1}{1 - (z^2)}$$

$$= -z^2 \sum_{k=0}^{\infty} z^{2k}$$

$$X(z) = -\sum_{k=0}^{\infty} z^{2(k+1)}$$

$$x[n] = -\sum_{k=0}^{\infty} \delta[n+2(k+1)]$$

(c) $X(z) = \cos(2z)$, $|z| < \infty \rightarrow$ anticausal

$$= \sum_{k=0}^{\infty} \frac{(2z)^{2k}}{(2k)!} (-1)^k$$

$$\therefore x[n] = \sum_{k=0}^{\infty} \frac{(-4)^k}{(2k)!} \delta[n+2k]$$

(d) $X(z) = \cos(z-2)$, $|z| > 0 \rightarrow$ causal

$$= \sum_{k=0}^{\infty} (-1)^k \frac{(z^{-2})^{2k}}{(2k)!}$$

$$= \sum_{k=0}^{\infty} \frac{(-z^{-4})^k}{(2k)!}$$

$$\therefore x[n] = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} \delta[n-4k]$$

(e) $X(z) = \ln(1+z^{-1})$, $|z| > 0 \rightarrow$ causal

$$= \sum_{k=1}^{\infty} \frac{(-1)^{k-1}(z^{-1})^k}{k}$$

$$\therefore x[n] = \sum_{k=1}^{\infty} \frac{(-1)^{k-1} \delta[n-k]}{k}$$

$\boxed{7.12}$

(a) $x[n] = \delta[n] + \frac{1}{4}\delta[n-1] - \frac{1}{8}\delta[n-2]$

$y[n] = \delta[n] - \frac{3}{4}\delta[n-1]$

$X(z) = 1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}$

$Y(z) = 1 - \frac{3}{4}z^{-1}$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$= \frac{1 - \frac{3}{4}z^{-1}}{1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$$

$$= \frac{-\frac{2}{3}}{1 - \frac{1}{4}z^{-1}} + \frac{\frac{5}{3}}{1 + \frac{1}{2}z^{-1}}$$

$\therefore h[n] = \frac{1}{3}\left(5\left(-\frac{1}{2}\right)^n - 2\left(\frac{1}{4}\right)^n\right)u[n]$

(b) $x[n] = \left(-\frac{1}{3}\right)^n u[n]$

$y[n] = 3(-1)^n u[n] + \left(\frac{1}{3}\right)^n u[n]$

$$X(z) = \frac{1}{1 + \frac{1}{3}z^{-1}}$$

$$Y(z) = \frac{3}{1 + z^{-1}} + \frac{1}{1 - \frac{1}{3}z^{-1}}$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$H(z) = \frac{3\left(1+\frac{1}{3}z^{-1}\right)}{1+z^{-1}}$$

$$+ \frac{1+\frac{1}{3}z^{-1}}{1-\frac{1}{3}z^{-1}}$$

$$= 2\left(\frac{1}{1+z^{-1}} + \frac{1}{1-\frac{1}{3}z^{-1}}\right)$$

$$\therefore h[n] = 2\left((-1)^n + \left(\tfrac{1}{3}\right)^n\right) u[n]$$

(c) $x[n] = (-3)^n \, u[n]$

$y[n] = 4\,(2)^n \, u[n] - \left(\tfrac{1}{2}\right)^n u[n]$

$$X(z) = \frac{1}{1+3z^{-1}} \quad , \quad Y(z) = \frac{4}{1-2z^{-1}} - \frac{1}{1-\frac{1}{2}z^{-1}}$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$= \frac{4\left(1+3z^{-1}\right)}{1-2z^{-1}} - \frac{1+3z^{-1}}{1-\frac{1}{2}z^{-1}}$$

$$= \frac{10}{1-2z^{-1}} - \frac{7}{1-\frac{1}{2}z^{-1}}$$

$$\therefore h[n] = \left(10\,(2)^n - 7\left(\tfrac{1}{2}\right)^n\right) u[n]$$

$\boxed{7.13}$ $h[n] = \left(\frac{1}{2}\right)^n u[n]$

$$H(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$$

(a) $y[n] = \delta[n-2]$

$$Y(z) = z^{-2}$$

$$X(z) = \frac{Y(z)}{H(z)}$$

$$= z^{-2}\left(1-\frac{1}{2}z^{-1}\right)$$

$$= z^{-2} - \frac{1}{2}z^{-3}$$

$$\therefore x[n] = \delta[n-2] - \frac{1}{2}\delta[n-3]$$

(b) $y[n] = \left(\frac{1}{2}\right)^n u[n] + \left(-\frac{1}{2}\right)^n u[n]$

$$Y(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1+\frac{1}{2}z^{-1}}$$

$$X(z) = \frac{Y(z)}{H(z)}$$

$$= 1 + \frac{1-\frac{1}{2}z^{-1}}{1+\frac{1}{2}z^{-1}}$$

$$= \frac{2}{1+\frac{1}{2}z^{-1}}$$

$$\therefore x[n] = 2\left(-\frac{1}{2}\right)^n u[n]$$

(c) $y[n] = \frac{1}{3}u[n] + \frac{2}{3}\left(-\frac{1}{2}\right)^n u[n]$

$$Y(z) = \frac{\frac{1}{3}}{1-z^{-1}} + \frac{\frac{2}{3}}{1+\frac{1}{2}z^{-1}}$$

$$X(z) = \frac{\frac{1}{3}\left(1-\frac{1}{2}z^{-1}\right)}{1-z^{-1}} + \frac{\frac{2}{3}\left(1-\frac{1}{2}z^{-1}\right)}{1+\frac{1}{2}z^{-1}}$$

$$= -\frac{1}{2} + \frac{\frac{1}{6}}{1-z^{-1}} + \frac{\frac{4}{3}}{1+\frac{1}{2}z^{-1}}$$

$$\therefore x[n] = -\frac{1}{2}\delta[n] + \frac{1}{6}u[n] + \frac{4}{3}\left(-\frac{1}{2}\right)^n u[n]$$

$\boxed{7.14}$

(a) $y[n] - \frac{1}{2}y[n-1] = 2x[n-1]$

$$\left(1-\frac{1}{2}z^{-1}\right)Y(z) = 2z^{-1}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$= \frac{2z^{-1}}{1-\frac{1}{2}z^{-1}}$$

$$h[n] = 2\left(\frac{1}{2}\right)^{n-1}u[n-1]$$

(b) $y[n] = x[n] - x[n-2] + x[n-4] - x[n-6]$

$$Y(z) = \left(1 - z^{-2} + z^{-4} - z^{-6}\right)X(z)$$

$$H(z) = 1 - z^{-2} + z^{-4} - z^{-6}$$

$$h[n] = \delta[n] - \delta[n-2] + \delta[n-4] - \delta[n-6]$$

(c) $y[n] - \frac{1}{4}y[n-1] - \frac{3}{8}y[n-2] = -x[n] + 2x[n-1]$

$$\left(1-\frac{1}{4}z^{-1}-\frac{3}{8}z^{-2}\right)Y(z) = \left(-1+2z^{-1}\right)X(z)$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$= \frac{-1 + 2z^{-1}}{1 - \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}}$$

$$= \frac{-2}{1+\frac{1}{2}z^{-1}} + \frac{1}{1-\frac{3}{4}z^{-1}}$$

$$\therefore h[n] = \left(-2\left(-\frac{1}{2}\right)^n + \left(\frac{3}{4}\right)^n\right) u[n]$$

(d) $y[n] - \frac{4}{5}y[n-1] - \frac{16}{25}y[n-2] = 2x[n] + x[n-1]$

$$\left(1 - \frac{4}{5}z^{-1} - \frac{16}{25}z^{-2}\right)Y(z) = \left(2 + z^{-1}\right)X(z)$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$= \frac{2 + z^{-1}}{1 - \frac{4}{5}z^{-1} - \frac{16}{25}z^{-2}}$$

$$= \frac{2 - \frac{8}{5}z^{-1} + \frac{13}{5}z^{-1}}{\left(1 - \frac{4}{5}z^{-1}\right)^2}$$

$$= \frac{2}{1-\frac{4}{5}z^{-1}} + \frac{\frac{13}{5}z^{-1}}{\left(1-\frac{4}{5}z^{-1}\right)^2}$$

$$\therefore h[n] = \left(2\left(\frac{4}{5}\right)^n + \frac{13}{4}n\left(\frac{4}{5}\right)^n\right)u[n]$$

$\boxed{7.15}$

(a) $h[n] = 3\left(\frac{1}{4}\right)^n u[n-1]$

$$= 3\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)^{n-1} u[n-1]$$

$$H(z) = \frac{3}{4} z^{-1} \frac{1}{1 - \frac{1}{4} z^{-1}}$$

$$= \frac{Y(z)}{X(z)} \xleftrightarrow{z} y[n] - \frac{1}{4} y[n-1] = \frac{3}{4} x[n-1]$$

(b) $h[n] = \left(\frac{1}{3}\right)^n u[n] + \left(\frac{1}{2}\right)^{n-2} u[n-1]$

$$= \left(\frac{1}{3}\right)^n u[n] + 2\left(\frac{1}{2}\right)^{n-1} u[n-1]$$

$$H(z) = \frac{1}{1 - \frac{1}{3} z^{-1}} - \frac{2 z^{-1}}{1 - \frac{1}{2} z^{-1}}$$

$$= \frac{1 + \frac{3}{2} z^{-1} - \frac{2}{3} z^{-2}}{1 - \frac{5}{6} z^{-1} + \frac{1}{6} z^{-2}}$$

$$= \frac{Y(z)}{X(z)}$$

$$\xleftrightarrow{z} y[n] - \frac{5}{6} y[n-1] + \frac{1}{6} y[n-2]$$

$$= x[n] + \frac{3}{2} x[n-1] - \frac{2}{3} x[n-2]$$

(c) $h[n] = 2\left(\frac{2}{3}\right)^n u[n-1] + \left(\frac{1}{4}\right)^n \left[\cos\left(\frac{\pi}{6} n\right) - 2 \sin\left(\frac{\pi}{6} n\right)\right]$

$$H(z) = \frac{\frac{4}{3} z^{-1}}{1 - \frac{2}{3} z^{-1}} + \frac{\left(1 - \frac{1}{4} z^{-1} \cos\left(\frac{\pi}{6}\right) - \frac{1}{2} z^{-1} \sin\left(\frac{\pi}{6}\right)\right)}{1 - z^{-1} \frac{1}{2} \cos\left(\frac{\pi}{6}\right) + \frac{1}{16} z^{-2}}$$

$$= \frac{\frac{4}{3} z^{-1}}{1 - \frac{2}{3} z^{-1}} + \frac{1 - \left(\frac{1}{8}\sqrt{3} + \frac{1}{4}\right) z^{-1}}{1 - \frac{1}{4}\sqrt{3}\, z^{-1} + \frac{1}{16} z^{-2}}$$

$$H(z) = \frac{1 + \left(\frac{5}{12} - \frac{1}{8}\sqrt{3}\right)z^{-1} + \left(\frac{1}{6} - \frac{1}{4}\sqrt{3}\right)z^{-2} + \frac{1}{12}z^{-3}}{1 - \left(\frac{2}{3} + \frac{1}{4}\sqrt{3}\right)z^{-1} + \left(\frac{1}{16} + \frac{1}{6}\sqrt{3}\right)z^{-2} - \frac{1}{24}z^{-3}}$$

$$= \frac{Y(z)}{X(z)}$$

$$\overset{z}{\longleftrightarrow} y[n] - \left(\frac{2}{3} + \frac{1}{4}\sqrt{3}\right)y[n-1] + \left(\frac{1}{16} + \frac{1}{6}\sqrt{3}\right)y[n-2] - \frac{1}{24}y[n-3]$$

$$= x[n] + \left(\frac{5}{12} - \frac{1}{8}\sqrt{3}\right)x[n-1] + \left(\frac{1}{6} - \frac{1}{4}\sqrt{3}\right)x[n-2] + \frac{1}{12}x[n-3]$$

(d) $h[n] = \delta[n] - \delta[n-5]$

$$H(z) = 1 - z^{-5}$$

$$= \frac{Y(z)}{X(z)} \quad \overset{z}{\longleftrightarrow} y[n] = x[n] - x[n-5]$$

$\boxed{7.16}$

(a) $\bar{A} = \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}, \ \bar{b} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \ \bar{c} = \begin{bmatrix} 1 & -1 \end{bmatrix}, \ \bar{D} = [1]$

$$H(z) = \bar{c}\,(z\bar{I} - \bar{A})^{-1}\,\bar{b} + \bar{D}$$

$$= \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{z+\frac{1}{2}} & 0 \\ 0 & \frac{1}{z-\frac{1}{2}} \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} + [1]$$

$$= \frac{z - \frac{5}{2}}{z - \frac{1}{2}}$$

(b) $\bar{A} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{4} \end{bmatrix}, \ \bar{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ \bar{c} = \begin{bmatrix} 2 & 1 \end{bmatrix}, \ \bar{D} = [0]$

$$H(z) = \bar{c}\,(z\bar{I} - \bar{A})^{-1}\,\bar{b} + \bar{D}$$

$$H(z) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} z-\frac{3}{4} & -\frac{1}{8} \\ \frac{7}{2} & z+\frac{1}{4} \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} \frac{1}{(z-\frac{1}{4})^2}$$

$$H(z) = \frac{2z + 7\frac{1}{2}}{(z-\frac{1}{4})^2}$$

$\boxed{7.17}$

(a) $$H(z) = \frac{2z+1}{z^2 + z - \frac{5}{16}}$$

$$= \frac{2(z+\frac{1}{2})}{(z+\frac{5}{4})(z-\frac{1}{4})}$$

zero : $z = -\frac{1}{2}$

pole : $z = -\frac{5}{4}, \frac{1}{4}$

(i) not all poles are inside $|z| = 1 \rightarrow$ NOT BOTH causal and stable

(ii) all zeros are inside $|z| = 1 \rightarrow$ minimum phase

(b) $$H(z) = \frac{1 + 2z^{-1}}{1 + \frac{14}{8}z^{-1} + \frac{49}{64}z^{-2}}$$

$$= \frac{z(z+2)}{(z+\frac{7}{8})^2}$$

zero : $z = 0$, $z = -2$

pole : $z = -\frac{7}{8}$ (double)

(i) Both causal and stable

(ii) Not minimum phase

(c) $y[n] - \frac{6}{5}y[n-1] - \frac{16}{25}y[n-2] = 2x[n] + x[n-1]$

$$\left[1 - \frac{6}{5}z^{-1} - \frac{16}{25}z^{-2}\right]Y(z) = (2 + z^{-1})X(z)$$

$$H(z) = \frac{2 + z^{-1}}{1 - \frac{6}{5}z^{-1} - \frac{16}{25}z^{-2}}$$

$$= \frac{z(2z+1)}{z^2 - \frac{6}{5}z - \frac{16}{25}}$$

$$= \frac{2z(z + 1/2)}{(z - 8/5)(z + 2/5)}$$

zeros: $z = 0$, $z = -1/2$

poles: $z = 8/5$ $z = -2/5$

(i) not both stable and causal

(ii) minimum phase

(d) $y[n] - 2y[n-2] = x[n] - \frac{1}{2}x[n-1]$

$$(1 - 2z^{-2})\,Y(z) = (1 - \tfrac{1}{2}z^{-1})\,X(z)$$

$$H(z) = \frac{1 - \frac{1}{2}z^{-1}}{1 - 2z^{-2}}$$

$$= \frac{z(z - \frac{1}{2})}{(z - \sqrt{2})(z + \sqrt{2})}$$

zero : $z = 0$, $z = \frac{1}{2}$
pole : $z = \pm\sqrt{2}$

(i) NOT both causal and stable
(ii) minimum phase

(e) $\bar{A} = \begin{bmatrix} -\frac{1}{4} & \frac{1}{8} \\ \frac{7}{2} & \frac{3}{4} \end{bmatrix}$, $\bar{b} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$, $\bar{c} = [1 \;\; 1]$, $\bar{D} = [0]$

$$H(z) = \bar{c}\,(z\bar{I} - \bar{A})^{-1}\,\bar{b} + \bar{D}$$

$$= [1 \;\; 1] \begin{bmatrix} z - \frac{3}{4} & -\frac{1}{8} \\ \frac{7}{2} & z + \frac{1}{4} \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} \frac{1}{(z - \frac{1}{4})^2}$$

$$H(z) = \frac{2z + \frac{1}{4}}{(z - \frac{1}{4})^2}$$

pole : $z = \frac{1}{4}$ (double)
zero : $z = -\frac{1}{8}$

(i) both stable and causal
(ii) minimum phase

$\boxed{7.18}$

(a) $$H(z) = \frac{1 - 4z^{-1} + 4z^{-2}}{1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}}$$

$$H^{-1}(z) = \frac{1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}}{1 - 4z^{-1} + 4z^{-2}}$$

$$= \frac{z^2 - \frac{1}{2}z + \frac{1}{4}}{z^2 - 4z + 4}$$

$$= \frac{(z - \frac{1}{4})^2 + \frac{3}{16}}{(z-2)^2}$$

pole : $z = 2$ (double) $\longrightarrow$ not inside $|z| = 1$

$h^{-1}[n]$ can <u>NOT</u> be both causal and stable

(b) $$H(z) = \frac{z^2 - \frac{49}{64}}{z^2 - 4}$$

$$H^{-1}(z) = \frac{z^2 - 4}{z^2 - \frac{49}{64}}$$

$$= \frac{(z-2)(z+2)}{(z - \frac{7}{8})(z + \frac{7}{8})}$$

pole : $z = \pm\frac{7}{8}$ $\longrightarrow$ inside $|z| = 1$, $|\pm\frac{7}{8}| < 1$

$h^{-1}[n]$ can be both causal and stable

(c) $$h[n] = 10\left(-\frac{1}{2}\right)^n u[n] - 9\left(-\frac{1}{4}\right)^n u[n]$$

$$H(z) = \frac{10}{1 + \frac{1}{2}z^{-1}} - \frac{9}{1 + \frac{1}{4}z^{-1}}$$

$$H(z) = \frac{1-2z^{-1}}{\left(1+\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)}$$

$$= \frac{z(z-2)}{\left(z+\frac{1}{2}\right)\left(z+\frac{1}{4}\right)}$$

$$H^{-1}(z) = \frac{\left(z+\frac{1}{2}\right)\left(z+\frac{1}{4}\right)}{z(z-2)}$$

pole : $z = 0, 2$, $|2| > 1$

$h^{-1}[n]$ can <u>NOT</u> be both causal and stable

(d) $h[n] = 24\left(\frac{1}{2}\right)^n u[n-1] - 30\left(\frac{1}{3}\right)^n u[n-1]$

$$= 12\left(\frac{1}{2}\right)^{n-1} u[n-1] - 10\left(\frac{1}{3}\right)^{n-1} u[n-1]$$

$$H(z) = \frac{12z^{-1}}{1-\frac{1}{2}z^{-1}} - \frac{10z^{-1}}{1-\frac{1}{3}z^{-1}}$$

$$= \frac{2z^{-1}+z^{-2}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)}$$

$$H^{-1}(z) = \frac{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)}{2z^{-1}+z^{-2}}$$

$$= \frac{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}{2\left(z+\frac{1}{2}\right)}$$

pole : $z = -\frac{1}{2}$, $\left|-\frac{1}{2}\right| < 1$

$h^{-1}[n]$ can be both causal and stable

(e) $y[n] - \frac{1}{4}y[n-2] = 6x[n] - 7x[n-1] + 3x[n-2]$

$$\left(1-\frac{1}{4}z^{-2}\right)Y(z) = \left(6-7z^{-1}+3z^{-2}\right)X(z)$$

$$H^{-1}(z) = \frac{X(z)}{Y(z)}$$

$$= \frac{1 - \frac{1}{4} z^{-2}}{6 - 7z^{-1} + 3z^{-2}}$$

$$= \frac{(z - \frac{1}{2})(z + \frac{1}{2})}{6z^2 + 7z + 3}$$

pole : $6z^2 \div 7z + 3 = 0$, $z_p = \frac{7 \pm j\sqrt{23}}{12}$

$\therefore h^{-1}[n]$ can be both stable and causal

(f) $y[n] - 2y[n-1] = x[n]$

$$(1 - 2z^{-1})\, Y(z) = X(z)$$

$$H^{-1}(z) = \frac{X(z)}{Y(z)}$$

$$= 1 - 2z^{-1}$$

$$= \frac{z - 2}{z}$$

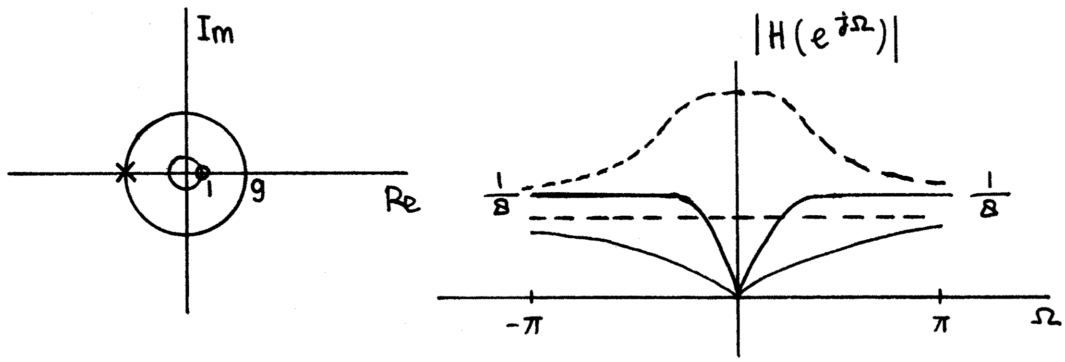
pole : $z = 0$

$\therefore h^{-1}[n]$ can be both stable and causal

$\boxed{7.19}$

(a) $H(z) = \frac{z - 1}{z + 9}$

$$H(e^{j\Omega}) = \frac{e^{j\Omega} - 1}{e^{j\Omega} + 9}$$

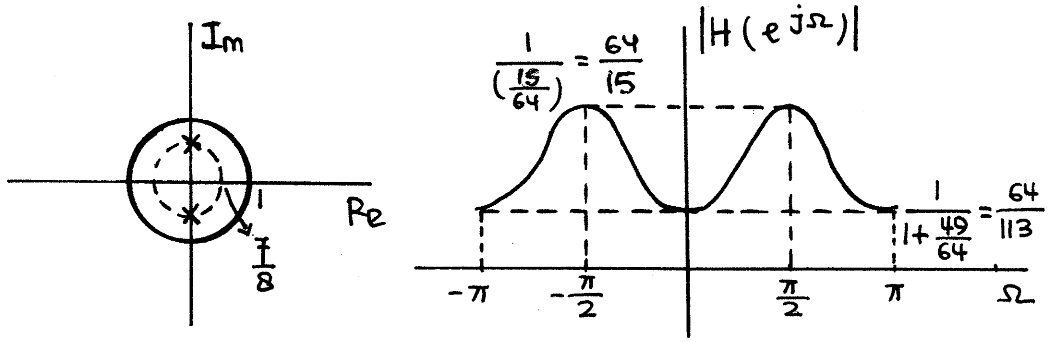


(b) $$H(z) = \frac{z^{-2}}{1+\frac{49}{64}z^{-2}}$$

$$= \frac{1}{z^2 + \frac{49}{64}}$$

$$= \frac{1}{(z+j\frac{7}{8})(z-j\frac{7}{8})}$$

$$H(e^{j\Omega}) = \frac{1}{(e^{j\Omega}+j\frac{7}{8})(e^{j\Omega}-j\frac{7}{8})}$$



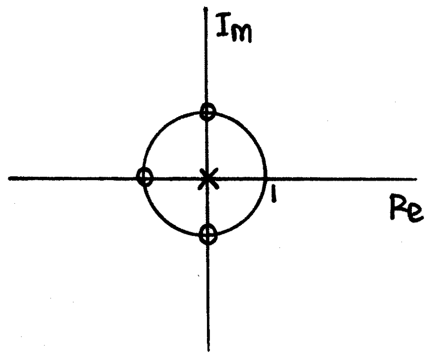
(c) $$H(z) = \frac{1+z^{-1}+z^{-2}+z^{-3}}{4}$$

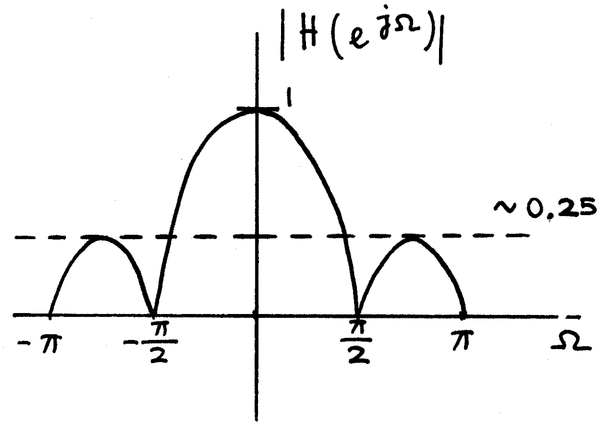
$$= \frac{z^3+z^2+z+1}{4z^3}$$

$$H(e^{j\Omega}) = \frac{(e^{j\Omega})^3+(e^{j\Omega})^2+(e^{j\Omega})+1}{4(e^{j\Omega})^3}$$

zeros : $z = e^{j\frac{\pi}{2}}, e^{j\pi}, e^{j\frac{3\pi}{2}}$

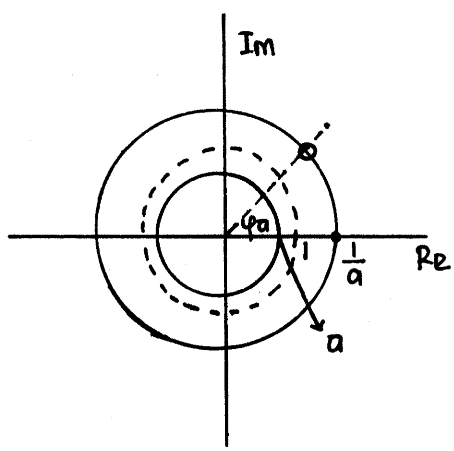
poles : $z = 0$ (triple)





$\boxed{7.20}$ $H(z) = \dfrac{1 - a^* z}{z - a}$, $|a| < 1$

(a)



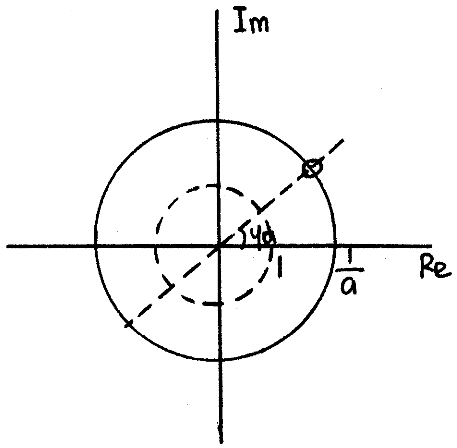
Let $a = |a| e^{j\varphi_a}$

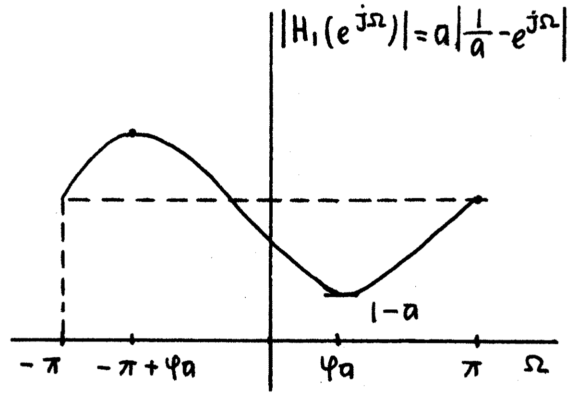
then $\dfrac{1}{a^*} = \dfrac{1}{|a|} e^{j\varphi_a}$

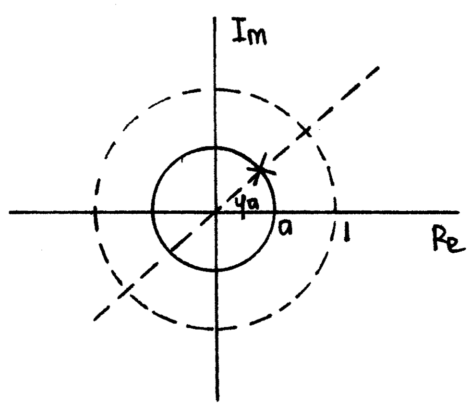
(b) $|H(e^{j\Omega})| = \left| \dfrac{1 - a^* e^{j\Omega}}{e^{j\Omega} - a} \right|$

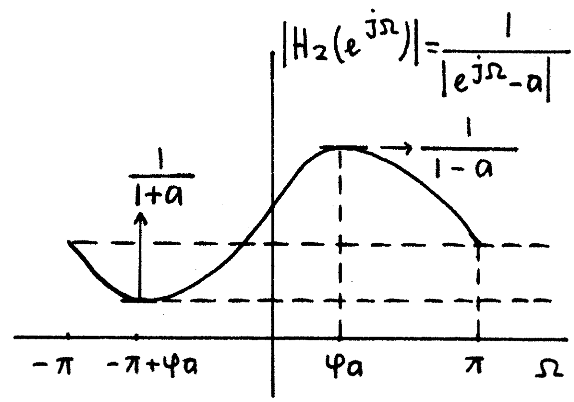
$= |1 - a^* e^{j\Omega}| \dfrac{1}{|e^{j\Omega} - a|}$

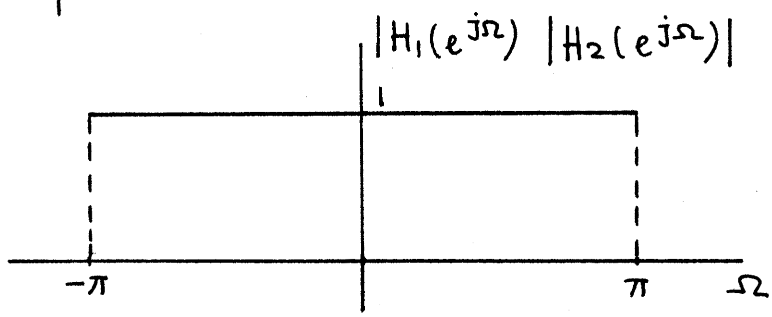
As shown below, the $|H(e^{j\Omega})| = 1$ for all Ω









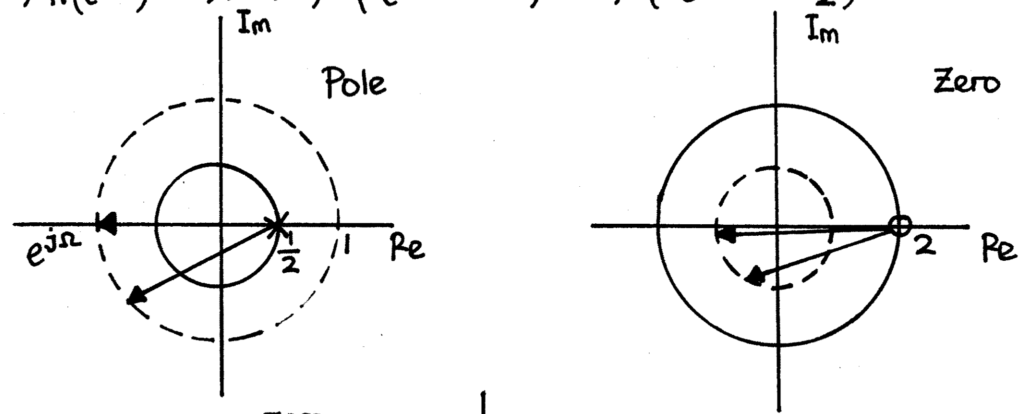


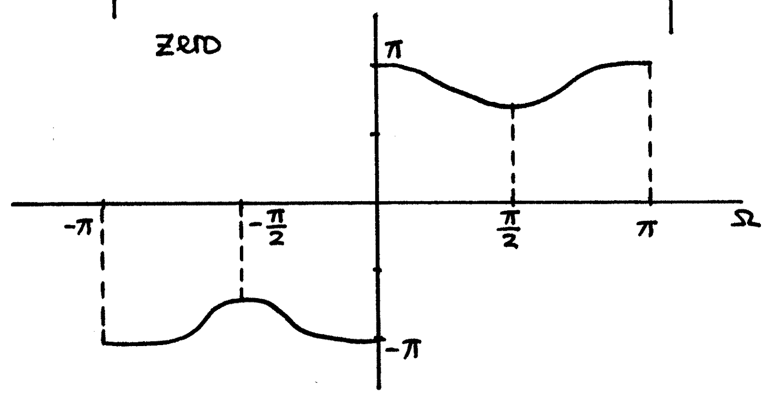
(c) $a = \frac{1}{2}$

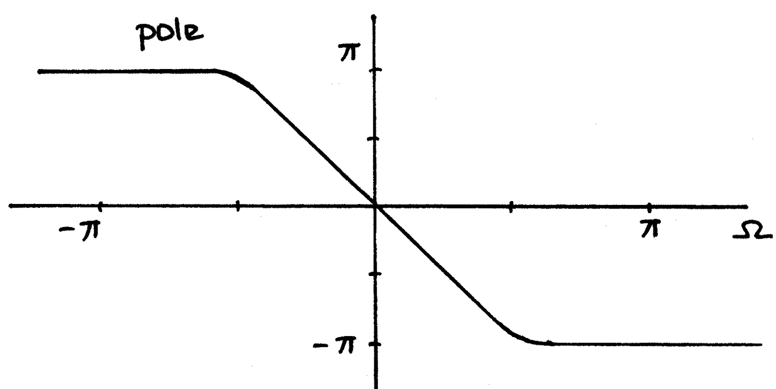
$$H(e^{j\Omega}) = \frac{1 - \frac{1}{2}e^{j\Omega}}{e^{j\Omega} - \frac{1}{2}}$$

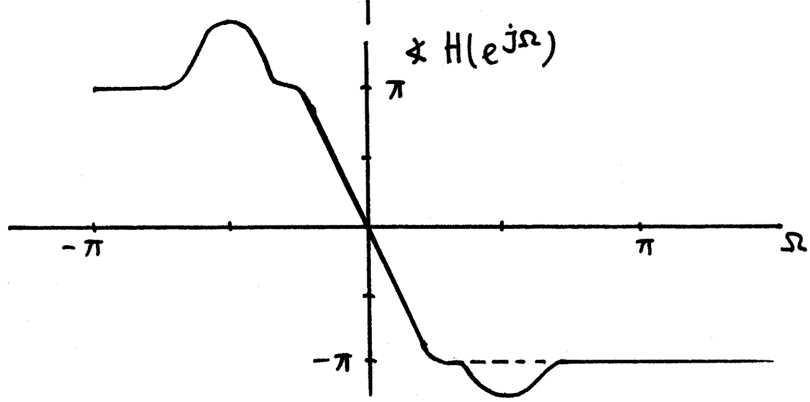
$$\measuredangle H(e^{j\Omega}) = \measuredangle\left(1 - \frac{1}{2}e^{j\Omega}\right) - \measuredangle\left(e^{j\Omega} - \frac{1}{2}\right)$$

$$\measuredangle H(e^{j\Omega}) = \pi + \measuredangle\left(e^{j\Omega} - 2\right) - \measuredangle\left(e^{j\Omega} - \tfrac{1}{2}\right)$$









(d) $$H(z) = \prod_{k=1}^{P} \frac{1 - a_k^* z}{z - a_k} \quad , |a_k| < 1$$

since $|a_k| < 1$, if the system is causal, then the system is stable

$$H(e^{j\Omega}) = \prod_{k=1}^{P} \frac{1 - a_k^* e^{j\Omega}}{e^{j\Omega} - a_k}$$

$$|H(e^{j\Omega})| = \left| \frac{1 - a_1^* e^{j\Omega}}{e^{j\Omega} - a_1} \cdot \frac{1 - a_2^* e^{j\Omega}}{e^{j\Omega} - a_2} \cdot \cdots \cdot \frac{1 - a_p^* e^{j\Omega}}{e^{j\Omega} - a_p} \right|$$

$$|H(e^{j\Omega})| = \left| \frac{1 - a_1^* e^{j\Omega}}{e^{j\Omega} - a_1} \right| \cdot \left| \frac{1 - a_2^* e^{j\Omega}}{e^{j\Omega} - a_2} \right| \cdot \cdots \cdot \left| \frac{1 - a_p^* e^{j\Omega}}{e^{j\Omega} - a_p} \right|$$

$|H(e^{j\Omega})| = 1$, all-pass system

(e) For a stable and causal all-pass system

$|a_k| < 1$ for all k

Let's use $P = 1$

So : $$H(z) = \frac{1 - a_1^* z}{z - a_1}$$

The zero is : $$z_z = \frac{1}{a_1^*}$$

which : $$|z_z| = \frac{1}{|a_1|} > 1$$

$\therefore$ This system can <u>NOT</u> also be minimum phase

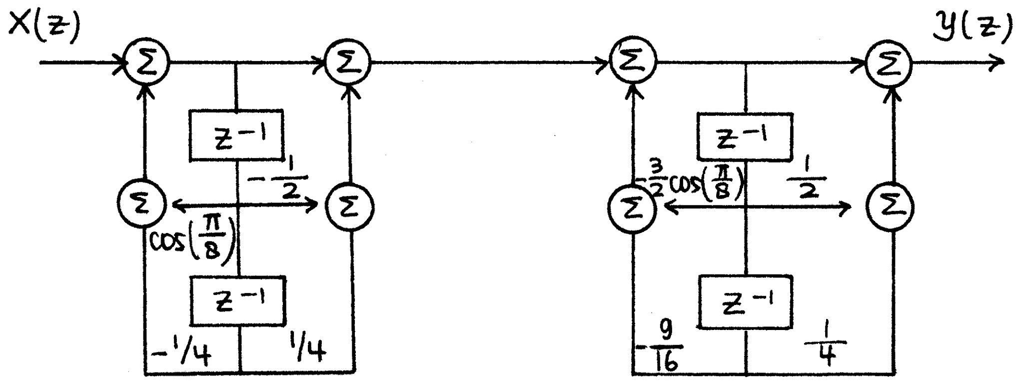
$\boxed{7.21}$

(a) $$H(z)=\frac{\left(1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}\right)\left(1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}\right)}{\left(1-\cos\left(\frac{\pi}{8}\right)z^{-1}+\frac{1}{4}z^{-2}\right)\left(1+\frac{3}{2}\cos\left(\frac{\pi}{8}\right)z^{-1}+\frac{9}{16}z^{-2}\right)}$$

$$H_1(z)=\frac{1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}{1-\cos\left(\frac{\pi}{8}\right)z^{-1}+\frac{1}{4}z^{-2}}$$

$$H_2(z)=\frac{1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}}{1+\frac{3}{2}\cos\left(\frac{\pi}{8}\right)z^{-1}+\frac{9}{16}z^{-2}}$$

$$H(z)=H_1(z)\cdot H_2(z)$$

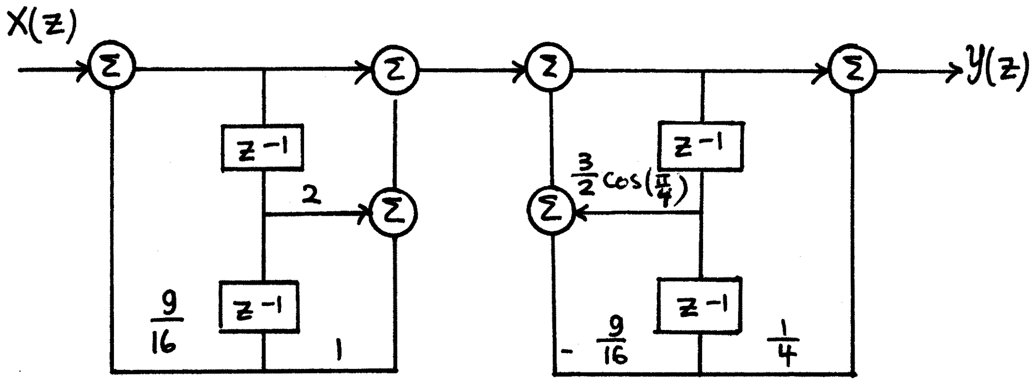


(b) $$H(z)=\frac{(1+z^{-1})^2\left(1+\frac{1}{4}z^{-2}\right)}{\left(1-\frac{9}{16}z^{-2}\right)\left(1-\frac{3}{4}e^{j\frac{\pi}{4}}z^{-1}\right)^2}$$

$$H_1(z)=\frac{1+2z^{-1}+z^{-2}}{1-\frac{9}{16}z^{-2}}$$

$$H_2(z)=\frac{1+\frac{1}{4}z^{-2}}{1-\frac{3}{2}\cos\frac{\pi}{4}z^{-1}+\frac{9}{16}z^{-2}}$$

$$H(z) = H_1(z) \cdot H_2(z)$$



$\boxed{7.22}$

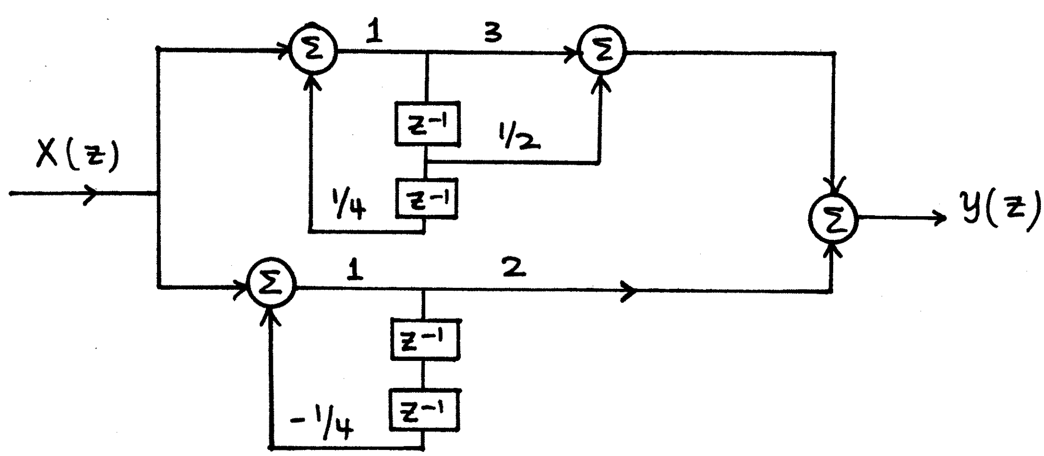
(a) $h[n] = 2\left(\frac{1}{2}\right)^n u[n] + \left(\frac{j}{2}\right)^n u[n] + \left(\frac{-j}{2}\right)^n u[n]$

$+ \left(-\frac{1}{2}\right)^n u[n]$

$$H(z) = \frac{2}{1-\frac{1}{2}z^{-1}} + \frac{1}{1-\frac{j}{2}z^{-1}} + \frac{1}{1+\frac{j}{2}z^{-1}} + \frac{1}{1+\frac{1}{2}z^{-1}}$$

$$H(z) = \frac{3+\frac{1}{2}z^{-1}}{1-\frac{1}{4}z^{-2}} + \frac{2}{1+\frac{1}{4}z^{-2}}$$

$$H(z) = H_1(z) + H_2(z)$$



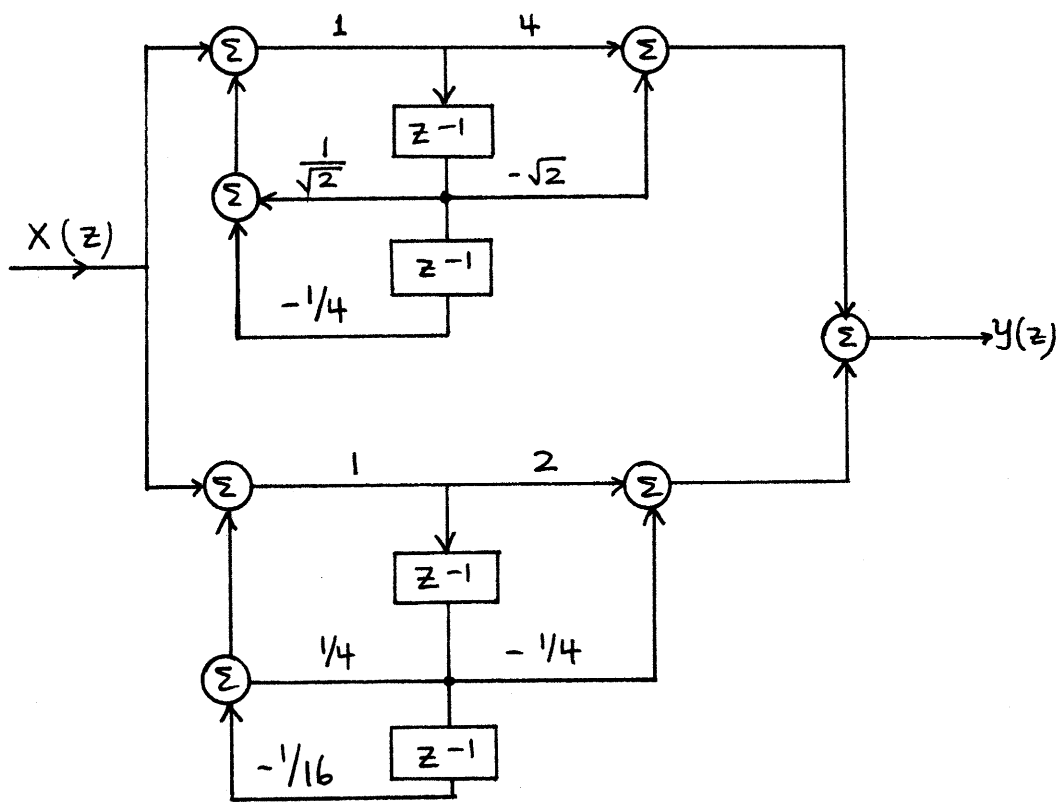
(b) $h[n] = 2\left(\frac{1}{2} e^{j\frac{\pi}{4}}\right)^n u[n] + \left(\frac{1}{4} e^{j\frac{\pi}{3}}\right)^n u[n]$

$+\left(\frac{1}{4} e^{-j\frac{\pi}{3}}\right)^n u[n] + 2\left(\frac{1}{2} e^{-j\frac{\pi}{4}}\right)^n u[n]$

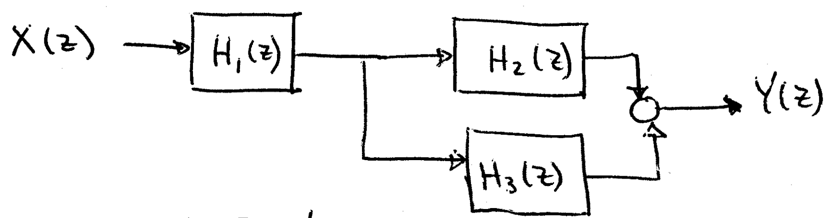
$$H(z) = \frac{2}{1-\frac{1}{2}e^{j\frac{\pi}{4}}z^{-1}} + \frac{1}{1-\frac{1}{4}e^{j\frac{\pi}{3}}z^{-1}} + \frac{1}{1-\frac{1}{4}e^{-j\frac{\pi}{3}}z^{-1}} + \frac{2}{1-\frac{1}{2}e^{-j\frac{\pi}{4}}z^{-1}}$$

$$H(z) = \frac{4-\sqrt{2}\,z^{-1}}{1-\frac{1}{\sqrt{2}}z^{-1}+\frac{1}{4}z^{-2}} + \frac{2-\frac{1}{4}z^{-1}}{1-\frac{1}{4}z^{-1}+\frac{1}{16}z^{-2}}$$

$$= H_1(z) + H_2(z)$$



$\boxed{7.23}$



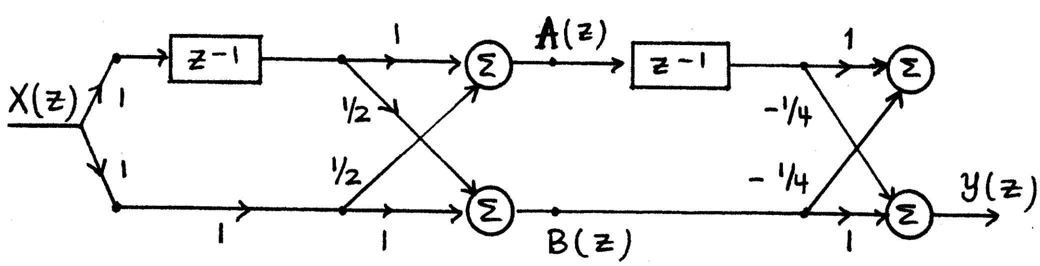
$$H_1(z) = \frac{1-2z^{-1}}{1-\frac{1}{2}z^{-2}} \qquad H_2(z) = \frac{z^{-2}}{1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}}$$

$$H_3(z) = \frac{1}{1-\frac{1}{3}z^{-1}}$$

$$H(z) = H_1(z)\,H_2(z) + H_1(z)\,H_3(z)$$

$\boxed{7.24}$

(a)



$$y(z) = -\frac{1}{4}A(z)z^{-1} + B(z)$$

$$A(z) = X(z).z^{-1} + \frac{1}{2}X(z)$$

$$= \left(z^{-1} + \frac{1}{2}\right)X(z)$$

$$B(z) = \frac{1}{2}.X(z).z^{-1} + X(z)$$

$$= \left(\frac{1}{2}z^{-1} + 1\right)X(z)$$

$$y(z) = -\frac{1}{4} z^{-1}\left(z^{-1}+\frac{1}{2}\right) X(z) + \left(\frac{1}{2} z^{-1}+1\right) X(z)$$

$$y(z) = \left(-\frac{1}{4} z^{-2} + \frac{3}{8} z^{-1} + 1\right) X(z)$$

$$H(z) = \frac{y(z)}{X(z)}$$

$$= 1 + \frac{3}{8} z^{-1} - \frac{1}{4} z^{-2}$$

(b) $\widetilde{H}_i(z) = \widetilde{H}_{i-1}(z) z^{-1} + C_i H_{i-1}(z)$

$H_i(z) = \widetilde{H}_{i-1}(z) \cdot z^{-1} C_i + H_{i-1}(z)$

$$\begin{bmatrix} \widetilde{H}_i(z) \\ H_i(z) \end{bmatrix} = \begin{bmatrix} z^{-1} & C_i \\ z^{-1} \cdot C_i & 1 \end{bmatrix} \begin{bmatrix} \widetilde{H}_{i-1}(z) \\ H_{i-1}(z) \end{bmatrix}$$

$$\therefore \overline{T}(z) = \begin{bmatrix} z^{-1} & C_i \\ z^{-1} C_i & 1 \end{bmatrix}$$

(c) $\widetilde{H}_i(z) = z^{-i} H_i(z^{-1})$

$\underline{i=1}$:
$$\begin{aligned} \widetilde{H}_{i-1}(z) &= H_{i-1}(z) = 1 \\ \widetilde{H}_1(z) &= z^{-1} + C_1 \\ H_1(z) &= z^{-1} C_1 + 1 \\ H_1(z^{-1}) &= 1 + z \cdot C_1 \end{aligned}$$

$$\widetilde{H}_1(z) = z^{-1} H_1(z^{-1})$$

$\underline{i=k}$: assume $\widetilde{H}_k(z) = z^{-k} H_k(z^{-1})$

$\underline{i=k+1}$: $\widetilde{H}_{k+1}(z) = z^{-1} \widetilde{H}_k(z) + C_{k+1} H_k(z)$

$$H_{k+1}(z) = z^{-1} C_{k+1} \widetilde{H}_k(z) + H_k(z)$$

substitute $\tilde{H}_k(z) = z^{-k} H_k(z^{-1})$

$$\tilde{H}_{k+1}(z) = z^{-1} z^{-k} H_k(z^{-1}) + C_{k+1} H_k(z)$$

$$= z^{-(k+1)} H_k(z^{-1}) + C_{k+1} H_k(z)$$

$$H_{k+1}(z) = z^{-1} C_{k+1} z^{-k} H_k(z^{-1}) + H_k(z)$$

$$= z^{-(k+1)} C_{k+1} H_k(z^{-1}) + H_k(z)$$

$$H_{k+1}(z^{-1}) = z^{(k+1)} C_{k+1} H_k(z) + H_k(z^{-1})$$

$$\tilde{H}_{k+1}(z) = z^{-(k+1)}\left(z^{(k+1)} C_{k+1} H_k(z) + H_k(z^{-1})\right)$$

$$\tilde{H}_{k+1}(z) = z^{-(k+1)} H_{k+1}(z^{-1})$$

$$\therefore \tilde{H}_i(z) = z^{-i} H_i(z^{-1})$$

(d) $$\begin{bmatrix} \tilde{H}_i \\ H_i \end{bmatrix} = \begin{bmatrix} z^{-1} & C_i \\ z^{-1}C_i & 1 \end{bmatrix} \begin{bmatrix} \tilde{H}_{i-1} \\ H_{i-1} \end{bmatrix}$$

$$H_i = z^{-1} C_i \tilde{H}_{i-1} + H_{i-1}$$

and

$$\tilde{H}_{i-1}(z) = z^{-(i-1)} + \ldots\ldots + C_{i-1}$$

The highest order of (z^{-1}) in $H_i(z)$ is (i) and the coefficient of z^{-i} is (C_i), since $H_{i-1}(z)$ does not contribute to z^{-i}

$\therefore$ The coefficient of z^{-i} in $H_i(z)$ is given by C_i

(e) $\begin{bmatrix} \tilde{H}_i(z) \\ H_i(z) \end{bmatrix} = \bar{T} \begin{bmatrix} \tilde{H}_{i-1}(z) \\ H_{i-1}(z) \end{bmatrix}$

$$\begin{bmatrix} \tilde{H}_{i-1}(z) \\ H_{i-1}(z) \end{bmatrix} = \bar{A} \begin{bmatrix} \tilde{H}_i(z) \\ H_i(z) \end{bmatrix} \qquad \ldots\ldots (1)$$

$$\bar{A} = \bar{T}^{-1} = \frac{1}{z^{-1}(1 - c_1^{\,2})} \begin{bmatrix} 1 & -c_i \\ -z^{-1}c_1 & z^{-1} \end{bmatrix}$$

Let : $H(z) = \sum_{k=0}^{M} b_k z^{-k}$

$= H_M(z)$, where $H(z)$ is given

From this, we have : $\boxed{c_M = b_M}$

This is the algorithm to obtain all c_is (pseudocode):

```
c_M = b_M
for i = M to 2   // descending
    compute H~_{i-1}(z) and H_{i-1}(z) from (1);
    get c_{i-1} from H_{i-1}(z);
end
```

Thus, we will have $c_1, c_2, \ldots\ldots\ldots\ldots, c_{M-1}, c_M$

7.25

$$y_1[n] = x[n] * h[n]$$
$$y_2[n] = y_1[-n] * h[n]$$
$$y[n] = y_2[-n]$$

(a) $y[n] = y_1[n] * h[n]$

$= (x[n] * h[n]) * h[-n]$

$$y[n] = x[n] * (h[n] * h[-n])$$
$$= x[n] * h_o[n]$$

$$\therefore h_o[n] = h[n] * h[-n]$$

(b) $h_o[-n] = h[-n] * h[n]$
$$= h[n] * h[-n]$$
$h_o[-n] = h_o[n]$

$\therefore$ $h_o[n]$ is an even signal
If $h_o[n]$ is even
The phase response can be found from the :

$$H_o(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} h_o[n]\, e^{-j\Omega n}$$

$$H_o^*(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} h_o[n]\, e^{j\Omega n}$$

$$= \sum_{n=-\infty}^{\infty} h_o[-n]\, e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{\infty} h_o[n]\, e^{-j\Omega n} \quad \because h_o[n] = h_o[-n]$$

$$H_o^*(e^{j\Omega}) = H_o(e^{j\Omega})$$

$$\therefore \measuredangle H_o(e^{j\Omega}) = 0$$

(c) $h_o[n] = h[n] * h[-n]$, so $H_o(z) = H(z) \cdot H(z^{-1})$

Let $H(z) = \dfrac{z - c}{z - \beta}$, then $H_o(z) = \dfrac{z - c}{z - \beta} \cdot \dfrac{z^{-1} - c}{z^{-1} - \beta}$

$$H_0(z) = \frac{(z-c)\ (1-cz)}{(z-\beta)(\frac{1}{\beta}-z)\beta}$$

$\therefore$ $H_0(z)$ has a pair of poles at $z=\beta$ and $\frac{1}{\beta}$

Let $H(z) = \dfrac{z-\beta}{z-p}$, then $H_0(z) = \dfrac{z-\beta}{z-p} \cdot \dfrac{z^{-1}-\beta}{z^{-1}-p}$

$$H_0(z) = \frac{z-\beta}{z-p} \cdot \frac{z-\frac{1}{\beta}}{z-\frac{1}{p}} \cdot \frac{\beta}{p}$$

$\therefore$ $H_0(z)$ has a pair of zeros at $z=\beta$ and $\frac{1}{\beta}$

$\boxed{7.26}$ $x[n] = \delta[n-2] + \delta[n+2]$

(a) $X(z) = z^{-2} + 0$
$\quad = z^{-2}$

(b) i) $w[n] = x[n-1]$
$W(z) = z^{-1} X(z)$
$\quad = z^{-3}$

ii) $w[n] = x[n-3]$
$W(z) = z^{-3} X(z) + z^{-1}(1)$
$\quad = z^{-5} + z^{-1}$

$\boxed{7.27}$

(a) $y[n] - \frac{1}{2} y[n-1] = 2x[n]$
$y[-1] = 3$, $x[n] = 2(-\frac{1}{2})^n u[n]$

$$Y(z) - \frac{1}{2}(z^{-1}Y(z) + 3) = 2X(z)$$

$$Y(z)\left(1-\frac{1}{2}z^{-1}\right) = \frac{3}{2} + \frac{4}{1+\frac{1}{2}z^{-1}} = \frac{11+\frac{3}{2}z^{-1}}{2\left(1+\frac{1}{2}z^{-1}\right)}$$

$$Y(z) = \frac{\frac{1}{2}\left(11+\frac{3}{2}z^{-1}\right)}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)}$$

$$= \frac{\frac{7}{2}}{1-\frac{1}{2}z^{-1}} + \frac{2}{1+\frac{1}{2}z^{-1}}$$

$$\therefore\ y[n] = \left(\frac{7}{2}\left(\frac{1}{2}\right)^n + 2\left(-\frac{1}{2}\right)^n\right)u[n]$$

(b) $y[n] - \frac{1}{9}\,y[n-2] = x[n-1]$

$y[-1] = 0$
$y[-2] = 1$
$x[n] = 3\,u[n]$

$$Y(z) - \frac{1}{9}\left(z^{-2}\,Y(z) + 1\right) = z^{-1}X(z) = \frac{3z^{-1}}{1-z^{-1}}$$

$$Y(z)\left(1-\frac{1}{9}z^{-2}\right) = \frac{1}{9} + \frac{3z^{-1}}{1-z^{-1}} = \frac{\frac{1}{9}\left(1+26z^{-1}\right)}{1-z^{-1}}$$

$$Y(z) = \frac{\frac{1}{9}\left(1+26z^{-1}\right)}{\left(1-\frac{1}{3}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)\left(1-z^{-1}\right)}$$

$$= \frac{-\frac{79}{36}}{1-\frac{1}{3}z^{-1}} + \frac{-\frac{77}{72}}{1+\frac{1}{3}z^{-1}} + \frac{\frac{27}{8}}{1-z^{-1}}$$

$$\therefore\ y[n] = \left(-\frac{79}{36}\left(\frac{1}{3}\right)^n - \frac{77}{72}\left(-\frac{1}{3}\right)^n + \frac{27}{8}\right)u[n]$$

(c) $y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n] + x[n-1]$

$y[-1] = 1$
$y[-2] = -1$
$x[n] = 3^n u[n]$

$$Y(z) = -\frac{1}{4}\left(z^{-1}Y(z)+1\right) - \frac{1}{8}\left(z^{-2}Y(z)+z^{-1}-1\right)$$

$$= X(z) + z^{-1}X(z) \quad , \quad X_z = \frac{1}{1-3z^{-1}}$$

$$\left(1-\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}\right)Y(z) = \frac{1}{8} + \frac{1}{8}z^{-1} + (1+z^{-1})\frac{1}{1-3z^{-1}}$$

$$= \frac{(1+z^{-1})\left(\frac{9}{8}-\frac{3}{8}z^{-1}\right)}{1-3z^{-1}}$$

$$Y(z) = \frac{\frac{3}{8}(1+z^{-1})(3-z^{-1})}{(1-3z^{-1})(1-\frac{1}{2}z^{-1})(1+\frac{1}{4}z^{-1})}$$

$$= \frac{\frac{96}{65}}{1-3z^{-1}} + \frac{-\frac{3}{20}}{1-\frac{1}{2}z^{-1}} + \frac{-\frac{21}{104}}{1+\frac{1}{4}z^{-1}}$$

$$\therefore y[n] = \left(\frac{96}{65}(3)^n - \frac{3}{20}\left(\frac{1}{2}\right)^n - \frac{21}{104}\left(-\frac{1}{4}\right)^n\right)u[n]$$

(d) $y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n]$

$y[-1] = 1$
$y[-2] = -1$
$x[n] = u[n] \quad \rightarrow \quad X(z) = \frac{1}{1-z^{-1}}$

$$Y(z) - \frac{3}{4}\left(z^{-1}Y(z) + 1\right) + \frac{1}{8}\left(z^{-2}Y(z) + z^{-1} - 1\right) = 2 \cdot \frac{1}{1 - z^{-1}}$$

$$Y(z)\left(1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right) = \frac{2}{1 - z^{-1}} + \frac{1}{8}\left(7 - z^{-1}\right)$$

$$Y(z) = \frac{\frac{1}{8}\left(23 - 8z^{-1} + z^{-2}\right)}{\left(1 - z^{-1}\right)\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)}$$

$$= \frac{\frac{16}{3}}{1 - z^{-1}} + \frac{-\frac{11}{4}}{1 - \frac{1}{2}z^{-1}} + \frac{\frac{7}{24}}{1 - \frac{1}{4}z^{-1}}$$

$$\therefore y[n] = \left(\frac{16}{3} - \frac{11}{4}\left(\frac{1}{2}\right)^n + \frac{7}{24}\left(\frac{1}{4}\right)^n\right)u[n]$$

$\boxed{7.28}$

(a) $y[n] + \frac{1}{2}y[n-1] = 2x[n]$

$y[-1] = 0$,
$y[n] = h[n]$,
$x[n] = \delta[n]$

$$H(z) + \frac{1}{2}\left(H(z)z^{-1}\right) = 2$$

$$H(z) = \frac{2}{1 + \frac{1}{2}z^{-1}}$$

$$h[n] = 2\left(-\frac{1}{2}\right)^n u[n]$$

(b) $y[n] - \frac{1}{4}y[n-2] = x[n-1]$

$y[-1] = y[-2] = 0$
$x[n] = \delta[n]$
$y[n] = h[n]$

$$H(z)\left(1-\frac{1}{4}z^{-2}\right)=z^{-1}$$

$$H(z)=\frac{z^{-1}}{1-\frac{1}{4}z^{-2}}$$

$$=\frac{1}{1-\frac{1}{2}z^{-1}}+\frac{-1}{1+\frac{1}{2}z^{-1}}$$

$$\therefore h[n]=\left(\left(\tfrac{1}{2}\right)^n-\left(-\tfrac{1}{2}\right)^n\right)u[n]$$

(c) $y[n]+y[n-1]+\frac{1}{4}y[n-2]=x[n]-x[n-1]$

$$H(z)\left(1+z^{-1}+\frac{1}{4}z^{-2}\right)=1-z^{-1}$$

$$H(z)=\frac{1-z^{-1}}{1+z^{-1}+\frac{1}{4}z^{-2}}$$

$$=\frac{\left(1+\frac{1}{2}z^{-1}\right)-\frac{3}{2}z^{-1}}{\left(1+\frac{1}{2}z^{-1}\right)^2}$$

$$=\frac{1}{1+\frac{1}{2}z^{-1}}+\frac{3\left(-\frac{1}{2}\right)z^{-1}}{\left(1+\frac{1}{2}z^{-1}\right)^2}$$

$$\therefore h[n]=\left(-\tfrac{1}{2}\right)^n(1+3n)\,u[n]$$

$\boxed{7.29}$ $y[n]=\left(1+\frac{r}{12}\right)y[n-1]-x[n]$

$$\xleftrightarrow{z}\ Y(z)\left[1-\left(1+\frac{r}{12}\right)z^{-1}\right]=\left(1+\frac{r}{12}\right)y[-1]-X(z)$$

$$x[n]=c\{u[n]-u[n-L]\}$$

$$\xleftrightarrow{z} X(z) = \frac{c(1-z^{-L})}{1-z^{-1}} \quad , \quad \text{ROC } |z|>1$$

(a) By inspection, the impulse response :

$$H(z) = \frac{-1}{1-(1+\frac{r}{12})z^{-1}}$$

The system is causal since $h[n]=0$ for $n<0$

Poles : $z_p = 1+\frac{r}{12} > 1$ since $r>0$

$\therefore$ The system is <u>not stable</u> since z_p is outside the unit circle

(b) Natural response : $x[n]=0 \Rightarrow X(z)=0$

$$y_n(z) = \frac{(1+\frac{r}{12})\,y[-1]}{1-(1+\frac{r}{12})z^{-1}}$$

$$y_n[n] = \left(1+\frac{r}{12}\right) y[-1] \left(1+\frac{r}{12}\right)^n u[n]$$

as $n \longrightarrow \infty$, $y_n[n] \longrightarrow \infty$, an indication of instability

(c) $$y(z) = \frac{(1+\frac{r}{12})\,y[-1] - X(z)}{1-(1+\frac{r}{12})z^{-1}}$$

$$X(z) = c \cdot \frac{1-z^{-L}}{1-z^{-1}}$$

$$= c\left(1+z^{-1}+z^{-2}+\ldots\ldots+z^{-L+2}+z^{-L+1}\right)$$

$$X(z) = c \cdot \sum_{n=0}^{L-1} z^{-n}$$

(can also be shown by geometric series sum)

$$\therefore\ y(z) = \frac{y[-1]\left(1+\frac{r}{12}\right) - c \cdot \sum_{n=0}^{L-1} z^{-n}}{1-\left(1+\frac{r}{12}\right) z^{-1}}$$

(d)
$$y(z) = \frac{y[-1]k - c\sum_{n=0}^{L-1} z^{-n}}{1-kz^{-1}}$$

$$= \sum_{n=0}^{L-1} y[n]z^{-n} + \sum_{n=L}^{\infty} y[n]z^{-n}, \quad k = 1+\frac{r}{12}$$

multiply both sides with z^{L-2} and simplify :

$$\frac{-c\sum_{n=0}^{L-2} z^{n} + \left(y[-1]k-c\right)z^{L-1}}{z-k} = \underbrace{\sum_{n=0}^{L-2} y[n]\,z^{(L-2-n)}}_{\text{first term (quotient)}} + \underbrace{\sum_{n=L-1}^{\infty} y[n]z^{(L-2-n)}}_{\text{second term (residual)}}$$

Zero balance after L payments :
$y[n] = 0$ for $n \geqslant L-1$

$$\Rightarrow \frac{-c\sum_{n=0}^{L-2} z^{n} + \left(y[-1]k-c\right)z^{L-1}}{z-k} = \sum_{n=0}^{L-2} y[n]z^{(L-2-n)} + 0$$

Let $f(z) = -c \sum_{n=0}^{L-2} z^n + \left(y[-1]k - c\right) z^{L-1}$

From polynomial theory, the second term is zero if $f(z=k)=0$, or $k = 1+\frac{r}{12}$ is a zero of $y(z)$.

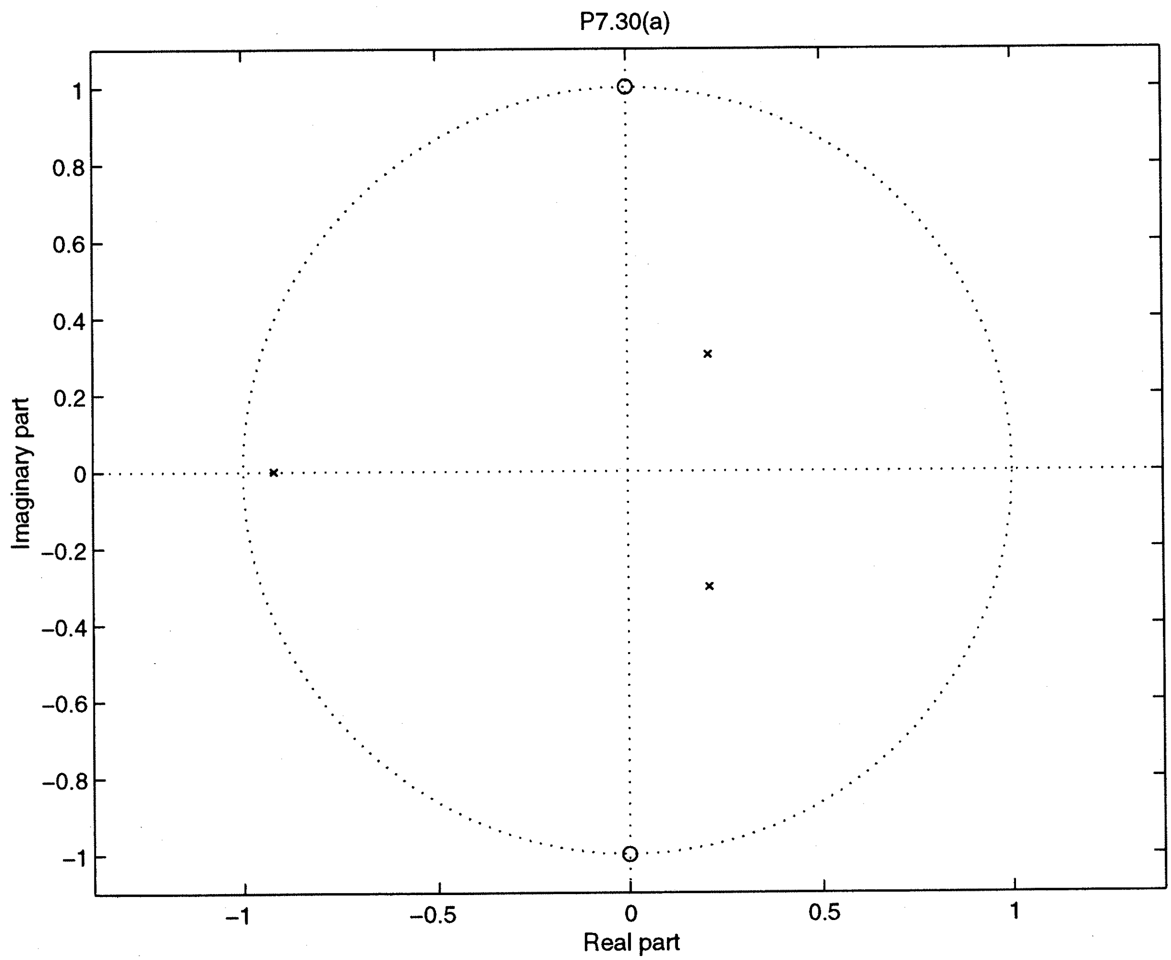
(e) $y[-1]\left(1+\frac{r}{12}\right) - c \sum_{n=0}^{L-1} \left(1+\frac{r}{12}\right)^{-n} = 0$

$$c \cdot \frac{1-\left(1+\frac{r}{12}\right)^{-L}}{1-\left(1+\frac{r}{12}\right)^{-1}} = y[-1]\left(1+\frac{r}{12}\right)$$

$$c = y[-1] \frac{\frac{r}{12}}{1-\left(1+\frac{r}{12}\right)^{-L}}$$

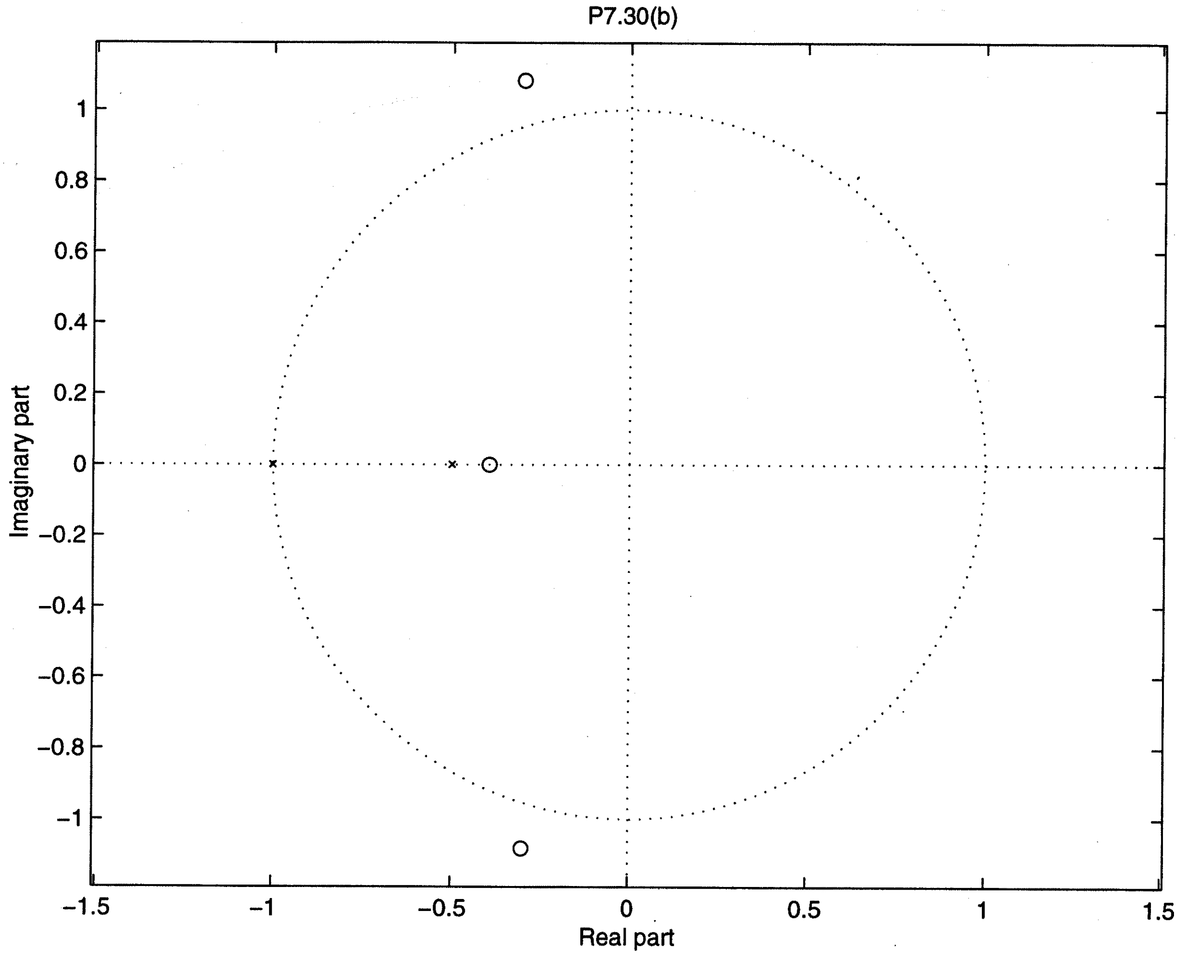
P 7.30

– Plot 1 of 2 –



P 7.30

– Plot 2 of 2 –



$\boxed{7.31}$

(d) $$X(z) = \frac{1-3z^{-1}}{1+\frac{3}{2}z^{-1}-z^{-2}} = \frac{2}{1+2z^{-1}} - \frac{1}{1-\frac{1}{2}z^{-1}}$$

(e) $$X(z) = \frac{11-3z^{-1}}{1-\frac{7}{12}z^{-1}+\frac{z^{-2}}{12}} = \frac{8}{1-\frac{1}{3}z^{-1}} + \frac{3}{1-\frac{1}{4}z^{-1}}$$

(f) $$X(z) = \frac{2+z^{-1}}{1-z^{-1}+\frac{1}{4}z^{-2}} = \frac{2(1+\frac{1}{2}z^{-1})}{(1-\frac{1}{2}z^{-1})^2} = \frac{2}{1-\frac{1}{2}z^{-1}} + \frac{2z^{-1}}{(1-\frac{1}{2}z^{-1})^2}$$

(g) $$X(z) = \frac{1+z^{-1}+\frac{3}{2}z^{-2}+\frac{1}{2}z^{-3}}{1+\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2}} = \frac{2}{1+z^{-1}} - \frac{1}{1+\frac{1}{2}z^{-1}} + z^{-1}$$

(h) $$X(z) = z\,\frac{1+z^{-1}+\frac{3}{2}z^{-2}+\frac{1}{2}z^{-3}}{1+\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2}} = z\left(\frac{2}{1+z^{-1}} - \frac{1}{1+\frac{1}{2}z^{-1}} + z^{-1}\right)$$

(i) $$X(z) = 2z^2\,\frac{1-z^{-1}-z^{-2}}{1-z^{-2}} = z^2\left(\frac{1}{1+z^{-1}} - \frac{1}{1-z^{-1}} + 2\right)$$

```
P7.31 :
=======
Part (d) :
==========
r =
     2
    -1

p =
   -2.0000
    0.5000

k =
    []

Part (e) :
==========
r =
    8.0000
    3.0000

p =
    0.3333
    0.2500

k =
    []

Part (f) :
==========
r =
   -2.0000
    4.0000

p =
    0.5000
    0.5000

k =
    []

Part (g) :
==========
r =
     2
    -1

p =
   -1.0000
   -0.5000

k =
     0     1

Part (h) :
==========
r =
     2
    -1

p =
   -1.0000
   -0.5000

k =
     0     1

Part (i) :
==========
r =
    0.5000
   -0.5000

p =
   -1.0000
    1.0000

k =
     1     0     0
```

7.32

```
P7.32 :
=======
Part (a) :
==========
A =
    0.5000

B =
     1

C =
     2

D =
     0

Part (b) :
==========
A =

     0     0     0     0     0     0
     1     0     0     0     0     0
     0     1     0     0     0     0
     0     0     1     0     0     0
     0     0     0     1     0     0
     0     0     0     0     1     0

B =
     1
     0
     0
     0
     0
     0

C =
     0    -1     0     1     0    -1

D =
     1

Part (c) :
==========
A =

    0.2500    0.3750
    1.0000         0

B =
     1
     0

C =
    -1     2

D =
     0

Part (d) :
==========


A =
    0.8000    0.6400
    1.0000         0

B =
     1
     0

C =
     2     1

D =
     0
```

$\boxed{7.33}$

(a) $$T(z) = \frac{1 - 2z^{-1} - 1.25z^{-2}}{1 - 0.25z^{-2}}$$

(b) $$T(z) = \frac{2z^{-1}}{1 - \frac{1}{4}z^{-1} - 0.375z^{-2}}$$

(c) $$T(z) = \frac{2z^{-1} - 6.5z^{-2}}{1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}}$$

```
P7.33 :

=======
Part (a) :
==========
Num =
    1.0000   -2.0000   -1.2500

Den =
    1.0000         0   -0.2500

Part (b) :
==========
Num =
     0     2     0

Den =
    1.0000   -0.2500   -0.3750

Part (c) :
==========
Num =
         0    2.0000   -6.5000

Den =
    1.0000   -0.5000    0.2500
```

,

$\boxed{7.34}$

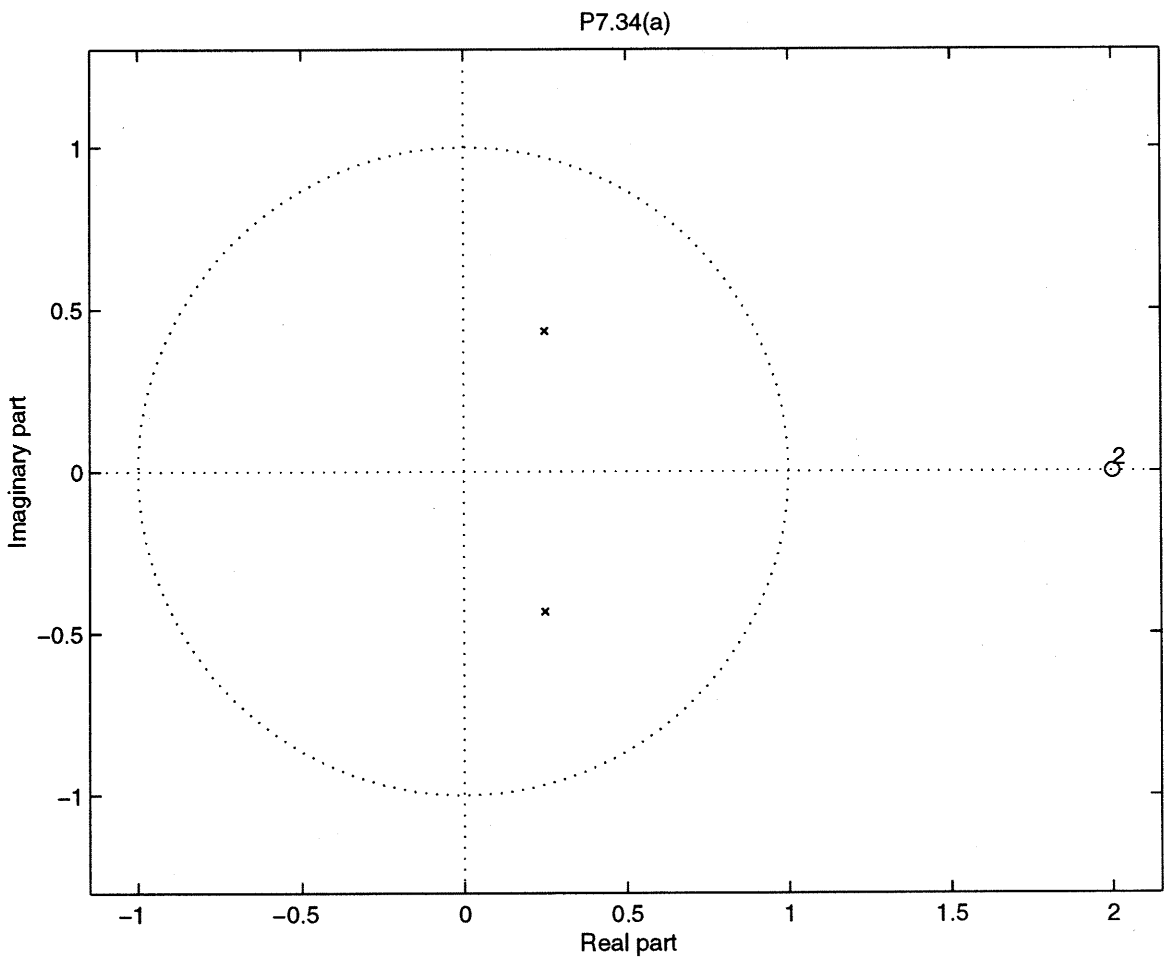
- If all poles are inside $|z| = 1$, the system can be both causal and stable, otherwise not

- If all zeros are inside $|z| = 1$, the system is minimum phase, otherwise not

(a) both causal and stable ; NOT minimum phase
(b) both causal and stable ; NOT minimum phase
(c) NOT both causal and stable ; minimum phase
(d) NOT both causal and stable ; minimum phase
(e) both causal and stable ; minimum phase

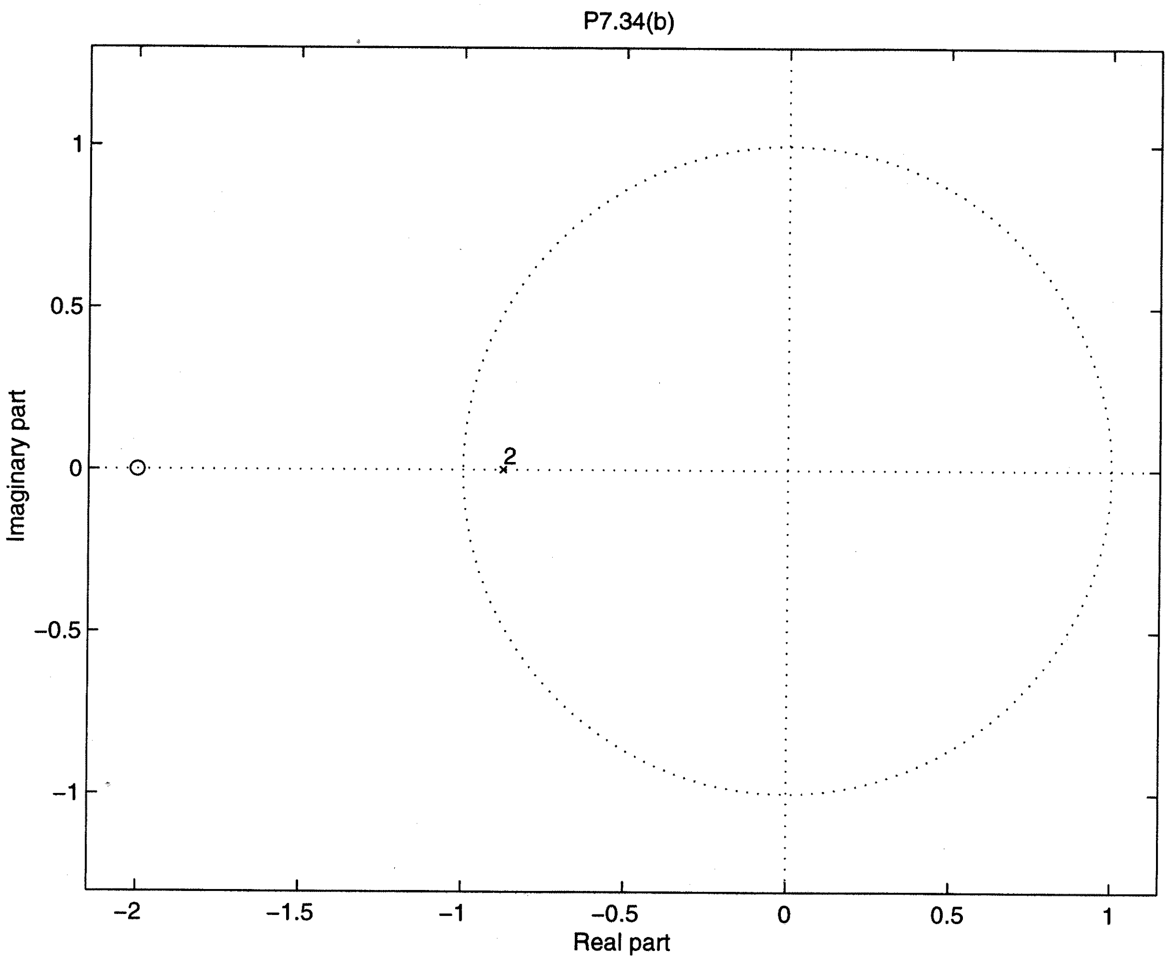
P 7.34

- Plot 1 of 5 -



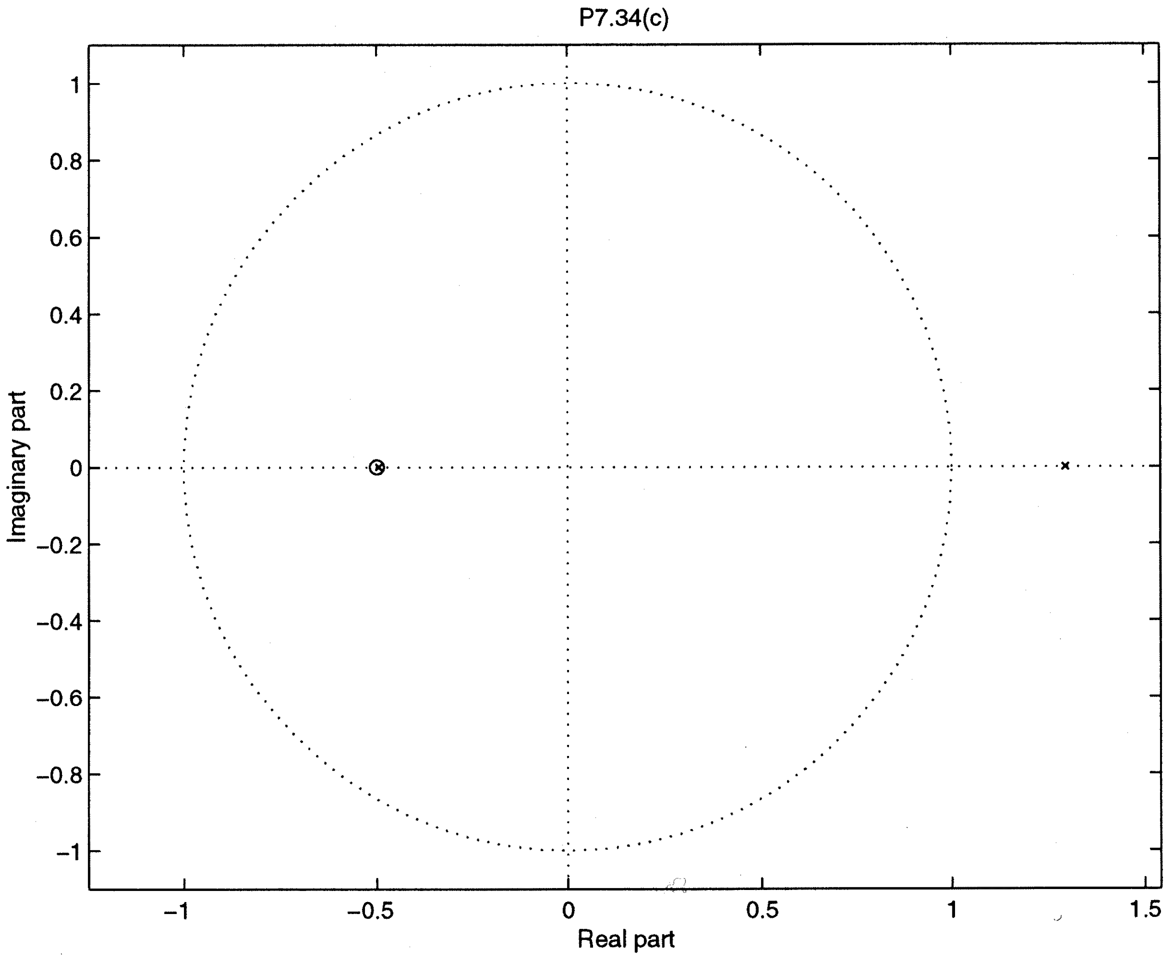
P7.34

- Plot 2 of 5 -



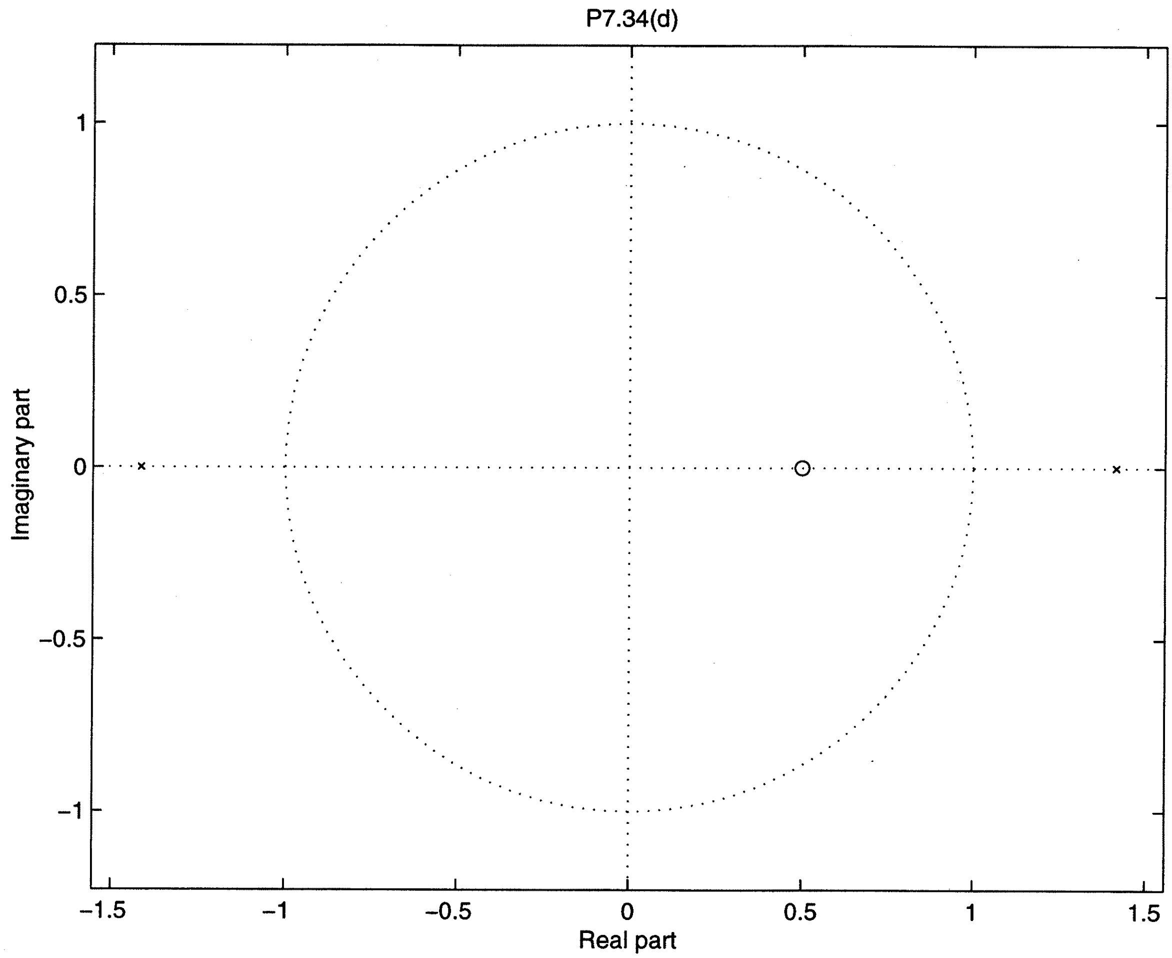
P 7.34

- Plot 3 of 5 -



P 7.34

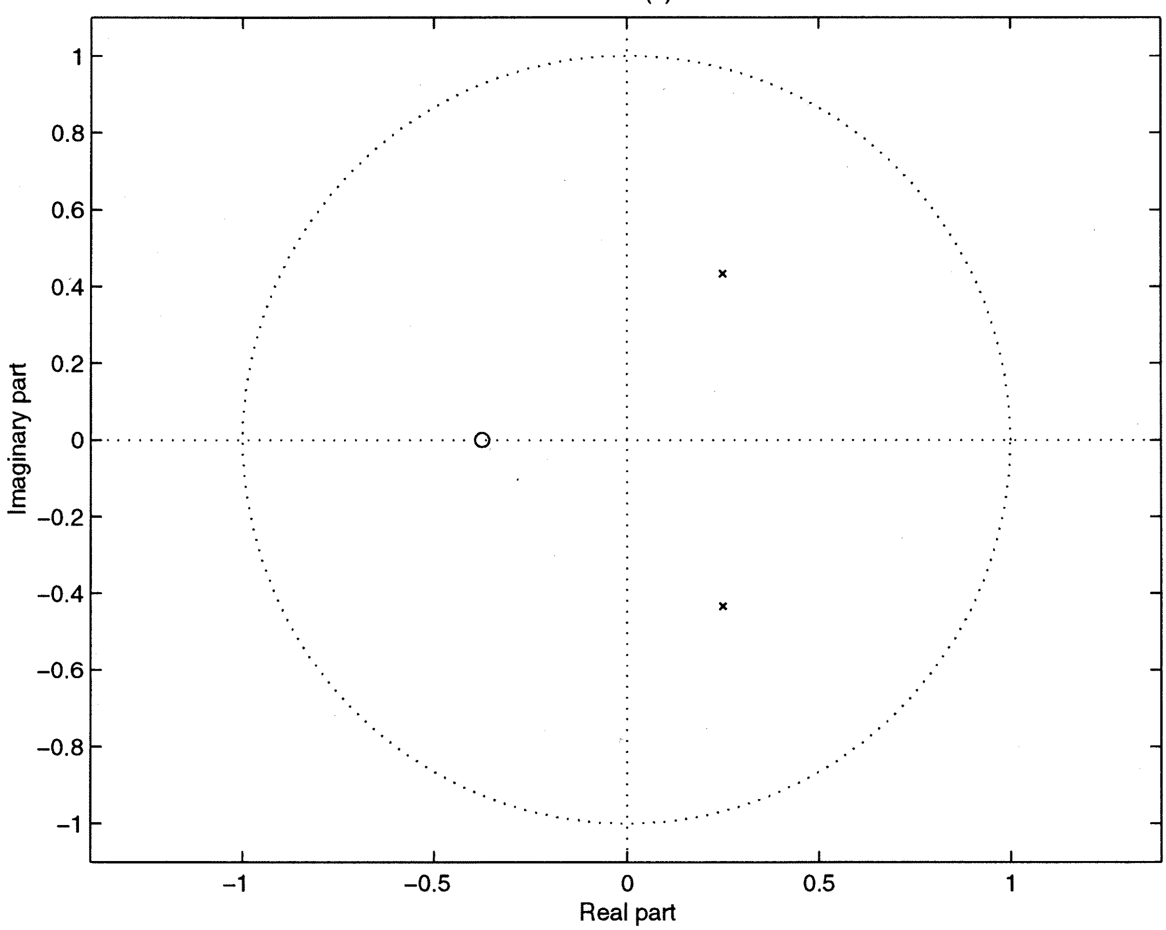
- Plot 4 of 5 -



P 7.34

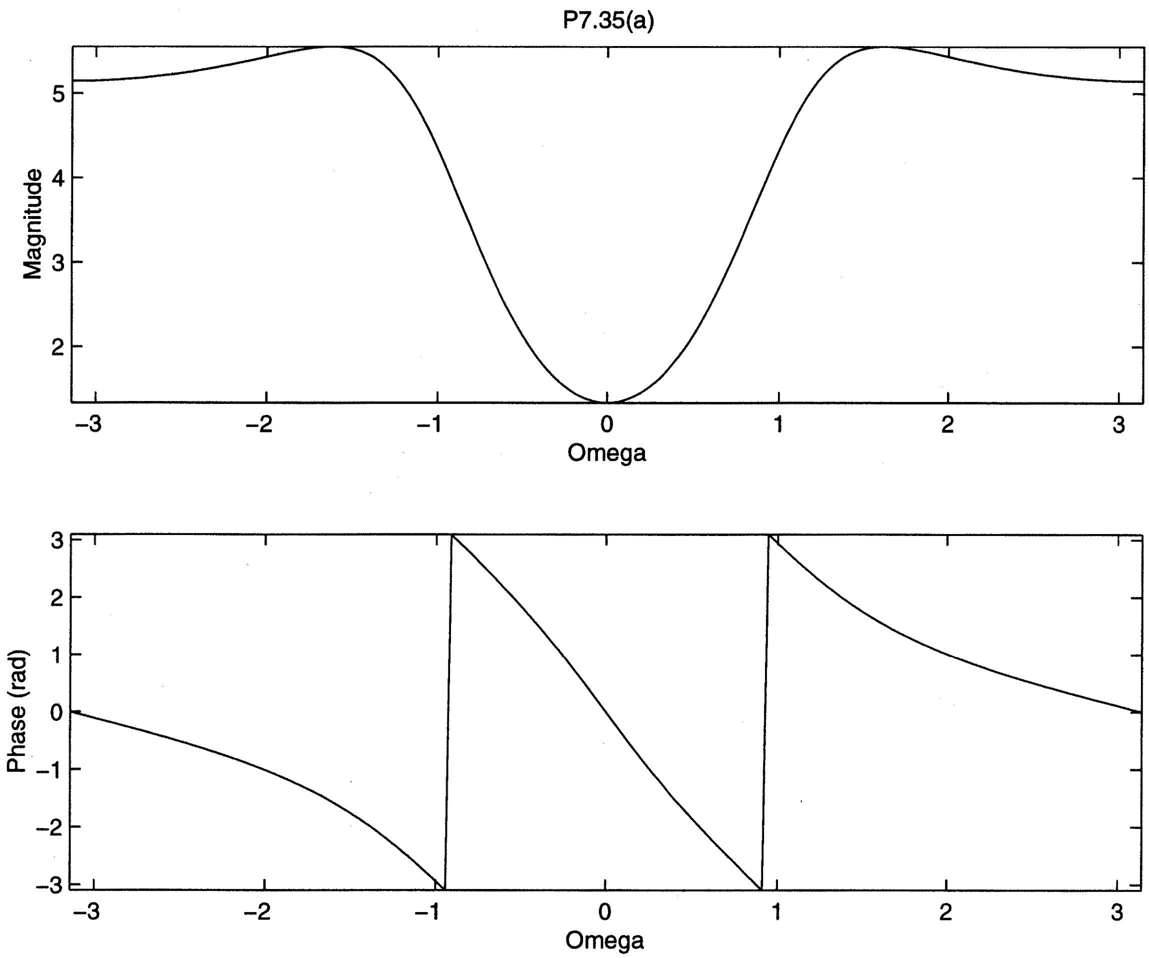
-Plot 5 of 5 -

P7.34(e)



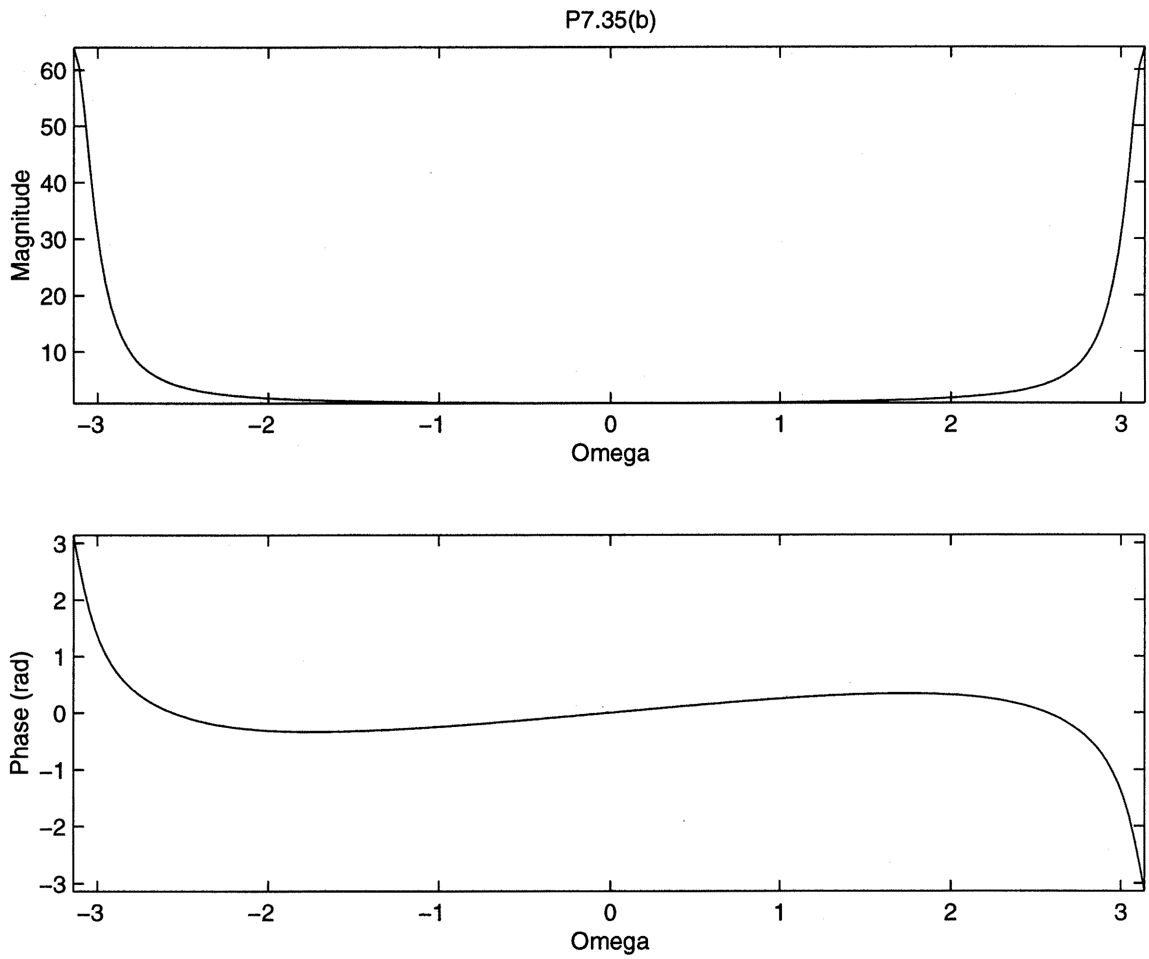
P 7.35

– Plot 1 of 5 –



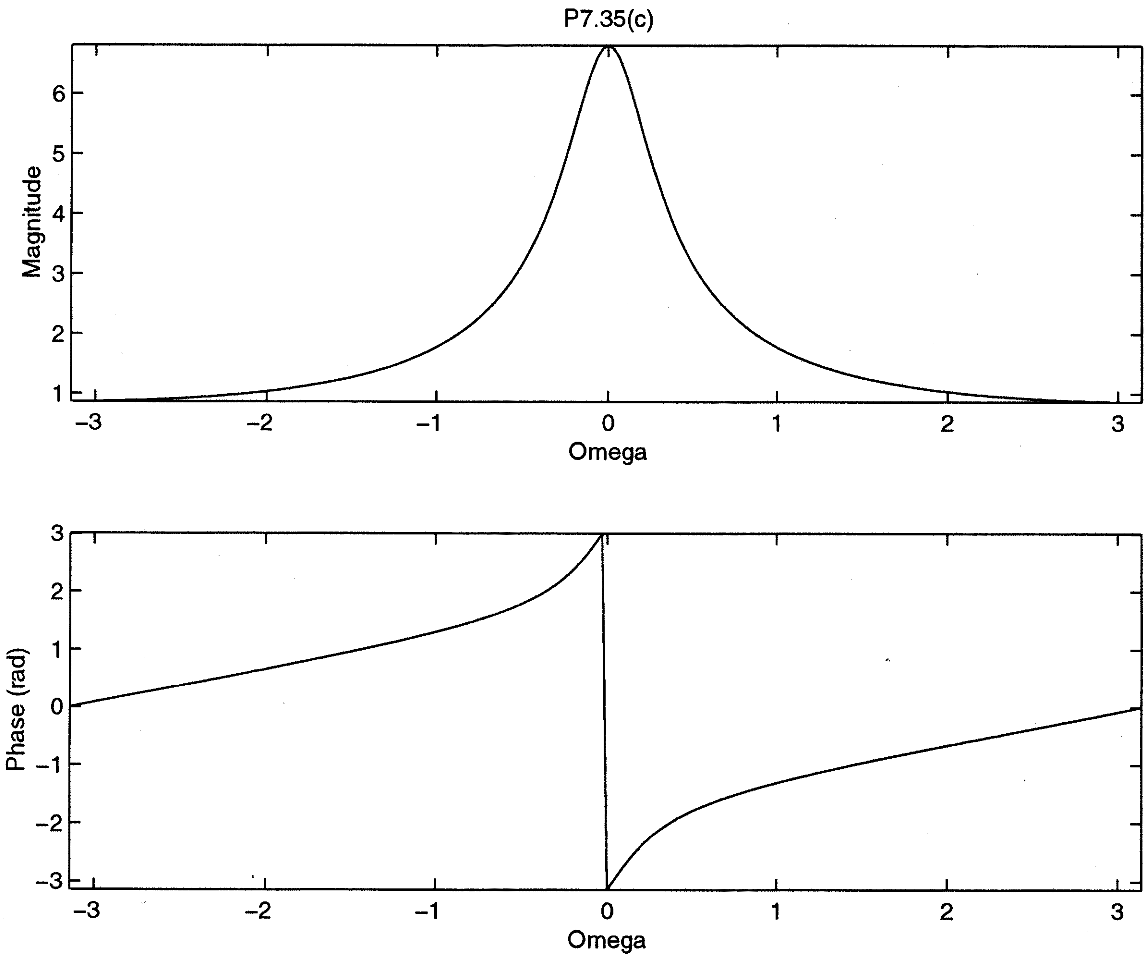
P 7.35

- Plot 2 of 5 -



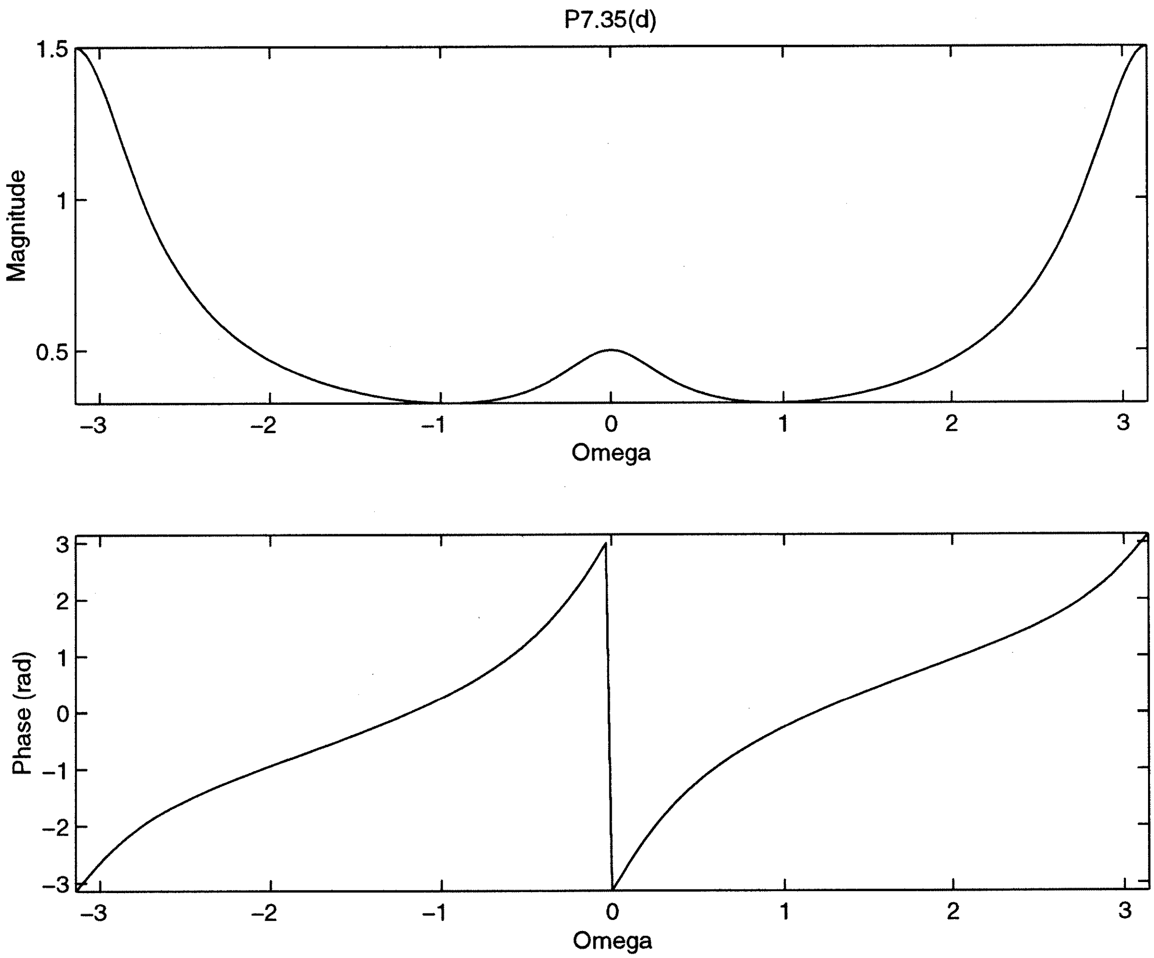
P 7.35

– Plot 3 of 5 –



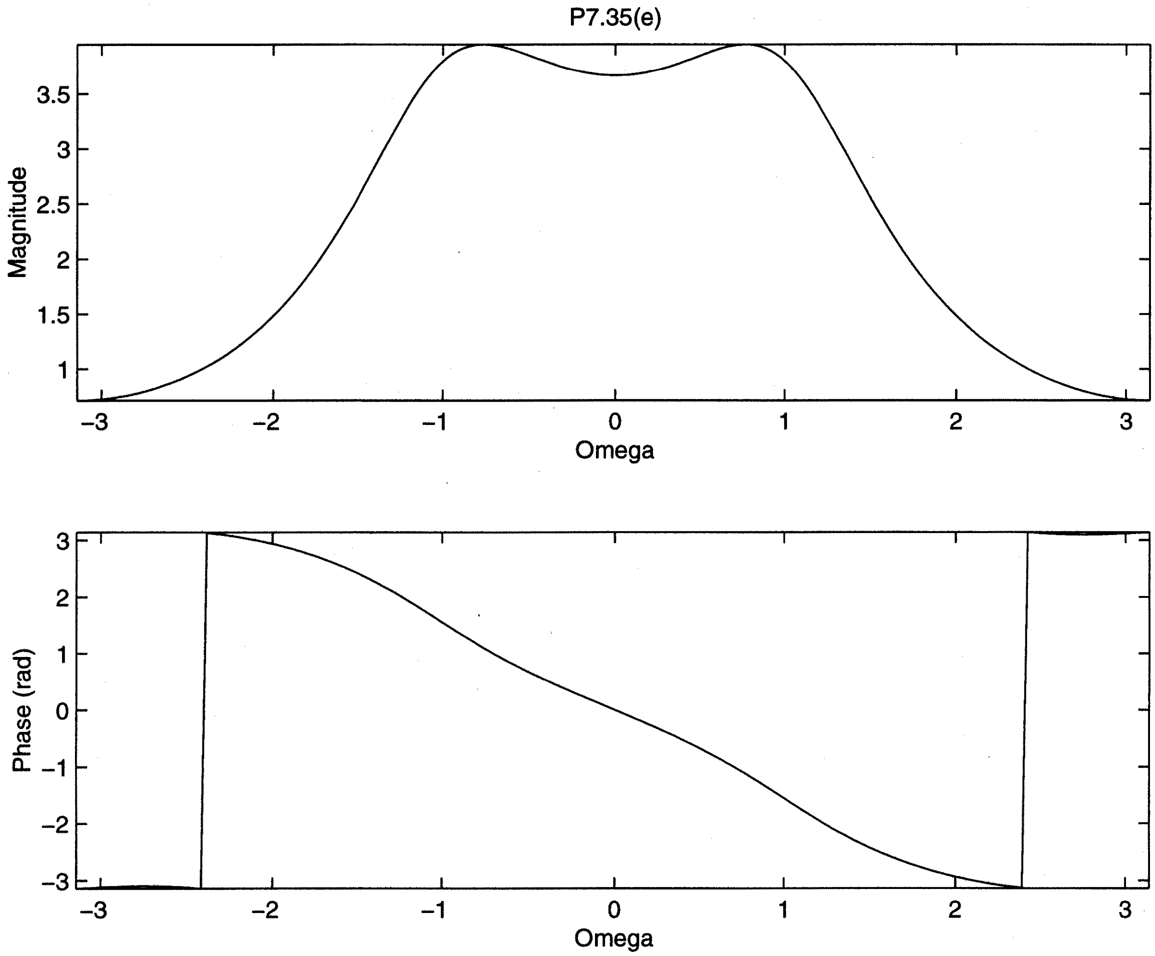
P 7.35

- Plot 4 of 5 -



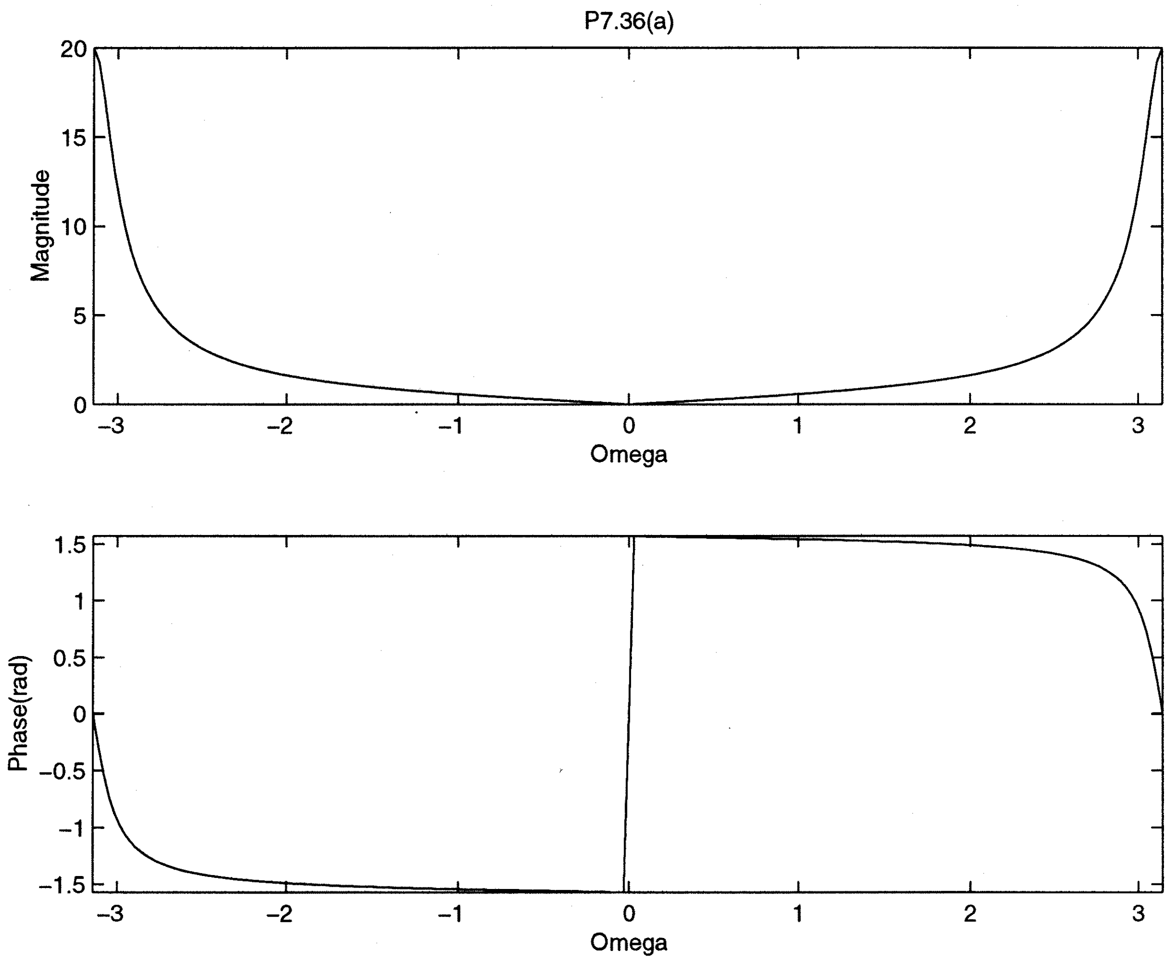
P 7.35

– Plot 5 of 5 –



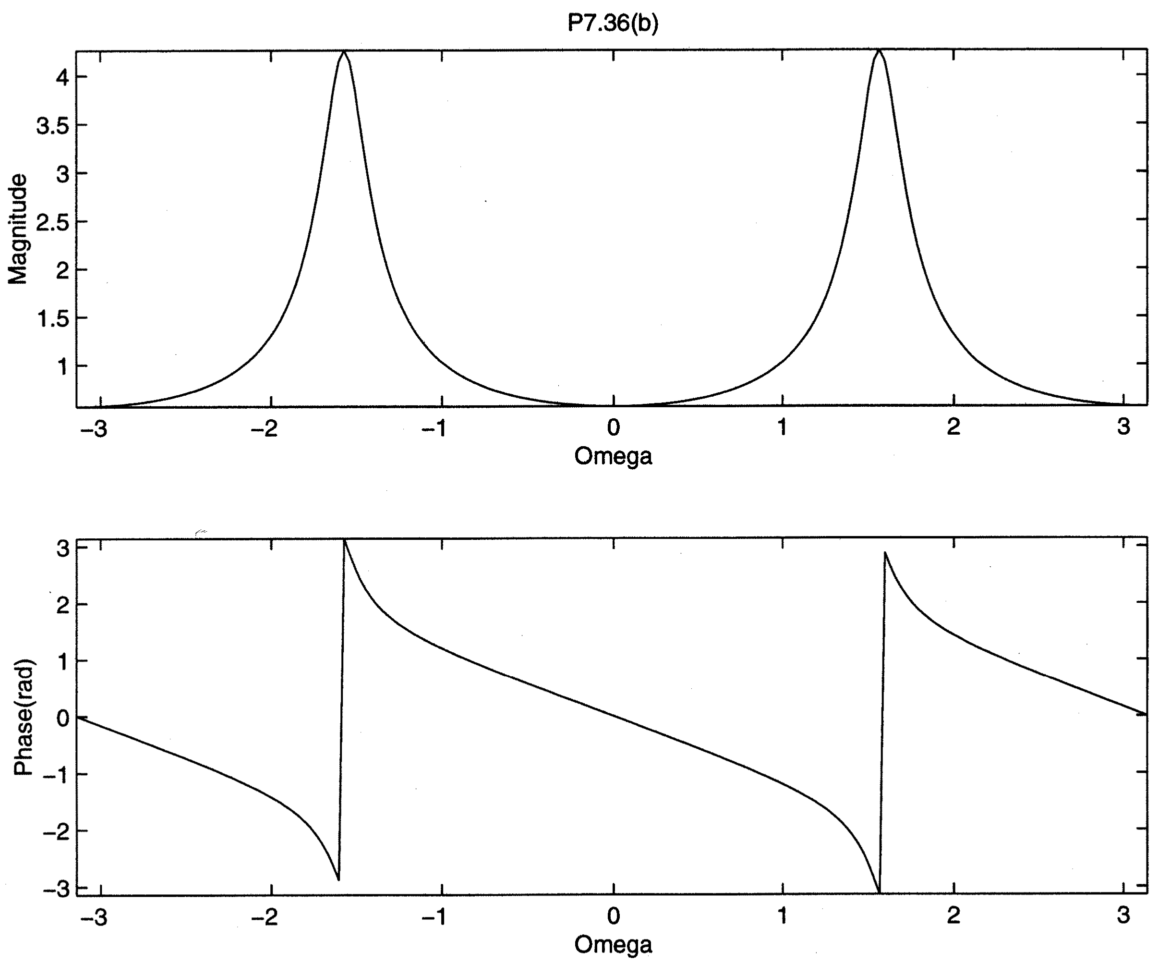
P 7.36

- Plot 1 of 3 -



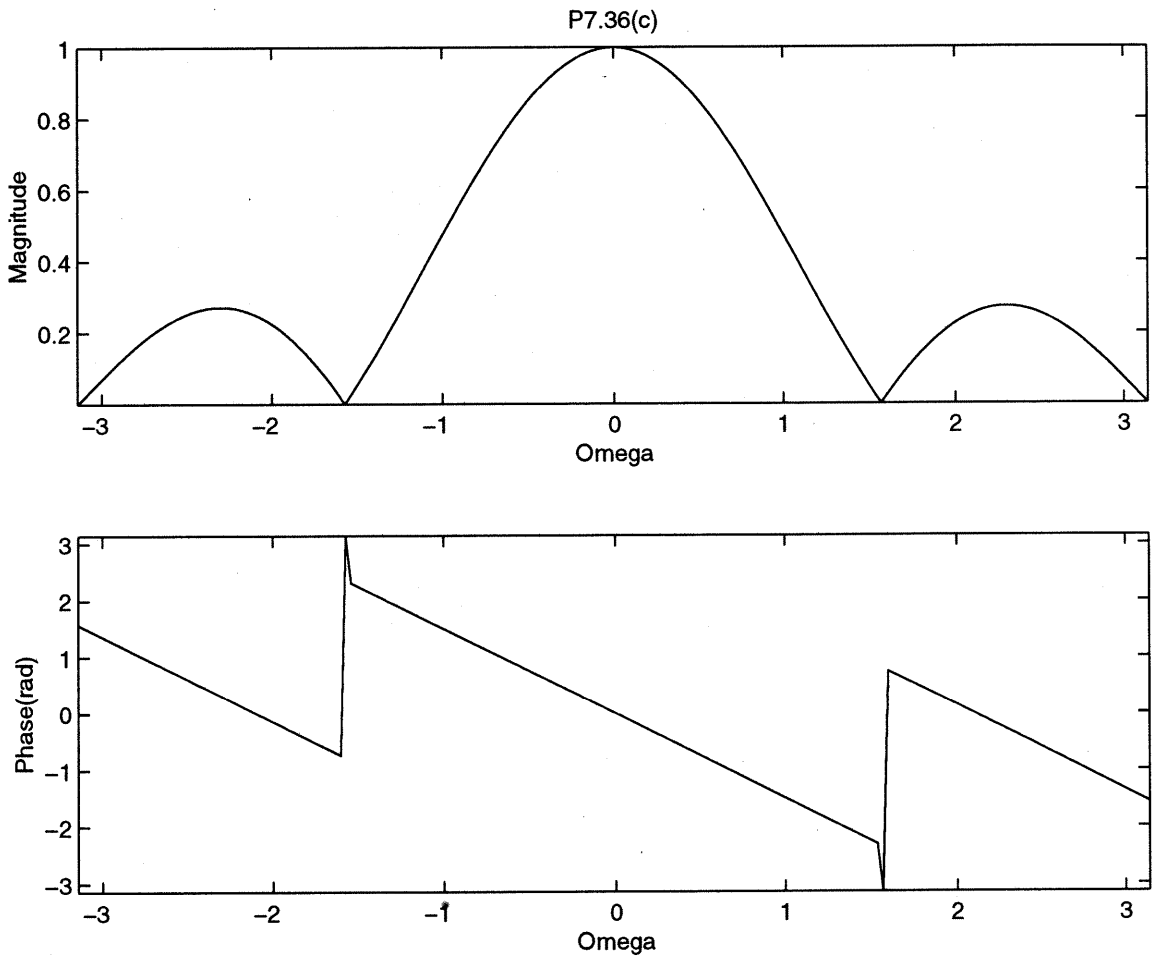
P 7.36

– Plot 2 of 3 –



P 7.36

- Plot 3 of 3



$\boxed{7.37}$ From problem 7.29 :

$$c = y[-1] \frac{\frac{r}{12}}{1-(1+\frac{r}{12})^{-L}}$$

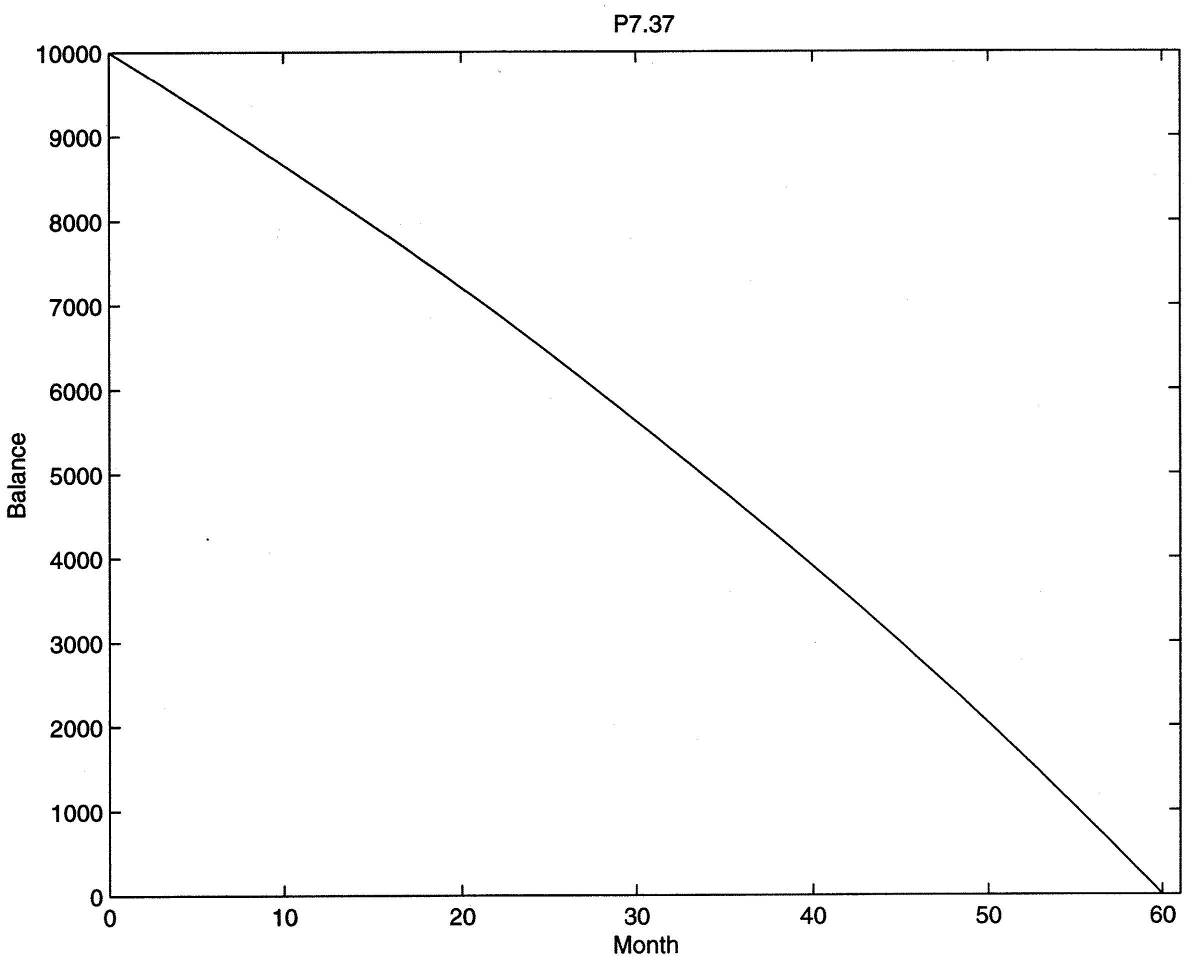
$y[-1] = 10{,}000$
$L = 60$
choose $r = 0.1$

Here $\underline{b} = [y[-1](1+\frac{r}{12}) - c \quad . - c * \text{ones}(1,59)]$

$\underline{a} = [1 - (1+\frac{r}{12})]$

P 7.37

– Plot 1 of 1 –

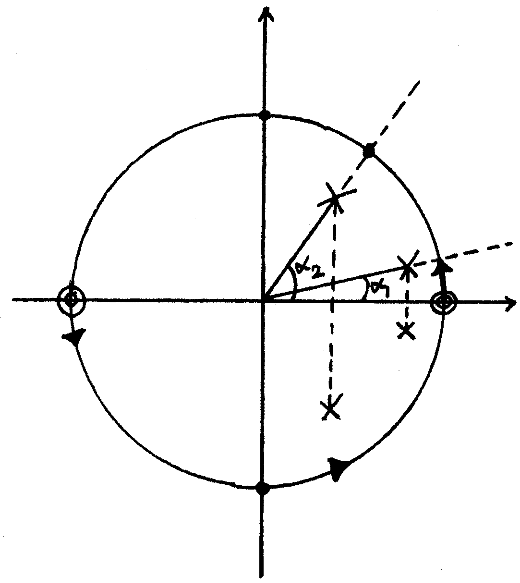


7.38

```
P7.38 :
=======

Part (a) :
==========

sos =

    0.4348   -0.2174    0.1087    1.0000   -0.9239    0.2500
    2.2998    1.1499    0.5749    1.0000    1.3858    0.5625

Part (b) :
==========

sos =

    0.1094    0.2188    0.1094    1.0000         0   -0.5625
    9.1429    0.0000    2.2857    1.0000   -1.0607    0.5625
```

$\boxed{7.39}$

(a)

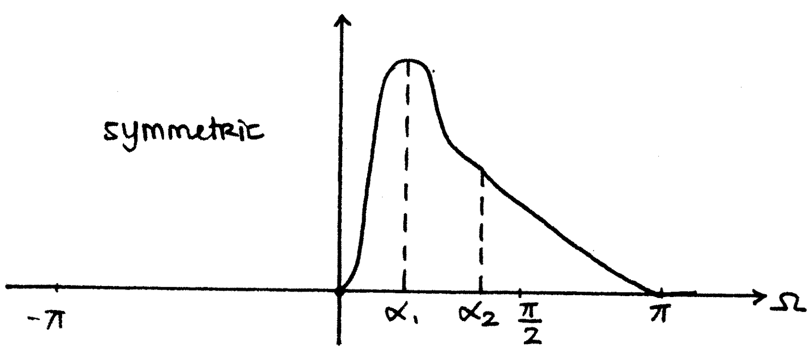


$$\alpha_1 = \tan^{-1} \frac{0.3338}{0.7686}$$

$$= 0.4097$$

$$\alpha_2 = \tan^{-1} \frac{0.5889}{0.3575}$$

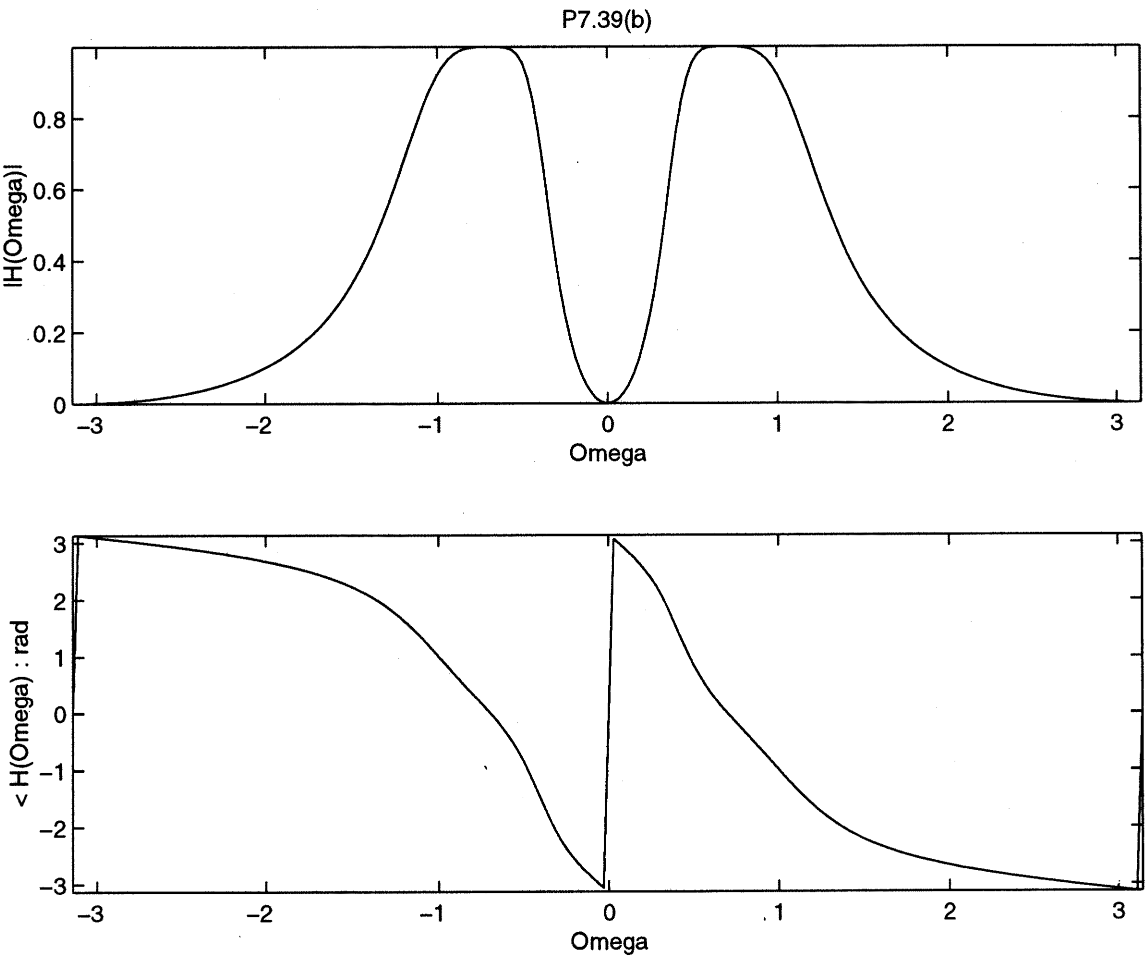
$$= 1.0252$$



$$\text{(c)} \quad H(z) = \frac{0.1413(1+2z^{-1}+z^{-2})}{1-0.715z^{-1}+0.4746z^{-2}} \cdot \frac{0.6907(1-2z^{-1}+z^{-2})}{1-1.5372z^{-1}+0.7022z^{-2}}$$

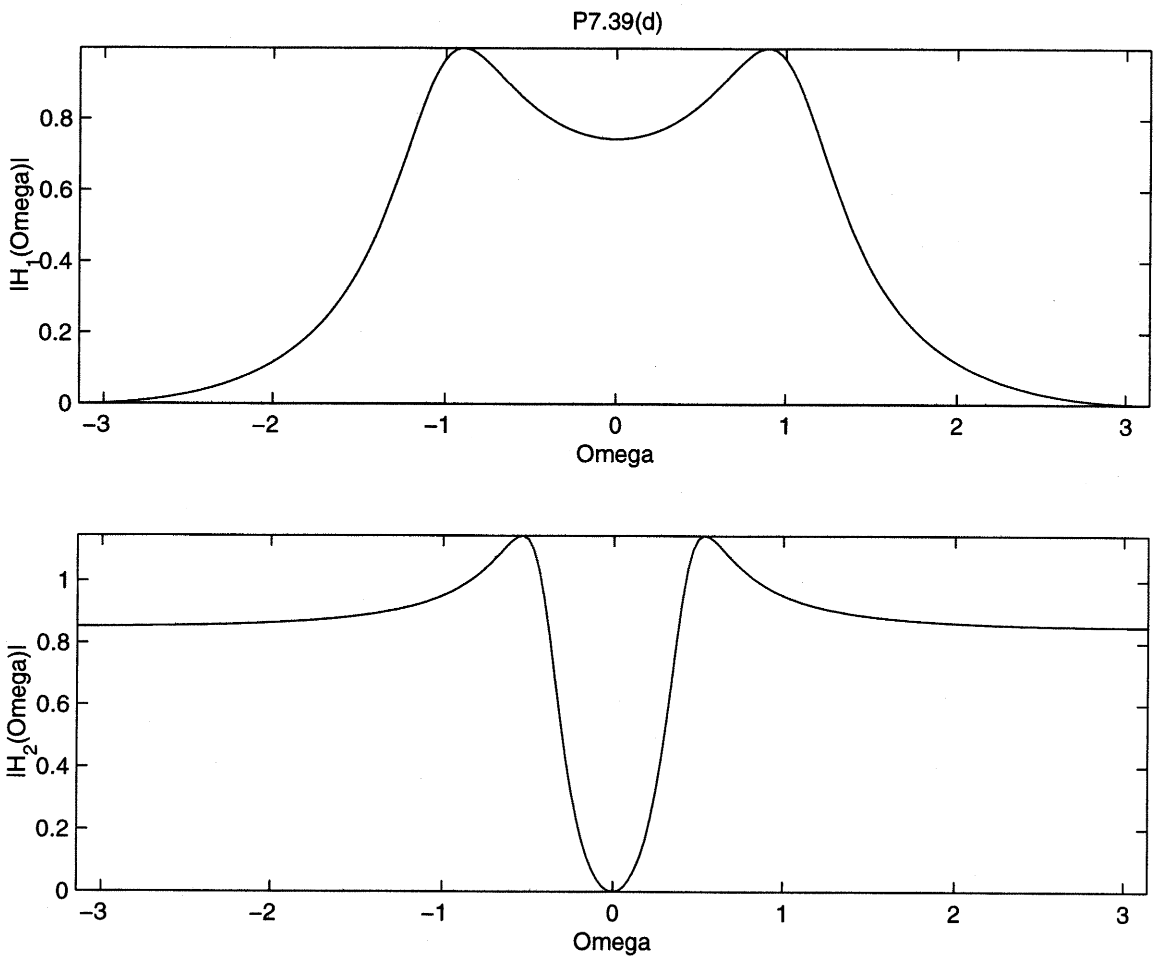
P 7.39

- Plot 1 of 3 -



P 7.39

- Plot 2 of 3 -



P 7.39

- Plot 3 of 3 -