Chapter 9

9.1

(a) The closed-loop gain of the feedback amplifier is given by

$$T = \frac{A}{1+\beta A} \qquad (1)$$

We are given the following values for the forward amplification A and feedback factor β:

$$A = 2{,}500$$

$$\beta = 0.01$$

Substituting these values in Eq. (1):

$$T = \frac{2500}{1 + 0.01 \times 2500} = \frac{2500}{26} = 92.15$$

(b) The sensitivity of the feedback amplifier to changes in A is given by

$$S_T^A = \frac{\Delta T/T}{\Delta A/A} = \frac{1}{1+\beta A} = \frac{1}{26}$$

With $(\Delta A/A) = 10\% = 0.10$, we thus have

$$\frac{\Delta T}{T} = S_T^A \left(\frac{\Delta A}{A}\right)$$

$$= \frac{1}{26}(0.10) = 0.0038 = 0.38\%$$

9.2

(a) The closed-loop gain of the feedback system is

$$T = \frac{G_a G_p}{1 + H G_a G_p}$$

(b) The return difference of the system is

$$F = 1 + H G_a G_p$$

Hence the sensitivity of T with respect to changes in G_p is

$$S_T^{G_p} = \frac{\Delta T / T}{\Delta G_p / G_p}$$

$$= \frac{1}{F}$$

$$= \frac{\quad}{1 + H G_a G_p}$$

(c) We are given: $H = 1$ and $G_p = 1.5$. Hence for $S_T^{G_p} = 1\% = 0.01$, we require

$$F = 100$$

The corresponding value of G_a is therefore

$$G_a = \frac{F - 1}{H G_p}$$

$$= \frac{99}{1.5} = 66$$

9.3 The local feedback around the motor has the closed-loop gain $G_p/(1+HG_p)$. The closed-loop gain of the whole system is therefore

$$T = \frac{K_f G_c G_p/(1+HG_p)}{1+K_f G_c G_p/(1+HG_p)}$$

$$= \frac{K_f G_c G_p}{1+HG_p+K_f G_c G_p}$$

9.4 The closed-loop gain of the operational amplifier is

$$\frac{V_2(s)}{V_1(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

We are given $Z_1(s) = R_1$ and $Z_2(s) = R_2$. The closed-loop gain or transfer function of the operational amplifier in Fig. P9.4 is therefore

$$\frac{V_2(s)}{V_1(s)} = -\frac{R_2}{R_1}$$

9.5

(a) From Fig. P9.5, we have

$$Z_1(s) = R_1 + \frac{1}{sC_1}$$

$$Z_2(s) = R_2$$

The transfer function of the operational amplifier is therefore

$$\frac{V_2(s)}{V_1(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

$$= -\frac{R_2}{R_1 + \dfrac{1}{sC_1}}$$

$$= -\frac{sC_1 R_2}{1 + sC_1 R_1} \qquad (1)$$

(b) For positive values of frequency ω that satisfy the condition

$$\frac{1}{\omega C_1} >> R_1$$

we may approximate Eq. (1) as

$$\frac{V_2(s)}{V_1(s)} \simeq -sC_1 R_2$$

That is, the operational amplifier acts as a differentiator.

9.6

(a) The "sensor" transfer function is

$$H(s) = \frac{1}{s}$$

Hence the closed-loop gain (i.e., transfer function) of the phase-locked loop model shown in Fig. P9.6 is

$$\frac{V(s)}{\Phi_1(s)} = \frac{G(s)}{1 + G(s)\,H(s)}$$

$$= \frac{K_0\,H(s)\,(1/K_v)}{1 + \frac{1}{s}\,K_0\,H(s)}$$

$$= \frac{s\,K_0\,H(s)\,(1/K_v)}{s + K_0\,H(s)} \qquad (1)$$

(b) For positive frequencies that satisfy the condition $|K_0\,H(j\omega)| \gg \omega$, we may approximate Eq. (1) as

$$\frac{V(s)}{\Phi_1(s)} \simeq \frac{s\,K_0\,H(s)\,(1/K_v)}{K_0\,H(s)}$$

$$= \frac{s}{K_v}$$

Correspondingly, we may write

$$v(t) \simeq \frac{1}{K_v}\,\frac{d}{dt}\,\phi_1(t)$$

9.7 Throughout this problem, $H(s) = 1$.

(a) We are given

$$G(s) = \frac{15}{(s+1)(s+3)}$$

The control system is therefore type 0. The steady-state error for unit step is

$$\epsilon_{ss} = \frac{1}{1 + K_p}$$

where

$$K_p = \lim_{s \to 0} G(s) H(s)$$

That is,

$$K_p = \lim_{s \to 0} \frac{15}{(s+1)(s+3)}$$

$$= \frac{15}{3} = 5$$

Hence,

$$\epsilon_{ss} = \frac{1}{1+5} = \frac{1}{6}$$

For both ramp and parabolic inputs, the steady-state error is infinitely large.

(b) For

$$G(s) = \frac{5}{s(s+1)(s+4)}$$

the control system is type 1. The steady-state error for a step input is zero. For a ramp of unit slope, the steady-state error is

$$\epsilon_{ss} = \frac{1}{K_v}$$

where

$$K_v = \lim_{s \to 0} \frac{5}{(s+1)(s+4)}$$

$$= \frac{5}{4}$$

The steady-state error is therefore

$$\epsilon_{ss} = \frac{4}{5}$$

For a parabolic input the steady-state error is infinitely large.

(c) For

$$G(s) = \frac{5(s+1)}{s^2(s+3)}$$

the control system is type 2. Hence the steady-state error is zero for both step and ramp inputs. For a unit parabolic input the steady-state error is

$$\epsilon_{ss} = \frac{1}{K_a}$$

where

$$K_a = \lim_{s \to 0} \frac{5(s+1)}{s+3}$$

$$= \frac{5}{3}$$

The steady-state error is therefore 3/5.

(d) For

$$G(s) = \frac{5(s+1)(s+2)}{s^2(s+3)}$$

the control system is type 2. The steady-state error is therefore zero for both step and ramp inputs. For a unit parabolic input the steady-state error is

$$\epsilon_{ss} = \frac{1}{K_a}$$

where

$$K_a = \lim_{s \to 0} \frac{5(s+1)(s+2)}{s+3}$$

$$= \frac{10}{3}$$

The steady-state error is therefore 3/10.

9.8 For the results on steady-state errors calculated in Problem 9.7 to hold, all the feedback control systems in parts (a) to (d) of the problem have to be stable.

(a) $$G(s)H(s) = \frac{15}{(s+1)(s+3)}$$

The characteristic equation is

$$A(s) = (s+1)(s+3) + 15$$

$$= s^2 + 4s + 18$$

both roots of which are in the left-half plane. The system is therefore stable.

(b) $$G(s)H(s) = \frac{5}{s(s+1)(s+4)}$$

$$= \frac{5}{s^3 + 5s^2 + 4s}$$

$$A(s) = s^3 + 5s^2 + 4s + 5$$

Applying the Routh-Hurwitz criterion:

$$\begin{array}{lll} s^3 & 1 & 4 \\ s^2 & 5 & 5 \\ s^1 & \frac{15}{5} & 0 \\ s^0 & 5 & 0 \end{array}$$

There are no sign changes in the first column coefficients of the array; the system is therefore stable.

(c) $G(s)\,H(s) = \dfrac{5(s+1)}{s^2(s+3)}$

$A(s) = s^3 + 3s^2 + 5s + 5$

The Routh-Hurwitz array is

$$\begin{array}{ccc} s^3 & 1 & 5 \\ s^2 & 3 & 5 \\ s^1 & \frac{10}{3} & 0 \\ s^0 & 5 & 0 \end{array}$$

Here again there are no sign changes in the first column of coefficients, and the control system is therefore stable.

(d) $G(s)\,H(s) = \dfrac{5(s+1)(s+2)}{s^2(s+3)}$

$A(s) = s^3 + 3s^2 + 5s^2 + 15s + 10$

$= s^3 + 8s^2 + 15s + 10$

The Routh-Hurwitz array is

s^3	1	15
s^2	8	10
s^1	$\frac{110}{8}$	0
s^0	10	0

Here again there are no sign changes in the first column of array coefficients, and the control system is therefore stable.

9.9

(a) $s^4 + 2s^2 + 1 = 0$

Equivalently, we may write

$$(s^2+1)^2 = 0$$

which has double roots at $s = \pm j$. The system is therefore on the verge of instability.

(b) $s^4 + s^3 + s + 0.5 = 0$

By inspection, we can say that this feedback control system is unstable because the term s^2 is missing from the characteristic equation.

We can verify this observation by constructing the Routh array:

s^4	1	0	0.5
s^3	1	1	0
s^2	-1	0.5	0
s^1	$+1.5$	0	
s^0	0.5	0	

There are two sign changes in the first column of array coefficients, indicating that the characteristic equation has a pair of complex-conjugate roots in the right half of the s-plane. The system is therefore unstable as previously observed.

(c) $$s^4 + 2s^3 + 2s^2 + 2s + 4 = 0$$

The Routh array is

s^4	1	3	4
s^3	2	2	0
s^2	2	4	0
s^1	-2	0	
s^0	4	0	

There are two sign changes in the first column of array coefficients, indicating the presence of two roots of the characteristic equation in the right half of the s-plane. The control system is therefore unstable.

9.10 The characteristic equation of the control system is

$$s^3 + s^2 + s + K = 0$$

Applying the Routh-Hurwitz criterion:

s^3	1	1
s^2	1	K
s^1	1 − K	0
s^0	K	0

The control system is therefore stable provided the the parameter K satisfies the condition:

$$0 < K < 1$$

9.11

(a) The characteristic equation of the feedback system is

$$a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0$$

Applying the Routh-Hurwitz criterion:

$$\begin{array}{lll} s^3 & a_3 & a_1 \\ s^2 & a_2 & a_0 \\ s^1 & \dfrac{a_2 a_1 - a_3 a_0}{a_2} & 0 \\ s^0 & a_0 & \end{array}$$

For the system to be stable, the following conditions must be satisfied

$$a_3 > 0, \quad a_2 > 0, \quad a_0 > 0 \quad \text{and}$$

$$\frac{a_2 a_1 - a_3 a_0}{a_2} > 0 \qquad (1)$$

(b) For the characteristic equation

$$s^3 + s^2 + s + K = 0$$

we have $a_3 = 1$, $a_2 = 1$, $a_1 = 1$, and $a_0 = K$. Hence, applying the condition (1):

$$\frac{1 \times 1 - 1 \times K}{1} > 0$$

or

$$K > 1$$

Also, we require that $K > 0$ since we must have $a_0 > 0$.

9.12

(a) We are given the loop transfer function

$$L(s) = \frac{K}{s(s^2 + s + 2)}, \qquad K > 0$$

$L(s)$ has three poles, one at $s = 0$ and the other two at $s = -\frac{1}{2} \pm j\frac{\sqrt{7}}{2}$. Hence, the root locus of $L(s)$ has 3 branches. It starts at these 3 pole locations and terminates at the zeros of $L(s)$ at $s = \infty$. The straight-line asymptotes of the root locus are defined by the angles (see Eq. (9.82))

$$\theta_k = \frac{(2k+1)\pi}{3}, \quad k = 0, 1, 2$$

or $\pi/3$, π, and $5\pi/3$. Their location point on the real axis of the s-plane is defined by (see Eq. (9.83))

$$\sigma_0 = \frac{0 + \left(-\frac{1}{2} + j\frac{\sqrt{7}}{2}\right) + \left(-\frac{1}{2} - j\frac{\sqrt{7}}{2}\right)}{3}$$

$$= -\frac{1}{3}$$

Since $L(s)$ has a pair of complex-conjugate poles, we need (in addition to the rules described in the text) a rule concerning the angle of which the root locus leaves a complex pole (i.e., angle of departure). Specifically, we wish to determine the angle θ_{dep} indicated in Fig. 1 for the problem at hand:

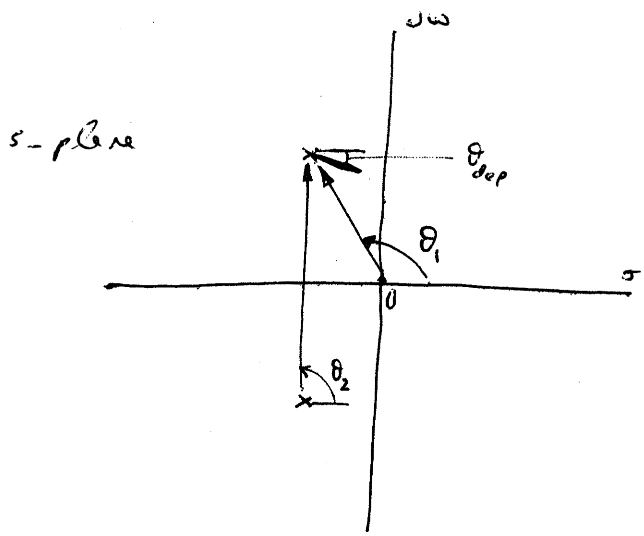


Figure 1

The angles θ_1 and θ_2 are defined by

$$\theta_1 = \tan^{-1}\left(\frac{\sqrt{7}/2}{-1/2}\right) = -\operatorname{Tan}^{-1}(2.6458) = 111.7^\circ$$

$$\theta_2 = 90^\circ$$

Applying the angle criterion (Eq. (9.)):

$$\theta_{dep} + 90^\circ + 111.7 = 180$$

Hence

$$\theta_{dep} = -21.70^\circ$$

We may thus sketch the root locus of the given system as in Fig. 2:

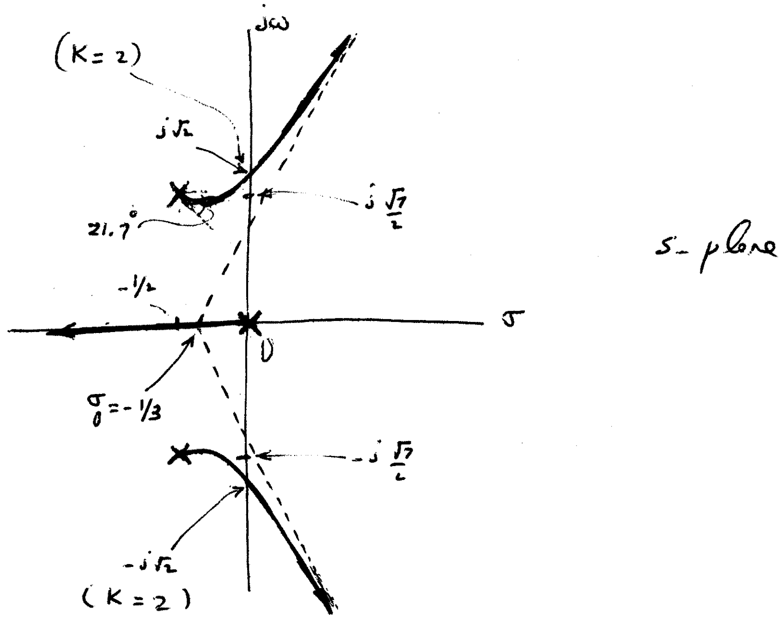


Figure 2

(b) The characteristic equation of the system is

$$s^3 + s^2 + 2s + K = 0$$

Applying the Routh-Hurwitz criterion:

s^3	1	2
s^2	1	K
s^1	2-K	0
s^0	K	0

The system is therefore on the verge of instability when $K = 2$. For this value of K the root locus intersects the $j\omega$-axis at $s^2 + K = 0$ or $s = \pm j\sqrt{K} = \pm j\sqrt{2}$, as indicated in Fig. 2

9.13 The loop transfer function of the feedback control system is

$$L(s) = G(s)\,H(s)$$

$$= \frac{0.2\,K_p}{(s+1)(s+3)}$$

The closed-loop transfer function of the system is

$$T(s) = \frac{L(s)}{1+L(s)}$$

$$= \frac{0.2\,K_p}{s^2 + 4s + (3 + 0.2\,K_p)} \qquad (1)$$

For a second-order system defined by

$$T(s) = \frac{T(0)\,\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad (2)$$

the damping factor is ζ and the natural frequency is ω_n. Comparing Eqs. (1) and (2):

$$2\zeta\omega_n = 4$$

$$\omega_n^2 = 3 + 0.2\,K_p$$

$$T(0)\,\omega_n^2 = 0.2\,K_p$$

Hence,

$$\omega_n = \sqrt{3 + 0.2\,K_p} \qquad (3)$$

$$\zeta = 2/\sqrt{3 + 0.2\,K_p} \qquad (4)$$

$$T(0) = 0.2\,K_p/(3 + 0.2\,K_p) \qquad (5)$$

We are required to have

$$\omega_n = 2 \text{ rad/s}$$

Hence the use of Eq. (3) yields

$$\sqrt{3 + 0.2\,K_p} = 2$$

That is,

$$K_p = \frac{4 - 3}{0.2} = 5$$

Next, the use of Eq. (4) yields

$$\zeta = \frac{2}{\sqrt{3 + 0.2 \times 5}}$$

$$= \frac{2}{\sqrt{4}} = 1$$

which means that the system is critically damped.

The time constant of a system is defined by

$$\tau = \frac{1}{\zeta\,\omega_n}$$

$$= \frac{1}{1 \times 2} = 0.5 \text{ seconds}$$

(Note : The use of Eq. (5) yields

$$T(0) = \frac{0.2 \times 5}{3 + 0.2 \times 5}$$

$$= \frac{1}{4} \Big)$$

9.14 The loop transfer function of the system is

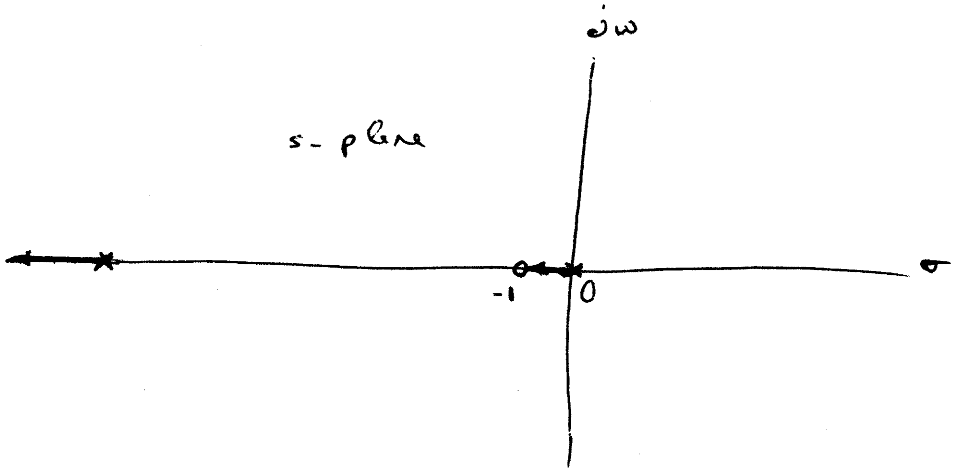
$$L(s) = \left(K_p + \frac{K_I}{s}\right)\left(\frac{0.25}{s+1}\right)$$

$$= 0.25\,K_p\left(1 + \frac{K_I/K_p}{s}\right)\left(\frac{1}{s+1}\right)$$

with $K_I/K_p = 0.1$ we have

$$L(s) = \frac{0.25\,K_p\,(s+0.1)}{s(s+1)} \tag{1}$$

The root locus of the system is therefore as shown below:



The closed-loop transfer function of the system is

$$T(s) = \frac{L(s)}{1+L(s)} \tag{2}$$

Substituting Eq. (1) in (2):

$$T(s) = \frac{0.25\, K_p (s + 0.1)}{s(s+1) + 0.25\, K_p (s + 0.1)} \qquad (3)$$

For a closed-loop at $s = -5$ we require the the denominator of $T(s)$ satisfy the following equation

$$(s+5)(s+a) = 0 \qquad (4)$$

where $s = -a$ is the other closed-loop pole of the system. Comparing the denominator of Eq. (3) with (4):

$$5 + a = 1 + 0.25\, K_p$$

$$5a = 0.1 \times 0.25\, K_p$$

Solving this pair of equations for K_p and a, we get

$$K_p = \frac{20}{1.225} = 16.33$$

$$a = 0.005 * K_p = 0.0816$$

9.15 The loop transfer function of the PD controller system is

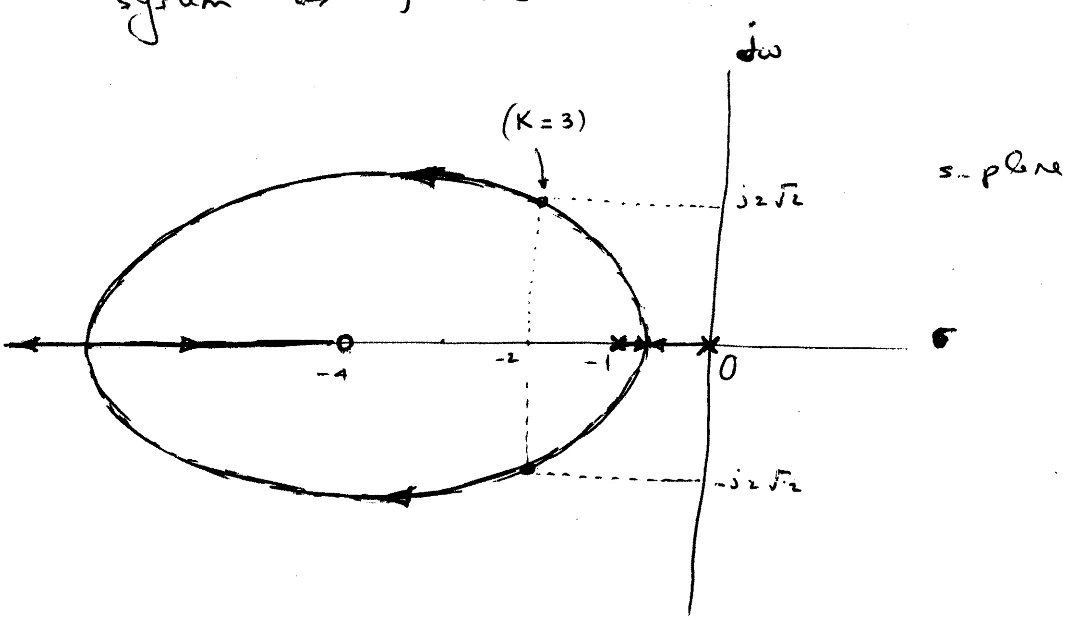
$$L(s) = (K_p + K_D s)\left(\frac{1}{s(s+1)}\right)$$

$$= K_D\left(s + \frac{K_P}{K_D}\right)\left(\frac{1}{s(s+1)}\right)$$

We are given $K_p / K_D = 4$. Hence

$$L(s) = \frac{K_D(s+4)}{s(s+1)}$$

We may therefore sketch the root locus of the system as follows:



The closed-loop transfer function of the system is

$$T(s) = \frac{L(s)}{1 + L(s)}$$

$$= \frac{K_D(s+4)}{s^2 + s + K_D(s+4)} \qquad (1)$$

We are required to choose K_D so as to locate the closed-loop poles at $s = -2 \pm j2\sqrt{2}$. That is, the characteristic equation of the system is to be

$$(s + 2 + j2\sqrt{2})(s + 2 - j2\sqrt{2}) = 0$$

or

$$s^2 + 4s + 12 = 0 \qquad (2)$$

Comparing the denominator of Eq. (1) with (2):

$$1 + K_D = 4$$

$$4K_D = 12$$

Both of these conditions are satisfied by choosing

$$K_D = 3$$

(Note: There is an error in the first printing of the book: the closed-loop poles are to be located at $s = -2 \pm j2\sqrt{2}$ and not $s = -2 \pm j2$.)

9.16

(a) For a feedback system using PI controller, the loop transfer function is

$$L(s) = \left(K_p + \frac{K_I}{s}\right) L'(s)$$

where $L'(s)$ is the uncompensated loop transfer function. For $s = j\omega$ we may write

$$K_p + \frac{K_I}{j\omega} = \frac{1}{j\omega}\left(K_I + jK_p\omega\right)$$

The contribution of the PI controller to the loop phase response of the feedback system is

$$-90^\circ + \tan^{-1}\left(\frac{K_p\omega}{K_I}\right)$$

For all positive values of K_p/K_I, the angle $\tan^{-1}(K_p\omega/K_I)$ is limited to the range $[0, 90^\circ]$, as indicated in Fig. 1. It follows therefore that the use of a PI controller introduces a phase lag into the loop phase response of the feedback system.

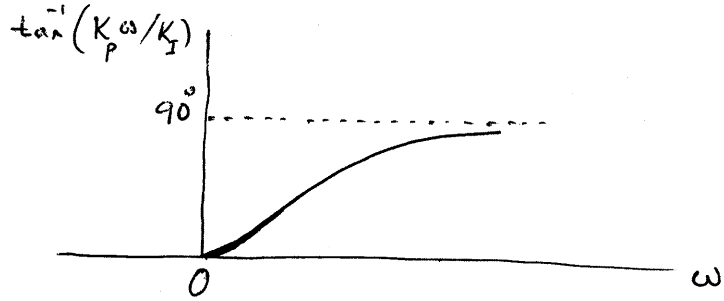


Figure 1

(b) For a feedback system using PD controller, the loop transfer function of the system may be expressed as

$$L(s) = (K_p + K_D s)\, L'(s)$$

where $L'(s)$ is the uncompensated loop transfer function. For $s = j\omega$, the contribution of the PD controller to the loop phase response of the system is $\tan^{-1}(K_D\omega/K_p)$. For all positive values of K_D/K_p, this contribution is limited to the range $[0, 90°]$, as indicated in Fig. 2. We therefore conclude that the use of PD controller has the effect of introducing a phase lead into the loop phase response of the system.

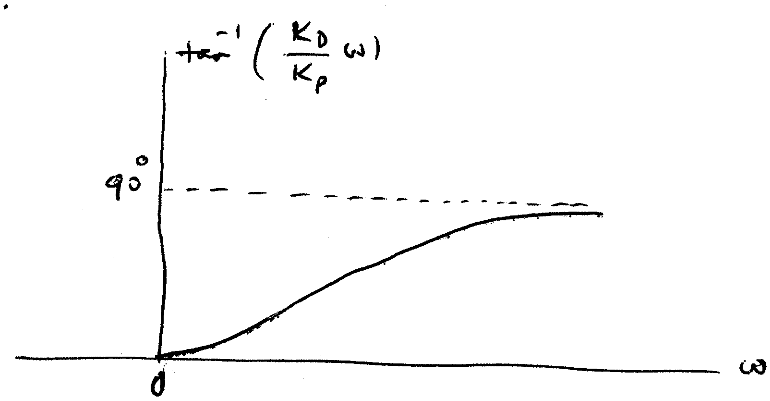


Figure 2

9.17 The transfer function of the PID controller is defined by $K_P + \frac{K_I}{s} + K_D s$. The requirement is to use this controller to introduce zeros at $s = -1 \pm j2$ into the loop transfer function of the feedback system. Hence

$$s^2 + (K_P/K_D)s + (K_I/K_D) = (s+1)^2 + (2)^2$$
$$= s^2 + 2s + 5$$

Comparing terms:

$$\frac{K_P}{K_D} = 2$$

$$\frac{K_I}{D} = 5$$

The loop transfer function of the compensated feedback system is therefore

$$L(s) = \frac{K_P + \frac{K_I}{s} + K_D s}{(s+1)(s+2)}$$

$$= \frac{K_D(s^2 + 2s + 5)}{s(s+1)(s+2)}$$

Figure 1 on the next page displays the root locus of $L(s)$ for varying K_D. From this figure we see that the root locus is confined to the left half of the s-plane for all positive values of K_D. The feedback system is therefore stable for $K_D > 0$.

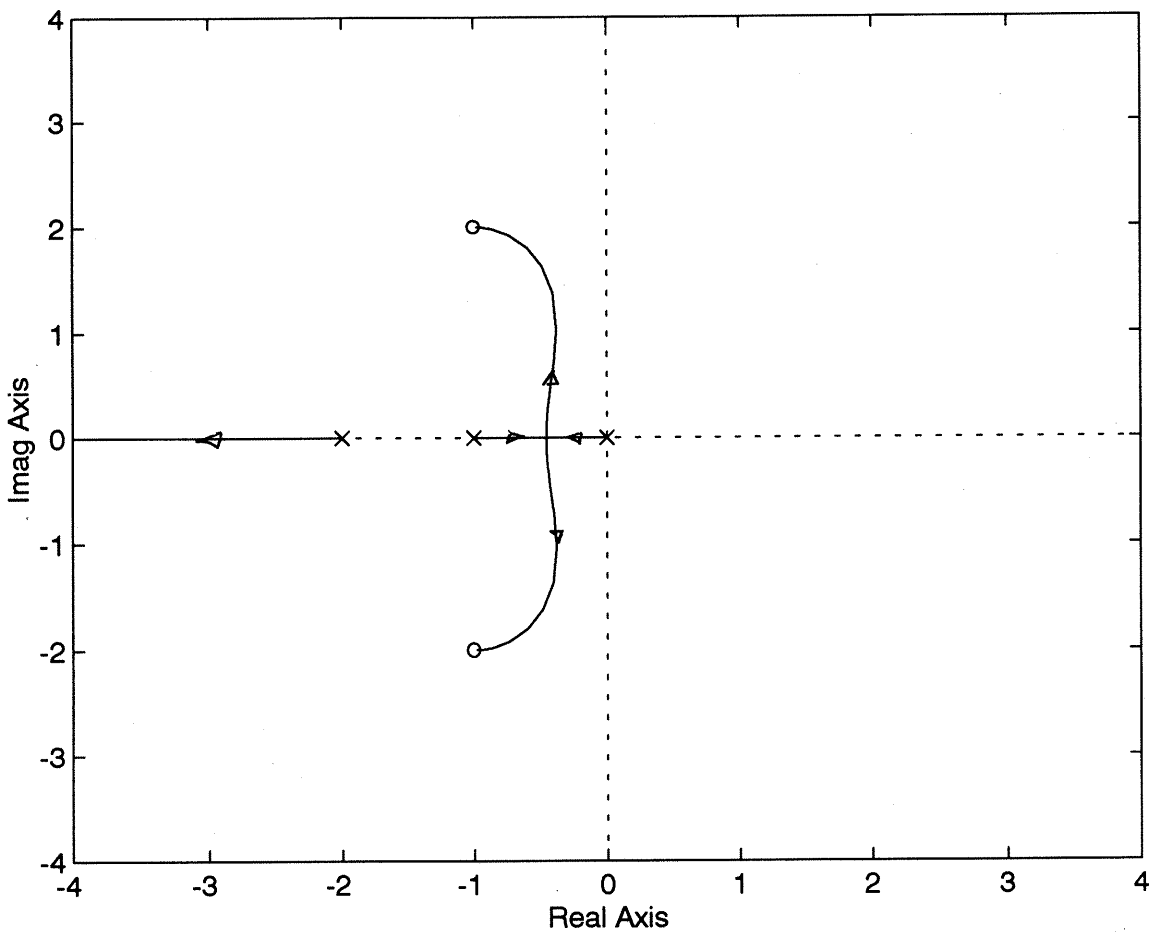


Figure 1

9.18 Let K_p denote the action of the proportional controller. We may then express the loop transfer function of the controlled inverted pendulum as

$$L(s) = \frac{K_p\,(s+3.1)(s-3.1)}{s^2\,(s+4.4)(s-4.4)}.$$

which has zeros at $s = \pm 3.1$ and poles at $s = 0$ (order 2) and $s = \pm 4.4$. The resulting root locus is sketched in Fig. 1. This diagram shows that for all positive values of K_p, the closed-loop transfer function of the system will have a pole in the right-half plane and a pair of poles on the $j\omega$-axis.

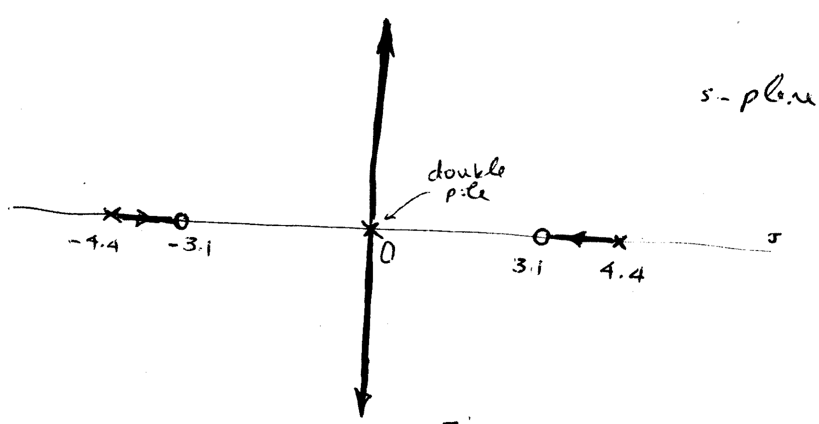


Figure 1

Accordingly, the inverted pendulum cannot be stabilized using a proportional controller for $K_p > 0$. The reader is invited to reach a similar conclusion for $K_p < 0$.

The stabilization of an inverted pendulum is made difficult by the presence of two factors in the loop transfer function $L(s)$:

1. A double pole at $s = 0$.
2. A zero at $s = 3.1$ in the right half plane.

We may compensate for (1) but nothing can be done about (2) if the compensator is itself to be stable. Moreover, we have to make sure that the transfer function of the compensator is proper for it to be realizable.

We may thus propose the use of a compensator (controller) whose transfer function is

$$C(s) = \frac{K s^2 (s - 4.4)}{(s+1)(s+2)(s+3.1)}$$

The compensated loop transfer function is therefore (after performing pole-zero cancellation)

$$L'(s) = C(s)\,L(s)$$

$$= \frac{K(s-3.1)}{(s+1)(s+2)(s+4.4)}$$

Figure 2 shows a sketch of the root locus of $L'(s)$. From this figure we see that the compensated system is stable provided that the gain factor K satisfies the condition

$$0 < K < \frac{4 \times 2}{3.1} = 2.839$$

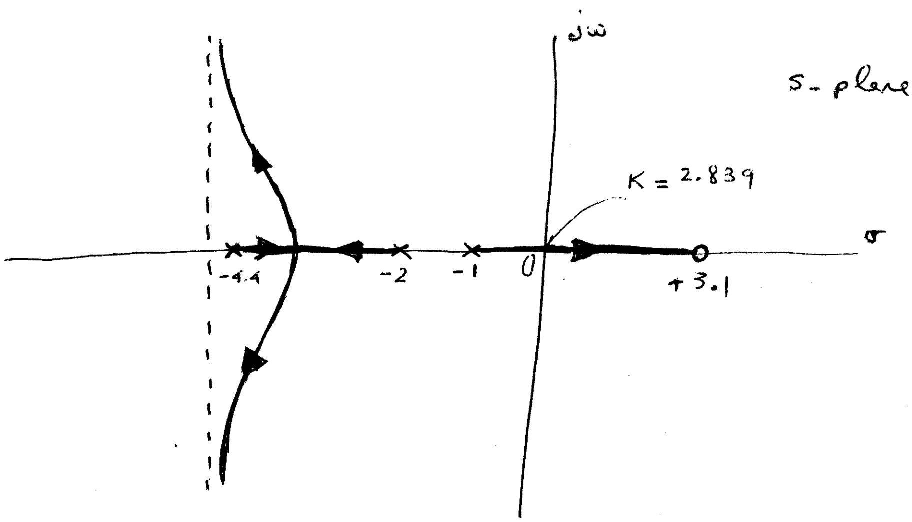


Figure 2

9.19 The closed-loop transfer function of a control system with unity feedback is

$$T(s) = \frac{L(s)}{1 + L(s)}$$

We are given

$$L(s) = \frac{K}{s(s+1)} = \frac{K}{s^2 + s}$$

Hence

$$T(s) = \frac{K}{s^2 + s + K}$$

(a) For $K = 0.1$ the system is overdamped:

$$T(s) = \frac{0.1}{s^2 + s + 0.1}$$

where $\omega_n = \sqrt{0.1}$ and $\zeta = 0.5/\sqrt{0.1}$.

(b) For $K = 0.25$ the system is critically damped:

$$T(s) = \frac{0.25}{s^2 + s + 0.25}$$

where $\omega_n = 0.5$ and $\zeta = 1$.

(c) For $K = 2.5$ the system is underdamped:

$$T(s) = \frac{2.5}{s^2 + s + 2.5}$$

with $\omega_n = \sqrt{2.5}$ and $\zeta = 0.5/\sqrt{2.5}$.

Figure 1, made up of parts (a), (b) and (c) on the next three pages, plots the respective step responses of these three special cases of the feedback system.

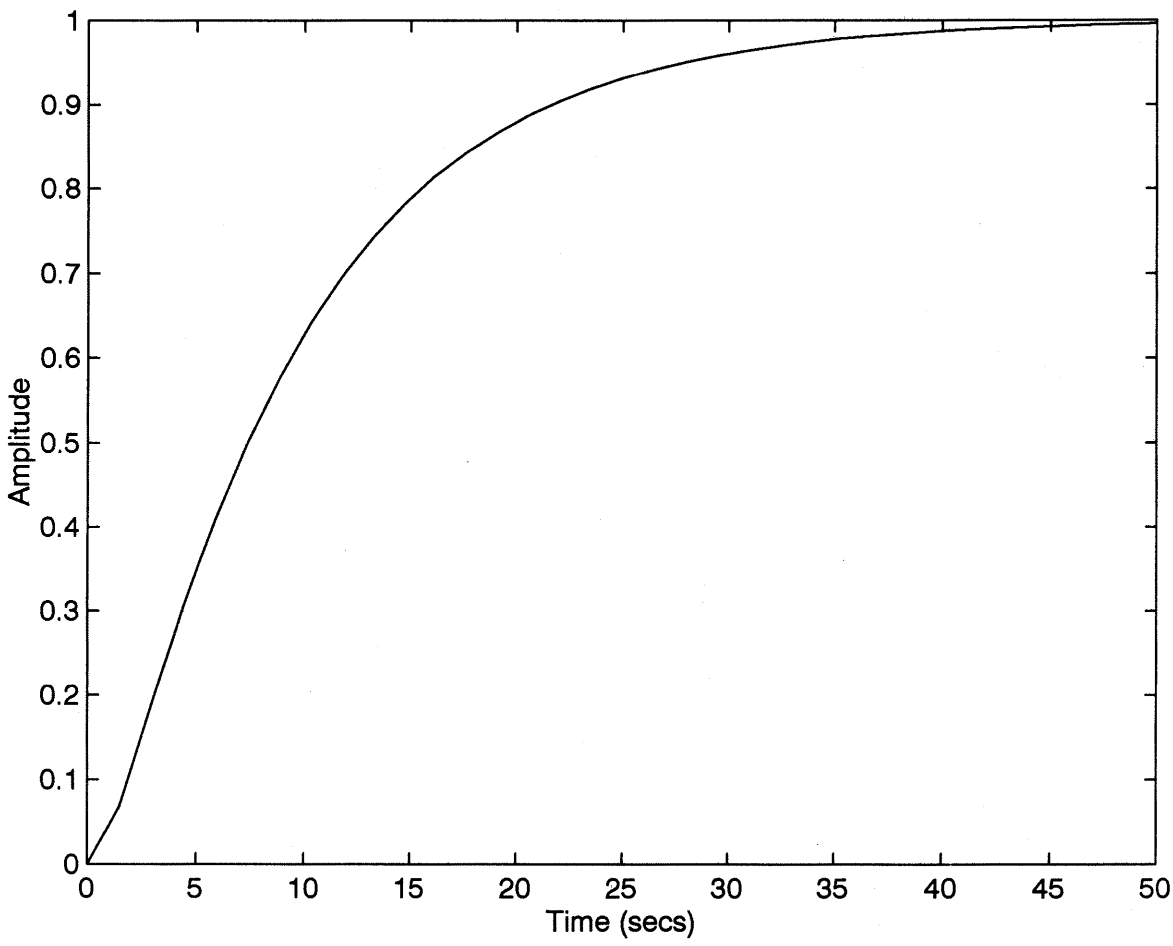


Figure 1 (a)

K = 0.1 (overdamped)

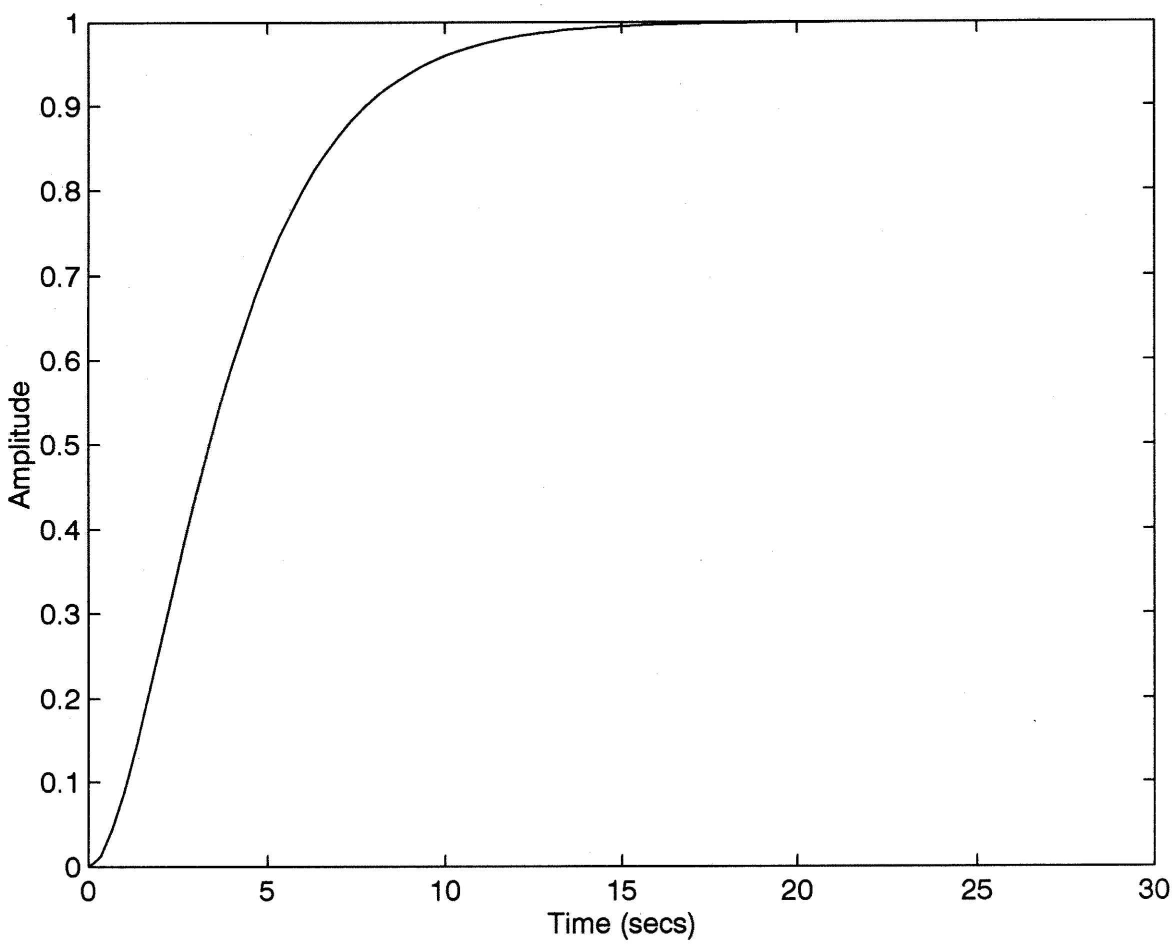


Figure 1 (b)

$K = 0.25$ (critically damped)

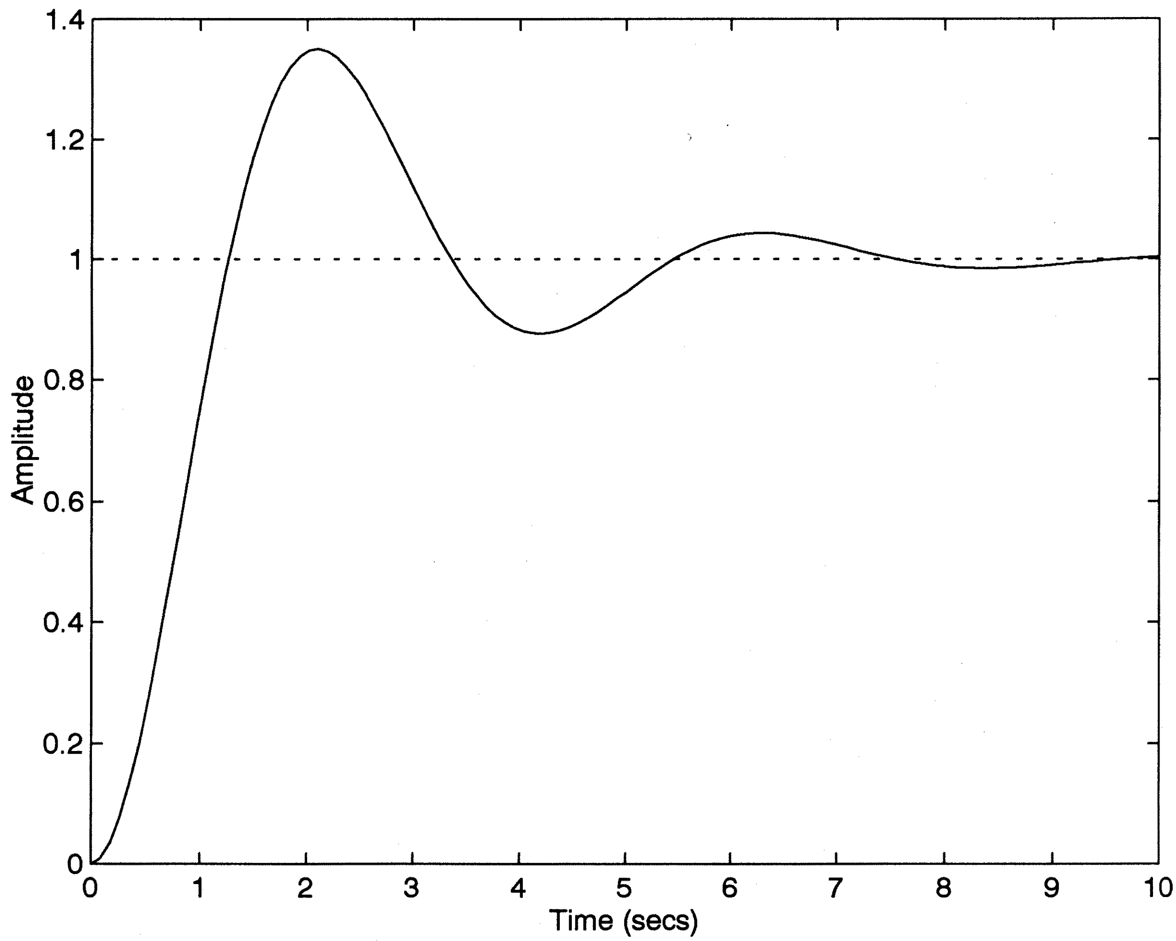


Figure 1 (c)

K = 2.5 (underdamped)

9.20 From Fig. P9.20 the closed-loop transfer function of the feedback system is

$$T(s) = \frac{K/(s^2+2s)}{1+K/(s^2+2s)}$$

$$= \frac{K}{s^2+2s+K} \qquad (1)$$

In general, we may express $T(s)$ in the form

$$T(s) = \frac{\omega_n^2\, T(0)}{s^2+2\zeta\omega_n s+\omega_n^2} \qquad (2)$$

Hence, comparing Eqs. (1) and (2):

$$T(0) = 1 \qquad (3)$$

$$\omega_n = \sqrt{K} \qquad (4)$$

$$\zeta = 1/\omega_n = 1/\sqrt{K} \qquad (5)$$

We are given $K = 20$, for which the use of Eqs. (4) and (5) yields

$$\omega_n = \sqrt{20} = 4.472 \text{ rad/second}$$

$$\zeta = 0.224$$

The time constant of the system is

$$\tau = \frac{1}{\zeta\omega_n} = 1 \text{ second}$$

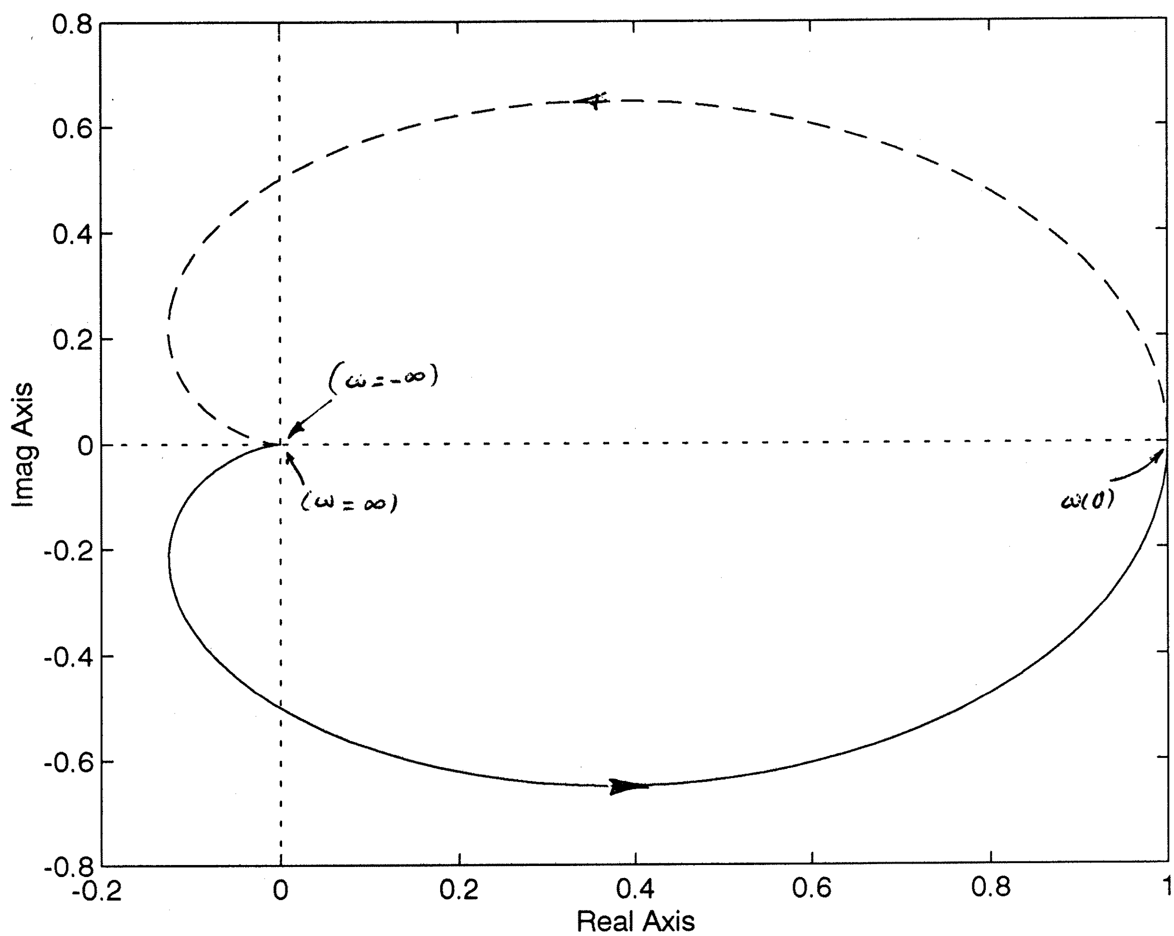
9.21 We are given the loop transfer function

$$L(s) = \frac{K}{(s+1)^2}$$

For $s = j\omega$,

$$L(j\omega) = \frac{K}{(j\omega+1)^2}$$

Figure 1 on the next page plots the Nyquist locus of the system for $-\infty \leq \omega \leq \infty$. From this figure we see that so long as $K > 0$, the Nyquist locus will never encircle the critical point $(-1, 0)$. Hence this feedback system is stable for all $K > 0$.


0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-0.2
0
0.2
0.4
0.6
0.8
1
Imag Axis
Real Axis
(ω = -∞)
(ω = ∞)
ω(0)

9.22 We are given the loop transfer function

$$L(s) = \frac{K}{(s+1)^2(s+5)}$$

Putting $s = j\omega$,

$$L(j\omega) = \frac{K}{(j\omega+1)^2(j\omega+5)}$$

Parts (a), (b) and (c) of Fig. 1 on the next page show plots of the Nyquist locus of the system for $K = 50$, 72, and 100. On the basis of these figures we can make the following statements:

1. For $K = 50$ the locus does not encircle the critical point $(-1, 0)$ and the system is stable.
2. For $K = 100$ the locus encloses the critical point $(-1, 0)$ and the system is unstable.
3. For $K = 72$ the Nyquist locus passes through the critical point $(-1, 0)$ and the system is on the verge of instability.

The critical value $K = 72$ is determined in accordance with the condition

$$|L(j\omega_p)| = \frac{K_{critical}}{(\omega_p^2+1)(\omega_p^2+25)^{1/2}}$$

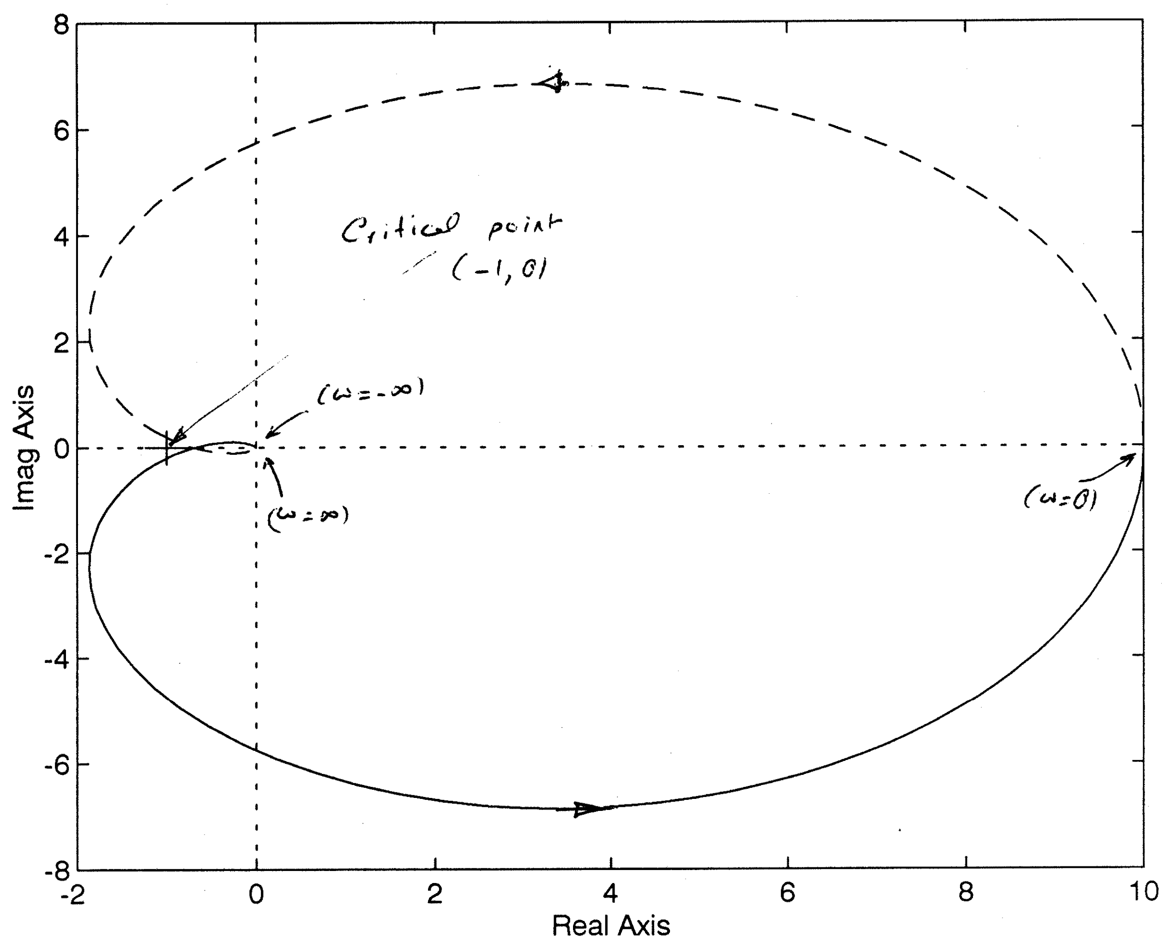


Figure 1(a)

K = 50

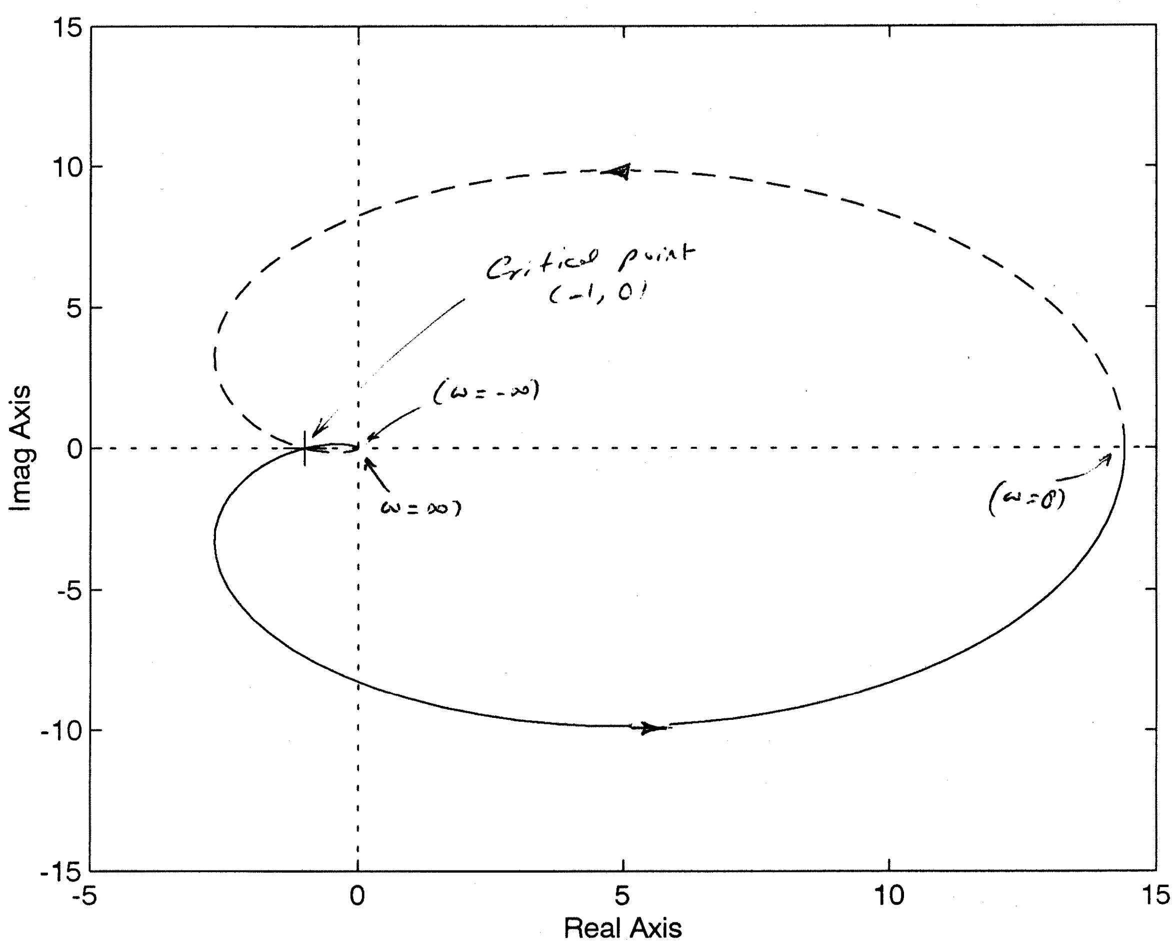


Figure 1 (b)

$K = 72$

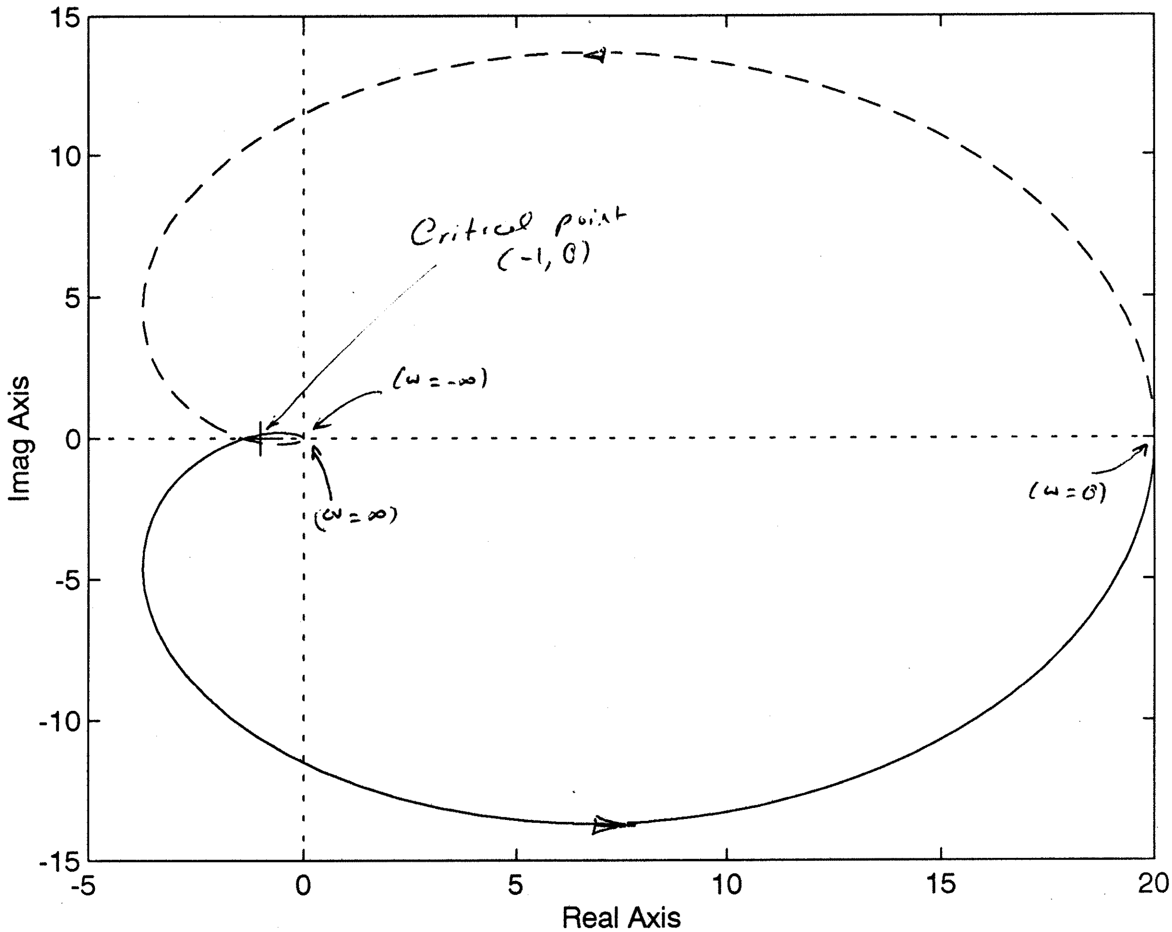


Figure 1 (c)

K = 100

where ω_p is the phase crossover frequency defined by

$$180^\circ = 2\tan^{-1}(\omega_p) + \tan^{-1}\left(\frac{\omega_p}{5}\right)$$

(b) For a gain margin of 5 dB:

$$20\log_{10} K = 20\log_{10} K_{critical} - 5$$

Hence

$$K = \frac{72}{\text{antilog}(0.25)} = \frac{72}{1.7783} = 40.49$$

(c) For $K = 40.49$ the gain-crossover frequency is

$$(40.49)^2 = (1 + \omega_g^2)^2(25 + \omega_g^2)$$

Let $\omega_g^2 = x$ and so rewrite this equation in the form of a cubic equation in x:

$$x^3 + 27x^2 + 51x - 1614 = 0$$

The only positive root of x is 6.2429. Hence

$$\omega_g = \sqrt{6.2429} = 2.4986$$

The phase of $L(j\omega)$ at $\omega = 2.4986$ is

$$2\tan^{-1}(2.4986) + \tan^{-1}\left(\frac{2.4986}{5}\right) = 2 \times 68.2^\circ + 26.5^\circ$$
$$= 162.9^\circ$$

The phase margin is therefore

$$\varphi_m = 180^\circ - 162.9^\circ = 17.1^\circ$$

9.23 We are given

$$L(s) = \frac{K}{s(s+1)}$$

For $s = j\omega$,

$$L(j\omega) = \frac{K}{j\omega(j\omega+1)}$$

from which we deduce:

$$|L(j\omega)| = \frac{K}{\omega(\omega^2+1)^{1/2}} \quad (1)$$

$$\arg\{L(j\omega)\} = -90^\circ - \tan^{-1}(\omega) \quad (2)$$

From Eq. (2) we observe that the phase response $\arg\{L(j\omega)\}$ is confined to the range $[-90^\circ, -180^\circ]$. The value of -180° is attained only at $\omega = \infty$. At this frequency we see from Eq. (1) that the magnitude response $|L(j\omega)|$ is zero. Hence the Nyquist locus will never encircle the critical point $(-1, 0)$ for all positive values of K. The feedback system is therefore stable for all $K > 0$.

9.24 We are given

$$L(s) = \frac{K}{s^2(s+1)}$$

For $s = j\omega$,

$$L(j\omega) = \frac{K}{-\omega^2(j\omega+1)}$$

from which we deduce:

$$|L(j\omega)| = \frac{K}{\omega^2(\omega^2+1)^{1/2}}$$

$$\arg\{L(j\omega)\} = -180^\circ - \tan^{-1}(\omega)$$

Hence the Nyquist locus will encircle the critical point $(-1, 0)$ for all $K > 0$; that is, the system is unstable for all $K > 0$.

We may also verify this result by applying the Routh-Hurwitz criterion:

s^3	1	0
s^2	1	K
s^1	$-K$	0
s^0	K	0

There are 2 sign changes in the first column of coefficients in the array, assuming $K > 0$. Hence the system is unstable for all $K > 0$.

9.25 We are given the loop transfer function

$$L(s) = \frac{K}{s(s+1)(s+2)}$$

For $s = j\omega$,

$$L(j\omega) = \frac{K}{j\omega(j\omega+1)(j\omega+2)}$$

Hence

$$|L(j\omega)| = \frac{K}{\omega(\omega^2+1)^{1/2}(\omega^2+4)^{1/2}} \qquad (1)$$

$$\arg\{L(j\omega)\} = -90^\circ - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right) \qquad (2)$$

Setting $K = 6$ in Eq. (1) under the condition $|L(j\omega_p)| = 1$:

$$36 = \omega_p^2(\omega_p^2+1)(\omega_p^2+4)$$

where ω_p is the phase-crossover frequency. Putting $\omega_p^2 = x$ and rearranging terms:

$$x^3 + 5x^2 + 4x - 36 = 0$$

Solving this cubic equation for x we find that it has only one positive root at $x = 2$. Hence

$$\omega_p = \sqrt{2}$$

Substituting this value in Eq. (2):

$$\arg\{L(j\omega_p)\} = -90^{\circ} - \tan^{-1}(\sqrt{2}) - \tan^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$= -90^{\circ} - 54.8^{\circ} - 35.2^{\circ} = -180^{\circ}$$

This result confirms ω_p as the phase-crossover frequency, and $K = 6$ is the critical value of the scaling factor for which the Nyquist locus of $L(j\omega)$ passes through the critical point $(-1, 0)$.

We may also verify this result by applying the Routh-Hurwitz criterion to the characteristic equation:

$$s^3 + 3s^2 + 2s + K = 0$$

Specifically, we write

$$\begin{array}{lll} s^3 & 1 & 2 \\ s^2 & 3 & K \\ s^1 & \frac{6-K}{3} & 0 \\ s^0 & K & \end{array}$$

The system is therefore on the verge of instability for $K = 6$.

(d) For $K=2$ the gain margin is

$$20\log_{10} 6 - 20\log_{10} 2 = 20\log_{10} 3 = 9.542 \text{ dB}$$

To calculate the phase margin for $K=2$ we need to know the gain-crossover frequency ω_g.

At $\omega = \omega_g$, $|L(j\omega)| = 1$. Hence for the problem at hand

$$1 = \frac{2}{\omega_g(\omega_g^2+1)^{1/2}(\omega_g^2+4)^{1/2}}$$

Let $\omega_g^2 = x$, for which we may then rewrite this equation as

$$x^3 + 5x^2 + 4x - 4 = 0$$

The only positive root of this equation is $x = 0.5616$.

That is,

$$\omega_g = \sqrt{0.5616} = 0.7494$$

The phase margin is therefore

$$180^\circ - \left(90^\circ + \tan^{-1}(0.7494) + \tan^{-1}(0.3749)\right)$$

$$= 90^\circ - 36.9^\circ - 20.5^\circ$$

$$= 32.6^\circ$$

(c) Let ω_g denote the gain-crossover frequency for the required phase margin $\varphi_m = 20°$. We may then write

$$|L(j\omega_g)| = 1$$

$$\arg\{L(j\omega_g)\} = -90° - \tan^{-1}(\omega_g) - \tan^{-1}\left(\frac{\omega_g}{2}\right)$$

With

$$\arg\{L(j\omega_g)\} = 180° - 20°$$

we thus have

$$70° = \tan^{-1}(\omega_g) + \tan^{-1}\left(\frac{\omega_g}{2}\right)$$

Through a process of trial and error, we obtain the solution

$$\omega_g = 0.972$$

For $|L(j\omega_g)| = 1$ we thus require

$$K = \omega_g(\omega_g^2+1)^{1/2}(\omega_g^2+4)^{1/2}$$

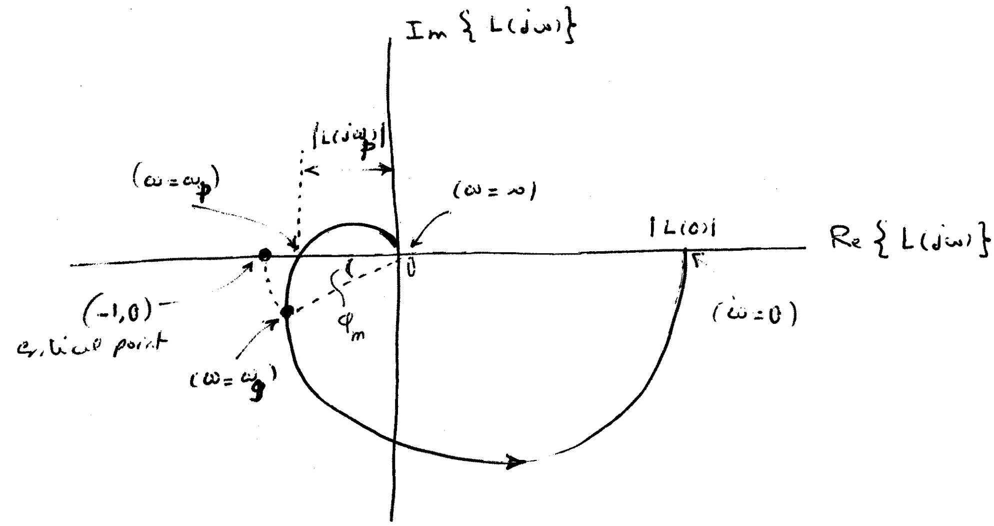
$$= 0.972(1.9448)^{1/2}(4.9448)^{1/2}$$

$$= 3.0143$$

The gain margin is therefore

$$20\log_{10}\left(\frac{6}{3.0143}\right) \simeq 20\log_{10}2 = 6\text{ dB}$$

9.26 For the purpose of illustration, suppose the loop frequency response $L(j\omega)$ has a finite magnitude at $\omega = 0$. Suppose also the feedback system represented by $L(j\omega)$ is stable. We may then sketch the Nyquist locus of the system for $0 \le \omega \le \infty$, including gain margin and phase margin provisions, as follows:



The phase margin of the system is φ_m (measured at the gain-crossover frequency ω_g). The gain margin is $20 \log_{10} \dfrac{1}{|L(j\omega_p)|}$ (measured at the phase crossover frequency).

9.27 We are given the loop frequency response

$$L(j\omega) = \frac{6K}{(j\omega+1)(j\omega+2)(j\omega+3)}$$

$$= \frac{6K}{(j\omega)^3 + 6(j\omega)^2 + 11(j\omega) + 6}$$

(a) Figure 1 on the next page plots the Bode diagram of $L(j\omega)$ for $K=1$. Changing the value of K merely shifts the loop gain response by a constant amount equal to $20 \log_{10} K$. This constant gain is tabulated as follows:

K	$20 \log_{10} K$, dB
7	16.90
8	18.06
9	19.08
10	20.00
11	20.83

Applying the constant gain adjustments (as shown here) to the loop gain response of Fig. 1, we

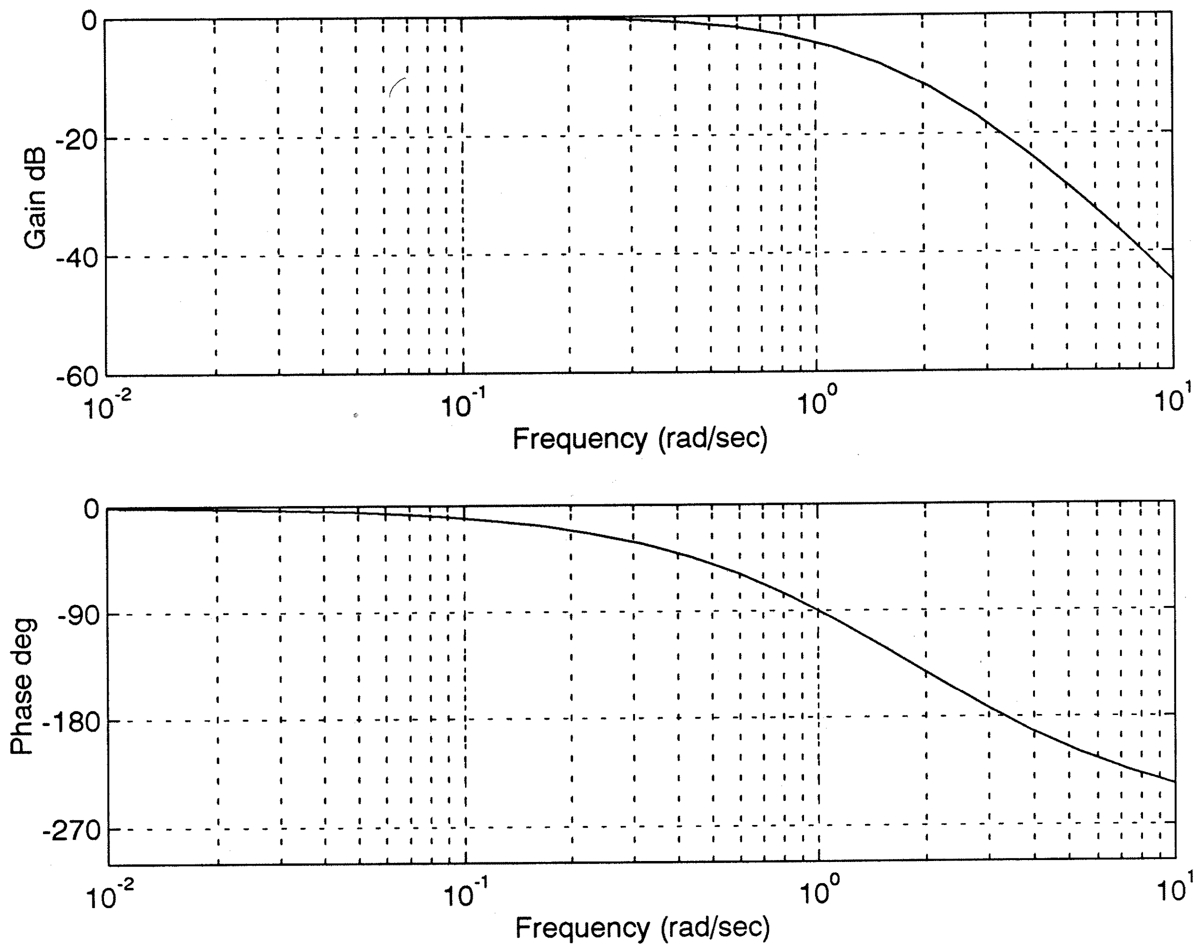

0
-20
-40
-60
Gain dB
10^{-2}
10^{-1}
10^{0}
10^{1}
Frequency (rad/sec)
0
-90
-180
-270
Phase deg
10^{-2}
10^{-1}
10^{0}
10^{1}
Frequency (rad/sec)

Figure 1
$(K = 1)$

may make the following observations:

1. The feedback system is stable for $K = 7, 8, 9$.

2. It is on the verge of instability for $K = 10$.

3. It is unstable for $K = 11$.

The critical value $K = 10$ may also be verified by applying the Routh–Hurwitz criterion to the characteristic equation:

$$s^3 + 6s^2 + 11s + 6(K+1) = 0$$

Specifically, we have

$$\begin{array}{llll}
s^3 & 1 & 11 \\
s^2 & 6 & 6(K+1) \\
s^1 & \dfrac{60-6K}{6} & 0 \\
s^0 & 6(K+1) & 0
\end{array}$$

Hence the system is on the verge of instability for $K = 10$.

(Note: In the first printing of the book the numerator of $L(j\omega)$ should read $6K$).

(b) For $K = 7$ the gain margin is

$$20 \log_{10}\left(\frac{10}{7}\right) = 3.098 \text{ dB}$$

To calculate the phase margin we need to know the gain-crossover frequency ω_g. At $\omega = \omega_g$ the following condition is satisfied

$$1 = \frac{42}{(\omega_g^2+1)^{1/2}(\omega_g^2+4)(\omega_g^2+9)^{1/2}}$$

Let $\omega_g^2 = x$ and so rewrite this equation as

$$x^3 + 14x^2 + 49x - 1728 = 0$$

The only positive root of this cubic equation is at $x = 7.8432$. Hence

$$\omega_g = \sqrt{7.8432} = 2.8006$$

The phase margin for $K = 7$ is therefore

$$180^\circ - \tan^{-1}(2.8006) - \tan^{-1}(1.4003) - \tan^{-1}(.9335)$$

$$= 180^\circ - 70.4^\circ - 54.4^\circ - 43^\circ$$

$$= 12.2^\circ$$

Proceeding in a similar fashion we get the following results:

- For $K = 8$, gain margin $= 1.938$ dB
  phase margin $= 7.39^\circ$

- For $K = 9$, gain margin $= 0.915$ dB
  phase margin $= 3.41^\circ$

9.28 Consider first the original system described by the transfer function

$$T(s) = \frac{48}{(s+5.7259)(s^2+0.2740\,s+9.4308)}$$

The corresponding Bode diagram, obtained by putting $s = j\omega$, is plotted in Fig. 1 on the next page.

Consider next the reduced-order model described by the transfer function

$$T'(s) = \frac{8.3832}{s^2+0.2740\,s+9.4308}$$

which is obtained from $T(s)$ by ignoring the distant pole at $s = -5.7259$ and readjusting the constant gain factor. Figure 2, on the page after the next one, plots the Bode diagram of the approximating system.

Comparing these two figures, we see that the frequency responses of the original feedback system and its reduced-order approximate model are close to each other over the frequency range $0 \le \omega \le 4.0$.

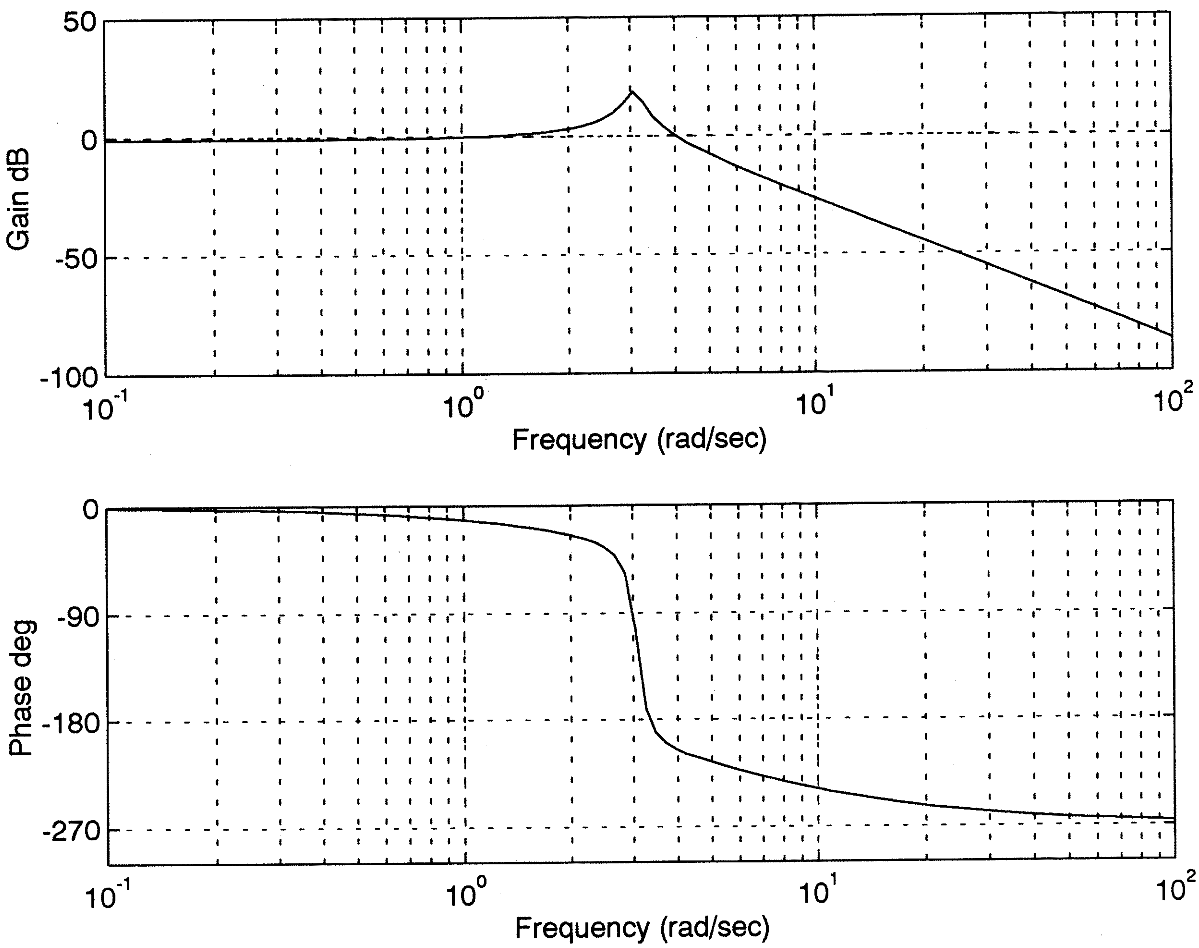


Figure 1
Frequency response of original system

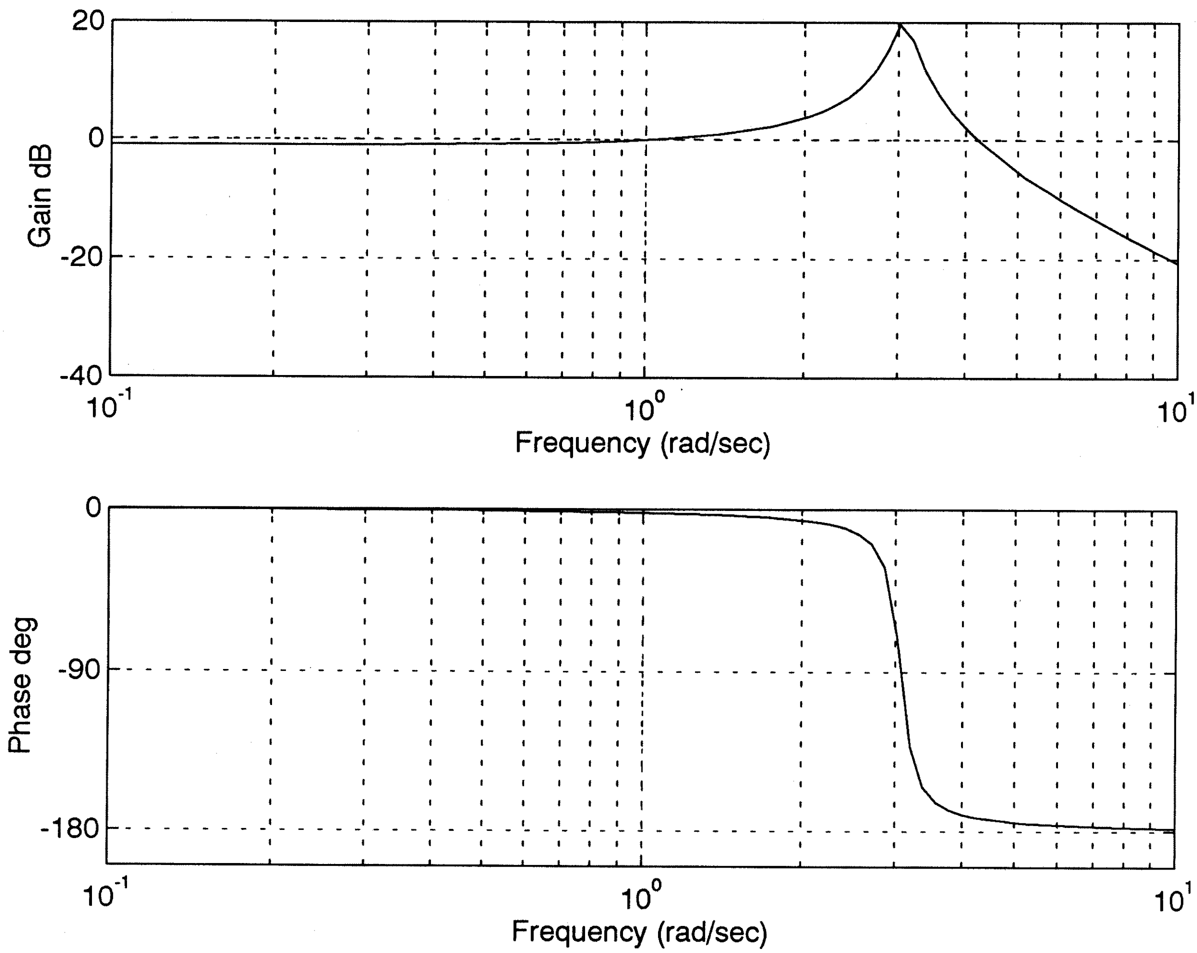


Figure 2

Frequency response of reduced-order model.

9.29

Figures 1, 2, 3, and 4 on the next four pages plot the Bode diagrams for the following loop transfer functions:

(a) $$L(s) = \frac{50}{(s+1)(s+2)}$$

(b) $$L(s) = \frac{10}{(s+1)(s+2)(s+5)}$$

(c) $$L(s) = \frac{5}{(s+1)^3}$$

(d) $$L(s) = \frac{10(s+0.5)}{(s+1)(s+2)(s+5)}$$

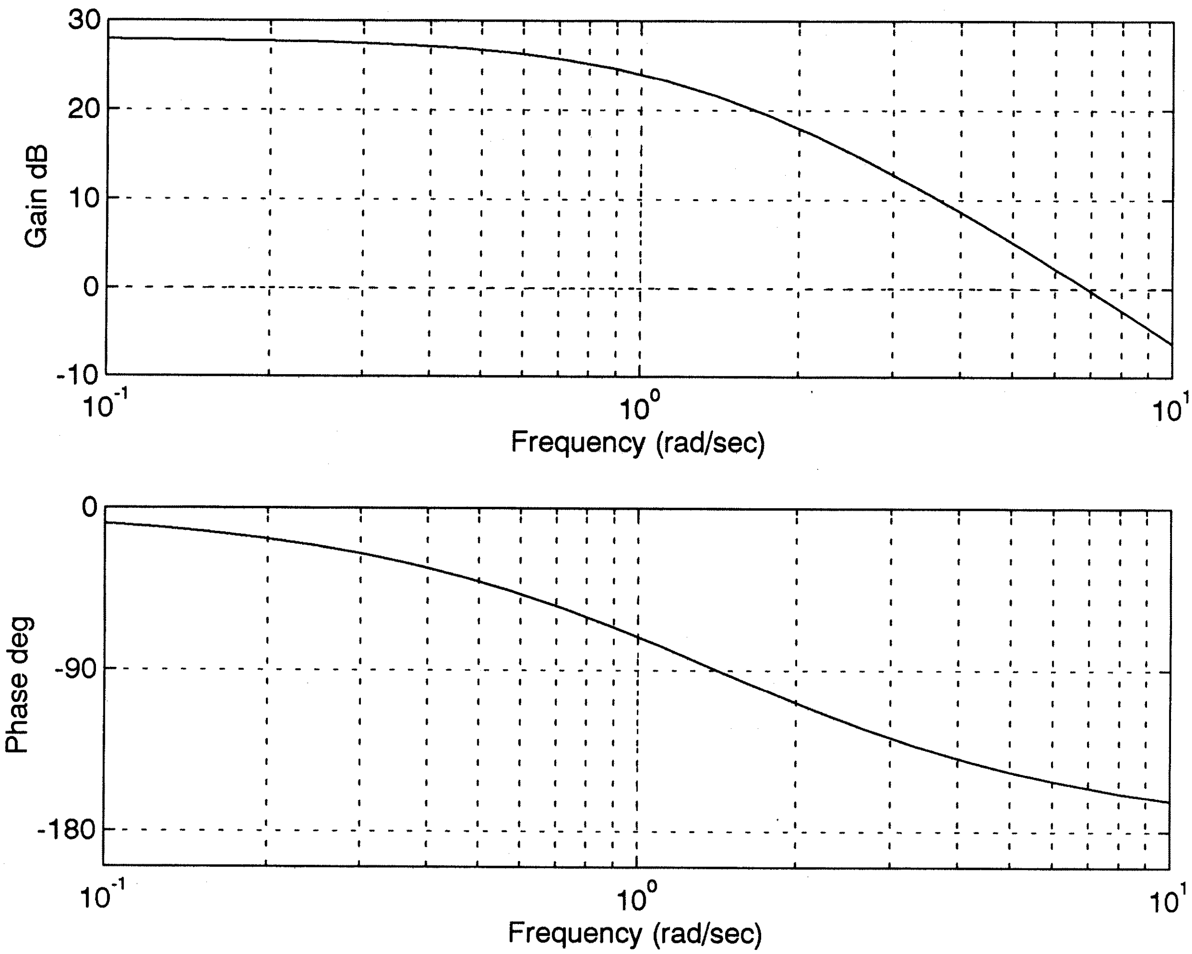


Figure 1

(a) $L(s) = \dfrac{50}{(s+1)(s+2)}$

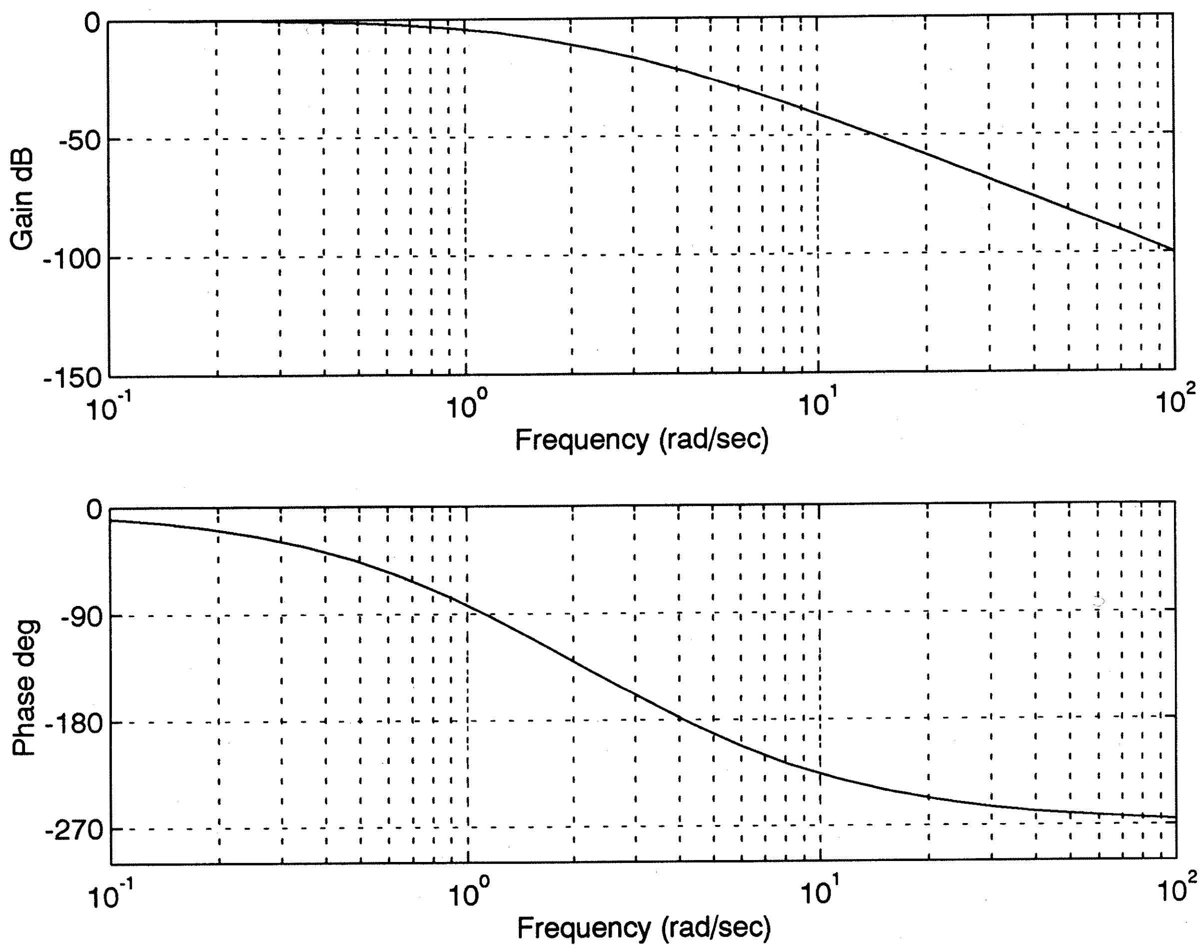


Figure 2

(b) $L(s) = \dfrac{10}{(s+1)(s+2)(s+5)}$

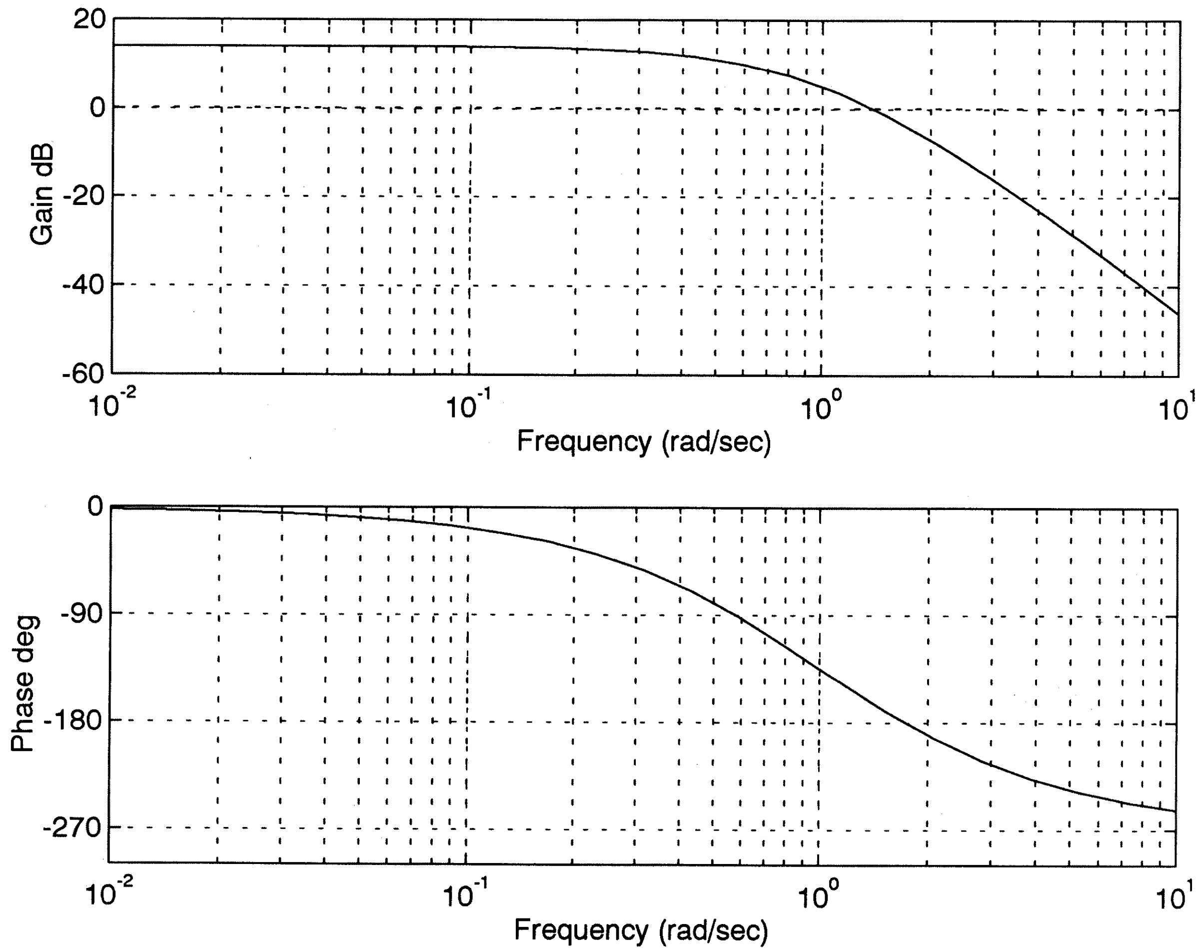


Figure 1 (c)

$$L(s) = \frac{5}{(s+1)^3}$$

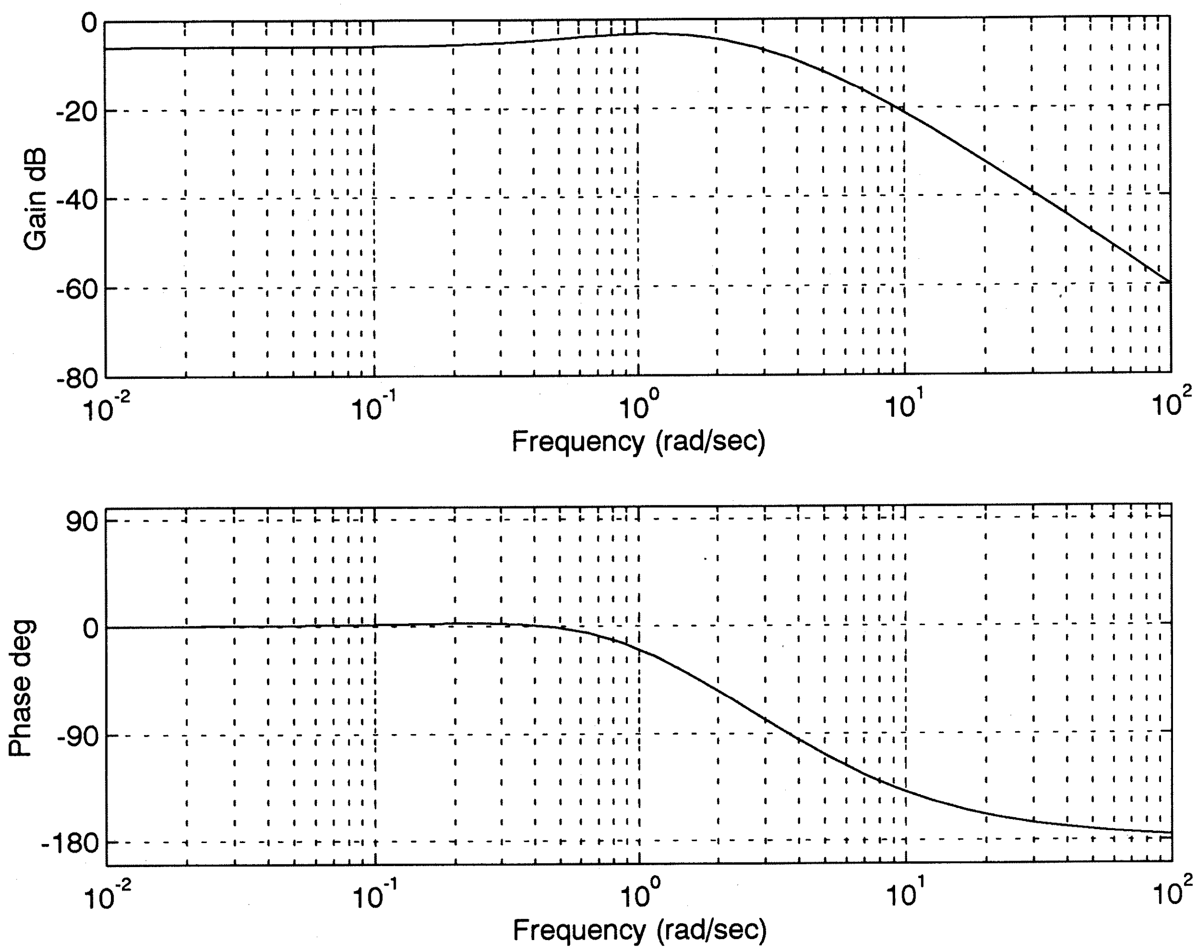


Figure 1

(d) $L(s) = \dfrac{10(s+0.5)}{(s+1)(s+2)(s+5)}$

9.30 From Example 9.11 we have the loop transfer function

$$L(s) = \frac{0.5K(s+2)}{s(s+12)(s-4)}$$

For $s = j\omega$,

$$L(j\omega) = \frac{0.5K(j\omega+2)}{j\omega(j\omega+12)(j\omega-4)}$$

Figures 1, 2, and 3 on the next three pages plot the Bode diagrams of $L(j\omega)$ for $K = 100, 128,$ and 160, respectively.

From these figures we may make the following observations:

(a) The phase-crossover frequency $\omega_p = 4$ for all K.

(b) For $K = 128$ the Bode diagram exactly satisfies the conditions $|L(j\omega)| = 1$ and $\arg\{L(j\omega)\} = 180°$. Hence the system is on the verge of instability for $K = 128$.

(c) For $K = 100$ the gain margin is -2.14 dB, which is negative. The system is therefore unstable for $K = 100$.

(d) For $K = 160$ the gain margin is 1.94 dB, which is positive. The system is therefore stable for $K = 160$.

These observations reconfirm the conclusions reached on the stability performance of $L(s)$ in Example 9.11

(Note: In the first printing of the book, reference should have been made to Example 9.11 and not 9.10.)

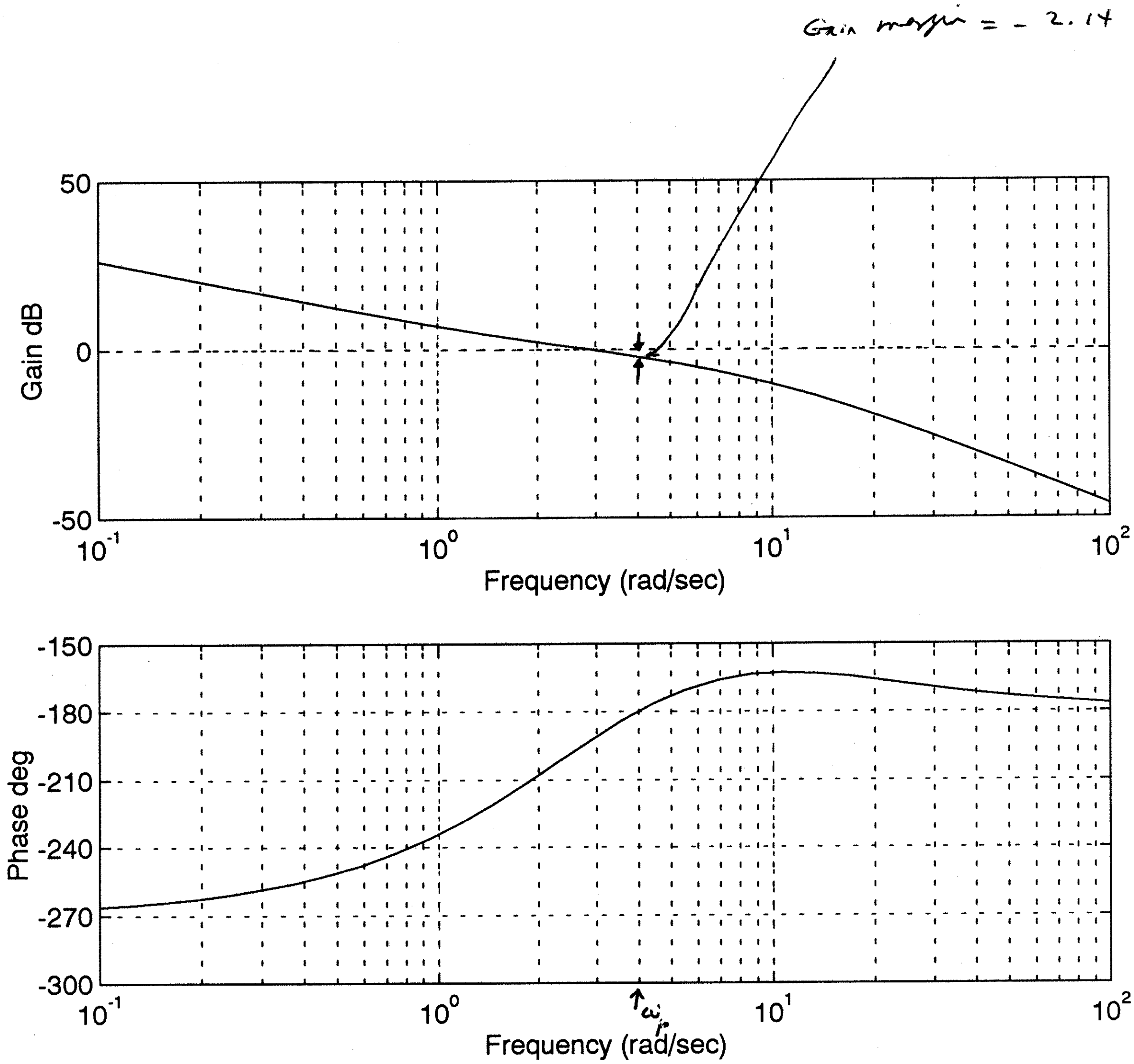


Figure 1

$K = 100$

(system is unstable)

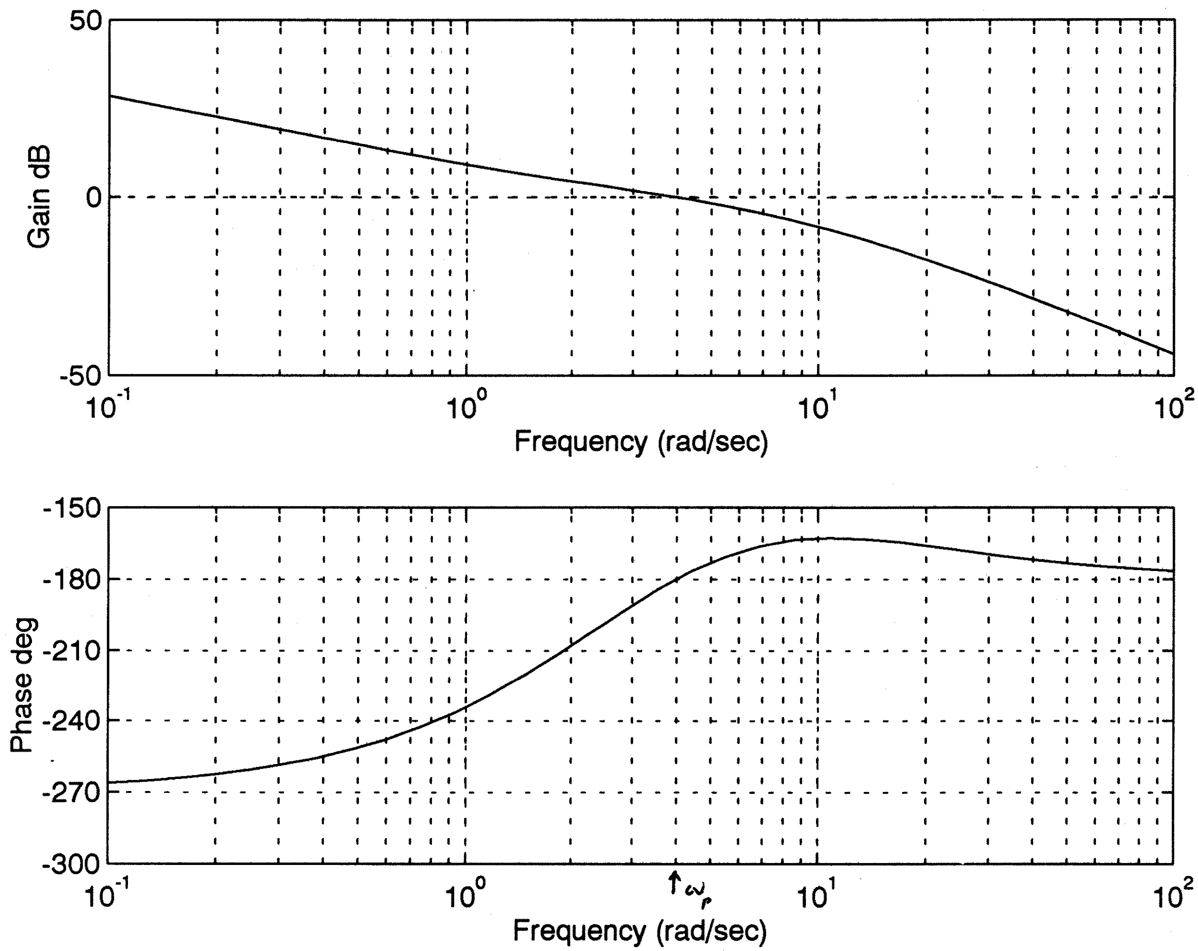


Figure 2

$K = 128$

(System is on the verge of instability)

Gain margin
= +1.94

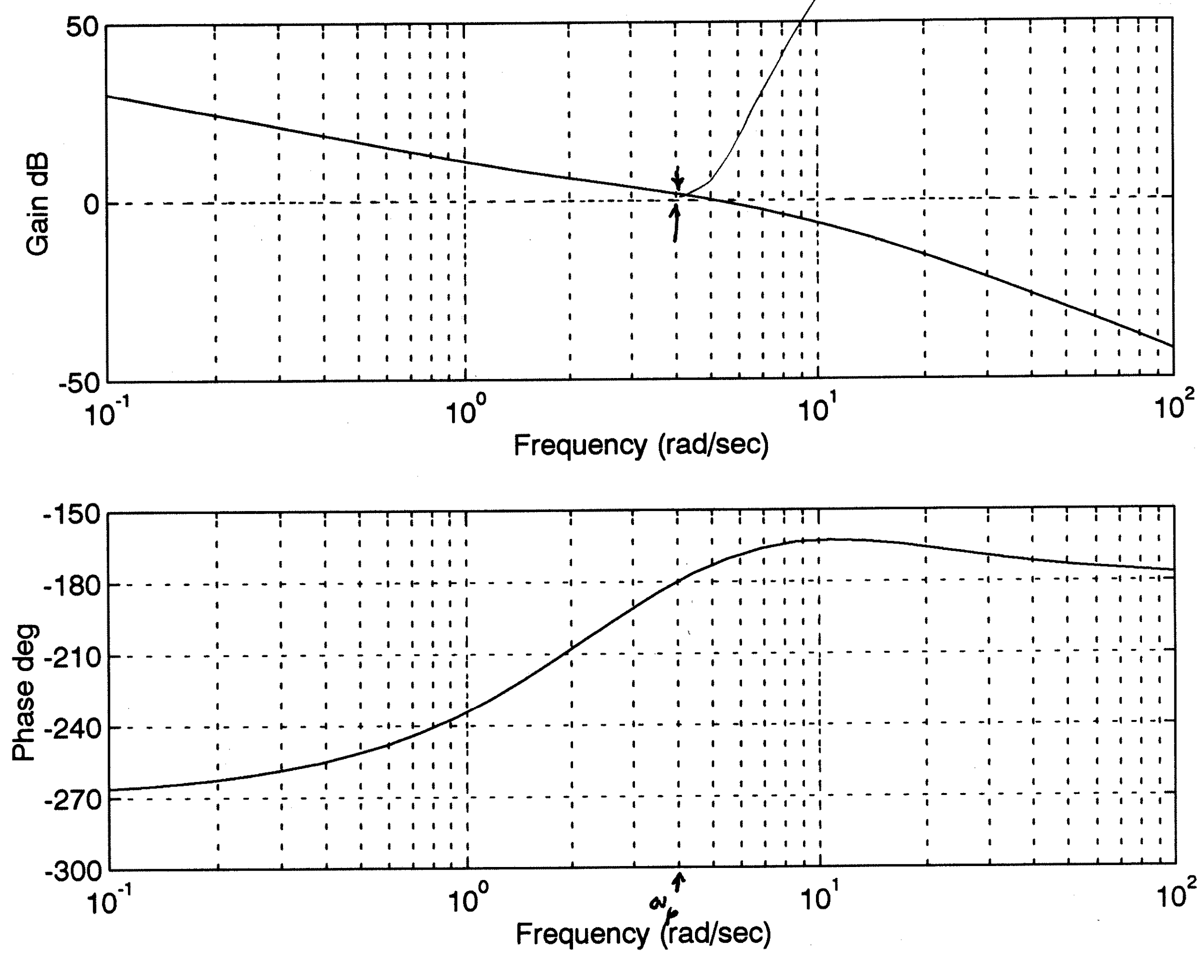


Figure 3

K = 160

(system is stable)

9.31 Let $r(t)$ denote the feedback (return) signal produced by the sensor $H(s)$. Then, following the material presented in Section 9.17, we may, insofar as the computation of the Laplace transform $R(s) = \mathcal{L}\{r(t)\}$ is concerned, we may replace the sampled-data system of Fig. P9.31, with that shown in Fig. 1 below:

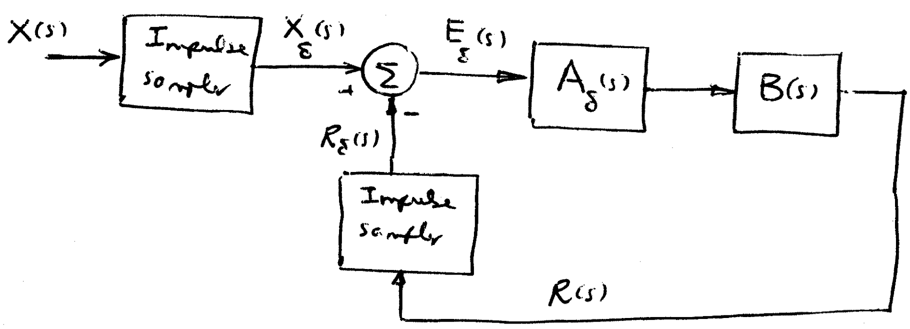


Figure 1

The blocks labeled $A_\delta(s)$ and $B(s)$ are defined as follows, respectively:

1. $A_\delta(s)$ = transfer function of the discrete-time components of the system

$$= D_\delta(s)\left(1 - e^{-Ts}\right)$$

where T is the sampling period and

$$D_\delta(s) = D(z)\Big|_{z = e^{Ts}}$$

2. $B(s)$ = transfer function of the continuous-time components of the system

$$= \frac{1}{s} G_c(s)\, G_p(s)\, H(s)$$

where $G_c(s)$, $G_p(s)$, and $H(s)$ are as indicated in Fig. P9.31.

Adapting Eq. (9.121) of the text to the problem at hand:

$$\frac{R_\delta(s)}{X_\delta(s)} = \frac{L_\delta(s)}{1 + L_\delta(s)} \qquad (1)$$

where

$$L_\delta(s) = A_\delta(s)\, B_\delta(s) \qquad (2)$$

The $B_\delta(s)$ is itself defined in terms of $B(s)$ by the formula

$$B_\delta(s) = \frac{1}{T} \sum_{k=-\infty}^{\infty} B(s - jk\omega_s)$$

where $\omega_s = 2\pi/T$.

From Fig. P9.31 we note that

$$E(s) = H(s)\, Y(s)$$

Hence,

$$E_\delta(s) = H_\delta(s)\, Y_\delta(s) \qquad (3)$$

where

$$H_\delta(s) = \frac{1}{T} \sum_{k=-\infty}^{\infty} H(s - jk\omega_s)$$

Eliminating $E_\delta(s)$ between Eqs. (1) and (3):

$$Y_\delta(s) = \frac{L_\delta(s) / H_\delta(s)}{1 + L_\delta(s)}\, X_\delta(s)$$

Changing this result to the z-transform:

$$Y(z) = \frac{L(z)/H(z)}{1 + L(z)}\, X(z)$$

where

$$Y(z) = Y_\delta(s) \Big|_{e^{Ts} = z}$$

and so on for $L(z)$, $H(z)$, and $X(z)$.

9.32 From the material presented in Section 9.17, we note that (see Eq. (9.121))

$$Y_\delta(s) = \frac{L_\delta(s)}{1 + L_\delta(s)} X_\delta(s) \qquad (1),$$

where

$$L_\delta(s) = A_\delta(s) B_\delta(s) \qquad (2)$$

For the problem at hand, $A_\delta(s)$ and $B_\delta(s)$ are defined as follows, respectively:

1. $$A_\delta(s) = (1 - e^{-Ts}) D_\delta(s) \qquad (3)$$

   where

   $$D_\delta(s) = D(z)\Big|_{z = e^{Ts}}$$

2. $$B_\delta(s) = \frac{1}{T} \sum_{k=-\infty}^{\infty} B(s - jk\omega_s), \qquad \omega_s = \frac{2\pi}{T}$$

   where

   $$B(s) = \frac{1}{s} G(s)$$
   $$= \frac{K}{s^3}$$

We are given

$$D(z) = K\left(1.5 + \frac{z-1}{z}\right)$$

$$= K(2.5 - z^{-1})$$

Hence

$$D_\delta(s) = K(2.5 - e^{-Ts})$$

The use of Eq. (3) thus yields

$$A_\delta(s) = K(1 - e^{-Ts})(2.5 - e^{-Ts})$$

$$= K\left(2.5 - 3.5\, e^{-Ts} + e^{-2Ts}\right)$$

Correspondingly, we may write

$$A(z) = K\left(2.5 - 3.5\, z^{-1} + z^{-2}\right)$$

Next, with $B(s) = K/s^3$ we may write

$$b(t) = \mathcal{L}^{-1}\{B(s)\}$$

$$= \frac{Kt^2}{2} u(t)$$

Hence,

$$B_\delta(s) = \sum_{k=-\infty}^{\infty} b[n]\, e^{-nTs}$$

$$= \frac{K}{2} \sum_{k=0}^{\infty} n^2 e^{-nTs}$$

which may be summed to yield

$$B(z) = B_\delta(s)\Big|_{e^{Ts}=z}$$

$$= \frac{KT^2}{4}\,\frac{z(z+1)}{(z-1)^3} = \frac{KT^2}{4}\,\frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$$

The use of Eq.(2) yields

$$L(z) = L_\delta(s)\Big|_{e^{Ts}=z}$$

$$= A(z)\,B(z)$$

$$= K^2(2.5 - 3.5z^{-1} + z^{-2})\,\frac{T^2}{4}\,\frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$$

$$= \frac{K^2T^2}{4}\,\frac{z^{-1}(2.5-z^{-1})(1+z^{-1})}{(1-z^{-1})^2}$$

Finally, the closed-loop transfer function of the system is

$$T(z) = \frac{L(z)}{1+L(z)}$$

where $L(z)$ is defined by Eq.(4).

9.33 Here again following the material presented presented in Section 9.17, we may state that

$$\frac{Y(z)}{X(z)} = \frac{L(z)}{1+L(z)}$$

where

$$L(z) = L_\delta(s)\Big|_{e^{Ts}=z}$$

$$L_\delta(s) = A_\delta(s)\,B_\delta(s)$$

$$A_\delta(s) = (1-e^{-Ts})$$

$$B_\delta(s) = \frac{1}{T}\sum_{k=-\infty}^{\infty} B(s-jk\omega_s), \qquad \omega_s = \frac{2\pi}{T}$$

$$B(s) = \frac{1}{s}G(s)$$

$$= \frac{5}{s^2(s+2)}$$

$$= -\frac{5/4}{s} + \frac{5/2}{s^2} + \frac{5/4}{s+2}$$

To calculate $B_\delta(s)$ we first note that

$$b(t) = -\frac{5}{4}u(t) + \frac{5}{2}t\,u(t) + \frac{5}{4}e^{-2t}u(t)$$

Hence,

$$B_\delta(s) = \sum_{n=-\infty}^{\infty} b[n]\, e^{-nTs}$$

$$= -\frac{5}{4}\sum_{n=0}^{\infty} e^{-nTs} + \frac{5}{2}\sum_{n=0}^{\infty} n\, e^{-nTs} + \frac{5}{4}\sum_{n=0}^{\infty} e^{-2nT} e^{-nTs}$$

Correspondingly, we may write

$$B(z) = -\frac{5}{4}\,\frac{1}{1-z^{-1}} + \frac{5}{2}\,\frac{T z^{-1}}{(1-z^{-1})^2} + \frac{5}{4}\,\frac{1}{1-e^{-2T} z^{-1}}$$

From the expression for $A_\delta(s)$:

$$A(z) = (1-z^{-1})$$

The loop transfer function of the system is therefore

$$L(z) = A(z)\, B(z)$$

$$= (1-z^{-1})\left[-\frac{5}{4}\,\frac{1}{1-z^{-1}} + \frac{5}{2}\,\frac{T z^{-1}}{(1-z^{-1})^2} + \frac{5}{4}\,\frac{1}{1-e^{-2T} z^{-1}}\right]$$

Putting this expression on a common denominator:

$$L(z) = \frac{5}{4}\,\frac{z^{-1}\left((-1+2T+e^{-2T}) + (1-e^{-2T}(1+2T))\, z^{-1}\right)}{(1-z^{-1})(1-e^{-2T} z^{-1})}$$

The closed-loop transfer function is

$$T(z) = \frac{L(z)}{1+L(z)}$$

(a) For sampling period $T = 0.1\,s$, the loop transfer function takes the value

$$L(z) = \frac{5}{4}\,\frac{z^{-1}\left((-1 + 0.2 + e^{-0.2}) + (1 - e^{-0.2}(1.2)\,z^{-1})\right)}{(1 - z^{-1})(1 - e^{-0.2}z^{-1})}$$

$$= \frac{5}{4}\,\frac{z^{-1}(0.019 + 0.017z^{-1})}{(1 - z^{-1})(1 - 0.819\,z^{-1})}$$

The closed-loop transfer function is therefore

$$T(z) = \frac{\dfrac{5}{4}\,\dfrac{z^{-1}(0.019 + 0.017\,z^{-1})}{(1 - z^{-1})(1 - 0.819\,z^{-1})}}{1 + \dfrac{5}{4}\,\dfrac{z^{-1}(0.019 + 0.017\,z^{-1})}{(1 - z^{-1})(1 - 0.819\,z^{-1})}}$$

$$= \frac{\frac{5}{4}z^{-1}(0.019 + 0.017\,z^{-1})}{(1 - z^{-1})(1 - 0.819\,z^{-1}) + \frac{5}{4}z^{-1}(0.019 + 0.017\,z^{-1})}$$

$$= \frac{\frac{5}{4}z^{-1}(0.019 + 0.017\,z^{-1})}{1 - 1.795\,z^{-1} + 0.840\,z^{-2}}$$

$T(z)$ has a zero at $z = -0.895$ and a pair of complex poles at $z = 0.898 \pm j\,0.186$. The poles are inside the unit circle and the system is therefore stable.

(b) For sampling period $T = 0.05$ s, $L(z)$ takes the value

$$L(z) = \frac{\frac{5}{4} z^{-1}\left((-1+0.1+e^{-0.1}) + (1-e^{-0.1}(1.1))z^{-1}\right)}{(1-z^{-1})(1-e^{-0.1}z^{-1})}$$

$$= \frac{5}{4}\,\frac{z^{-1}(0.005+0.005\,z^{-1})}{(1-z^{-1})(1-0.996\,z^{-1})}$$

The closed-loop transfer function is therefore

$$T(z) = \frac{\frac{5}{4}\,\frac{z^{-1}(0.005+0.005\,z^{-1})}{(1-z^{-1})(1-0.996\,z^{-1})}}{1+\frac{5}{4}\,\frac{z^{-1}(0.005+0.005\,z^{-1})}{(1-z^{-1})(1-0.996\,z^{-1})}}$$

$$= \frac{\frac{5}{4} z^{-1}(0.005+0.005\,z^{-1})}{(1-z^{-1})(1-0.996\,z^{-1}) + \frac{5}{4} z^{-1}(0.005+0.005\,z^{-1})}$$

$$= \frac{0.006\,z^{-1}(1+z^{-1})}{1-1.989\,z^{-1}+1.001\,z^{-2}}$$

$T(z)$ has a zero at $z \simeq -1$ and a pair of complex poles at $z = 0.995 \pm j0.109$. The poles lie inside the unit circle in the z-plane and the sampled-data system remains stable. Note, however, the reduction in the sampling period has pushed the closed-loop poles much closer to the unit circle.

9.34 In both two-degrees-of-freedom controllers shown in Fig. 9.34, we have forward compensation in addition to the compensation provided by the controller placed inside the feedback loop. In both configurations, the key idea to the forward compensation is that since it is outside the feedback loop, it does not affect the roots of the characteristic equation of the original feedback system. The poles and zeros of the forward compensator (controller) may therefore be selected so as to add or cancel the poles and zeros of the closed-loop transfer function of the system.

A possible disadvantage of the configuration shown in Fig. P9.34(b) is that the forward compensator may feed additional noise inside the feedback loop due to perturbations produced at the input to the system.

9.35 We are given

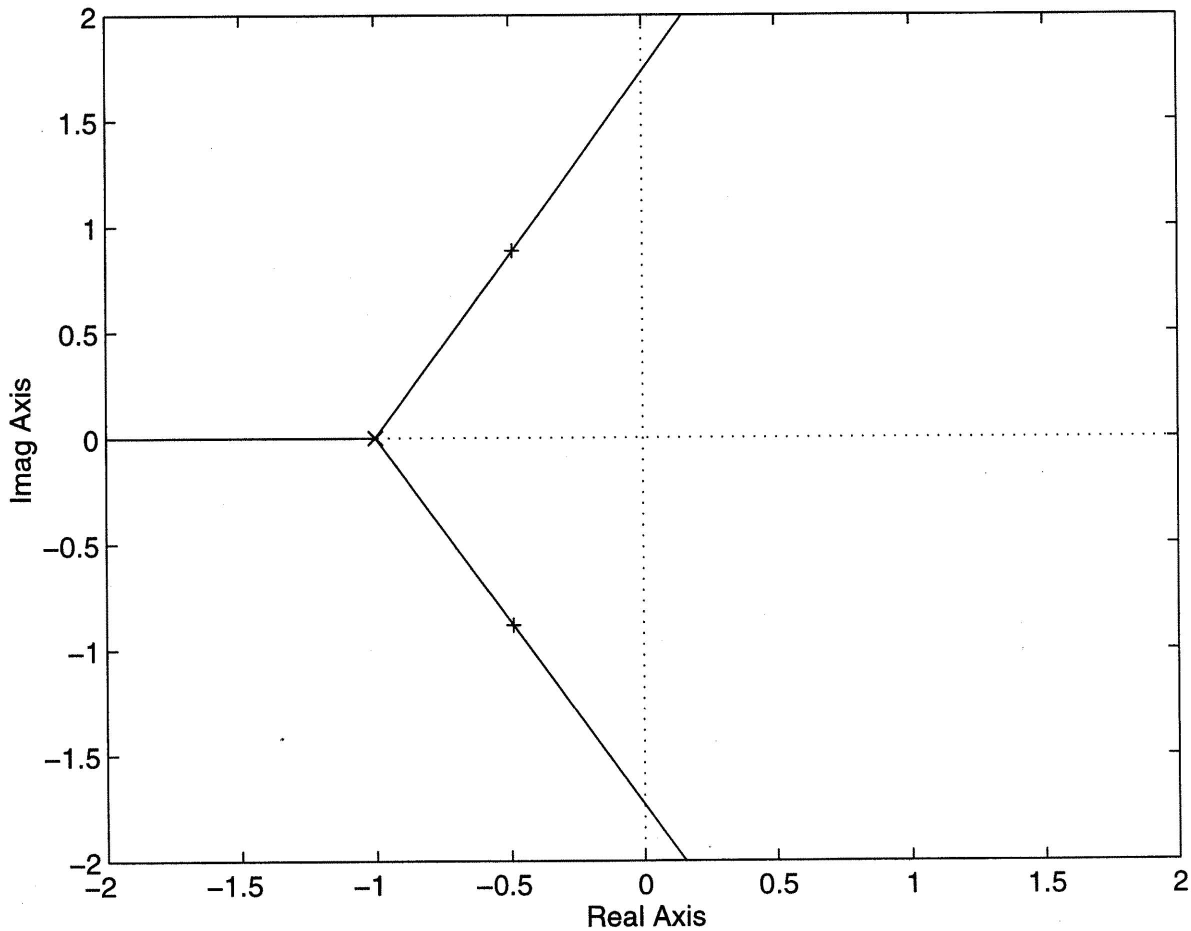
$$L(s) = \frac{K}{(s+1)^3}$$

The MATLAB command for finding the value of K corresponding to a desired pair of closed-loop poles, and therefore damping factor ζ, is given on the next page.

The root locus of the unity feedback system is presented in Fig. 1. A damping factor $\zeta = 0.5$ corresponds to a pair of dominant closed-loop poles lying on the angles $\pm 120°$. Hence using the command rlocfind, we obtain the value $K = 1$.

The corresponding values of the complex closed-loop poles are $s = -0.5 \pm j\,0.866$. Hence the natural frequency $\omega_n = \sqrt{0.5^2 + .866^2} = 1$

```
num=1;
den=[1 3 3 1];
rlocus(num,den);
grid on
K=rlocfind(num,den)
```



2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
Imag Axis
−2
−1.5
−1
−0.5
0
0.5
1
1.5
2
Real Axis

9.36 The MATLAB command for this experiment is presented on the page after the next one.

Figure 1 displays the root locus for the loop transfer function

$$L(s) = \frac{K}{s(s^2 + s + 2)}$$

The value of K, corresponding to a pair of complex poles with damping factor $\zeta = 0.0707$, is computed to be $K = 1.46$ using rlocfind.

Figure 2 presents the Bode diagram of $L(s)$ for $K = 1.5$. From this figure we find the following:

(1) Gain margin = 6.1 dB

Phase crossover frequency = 1.4 rad/sec

(2) Phase margin = 71.3°

Gain crossover frequency = 0.6 rad/sec.

(Note: In part b of the problem should read 0.0707 and not 0.707 as in the first printing of the book).

```
num=1;
den=[1 1 2 0];
rlocus(tf(num,den));
grid on
K=rlocfind(num,den)
figure
margin(tf(num,den));
```

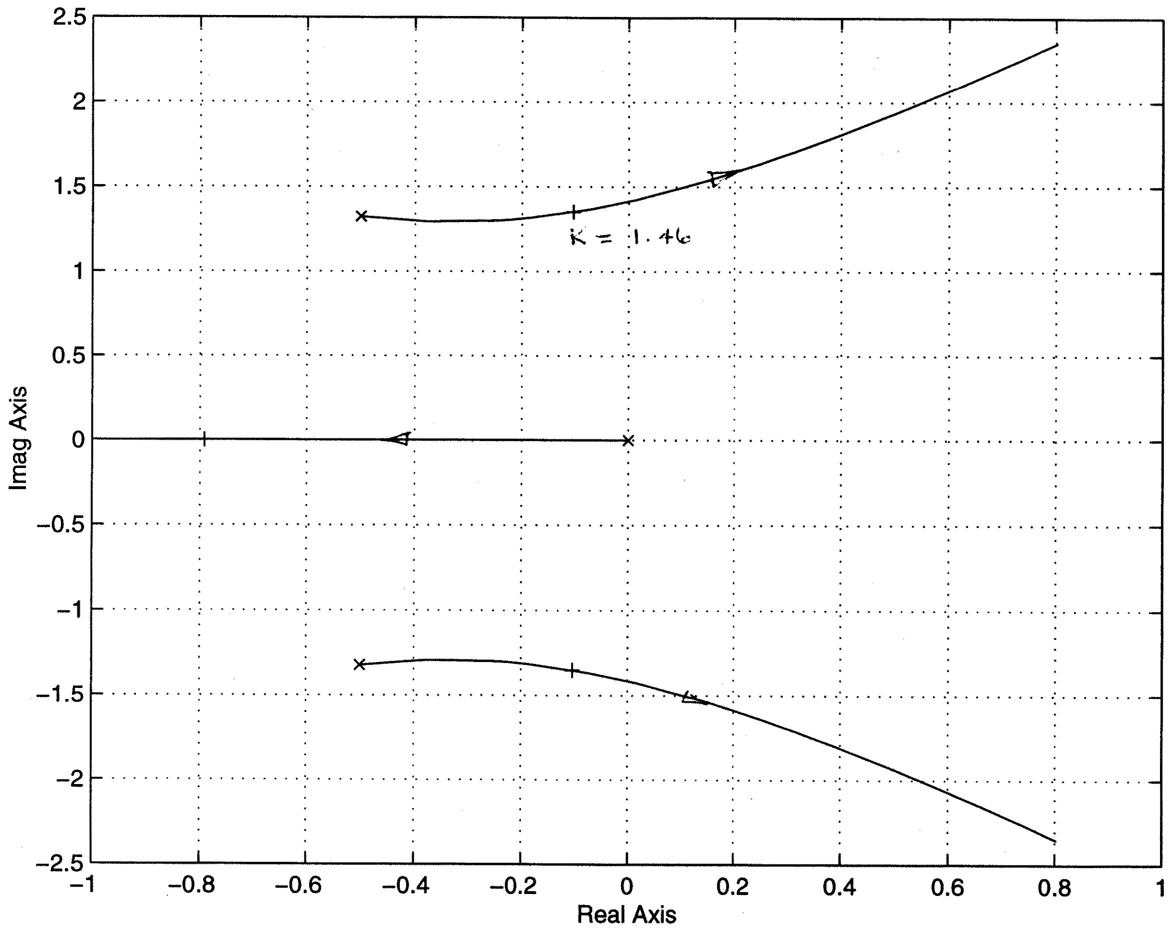

2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2.5
Imag Axis
K = 1.46
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
Real Axis

Figure 1

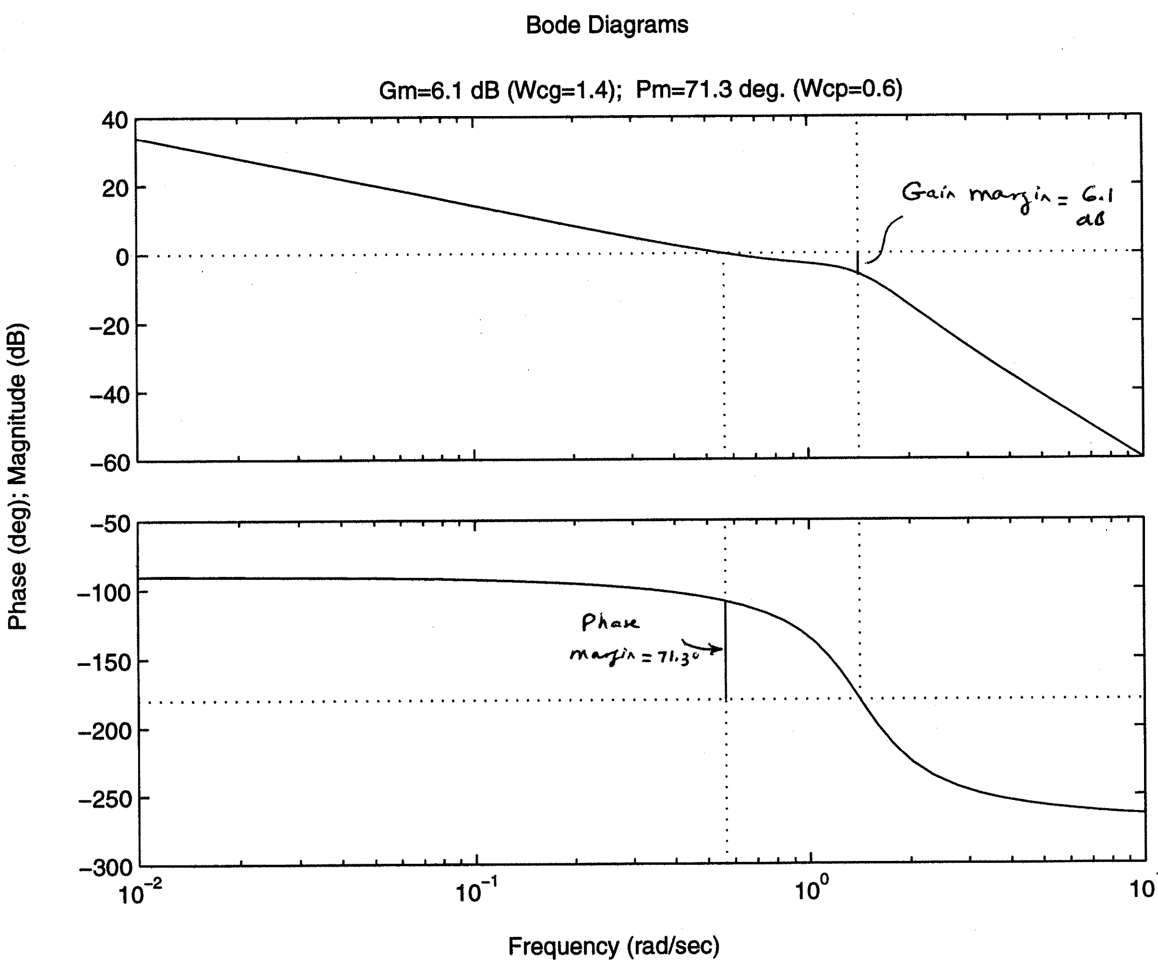

Bode Diagrams
Gm=6.1 dB (Wcg=1.4); Pm=71.3 deg. (Wcp=0.6)
Phase (deg); Magnitude (dB)
Frequency (rad/sec)
Gain margin = 6.1 dB
Phase margin = 71.3°
40
20
0
−20
−40
−60
−50
−100
−150
−200
−250
−300
10^{-2}
10^{-1}
10^{0}
10^{1}

Figure 2

9.37 The MATLAB code for this experiment is presented on the next page.

Figure 1 displays the root locus of the feedback system defined by the loop transfer function

$$L(s) = \frac{K(s-1)}{(s+1)(s^2+s+1)}$$

Using the command rlocfind, we find that the value of K for which the only real closed-loop pole lies on the $j\omega$-axis (i.e., the system is on the verge of instability) is $K=1$. We may verify this result by constructing the Routh array for the characteristic equation

$$(s+1)(s^2+s+1) + K(s-1) = 0$$

as shown by

s^3	1	$2+K$
s^2	2	$-K$
s^1	$\frac{4+3K}{2}$	0
s^0	$-K$	0

For $K = 1$ there is only one sign change in the first column of array coefficients.

Figure 2 displays the Nyquist locus of $L(j\omega)$ for $K = 0.8$. We see that the critical point is not enclosed, and therefore the feedback system is stable.

Figure 3 displays the Bode diagram of $L(j\omega)$ for $K = 0.8$. From this figure we obtain the following values:

Gain margin = 2.0 dB

Phase - crossover frequency = 0 rad/sec.

Note that the gain margin is exactly equal to

$$-20 \log_{10} K = 20 \log_{10} \frac{1}{0.8} = 2 \text{ dB}$$

```
num=[1 -1];
den=[1 2 2 1];
rlocus(num,den);
K=rlocfind(num,den)
figure
num=.8*num;
nyquist(num,den);
figure
margin(num,den);
```

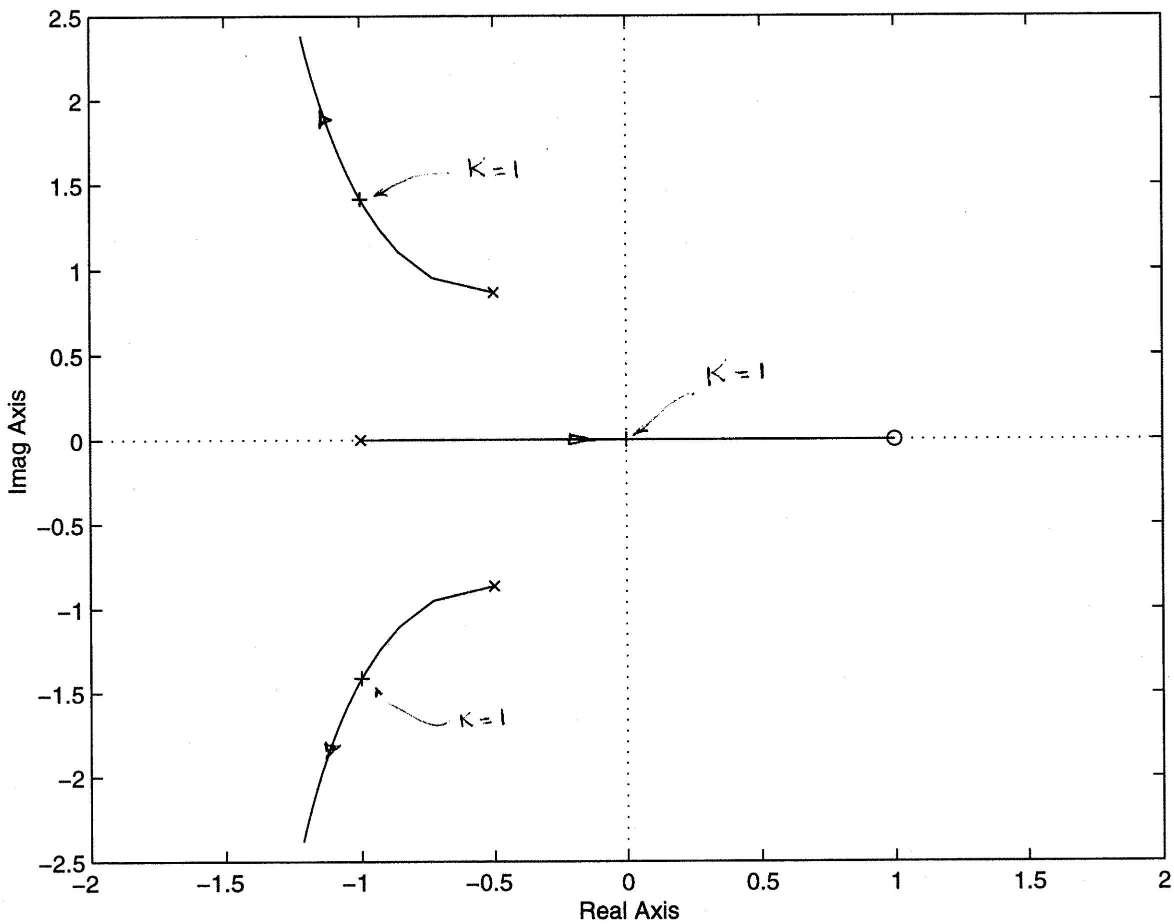

Imag Axis
Real Axis
2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2.5
−2
−1.5
−1
−0.5
0
0.5
1
1.5
2
K=1
K=1
K=1

Figure 1

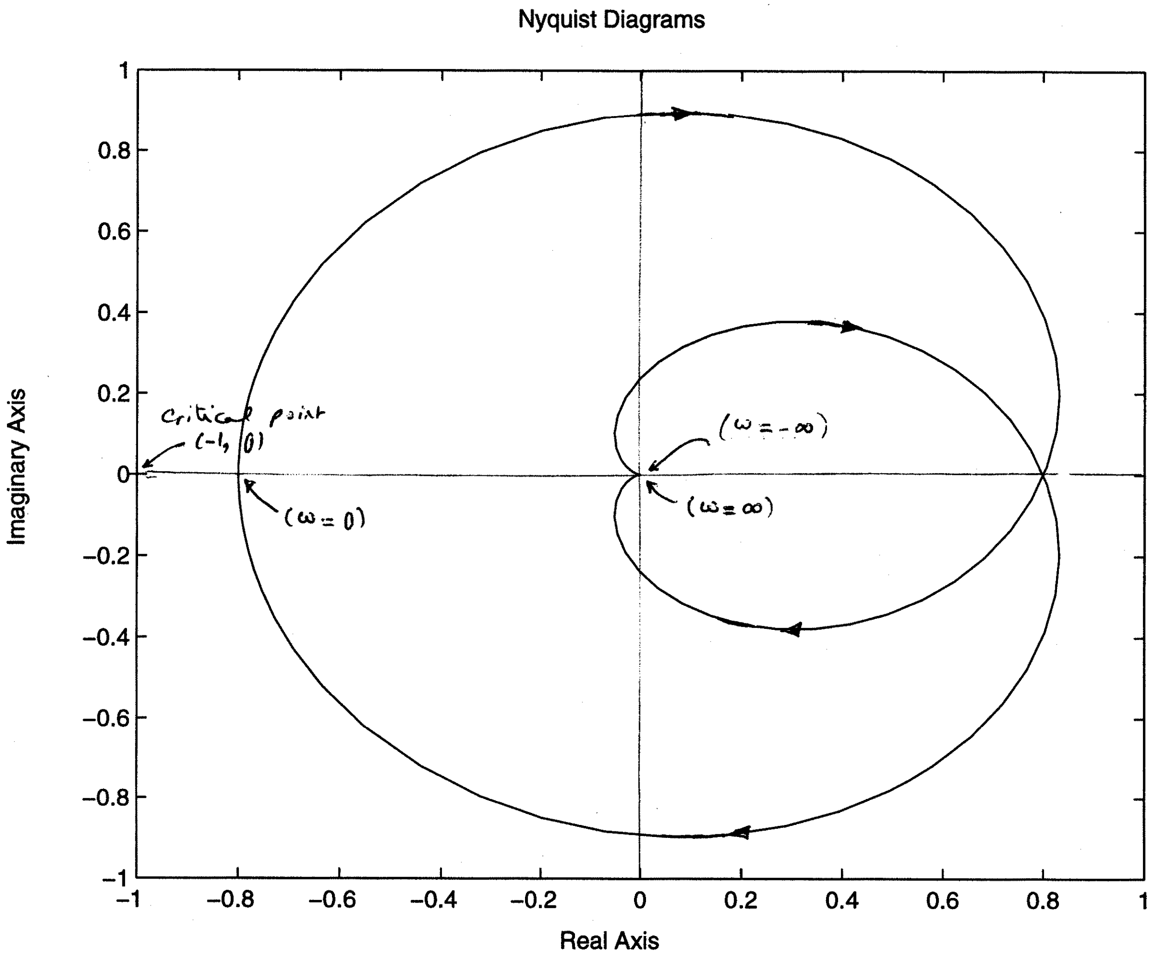


Figure 2

Nyquist diagram for $K = 0.8$

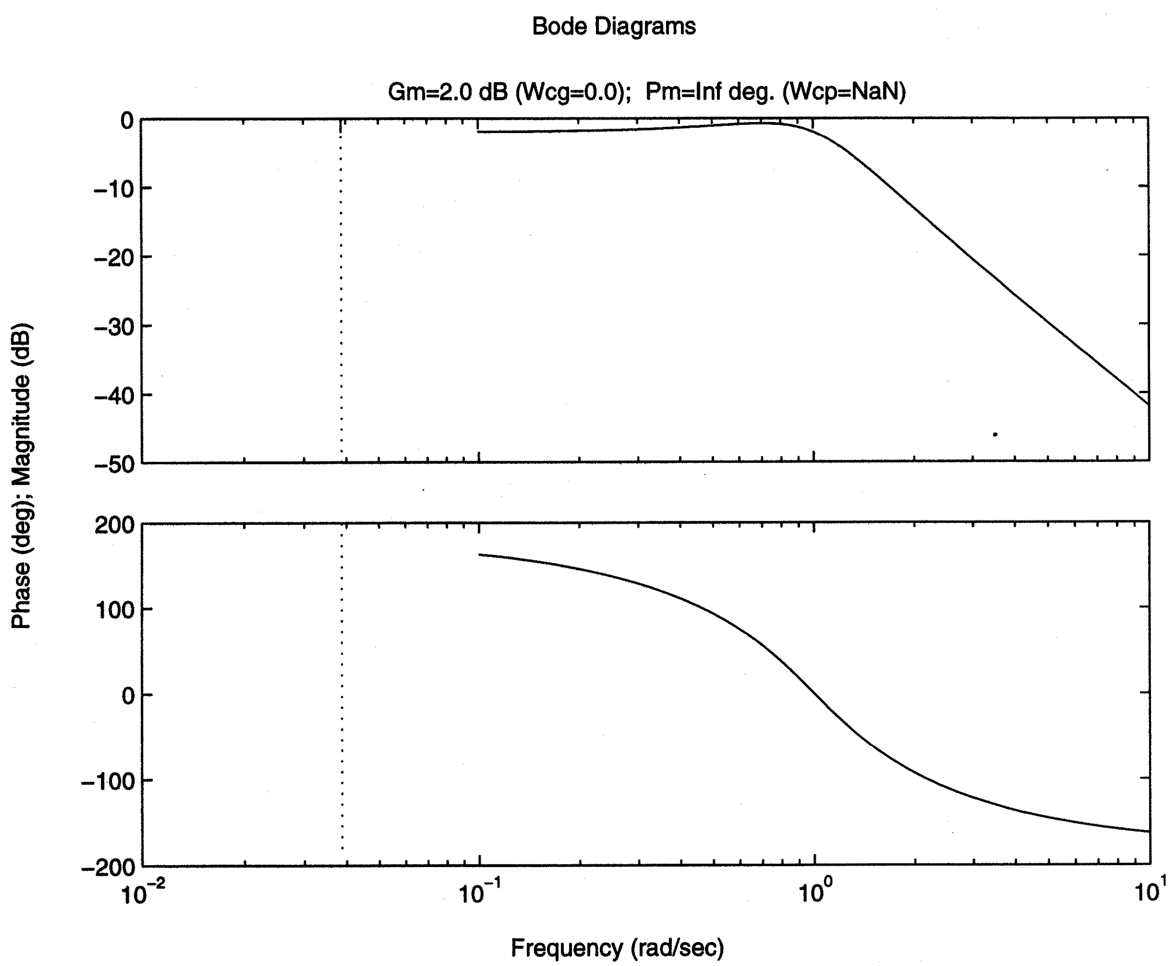


Figure 3
Bode diagram for $K = 0.8$

9.38 The MATLAB code for this experiment is attached herewith.

We are given a feedback system with the loop transfer function

$$L(s) = \frac{K(s+1)}{s^4 + 5s^3 + 6s^2 + 2s - 8}$$

This $L(s)$ is representative of a conditionally stable system in that for it to be stable, K must lie inside a certain range of values.

The characteristic equation of the system is

$$s^4 + 5s^3 + 6s^2 + (K+2)s + (K-8) = 0$$

Constructing the Routh array:

s^4	1	6	$K-8$
s^3	5	$K+2$	0
s^2	$5.6-0.2K$	$K-8$	0
s^1	$\dfrac{51.2+0.2K-0.2K^2}{5.6-0.2K}$	0	
s^0	$K-8$	0	

From this array we can make the following deductions:

1. For stability, $K > 8$, which follows from the last entry of the first column of array coefficients.

   For $K = 8$ the system is on the verge of instability. From the fourth row, it follows that this occurs when $s = 0$.

2. From the fourth row of the array, it also follows that K must satisfy the condition

$$51.2 + 0.2K - 0.2K^2 > 0$$

for the system to be stable. That is,

$$(K - 16.508)(K + 15.508) < 0$$

from which it follows that for stability

$$K < 16.508$$

If this condition is satisfied, then $5.6 - 0.2K > 0$.

When $K = 16.508$ the system is again on the verge of instability. From the third row of the array, this occurs when

$$(5.6 - 0.2K)s^2 + (K - 8) = 0$$

or

$$s = \pm j\sqrt{\frac{8.508}{2.302}} = \pm j1.923$$

Accordingly, we may state that the feedback system is stable provided that K satisfies the condition

$$8 < K \; 16.508$$

This result is confirmed using the following constructions:

(a) Root locus. Figure 1 shows the root locus of the system. Using the command rlocfind, we obtain the following limiting values of K:

$$K_{min} = 8$$

$$K_{max} = 16.5$$

For $K < 8$, the system has a single closed-loop pole in the right-half plane. For $K > 16.5$, the complex closed-loop poles of the system move to the right-half plane. Hence for stability, we must have $8 < K < 16.5$.

(b) <u>Bode diagram</u>. Figure 2(a) shows the Bode diagram of $L(s)$ for

$$K = \frac{K_{max} + K_{min}}{2}$$

$$= \frac{16.5 + 8}{2}$$

$$= 12.25$$

From this figure we obtain the following stability margins:

Gain margin $= -3.701$ dB; phase-crossover frequency $= 0$ rad/s

Phase Margin $= 14.58^\circ$, gain-crossover frequency $= 1.54$ rad/s

Figure 2(b) shows the Bode diagram for $K = 8$. From this second Bode diagram we see that the gain margin is zero, and the system is therefore on the verge of instability. Figure 2(c) shows the Bode diagram for $K = 16.508$. From this third Bode diagram we see that the phase margin is zero, and the feedback system is again on the verge of instability.

(c) Nyquist diagram. Figure 3(a) shows the Nyquist diagram of $L(j\omega)$ for $K = 12.25$. The critical point $(-1, 0)$ is not encircled for this value of K and the system is therefore stable. Figure 3(b) shows the Nyquist locus of $L(j\omega)$ for $K = 8$. The locus of Fig. 3(b) passes through the critical point $(-1, 0)$ exactly and the system is therefore on the verge of instability. Figure 3(c) shows the Nyquist locus for $K < 8$, namely, $K = 5$. The locus of Fig. 3(c) encircles the critical point $(-1, 0)$ and the system is therefore unstable. Figure 3(d) shows the Nyquist locus for $K > 16.508$, namely, $K = 20$. The locus of Fig. 3(d) encircles the critical point $(-1, 0)$ and ~~the~~ system is therefore unstable. Finally, Fig. 3(e) shows the Nyquist locus for $K = 16.508$. Here again we see that the locus passes through the critical point $(-1, 0)$ exactly, and so the system is on the verge of instability.

Care has to be exercised in the interpretation of these Nyquist loci, hence the reason for the use of shading.

```
num=[1 1];
den=[1 5 6 2 -8];
rlocus(num,den);
K=rlocfind(num,den)
K=rlocfind(num,den)
figure
num=K*num;
nyquist(num,den);
figure
margin(num,den);
```

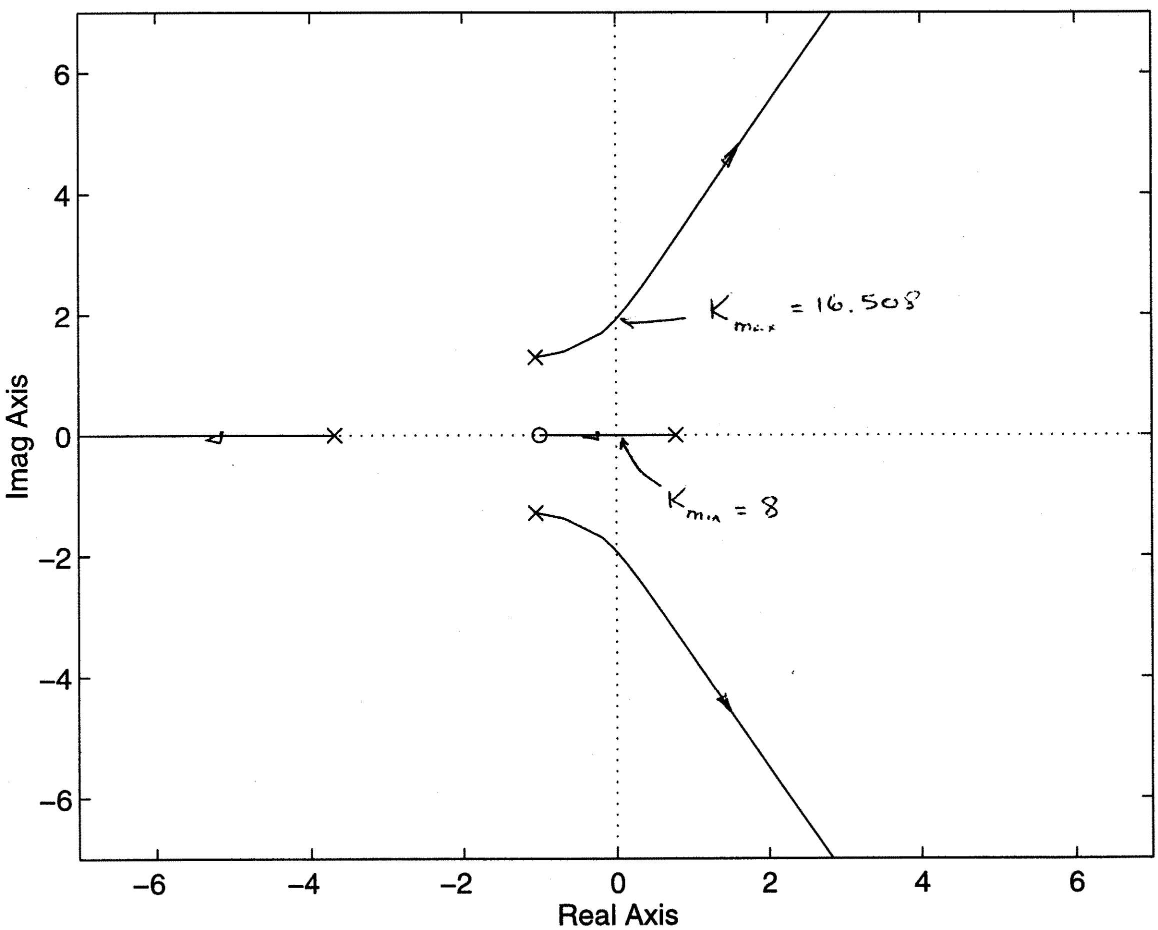


Figure 1
Root Locus

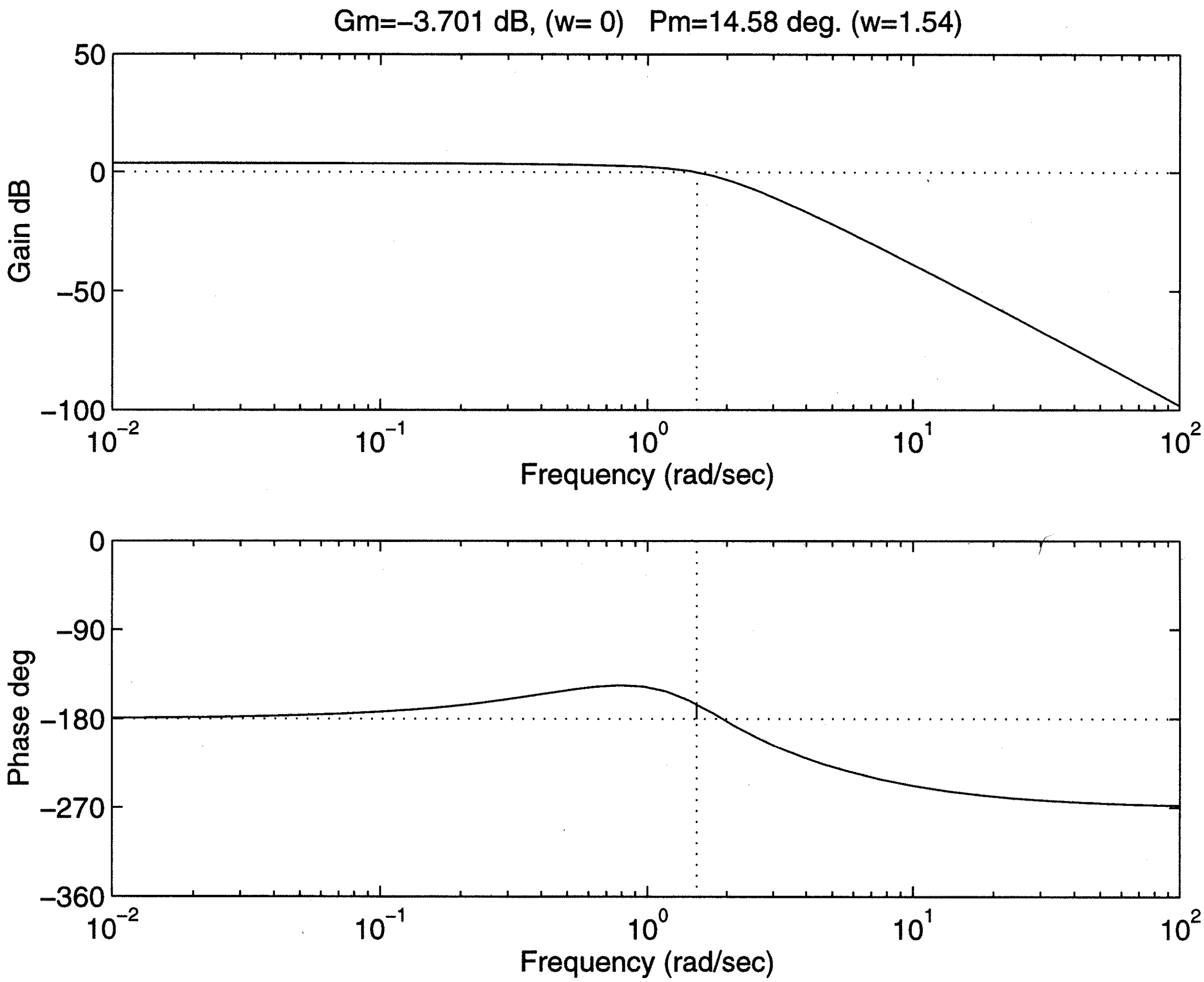


Figure 2(a)
Bode diagram for $K = 12.25$

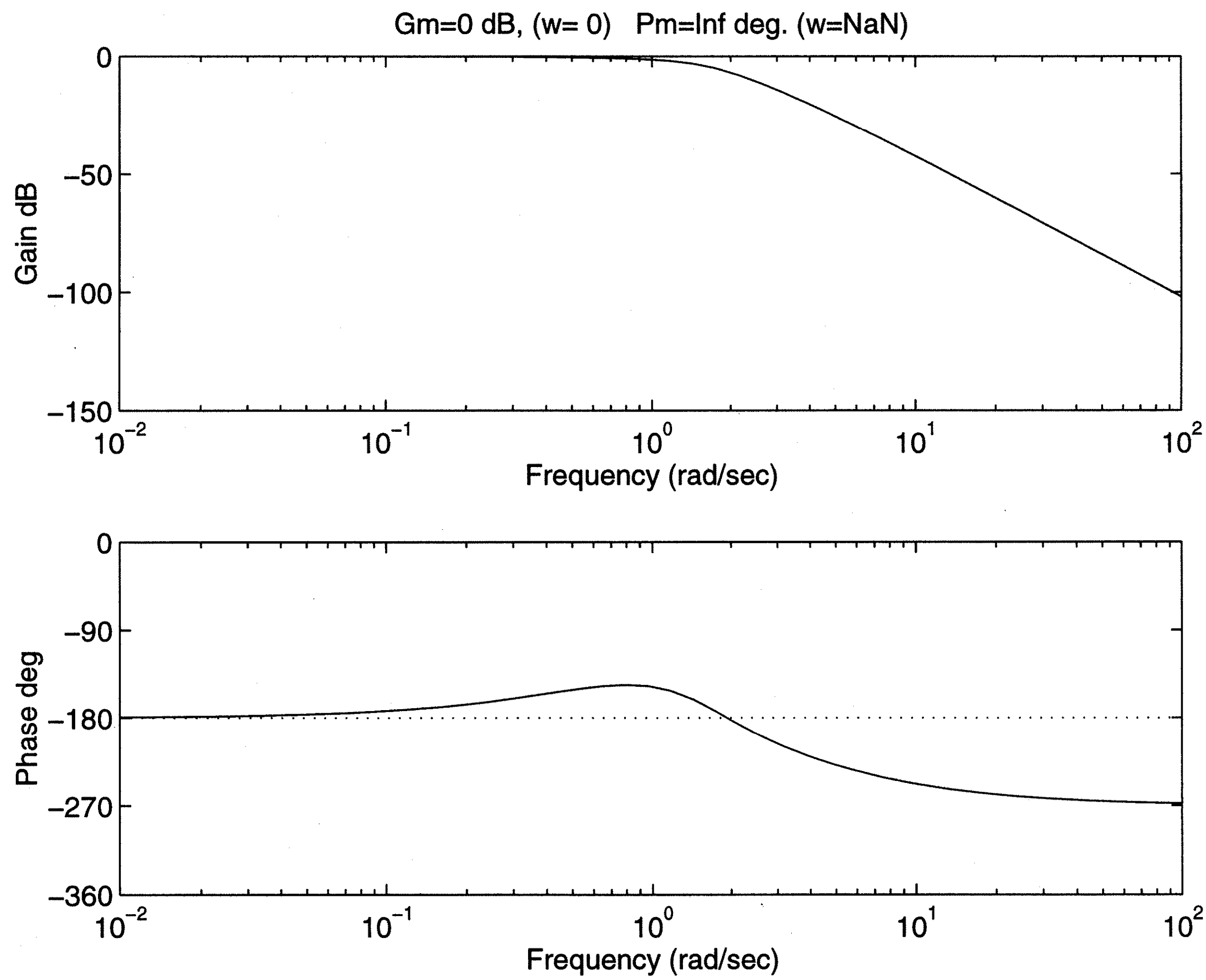


Figure 2(b)

Bode diagram for $K = 8$

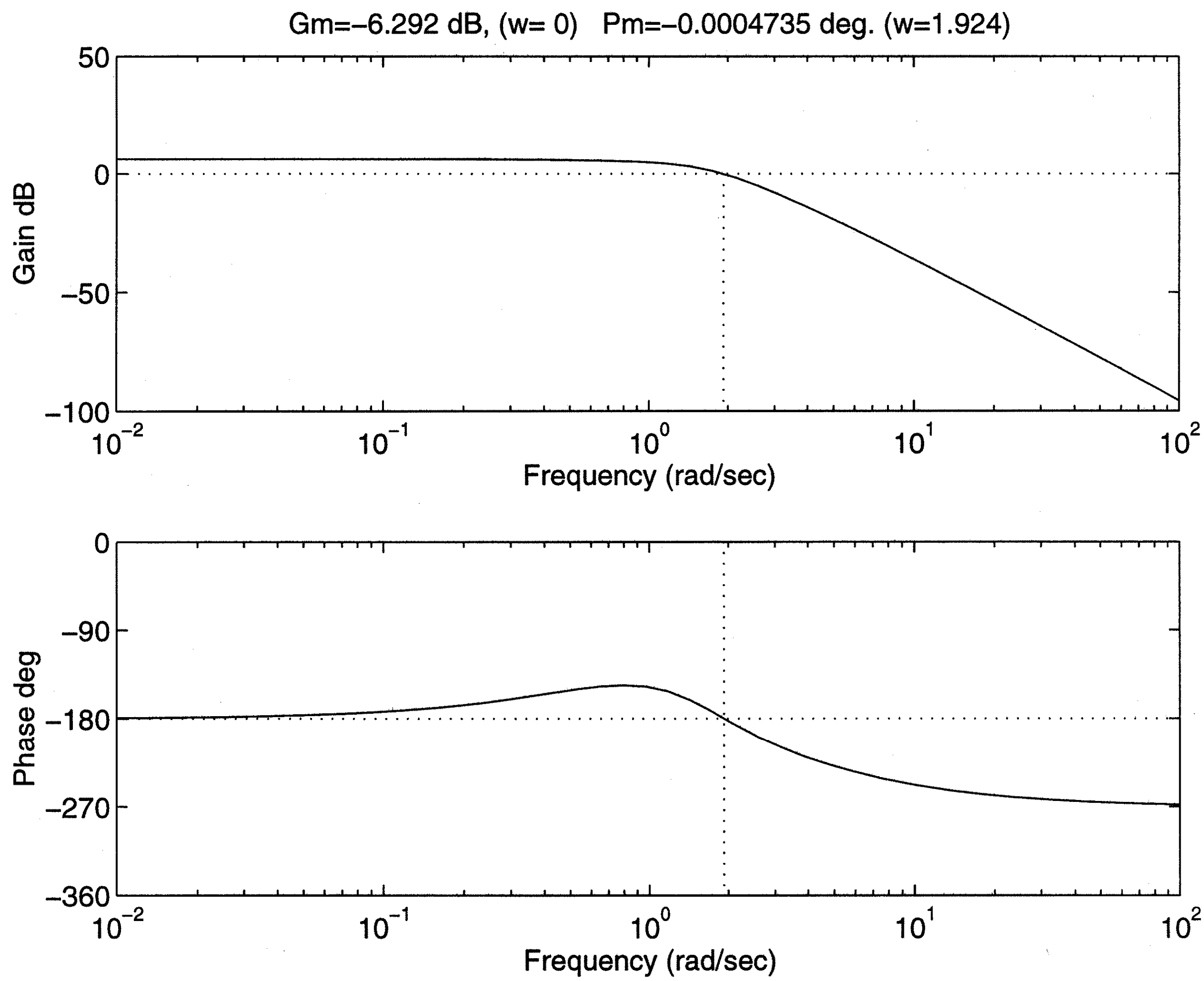


Figure 2(c)
Bode diagram for K = 16.508

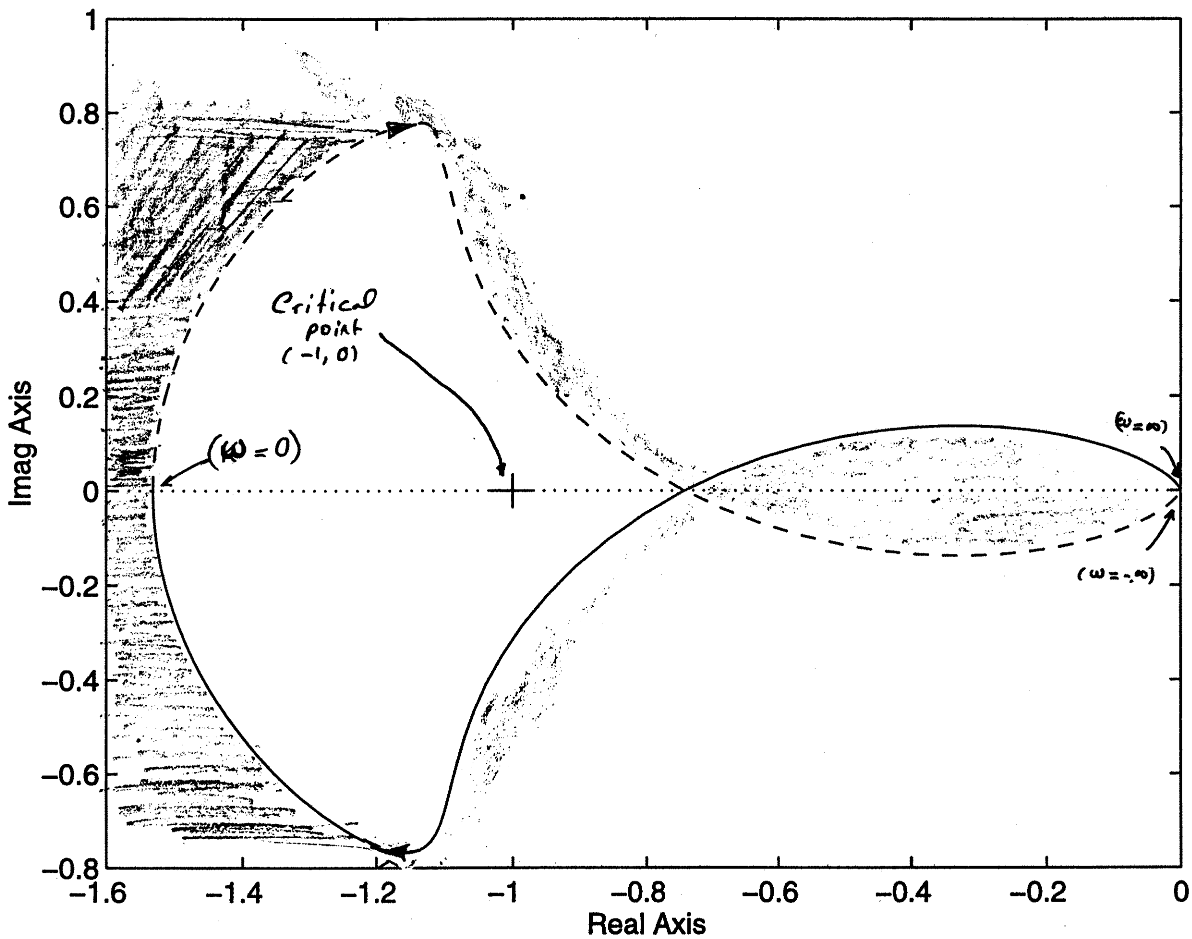


Figure 3(c)

Nyquist locus for K = 12.25

(stable system)

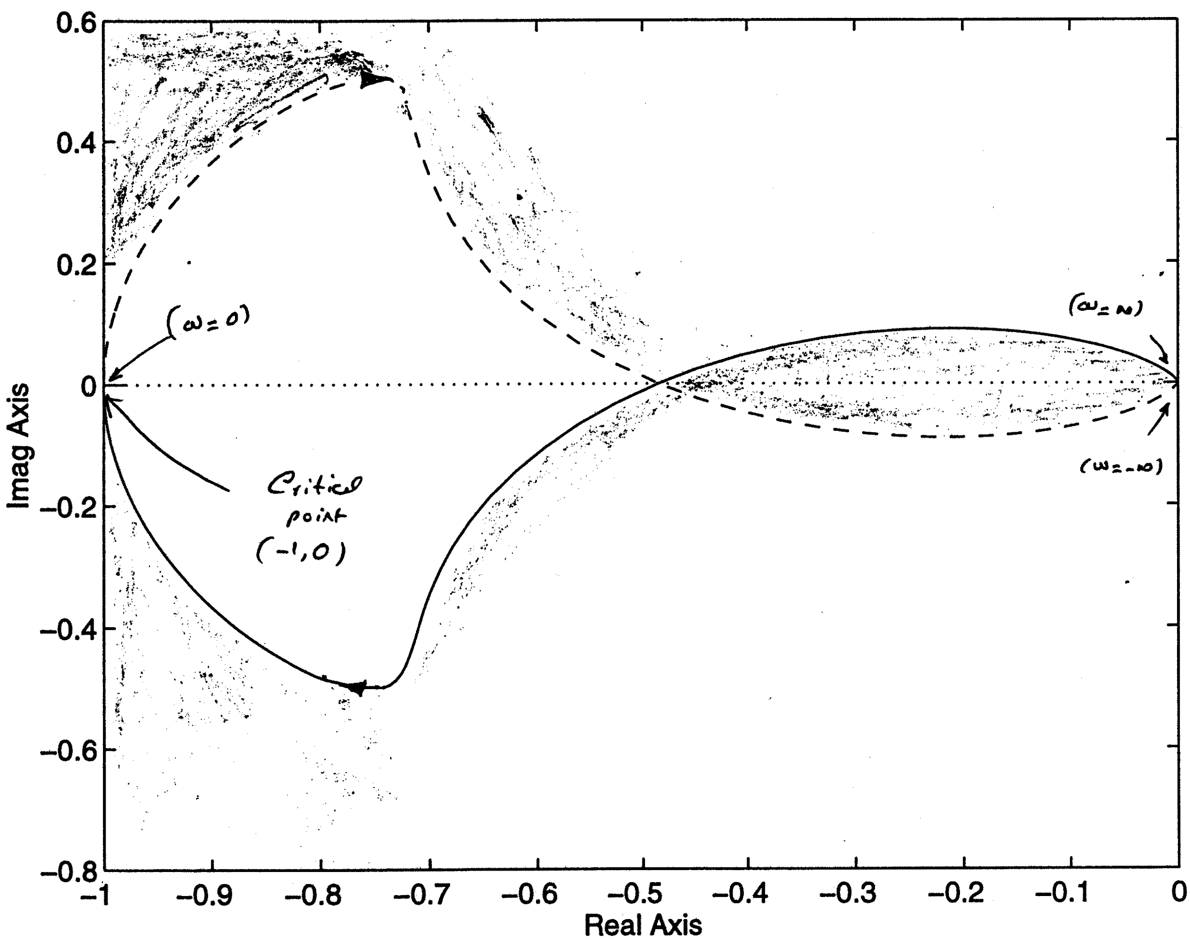


Figure 3(a)
Nyquist locus for $K = 8$
(system on the verge of instability)

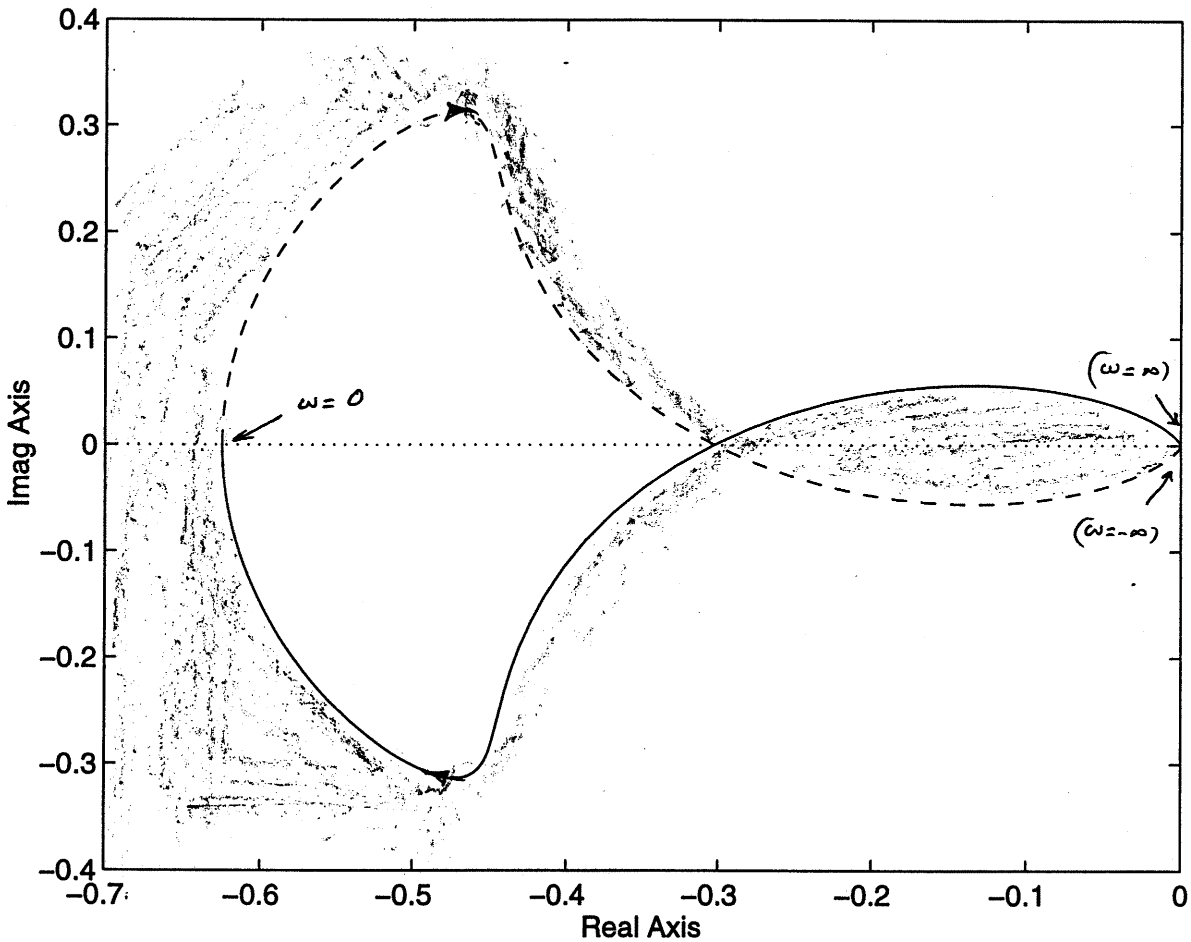


Figure 3(c)
Nyquist locus for K = 5
(Critical point (-1, 0) is encircled by the locus and the system is unstable)

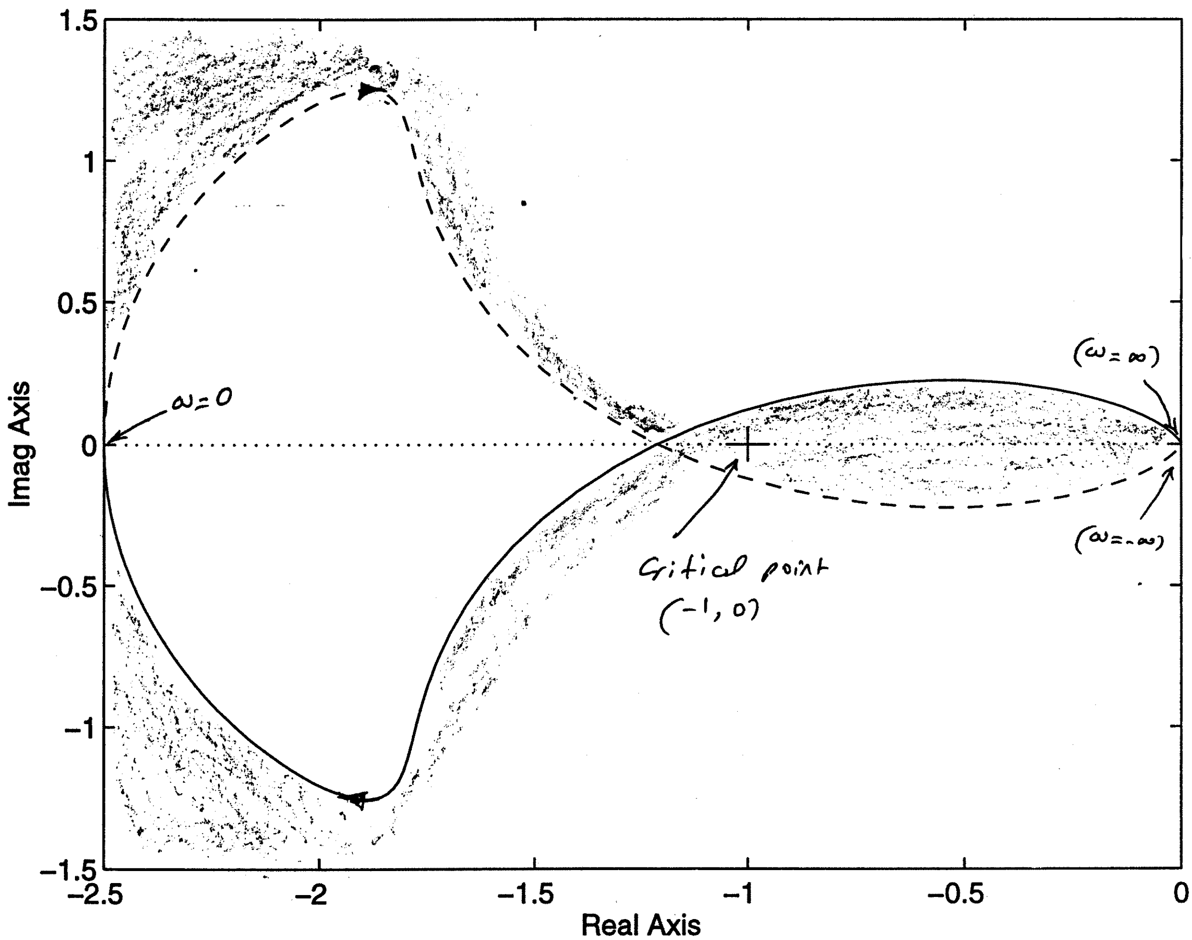


Figure 3(d)

Nyquist locus for K = 20

(unstable system)

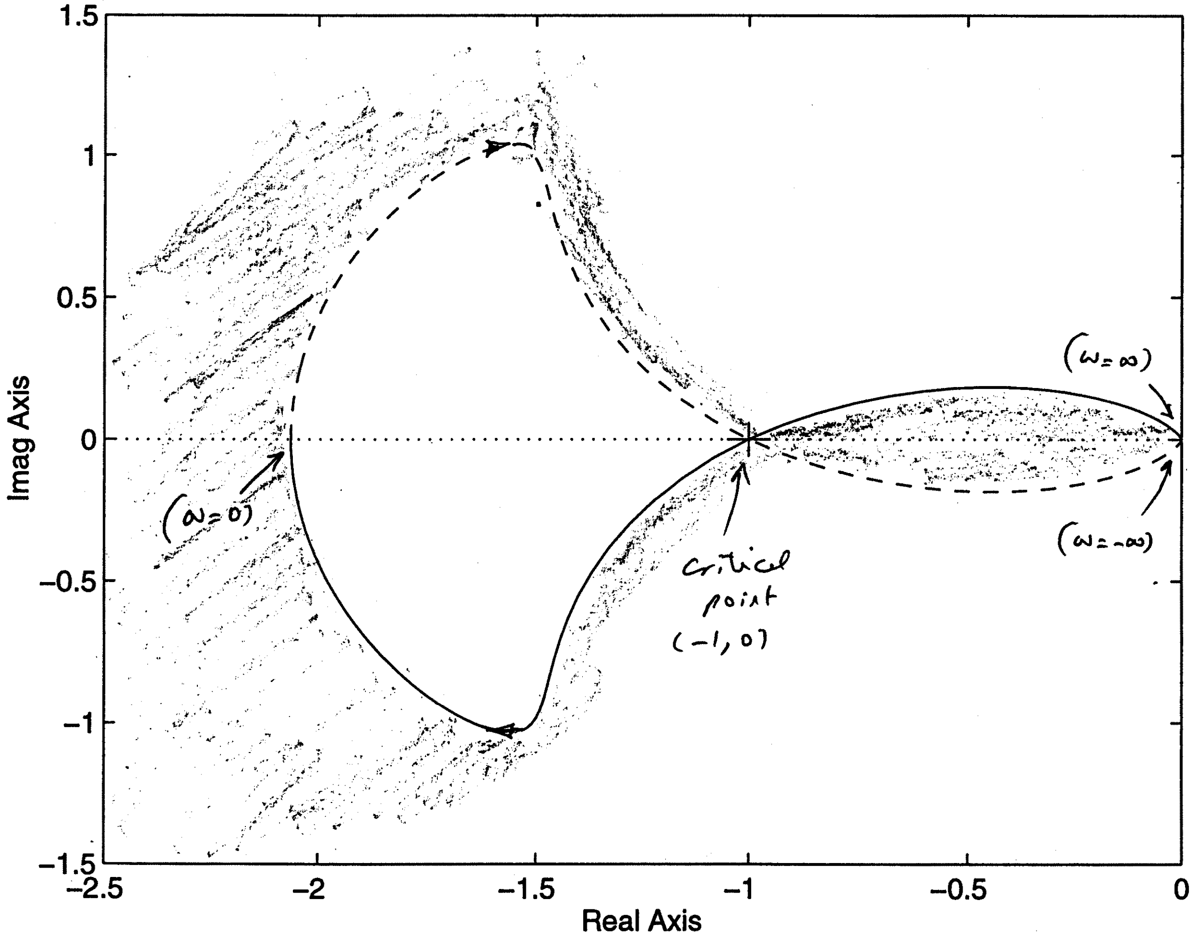


Figure 3(e)

Nyquist locus for $K = 16.508$

(System is on the verge of instability)

9.39 We are given a unity feedback system with loop transfer function

$$L(s) = \frac{K}{s(s+1)}$$

$$= \frac{K}{s^2 + s} \qquad (1)$$

The closed-loop transfer function of the system is

$$T(s) = \frac{L(s)}{1 + L(s)}$$

$$= \frac{K}{s^2 + s + K} \qquad (2)$$

(a) With $K = 1$, $T(s)$ takes the value

$$T(s) = \frac{1}{s^2 + s + 1} \qquad (3)$$

In general, the transfer function of a second-order system is described as

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta_n\omega_n s + \omega_n^2} \qquad (4)$$

Comparing Eqs.(3) and (4):

$\omega_n = 1$

$\zeta_n = 0.5$

The closed-loop poles of the uncompensated system with $K = 1$ are located at $s = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$. Figure 1 shows the root locus of the uncompensated system.

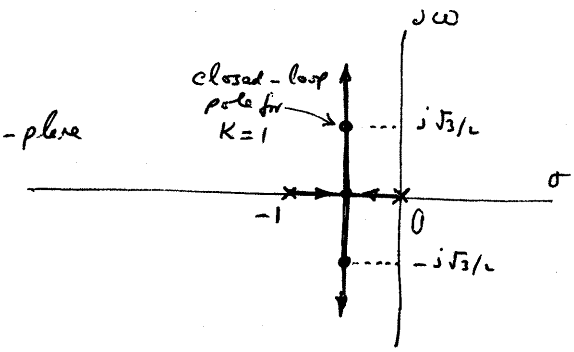


Figure 1

(b) The feedback system is to be compensated so as to produce a dominant pair of closed-loop poles with $\omega_n = 2$ and $\zeta_n = 0.5$. That is, the damping factor is left unchanged but the natural frequency is doubled. This set of specifications is equivalent to pole locations at $s = -1 \pm j\sqrt{3}$, as indicated in Fig. 2, shown on the next page.

It is clear from the root locus of Fig. 1 that this requirement cannot be satisfied by

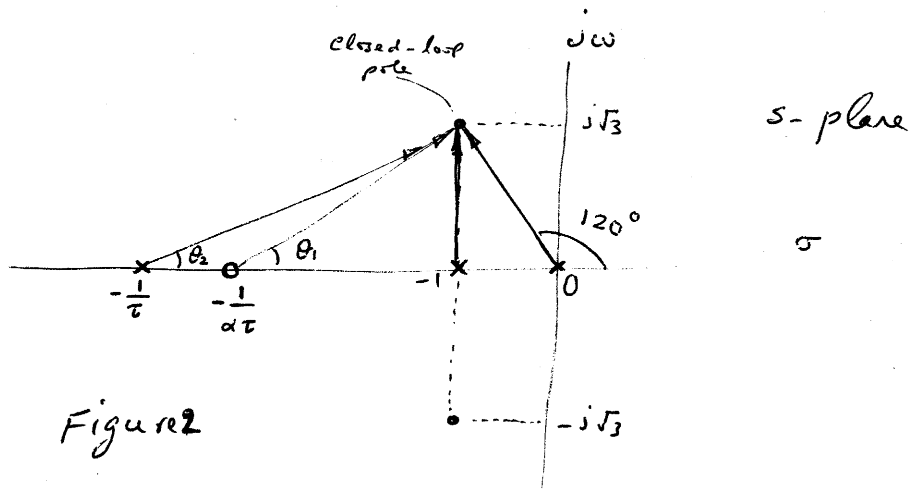


Figure 2

a change in the gain K of the uncompensated loop transfer function L(s). Rather, we may have to use a phase-lead compensator as indicated in the problem statement.

We may restate the design requirement:

- Design a phase-lead compensated system so that the resulting root locus has a dominant pair of closed-loop poles at $s = -1 \pm j\sqrt{3}$ as indicated in Fig. 2.

For this to happen, the angle criterion of the root

root locus must be satisfied at $s = -1 \pm j\sqrt{3}$. With the open-loop poles of the uncompensated system at $s = 0$ and $s = -1$, we readily see from Fig. 2 that the sum of contributions of these two poles to the angle criterion is

$$-120^\circ - 90^\circ = -210^\circ$$

We therefore require a phase advance of 30° to satisfy the angle criterion.

Figure 2 also includes the pole-zero pattern of the phase-lead compensator, where the angles θ_1 and θ_2 are defined by

$$\theta_1 = \tan^{-1}\left(\frac{\sqrt{3}}{\frac{1}{\alpha\tau} - 1}\right) \tag{5}$$

$$\theta_2 = \tan^{-1}\left(\frac{\sqrt{3}}{\frac{1}{\tau} - 1}\right) \tag{6}$$

where the parameters α and τ pertain to the compensator.

To realize a phase advance of 30°, we require

$$\theta_1 - \theta_2 = 30^\circ$$

or

$$\theta_1 = 30^\circ + \theta_2$$

Taking the tangents of both sides of this equation:

$$\tan\theta_1 = \frac{\tan 30^\circ + \tan\theta_2}{1 - \tan 30^\circ \tan\theta_2}$$

$$= \frac{\frac{1}{\sqrt{3}} + \tan\theta_2}{1 - \frac{1}{\sqrt{3}}\tan\theta_2} \tag{7}$$

Using Eqs. (5) and (6) in (7) and rearranging terms, we find that with α the time constant τ is constrained by the equation

$$\tau^2 - 0{,}95\,\tau + 0{,}025 = 0$$

Solving this equation for τ:

$$\tau = 0.923 \quad \text{or} \quad \tau = 0.027$$

The solution $\tau = 0.923$ corresponds to a compensator whose transfer function has a zero to the right of the desired dominant poles and a pole on their left. On the other hand, for the solution $\tau = 0.027$ both the pole and the zero of the compensator lie to the left of the dominant closed-loop poles, which conforms to the picture portrayed in Fig. 2. So we choose $\tau = 0.027$, as suggested in the problem statement.

(c) The loop transfer function of the compensated feedback system is

$$L_c(s) = G_c(s)\, L(s)$$

$$= \frac{\alpha \tau s + 1}{\tau s + 1} \; \frac{K}{s(s+1)}$$

$$= \frac{K(0.27s + 1)}{s(s+1)(0.027s + 1)}$$

Figure 3a shows a complete root locus of $L(s)$. An expanded version of the root locus around the origin is shown in Fig. 3b. Using the RLOCFIND command of MATLAB, the point $-1 + j\sqrt{3}$ is located on the root locus. The value of K corresponding to this closed-loop pole is 3.846.

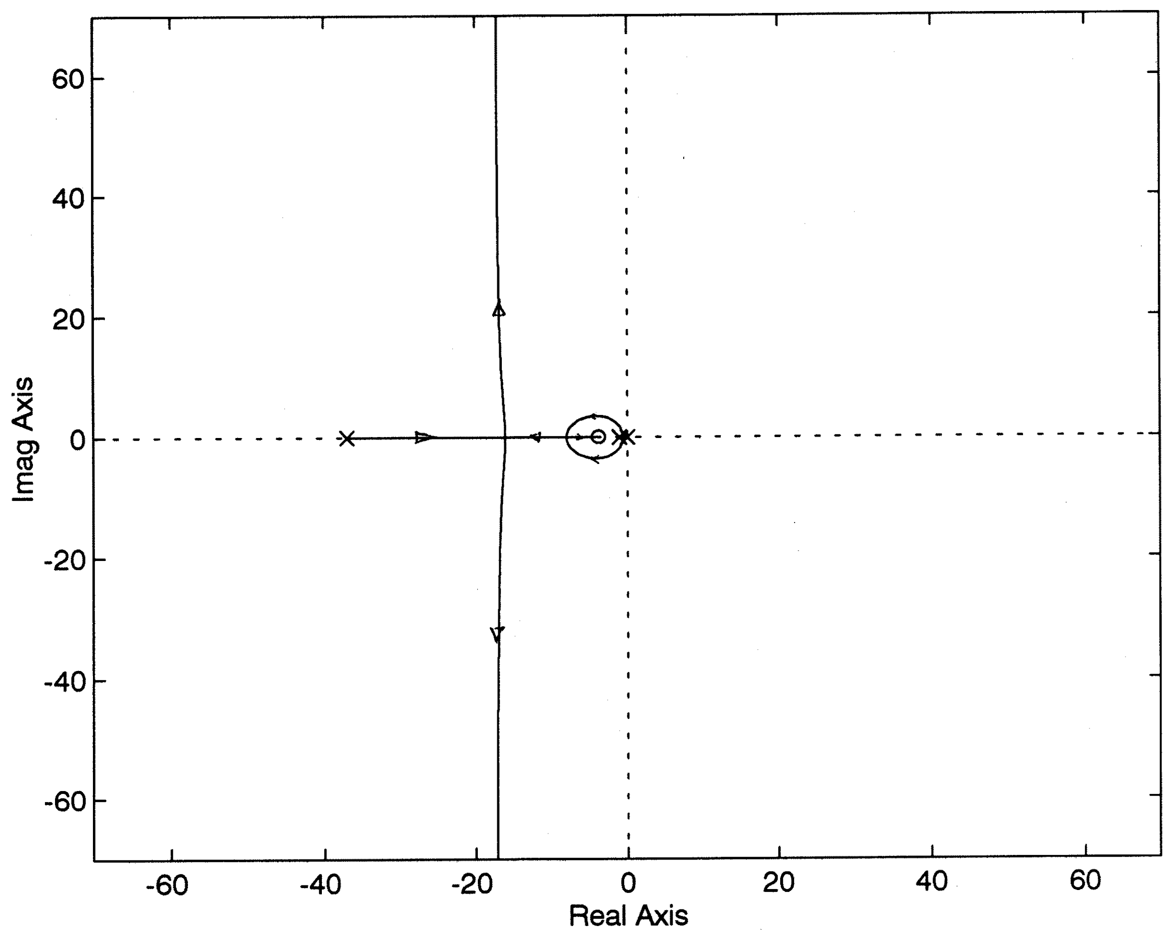


Figure 3(a)

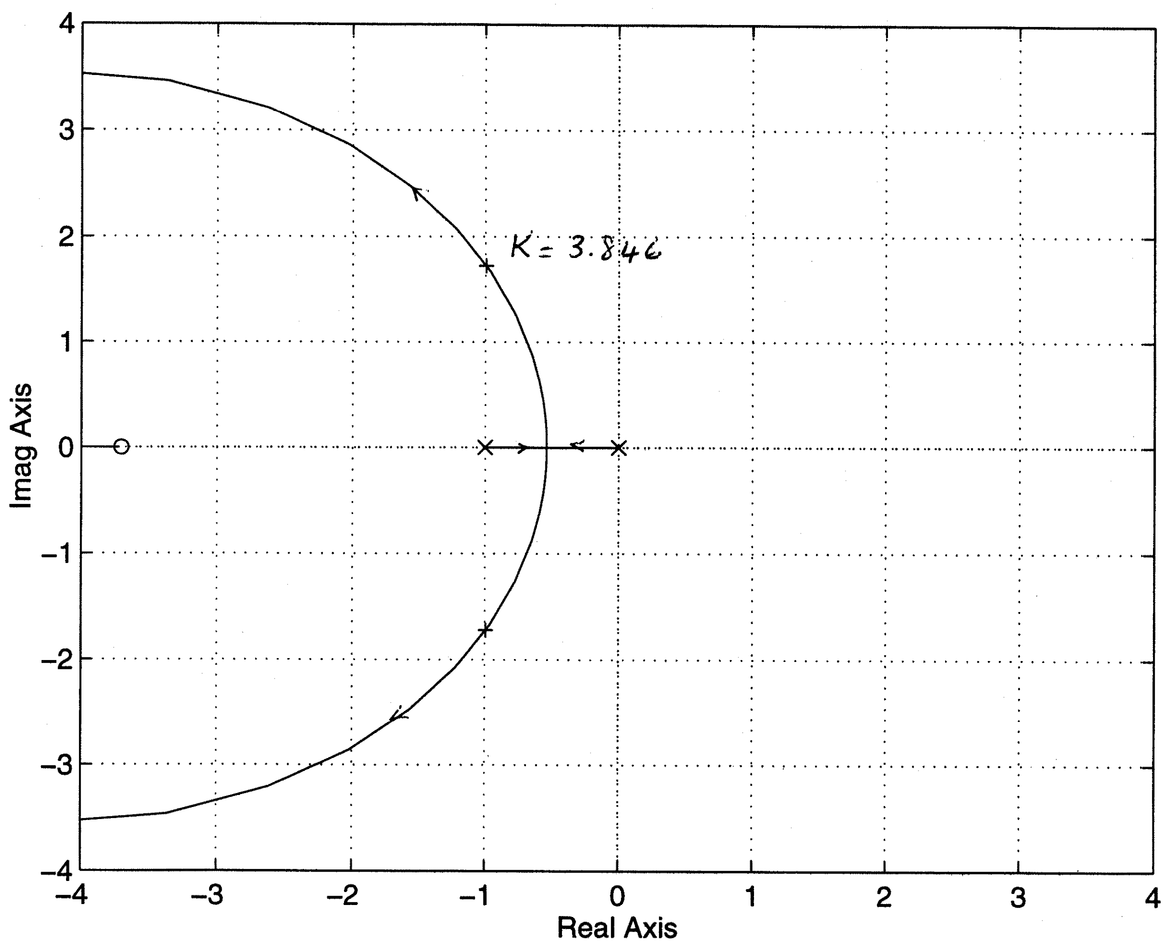


Figure 31(b)

9.40 The loop transfer function of the compensated feedback system is

$$L(s) = G(s)\,H(s)$$

$$= \frac{10K(\alpha\tau s + 1)}{s(0.2s+1)(\tau s+1)}, \qquad \alpha < 1 \qquad (1)$$

We start the system design with the requirement for a steady-state error of 0.1 to a ramp input of unit slope.

The system under study is a type-1 system. The velocity error constant of such a system is given by

$$K_v = \lim_{s \to 0} s\,G(s)\,H(s)$$

Hence,

$$K_v = \lim_{s \to 0} \frac{10K(\alpha\tau s+1)}{(0.2s+1)(\tau s+1)}$$

$$= 10K$$

From the definition of K_v:

$$\frac{1}{K_v} = 0.1$$

Hence $K = 1$.

Next, we use the prescribed value of percentage overshoot to calculate the minimum permissible phase margin. We do this in two steps:

1. We use the relation between percentage overshoot P.O. and damping ratio ζ (see Example 9.5)

$$P.O. = 100\, e^{-\pi\zeta/\sqrt{1-\zeta^2}} \qquad (2)$$

With P.O. = 10% in response to a step input, the use of Eq. (2) yields

$$\frac{\zeta}{\sqrt{1-\zeta^2}} = \frac{1}{\pi}\log_e 10 = 0.7329$$

Hence, solving for ζ:

$$\zeta = 0.5911$$

We may thus set $\zeta = 0.6$.

2. We use the relation between phase margin φ_m and damping ratio ζ (see Eq. (9.104))

$$\varphi_m = \tan^{-1}\left(\frac{2\zeta}{\sqrt{\sqrt{4\zeta^4+1}-2\zeta^2}}\right)$$

Using the value $\zeta = 0.6$:

$$\varphi_m = \tan^{-1}\left(\frac{1.2}{\sqrt{\sqrt{4\times 0.6^4+1}-2\times 0.6^2}}\right)$$

$$= \tan^{-1}(1.6767)$$

$$= 59.19°$$

Hence we may set $\varphi_m = 59°$

The next step in the design is to calculate the minimum value of the gain-crossover frequency ω_g. To do this, we use the requirement that the 5% settling time of the step response should be less than 2s. We may again proceed in two stages:

1. Using the formula for the settling time T_s with $\delta = 5\% = 0.05$ (see Example 9.5)

$$e^{-\zeta\omega_n T_s} = 0.05$$

Solving this equation for ω_n with $T_s = 2$ seconds and $\zeta = 0.6$:

$$\omega_n = \frac{\log_e 20}{2 \times 0.6}$$

$$= 2.5 \text{ rad/s}$$

2. Using Eq. (9.103) for the gain-crossover frequency:

$$\omega_g = \omega_n \sqrt{\sqrt{4\zeta^4 + 1} - 2\zeta^2}$$

$$= 2.5 \sqrt{\sqrt{4 \times 0.6^4 + 1} - 2 \times 0.6^2}$$

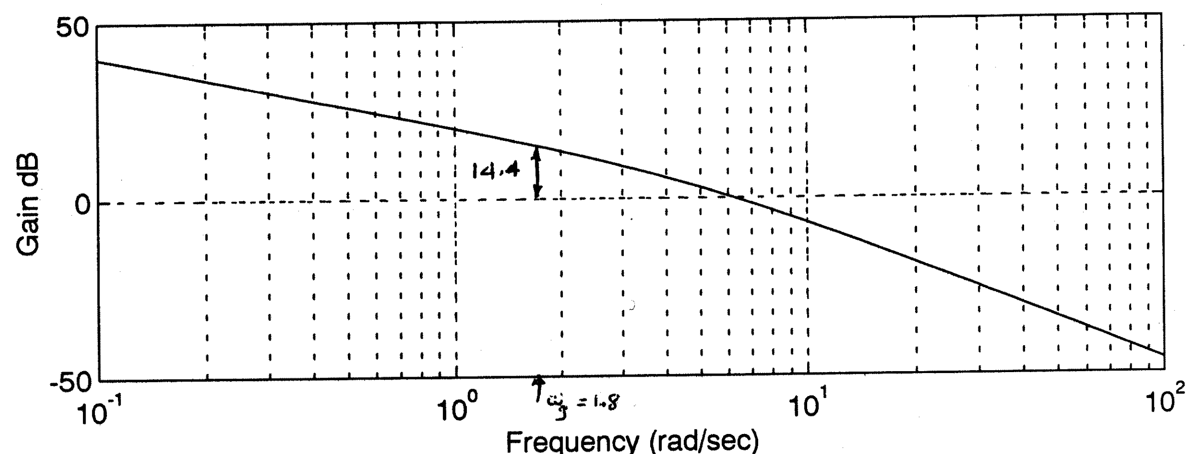
$$= 1.79 \text{ rad/s}$$

So we may set $\omega_g = 1.8$ rad/s.

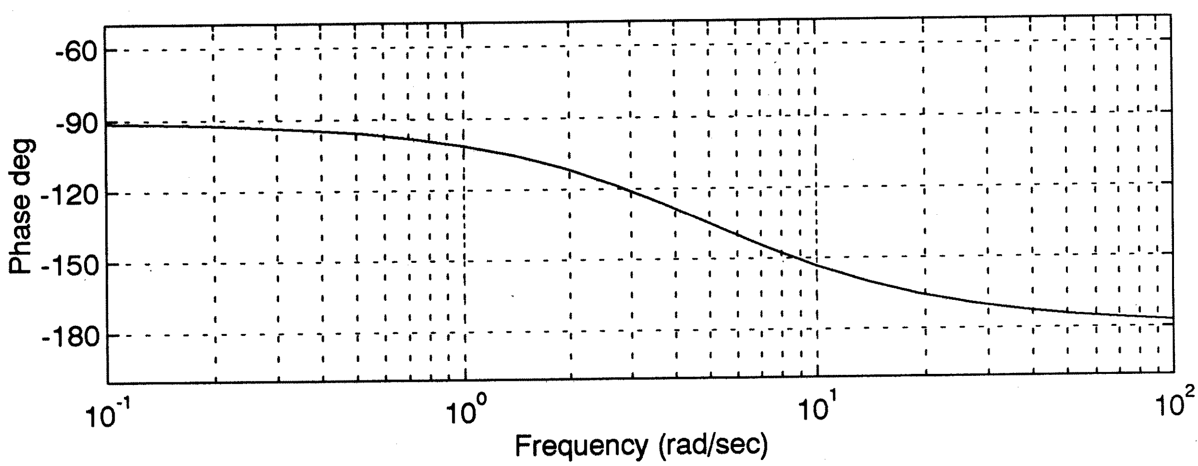
The stage is now set for calculating the parameters of the compensating network. Figures 1(a) and 1(b) show the uncompensated loop gain response and loop phase response, respectively. From Fig. 1(a) we see that at $\omega = \omega_g = 1.8$ rad/s, the uncompensated loop gain is 14.4 dB, which corresponds to the numerical value 5.23. Hence

$$\alpha = \frac{1}{5.23} = 0.19$$

We also note that the corner frequency $1/\alpha\tau$ should coincide with $\omega_g/10$, and so we may set



(a)



(b)

Figure 1

Uncompensated loop response

$$\frac{\omega_g}{10} = \frac{1}{\alpha \tau}$$

With $\omega_g = 1.8$ rad/s and $\alpha = 0.19$, we thus get

$$\tau = \frac{10}{\alpha \omega_g}$$

$$= \frac{10}{0.19 \times 1.8}$$

$$= 29 \text{ seconds}$$

The transfer function of the compensator is therefore

$$H(s) = K\left(\frac{\alpha \tau s + 1}{\tau s + 1} \right)$$

$$= \frac{5.6 s + 1}{29 s + 1}$$

Performance evaluation and Fine Tuning

The compensated loop transfer function is

$$L(s) = G(s)\, H(s)$$

$$= \frac{10(5.6 s + 1)}{s(0.2 s + 1)(29 s + 1)}$$

Figure 2 shows the compensated loop response of the feedback system. According to this figure, the phase margin ϕ_m is $65.46°$ and the gain-crossover frequency ω_g is 1.823 rad/s, both of which are within the design calculations.

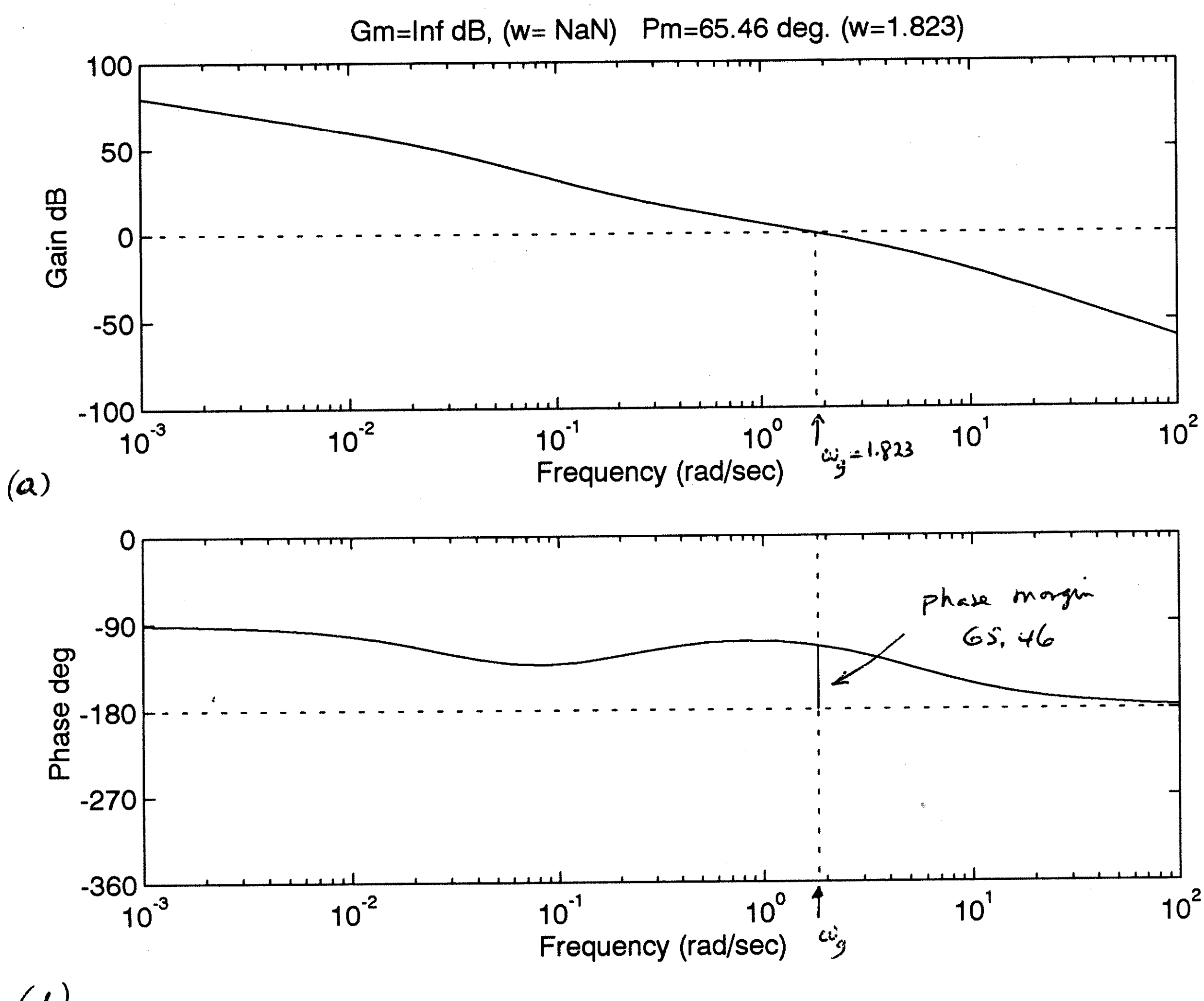


Figure 2

Compensated loop response

$$H(s) = \frac{5.6s + 1}{29s + 1}$$

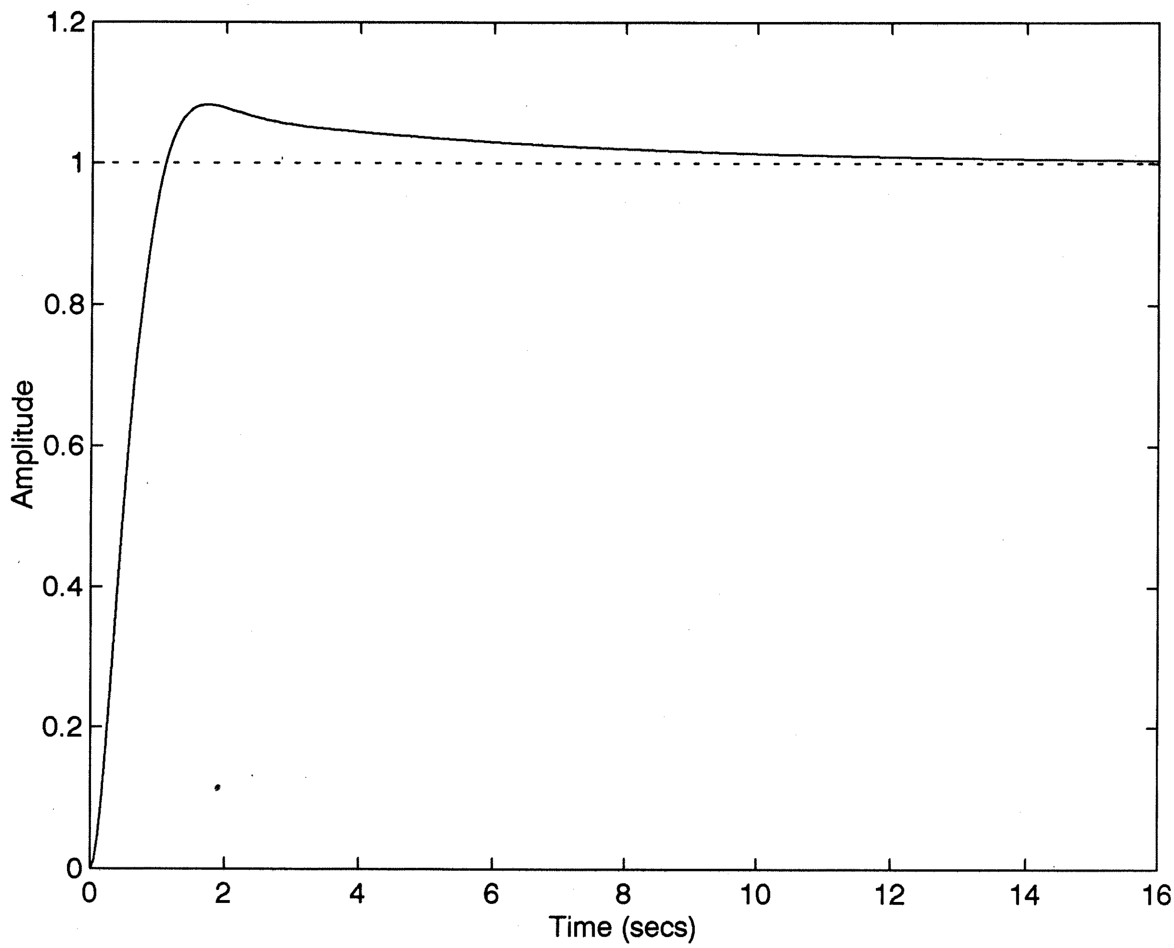


Figure 3

Step response of compensated system.

$$H(s) = \frac{5.6s + 1}{24s + 1}$$

Figure 3 shows the response of the closed-loop system to a step input. The overshoot is less than the prescribed value of 10%. But the 5% settling time is greater than the prescribed value of 2 seconds. We therefore need to fine tune the compensator design.

We propose to move the pole-zero pattern of the compensator's transfer function $H(s)$ away from the $j\omega$-axis so as to reduce the settling time. Specifically, we modify $H(s)$ as

$$H(s) = \frac{5.2s + 1}{20s + 1}$$

Hence the modified loop transfer function is

$$L(s) = \frac{10(5.2s+1)}{s(0.2s)(20s+1)} \qquad (3)$$

Figures 4 and 5 show the modified loop frequency response and closed-loop step response of the feedback system, respectively. From these figures we observe the following:

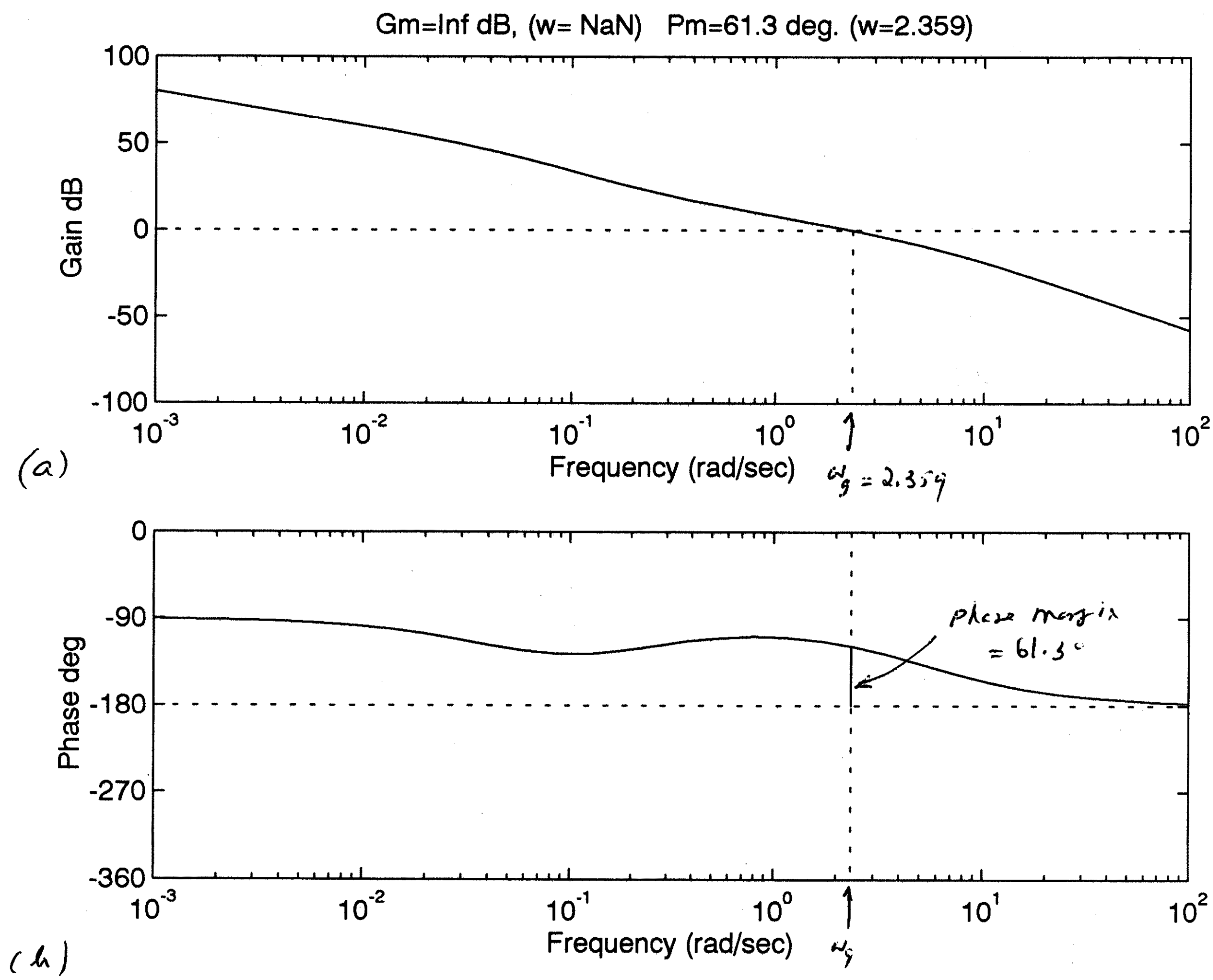


Figure 4
Loop frequency response with fine-tuned compensator:

$$H(s) = \frac{5.2s+1}{20s+1}$$

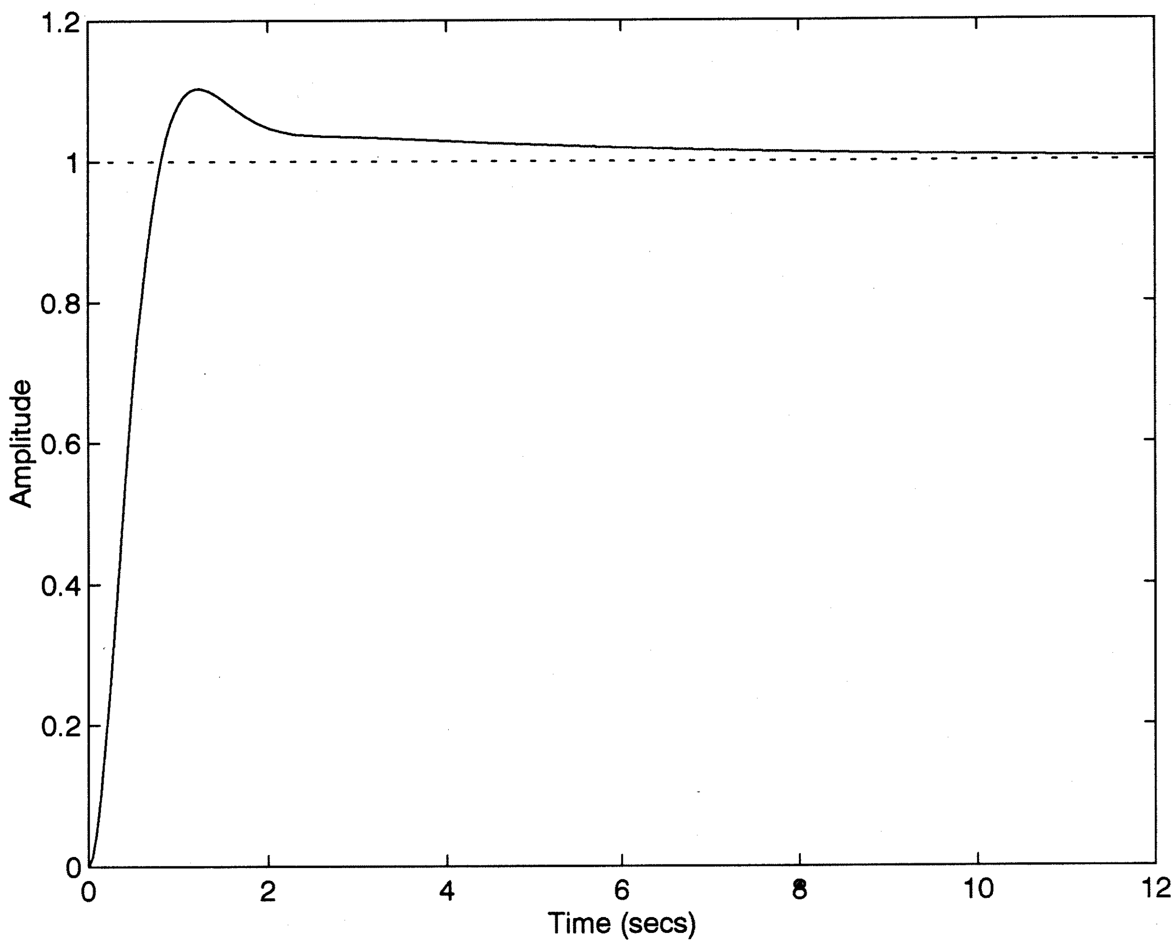


Figure 5 step response with fine-tuned compensator:

$$H(s) = \frac{5.2s + 1}{20s + 1}$$

1. The phase margin is 61.3° and the gain-crossover frequency is 2.359, both of which are within the design calculations.

2. The percentage overshoot of the step response is just under 10% and the 5 percent settling time is just under 2 seconds.

We can therefore say that the modified transfer function of Eq. (3) does indeed meet all the prescribed design specifications.

<u>Design of the Compensator</u>

Figure 6 shows an operational amplifier circuit for implementing the phase-lead compensator, characterized by the transfer function

$$H(s) = \frac{5.2\,s + 1}{20\,s + 1}$$

The transfer function of this circuit is

$$\frac{V_2(s)}{V_1(s)} = \frac{R_2}{R_3}\left(\frac{R_1 C_1 s + 1}{(R_1 + R_2)\, C_1 s + 1}\right)$$

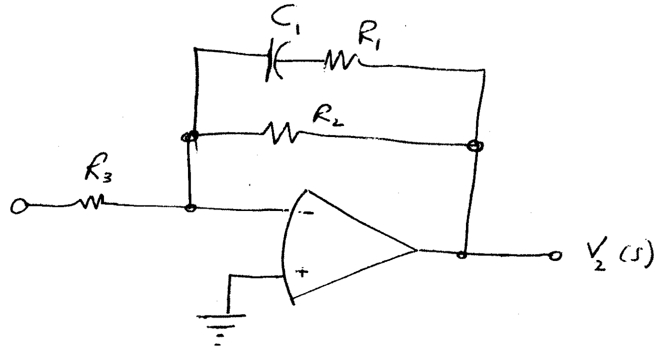


Figure 6

Hence,

$$\frac{R_2}{R_3} = 1$$

$$R_1 C_1 = 5.2$$

$$(R_1 + R_2) C_1 = 20$$

Choose $C_1 = 10\,\mu F$. We may then solve for the resistive elements of the circuit:

$$R_1 = 0.52\ M\Omega$$

$$R_2 = 1.48\ M\Omega$$

$$R_3 = 1.48\ M\Omega$$

9.41 The MATLAB code is attached.

(a) We are given the loop transfer function

$$L(s) = \frac{K(s-1)}{(s+1)(s^2+s+1)}$$

This is the same as the $L(s)$ considered in Problem 9.38, except for the fact that this time the gain factor K is negative. Let

$$K = -K', \qquad K' \geq 0$$

We may then rewrite $L(s)$ as

$$L(s) = \frac{K'(-s+1)}{(s+1)(s^2+s+1)}$$

Now we can proceed in exactly the same way as before, treating K' as a positive gain factor.

Figure 1 shows the root locus of $L(s)$. Using the command rlocfind, we find that the system is on the verge of instability (i.e., the closed-loop poles reside exactly on the $j\omega$-axis) when $K' = 1$ or $K = -1$.

We may verify this special value of K' using the Routh-Hurwitz criterion. The characteristic equation of the system is

$$s^3 + 2s^2 + (2-K')s + (1+K') = 0$$

Hence constructing the Routh array:

$$\begin{array}{lll}
s^3 & 1 & 2-K' \\
s^2 & 2 & 1+K' \\
s^1 & \frac{3(1-K')}{2} & 0 \\
s^0 & 1+K' & 0
\end{array}$$

The third element of the first column of array coefficients is zero when $K'=1$ or $K=-1$. This occurs when

$$2s^2 + (1+K') = 0$$

or

$$s = \pm j$$

as shown on the root locus in Fig. 1.

```
num=-1*[1 -1];
den=[1 2 2 1];
rlocus(num,den);
figure

K=[-3 -2 -1 -.4 -.2 -.1];
for i=1:length(K)
  num=K(i)*[1 1];
  den=[1 5 6 2 -8];
  subplot(2,3,i);
  rlocus(num,den);
  title(['K = 'num2str(K(i))])
end
```

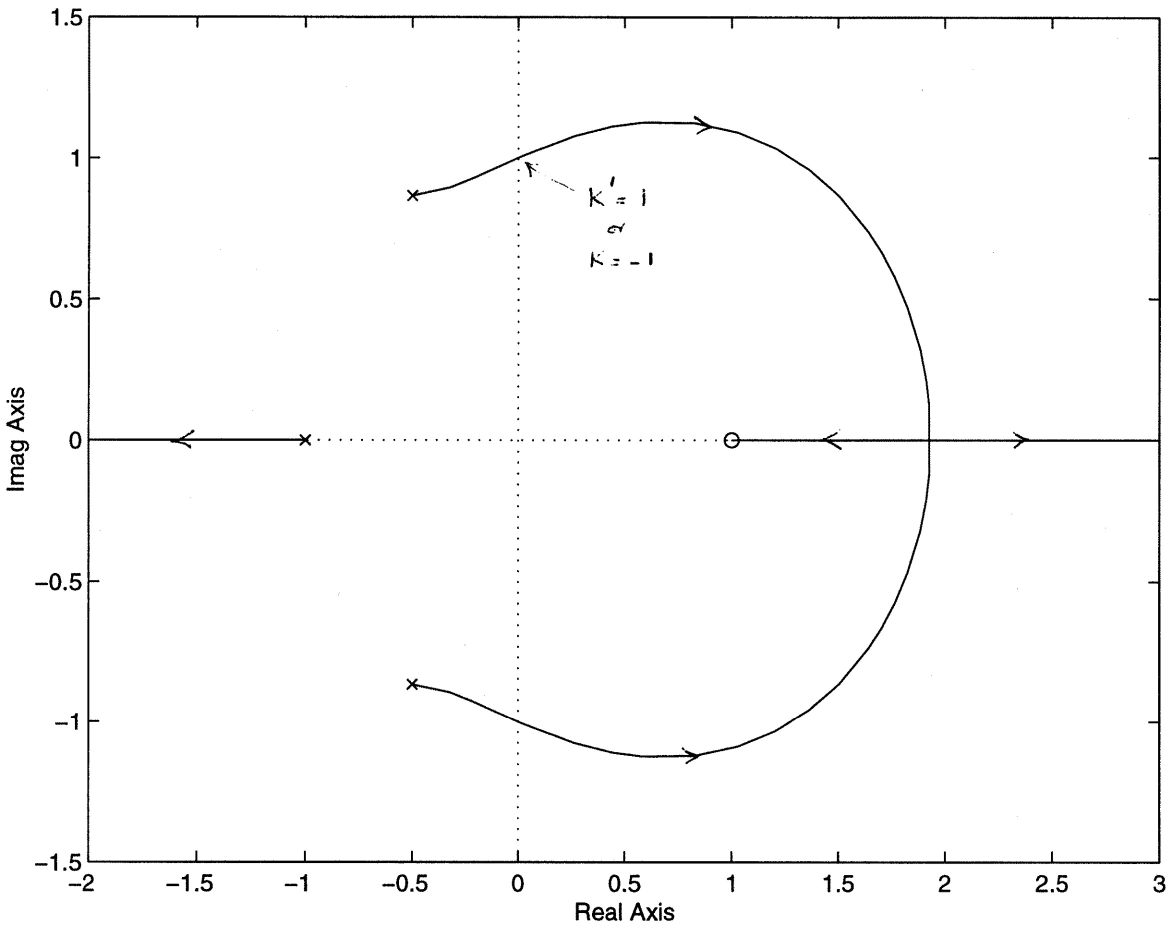


Figure 1

(b) We are given the loop transfer function

$$L(s) = \frac{K(s+1)}{s^4 + 5s^3 + 6s^2 + 2s - 8}$$

which is the same as the $L(s)$ considered in Problem 9.38, except this time K is negative.

Set $K = -K'$, where K' is nonnegative. Hence we may rewrite $L(s)$ as

$$L(s) = \frac{-K'(s+1)}{s^4 + 5s^3 + 6s^2 + 2s - 8}$$

and thus proceed in the same way as before. Figure 3 shows the root locus of $L(s)$ for six different values of K, namely, $K = -1, -2, -3$ and $K = -0.1, -0.2, -0.3$. We see that for all these values of K or the corresponding values of K', we see that the root locus has a pole in the right-half plane. Indeed, the feedback system described here will always have a closed-loop pole in the right-half plane. Hence, the system is unstable for all $K < 0$.

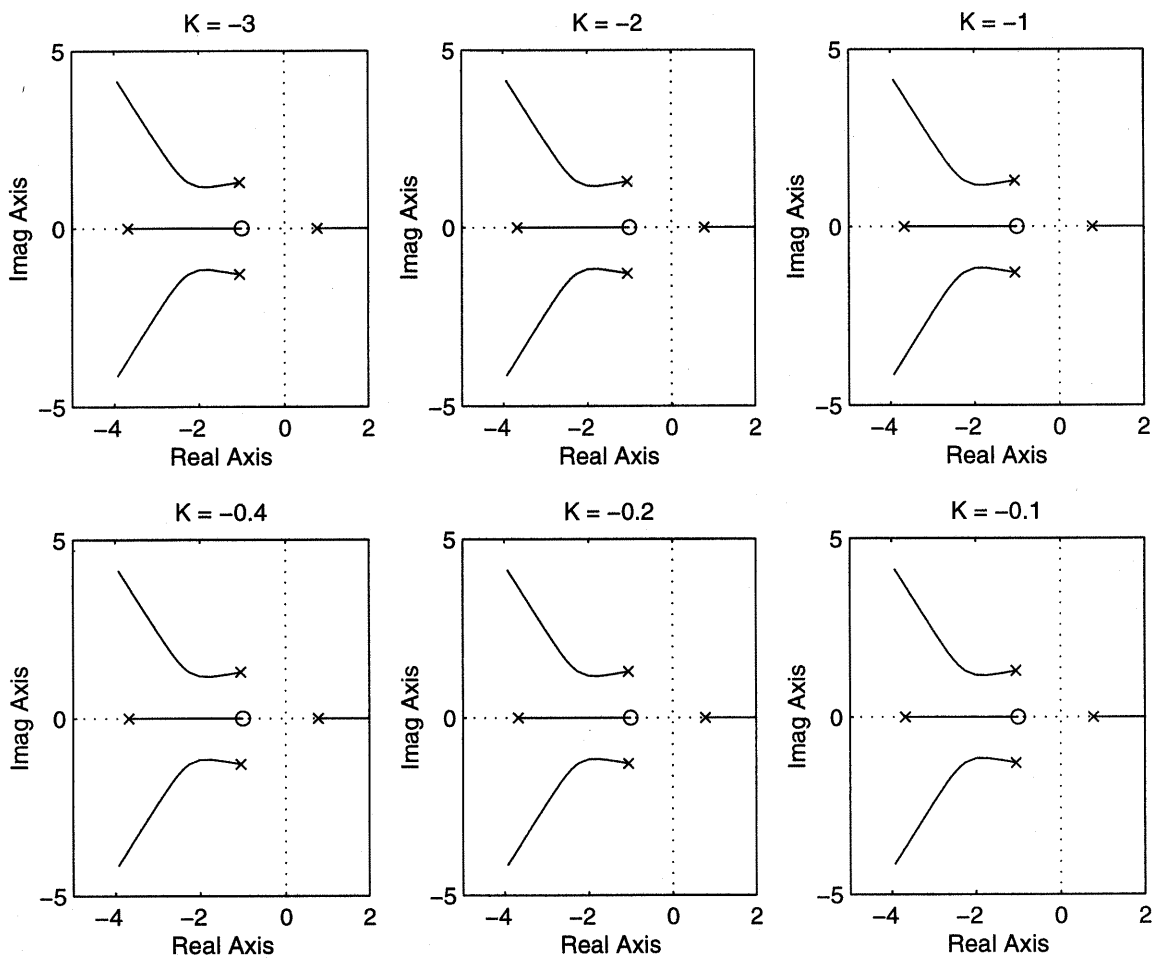


Figure 2

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