

Transformada de Fourier - Tabela e Propriedades

$f(t)$	$F(\omega), F = \mathcal{F}(f)$
$e^{-at} u(t)$	$\frac{1}{a + i\omega}, a > 0$
$e^{-a t }$	$\frac{2a}{a^2 + \omega^2}, a > 0$
$\delta(t)$	1
1	$2\pi \delta(\omega)$
$\cos(at)$	$\pi [\delta(\omega - a) + \delta(\omega + a)]$
$\sin(at)$	$\pi [\delta(\omega - a) - \delta(\omega + a)]$
$u(t)$	$\pi \delta(\omega) + \frac{1}{i\omega}$
$\text{sgn}(t)$	$\frac{2}{i\omega}$
$\text{rect}\left(\frac{t}{\tau}\right)$	$\tau \text{sinc}\left(\frac{\omega\tau}{2}\right)$
$\frac{\tau}{\pi} \text{sinc}(\tau t)$	$\text{rect}\left(\frac{\omega}{2\tau}\right)$
$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2} \text{sinc}^2\left(\frac{\omega\tau}{4}\right)$
$\frac{\tau}{2\pi} \text{sinc}^2\left(\frac{\tau t}{2}\right)$	$\Delta\left(\frac{\omega}{2\tau}\right)$
$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi} e^{-\sigma^2\omega^2/2}$

Identidade de Parseval:

$$\|f\|_2^2 = \int_{\mathbb{R}} |f(x)|^2 dx = \frac{1}{2\pi} \int_{\mathbb{R}} |F(\omega)|^2 d\omega = \frac{1}{2\pi} \|F\|_2^2 = \text{energia do sinal } f, F = \mathcal{F}(f)$$

$f(t)$	$F(\omega), F = \mathcal{F}(f)$	propriedade
$F(t)$	$2\pi f(-\omega)$	simetria
$f(at), a \in \mathbb{R}$	$\frac{1}{ a } F\left(\frac{\omega}{a}\right)$	escalonamento
$f(t - a)$	$F(\omega) e^{-i\omega a}$	deslocamento no tempo
$e^{iat} f(t)$	$F(\omega - a)$	deslocamento na frequência
$(f * g)(t)$	$F(\omega) G(\omega)$	convolução no tempo
$f(t) g(t)$	$\frac{1}{2\pi} F(\omega) \star G(\omega)$	convolução na frequência
$f^{(n)}(t)$	$i^n \omega^n F(\omega)$	derivação no tempo
$t^n f(t)$	$i^n F^{(n)}(\omega)$	derivação na frequência
$\int_{-\infty}^t f(x) dx$	$\frac{F(\omega)}{i\omega} + \pi F(0) \delta(\omega)$	integração no tempo

Transformadas de Fourier

$$\hat{f}(\omega) \equiv F(\omega) \equiv \mathcal{F}\{f(t)\} \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \quad \text{Transformada de Fourier do domínio do tempo}$$

$$f(t) \equiv \mathcal{F}^{-1}\{F(\omega)\} \stackrel{\text{def}}{=} \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega \quad \omega \equiv 2\pi f \text{ (Frequência angular)}$$

Forma Trigonométrica (senos e cossenos)

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(2\pi n f_0 t) + b_n \sin(2\pi n f_0 t),$$

$$a_n = \frac{\int_{t_1}^{t_1+T_0} g(t) \cos(2\pi n f_0 t) dt}{\int_{t_1}^{t_1+T_0} \cos^2(2\pi n f_0 t) dt} = \begin{cases} \frac{1}{T_0} \int_{t_1}^{t_1+T_0} g(t) dt, & n=0 \\ \frac{2}{T_0} \int_{t_1}^{t_1+T_0} g(t) \cos(2\pi n f_0 t) dt, & n=1,2,3,\dots \end{cases}$$

$$b_n = \frac{\int_{t_1}^{t_1+T_0} g(t) \sin(2\pi n f_0 t) dt}{\int_{t_1}^{t_1+T_0} \sin^2(2\pi n f_0 t) dt} = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} g(t) \sin(2\pi n f_0 t) dt, \quad n=1,2,3,\dots$$

Fase e Magnitude:

$$C_n = \sqrt{a_n^2 + b_n^2} \quad \theta_n = -\arctg\left(\frac{b_n}{a_n}\right), \quad n \geq 1$$

Exponencial:

$$g(t) = \sum_{n=-\infty}^{\infty} D_n e^{j2\pi n f_0 t}$$

Tabela de Relações Trigonométricas

01) $\sin^2 x + \cos^2 x = 1$	02) $1 + \operatorname{tg}^2 x = \sec^2 x$
03) $1 + \cotg^2 x = \operatorname{cosec}^2 x$	04) $\sin(-x) = -\sin x$
05) $\cos(-x) = \cos x$	06) $\operatorname{tg}(-x) = -\operatorname{tg} x$
07) $\operatorname{cosec} x = \frac{1}{\sin x}$	08) $\sec x = \frac{1}{\cos x}$
09) $\cotg x = \frac{1}{\operatorname{tg} x}$	10) $\operatorname{tg} x = \frac{\sin x}{\cos x}$
11) $\cotg x = \frac{\cos x}{\sin x}$	12) $\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$
13) $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$	14) $\operatorname{tg}(a + b) = \frac{\operatorname{tg} a + \operatorname{tg} b}{1 - \operatorname{tg} a \operatorname{tg} b}$
15) $\operatorname{tg}(a - b) = \frac{\operatorname{tg} a - \operatorname{tg} b}{1 + \operatorname{tg} a \operatorname{tg} b}$	16) $\cos^2 x = \frac{1}{2} (1 + \cos 2x)$
17) $\sin^2 x = \frac{1}{2} (1 - \cos 2x)$	18) $\sin 2x = 2 \sin x \cos x$
19) $\cos 2x = \cos^2 x - \sin^2 x = 1 - 2 \sin^2 x = 2 \cos^2 x - 1$	20) $\operatorname{tg} 2x = \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x}$
21) $\left \sin \frac{x}{2} \right = \sqrt{\frac{1 - \cos x}{2}}$	22) $\left \cos \frac{x}{2} \right = \sqrt{\frac{1 + \cos x}{2}}$
23) $\operatorname{tg} \frac{x}{2} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$	24) $\sin x \cos y = \frac{1}{2} [\sin(x - y) + \sin(x + y)]$
25) $\sin x \sin y = \frac{1}{2} [\cos(x - y) - \cos(x + y)]$	26) $\cos x \cos y = \frac{1}{2} [\cos(x - y) + \cos(x + y)]$
27) $\cos x \sin y = \frac{1}{2} [\sin(x + y) - \sin(x - y)]$	28) $\sin x - \sin y = 2 \sin\left(\frac{x - y}{2}\right) \cos\left(\frac{x + y}{2}\right)$
29) $\sin x \cos x = \frac{1}{2} \sin 2x$	30) $1 - \cos x = 2 \sin^2 \frac{x}{2}$
31) $1 + \cos x = 2 \cos^2 \frac{x}{2}$	32) $1 \pm \sin x = 1 \pm \cos\left(\frac{\pi}{2} - x\right)$
33) $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$	

