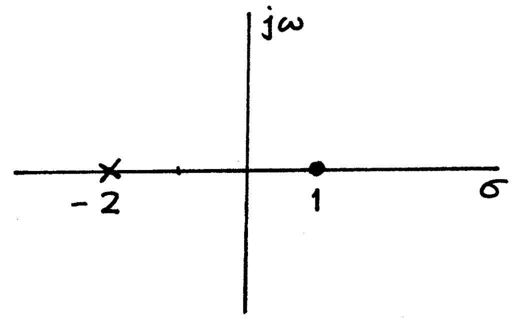
CHAPTER 6

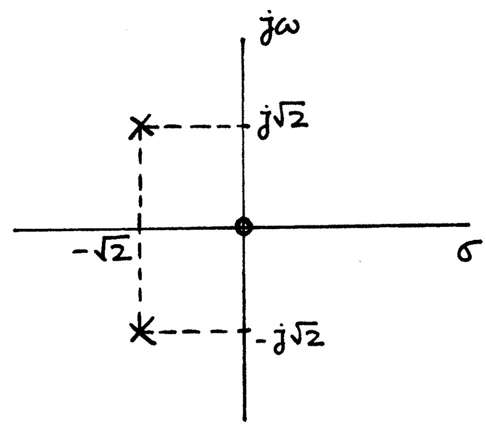
$\boxed{6.1}$

(a) $$X(s) = \frac{s^2 - 1}{s^2 + 3s^2 + 2} = \frac{(s-1)(s+1)}{(s+2)(s+1)} = \frac{s-1}{s+2}$$



$$X(j\omega) = X(s)\Big|_{s=0+j\omega} = \frac{j\omega - 1}{j\omega + 2}$$

poles : $s = -2$
zeros : $s = 1$



(b) $$X(s) = \frac{2s^2}{s^2 + 2\sqrt{2}\,s + 4} = \frac{2s^2}{(s+\sqrt{2})^2 + (\sqrt{2})^2}$$

$$X(j\omega) = X(s)\Big|_{s=0+j\omega} = \frac{-2\omega^2}{j\omega 2\sqrt{2} + 4 - \omega^2}$$

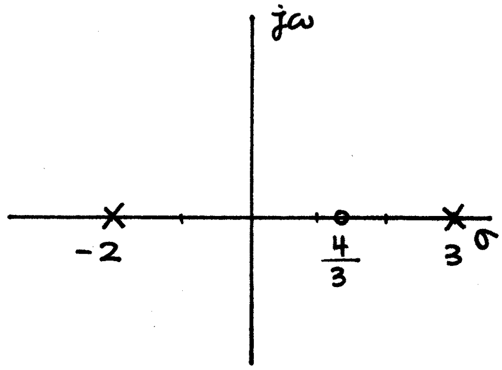
poles : $s = -\sqrt{2} \pm j\sqrt{2}$
zeros : $s = 0$ (double)

(c) $$X(s) = \frac{1}{s-3} + \frac{2}{s+2} = \frac{3s-4}{(s-3)(s+2)}$$

$$= 3\,\frac{s-\frac{4}{3}}{(s-3)\,(s+2)}$$

$$X(j\omega) = X(s)\Big|_{s=0+j\omega}$$

$$= \frac{1}{j\omega-3} + \frac{2}{j\omega+2}$$



poles : $s = 3,\ s = -2$
zeros : $s = \frac{4}{3}$

$\boxed{6.2}$

(a) $x(t) = u(t-2)$

$$X(s) = \int_{-\infty}^{\infty} u(t-2)\,e^{-st}\,dt$$

$$X(s) = \int_{2}^{\infty} e^{-st}\,dt$$

$$X(s) = \frac{e^{-2s}}{s}$$

ROC : $\mathrm{Re}(s) > 0$

(b) $x(t) = e^{2t}\,u(-t+2)$

$$X(s) = \int_{-\infty}^{2} e^{2t}\cdot e^{-st}\,dt$$

$$= \frac{-1}{s-2}\,e^{-(s-2)t}\Big|_{-\infty}^{2}$$

$$= \frac{-e^{-2(s-2)}}{s-2}$$

ROC : $\mathrm{Re}(s) < 2$

(c) $x(t) = \delta(t - t_0)$

$$X(s) = \int_{-\infty}^{\infty} \delta(t - t_0) e^{-st} dt$$

$$= e^{-st_0}$$

ROC : entire s plane

(d) $x(t) = \cos(2t)\, u(t)$

$$X(s) = \int_0^{\infty} \frac{e^{j2t} + e^{-j2t}}{2} \cdot e^{-st} dt$$

$$= \left(\frac{1}{j2 + s} - \frac{1}{j2 - s} \right) \frac{1}{2}$$

$$X(s) = \frac{s}{s^2 + 4}$$

ROC : $\mathbb{R}e\{s\} > 0$

$\boxed{6.3}$

(a) $x(t) = u(t - 1)$

$$X(s) = \int_0^{\infty} u(t - 1) e^{-st} dt$$

$$= \int_1^{\infty} e^{-st} dt$$

$$= \frac{e^{-s}}{s}$$

(b) $x(t) = u(t + 1)$

$$X(s) = \int_0^{\infty} e^{-st} dt = \frac{1}{s}$$

(c) $x(t) = e^{-t+2}\, u(t)$

$$X(s) = \int_0^\infty e^{-t+2}\, e^{-st}\, dt$$

$$= \frac{e^2}{s+1}$$

(d) $x(t) = \cos(\omega_0 t)\, u(t-3)$

$$X(s) = \int_3^\infty \frac{e^{j\omega t} + e^{-j\omega_0 t}}{2}\, e^{-st}\, dt$$

$$= \frac{e^{-3s}\left(s.\cos(3\omega_0) - \omega_0 . \sin(3\omega_0)\right)}{s^2 + \omega_0^2}$$

(e) $x(t) = \sin(\omega_0 . t)\, u(t+2)$

$$X(s) = \int_0^\infty \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{j2}\, e^{-st}\, dt$$

$$= \frac{\omega_0^2}{s^2 + \omega_0^2}$$

(f) $x(t) = e^{2t}\left[u(t) - u(t-4)\right]$

$$X(s) = \int_0^4 e^{2t}\, e^{-st}\, dt$$

$$= \frac{1 - e^{-4(s-2)}}{s-2}$$

(g)

$$x(t) = \begin{cases} \sin(\pi t) & , 0 < t < 1 \\ 0 & , \text{otherwise} \end{cases}$$

$$X(s) = \int_0^1 \frac{e^{j\pi t} - e^{-j\pi t}}{j2} . e^{-st}\, dt$$

$$X(s) = \frac{\pi(1+e^{-s})}{(s^2+\pi^2)}$$

$\boxed{6.4}$

(a) $x(t) = t^2 e^{-2t} u(t)$

$$t^2 s(t) \overset{\mathcal{L}u}{\longleftrightarrow} \frac{d^2}{ds^2} S(s)$$

$$e^{-2t}u(t) \overset{\mathcal{L}u}{\longleftrightarrow} \frac{1}{s+2}$$

$$\therefore X(s) = \frac{d^2}{ds^2}\left(\frac{1}{s+2}\right)$$

$$X(s) = \frac{2}{(s+2)^3}$$

(b) $x(t) = e^{-t} u(t) * \sin(3\pi t)\, u(t)$

$$s_1(t) * s_2(t) \overset{\mathcal{L}u}{\longleftrightarrow} S_1(s) \cdot S_2(s)$$

$$e^{-t} u(t) \overset{\mathcal{L}u}{\longleftrightarrow} \frac{1}{s+1}$$

$$\sin(3\pi t)\, u(t) \overset{\mathcal{L}u}{\longleftrightarrow} \frac{3\pi}{s^2+9\pi^2}$$

$$\therefore X(s) = \frac{3\pi}{(s+1)(s^2+9\pi^2)}$$

(c) $x(t) = \frac{d}{dt}\{t\, u(t)\}$

$$\frac{d}{dt} s(t) \overset{\mathcal{L}u}{\longleftrightarrow} s.S(s) - s(0^+)$$

$$t\, u(t) \overset{\mathcal{L}u}{\longleftrightarrow} \frac{1}{s^2}$$

$$\therefore X(s) = \frac{1}{s} - 0$$

$$X(s) = \frac{1}{s}$$

(d) $x(t) = t.u(t) - (t-1)\,u(t-1) - (t-2)\,u(t-2) + (t+3)\,u(t-3)$

$$s(t-\tau) \xleftrightarrow{\mathcal{L}_u} e^{-s\tau}\, S(s)$$

$$\therefore X(s) = \left(1 - e^{-s} - e^{-2s} + e^{-3s}\right)\frac{1}{s^2}$$

(e) $$x(t) = \int_0^t e^{-2\tau} \cos(3\tau)\, d\tau$$

$$= \int_{-\infty}^{t} e^{-2\tau} \cos(3\tau)\, u(t)\, d\tau$$

$$\int_{-\infty}^{t} s(\tau)\, d\tau \xleftrightarrow{\mathcal{L}_u} \frac{1}{s}\int_{-\infty}^{0^+} s(\tau)\, d\tau + \frac{S(s)}{s}$$

$$\therefore X(s) = \frac{1}{s} \cdot \frac{s+2}{(s+2)^2+9}$$

(f) $$x(t) = 2\,t\,\frac{d}{dt}\left(e^{-t} \sin(t) \, . \, u(t)\right)$$

$$t \,.\, \ell(t) \xleftrightarrow{\mathcal{L}_u} -\frac{d}{ds} L(s)$$

$$\frac{d}{dt}\ell(t) \xleftrightarrow{\mathcal{L}_u} s\,L(s) - \ell(0+)$$

$$\therefore X(s) = -2\frac{d}{ds}\left(\frac{s}{(s+1)^2+1} - 0\right)$$

$$= -2\left(\frac{1}{s^2+2s+2} - \frac{s(2s+2)}{(s^2+2s+2)^2}\right)$$

$$X(s) = \frac{2(s^2-2)}{(s^2+2s+2)^2}$$

$\boxed{6.5}$

(a) $X(s) = \left(\frac{1}{s}\right)\frac{1}{s+1}$

$$\frac{L(s)}{s} \longleftrightarrow \int_{-\infty}^{t} \ell(\tau)\,d\tau$$

$$\frac{1}{s+1} \longleftrightarrow e^{-t}u(t)$$

$$\therefore x(t) = \int_{0}^{t} e^{-\tau}\,d\tau$$

$$= (1-e^{-t})\,u(t)$$

(b) $X(s) = \frac{d}{ds}\left(e^{-2s}\frac{1}{(s+2)^2}\right)$

$$\frac{d}{ds}L(s) \longleftrightarrow -t\,\ell(t)$$

$$e^{-\tau.s}L(s) \longleftrightarrow \ell(t-\tau)$$

$$\frac{1}{(s+2)^2} \longleftrightarrow t.e^{-2t}u(t)$$

$$\therefore x(t) = -t\left((t-2)e^{-2(t-2)}u(t-2)\right)$$

$$= (2t-t^2)e^{-2(t-2)}u(t-2)$$

(c) $X(s) = \frac{1}{(2s+1)^2+4}$

$$X\left(\frac{s}{a}\right) \longleftrightarrow a\,x(at)$$

$$\frac{1}{(s+1)^2+4} \longleftrightarrow \frac{1}{2}\left(e^{-t}\sin(2t)\right)u(t)$$

$$\therefore\ x(t) = \frac{1}{4}\left(e^{-\frac{1}{2}t}\sin(t)\right)\ u(t)$$

(d) $$X(s) = s.\frac{d^2}{ds^2}\left(\frac{4}{s^2+4}\right)$$

$$\frac{d^2}{ds^2}L(s) \longleftrightarrow t^2\, \ell(t)$$

$$s\,F(s) \longleftrightarrow \frac{d}{dt}f(t),\ f(0^+)=0$$

$$\frac{4}{s^2+4} \longleftrightarrow 2(\sin(2t))\,u(t)$$

$$\therefore\ x(t) = \frac{d}{dt}\left(2t^2\sin(2t)\,u(t)\right)$$

$$= \left[4t\sin(2t) + 4t^2\cos(2t)\right]u(t)$$

$\boxed{6.6}$

$$\cos(2t)\,u(t) \xleftrightarrow{\mathcal{L}u} X(s) \quad ,\ x(0^+)=1$$

(a) $s\,X(s) - 1 \longleftrightarrow \frac{d}{dt}x(t)$

signal $= -2\sin(2t)\,u(t)$

(b) $X(2s) \longleftrightarrow \frac{1}{2}x\left(\frac{1}{2}t\right)$

signal $= \frac{1}{2}\cos(t).\,u(t)$

(c) $X(s+1) \longleftrightarrow e^{-t}x(t)$

signal $= e^{-t}\cos(2t).\,u(t)$

(d) $s^{-1} X(s) \longleftrightarrow \int_{-\infty}^{t} x(\tau)\, d\tau$

$$\text{signal} = \int_{-\infty}^{t} \cos(2\tau).\, u(\tau)\, d\tau$$

$$= \frac{1}{2} \sin(2t).u(t)$$

(e) $\frac{d}{ds}\left(e^{-2s} X(s)\right) \longleftrightarrow -t\left(x(t-2)\right)$

$$\text{signal} = -t\left(\cos(2(t-2))\, u(t-2)\right)$$

$$= -t \cos(2t-4) \,.\, u(t-2)$$

$\boxed{6.7}$ $x(t) \xleftrightarrow{\mathcal{L}u} \frac{2s}{s^2-2}$

(a) $x(2t) \longleftrightarrow \frac{1}{2} X\left(\frac{s}{2}\right)$

$$LT = \frac{1}{2} \frac{s}{\left(\frac{s}{2}\right)^2 - 2}$$

(b) $x(t-3) \longleftrightarrow e^{-3s} X(s)$

$$LT = \frac{2s e^{-3s}}{s^2-2}$$

(c) $x(t) * t\, x(t) \longleftrightarrow X(s).\left(-\frac{d}{ds} X(s)\right)$

$$LT = \frac{-2s}{s^2-2} \cdot \frac{d}{ds}\left(\frac{2s}{s^2-2}\right)$$

$$= \frac{4s(s^2+2)}{(s^2-2)^3}$$

(d) $e^{-2t}\, x(t) \longleftrightarrow X(s+2)$

$$\mathcal{L}T = \frac{2(s+2)}{(s+2)^2 - 2}$$

(e) $2 \cdot \frac{d}{dt} x(t) \longleftrightarrow 2\left(s X(s) - x(0^+)\right)$

$$x(0^+) = \lim_{s \to \infty} s X(s) = 2$$

$$\mathcal{L}T = 2\left(\frac{2s^2}{s^2 - 2} - 2\right)$$

(f) $\int_0^t x(2\tau)\, d\tau \longleftrightarrow \frac{1}{s} \cdot \frac{1}{2} X\left(\frac{s}{2}\right)$ provided that $x(t) = 0,\ t < 0$

$$\mathcal{L}T = \frac{2}{\left(\frac{s}{2}\right)^2 - 2}$$

$\boxed{6.8}$ $e^{-at}\, u(t) \xleftrightarrow{\mathcal{L}u} \frac{1}{s+a}$

$$x(t) = e^{-at} \cos(\omega_1 . t)\, u(t)$$

$$= \frac{1}{2} e^{-at} \left(e^{j\omega_1 t} + e^{-j\omega_1 t}\right) u(t)$$

Using s-domain shift property :

$$X(s) = \frac{1}{2}\left(\frac{1}{(s - j\omega_1) + a} + \frac{1}{(s + j\omega_1) + a}\right)$$

$$= \frac{1}{2} \cdot \frac{2(s+a)}{(s+a)^2 + \omega_1^2}$$

$$\therefore X(s) = \frac{s+a}{(s+a)^2 + \omega_1^2}$$

$\boxed{6.9}$

(a) Linearity

$$z(t) = a\,x(t) + b\,y(t)$$

$$Z(s) = \int_0^\infty z(t)\,e^{-st}\,dt$$

$$= \int_0^\infty (a\,x(t) + b\,y(t))\,e^{-st}\,dt$$

$$= \int_0^\infty (a\,x(t)e^{-st} + b\,y(t)e^{-st})\,dt$$

$$= a\left[\int_0^\infty x(t)e^{-st}\,dt\right] + b\left[\int_0^\infty y(t)\,e^{-st}\,dt\right]$$

$$Z(s) = a.X(s) + b.Y(s)$$

(b) Scaling

$$z(t) = x(at)$$

$$Z(s) = \int_0^\infty x(at)\,e^{-st}\,dt$$

$$= \frac{1}{a}\int_0^\infty x(\mu)\,e^{-\frac{s}{a}\mu}\,d\mu$$

$$Z(s) = \frac{1}{a}X\left(\frac{s}{a}\right)$$

(c) Time shift

$$z(t) = x(t-\tau)$$

$$Z(s) = \int_0^\infty x(t-\tau)\,e^{-st}\,dt \quad , \quad \eta = t-\tau$$

$$= \int_{-\tau}^\infty x(\eta)\,e^{-s(\eta+\tau)}\,d\eta$$

If $x(t-\tau)\,u(t) = x(t-\tau)\,u(t-\tau)$

$$Z(s) = \int_0^\infty x(\eta)\,e^{-s\eta}.\,e^{-s\tau}\,d\eta = e^{-s\tau}.X(s)$$

(d) s-domain shift

$$z(t) = e^{s_o.t}\, x(t)$$

$$Z(s) = \int_0^\infty x(t)\, e^{s_o.t} . e^{-st}\, dt$$

$$= \int_0^\infty x(t)\, e^{-(s-s_o)t}\, dt$$

$$Z(s) = X(s-s_o)$$

(e) convolution

$$z(t) = x(t) * y(t)$$

$$= \int_0^\infty x(\tau)\, y(t-\tau)\, d\tau \; ; \; \underset{\text{causal}}{x(t), y(t)}$$

$$Z(s) = \int_0^\infty \left(\int_0^\infty x(\tau) . y(t-\tau)\, d\tau \right) e^{-st}\, dt$$

$$= \int_0^\infty \left(\int_0^\infty x(\tau) . y(\mu)\, d\tau \right) e^{-s\mu} . e^{-s\tau}\, d\mu$$

$$= \left(\int_0^\infty x(\tau) . e^{-s\tau}\, d\tau \right) \left(\int_0^\infty y(\mu)\, e^{-s\mu}\, d\mu \right)$$

$$Z(s) = X(s) . Y(s)$$

(f) differentiation in s-domain

$$z(t) = -t\, x(t)$$

$$Z(s) = \int_0^\infty -t\, x(t)\, e^{-st}\, dt$$

$$= \int_0^\infty x(t) \frac{d}{ds}\left(e^{-st}\right) dt$$

$$= \int_0^\infty \frac{d}{ds}\left(x(t)\, e^{-st}\right) dt$$

Assume : $\int_0^\infty (\cdot)\, dt$ and $\frac{d}{ds}(\cdot)$ are interchangeable

$$Z(s) = \frac{d}{ds} \int_0^\infty x(t)\, e^{-st}\, dt$$

$$Z(s) = \frac{d}{ds} X(s)$$

$\boxed{6.10}$

(a) $X(s) = \dfrac{3}{s^2+5s-1}$

$$x(0^+) = \lim_{s\to\infty} \frac{3s}{s^2+5s-1}$$
$$= 0$$

(b) $X(s) = \dfrac{2s+3}{s^2+5s-6}$

$$x(0^+) = \lim_{s\to\infty} \frac{2s^2+3s}{s^2+5s-6}$$
$$= 2$$

(c) $X(s) = e^{-5s} \dfrac{3s^2+2s}{s^2+s-1}$

$$x(0^+) = \lim_{s\to\infty} e^{-5s} \frac{3s^3+2s^2}{s^2+s-1}$$
$$= 0$$

$\boxed{6.11}$

(a) $$X(s) = \frac{2s^2+3}{s^2+5s+1}$$

$$x(\infty) = \lim_{s\to 0} s \cdot \frac{2s^2+3}{s^2+5s+1}$$

$$= 0$$

(b) $$X(s) = \frac{2s+3}{s^3+5s^2+6s}$$

$$x(\infty) = \lim_{s\to 0} \frac{2s+3}{s^2+5s+6}$$

$$= \frac{1}{2}$$

(c) $$X(s) = \frac{2s-1}{s^2+2s+1}$$

$$x(\infty) = \lim_{s\to 0} s \cdot \frac{2s-1}{s^2+2s+1}$$

$$= 0$$

$\boxed{6.12}$

(a) $$X(s) = \frac{-s-4}{s^2+3s+2}$$

$$= \frac{-3}{s+1} + \frac{+2}{s+2}$$

$$x(t) = (-3e^{-t} + 2e^{-2t})\, u(t)$$

(b) $$X(s) = \frac{s}{s^2+5s+6}$$

$$= \frac{-2}{s+2} + \frac{3}{s+3}$$

$$x(t) = \left(-2e^{-2t} + 3e^{-3t}\right) u(t)$$

(c) $$X(s) = \frac{2s-1}{s^2+2s+1}$$

$$= \frac{2}{s+1} + \frac{-3}{(s+1)^2}$$

$$x(t) = \left(2e^{-t} - 3te^{-t}\right) u(t)$$

(d) $$X(s) = \frac{5s+4}{s^3+3s^2+2s}$$

$$= \frac{2}{s} + \frac{1}{s+1} + \frac{-3}{s+2}$$

$$x(t) = \left(2 + e^{-t} - 3e^{-2t}\right) u(t)$$

(e) $$X(s) = \frac{3s^2+8s+5}{(s+2)(s^2+2s+1)}$$

$$= \frac{1}{s+2} + \frac{2}{s+1} + \frac{0}{(s+1)^2}$$

$$x(t) = \left(e^{-2t} + 2e^{-t}\right) u(t)$$

(f) $$X(s) = \frac{3s+2}{s^2+4s+5}$$

$$= \frac{3s+6}{(s+2)^2+1} - \frac{4}{(s+2)^2+1}$$

$$x(t) = \left(3e^{-2t}\cos(t) - 4e^{-2t}\sin(t)\right) u(t)$$

(g) $$X(s) = \frac{4s^2+8s+10}{(s+2)(s^2+2s+10)}$$

$$= \frac{1}{s+2} + \frac{3(s+1)}{(s+1)^2+3^2} + \frac{-3}{(s+1)^2+3^2}$$

$$x(t) = \left(e^{-2t} + 3e^{-t}\cos(3t) - e^{-t}\sin(3t)\right)u(t)$$

(h) $X(s) = \dfrac{-9}{(s+1)(s^2+2s+10)}$

$$= \frac{-1}{s+1} + \frac{s+1}{(s+1)^2+3^2}$$

$$x(t) = \left(-e^{-t} + e^{-t}\cos(3t)\right)u(t)$$

(i) $X(s) = \dfrac{s+4+e^{-2s}}{s^2+5s+6}$

$$= \frac{2}{s+2} + \frac{-1}{s+3} + e^{-2s}\left(\frac{1}{s+2} + \frac{-1}{s+3}\right)$$

$$x(t) = \left(2e^{-2t} - e^{-3t}\right)u(t) + \left(e^{-2(t-2)} - e^{-3(t-2)}\right) u(t-2)$$

$\boxed{6.13}$

(a) $5\dfrac{d}{dt}y(t) + 10\,y(t) = 2x(t)$, $y(0^+) = 2$, $x(t) = u(t)$

$$\xleftrightarrow{\mathcal{L}_u} 5\left(s\,Y(s) - 2\right) + 10\,Y(s) = \frac{2}{s}$$

$$[5s+10]\,Y(s) = \frac{2}{s} + 10$$

$$Y_s = \left(\frac{2}{5s(s+2)}\right) + \left(\frac{2}{s+2}\right)$$

$$= Y^f(s) + Y^n(s)$$

$$Y^f(s) = \frac{1}{5}\left(\frac{1}{s} - \frac{1}{s+2}\right) \Rightarrow y^f(t) = \frac{1}{5}\left(1 - e^{-2t}\right)u(t)$$

$$Y^n(s) = \frac{2}{s+2} \Rightarrow y^n(t) = 2e^{-2t}\,u(t)$$

(b) $\frac{d^2}{dt^2} y(t) + 5\frac{d}{dt} y(t) + 6y(t) = -4\,x(t) - 3\frac{d}{dt}x(t)$

$y(0^+) = -1 \quad , \dot{y}(0^+) = 5 \quad , x(t) = e^{-t}u(t)$

$\overset{\mathcal{L}u}{\longleftrightarrow} [s^2 + 5s + 6]\, Y(s) - 5 \quad + s + 5$

$= (-4 - 3s)\frac{1}{s+1} + 3$

$Y(s) = \left(\frac{-1}{(s+1)(s+2)(s+3)}\right) + \left(\frac{s}{(s+2)(s+3)}\right)$

$= Y^f(s) + Y^n(s)$

$Y^f(s) = \frac{-\frac{1}{2}}{s+1} + \frac{1}{s+2} + \frac{-\frac{1}{2}}{s+3} \Rightarrow y^f(t) = \left(-\frac{1}{2}e^{-t} + e^{-2t} - \frac{1}{2}e^{-3t}\right)$
$.u(t)$

$Y^n(s) = \frac{-2}{s+2} + \frac{3}{s+3} \Rightarrow y^n(t) = (-2\,e^{-2t} + 3\,e^{-3t}).u(t)$

(c) $\frac{d^2}{dt^2} y(t) + 4\,y(t) = 8\,x(t)$, $y(0^+) = 1$,

$\frac{d}{dt} y(t)\Big|_{t=0+} = 2$,

$x(t) = u(t)$

$\overset{\mathcal{L}u}{\longleftrightarrow} (s^2 + 4)\, Y(s) - 2 \; - s \quad = \frac{8}{s}$

$Y(s) = \left(\frac{8}{s(s^2+4)}\right) + \left(\frac{2 + s}{s^2+4}\right)$

$= Y^f(s) + Y^n(s)$

$Y^f(s) = \frac{8}{s(s^2+4)}$

$$y^f(s) = \frac{2}{s} - \frac{2s}{s^2+4} \Rightarrow y^f(t) = 2\left(1-\cos(2t)\right) u(t)$$

$$y^n(s) = \frac{s}{s^2+4} + \frac{2}{s^2+4} \Rightarrow y^n(t) = \left(\cos(2t) + \sin(2t)\right)u(t)$$

(d) $$\frac{d^2}{dt^2} y(t) + 2\frac{d}{dt} y(t) + 5y(t) = \frac{d}{dt} x(t)$$

$$y(0^+) = 2, \quad \frac{d}{dt} y(t)\Big|_{t=0^+} = 0, \quad x(t) = e^{-t}u(t)$$

$$\left[s^2+2s+5\right] Y(s) - 2s - 4 = \frac{s}{s+1} - 1$$

$$Y(s) = \left(\frac{-1}{(s+1)\left((s+1)^2+2^2\right)}\right) + \left(\frac{+2s+4}{(s+1)^2+2^2}\right)$$

$$= Y^f(s) + Y^n(s)$$

$$Y^f(s) = \frac{-\frac{1}{4}}{s+1} + \frac{\frac{1}{4}s + \frac{1}{4}}{(s+1)^2+2^2}$$

$$= -\frac{1}{4}\cdot\frac{1}{s+1} + \frac{1}{4}\left(\frac{s+1}{(s+1)^2+2^2}\right)$$

$$\Rightarrow y^f(t) = \frac{1}{4}\left(-e^{-t} + e^{-t}\cos(2t)\right)u(t)$$

$$Y^n(s) = \frac{2(s+1)}{(s+1)^2+2^2} + \frac{2}{(s+1)^2+2^2} \quad y^n(t) = -2e^{-t}\cos(2t)u(t) - e^{-t}\sin(2t)u(t)$$

$\boxed{6.14}$ $x(t) = e^{-t}u(t) \qquad R = 1\Omega, \; L = \frac{1}{2}+1, \; y(0^+) = 2A$

$$x(t) = R\cdot y(t) + L\cdot\frac{d}{dt} y(t)$$

$$\overset{\mathcal{L}u}{\longleftrightarrow} X(s) = (R + Ls)\,Y(s) - Ly(0^+)$$

$$Y(s) = \frac{X(s)}{R+Ls} + \frac{Ly(0^+)}{R+Ls} \quad ; \; X(s) = \frac{1}{s+1}$$

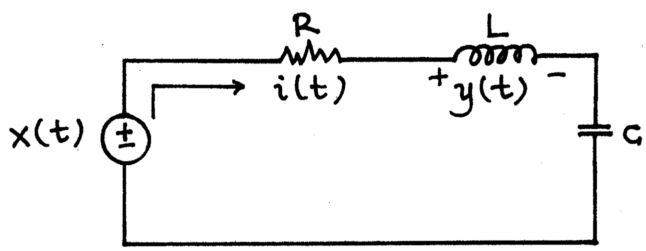
$$Y(s) = \frac{1}{L}\,\frac{1}{(s+1)(s+\frac{R}{L})} + \frac{y(0^+)}{s+\frac{R}{L}}$$

$$= \frac{1}{(R-L)}\left(\frac{1}{s+1} - \frac{1}{s+\frac{R}{L}}\right) + y(0^+)\,\frac{1}{s+\frac{R}{L}}$$

$$y(t) = \frac{1}{R-L}\left(e^{-t} - e^{-\frac{R}{L}t}\right)u(t) + y(0^+)e^{-\frac{R}{L}t}.u(t)$$

$$y(t) = 2\left(e^{-t} - e^{-2t}\right)u(t) + 2e^{-2t}u(t)$$
$$= 2e^{-t}.u(t)$$

$\boxed{6.15}$



$$x(t) = R\,.\,i(t) + L\,\frac{di(t)}{dt} + \frac{1}{c}\int_{-\infty}^{t} i(\tau)\,d\tau$$

differentiate once :

$$\frac{d}{dt}x(t) = R\,\frac{d}{dt}\,i(t) + L\,\frac{d^2}{dt^2}\,i(t) + \frac{1}{C}\,i(t) \quad \ldots(1)$$

$$y(t) = L\,\frac{d}{dt}\,i(t) \quad \ldots(2)$$

Given Initial Conditions :

$i_L(0^+)$, $V_C(0^+)$

$$L \cdot \frac{d}{dt} \cdot i_L(0^+) = x(0^+) - i_L(0^+)R - V_C(0^+)$$

$$\Rightarrow \boxed{\dot{i}_L(0^+) = \frac{x(0^+) - i_L(0^+) \cdot R - V_C(0^+)}{L}}$$

So, we have 2 Initial conditions for $i(t)$

To solve for $y(t)$, note that (2):

$$\boxed{\begin{aligned} Y(s) &= L\,(s\,I(s) - i(0^+)) \\ y(t) &= \mathcal{L}_u^{-1}\{Y(s)\} \end{aligned}}$$

(1). $(Ls^2 + Rs + \frac{1}{C})\,I(s) - L(\,\dot{i}(0^+) + s\,i(0^+)) - R\,i(0^+)$
$= s\,X(s) - x(0^+)$

$$I(s) = \left(\frac{s.X(s) - x(0^+)}{Ls^2 + Rs + \frac{1}{C}}\right) + \left(\frac{L.\dot{i}(0^+) + (sL+R)\,i(0^+)}{Ls^2 + Rs + \frac{1}{C}}\right)$$

$$I(s) = I^f(s) + I^n(s) \Rightarrow \begin{aligned} Y^f(s) &= Ls\,I^f(s) \\ Y^n(s) &= Ls\,I^n(s) - L\,i(0^+) \end{aligned}$$

$$\boxed{\begin{aligned} I^f(s) &= \frac{s.X(s) - x(0^+)}{Ls^2 + Rs + \frac{1}{C}} \\ I^n(s) &= \frac{L\,\dot{i}(0^+) + (sL+R)\,i(0^+)}{Ls^2 + Rs + \frac{1}{C}} \end{aligned}}$$

(a) $R = 3\,\Omega$, $L = 1\,H$, $C = \frac{1}{2}\,F$, $x(t) = e^{-3t}u(t)$
$i_L(0^+) = 2\,A$, $V_C(0^+) = 1\,V$

$X(s) = \frac{1}{s}$, $x(0^+) = 1\,V$

$i(0^+) = 2\ A$

$\dot{i}(0^+) = \dfrac{1 - 2(3) - 1}{1} = -6\ \dfrac{A}{s}$

$y^f(s) = \dfrac{s^2 - s}{(s+3)(s+2)(s+1)} = \dfrac{6}{s+3} + \dfrac{1}{s+1} - \dfrac{6}{s+2}$

$I^n(s) = \dfrac{2s}{s^2 + 3s + 2}$

$y^n(s) = \dfrac{2s^2}{(s+1)(s+2)} - 2$

$= \dfrac{-6s - 4}{(s+1)(s+2)}$

$y^n(s) = \dfrac{2}{s+1} + \dfrac{-8}{s+2}$

$\therefore\ y^f(t) = \left(6e^{-3t} + e^{-t} - 6e^{-2t}\right)u(t)$

$y^n(t) = \left(2e^{-t} - 8e^{-2t}\right)u(t)$

(b) $R = 2\,\Omega$, $L = 1\ H$, $C = \frac{1}{5}\ F$, $x(t) = u(t)$,
$i_L(0^+) = 2\ A$, $V_C(0^+) = 1\ V$

$X(s) = \dfrac{1}{s}$, $x(0^+) = 1\ V$, $i(0^+) = 2\ A$,

$\dot{i}(0^+) = \dfrac{1 - 4 - 1}{1}$

$= -4\ \dfrac{A}{s}$

$y^f(s) = \dfrac{2(s^2 - s)}{(s+1)(s^2+2s+5)} = \dfrac{1}{s+1} - \dfrac{2(s+1)}{(s+1)^2 + 2^2} - \dfrac{3/2\,(2)}{(s+1)^2 + 2^2}$

$I^n(s) = \dfrac{-4 + 2(s+2)}{s^2 + 2s + 5}$

$$y^n(s) = \frac{2s^2}{s^2+2s+5} - 2$$

$$= \frac{-4s-10}{(s+1)^2+2^2}$$

$$y^n(s) = \frac{-4(s+1)}{(s+1)^2+2^2} + \frac{-3(2)}{(s+1)^2+2^2}$$

$$\therefore\ y^f(t) = \left(e^{-t} - 2e^{-t}\cos 2t - \tfrac{3}{2}e^{-t}\sin 2t\right)u(t)$$

$$y^n(t) = (-4\cos(2t) - 3\sin(2t))e^{-t}u(t)$$

$\boxed{6.16}$

(a) $x(t) = e^{-2t}u(t) + e^{-t}u(t) + e^{t}u(-t)$

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt$$

$$= \int_0^{\infty} e^{-2t}e^{-st}\,dt + \int_0^{\infty} e^{-t}e^{-st}\,dt + \int_{-\infty}^{0} e^{t}e^{-st}\,dt$$

$$X(s) = \frac{1}{s+2} + \frac{1}{s+1} - \frac{1}{s-1}$$

ROC : $(\mathrm{Re}\{s\} > -2) \cap (\mathrm{Re}\{s\} > -1) \cap (\mathrm{Re}\{s\} < 1)$

$= -1 < \mathrm{Re}\{s\} < 1$

*. From now on, we use table and properties

(b) $x(t) = e^{2t}\cos(2t)u(-t) + e^{-t}u(t) + e^{t}u(t)$

$$X(s) = -\frac{s-2}{(s-2)^2+4} + \frac{1}{s+1} + \frac{1}{s-1}$$

$$\text{ROC}: (\mathrm{Re}\{s\} < 2) \cap (\mathrm{Re}\{s\} > -1) \cap (\mathrm{Re}\{s\} > 1)$$

$$= 1 < \mathrm{Re}\{s\} < 2$$

(c) $$x(t) = e^{2t+4}\, u(t+2)$$

$$= e^{2(t+2)}\, u(t+2)$$

$$\ell(t+2) \xleftrightarrow{\mathcal{L}} e^{2s} L(s)$$

$$X(s) = \frac{e^{2s}}{s-2}$$

$$\text{ROC}: \mathrm{Re}\{s\} > -2$$

(d) $$x(t) = \cos(3t)\, u(-t) * e^{-t} u(t)$$

$$\ell_1(t) * \ell_2(t) \xleftrightarrow{\mathcal{L}} L_1(s) \cdot L_2(s)$$

$$X(s) = -\frac{s}{s^2+9} \cdot \frac{1}{s+1}$$

$$\text{ROC}: (\mathrm{Re}\{s\} < 0) \cap (\mathrm{Re}\{s\} > -1)$$

$$= -1 < \mathrm{Re}\{s\} < 0$$

(e) $$x(t) = e^{t} \sin(2t+4)\, u(t+2)$$

$$= e^{-2} \cdot e^{(t+2)} \sin(2(t+2))\, u(t+2)$$

$$\ell(t+2) \longleftrightarrow L(s)\, e^{2s}$$

$$X(s) = e^{2s} \quad \frac{2}{(s-1)^2+4}$$

ROC : $\mathrm{Re}\{s\} > 1$

(f) $x(t) = e^{-t} \frac{d}{dt}\left(e^{-t} u(t+1)\right)$

$$= e^{-t}\, e \, \frac{d}{dt}\left(e^{-(t+1)} \quad u(t+1)\right)$$

$$\frac{d}{dt}\left(e^{-(t+1)} u(t+1)\right) \longleftrightarrow \frac{s \cdot e^{s}}{s+1}$$

$$e^{-t} \quad \ell(t) \longleftrightarrow L(s+1)$$

$$\therefore \; X(s) = \frac{e(s+1)\, e^{s+1}}{s+2}$$

ROC : $\{\mathrm{Re}\{s\} > -1\} - \{\mathrm{Re}\{-1\}\} = \mathrm{Re}\{s\} > -1$

(g) $x(t) = \int_{-\infty}^{t} e^{2\tau} \sin(\tau)\, u(-\tau)\, d\tau$

$$\int_{-\infty}^{t} \ell(\tau)\, d\tau \longleftrightarrow \frac{L(s)}{s}$$

$$X(s) = \frac{1}{s} \cdot \frac{1}{(s-2)^2+1}$$

ROC : $(\mathrm{Re}\{s\} < 2) \cap \{\mathrm{Re}\{s\} > 0\}$

$= 0 < \mathrm{Re}\{s\} < 2$ (at least)

$\boxed{6.17}$

(a) $X(s) = e^{5s} \dfrac{1}{s+2}$, ROC : $\mathrm{Re}\{s\} > -2$

causal (right - sided)

$x(t) = e^{-2(t+5)} \, u(t+5)$

(b) $X(s) = \dfrac{d^2}{ds^2}\left(\dfrac{1}{s-1}\right)$, ROC : $\mathrm{Re}\{s\} < 1$

anticausal (left-sided)

$x(t) = -t^2 e^t \, u(-t)$

(c) $X(s) = s\left(\dfrac{1}{s^2} - \dfrac{e^{-s}}{s} - \dfrac{e^{-2s}}{s}\right)$

ROC : $\mathrm{Re}\{s\} < 0$

anticausal (left - sided)

$x(t) = \dfrac{d}{dt}\left(-t u(-t) + u(-t-1) + u(-t-2)\right)$

$x(t) = -u(-t) - \delta(t+1) - \delta(t+2)$

(d) $X(s) = s^{-1} \dfrac{d}{ds}\left(\dfrac{e^{-2s}}{s}\right)$, ROC : $\mathrm{Re}\{s\} > 0$

causal (right - sided)

$$\frac{e^{-2s}}{s} \xleftrightarrow{\mathcal{L}} u(t-2)$$

$x(t) = \int_{-\infty}^{t} -\tau \, u(\tau-2) \, d\tau$

$x(t) = \int_{2}^{t} -\tau \, d\tau = -\frac{1}{2}(t^2 - 4)$

$\boxed{6.18}$

(a) $$X(s) = \frac{s+3}{s^2+3s+2} = \frac{2}{s+1} + \frac{-1}{s+2}$$

(i) ROC : $\mathrm{Re}\{s\} < -2$ anticausal

$$x(t) = \left(-2e^{-t} + e^{-2t}\right) u(-t)$$

(ii) ROC : $\mathrm{Re}\{s\} > -1$ causal

$$x(t) = \left(2e^{-t} - e^{-2t}\right) u(t)$$

(iii) ROC : $-2 < \mathrm{Re}\{s\} < -1$

two sided $\begin{cases} e^{-2t}, & \text{causal} \\ e^{-t}, & \text{anticausal} \end{cases}$

$$x(t) = -e^{-2t} u(t) - 2e^{-t} u(-t)$$

(b) $$X(s) = \frac{s^2+7}{(s+1)(s^2-2s+4)} = \frac{\frac{8}{7}}{s+1} + \left(\frac{s-1}{(s-1)^2+3} + \frac{-\frac{16}{\sqrt{3}} \cdot \sqrt{3}}{(s-1)^2+3}\right) \cdot \frac{-1}{7}$$

(i) ROC : $\mathrm{Re}\{s\} < -1$ anticausal

$$x(t) = \left(-\frac{8}{7}e^{-t} + \frac{1}{7}e^{t}\cos(\sqrt{3}t) - \frac{16}{7\sqrt{3}}e^{t}\sin(\sqrt{3}t)\right) \cdot u(-t)$$

(ii) ROC : $\mathbb{R}e\{s\} > 1$ causal

$$x(t) = \left(\frac{8}{7}e^{-t} - \frac{1}{7}e^{t}\cos(\sqrt{3}t) + \frac{16}{7\sqrt{3}}e^{t}\sin(\sqrt{3}t)\right)u(t)$$

(iii) ROC : $-1 < \mathbb{R}e\{s\} < 1$ double-sided $\begin{cases} e^{-t} \text{ causal} \\ e^{t} \text{ anticausal} \end{cases}$

$$x(t) = \frac{8}{7}e^{-t}u(t) + \left(\frac{1}{7}\cos\sqrt{3}t - \frac{16}{7\sqrt{3}}\sin(\sqrt{3}t)\right)e^{t}$$

$$\cdot\, u(-t)$$

(c) $$X(s) = \frac{2s^2 + 4s + 2}{s^2 + 2s}$$

$$= 2 + \frac{1}{s} - \frac{1}{s+2}$$

(i) ROC : $\mathbb{R}e(s) < -2$ left-sided (anticausal)

$$\therefore x(t) = 2\delta(t) - [1 - e^{-2t}]u(-t)$$

(ii) ROC : $\mathbb{R}e(s) > 0$ right-sided (causal)

$$\therefore x(t) = 2\delta(t) + [1 - e^{-2t}]u(t)$$

(iii) ROC : $-2 < \mathbb{R}e(s) < 0$

$$\therefore x(t) = 2\delta(t) - u(-t) - e^{-2t}u(t)$$

(d) $$X(s) = \frac{s^2 + 3s + 4}{s^2 + 2s + 1}$$

$$= 1 + \frac{1}{s+1} + \frac{2}{(s+1)^2}$$

ROC : $\mathrm{Re}(s) < -1 \longrightarrow$ anticausal

$$\therefore x(t) = \delta(t) - \left[e^{-t} + 2te^{-t}\right] u(-t)$$

$\boxed{6.19}$

(a) $$H(s) = \frac{3s-1}{s^2-1}$$

$$= \frac{2}{s+1} + \frac{1}{s-1}$$

(i) causal : $h(t) = (2e^{-t} + e^{t})\, u(t)$

(ii) stable : ROC must include $j\omega$ axis

$$h(t) = 2e^{-t} u(t) - e^{t} u(-t)$$

(b) $$H(s) = \frac{5s+7}{s^2+3s+2}$$

$$= \frac{2}{s+1} + \frac{3}{s+2}$$

(i) causal : $h(t) = (2e^{-t} + 3e^{-2t})\, u(t)$

(ii) stable : $h(t) = 2e^{-t} u(t) + 3e^{-2t} u(t)$

(c) $$H(s) = \frac{s^2+5s-9}{(s+1)(s^2-2s+10)}$$

$$= \frac{-1}{s+1} + \frac{2s+1}{(s-1)^2+3^2}$$

$$= \frac{-1}{s+1} + \frac{2(s-1)}{(s-1)^2+3^2} + \frac{3}{(s-1)^2+3^2}$$

(i) causal :

$$h(t) = \left(-e^{-t} + e^{t}\left(2\cos(3t) + 3\sin(3t)\right)\right).u(t)$$

(ii) stable :

$$h(t) = -e^{-t}.\,u(t) - e^{t}\left(2\cos(3t) + 3\sin(3t)\right).u(-t)$$

$\boxed{\text{STABLE}}$

$\boxed{6.20}$

(a) $x(t) = u(t)$, $y(t) = e^{-t}\cos(2t)\,u(t)$

$$X(s) = \frac{1}{s}$$

$$Y(s) = \frac{s+1}{(s+1)^2+4}$$

$$H(s) = \frac{Y(s)}{X(s)}$$

$$= \frac{s(s+1)}{(s+1)^2+4}$$

$$= 1 - \left(\frac{s+1}{(s+1)^2+4} + \frac{4}{(s+1)^2+4}\right)$$

$$h(t) = \delta(t) - \left[e^{-t}\cos(2t) - 2e^{-t}\sin(2t)\right]u(t)$$

(b) $x(t) = e^{-2t}\,u(t)$,
$y(t) = -2e^{-t}\,u(t) + 2e^{-3t}\,u(t)$

$$X(s) = \frac{1}{s+2}$$

$$Y(s) = \frac{-2}{s+1} + \frac{2}{s+3}$$

$$H(s) = \frac{-2(s+2)}{s+1} + \frac{2(s+2)}{s+3}$$

$$= -2\left[\frac{1}{s+1} + \frac{1}{s+3}\right]$$

$$h(t) = -2\left(e^{-t} + e^{-3t}\right) u(t)$$

$\boxed{6.21}$

(a) $$5\frac{d}{dt}\, y(t) + 10\, y(t) = 2x(t)$$

$$(5s + 10)\, Y(s) = 2X(s)$$

$$H(s) = \frac{Y(s)}{X(s)}$$

$$= \frac{2}{5(s+2)}$$

$$h(t) = \frac{2}{5} e^{-2t} u(t)$$

(b) $$\frac{d^2}{dt^2}\, y(t) + 5\frac{d}{dt}\, y(t) + 6\, y(t) = x(t) + \frac{d}{dt}\, x(t)$$

$$(s^2 + 5s + 6).Y(s) = (s+1).X(s)$$

$$H(s) = \frac{s+1}{(s+2)(s+3)}$$

$$= \frac{-1}{s+2} + \frac{2}{s+3}$$

$$h(t) = \left(-e^{-2t} + 2e^{-3t}\right) u(t)$$

(c) $\frac{d^2}{dt^2} y(t) - 2\frac{d}{dt} y(t) + 10y(t) = x(t) + 2\frac{d}{dt}x(t)$

$$(s^2 - 2s + 10)\, Y(s) = (2s+1)\, X(s)$$

$$H(s) = \frac{2s+1}{(s-1)^2 + 3^2}$$

$$= \frac{2(s-1)}{(s-1)^2+3^2} + \frac{3}{(s-1)^2+3^2}$$

$$h(t) = e^t\left(2\cos(3t) + \sin(3t)\right)u(t)$$

$\boxed{6.22}$

(a) $H(s) = \frac{2s+1}{s(s+2)}$

$$H(s) = \frac{Y(s)}{X(s)}$$

$$\frac{d^2}{dt^2} y(t) + 2\frac{d}{dt} y(t) = x(t) + 2\frac{d}{dt} x(t)$$

(b) $H(s) = \frac{3s}{s^2+2s+10}$

$$H(s) = \frac{Y(s)}{X(s)}$$

$$\frac{d^2}{dt^2} y(t) + 2\frac{d}{dt} y(t) + 10y(t) = 3\frac{d}{dt} x(t)$$

(c) $H(s) = \frac{2(s+1)(s-2)}{(s+1)(s+2)(s+3)}$

$$= \frac{2(s^2-s-2)}{s^3+6s^2+11s+6}$$

$$= \frac{Y(s)}{X(s)}$$

$$\frac{d^3}{dt^3} y(t) + 6\frac{d^2}{dt^2} y(t) + 11\frac{d}{dt} y(t) + 6y(t)$$

$$= 2\left(-2x(t) - \frac{d}{dt}x(t) + \frac{d^2}{dt^2}x(t)\right)$$

$\boxed{6.23}$

(a) $\bar{A} = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix}$, $\bar{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\bar{c} = [1 \ \ -1]$, $D = [0]$

$$H(s) = \bar{c}(s\bar{I} - \bar{A})^{-1}\bar{b} + D$$

$$= [1 \ \ -1]\begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+3} \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} + [0]$$

$$H(s) = \frac{1-s}{(s+1)(s+3)}$$

(b) $\bar{A} = \begin{bmatrix} 1 & 2 \\ 1 & -6 \end{bmatrix}$, $\bar{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\bar{c} = [0 \ \ 1]$, $D = [0]$

$$H(s) = \bar{c}(s\bar{I} - \bar{A})^{-1}\bar{b} + \bar{D}$$

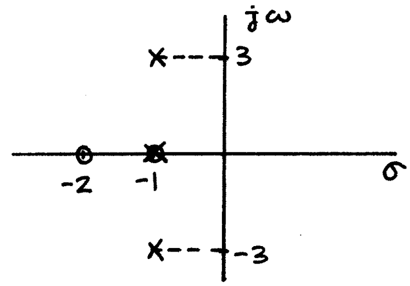
$$= [0 \ \ 1]\begin{bmatrix} s+6 & 2 \\ 1 & s-1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}\frac{1}{s^2+5s-6} + 0$$

$$H(s) = \frac{2s-1}{s^2+5s-6}$$

6.24

(a) $$H(s) = \frac{(s+1)\,(s+2)}{(s+1)\{(s+1)^2+3^2\}}$$

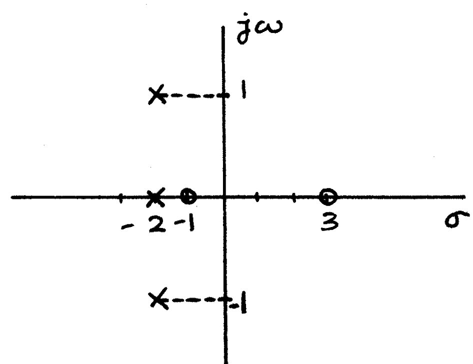
Pole Zero Plot :



(i) All poles are in LHP, and with ROC : $\mathrm{Re}\{s\} > -1$ the system is both stable and causal

(ii) All zeros are in LHP, so a stable and causal inverse system exists

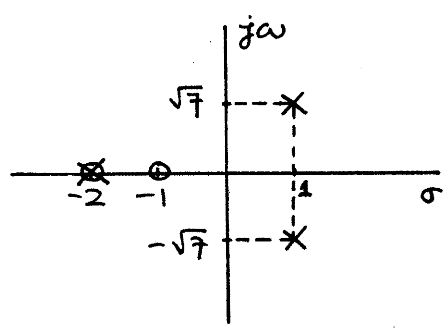
(b) $$H(s) = \frac{s^2-2s-3}{(s+2)(s^2+4s+5)} = \frac{(s-3)\,(s+1)}{(s+2)\{(s+2)^2+1\}}$$



(i) All poles are in LHP, with ROC : $\mathrm{Re}\{s\} > -2$, the system is both stable and causal

(ii) Not all the zeros are in LHP, no stable and causal inverse system exists

(c) $$H(s) = \frac{s^2+3s+2}{(s+2)(s^2-2s+8)} = \frac{(s+2)\,(s+1)}{(s+2)\{(s-1)^2+7\}}$$



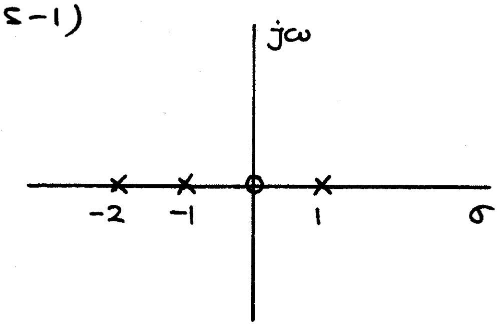
(i) No (poles are in RHP)

(ii) Yes (all zeros are in LHP)

(d) $$H(s) = \frac{s^2 + 2s}{(s^2+3s+2)(s^2+s-2)}$$

$$= \frac{s(s+2)}{(s+1)(s+2)(s+2)(s-1)}$$

$$= \frac{s}{(s+1)(s+2)(s-1)}$$



(i) No
(ii) Yes

$\boxed{6.25}$

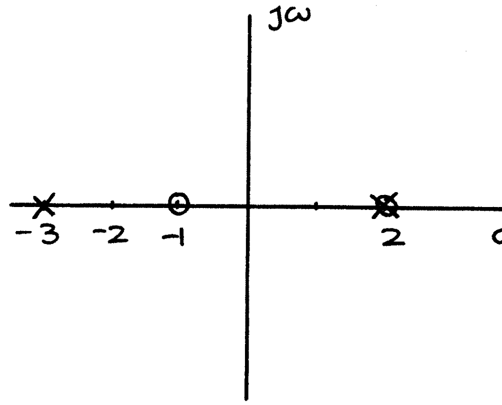
(a) $(s^2 + s - 6)\,Y(s) = (s^2 - s - 2)\,X(s)$

$$H(s) = \frac{Y(s)}{X(s)}$$

$$= \frac{s^2 - s - 2}{s^2 + s - 6}$$

$$= \frac{(s-2)(s+1)}{(s-2)(s+3)}$$

$$= \frac{s+1}{s+3}$$



Yes, after zero pole cancellation, the system only has 1 pole and 1 zero, and all are in left Half Plane

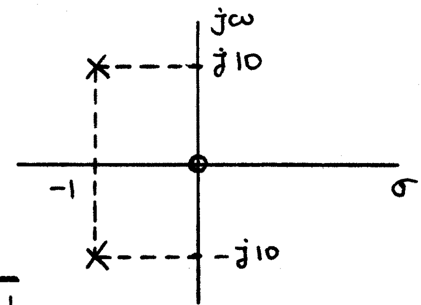
(b) $$H^{-1}(s) = \frac{1}{H(s)} = \frac{s^2 + s - 6}{s^2 - s - 2} = \frac{Y(s)}{X(s)}$$

inverse system :

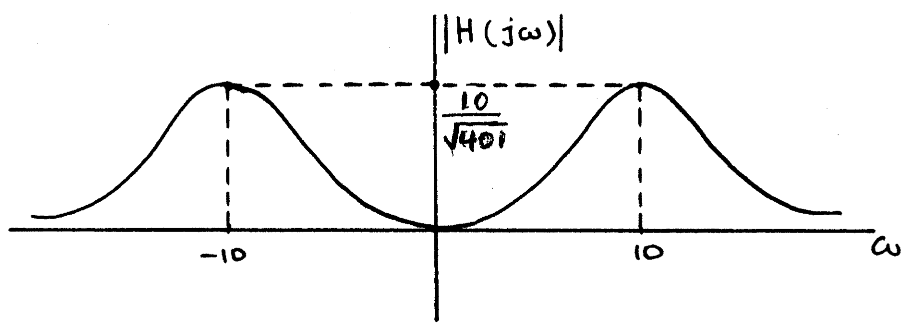
$$\frac{d^2}{dt^2}y(t) - \frac{d}{dt}y(t) - 2y(t) = \frac{d^2}{dt^2}x(t) + \frac{d}{dt}x(t) - 6x(t)$$

6.26

(a) $$H(s) = \frac{s}{s^2+2s+101} = \frac{s}{(s+1)^2+10^2}$$

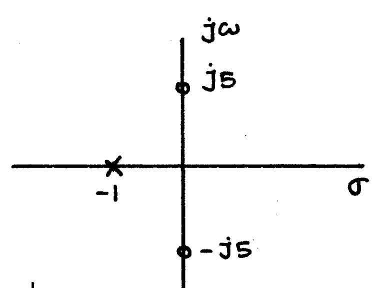


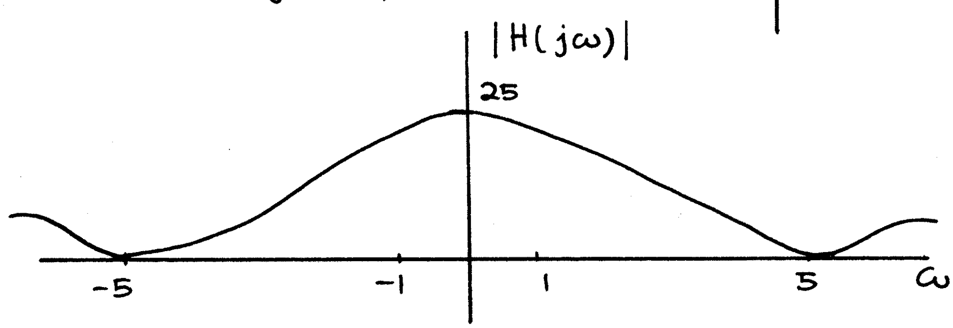
$$|H(j\omega)| = \frac{\omega}{|j\omega+1-j10||j\omega+1+j10|}$$



(b) $$H(s) = \frac{s^2+25}{s+1}$$

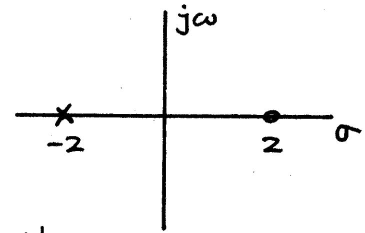
$$|H(j\omega)| = \frac{|j\omega+j5||j\omega-j5|}{|j\omega+1|}$$

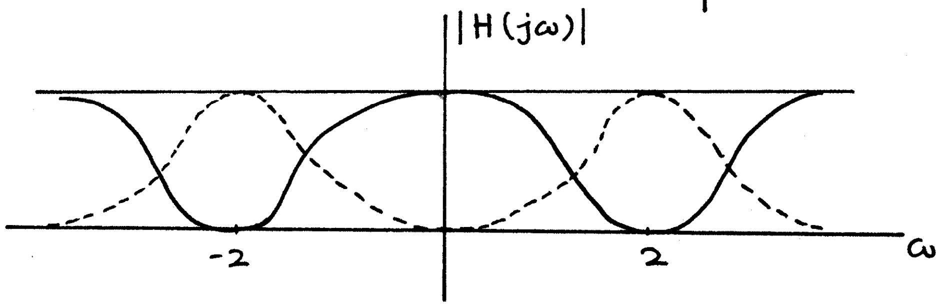




(c) $H(s) = \dfrac{s-2}{s+2}$

$|H(j\omega)| = \dfrac{|j\omega - 2|}{|j\omega + 2|}$





$\boxed{6.27}$ M poles : $d_k = \alpha_k + j\beta_k$

M zeros : $c_k = -\alpha_k + j\beta_k$

(a) $H(s) = \dfrac{\prod_{k=1}^{M}(s - c_k)}{\prod_{k=1}^{M}(s - d_k)}$ and $|H(j\omega)| = \dfrac{\prod_{k=1}^{M}|j\omega - \alpha_k - j\beta_k|}{\prod_{k=1}^{M}|j\omega + \alpha_k - j\beta_k|}$

$$= \frac{\prod_{k=1}^{M}|j(\omega - \beta_k) - \alpha_k|}{\prod_{k=1}^{M}|j(\omega - \beta_k) + \alpha_k|}$$

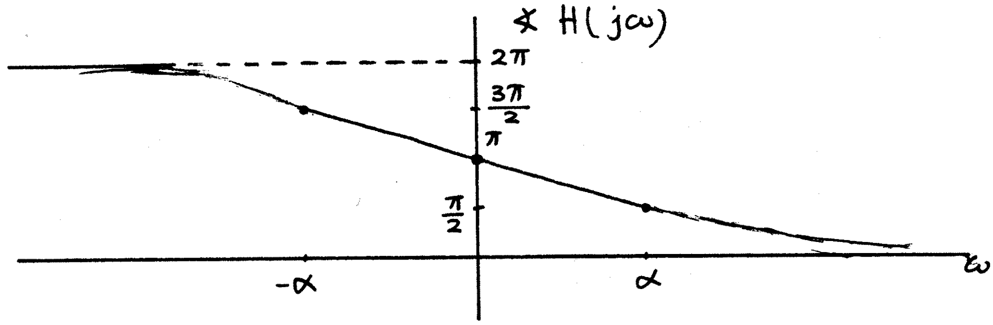
$$= \frac{\prod_{k=1}^{M}\sqrt{\alpha_k^2 + (\omega - \beta_k)^2}}{\prod_{k=1}^{M}\sqrt{\alpha_k^2 + (\omega - \beta_k)^2}}$$

$H(s) = 1$

(b) For $H(s) = \dfrac{s - \alpha}{s + \alpha}$, $H(j\omega) = \dfrac{j\omega - \alpha}{j\omega + \alpha}$

$\measuredangle H(j\omega) = \pi - \tan^{-1}\left(\frac{\omega}{\alpha}\right) - \tan^{-1}\left(\frac{\omega}{\alpha}\right)$

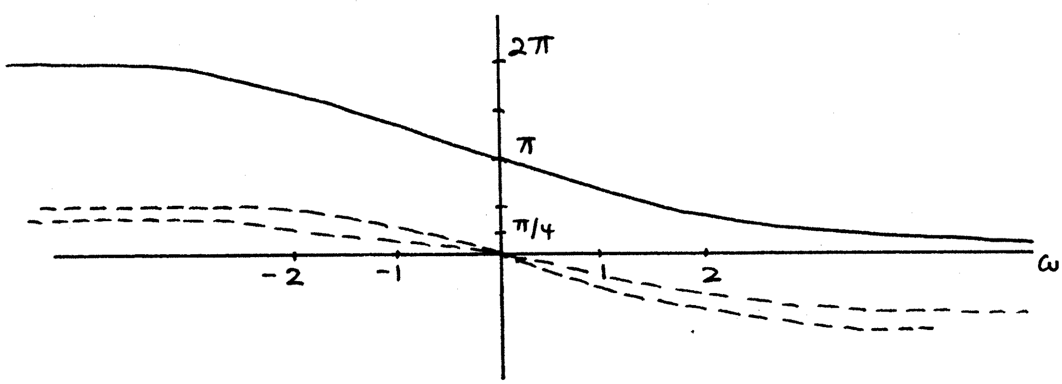
$$\measuredangle H(j\omega) = \pi - 2\tan^{-1}\left(\frac{\omega}{\alpha}\right)$$



$\boxed{6.28}$

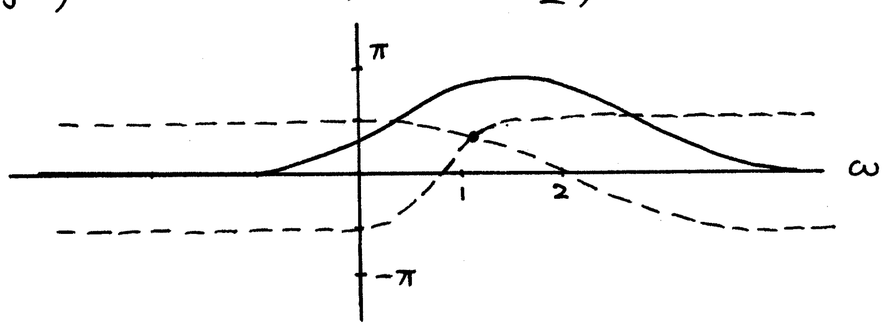
(a) $$H(s) = \frac{s-1}{s+2}, \quad H(j\omega) = \frac{j\omega - 1}{j\omega + 1}$$

$$\measuredangle H(j\omega) = \pi - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right)$$
$$= \pi - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right)$$



(b) $$H(s) = \frac{s+1}{s+2}$$

$$\measuredangle H(j\omega) = \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right)$$



(c) $$H(s) = \frac{1}{s^2+2s+101}$$

$$= \frac{1}{(s+1)^2+10^2}$$

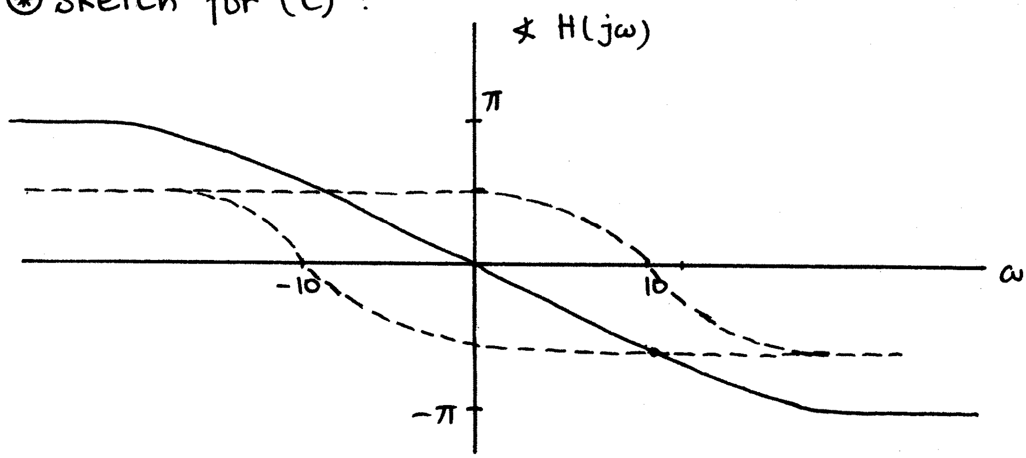
$$H(j\omega) = \frac{1}{(j\omega+1+j10)(j\omega+1-j10)}$$

$$\sphericalangle H(j\omega) = -\left(\tan^{-1}\left(\frac{\omega+10}{1}\right)+\tan^{-1}\left(\frac{\omega-10}{1}\right)\right)$$

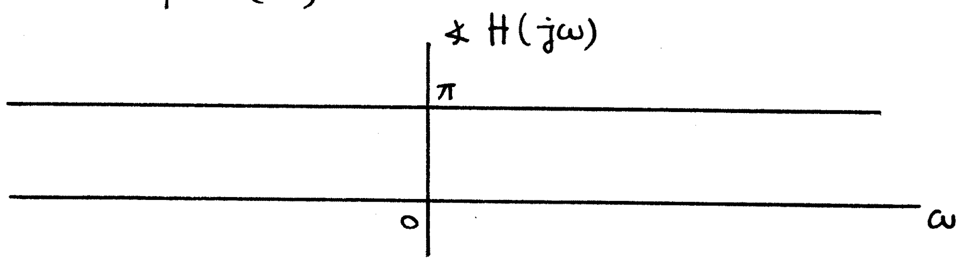
(d) $H(s) = s^2 \implies H(j\omega) = -\omega^2$

$$\sphericalangle H(j\omega) = \pi$$

⊛ Sketch for (c) :



⊛ Sketch for (d) :



$\boxed{6.29}$

$$H(s) = \frac{(s+2)(s-1)}{(s+4)(s+3)(s+5)} \qquad \text{non-minimum phase}$$

(a) The zeros of $H(s)$: $s = -2, 1$
Since one of them is in the right half plane, the <u>inverse system</u> can <u>not</u> be stable and causal.

(b) $$H_{min}(s) = \frac{(s+2)(s+1)}{(s+4)(s+3)(s+5)}$$

$$H_{ap}(s) = \frac{s-1}{s+1}$$

$$H(s) = H_{min}(s) \, . \, H_{ap}(s)$$

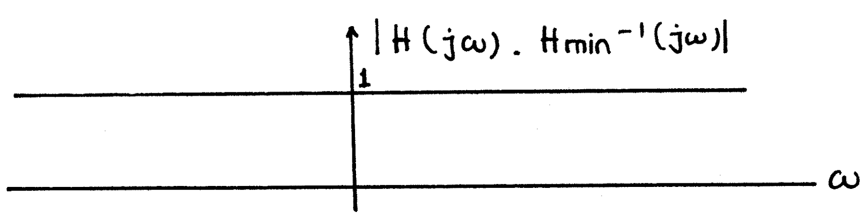
(c) $$H_{min}^{-1}(s) = \frac{(s+4)(s+3)(s+5)}{(s+1)(s+2)}$$

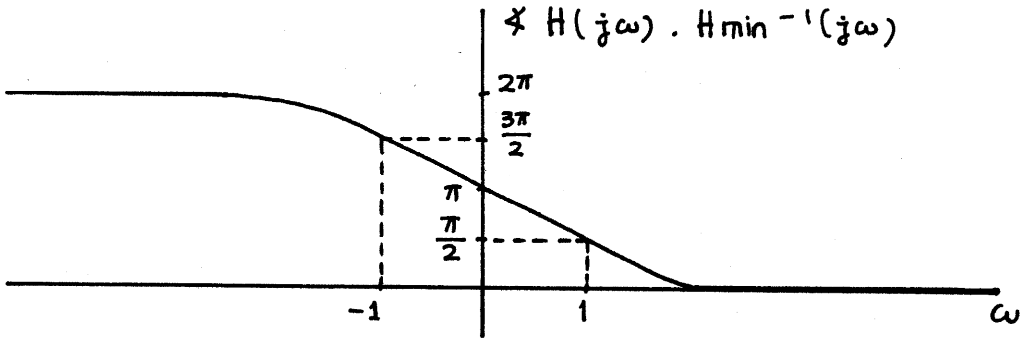
The poles of $H_{min}^{-1}(s)$ are : $s = -1, -2$.
All are in the left half plane.
So $h^{-1}_{min}(t)$ <u>can be</u> both stable and causal

(d) $$H(s) \, . \, H_{min}^{-1}(s) = \frac{s-1}{s+1} = H_{ap}(s)$$

$$H_{ap}(j\omega) = \frac{j\omega - 1}{j\omega + 1} \quad , \quad |H_{ap}(j\omega)| = 1$$

$$\measuredangle H_{ap}(j\omega) = \pi - \tan^{-1}(\omega) - \tan^{-1}(\omega)$$
$$= \pi - 2\tan^{-1}(\omega)$$





$\boxed{6.29}$

(e) Generalization for $H(s) = H'(s).(s-c)$
where $H'(s)$ is a minimum phase part, $\boxed{Re(c)>0}$

$$H_{min}(s) = H'(s)\ (s+c)$$

$$H_{ap}(s) = \frac{s-c}{s+c}$$

$$H_{min}^{-1}(s) = \frac{1}{H'(s)\ (s+c)}$$

$$H(s).\ H^{-1}_{min}(s) = H_{ap}(s)$$

$$H_{ap}(j\omega) = \frac{j\omega - c}{j\omega + c}$$

$$\left| H_{ap}(j\omega) \right| = 1$$

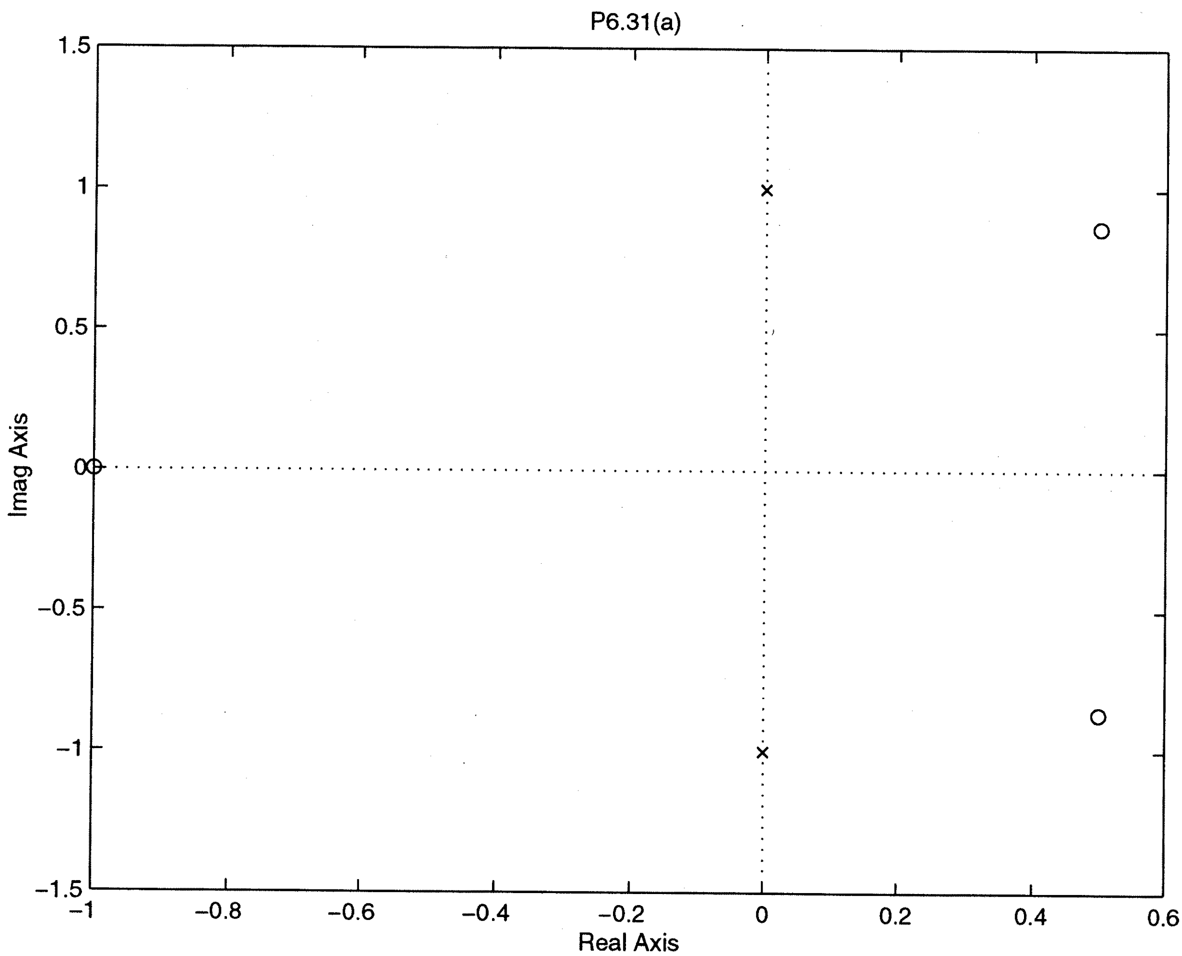
$$\measuredangle H_{ap}(j\omega) = \pi - 2\tan^{-1}\left(\frac{\omega}{c}\right)$$

6.30


```
P6.30 :
=======
Part (a) :
----------

ans =

        0 + 1.4142i
        0 - 1.4142i


ans =

  -2.5468
   0.2734 + 0.5638i
   0.2734 - 0.5638i

Part (b) :
----------

ans =

  -1.0000
   0.5000 + 0.8660i
   0.5000 - 0.8660i


ans =

  -0.0000 + 1.0000i
  -0.0000 - 1.0000i
   0.0000 + 1.0000i
   0.0000 - 1.0000i

Part (c) :
----------

ans =

  -1.0000 + 1.2247i
  -1.0000 - 1.2247i


ans =

  -1.0000 + 2.0000i
  -1.0000 - 2.0000i
  -2.0000
```

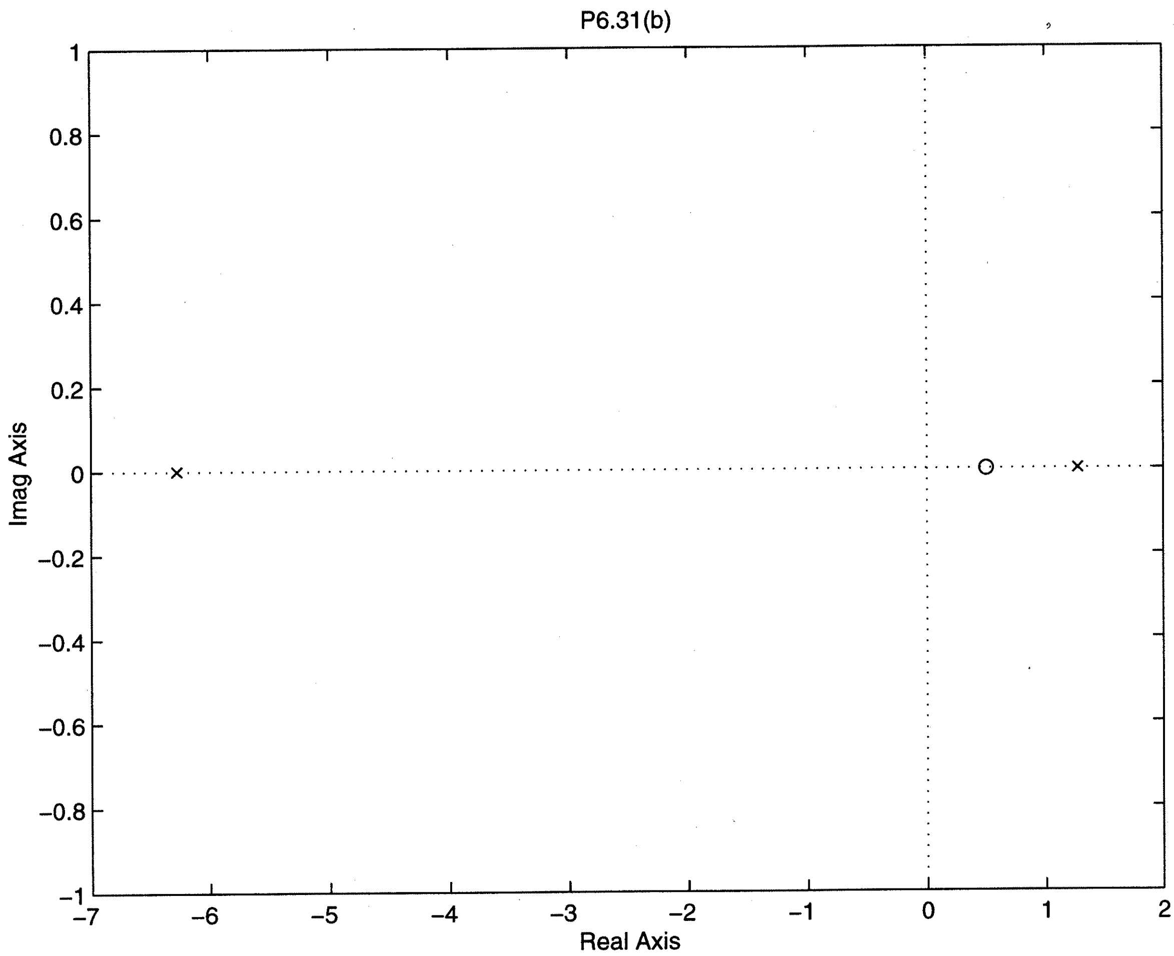
P 6.31

- Plot 1 of 2 -



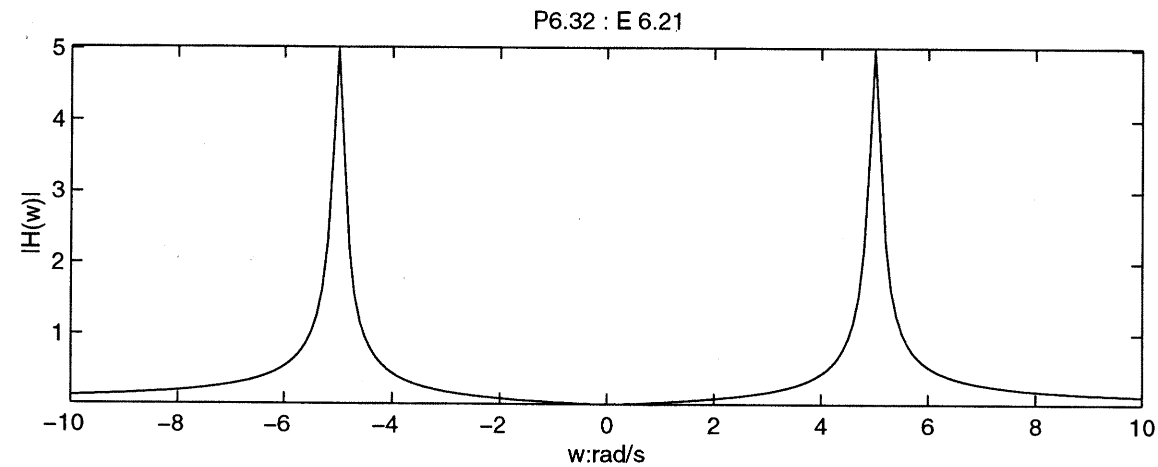
P 6.31

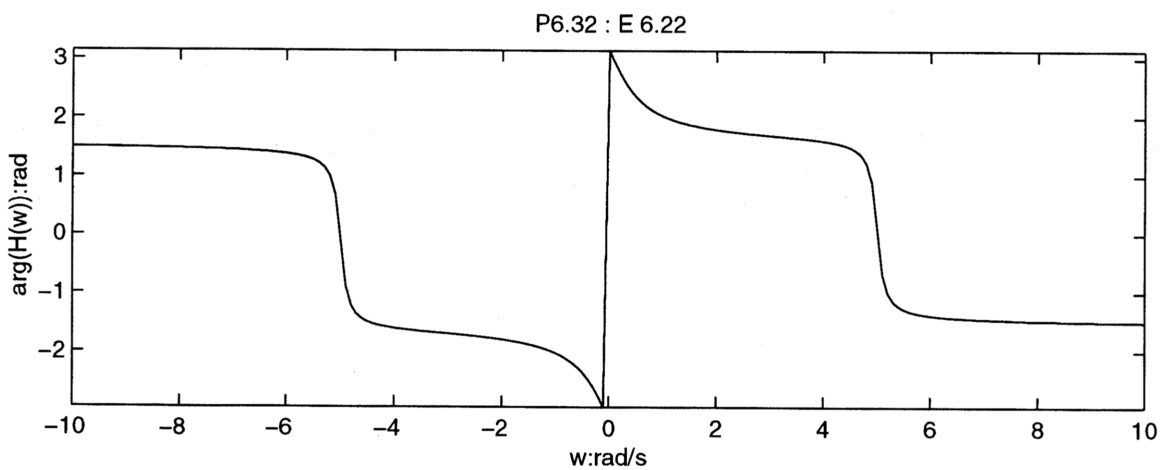
- Plot 2 of 2 -



P 6.32

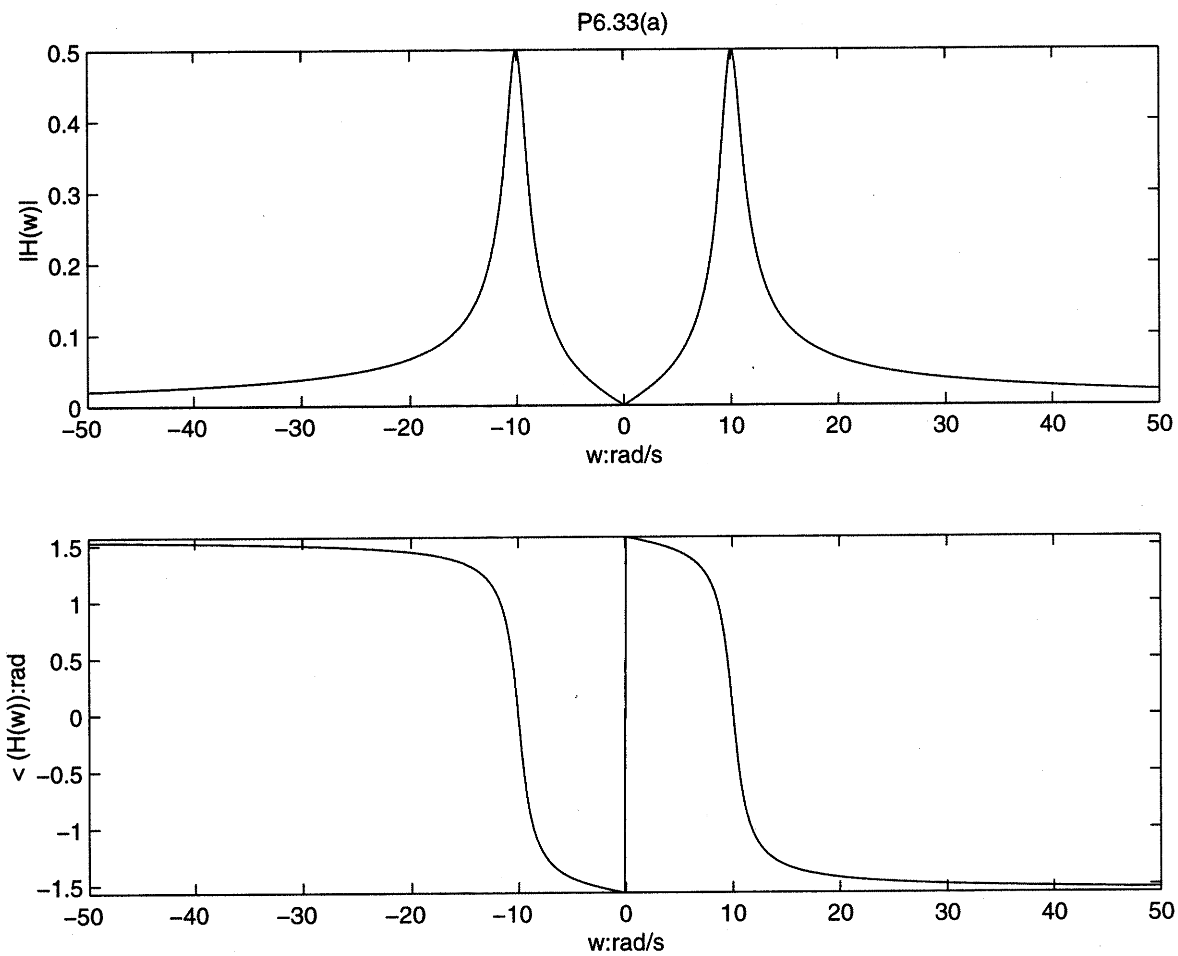
- Plot 1 of 1 -





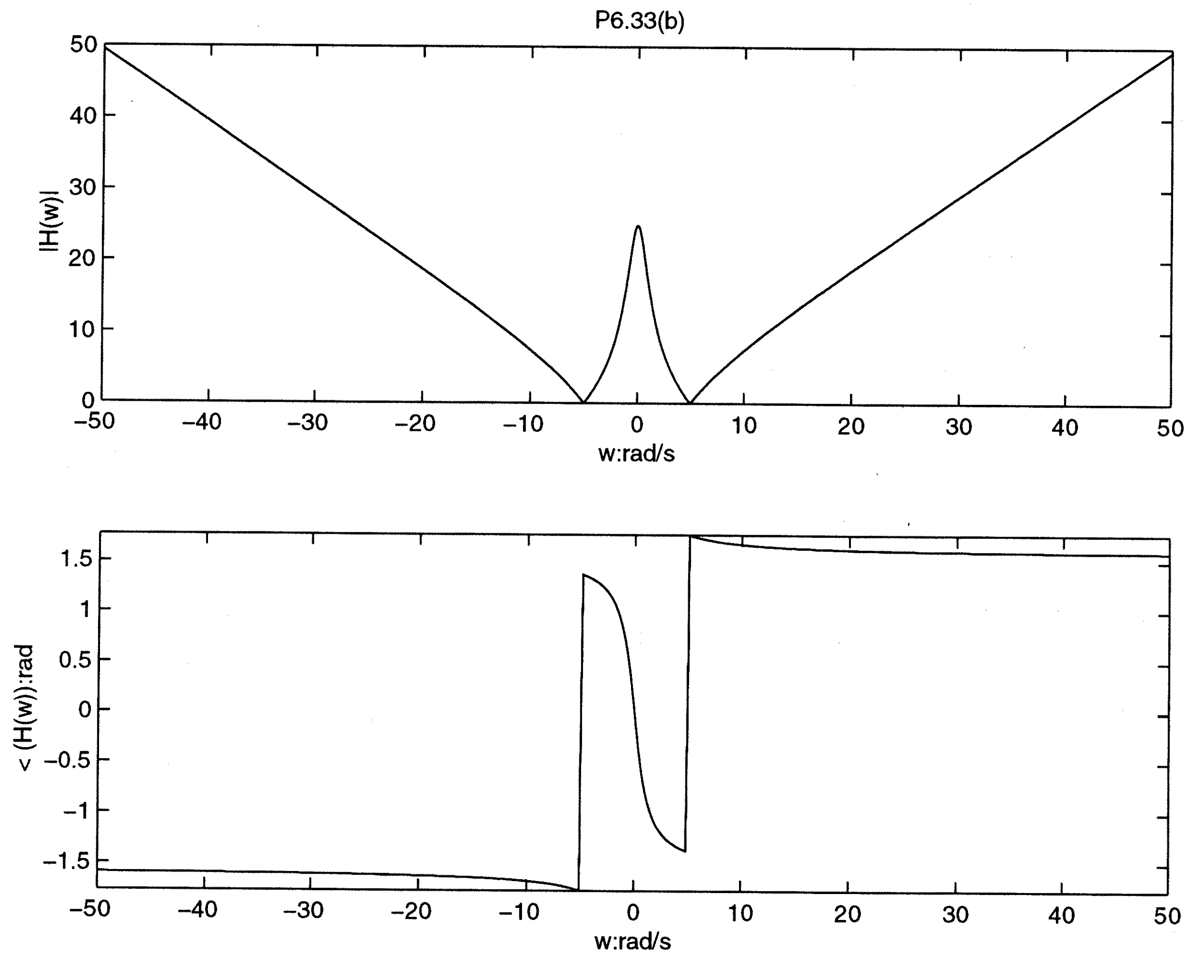
P 6.33

- Plot 1 of 3 -



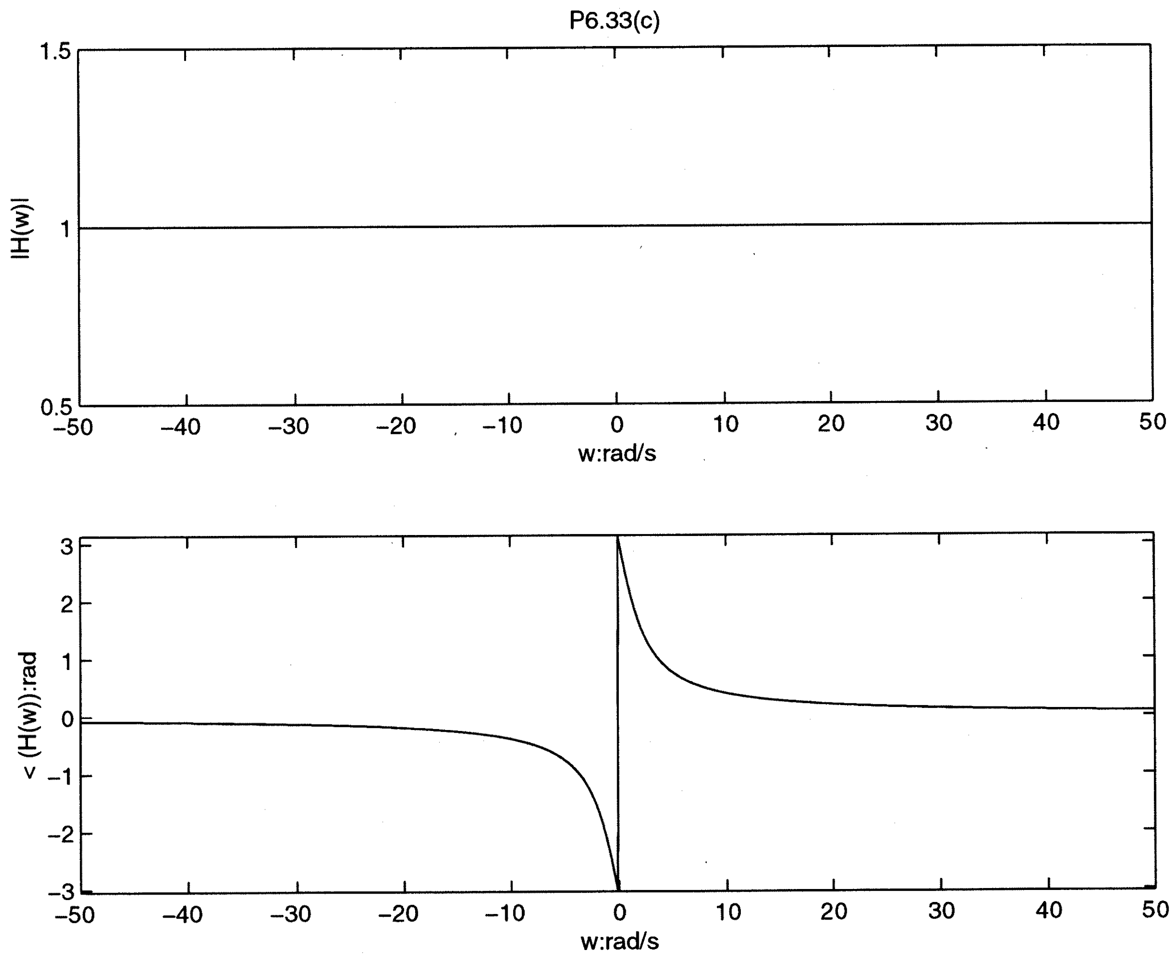
P 6.33

– Plot 2 of 3 –



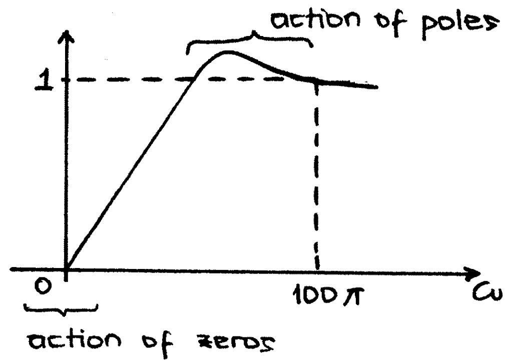
P 6.33

- Plot 3 of 3 -



$\boxed{6.34}$

(a) Need to have at least 1 zero @ $\omega = 0$

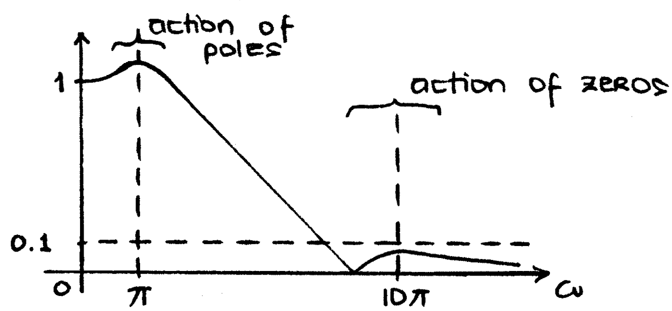


2 conjugate poles are needed around the transition

One possible solution is :

$$H(s) = \frac{s^2}{(s + 25 + j10\pi)(s + 25 - j10\pi)}$$

(b)

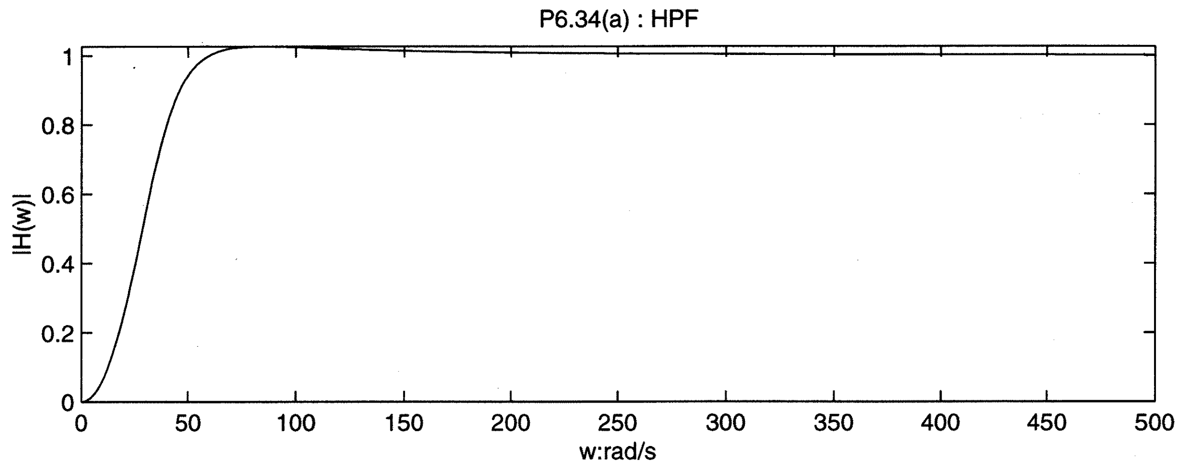


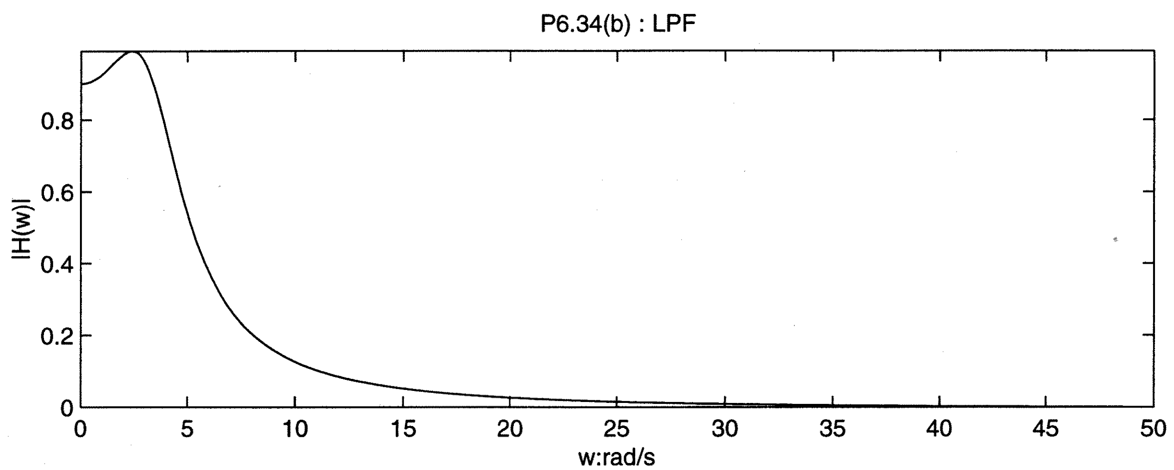
One possible solution is :

$$H(s) = \frac{(s - j50)(s + j50)}{(s + 2 - j\pi)(s + 2 + j\pi)}$$

P 6.34

- Plot 1 of 1 -





6.35

```
P6.35 :
=======
Part (a) :
==========

a =
                    x1            x2
        x1    -2.00000             0
        x2     0.50000             0

b =
                    u1
        x1     2.00000
        x2           0

c =
                    x1            x2
        y1     1.00000       1.00000

d =
                    u1
        y1           0

Continuous-time system.
Part (b) :
==========

a =
                    x1            x2
        x1    -2.00000      -2.50000
        x2     4.00000             0

b =
                    u1
        x1     2.00000
        x2           0

c =
                    x1            x2
        y1     1.50000             0

d =
                    u1
        y1           0

Continuous-time system.
Part (c) :
==========

a =
                    x1            x2            x3
        x1    -3.00000      -1.00000      -0.50000
        x2           0      -3.00000      -1.00000
        x3           0       2.00000             0

b =
                    u1
        x1     0.70711
        x2     2.82843
        x3           0

c =
                    x1            x2            x3
        y1     2.82843             0             0

d =
                    u1
        y1           0

Continuous-time system.
```

6.36

```
P6.36 :
=======
Part (a) :
==========

Transfer function:
  -2 s - 2
-------------
s^2 + 4 s + 3

Part (b) :
==========

Transfer function:
   2 s - 1
-------------
s^2 + 5 s - 8
```