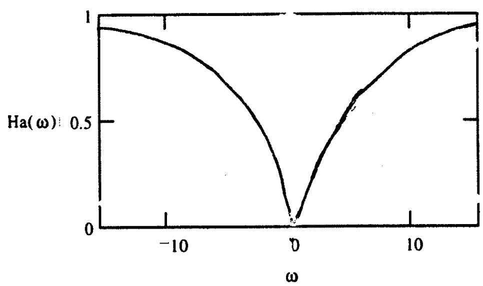
CHAPTER 4

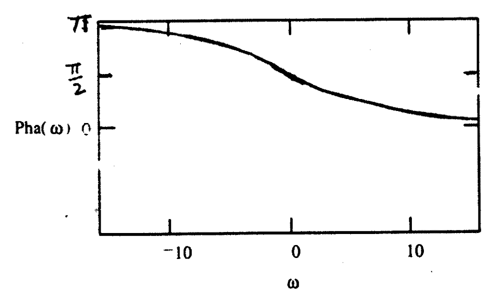
$\boxed{4.1}$

(a) $h(t) = \delta(t) - 2e^{-2t}$

$$H(j\omega) = 1 - \frac{2}{2+j\omega} = \frac{j\omega}{2+j\omega}$$

high pass



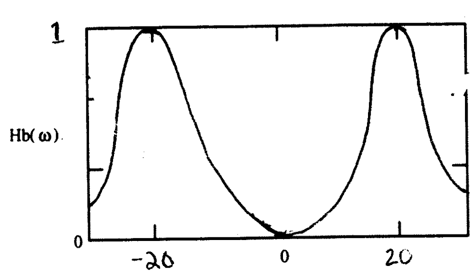


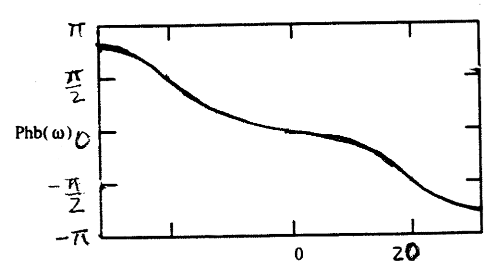
(b) $h(t) = 4e^{-2t}\cos(20t)\,u(t)$

$$H(j\omega) = 2\left(\frac{1}{2+j(\omega+20)} + \frac{1}{2+j(\omega-20)}\right)$$

attenuates components except at $\omega = \pm 20$

Bandpass

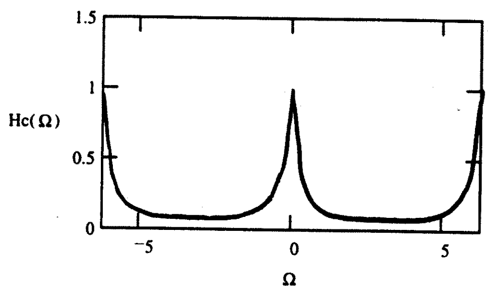


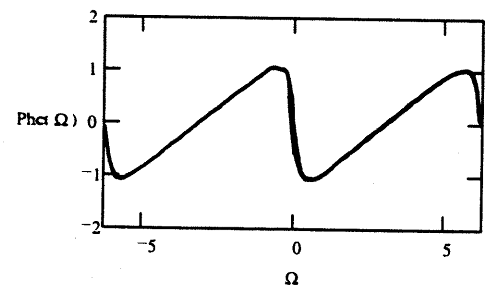


(c) $h[n] = \frac{1}{8}\left(\frac{7}{8}\right)^n u[n]$

$$H(e^{j\Omega}) = \frac{1}{8} \cdot \frac{1}{1 - \frac{7}{8}e^{-j\Omega}}$$

$$\left.\begin{array}{l} H(e^{j0}) = 1 \\ H(e^{j\pi}) = \frac{1}{8} \end{array}\right\} \text{lowpass}$$



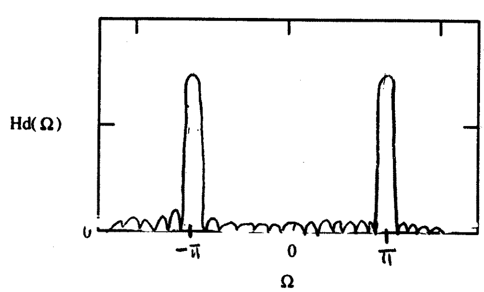


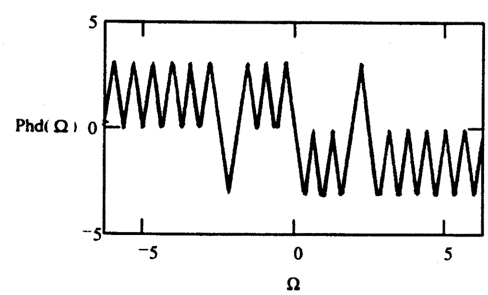
(d) $h[n] = \begin{cases} (-1)^n & |n| \leq 10 \\ 0 & \text{otherwise} \end{cases}$

$$H(e^{j\Omega}) = \sum_{n=-10}^{10} (-e^{-j\Omega})^n = (-e^{-j\Omega})^{-10} \frac{1-(-e^{-j\Omega})^{21}}{1-(-e^{-j\Omega})}$$

$$H(e^{j\Omega}) = \frac{\cos(\frac{21}{2}\Omega)}{\cos(\Omega/2)}$$

This has a highpass profile





$\boxed{4.2}$

(a) $x(t) = e^{-t}u(t)$
$y(t) = e^{-2t}u(t) + e^{-3t}u(t)$

$$X(j\omega) = \frac{1}{j\omega+1}$$

$$Y(j\omega) = \frac{1}{2+j\omega} + \frac{1}{3+j\omega}$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

$$\therefore H(j\omega) = \frac{j\omega+1}{2+j\omega} + \frac{j\omega+1}{3+j\omega}$$

$$= 2 - \frac{1}{2+j\omega} - \frac{2}{3+j\omega}$$

$$h(t) = 2\delta(t) - e^{-2t}u(t) - 2e^{-3t}u(t).$$

(b) $x(t) = e^{-3t}u(t)$, $y(t) = e^{-3(t-2)}u(t-2)$

$$X(j\omega) = \frac{1}{3+j\omega}, \quad Y(j\omega) = \frac{1}{3+j\omega}e^{-j2\omega}$$

$H(j\omega) = e^{-j2\omega}$
$h(t) = \delta(t-2)$

(c) $x(t) = e^{-2t}$
$y(t) = 2te^{-2t}u(t)$

$$X(j\omega) = \frac{1}{2+j\omega} \quad , \quad Y(j\omega) = \frac{2}{(2+j\omega)^2}$$

$$H(j\omega) = \frac{2}{2+j\omega}$$

$$h(t) = 2e^{-2t}u(t)$$

(d) $x[n] = \left(\frac{1}{2}\right)^n u[n]$

$$y[n] = \frac{1}{4}\left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{4}\right)^n u[n]$$

$$X(e^{j\Omega}) = \frac{1}{1-\frac{1}{2}e^{-j\Omega}} \quad , \quad Y(e^{j\Omega}) = \frac{\frac{1}{4}}{1-\frac{1}{2}e^{-j\Omega}} + \frac{1}{1-\frac{1}{4}e^{-j\Omega}}$$

$$H(e^{j\Omega}) = \frac{1}{4} + \frac{1-\frac{1}{2}e^{-j\Omega}}{1-\frac{1}{4}e^{-j\Omega}}$$

$$= \frac{9}{4} - \frac{1}{1-\frac{1}{4}e^{-j\Omega}}$$

$$h[n] = \frac{9}{4}\delta[n] - \left(\frac{1}{4}\right)^n u[n]$$

(e) $x[n] = \left(\frac{1}{4}\right)^n u[n]$, $y[n] = \left(\frac{1}{4}\right)^n u[n] - \left(\frac{1}{4}\right)^{n-1} u[n-1]$

$$X(e^{j\Omega}) = \frac{1}{1-\frac{1}{4}e^{-j\Omega}} \quad , \quad Y(e^{j\Omega}) = \frac{1}{1-\frac{1}{4}e^{-j\Omega}} - \frac{e^{-j\Omega}}{1-\frac{1}{4}e^{-j\Omega}}$$

$$H(e^{j\Omega}) = 1 - e^{-j\Omega}$$

$$h[n] = \delta[n] - \delta[n-1]$$

$\boxed{4.3}$

(a) $\frac{d}{dt}y(t) + 3y(t) = x(t)$

$$[j\omega + 3]\,Y(j\omega) = X(j\omega)$$

$$H(j\omega) = \frac{1}{3 + j\omega}$$

$$h(t) = e^{-3t}u(t)$$

(b) $\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 6y(t) = -\frac{d}{dt}x(t)$

$$[(j\omega)^2 + 5j\omega + 6]\,Y(j\omega) = -j\omega\, X(j\omega)$$

$$H(j\omega) = \frac{-j\omega}{(j\omega)^2 + 5j\omega + 6}$$

$$= \frac{-j\omega}{(j\omega + 2)(j\omega + 3)}$$

$$H(j\omega) = \frac{2}{j\omega + 2} - \frac{3}{j\omega + 3}$$

$$h(t) = (2e^{-2t} - 3e^{-3t})u(t)$$

(c) $y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = 3x[n] - \frac{3}{4}x[n-1]$

$$\left(1 - \frac{1}{4}e^{-j\Omega} - \frac{1}{8}e^{-j2\Omega}\right)Y(e^{j\Omega}) = \left(3 - \frac{3}{4}e^{-j\Omega}\right)X(e^{j\Omega})$$

$$H(e^{j\Omega}) = \frac{3 - \frac{3}{4}e^{-j\Omega}}{1 - \frac{1}{4}e^{-j\Omega} - \frac{1}{8}e^{-j2\Omega}}$$

$$= \frac{3 - \frac{3}{4}e^{-j\Omega}}{\left(1 - \frac{1}{2}e^{-j\Omega}\right)\left(1 + \frac{1}{4}e^{-j\Omega}\right)}$$

$$H(e^{j\Omega}) = \frac{1}{1-\frac{1}{2}e^{-j\Omega}} + \frac{2}{1+\frac{1}{4}e^{-j\Omega}}$$

$$h(t) = \left[\left(\frac{1}{2}\right)^n + 2\left(-\frac{1}{4}\right)^n\right] u[n]$$

(d) $y[n] + \frac{1}{2} y[n-1] = x[n] - 2x[n-1]$

$$\left[1+\frac{1}{2}e^{-j\Omega}\right] Y(e^{j\Omega}) = \left[1-2e^{-j\Omega}\right] X(e^{j\Omega})$$

$$H(e^{j\Omega}) = \frac{1-2e^{-j\Omega}}{1+\frac{1}{2}e^{-j\Omega}}$$

$$= -4 + \frac{5}{1+\frac{1}{2}e^{-j\Omega}}$$

$$h[n] = -4\,\delta[n] + 5\left(-\frac{1}{2}\right)^n u[n]$$

$\boxed{4.4}$

(a) $h(t) = \frac{1}{a} e^{-\frac{t}{a}} u(t)$

$$H(j\omega) = \frac{1}{a}\,\frac{1}{\frac{1}{a}+j\omega}$$

$$= \frac{1}{1+ja\omega}$$

$$= \frac{Y(j\omega)}{X(j\omega)}$$

$$\Longleftrightarrow a\frac{d}{dt} y(t) + y(t) = x(t)$$

(b) $h(t) = 2e^{-2t}u(t) - 2te^{-2t}u(t)$

$$H(j\omega) = \frac{2}{2+j\omega} - \frac{2}{(2+j\omega)^2}$$

$$H(j\omega) = \frac{2j\omega + 2}{(2 + j\omega)^2}$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

$$\Leftrightarrow \frac{d^2}{dt^2} y(t) + 4\frac{d}{dt} y(t) + 4y(t)$$

$$= 2\left(\frac{d}{dt} x(t) + x(t)\right)$$

(c) $h[n] = \alpha^n u[n] \quad , |\alpha| < 1$

$$H(e^{j\Omega}) = \frac{1}{1 - \alpha e^{-j\Omega}}$$

$$= \frac{Y(e^{j\Omega})}{X(e^{j\Omega})}$$

$$\Leftrightarrow -\alpha x[n-1] + x[n] = x[n]$$

(d) $h[n] = \delta[n] + 2\left(\frac{1}{2}\right)^n u[n] + \left(-\frac{1}{2}\right)^n u[n]$

$$H(e^{j\Omega}) = 1 + \frac{2}{1 - \frac{1}{2}e^{-j\Omega}} + \frac{1}{1 + \frac{1}{2}e^{-j\Omega}}$$

$$= \frac{-\frac{1}{4}e^{-j2\Omega} + \frac{1}{2}e^{-j\Omega} + 4}{1 - \frac{1}{4}e^{-j2\Omega}}$$

$$= \frac{Y(e^{j\Omega})}{X(e^{j\Omega})}$$

$$\Leftrightarrow -\frac{1}{4}y[n-2] + y[n] = -\frac{1}{4}x[n-2] + \frac{1}{2}x[n-1] + 4x[n]$$

$\boxed{4.5}$

(a) $H(j\omega) = \dfrac{2 + 3j\omega - 3(j\omega)^2}{1 + 2j\omega} = \dfrac{Y(j\omega)}{X(j\omega)}$

$$\Leftrightarrow 2\frac{d}{dt}y(t) + y(t) = -3\frac{d^2}{dt^2}x(t) + 3\frac{d}{dt}x(t) + 2x(t)$$

(b) $$H(j\omega) = \frac{1 - j\omega}{-\omega^2 - 4} = \frac{1 - j\omega}{(j\omega)^2 - 4} = \frac{Y(j\omega)}{X(j\omega)}$$

$$\Leftrightarrow \frac{d^2}{dt^2}y(t) - 4y(t) = -\frac{d}{dt}x(t) + x(t)$$

(c) $$H(j\omega) = \frac{1 + j\omega}{(j\omega+2)(j\omega+1)} = \frac{1 + j\omega}{(j\omega)^2 + 3j\omega + 2} = \frac{Y(j\omega)}{X(j\omega)}$$

$$\Leftrightarrow \frac{d^2}{dt^2}y(t) + 3\frac{d}{dt}y(t) + 2y(t) = \frac{d}{dt}x(t) + x(t)$$

(d) $$H(e^{j\Omega}) = \frac{1 + e^{-j\Omega}}{e^{-j2\Omega} + 3} = \frac{Y(e^{j\Omega})}{X(e^{j\Omega})}$$

$$\Leftrightarrow y[n-2] + 3y[n] = x[n-1] + x[n]$$

(e) $$H(e^{j\Omega}) = 1 + \frac{e^{-j\Omega}}{(1 - \frac{1}{2}e^{-j\Omega})(1 + \frac{1}{4}e^{-j\Omega})}$$

$$= \frac{1 + \frac{3}{4} e^{-j\Omega} - \frac{1}{8} e^{-j2\Omega}}{1 - \frac{1}{4} e^{-j\Omega} - \frac{1}{8} e^{-j2\Omega}}$$

$$H(e^{j\Omega}) = \frac{Y(e^{j\Omega})}{X(e^{j\Omega})}$$

$$\Leftrightarrow -\frac{1}{8} y[n-2] - \frac{1}{4} y[n-1] + y[n]$$

$$= -\frac{1}{8} x[n-2] + \frac{3}{4} x[n-1] + x[n]$$

$\boxed{4.6}$

(a) $\bar{A} = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}$, $\bar{b} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$, $\bar{c} = [1 \quad 1]$, $\bar{D} = [0]$

$$H(j\omega) = \bar{c}(j\omega \bar{I} - \bar{A})^{-1} \bar{b} + \bar{D}$$

$$= [1 \quad 1] \frac{1}{(j\omega+1)(j\omega+2)} \begin{bmatrix} j\omega+1 & 0 \\ 0 & j\omega+2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} + 0$$

$$H(j\omega) = \frac{2}{j\omega+1}$$

$$h(t) = 2e^{-t} u(t)$$

$$\frac{2}{j\omega+1} = \frac{Y(j\omega)}{X(j\omega)} \Leftrightarrow \frac{d}{dt} y(t) + y(t) = 2x(t)$$

(b) $\bar{A} = \begin{bmatrix} 1 & 2 \\ -3 & -4 \end{bmatrix}$, $\bar{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\bar{c} = [0 \quad 1]$, $\bar{D} = [0]$

$$H(j\omega) = \bar{c}(j\omega \bar{I} - \bar{A})^{-1} \bar{b} + \bar{D}$$

$$H(j\omega) = \frac{-5 + j2\omega}{(j\omega)^2 + 3j\omega + 2} = \frac{-5 + j2\omega}{(j\omega+1)(j\omega+2)}$$

$$\Longrightarrow H(j\omega) = \frac{-7}{j\omega+1} + \frac{9}{j\omega+2}$$

$$h(t) = (-7e^{-t} + 9e^{-2t})\,u(t)$$

$$\frac{-5 + j2\omega}{(j\omega)^2 + 3j\omega + 2} = \frac{Y(j\omega)}{X(j\omega)}$$

$$\Longleftrightarrow \frac{d^2}{dt^2}y(t) + 3\frac{d}{dt}y(t) + 2y(t) = 2\frac{d}{dt}x(t) - 5x(t)$$

$\boxed{4.7}$

(a) $\bar{A} = \begin{bmatrix} -1/2 & 1 \\ 0 & 1/4 \end{bmatrix}$, $\bar{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\bar{c} = [1 \quad 0]$, $\bar{D} = [1]$

$$H(e^{j\Omega}) = \bar{c}\,(e^{j\Omega}\bar{I} - \bar{A})^{-1}\,\bar{b} + \bar{D}$$

$$= \frac{1}{(e^{j\Omega} - \frac{1}{4})(e^{j\Omega} + \frac{1}{2})}[1 \quad 0]\begin{bmatrix} e^{j\Omega} - 1/4 & 1 \\ 0 & e^{j\Omega} + 1/2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} + 1$$

$$H(e^{j\Omega}) = \frac{1}{(e^{j\Omega} - \frac{1}{4})(e^{j\Omega} + \frac{1}{2})} + 1$$

$$= \frac{(e^{j\Omega})^2 + \frac{1}{4}(e^{j\Omega}) + \frac{7}{8}}{(e^{j\Omega})^2 + \frac{1}{4}(e^{j\Omega}) - \frac{1}{8}}$$

$$= \frac{1 + \frac{1}{4}(e^{-j\Omega}) + \frac{7}{8}(e^{-j2\Omega})}{1 + \frac{1}{4}(e^{-j\Omega}) - \frac{1}{8}(e^{-j2\Omega})}$$

$$= \frac{Y(e^{j\Omega})}{X(e^{j\Omega})}$$

$$\overset{DTFT}{\longleftrightarrow} -\frac{1}{8}y[n-2]+\frac{1}{4}y[n-1]+y[n]$$

$$=\frac{7}{8}x[n-2]+\frac{1}{4}x[n-1]+x[n].$$

$$H(e^{j\Omega})=e^{-j2\Omega}\cdot\frac{1}{(1-\frac{1}{4}e^{-j\Omega})(1+\frac{1}{2}e^{-j\Omega})}+1$$

$$=e^{-j2\Omega}\left(\frac{1/3}{1-\frac{1}{4}e^{-j\Omega}}+\frac{2/3}{1+\frac{1}{2}e^{-j\Omega}}\right)+1$$

$$\overset{DTFT}{\longleftrightarrow} h[n]=\delta[n]+\left(\frac{1}{3}\left(\frac{1}{4}\right)^{n-2}+\frac{2}{3}\left(-\frac{1}{2}\right)^{n-2}\right)u[n-2]$$

(b) $\bar{A}=\begin{bmatrix}1/4 & 3/4\\ 1/4 & -1/4\end{bmatrix}$, $\bar{b}=\begin{bmatrix}1\\1\end{bmatrix}$, $\bar{c}=\begin{bmatrix}0 & 1\end{bmatrix}$, $\bar{D}=[0]$

$$H(e^{j\Omega})=\bar{c}\left(e^{j\Omega}\bar{I}-\bar{A}\right)^{-1}\bar{b}+\bar{D}$$

$$=\frac{1}{(e^{j\Omega}-\frac{1}{4})(e^{j\Omega}+\frac{1}{4})-\frac{3}{16}}\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}e^{j\Omega}+\frac{1}{4} & \frac{3}{4}\\ \frac{1}{4} & -\frac{1}{4}\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}+0$$

$$H(e^{j\Omega})=\frac{e^{j\Omega}}{(e^{j\Omega})^2-\frac{1}{4}}$$

$$H(e^{j\Omega})=\frac{e^{j\Omega}}{(e^{j\Omega}+\frac{1}{2})(e^{j\Omega}-\frac{1}{2})}$$

$$H(e^{j\Omega})=\frac{e^{-j\Omega}}{(1+\frac{1}{2}e^{-j\Omega})(1-\frac{1}{2}e^{-j\Omega})}$$

$$H(e^{j\Omega}) = \frac{-1}{1+\frac{1}{2}e^{-j\Omega}} + \frac{1}{1-\frac{1}{2}e^{-j\Omega}}$$

$$h(t) = \left(\left(\frac{1}{2}\right)^n - \left(-\frac{1}{2}\right)^n\right) u[n]$$

$$H(e^{j\Omega}) = \frac{Y(e^{j\Omega})}{X(e^{j\Omega})} \xleftrightarrow{\text{DTFT}} -\frac{1}{4}y[n-2] + y[n] = x[n-1]$$

$\boxed{4.8}$

$$\bar{A} = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix}, \quad \bar{b} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad \bar{c} = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad \bar{D} = [0]$$

$$\bar{T} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

Before transformation : $H(j\omega) = \bar{c}\left[j\omega \bar{I} - \bar{A}\right]^{-1}\bar{b} + \bar{D}$

$$H(j\omega) = \frac{1}{(j\omega+1)(j\omega+3)}\begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} j\omega+3 & 0 \\ 0 & j\omega+1 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \end{bmatrix} + 0$$

$$H(j\omega) = \frac{2}{j\omega+3} \qquad \ldots.(1)$$

With transformation : $\bar{A}' = \bar{T}\bar{A}\bar{T}^{-1} = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$

$$\bar{b}' = T\bar{b}^{-1} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$$\bar{c}' = \bar{c}\,T^{-1} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\bar{D}' = \bar{D} = [0]$$

$$H'(j\omega) = \bar{c} - 1\left[j\omega \bar{I} - \bar{A}'\right]^{-1} \bar{b}' + \bar{D}'$$

$$= \frac{1}{(j\omega+2)^2 - 1}\begin{bmatrix} -\frac{1}{2} & \frac{1}{2}\end{bmatrix}\begin{bmatrix} j\omega+2 & 1 \\ 1 & j\omega+2\end{bmatrix}\begin{bmatrix} -2 \\ 2\end{bmatrix} + 0$$

$$= \frac{2(j\omega+1)}{(j\omega+1)(j\omega+3)}$$

$$H'(j\omega) = \frac{2}{j\omega+3} \qquad \ldots\ldots(2)$$

Note that $H(j\omega) = H'(j\omega)$

$\boxed{4.9}$

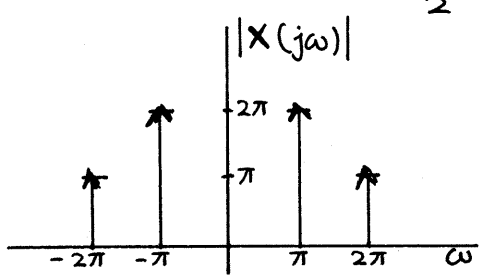
(a) $x(t) = 2\sin(\pi t) + \cos(2\pi t)$

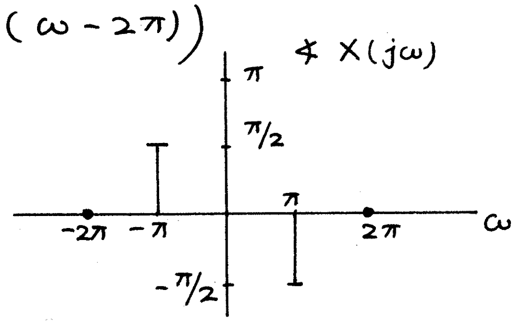
$$x(t) = -j\left(e^{j\pi t} - e^{-j\pi t}\right) + \frac{1}{2}\left(e^{j2\pi t} + e^{-j2\pi t}\right)$$

$$\omega_0 = gcd(\pi, 2\pi) = \pi$$

$$\therefore\ x[1] = -j$$
$$x[-1] = j$$
$$x[2] = x[-2] = \frac{1}{2}$$

$$\therefore\ X(j\omega) = 2\pi \sum X[k]\,\delta(\omega - k\omega_0)$$
$$= 2\pi\left(\frac{1}{2}\delta(\omega+2\pi) + j\delta(\omega+\pi) - j\delta(\omega-\pi) + \frac{1}{2}\delta(\omega-2\pi)\right)$$



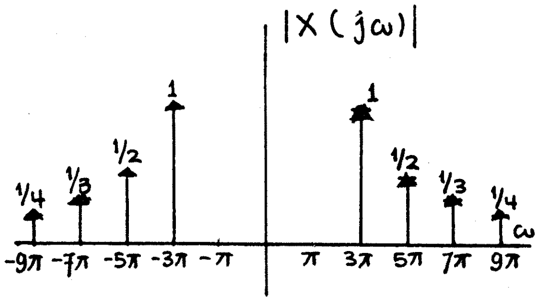


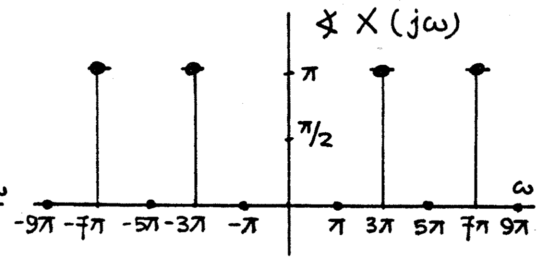
(b) $x(t) = \sum_{k=1}^{4} \frac{(-1)^k}{2k} \cos\left((2k+1)\pi t\right)$

$$= \sum_{k=1}^{4} \frac{(-1)^k}{k} \left(e^{j\pi(2k+1)t} + e^{-j\pi(2k+1)t}\right)$$

$\omega_0 = \pi$

$$\therefore X(j\omega) = 2\pi \sum_{k=1}^{4} \frac{(-1)^k}{k} \Big(\delta(\omega - \pi(2k+1)) + \delta(\omega + \pi(2k+1))\Big)$$





(c) $x(t) = |\sin(\pi t)|$ $\qquad \omega_0 = 2\pi$, $T = 1$

$$X[k] = \int_0^1 \sin(\pi t) e^{-jk2\pi t} dt$$

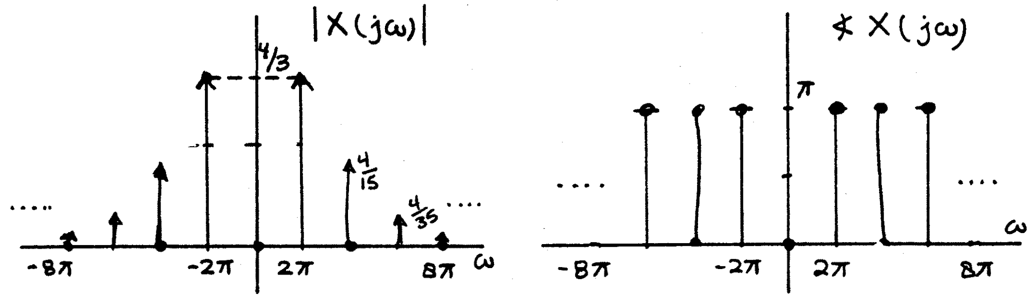
$$= -\frac{1}{2\pi}\left[\frac{e^{j\pi(1-2k)} - 1}{(1-2k)} + \frac{e^{-j\pi(1+2k)} - 1}{(1+2k)}\right]$$

$$\therefore X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi \, X[k] \, \delta(\omega - k2\pi)$$

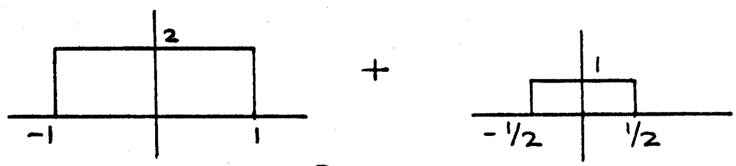
where $X[k]$ is defined above. Since $e^{j\pi(1-2k)} = -1$,

$$X[k] = \frac{1}{\pi}\left[\frac{1}{(1-2k)} + \frac{1}{(1+2k)}\right]$$

$$\therefore X(j\omega) = \sum_{k=-\infty}^{\infty} \left(\frac{2}{1-2k} + \frac{2}{1+2k}\right) \delta(\omega - k2\pi)$$



(d) $x(t) =$

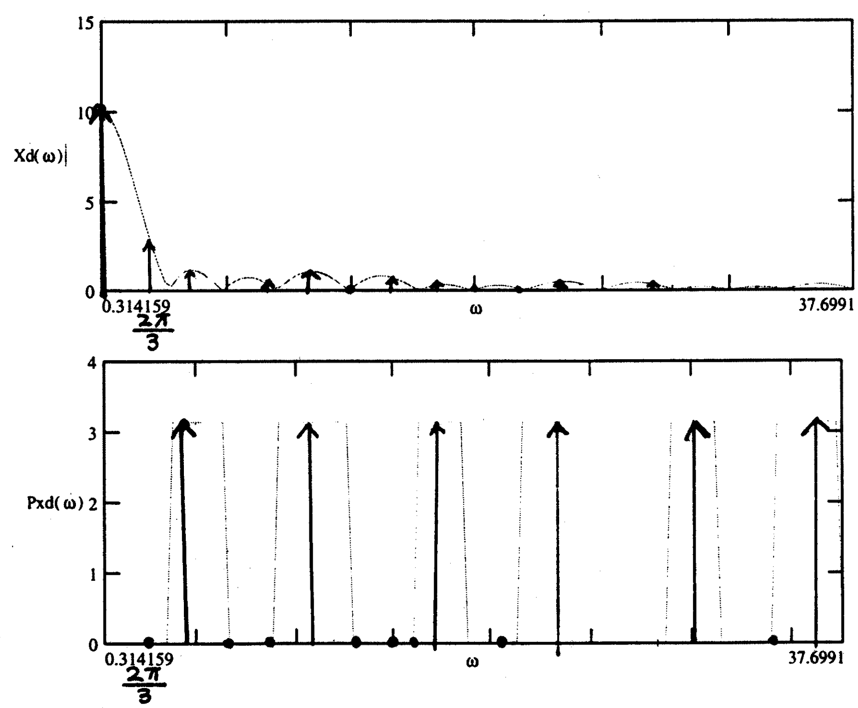


$T = 3 \quad , \quad \omega_0 = \dfrac{2\pi}{3}$

$$X[k] = \frac{2 \sin\left(k\frac{2\pi}{3}\right)}{k\pi} + \frac{\sin\left(k\frac{2\pi}{3}\right)}{k\pi} \quad , \; k \neq 0$$

$$X[0] = \frac{4}{3} + \frac{1}{3} = \frac{5}{3}$$

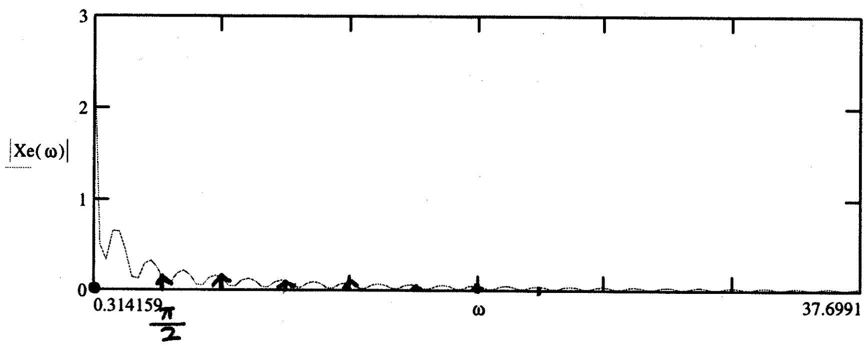
$$\therefore X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi X[k]\, \delta\left(\omega - k\frac{2\pi}{3}\right)$$ where $X[k]$ is defined above

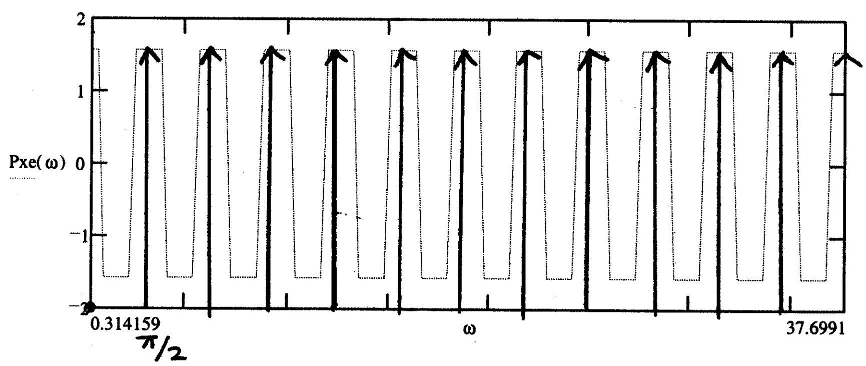


(e) $T = 4 \quad , \quad \omega_0 = \frac{\pi}{2}$

$$X[k] = \frac{1}{4}\int_{-2}^{2} t e^{-j\frac{\pi}{2}kt}\,dt = \begin{cases} 0 & , k=0 \\ \frac{j2\cos(\pi k)}{\pi k} & , k \neq 0 \end{cases}$$

$$\therefore X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi . X[k]\,\delta\left(\omega - \frac{\pi}{2}k\right) \text{ where } X[k] \text{ is above}$$





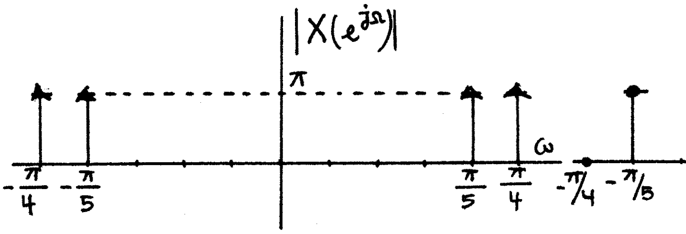
$\boxed{4.10}$

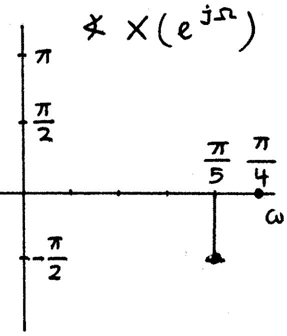
(a) $x[n] = \cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{5}n\right)$

$$= \frac{1}{2}\left(e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n}\right) + \frac{1}{2j}\left(e^{j\frac{\pi}{5}n} - e^{-j\frac{\pi}{5}n}\right)$$

$$\Omega_0 = \gcd\left(\frac{\pi}{5}, \frac{\pi}{4}\right) = \frac{\pi}{20}$$

$$\therefore X(e^{j\Omega}) = 2\pi\left[\frac{1}{2}\,\delta\left(\Omega+\frac{\pi}{4}\right)-\frac{1}{2j}\,\delta\left(\Omega+\frac{\pi}{5}\right)+\frac{1}{2j}\,\delta\left(\Omega-\frac{\pi}{5}\right)\right.$$
$$\left.+\frac{1}{2}\,\delta\left(\Omega-\frac{\pi}{4}\right)\right]$$



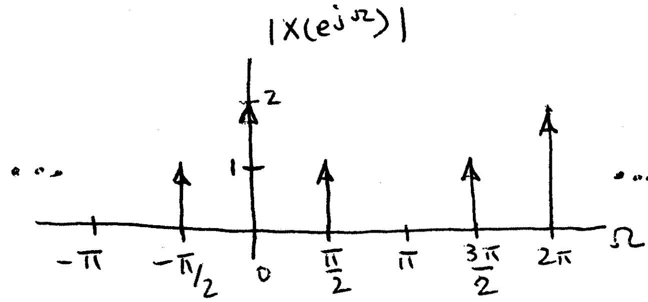


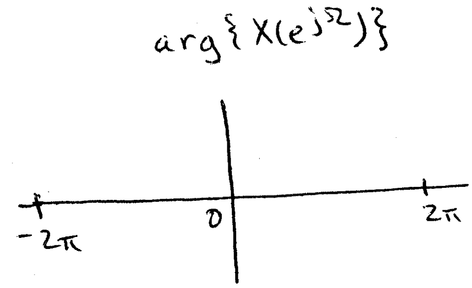
(b) $x[n] = 1 + \sum_{m=-\infty}^{\infty} \cos\left(\frac{\pi}{2}m\right)\delta[n-m]$

$N=4 \quad \Omega_o = \frac{\pi}{2}$ One period: $x[-1]=1$, $x[0]=2$, $x[1]=1$, $x[2]=0$

$$X[k] = \frac{1}{4}\left[e^{jk\pi/2} + 2 + e^{-jk\frac{\pi}{2}}\right] = \frac{1}{2} + \frac{1}{2}\cos k\frac{\pi}{2}$$

$$X(e^{j\Omega}) = \pi \sum_{k=-\infty}^{\infty}\left(1+\cos\left(k\frac{\pi}{2}\right)\right)\delta\left(\Omega - k\frac{\pi}{2}\right)$$





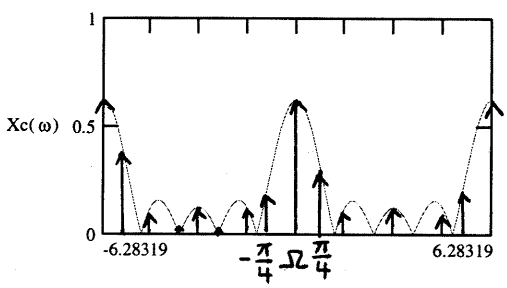
phase is zero

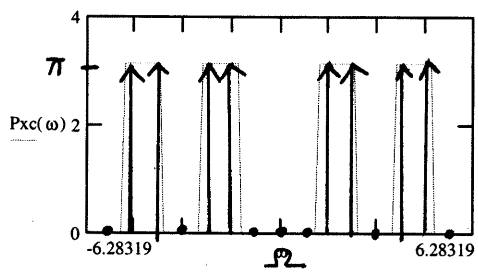
(c) $N=8$, $\Omega_0 = \frac{\pi}{4}$ from table : $M = 2$

$$X[k] = \frac{\sin\left[k\frac{\pi}{8}(5)\right]}{8\sin\left[k\frac{\pi}{8}\right]} \quad , k \neq 0$$

$$X[0] = \frac{5}{8}$$

$$\therefore X(e^{j\Omega}) = 2\pi \sum_{k=-\infty}^{\infty} X[k]\, \delta\left(\Omega - k.\frac{\pi}{4}\right)$$



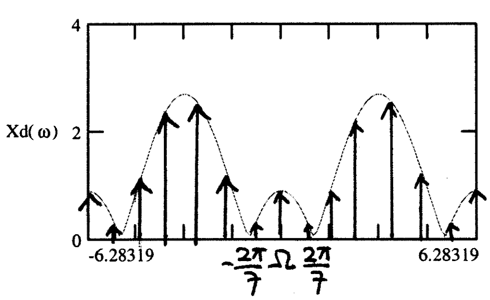


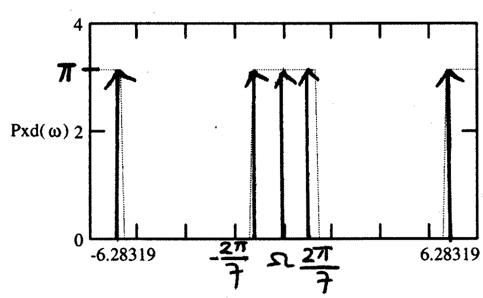
(d) $N = 7 \quad , \Omega_0 = \frac{2\pi}{7}$

$$X[k] = \frac{1}{7}\left(1 - e^{jk\frac{2\pi}{7}} - e^{-jk\frac{2\pi}{7}}\right)$$

$$= \frac{1}{7}\left(1 - 2\cos\left(\frac{2\pi}{7}k\right)\right)$$

$$\therefore X(e^{j\Omega}) = 2\pi \sum_{k=-\infty}^{\infty} X[k]\, \delta\left(\Omega - \frac{2\pi}{7}k\right)$$

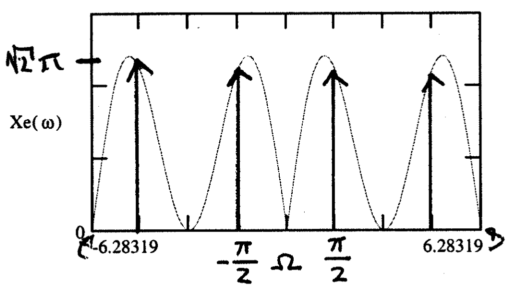


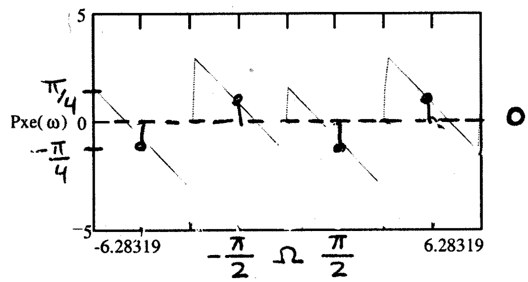


(e) $N = 4 \quad , \quad \Omega_0 = \frac{\pi}{2}$

$$X[k] = \frac{1}{4}\left(1 + e^{-jk\frac{\pi}{2}} - e^{-jk\pi} - e^{-jk\frac{3\pi}{2}}\right)$$

$$\therefore X(e^{j\Omega}) = 2\pi \sum_{k=-\infty}^{\infty} X[k]\,\delta(\Omega - k\tfrac{\pi}{2})$$





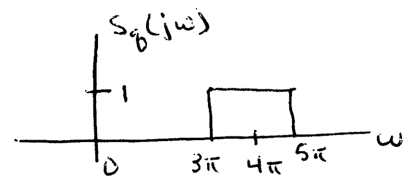
$\boxed{4.11}$

$$h(t) = 2\,\frac{\sin(\pi t)}{\pi t}\cos(4\pi t)$$

$$H(j\omega) = 2\left(\frac{1}{2\pi}\right) Sq(j\omega) * \pi\big(\delta(\omega - 4\pi) + \delta(\omega + 4\pi)\big)$$

$$= Sq(j(\omega - 4\pi)) + Sq(j(\omega + 4\pi))$$

where $Sq(j\omega) = \begin{cases} 1, & |\omega| < \pi \\ 0, & \text{otherwise} \end{cases}$



(a) $x(t) = 1 + \cos(\pi t) + \sin(4\pi t)$

$$X(j\omega) = 2\pi\,\delta(\omega) + \pi\big(\delta(\omega - \pi) + \delta(\omega + \pi)\big) + \frac{\pi}{j}\big(\delta(\omega - 4\pi) + \delta(\omega + 4\pi)\big)$$

$$Y(j\omega) = X(j\omega) \cdot H(j\omega)$$

$$= \frac{\pi}{j}\big(\delta(\omega - 4\pi) + \delta(\omega + 4\pi)\big)$$

$$\therefore y(t) = \sin(4\pi t)$$

(b) $$x(t) = \sum_{m=-\infty}^{\infty} \delta(t-m)$$

$$X(j\omega) = \sum_{m=-\infty}^{\infty} \delta(\omega - m)$$

$$y(j\omega) = X(j\omega) \,.\, H(j\omega)$$

$$= \sum_{k=10}^{15} \delta(\omega-k) + \delta(\omega+k)$$

$$\therefore\ y(t) = \frac{1}{\pi} \sum_{k=10}^{15} \cos(kt)$$

(c) $T = 1 \quad , \quad \omega_0 = 2\pi$

$$X(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} \left(\frac{\sin(k\frac{\pi}{4})}{k\pi} (1 - e^{-jk\pi}) \right) \delta(\omega - k2\pi)$$

$$y(j\omega) = X(j\omega) \,.\, H(j\omega)$$

$$= 2\pi \left(\frac{1}{2\pi}(1 - e^{-j2\pi}) + \frac{1}{2\pi}(1 - e^{j2\pi}) \right)$$

$$= 0$$

$$\therefore\ y(t) = 0$$

(d) $T = \frac{1}{4} \quad , \quad \omega_0 = 8\pi$

$$X[k] = 4 \int_{1/8}^{1/8} 16\, t\, e^{-jk8\pi t}\, dt$$

As we see, $X(j\omega) = 2\pi \sum X[k]\, \delta(\omega - k8\pi)$

$Y(j\omega) = X(j\omega) \cdot H(j\omega)$ will not have any intersect
thus $Y(j\omega) = 0$

$\therefore\ y(t) = 0$

(e) $T = 1 \quad , \quad \omega_0 = 2\pi$

$$X[k] = \int_0^1 e^{-t} \cdot e^{-jk2\pi t}\, dt$$

$$= \frac{1 - e^{-(1+jk2\pi)}}{1 + jk2\pi} = \frac{1 - e^{-1}}{1 + jk2\pi}$$

$$X(j\omega) = 2\pi \sum_k X[k]\, \delta(\omega - k2\pi)$$

$$Y(j\omega) = X(j\omega) \cdot H(j\omega)$$

$$Y(j\omega) = 2\pi\left(\frac{1 - e^{-1}}{1 + j4\pi}\delta(\omega - 4\pi) + \frac{1 - e^{-1}}{1 - j4\pi}\delta(\omega + 4\pi)\right)$$

$$Y(j\omega) = 2\pi\left(1 - e^{-1}\right)\left(\frac{\delta(\omega - 4\pi)}{1 + j4\pi} + \frac{\delta(\omega + 4\pi)}{1 - j4\pi}\right)$$

$$\therefore\ y(t) = \left(1 - e^{-1}\right)\left(\frac{e^{j4\pi t}}{1 + j4\pi} + \frac{e^{-j4\pi t}}{1 - j4\pi}\right)$$

$$= (1 - e^{-1})\, 2\, \mathrm{Re}\{ e^{j4\pi t}/(1 + j4\pi) \}$$

$\boxed{4.12}$

(a) $$H(j\omega) = \frac{Y(j\omega)}{G(j\omega)}$$

In time domain : $g(t) - y(t) = RC \dfrac{dy(t)}{dt}$

$$\xleftrightarrow{FT} G(j\omega) = (1 + j\omega CR)\, Y(j\omega)$$

$$\therefore H(j\omega) = \frac{1}{1 + j\omega CR}$$

(b) $x(t) = \cos(120\pi t)$

(i) $g(t) = |x(t)|$, $\omega_0 = 240\pi$, $T = \frac{1}{120}$

$$G[k] = 120 \int_{-1/240}^{1/240} \tfrac{1}{2}\left(e^{j120\pi t} + e^{-j120\pi t}\right) e^{-jk240\pi t}\,.dt$$

$$G[k] = \frac{(-1)^k}{\pi} \cdot \frac{2}{1-4k^2}$$

$$\therefore G(j\omega) = 4 \sum_{k=-\infty}^{\infty} \frac{(-1)^k}{1-4k^2}\, \delta(\omega - k240\pi)$$

(ii) $Y(j\omega) = H(j\omega)\, G(j\omega)$

$$= 4 \sum_{k=-\infty}^{\infty} \frac{(-1)^k}{1-4k^2}\, \frac{1}{1+jk240\pi CR}\, \delta(\omega - k240\pi)$$

(iii) Use first harmonic only :

$$Y(j\omega) \simeq 4\left[\delta(\omega) + \frac{1}{3}\left(\frac{\delta(\omega - 240\pi)}{1+j240\pi CR} + \frac{\delta(\omega+240\pi)}{1-j240\pi CR}\right)\right], \tau = CR$$

$$y(t) = \underbrace{4}_{\text{av.}} + \underbrace{\frac{4}{3}\left[\frac{e^{j240\pi t}}{1+j240\pi\tau} + \frac{e^{-j240\pi t}}{1-j240\pi\tau}\right]}_{\text{ripple}}$$

$$|\text{ripple}| = \frac{4}{3}\left[\frac{2}{\sqrt{1+(240\pi\tau)^2}}\right] < 0.01\,(4)$$

$$240\,\pi\,\tau > 66.659$$

$$\therefore\ \tau > 0.0884 \text{ s}$$

$\boxed{4.13}$

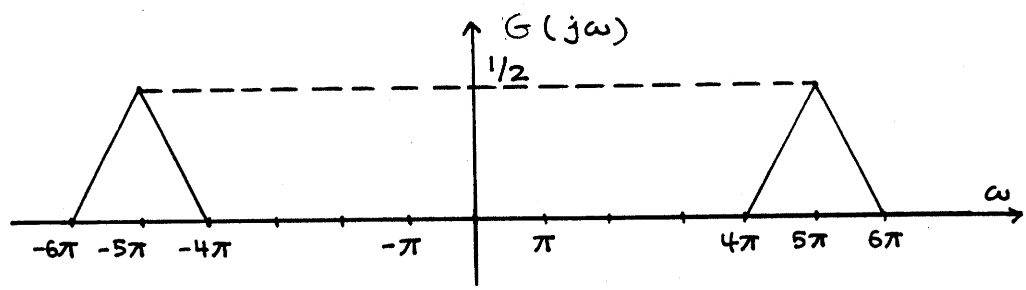
$$g(t) = x(t)\,.\,w(t)$$
$$y(t) = (g(t) * h(t))\cos(5\pi t)$$

(a) $w(t) = \cos(5\pi t)$, $h(t) = \dfrac{\sin(6\pi t)}{\pi t}$

$$G(j\omega) = \frac{1}{2\pi}X(j\omega) * W(j\omega)$$

$$W(j\omega) = \pi\left(\delta(\omega-5\pi) + \delta(\omega+5\pi)\right)$$

Thus : $G(j\omega) = \frac{1}{2}\left(X(j(\omega-5\pi)) + X(j(\omega+5\pi))\right)$



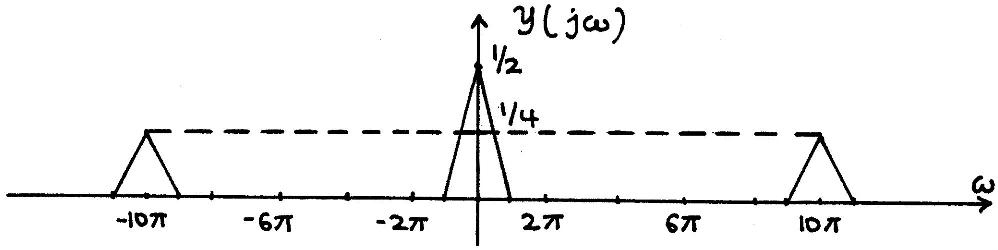
$$H(j\omega) = \begin{cases} 1 & , |\omega| < 6\pi \\ 0 & , \text{otherwise} \end{cases}$$

$$Y(j\omega) = \frac{1}{2\pi}\,C(j\omega) * \left(\pi\left(\delta(\omega-5\pi) + \delta(\omega+5\pi)\right)\right)$$

$$C(j\omega) = H(j\omega)\,G(j\omega) = G(j\omega)$$

$$\therefore Y(j\omega) = \frac{1}{2}\Big(G(j(\omega - 5\pi)) + G(j(\omega + 5\pi))\Big)$$

$$Y(j\omega) = \frac{1}{4}\Big(X(j(\omega - 10\pi)) + 2X(j\omega) + X(j(\omega + 10\pi))\Big)$$



(b) $w(t) = \cos(5\pi t)$, $h(t) = \dfrac{\sin(5\pi t)}{\pi t}$

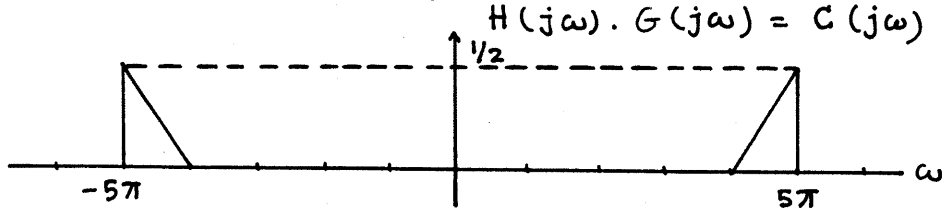
$G(j\omega)$ same as (a)

$$H(j\omega) = \begin{cases} 1, & |\omega| < 5\pi \\ 0, & \text{otherwise} \end{cases}$$

$$Y(j\omega) = \frac{1}{2\pi}\Big(H(j\omega) . G(j\omega)\Big) * \Big(\pi\big(\delta(\omega - 5\pi) + \delta(\omega + 5\pi)\big)\Big)$$

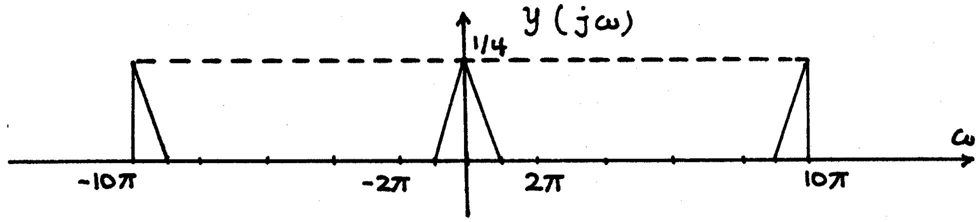
- $G(j\omega)$ the same as (a) !

$H(j\omega) . G(j\omega) = C(j\omega)$



$H(j\omega) . G(j\omega)$ is sketched
Let $C(j\omega) = H(j\omega) . G(j\omega)$

$$Y(j\omega) = \frac{1}{2}\Big(C(j(\omega - 5\pi)) + C(j(\omega + 5\pi))\Big)$$

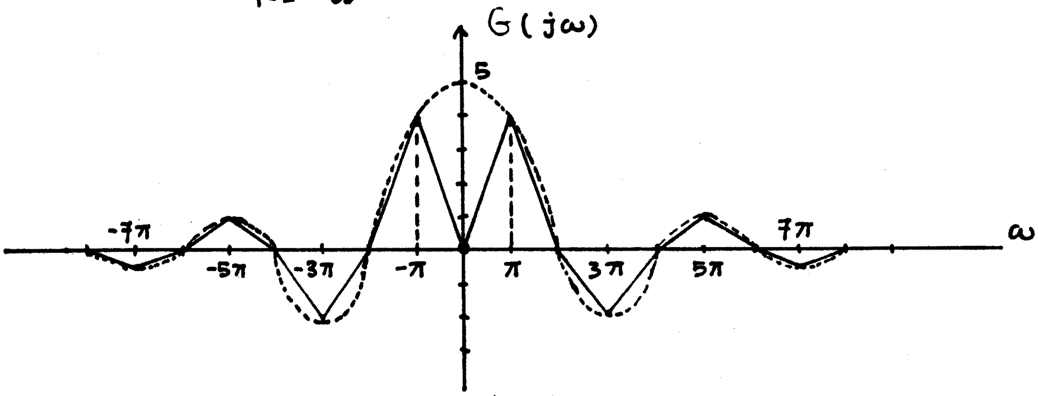


(c) $W[k] = -5\,\delta[k] + \dfrac{10\sin(k\frac{\pi}{2})}{k\pi}$, $\omega_0 = \pi$

$$\Rightarrow W(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} W[k]\,\delta(\omega - k.\pi)$$

$$G(j\omega) = \frac{1}{2\pi} X(j\omega) * W(j\omega)$$

$$G(j\omega) = \sum_{k=-\infty}^{\infty} W[k] . X(j(\omega - k\pi))$$

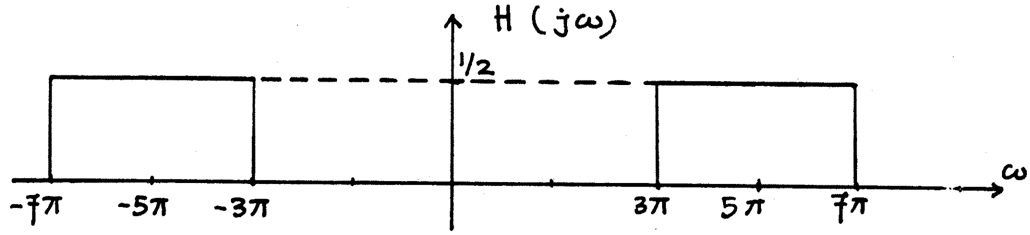


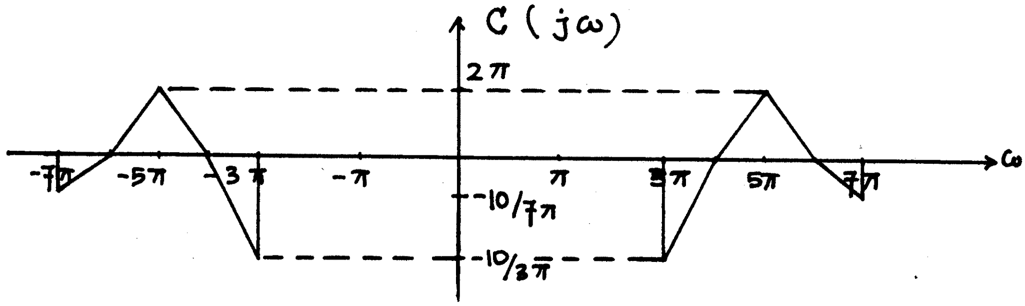
$$h(t) = \frac{\sin(2\pi t)}{\pi t} \cos(5\pi t)$$

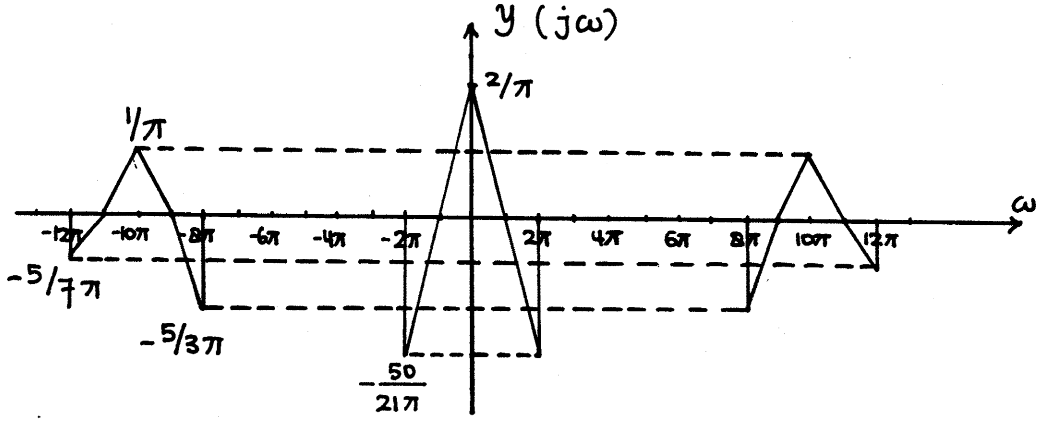
$$\Rightarrow H(j\omega) \begin{cases} 1/2 & , \ |\omega \pm 5\pi| < 2\pi \\ 0 & , \ \text{otherwise} \end{cases}$$

$$C(j\omega) = H(j\omega)\,G(j\omega)$$

$$= \sum_{k=3}^{7} W[k]\,X(j(\omega - k\pi)) + \sum_{k=-3}^{-7} W[k] X(j(\omega - k\pi))$$







$$Y(j\omega) = \frac{1}{2}\left(C(j(\omega+5\pi)) + C(j(\omega-5\pi))\right)$$

$\boxed{4.14}$

$$y(t) = \left[(x(t) * h(t)) \cdot (g(t) * h(t)) \right] * h(t)$$

$$= \left[x_h(t) \cdot g_h(t) \right] * h(t)$$

$$= m(t) * h(t)$$

$$h(t) = \frac{\sin(10\pi t)}{\pi t} \xleftrightarrow{FT} H(j\omega) = \begin{cases} 1 & , |\omega| \leqslant 10\pi \\ 0 & , \text{otherwise} \end{cases}$$

$$x(t) = \sum_{k=1}^{\infty} \frac{1}{k} \cos(k 4\pi t) \xleftrightarrow{FT} X(j\omega) = \pi \sum_{k=1}^{\infty} \frac{1}{k}\Big(\delta(\omega - k4\pi) + \delta(\omega + k4\pi)\Big)$$

$$g(t) = \sum_{k=1}^{10} \cos(k 8\pi t) \xleftrightarrow{FT} G(j\omega) = \pi \sum_{k=1}^{10} \delta(\omega - k8\pi) + \delta(\omega + k8\pi)$$

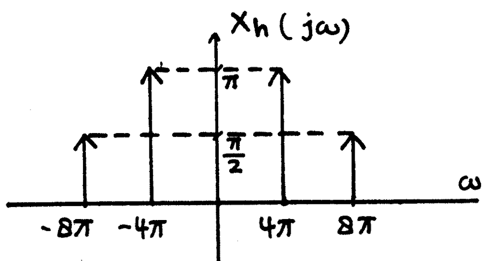
$$X_h(j\omega) = X(j\omega) . H(j\omega)$$

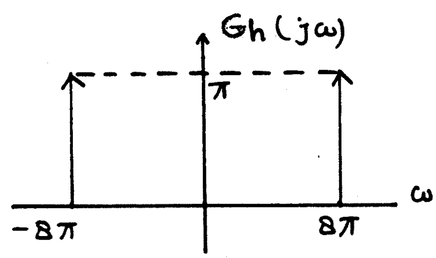
$$= \pi \sum_{k=1}^{2} \frac{1}{k}\Big(\delta(\omega - k4\pi) + \delta(\omega + k4\pi)\Big)$$

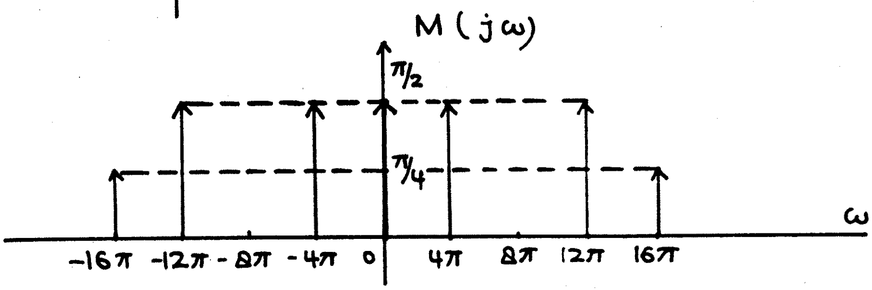
$$G_h(j\omega) = G(j\omega) . H(j\omega)$$

$$= \pi\big(\delta(\omega - 8\pi) + \delta(\omega + 8\pi)\big)$$

$$M(j\omega) = \frac{1}{2\pi} X_h(j\omega) . G_h(j\omega)$$







$$Y(j\omega) = M(j\omega) . H(j\omega)$$

$$Y(j\omega) = \frac{\pi}{2}\left(\delta(\omega) + \delta(\omega - 4\pi) + \delta(\omega + 4\pi)\right)$$

$$\therefore\ y(t) = \frac{1}{4} + \frac{1}{2}\cos(4\pi t)$$

$\boxed{4.15}$

$$x[n] = \cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{3\pi}{4}n\right)$$

$$X(e^{j\Omega}) = \pi\left(\delta\left(\Omega - \frac{\pi}{4}\right) + \delta\left(\Omega + \frac{\pi}{4}\right)\right) + \frac{\pi}{j}\left(\delta\left(\Omega - \frac{3\pi}{4}\right) - \delta\left(\Omega + \frac{3\pi}{4}\right)\right)$$

(a) $$h[n] = \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n}$$

$$H(e^{j\Omega}) = \begin{cases} 1, & |\Omega| < \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < |\Omega| < \pi \end{cases} \quad 2\pi \text{ periodic}$$

$$Y(e^{j\Omega}) = X(e^{j\Omega}) \cdot H(e^{j\Omega}) = \pi\left(\delta\left(\Omega - \frac{\pi}{4}\right) + \delta\left(\Omega + \frac{\pi}{4}\right)\right)$$

$$\therefore\ y[n] = \cos\left(\frac{\pi}{4}n\right)$$

(b) $$h[n] = (-1)^n \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n} = e^{j\pi n}\frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n}$$

$$H(e^{j\Omega}) = \begin{cases} 1, & |\Omega - \pi| < \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < |\Omega - \pi| < \pi \end{cases} \quad 2\pi \text{ periodic}$$

$$Y(e^{j\Omega}) = \frac{\pi}{j}\left(\delta\left(\Omega - \frac{3\pi}{4}\right) - \delta\left(\Omega + \frac{3\pi}{4}\right)\right)$$

$$\therefore\ y[n] = \sin\left(\frac{3\pi}{4}\right)$$

(c) $h[n] = \cos\left(\frac{\pi}{2}n\right) \cdot \frac{\sin\left(\frac{\pi}{8}n\right)}{\pi \cdot n}$

$$H(e^{j\Omega}) = \left(\begin{cases} 1/2 & , |\Omega - \frac{\pi}{2}| < \frac{\pi}{8} \\ 0 & , \frac{\pi}{8} < |\Omega - \frac{\pi}{2}| < \pi \end{cases}\right) + \left(\begin{cases} 1/2 & , |\Omega + \frac{\pi}{2}| < \frac{\pi}{8} \\ 0 & , \frac{\pi}{8} < |\Omega + \frac{\pi}{2}| < \pi \end{cases}\right)$$

$Y(e^{j\Omega}) = 0$

$\therefore\ y[n] = 0$

$\boxed{4.16}$ $y[n] = (x[n] \cdot w[n]) * h[n]$
$= g[n] * h[n]$

$$h[n] = \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n} \xleftrightarrow{\text{DTFT}} H(e^{j\Omega}) = \begin{cases} 1 & , |\Omega| < \frac{\pi}{2} \\ 0 & , \frac{\pi}{2} < |\Omega| < \pi \end{cases}$$

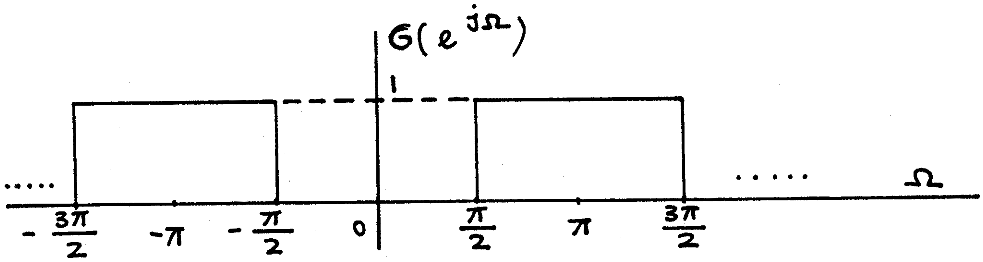
2π periodic

(a) $$x[n] = \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n} \longleftrightarrow X(e^{j\Omega}) = \begin{cases} 1 & , |\Omega| < \frac{\pi}{2} \\ 0 & , \frac{\pi}{2} < |\Omega| < \pi \end{cases}$$

$w[n] = (-1)^n = e^{j\pi n} \longleftrightarrow W(e^{j\Omega}) = 2\pi\,\delta(\Omega - \pi)$

$G(e^{j\Omega}) = \frac{1}{2\pi} X(e^{j\Omega}) * W(e^{j\Omega})$

$$G(e^{j\Omega}) = \begin{cases} 1 & , |\Omega - \pi| < \frac{\pi}{2} \\ 0 & , \frac{\pi}{2} < |\Omega - \pi| < \pi \end{cases}$$



$$\Rightarrow g[n] = e^{j\pi n} \frac{\sin(\frac{\pi}{2}n)}{\pi n}$$
$$= (-1)^n \frac{\sin(\frac{\pi}{2}n)}{\pi n}$$

$$Y(e^{j\Omega}) = G(e^{j\Omega})\, H(e^{j\Omega})$$
$$= 0$$

$$\therefore y[n] = 0$$

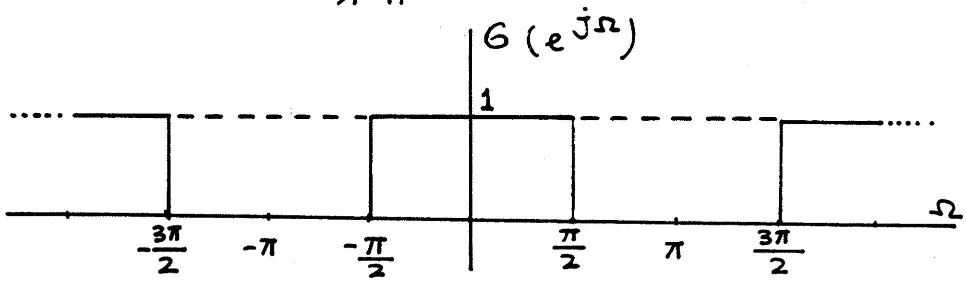
(b) $x[n] = \delta[n] - \dfrac{\sin(\frac{\pi}{2}n)}{\pi n}$, $w[n] = (-1)^n$

$$X(e^{j\Omega}) = \begin{cases} 0 , & |\Omega| < \frac{\pi}{2} \\ 1 , & \frac{\pi}{2} < |\Omega| < \pi \end{cases}$$

$$G(e^{j\Omega}) = \frac{1}{2} X(e^{j\Omega}) * W(e^{j\Omega}) = \begin{cases} 0 , & |\Omega - \pi| < \frac{\pi}{2} \\ 1 , & \frac{\pi}{2} < |\Omega - \pi| < \pi \end{cases}$$

$$= \begin{cases} 1 , & |\Omega| < \frac{\pi}{2} \\ 0 , & \frac{\pi}{2} < |\Omega| < \pi \end{cases}$$

$$\Rightarrow g[n] = \frac{\sin(\frac{\pi}{2}n)}{\pi n}$$



$$Y(e^{j\Omega}) = G(e^{j\Omega})\, H(e^{j\Omega}) = G(e^{j\Omega})$$
$$\therefore y[n] = \frac{\sin(\frac{\pi}{2}n)}{\pi n}$$

(c) $x[n] = \dfrac{\sin(\frac{\pi}{2}n)}{\pi n}$, $w[n] = \cos(\frac{\pi}{2}n)$

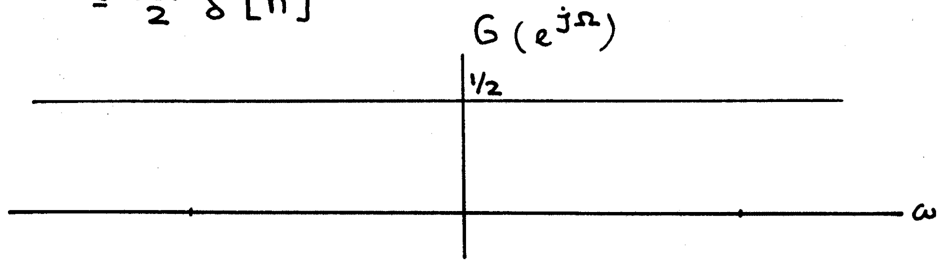
$W(e^{j\Omega}) = \pi\left(\delta(\Omega - \frac{\pi}{2}) + \delta(\Omega + \frac{\pi}{2})\right)$, 2π periodic

$$G(e^{j\Omega}) = \frac{1}{2\pi} X(e^{j\Omega}) * W(e^{j\Omega})$$

$$G(e^{j\Omega}) = \begin{cases} \frac{1}{2}, & |\Omega - \frac{\pi}{2}| < \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < |\Omega - \frac{\pi}{2}| < \pi \end{cases} + \begin{cases} \frac{1}{2}, & |\Omega + \frac{\pi}{2}| < \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < |\Omega + \frac{\pi}{2}| < \pi \end{cases}$$

$$g[n] = \frac{1}{2} \frac{\sin(\frac{\pi}{2}n)}{\pi n} \left(e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}\right)$$

$$\begin{aligned} g[n] &= \frac{\sin(\frac{\pi}{2}n)}{\pi n} \cos(\frac{\pi}{2}n) \\ &= \frac{\sin(\pi n)}{2\pi n} \\ &= \frac{1}{2}\delta[n] \end{aligned}$$



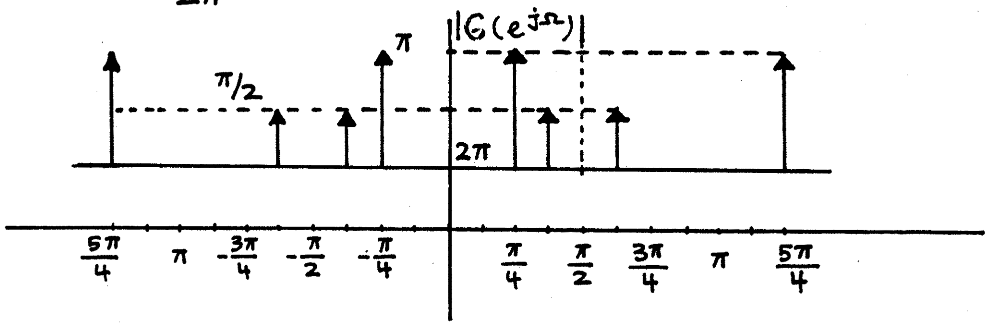
$$\begin{aligned} Y(e^{j\Omega}) &= G(e^{j\Omega}) \cdot H(e^{j\Omega}) \\ &= H(e^{j\Omega}) \end{aligned}$$

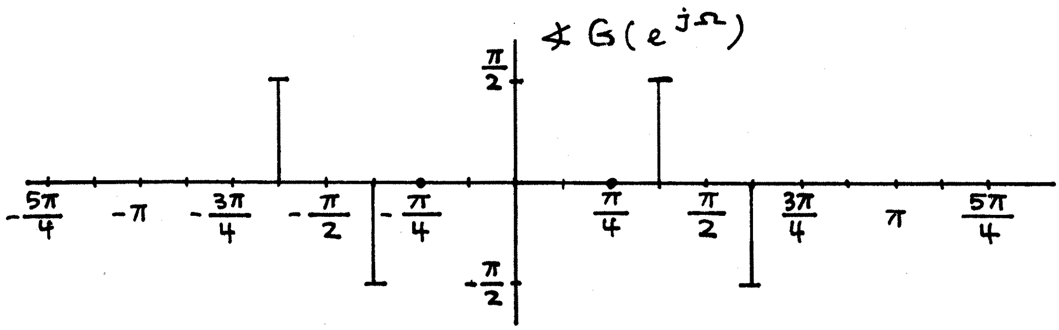
$$\therefore\ y[n] = \frac{\sin(\frac{\pi}{2}n)}{\pi n}$$

(d) $x[n] = 1 + \sin(\frac{\pi}{8}n) + 2\cos(\frac{3\pi}{4}n)$, $w[n] = \cos(\frac{\pi}{2}n)$

$$X(e^{j\Omega}) = 2\pi + \frac{\pi}{j}\left(\delta(\Omega-\tfrac{\pi}{8}) - \delta(\Omega+\tfrac{\pi}{8})\right) + 2\pi\left(\delta(\Omega-\tfrac{3\pi}{4}) + \delta(\Omega+\tfrac{3\pi}{4})\right)$$

$$G(e^{j\Omega}) = \frac{1}{2\pi} X(e^{j\Omega}) * W(e^{j\Omega})$$





$$Y(e^{j\Omega}) = G(e^{j\Omega})\, H(e^{j\Omega})$$

$$Y(e^{j\Omega}) = \pi\left(\delta(\Omega-\tfrac{\pi}{4}) + \delta(\Omega+\tfrac{\pi}{4})\right) + \frac{\pi}{2} j\left(\delta(\Omega-\tfrac{3\pi}{8}) - \delta(\Omega+\tfrac{3\pi}{8})\right) + 2\pi \cdot H(e^{j\Omega})$$

$$\therefore y[n] = \cos\left(\frac{\pi}{2}n\right) - \frac{1}{2}\sin\left(\frac{3\pi}{8}n\right) + \frac{2\sin\left(\frac{\pi}{2}n\right)}{n}$$

$\boxed{4.17}$

(a) $x[n] = \dfrac{\sin\left(\frac{\pi}{4}n\right)}{\pi n}$, $\tau = 1$

$$X_\delta(j\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega \tau n}$$

$$= X(e^{j\Omega})\Big|_{\Omega = \omega(1)}$$

$$= \begin{cases} 1, & |\omega| < \frac{\pi}{4} \\ 0, & \frac{\pi}{4} < |\omega| < \pi, \quad \frac{2\pi}{(1)} = 2\pi \text{ periodic} \end{cases}$$

(b) $x[n] = \dfrac{\sin(\frac{\pi}{4}n)}{\pi n}, \quad \tau = \frac{1}{4}$

$$X_\delta(j\omega) = X(e^{j\Omega})\Big|_{\Omega = \omega(\frac{1}{4})} \begin{cases} 1, & |\omega| < 4 \times \pi/4 \\ 0, & \frac{\pi}{4} \times 4 < |\omega| < \pi \times 4 \end{cases}$$

$$= \begin{cases} 1, & |\omega| < \pi \\ 0, & \pi < |\omega| < 4\pi \end{cases}, \quad 8\pi \text{ periodic}$$

(c) $x[n] = \cos(\frac{\pi}{2}n)\dfrac{\sin(\frac{\pi}{4}n)}{\pi n}, \quad \tau = 2$

$$X_\delta(j\omega) = X(e^{j\Omega})\Big|_{\Omega = 2\omega}$$

$$= \begin{cases} \frac{1}{2}, & |2\omega - \frac{\pi}{2}| < \frac{\pi}{4} \\ 0, & \frac{\pi}{4} < |2\omega - \frac{\pi}{2}| < \pi \end{cases} + \begin{cases} \frac{1}{2}, & |2\omega + \frac{\pi}{2}| < \frac{\pi}{4} \\ 0, & \frac{\pi}{4} < |2\omega + \frac{\pi}{2}| < \pi \end{cases}$$

$$\therefore X_\delta(j\omega) = \begin{cases} 1/2, & \frac{\pi}{8} < \omega < \frac{3\pi}{8} \\ 0, & \text{otherwise} \end{cases} + \begin{cases} \frac{1}{2}, & -\frac{3\pi}{8} < \omega < -\frac{\pi}{8} \\ 0, & \text{otherwise} \end{cases}$$

π periodic

(d) $\mathcal{T} = 4$

DTFS : $N = 8$, $\Omega_0 = \frac{\pi}{4} \Rightarrow X[k] = \dfrac{\sin\left(k\frac{5\pi}{8}\right)}{8\sin\left(k\frac{\pi}{8}\right)}$, $k \in [-3, 4]$

DTFT : $X(e^{j\Omega}) = 2\pi \sum_k X[k]\, \delta\left(\Omega - k\frac{\pi}{4}\right)$

FT : $X_\delta(j\omega) = X(e^{j\Omega})\Big|_{\Omega = 4\omega}$

$$= \frac{\pi}{4} \sum_{k=-\infty}^{\infty} \frac{\sin\left(k\frac{5\pi}{8}\right)}{\sin\left(k\frac{\pi}{8}\right)} \cdot \delta\left(4\omega - k\frac{\pi}{4}\right)$$

$$X_\delta(j\omega) = \frac{\pi}{16} \sum_{k=-\infty}^{\infty} \frac{\sin\left(k\frac{5\pi}{8}\right)}{\sin\left(k\frac{\pi}{8}\right)} \cdot \delta\left(\omega - k \cdot \frac{\pi}{16}\right)$$

$\frac{\pi}{2}$ periodic

(e) $x[n] = \sum_{p=-\infty}^{\infty} \delta[n - 4p]$ $\qquad \mathcal{T} = \frac{1}{8}$

DTFS : $N = 4$, $\Omega_0 = \frac{\pi}{2} \Rightarrow X[k] = \frac{1}{4}$, $k \in [0, 3]$

DTFT : $X(e^{j\Omega}) = \frac{\pi}{2} \sum_{k=-\infty}^{\infty} \delta\left(\Omega - k\frac{\pi}{2}\right)$

FT : $X_\delta(j\omega) = \frac{\pi}{2} \sum_{k=-\infty}^{\infty} \delta\left(\omega \mathcal{T} - k\frac{\pi}{2}\right)$

$$= 4\pi \sum_{k=-\infty}^{\infty} \delta(\omega - k4\pi)$$, 16π periodic

$\boxed{4.18}$ $x(t) = \frac{1}{\pi t} \sin(\pi t)$

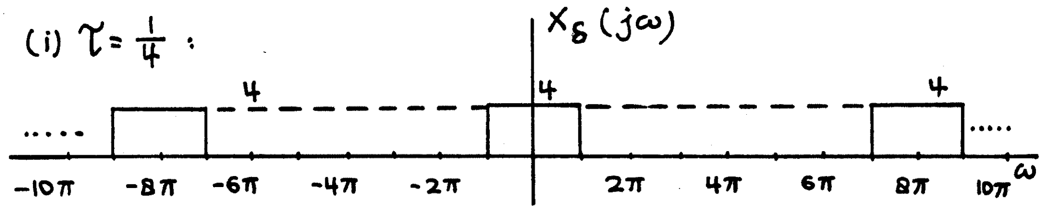
sampling rate $\frac{1}{\tau}$

$$X_\delta(j\omega) = \frac{1}{\tau} \sum_{k=-\infty}^{\infty} X\left(j\left(\omega - k\frac{2\pi}{\tau}\right)\right)$$

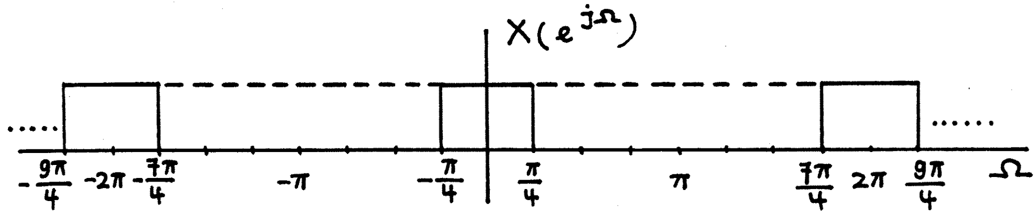
$$\text{, where } X(j\omega) = \begin{cases} 1 & , |\omega| < \pi \\ 0 & , \text{otherwise} \end{cases}$$

(i) $\tau = \frac{1}{4}$:

(a)

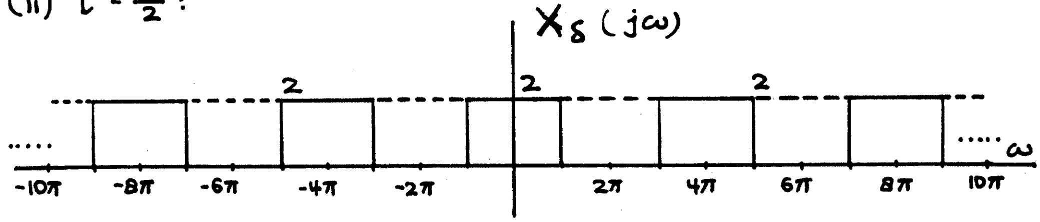


(b)

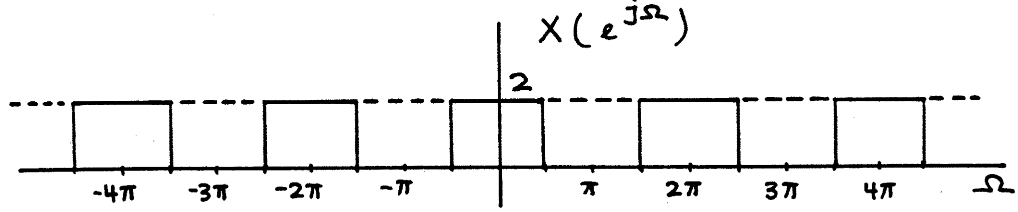


(ii) $\tau = \frac{1}{2}$:

(a)

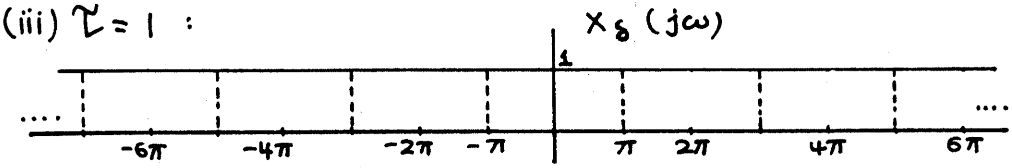


(b)

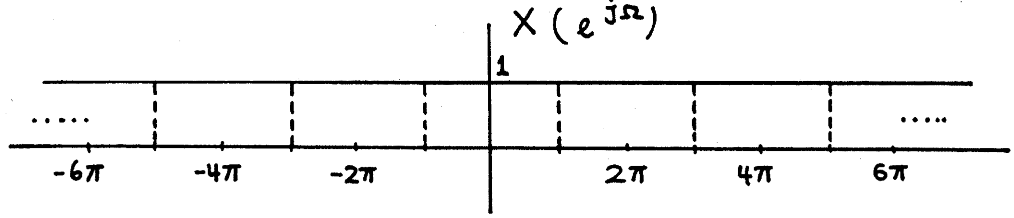


(iii) $\tau = 1$:

(a)

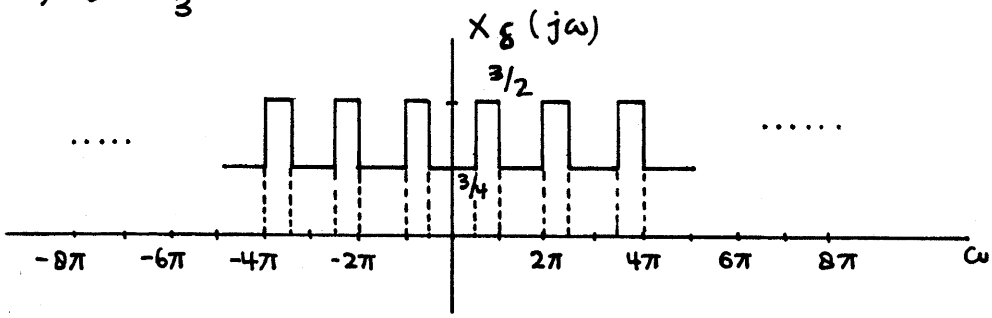


(b)

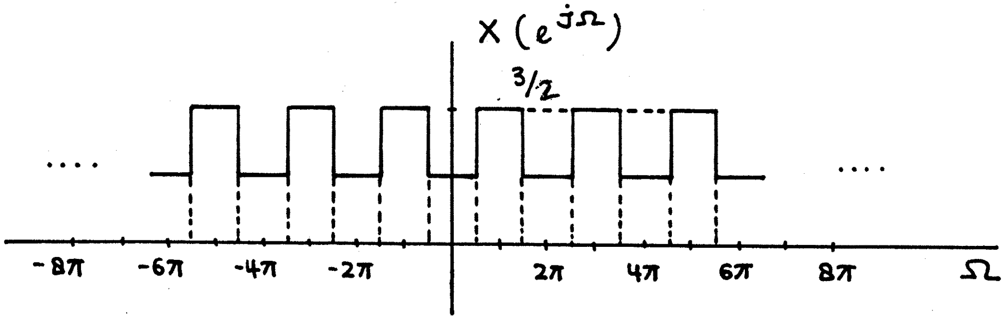


(iv) $\tau = \frac{4}{3}$

(a)



(b)

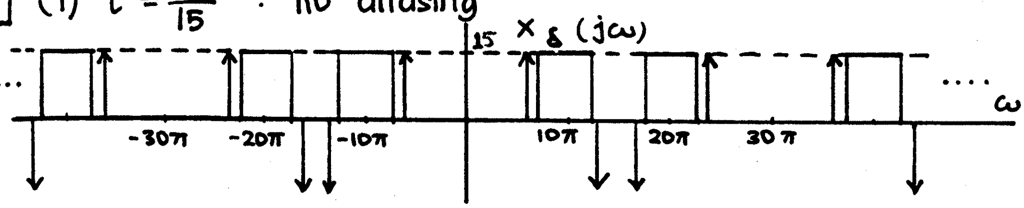


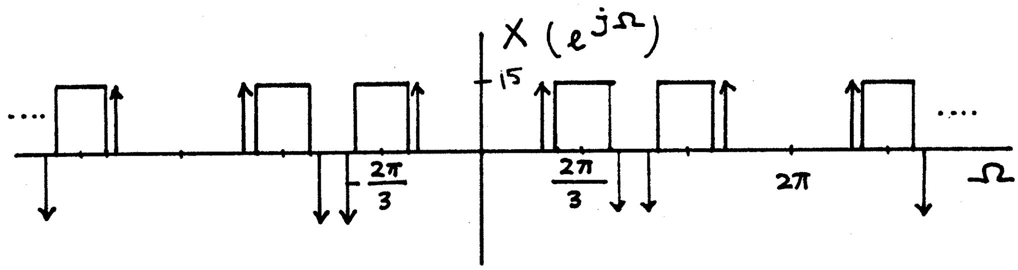
(b) $x[n] = x[n\tau]$

$$= \frac{1}{\pi n \tau} \sin(n\tau\pi) \xleftrightarrow{\text{DTFT}} \tilde{X}(e^{j\Omega})$$

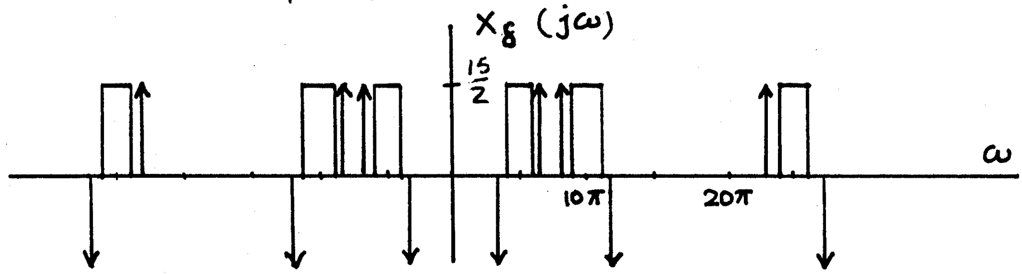
$$\tilde{X}(e^{j\Omega}) = \begin{cases} \frac{1}{\tau}, & |\Omega| < \tau\pi \\ 0, & \tau\pi < |\Omega| < \pi \end{cases} \quad 2\pi \text{ periodic}$$

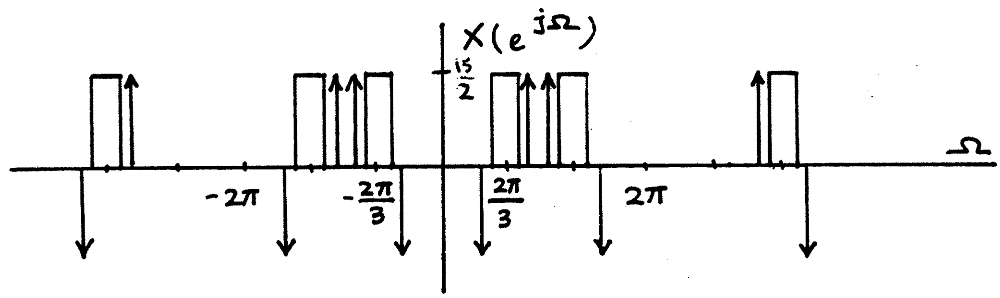
$\boxed{4.19}$ (i) $\tau = \frac{1}{15}$: no aliasing



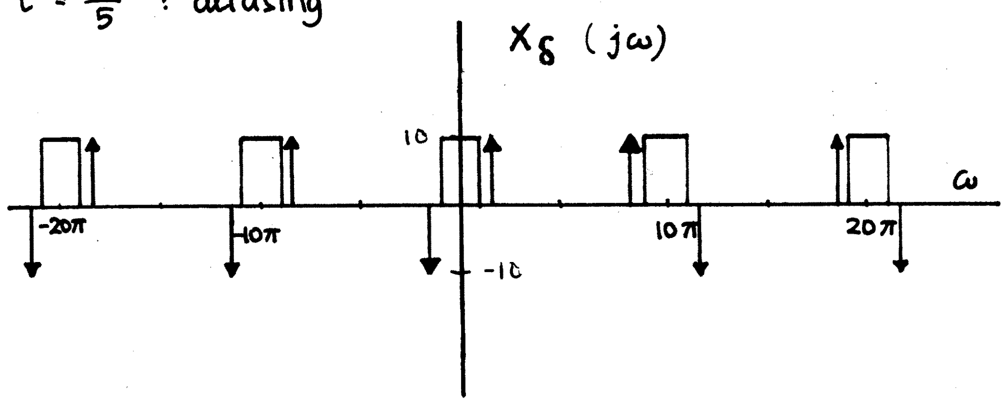


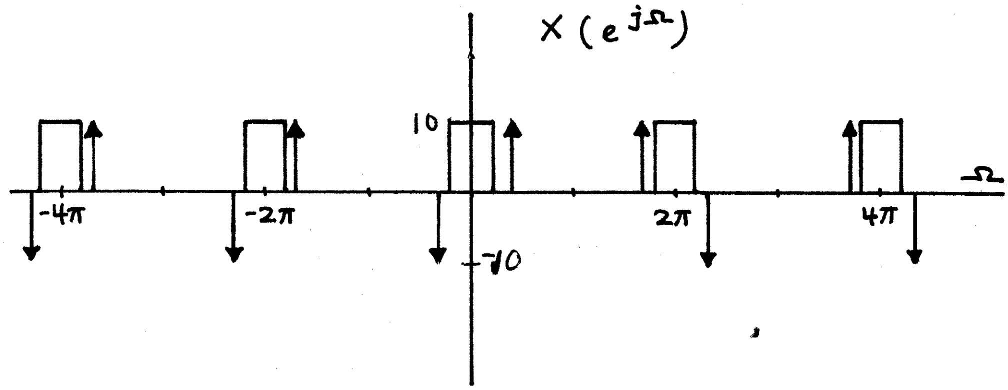
(ii) $\tau = \frac{2}{15}$: aliasing





(iii) $\tau = \frac{1}{5}$: aliasing



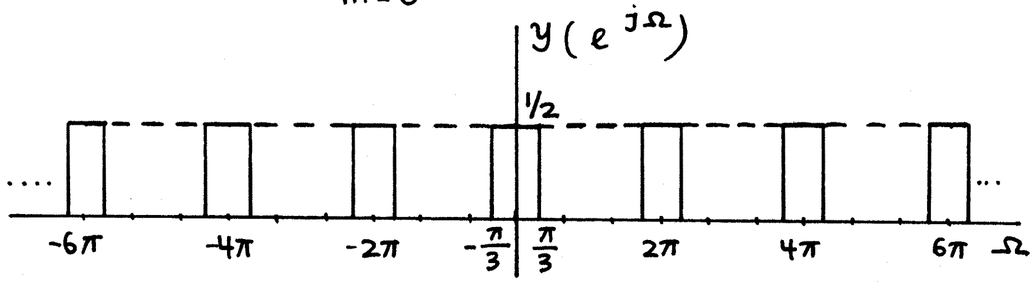


$\boxed{4.20}$ $x[n] = \dfrac{\sin(\frac{\pi}{6}n)}{\pi n}$; $X(e^{j\Omega}) = \begin{cases} 1 & , |\Omega| < \frac{\pi}{6} \\ 0 & , \frac{\pi}{6} < |\Omega| < \pi \text{ , } 2\pi \text{ periodic} \end{cases}$

$q[n] = x[qn]$; $Y(e^{j\Omega}) = \dfrac{1}{q} \displaystyle\sum_{m=0}^{q-1} X\left(e^{j\frac{1}{q}(\Omega - m2\pi)}\right)$

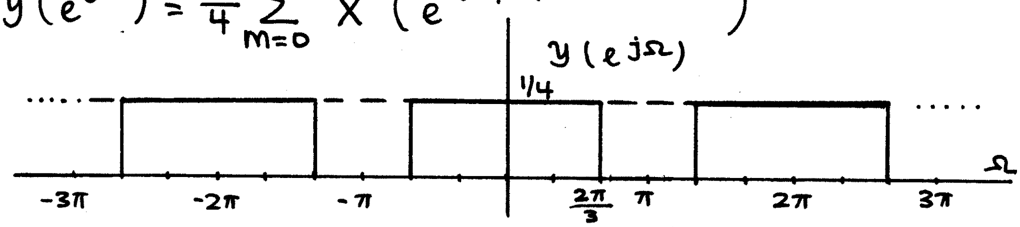
(a) $q = 2$

$$Y(e^{j\Omega}) = \frac{1}{2} \sum_{m=0}^{1} X\left(e^{j\frac{1}{2}(\Omega - m2\pi)}\right)$$



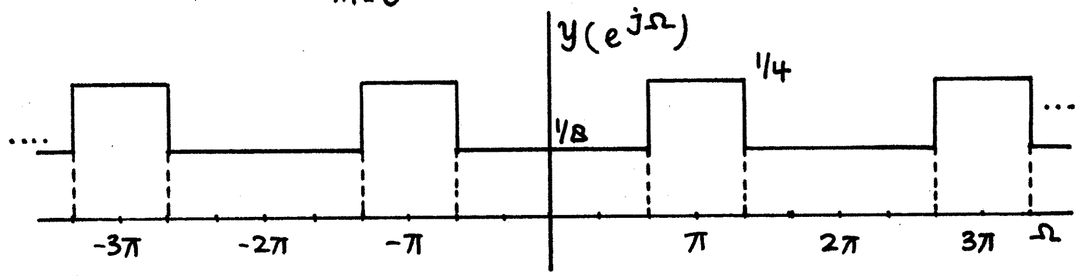
(b) $q = 4$

$$Y(e^{j\Omega}) = \frac{1}{4} \sum_{m=0}^{3} X\left(e^{j\frac{1}{4}(\Omega - m2\pi)}\right)$$



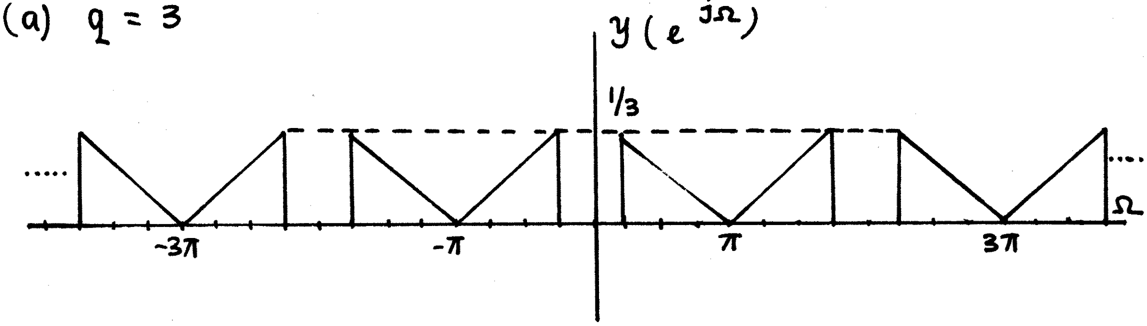
(c) $q = 8$

$$y(e^{j\Omega}) = \frac{1}{8}\sum_{m=0}^{7} X\left(e^{j\frac{1}{8}(\Omega - m2\pi)}\right)$$

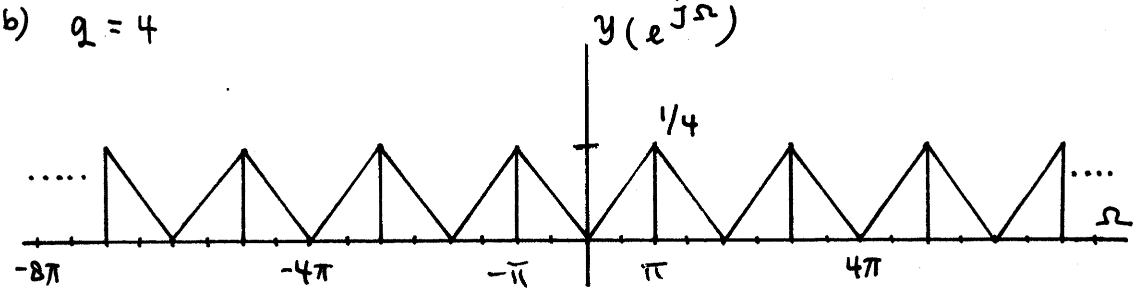


$\boxed{4.21}$

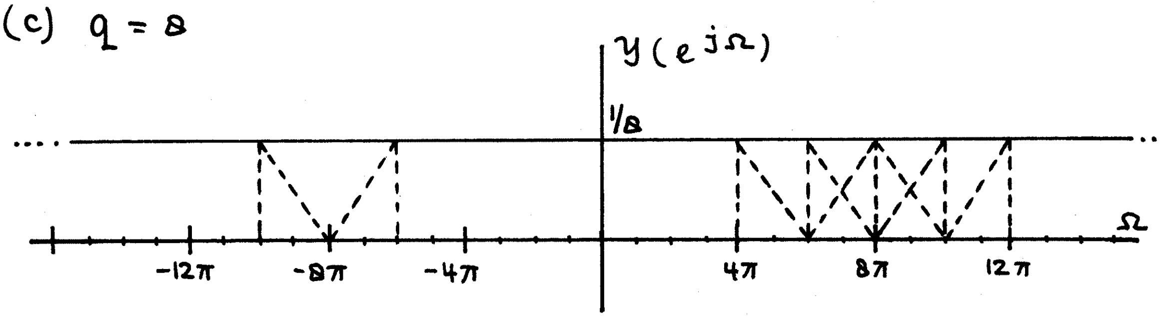
(a) $q = 3$



(b) $q = 4$



(c) $q = 8$



$\boxed{4.22}$

(a) $x(t) = \frac{1}{t}\sin(\pi t) + \cos(2\pi t)$

$$X(j\omega) = \begin{cases} \frac{1}{\pi}, & |\omega| < \pi \\ 0, & \text{otherwise} \end{cases} + \pi\left(\delta(\omega-2\pi) + \delta(\omega+2\pi)\right)$$

$\Rightarrow \omega\ max = 2\pi$, thus $B = 1\ Hz$

$\therefore T < \frac{1}{2B}$ or $T < 0.5\ s$

(b) $x(t) = \cos(10\pi t)\frac{\sin(\pi t)}{2t}$

$$X(j\omega) = \begin{cases} \frac{1}{4\pi}, & |\omega - 10\pi| < \pi \\ 0, & \text{otherwise} \end{cases} + \begin{cases} \frac{1}{4\pi}, & |\omega + 10\pi| < \pi \\ 0, & \text{otherwise} \end{cases}$$

$\Rightarrow \omega\ max = 11\pi$, thus $B = 5.5\ Hz$

$\therefore T < \frac{1}{11}\ s$

(c) $x(t) = e^{-4t}u(t) * \frac{\sin(Wt)}{\pi t}$

$$X(j\omega) = \frac{1}{4 + j\omega}\left(u(\omega+W) - u(\omega-W)\right)$$

$\rightarrow \omega\ max = W$, thus $B = \frac{W}{2\pi}$

$\therefore T < \frac{\pi}{W}$

(d) $x(t) = w(t) \cdot g(t)$

$X(j\omega) = \frac{1}{2\pi} W(j\omega) * G(j\omega)$

By inspection, $\omega_{max} = 5\pi + \omega_a$

So, $B = \dfrac{5\pi + \omega_a}{2\pi}$

$\therefore T < \dfrac{\pi}{5\pi + \omega_a}$

$\boxed{4.23}$ $X(j\omega)$ bandlimited to 5π

$$s(t) = x(t) + A\sin(120\pi t)$$
$$s[n] = s(nT) = x[n] + A\sin\left(\frac{240\pi}{13}n\right)$$
$$= x[n] + A\sin\left(9(2\pi)n + \frac{6\pi}{13}n\right)$$
$$s[n] = x[n] + A\sin\left(\frac{6\pi}{13}n\right)$$

$$\Omega_{sin} = \frac{6\pi}{13} \qquad \omega = \frac{\Omega}{T} = \left(\frac{6\pi}{13}\right)\Big/\left(2/13\right) = 3\pi$$

(a) The sinusoid appears at $\omega = 3\pi$ rads/sec in $S_s(j\omega)$

(b) Before the sampling, $s(t)$ is passed to the LPF

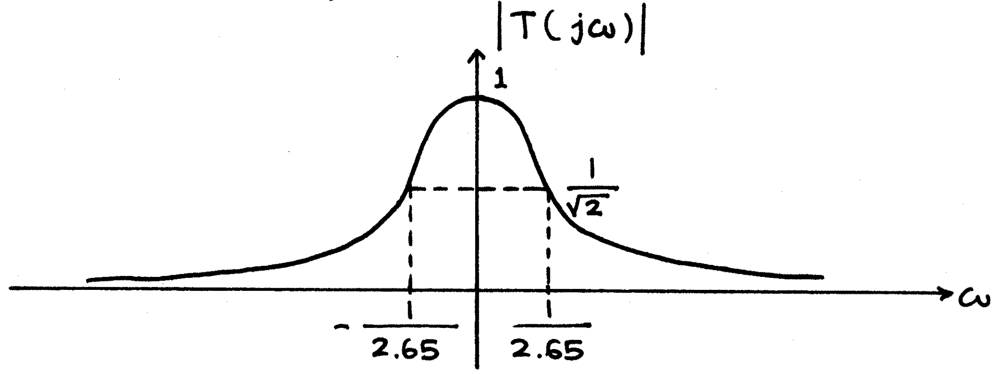
$$T(j\omega) = \frac{\frac{1}{j\omega c}}{R + \frac{1}{j\omega c}} = \frac{1}{1 + j\omega cR} = \frac{1}{1 + j\omega\tau}$$

$$|T(j\omega)| = \frac{1}{\sqrt{1+\omega^2\tau^2}} = \frac{1}{1000}\Bigg|_{\omega = 120\pi}$$

$\tau \simeq 2.65$ s

(c) $$T(j\omega) = \frac{1}{1 + j\omega(2.65)}$$

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \omega^2\, 7.04}}$$



$\boxed{4.24}$ $$y(t) = x(t) \cdot w(t)$$

$$|X(j\omega)| = 0 \text{ for } |\omega| > \omega_m$$

For reconstruction, need to have $\omega_s > 2 W_{max}$

or $$T < \frac{1}{2 \times \frac{\omega_m}{2\pi}} \Rightarrow T < \frac{\pi}{\omega_m}$$

Finite duty cycle results in distortion

$$W(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} X[k] \cdot \delta\left(\omega - k \cdot \frac{2\pi}{T}\right)$$

where $$X[k] = \frac{\sin\left(\frac{\pi}{2}k\right)}{k\pi} e^{-j\frac{\pi}{2}k}$$

After sampling :

$$Y(j\omega) = \sum_{k=-\infty}^{\infty} \frac{\sin\left(\frac{\pi}{2}k\right)}{\pi k} e^{-j\frac{\pi}{2}k} X\left(j\left(\omega - k\frac{2\pi}{T}\right)\right)$$

To reconstruct ,

$$H_r(j\omega) \cdot Y(j\omega) = X(j\omega) \,, |\omega| < \omega_m$$

$$\frac{2\pi}{T} > 2\omega_m$$

<u>k = 0</u> : $H_r(j\omega) \cdot \frac{1}{2} X(j\omega) = X(j\omega)$

$$\therefore H_r(j\omega) = \begin{cases} 2 & , |\omega| < \omega_m \\ \text{don't care} & , \omega_m < |\omega| < \frac{2\pi}{T} - \omega_m \\ 0 & , |\omega| > \frac{2\pi}{T} - \omega_m \end{cases}$$

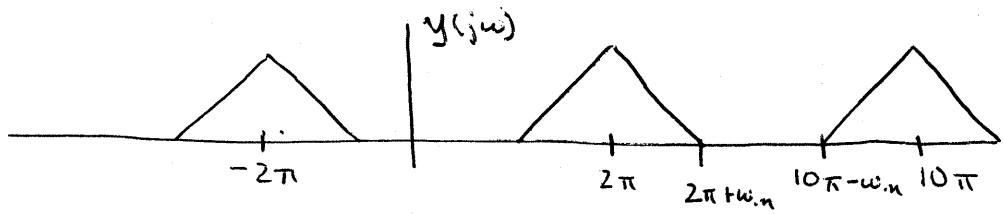
$\boxed{4.25}$ $|X(j\omega)| = 0 \quad , \quad |\omega| > \omega_m$

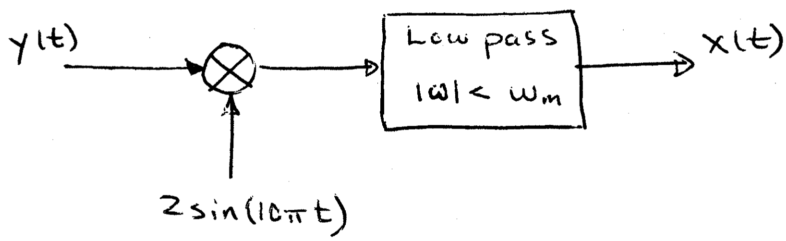
$$y(t) = x(t)\left[\cos(2\pi t) + \sin(10\pi t)\right]$$

$$Y(j\omega) = \frac{1}{2}\Big(X(j(\omega - 2\pi)) + X(j(\omega + 2\pi)) - j X(j(\omega - 10\pi))$$
$$+ j X(j(\omega + 10\pi))\Big)$$

$x(t)$ can be reconstructed from $y(t)$ if there is no overlap among the four shifted $X(j\omega)$.
Yet, $x(t)$ can still be reconstructed when overlap occurs , provided that there is at least one shifted $X(j\omega)$ that is not contaminated

$$\therefore \omega_m \max = \frac{10\pi - 2\pi}{2} = 4\pi$$

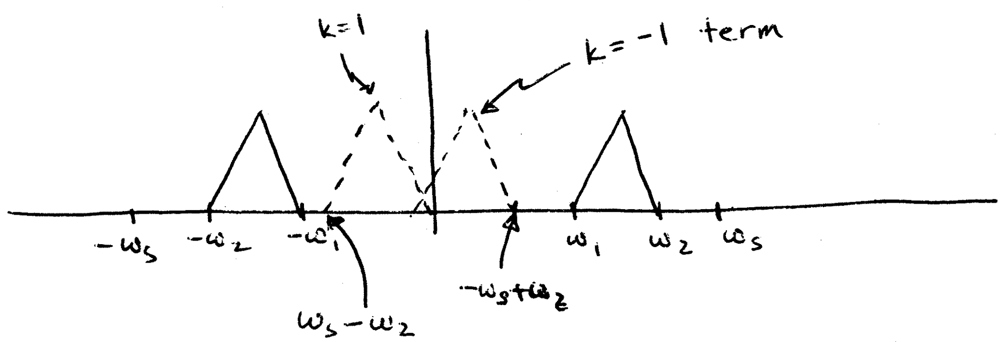




4.26 $|X(j\omega)| = 0$, for $|\omega| < \omega_1$, $|\omega| > \omega_2$

$\omega_1 > \omega_2 - \omega_1$

Can tolerate aliasing, as long as there is no overlap on $\omega_1 \le |\omega| \le \omega_2$



require $\omega_s - \omega_2 \ge -\omega_1 \Rightarrow \omega_s \ge \omega_2 - \omega_1$

$\Rightarrow T \le \dfrac{2\pi}{\omega_2 - \omega_1}$

$$H_r(j\omega) = \begin{cases} T & \omega_1 < |\omega| < \omega_2 \\ 0 & \text{otherwise} \end{cases}$$

$\boxed{4.27}$

(a) $X[k] = \begin{cases} \left(\frac{3}{4}\right)^k, & |k| \leq 4 \\ 0, & \text{otherwise} \end{cases} \qquad T = 1$

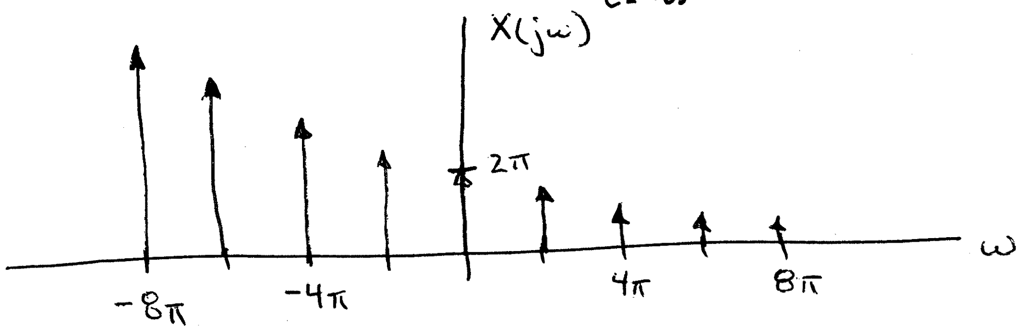
$$X(j\omega) = 2\pi \sum_{k=-4}^{4} \left(\frac{3}{4}\right)^k \delta(\omega - k \cdot 2\pi)$$

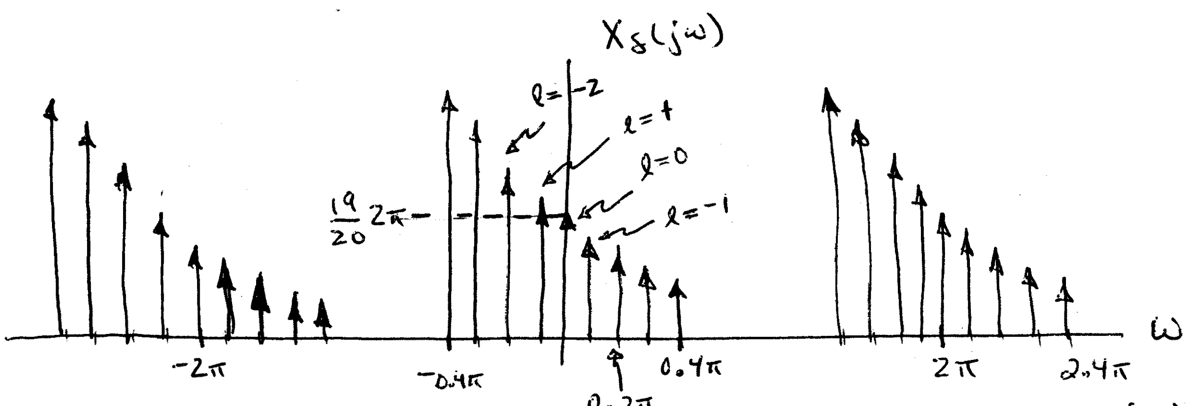
We can see that $\omega_{max} = 8\pi$

$$\frac{2\pi}{T_s} > 2 \cdot (8\pi) \longrightarrow T_{s\,min} = \frac{1}{8}$$

(b) $T = \frac{20}{19}$

Then : $$X_\delta(j\omega) = \frac{19}{20} \sum_{\ell=-\infty}^{\infty} X(j(\omega - \ell\, 1.9\pi))$$





Aliasing produces a "frequency scaled" replica of $X(j\omega)$ centered on zero. The scaling is by a factor of 20, from $\omega_0 = 2\pi$ to $\omega_0' = 0.1\pi$. Applying the LPF ($|\omega| < \pi$) gives $x(t/20)$.

And $X_{(\text{reconstructed})}(t) = \frac{19}{400} \, x\left(\frac{t}{20}\right)$

scaling factor $= \frac{1}{20}$

(c) The choice τ is such that no overlap (aliasing) occurs.

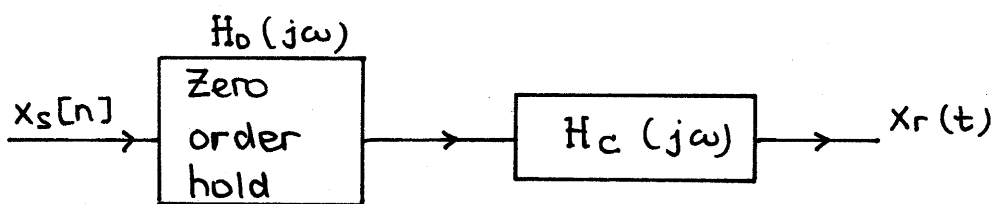
First : $\frac{2\pi}{\tau} < 2\pi \longrightarrow \tau > 1$ (period of original signal)

Second : $\left(2\pi - \frac{2\pi}{\tau}\right) . 4 < \frac{1}{2} . \frac{2\pi}{\tau}$

$$\tau < \frac{9}{8}$$

$$\therefore \ 1 < \tau < \frac{9}{8}$$

$\boxed{4.28}$ $x(t)$ bandlimited to ω_m



$x_s[n] = x(n\tau)$

(1) $0.99 < |H_0(j\omega)|.|H_c(j\omega)| < 1.01$, $-\omega_m \leq \omega \leq \omega_m$

Thus : $|H_c(j\omega)| > \dfrac{0.99 \, . \, \omega}{2\sin(\omega \, . \, \frac{\tau}{2})}$ ....(1)

$|H_c(j\omega)| < \dfrac{1.01 \, \omega}{2\sin(\omega \, \frac{\tau}{2})}$ ....(2)

Passband constraint for each case:

$$\frac{0.99\,\omega}{2\sin(\omega \frac{T}{2})} < |H_c(j\omega)| < \frac{1.01\,\omega}{2\sin(\omega\frac{T}{2})}$$

Stopband: $|H_0(j\omega)||H_c(j\omega)| < 10^{-4}$

at worst case $\tilde{\omega} = \frac{2\pi}{T} - \omega_m$

$$|H_c(j\tilde{\omega})| < \left|\frac{10^{-4}\,\tilde{\omega}}{2\sin(\tilde{\omega}\frac{T}{2})}\right|$$

a) $\omega_m = 10\pi$, $T = 0.08$

$\tilde{\omega} = 15\pi$ $\qquad |H_c(j\tilde{\omega})| < 0.00248$

b) $\omega_m = 10\pi$ $\quad T = 0.05$

$\tilde{\omega} = 30\pi$ $\qquad |H_c(j\tilde{\omega})| < 0.00666$

c) $\omega_m = 10\pi$ $\quad T = 0.01$

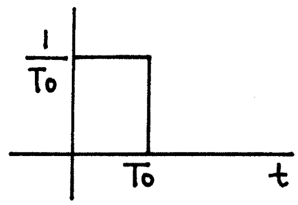
$\tilde{\omega} = 190\pi$ $\qquad |H_c(j\tilde{\omega})| < 0.191$

d) $\omega_m = 2\pi$ $\quad T = 0.08$

$\tilde{\omega} = 23\pi$ $\qquad |H_c(j\tilde{\omega})| < 0.0145$

$\boxed{4.29}$ $x[n] = x(n\tau)$

$$x_p(t) = \sum_{n=-\infty}^{\infty} x[n]\, h_p(t - n\tau)$$



$$X_p(j\omega) = H_p(j\omega) \cdot X_\Delta(j\omega)$$

$$H_p(j\omega) = \frac{2\sin\left(\omega \frac{T_0}{2}\right)}{T_0\,\omega} \cdot e^{-j\omega \frac{T_0}{2}}$$

constraints :

(1) Passband

$0.99 < |H_p(j\omega)| \cdot |H_c(j\omega)| < 1.01$, using $\omega_m = 10\pi$

$$\frac{0.99\, T_0\, \omega}{2\sin\left(\omega \cdot \frac{T_0}{2}\right)} < |H_c(j\omega)| < \frac{1.01\, T_0\,\omega}{2\sin\left(\omega \cdot \frac{T_0}{2}\right)}$$

(2) In the image location :

$|H_p(j\tilde{\omega})| < \dfrac{10^{-4} \cdot T_0\,\tilde{\omega}}{2\sin\left(\tilde{\omega} \cdot \frac{T_0}{2}\right)}$, where $\tilde{\omega} = \dfrac{2\pi}{\tau} - 10\pi$

(a) $\tau = 0.08$, $T_0 = 0.04$ $\tilde{\omega} = 15\pi$

$|H_c(j\tilde{\omega})| < 1.165 \times 10^{-4}$

(b) $\tau = 0.08$, $T_0 = 0.02$ $\tilde{\omega} = 15\pi$

$|H_c(j\tilde{\omega})| < 1.038 \times 10^{-4}$

(c) $\tau = 0.04$ $T_0 = 0.02$, $\tilde{\omega} = 40\pi$

$|H_c(j\tilde{\omega})| < 1.038 \times 10^{-4}$

(d) $\tau = 0.04$, $T_0 = 0.01$, $\tilde{\omega} = 40\pi$

$|H_c(j\tilde{\omega})| < 1.609 \times 10^{-4}$

$\boxed{4.30}$

(a) $x_1(t) = \sum_{n=-\infty}^{\infty} x[n]\; h_1(t - nT)$

$$= h_0(t) * h_0(t) * \sum_{n=-\infty}^{\infty} x[n]\, \delta(t - nT)$$

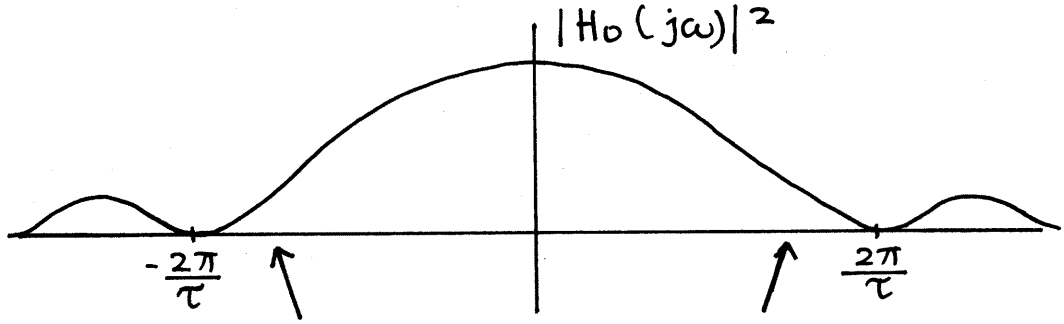
Thus : $X_1(j\omega) = H_0(j\omega) \cdot H_0(j\omega) \cdot X_\Delta(j\omega)$

$$\Rightarrow H_1(j\omega) = e^{-j\omega T} \frac{4 \sin^2(\omega \frac{T}{2})}{\omega^2}$$

Distortions :

(1) A linear phase shift corresponding to a time delay of T seconds (a unit of sampling time)

(2) $\sin^2(\cdot)$ term. introduces more distortion to the portion of $X_\Delta(j\omega)$, especially the higher freouency part is more severely attenuated than the lower one.



in the main lobe, between $-\omega_m$ and ω_m

(3) distorted and attenuated versions of $X(j\omega)$ still remain centered at non zero multiplies of ω_m, yet it is lower than the case of zero order hold

(b) $X_\Delta(j\omega) \cdot H_1(j\omega) \cdot H_c(j\omega) = X(j\omega)$

$$H_c(j\omega) = \frac{e^{j\omega T}\omega^2}{4\sin^2(\omega\frac{T}{2})} \times T \times (\text{ideal LPF})$$

$$\therefore H_c(j\omega) \begin{cases} \dfrac{T e^{j\omega T}\omega^2}{4\sin^2(\omega\frac{T}{2})} & , |\omega| < \omega_m \\ \text{don't care} & , \omega_m < |\omega| < \frac{2\pi}{T} - \omega_m \\ 0 & , |\omega| > \frac{2\pi}{T} - \omega_m \end{cases}$$

Assuming $X(j\omega) = 0$ for $|\omega| > \omega_m$

(c) <u>Constraints :</u>

(1) In the passband :

$$0.99 < |H_1(j\omega)|\,|H_c(j\omega)| < 1.01$$

$$\frac{0.99\,\omega^2}{4\sin^2(\omega\frac{T}{2})} < |H_c(j\omega)| < \frac{1.01\,\omega^2}{4\sin^2(\omega T/2)}$$

(2) In the image region : $\tilde{\omega} = \frac{2\pi}{T} - \omega_m$

$$|H_1(j\tilde{\omega})|\,|H_c(j\tilde{\omega})| < 10^{-4}$$

$$\Rightarrow |H_c(j\tilde{\omega})| < \frac{(10^{-4})(\tilde{\omega})^2}{4\sin^2(\tilde{\omega} \times \frac{T}{2})}$$

(i) $T = 0.08$

(2) $\tilde{\omega} = 15\pi$:

$|H_c(j15\pi)| < 0.0614$

or

$|H_c(j15\pi)| < 0.00491\, T^{-1}$

(ii) $T = 0.04$

(2) $\tilde{\omega} = 40\pi$:

$|H_c(j40\pi)| < 1.143$

or

$|H_c(j40\pi)| < 0.0457\, T^{-1}$

$\boxed{4.31}$

(a) $x[n] = \int_{(n-1)T}^{nT} x(t)\,dt = y(nT)$

so, $y(t) = \int_{t-T}^{t} x(\tau)\,d\tau$ by inspection

$$= \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\,d\tau$$

$$= x(t) * h(t)$$

choose $h(t) = \begin{cases} 1 & , 0 \leq t \leq T \\ 0 & , \text{elsewhere} \end{cases}$

so that

$$h(t-\tau) = \begin{cases} 1 & , t-\tau \leq \tau \leq t \\ 0 & , \text{elsewhere} \end{cases}$$

$$\therefore h(t) = u(t) - u(t-T)$$

(b) $Y(j\omega) = X(j\omega).H(j\omega)$

$$y(nT) = \xleftrightarrow{FT} \frac{1}{\tau} \sum_{k=-\infty}^{\infty} Y\left(j\left(\omega - k\frac{2\pi}{\tau}\right)\right)$$

$$\text{So : } FT\{x[n]\} = \frac{1}{T} \sum_{k=-\infty}^{\infty} X\left(j\left(\omega - k.\frac{2\pi}{\tau}\right)\right).H\left(j\left(\omega - k\frac{2\pi}{\tau}\right)\right)$$

(c) $x(t)$ is bandlimited to $|\omega| < \frac{3\pi}{4T} < \frac{2\pi}{T}$

we can use : $$H_r(j\omega) = \begin{cases} \dfrac{T}{H(j\omega)} & , |\omega| \leq \dfrac{3\pi}{4\tau} \\ 0 & , \text{elsewhere} \end{cases}$$

$$h(t) = \begin{cases} 1 & , 0 \leq t \leq \tau \\ 0 & , \text{elsewhere} \end{cases}$$

$$h\left(t + \frac{\tau}{2}\right) \xleftrightarrow{FT} \frac{2 \sin\left(\frac{\omega\tau}{2}\right)}{\omega}$$

$$\text{so : } H(j\omega) = \frac{2 \sin\left(\frac{\omega\tau}{2}\right)}{\omega} . e^{-j\frac{\omega.\tau}{2}}$$

$$\therefore H_r(j\omega) = \begin{cases} \dfrac{\omega.\tau . e^{\frac{j\omega\tau}{2}}}{2 \sin\left(\frac{\omega\tau}{2}\right)} & , |\omega| \leq \dfrac{3\pi}{4\tau} \\ 0 & , \text{elsewhere} \end{cases}$$

$\boxed{4.32}$ $$H(e^{j\Omega}) = \begin{cases} 1 & , |\Omega| < \frac{\pi}{4} \\ 0 & , \text{otherwise} \end{cases} \quad , \quad 2\pi \text{ periodic}$$

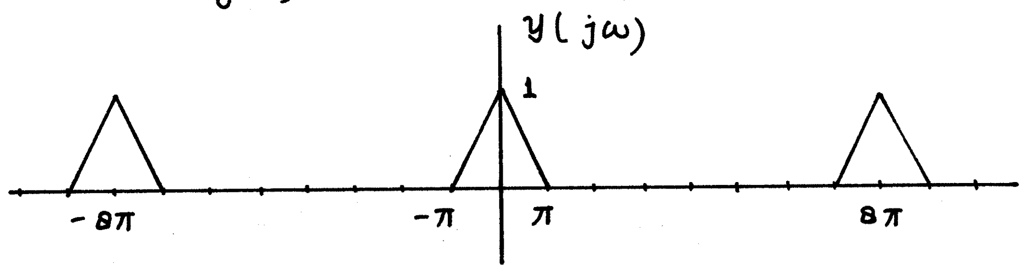
$$X_a(j\omega) = X(j\omega) \cdot H_a(j\omega)$$

$$X_{a_\delta}(j\omega) = \frac{1}{T} \sum_k X_a\left(j\left(\omega - k\frac{2\pi}{T}\right)\right)$$

To discard the high frequency component of $X(j\omega)$ and anticipate $\frac{1}{T}$, use :

$$H_a(j\omega) = \begin{cases} T & , |\omega| < \pi \\ 0 & , \text{otherwise} \end{cases}$$

Given , $y(e^{j\Omega})$, we can conclude that $T = \frac{1}{4}$ since $y(j\omega)$ is :



since the BW of the $x(t)$ should not change

$$\therefore \quad \omega_s = 8\pi$$

$$H_a(j\omega) = \begin{cases} \frac{1}{4} & , |\omega| < \pi \\ 0 & , \text{otherwise} \end{cases}$$

$\boxed{4.33}$ look at the eq. (4.35)

$$y(e^{j\Omega}) = \frac{1}{q} \sum_{m=0}^{q-1} X\left(e^{j\frac{1}{q}(\Omega - m2\pi)}\right)$$

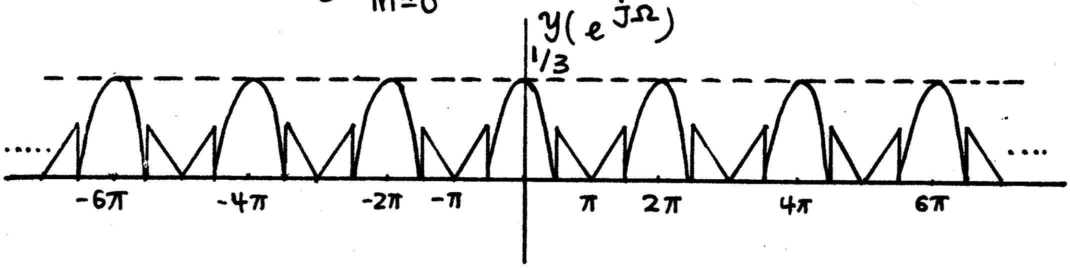
For bandlimited signal, overlap starts when:

$$2q \cdot W > 2\pi$$

Thus $q_{max} = \frac{\pi}{W} = 3$

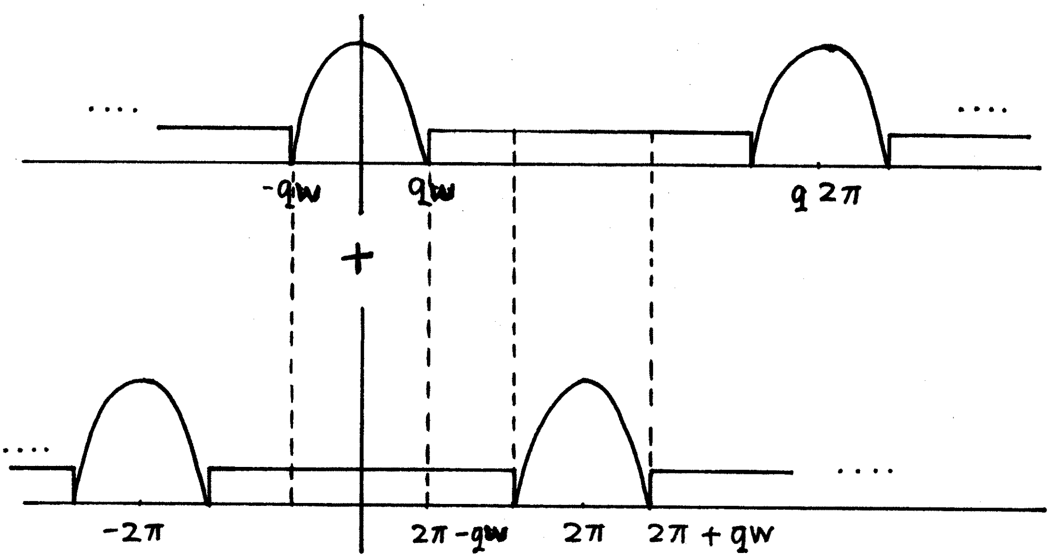
After decimated:

$$Y(e^{j\Omega}) = \frac{1}{3}\sum_{m=0}^{2} X\left(e^{j\frac{1}{3}(\Omega - m2\pi)}\right)$$



4.34

$$Y(e^{j\Omega}) = \frac{1}{q}\sum_{m=0}^{q-1} X\left(e^{j\frac{1}{q}(\Omega - m2\pi)}\right)$$

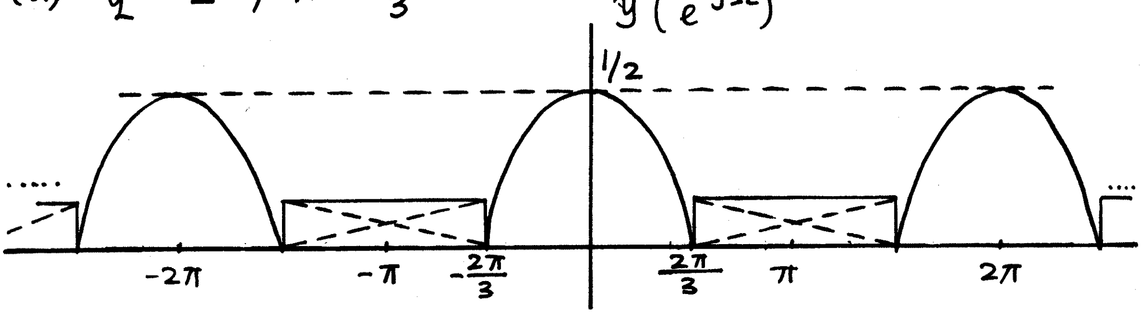


From the figure above, we can see that to preserve the shape within $|\Omega| < W$ (original signal), we need:

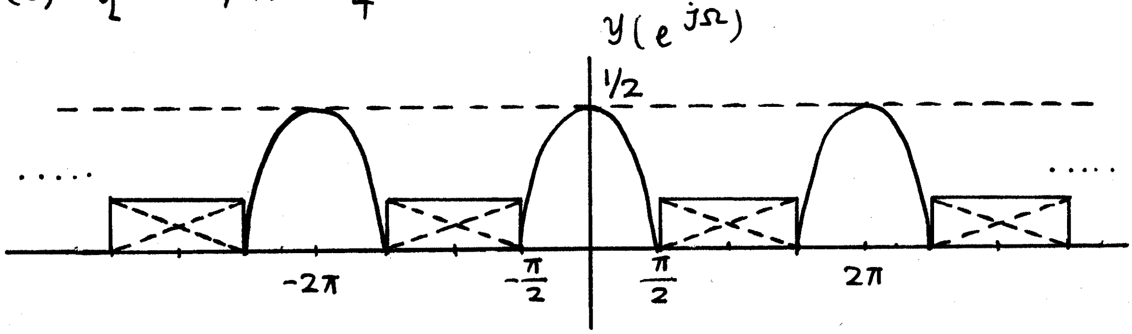
$$\Omega_p \min = \frac{qW}{q} = W$$

$$\Omega_s \max = \frac{2\pi - qW}{q} = \frac{2\pi}{q} - W$$

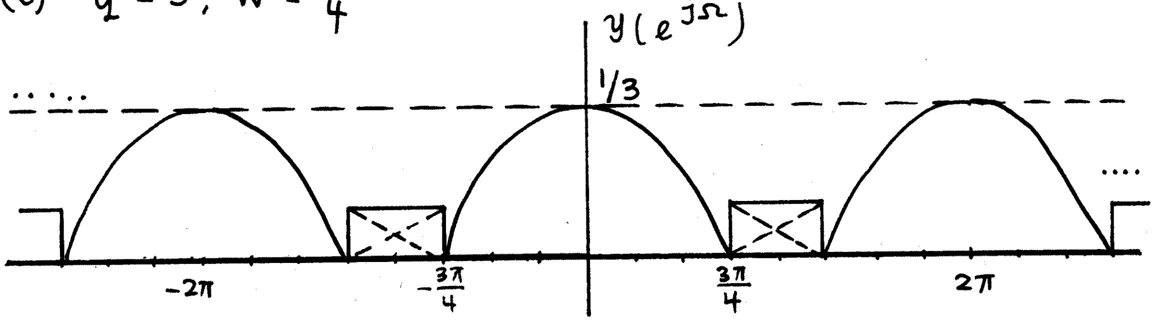
(a) $q = 2$, $W = \frac{\pi}{3}$



(b) $q = 2$, $W = \frac{\pi}{4}$



(c) $q = 3$, $W = \frac{\pi}{4}$



4.35

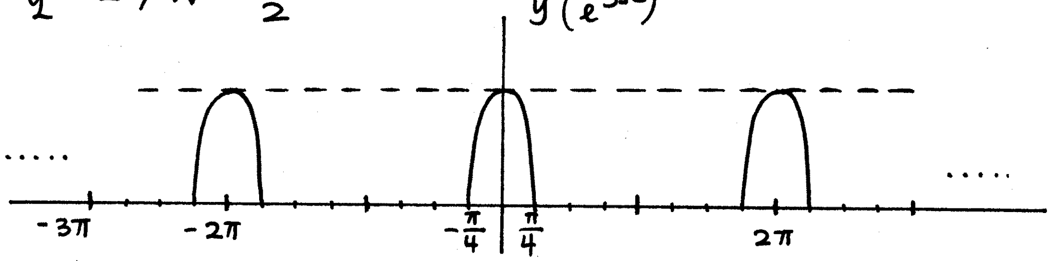
$$X_z(e^{j\Omega}) = X(e^{j\Omega q})$$

For ideal interpolation, additional

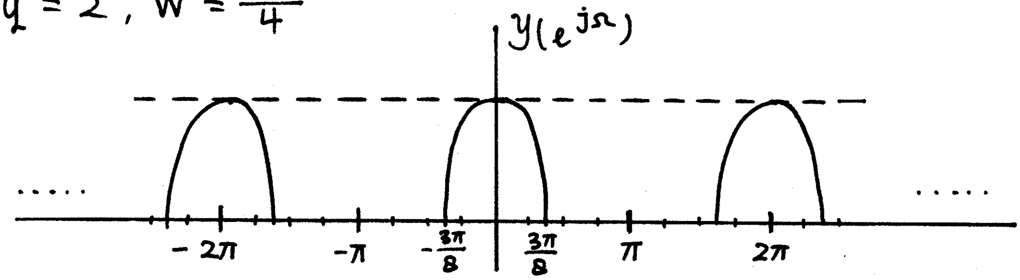
$$\Omega_p \min = \frac{W}{q}$$

$$\Omega_s \max = 2\pi - \frac{W}{q}$$

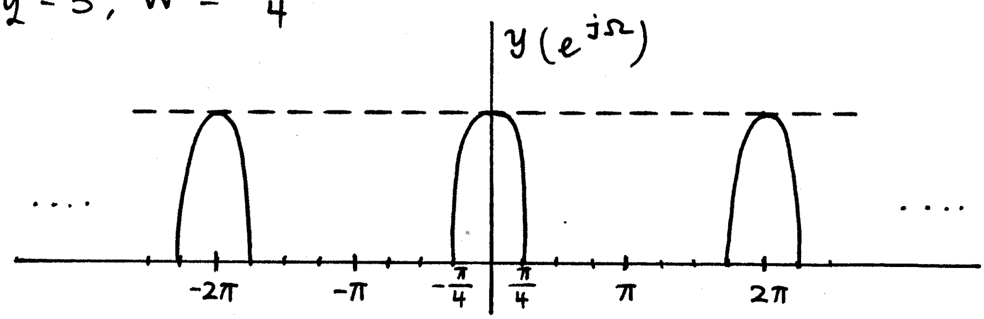
(a) $q = 2$, $W = \frac{\pi}{2}$



(b) $q = 2$, $W = \frac{3\pi}{4}$



(c) $q = 3$, $W = \frac{3\pi}{4}$



$\boxed{4.36}$ $x_o[n] = x_z[n] * h_o[n]$

$$x_z[n] = \begin{cases} x[\frac{n}{q}] & , \frac{n}{q} \text{ integer} \\ 0 & , \text{otherwise} \end{cases}$$

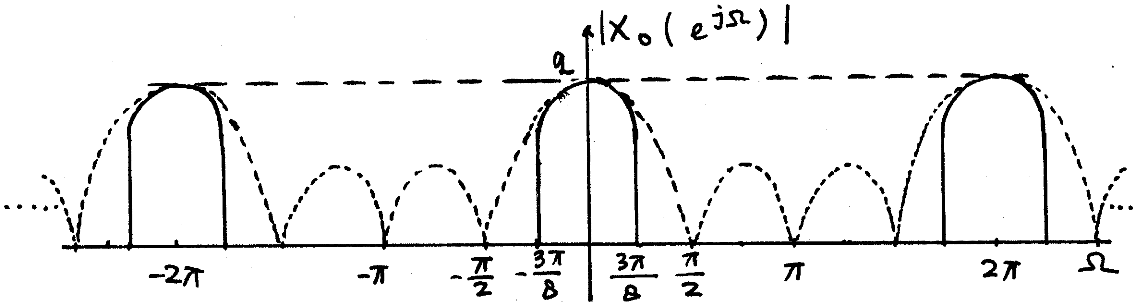
Thus : $X_z(e^{j\Omega}) = X(e^{jq\Omega})$

$$h_o[n] = \begin{cases} 1 & , 0 \leq n \leq q-1 \\ 0 & , \text{otherwise} \end{cases}$$

(a) $X_o(e^{j\Omega}) = X(e^{jq\Omega}) \cdot H_o(e^{j\Omega})$

If $x[n] = \dfrac{\sin(\frac{3\pi}{4}n)}{\pi n}$, $X(e^{j\Omega}) = \begin{cases} 1 , & |\Omega| < \frac{3\pi}{4} \\ 0 , & \frac{3\pi}{4} < |\Omega| < \pi \end{cases}$
2π periodic

$$|X_o(e^{j\Omega})| = |X(e^{jq\Omega})| \cdot \left| \frac{\sin(\Omega\frac{q}{2})}{\sin(\frac{\Omega}{2})} \right|$$



(b) For ideal interpolation, discard components other than the ones centered at multiple of 2π.
Also, need some magnitude and phase distortion correction

$$\therefore H(e^{j\Omega}) = \begin{cases} \dfrac{\sin(\frac{\Omega}{2})}{\sin(\Omega\frac{q}{2})} \cdot e^{j\Omega\frac{q}{2}} & , |\Omega| < \frac{W}{q} \\ 0 & , \frac{W}{q} < |\Omega| < 2\pi - \frac{W}{q} \text{ , } 2\pi \text{ periodic} \end{cases}$$

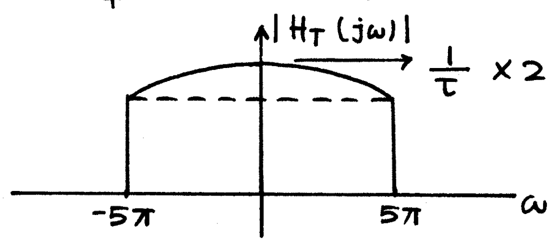
(i) $q = 2,\ W = \frac{3\pi}{4} \longrightarrow H(e^{j\Omega}) = \begin{cases} \dfrac{e^{j\Omega}}{2\cos(\frac{\Omega}{2})} & , |\Omega| < \frac{3\pi}{8} \\ 0 & , \frac{3\pi}{8} < |\Omega| < \frac{13\pi}{8} \end{cases}$

(ii) $q = 4,\ W = \frac{3\pi}{4} \longrightarrow H(e^{j\Omega}) = \begin{cases} \dfrac{\sin(\frac{\Omega}{2})\, e^{j2\Omega}}{\sin(2\Omega)} & , |\Omega| < \frac{3\pi}{16} \\ 0 & , \frac{3\pi}{16} < |\Omega| < \frac{29}{16}\pi \end{cases}$

$\boxed{4.37}$ $|H_T(j\omega)| = |H_a(j\omega)| \cdot \overbrace{\frac{1}{T}}^{20} |H(e^{j\Omega})| \left| \frac{2\sin(\omega \frac{T}{2})}{\omega} \right| |H_c(j\omega)|$

(a) $\Omega_1 = \frac{\pi}{4}$, $W_c = 20\pi$

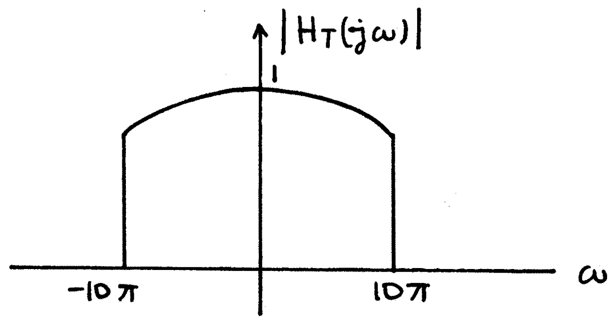
$\omega_m = \min(10\pi,\ \frac{\pi}{4} \times 20,\ 20\pi) = 5\pi$



$\frac{1}{T} \times 2 \times \frac{T}{2} = 1$

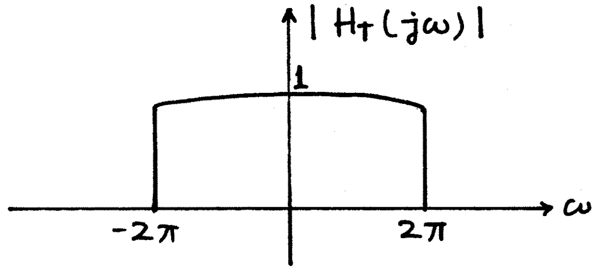
(b) $\Omega_1 = \frac{3\pi}{4}$, $W_c = 20\pi$

$\omega_m = \min(10\pi,\ \frac{3\pi}{4} \times 20,\ 20\pi) = 10\pi$



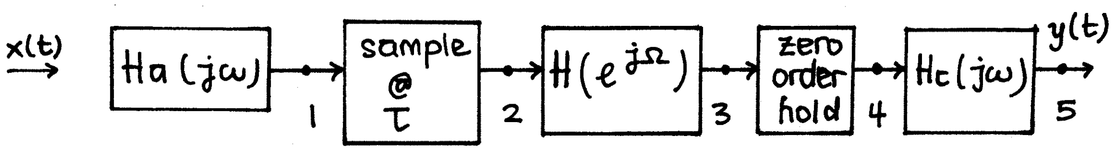
(c) $\Omega_1 = \frac{\pi}{4}$, $W_c = 2\pi$

$\omega_M = \min\left(10\pi\ ,\ \frac{\pi}{4} \times 20\ ,\ 2\pi\right) = 2\pi$



$\boxed{4.38}$ Specification : $0.9 < |G(j\omega)| < 1.1$: $100\pi < \omega < 200\pi$

$G(j\omega) = 0$ elsewhere



<u>3</u> : Passband : $100\pi < \omega < 200\pi$

Thus : $\Omega_a = 100\pi T$

$\Omega_b = 200\,T$

<u>4</u> : $|H_0(j\omega)| = \left| \dfrac{2\sin\left(\frac{\omega T}{2}\right)}{\omega} \right|$

at $\omega = 100\pi \longrightarrow \dfrac{2\sin(50\pi T)}{T\,100\pi} < 1.1$

$\dfrac{\sin(50\pi T)}{50\pi T} < 1.1$ always

at $\omega = 200\pi \longrightarrow \dfrac{2\sin(100\pi T)}{T\,200\pi} > 0.9$

$\dfrac{\sin(100\pi T)}{100\pi T} > 0.9 \longrightarrow T(100\pi) < 0.785$

$\therefore T_{max} = 0.0025$

$\underline{5}$: W_3 min : 200π
W_4 max : $\frac{2\pi}{\tau} - 200\pi = 600\pi$

$\underline{3}$: $\Omega_a = 0.25\pi$
$\Omega_b = 0.5\pi$

$\underline{1}$ and $\underline{2}$: W_1 min $= 200\pi$
W_2 max $= \frac{1}{2} \cdot \frac{2\pi}{\tau} = 400\pi$ (no overlap)

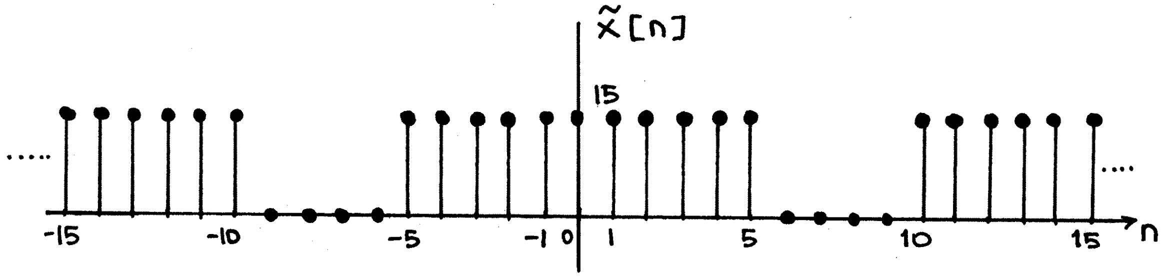
$\boxed{4.39}$ $X(e^{j\Omega}) = \dfrac{\sin\left(\frac{11\Omega}{2}\right)}{\sin\left(\frac{\Omega}{2}\right)}$; $\tilde{X}[k] = X(e^{jk\Omega_0})$

$$\tilde{X}[k] = \frac{\sin\left(\frac{11k\Omega_0}{2}\right)}{\sin\left(\frac{k\Omega_0}{2}\right)} \longleftrightarrow \tilde{x}[n] = N \times \begin{cases} 1 & , |n| \leq 5 \\ 0 & , 5 < |n| < \frac{N}{2} \end{cases}$$

period $= N$

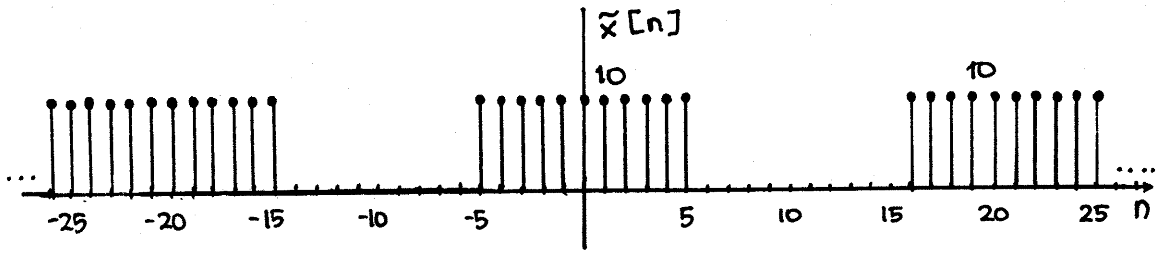
(a) $\Omega_0 = \frac{2\pi}{15}$, $N = 15$

$$\therefore \tilde{x}[n] = \begin{cases} 15 & , |n| \leq 5 \\ 0 & , 5 < |n| \leq 7 \end{cases}$$

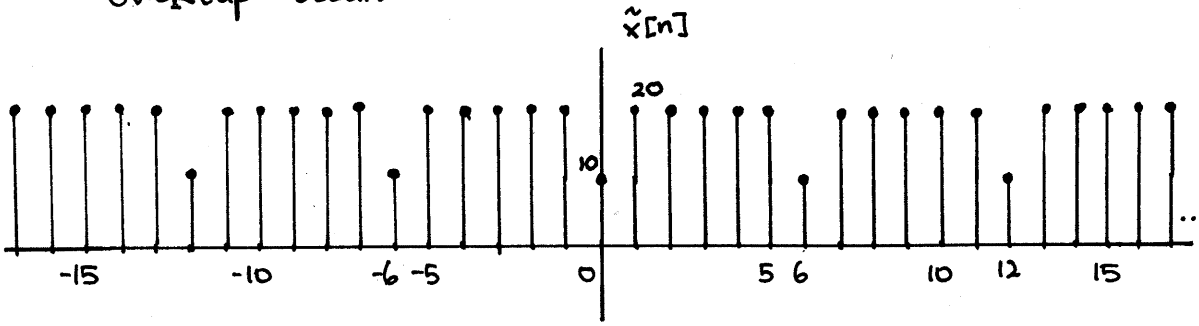


(b) $\Omega_0 = \frac{\pi}{10}$, $N = 20$

$$\therefore \tilde{x}[n] = \begin{cases} 10 & , |n| \leq 5 \\ 0 & , 5 < |n| < 10 \end{cases}$$



(c) $\Omega_0 = \frac{\pi}{3}$, $N = 6$
overlap occurs

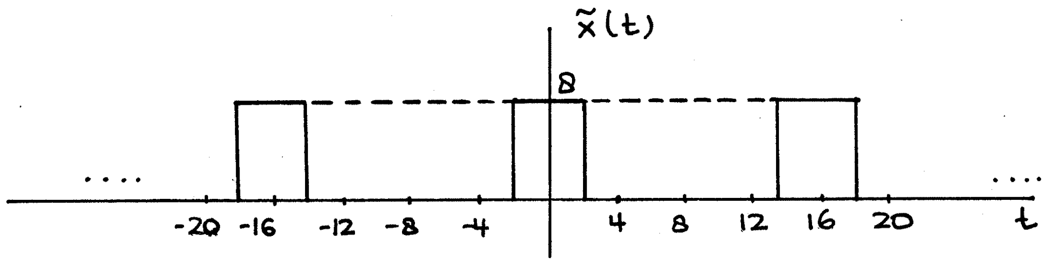


$\boxed{4.40}$ $X(j\omega) = \frac{\sin(2\omega)}{\omega}$

$\tilde{X}[k] = X(jk\omega_0) = \frac{\sin(2k\omega_0)}{k\omega_0}$

(a) $\omega_0 = \frac{\pi}{8}$, $\tilde{X}[k] = \frac{\sin(\frac{\pi}{4}k)}{\frac{\pi}{8}k}$, $T_s = 2$
$T = 16$

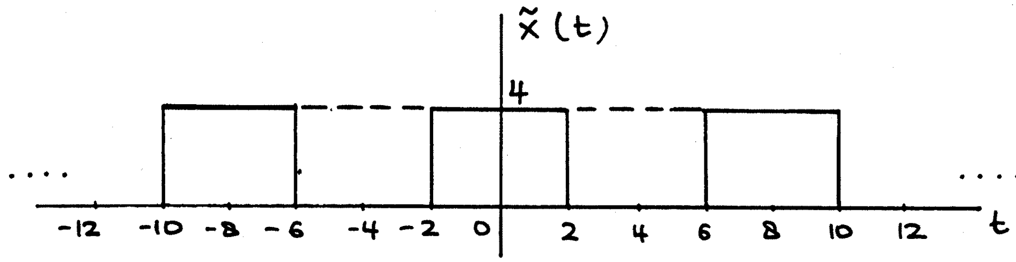
$$\tilde{x}(t) = \begin{cases} 8 & , \ |t| < 2 \\ 0 & , \ 2 < |t| < 8 \end{cases}$$



(b) $\omega_0 = \frac{\pi}{4}$

$$\tilde{x}[k] = \frac{\sin\left(\frac{\pi}{2}k\right)}{\frac{\pi}{2}k} \quad , \quad T_s = 2 \, , \; T = 8$$

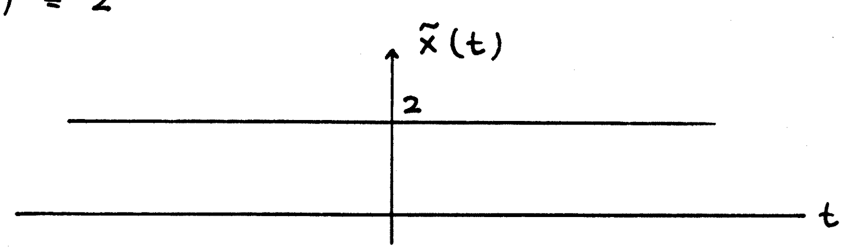
$$\tilde{x}(t) = \begin{cases} 4 & , \quad |t| < 2 \\ 0 & , \quad 2 < |t| < 4 \end{cases}$$



(c) $\omega_0 = \frac{\pi}{2}$

$$\tilde{x}[k] = \frac{\sin(\pi . k)}{\frac{\pi}{2}k} = 2\,\delta[k]$$

$$\tilde{x}(t) = 2$$



$\boxed{4.41}$ $x[n] = x[n+N]$

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} x[n]\,\delta(t - n\tau)$$

(a) $$x_\delta(t+\tau) = \sum_{n=-\infty}^{\infty} x[n]\,\delta(t + T - n\tau)$$

$$= \sum_{n=-\infty}^{\infty} x[n-N]\,\delta(t + T - n\mathcal{T})$$

$$= \sum_{n=-\infty}^{\infty} x[n]\,\delta(t - n\mathcal{T})$$

It is clear that if $\boxed{T = N.\mathcal{T}}$, the equality is satisfied

$\therefore\ x_\delta(t)$ is periodic with $T = N.\mathcal{T}$

(b) $$X_\delta[k] = \frac{1}{T}\int_{\langle T\rangle} x_\delta(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{N\mathcal{T}}\int_{\langle T\rangle} \sum_{n=-\infty}^{\infty} x[n]\,\delta(t - n\mathcal{T})\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{N\mathcal{T}} \sum_{n=-\infty}^{\infty} x[n] \int_{\langle T\rangle} \delta(t - n\mathcal{T})\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{N\mathcal{T}} \sum_{n=-\infty}^{\infty} x[n] \,.\, e^{-jk\omega_0 n\mathcal{T}}$$

$$= \frac{1}{\mathcal{T}}\left(\frac{1}{N}\sum_{n=-\infty}^{\infty} x[n]\; e^{-jk\frac{2\pi}{N}n}\right)$$

$$= \frac{1}{\mathcal{T}}\, X[k]$$

$\boxed{4.42}$ $x(t)$: $\underline{\mathcal{T} = 0.01}$ 100 samples $\underline{M = 100}$

use $\underline{N = 200}$ DTFS to approximate $X(j\omega)$

$|X(j\omega)| \approx 0$, $|\omega| > 120\pi$, $\underline{\omega_M = 120\pi}$

$$T < \frac{2\pi}{\omega_m + \omega_a} \quad , \quad \omega_a < \left(\frac{2\pi}{T} - \omega_m\right)$$

$$\therefore \quad \omega_a < 80\,\pi$$

$$MT > \frac{2\pi}{\omega_r} \quad , \quad \omega_r > \frac{2\pi}{MT}$$

$$\therefore \quad \omega_r > 2\pi$$

$$N > \frac{\omega_s}{\Delta\omega} \quad , \quad \Delta\omega > \frac{\omega_s}{N} \quad \text{or} \quad \Delta\omega = \frac{2\pi}{NT}$$

$$\therefore \quad \Delta\omega > \pi$$

$\boxed{4.43}$

(a) Lowpass

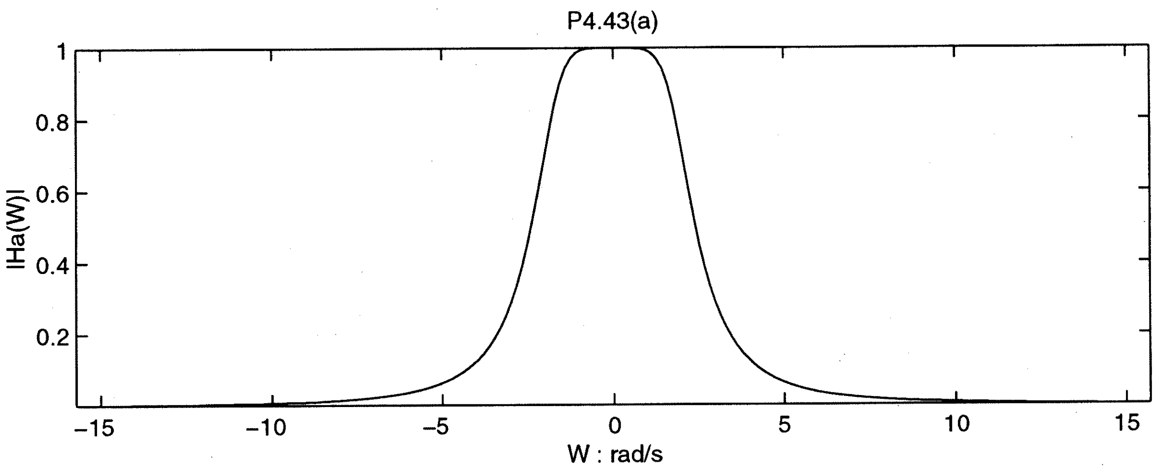
(b) Highpass

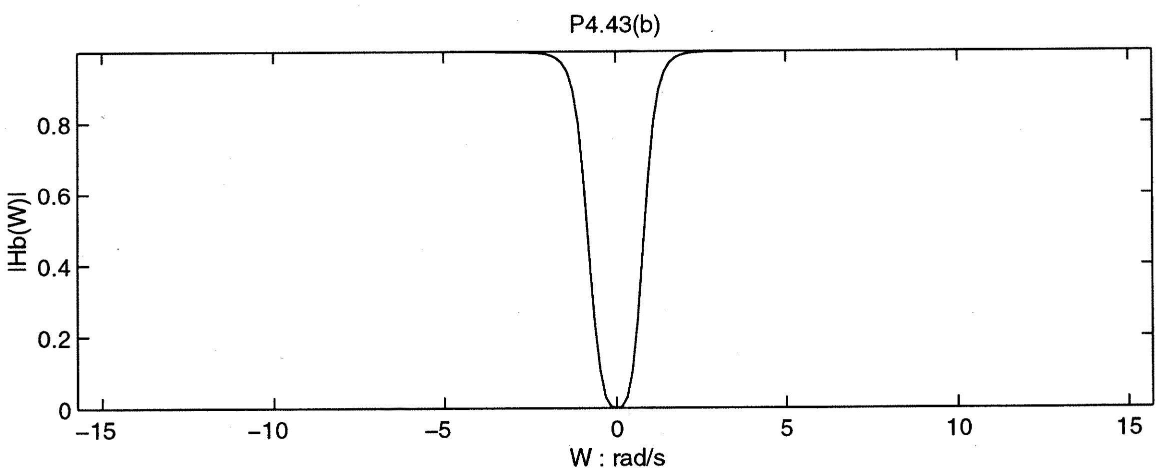
(c) Bandpass

(d) Lowpass

P 4.43

- Plot 1 of 2

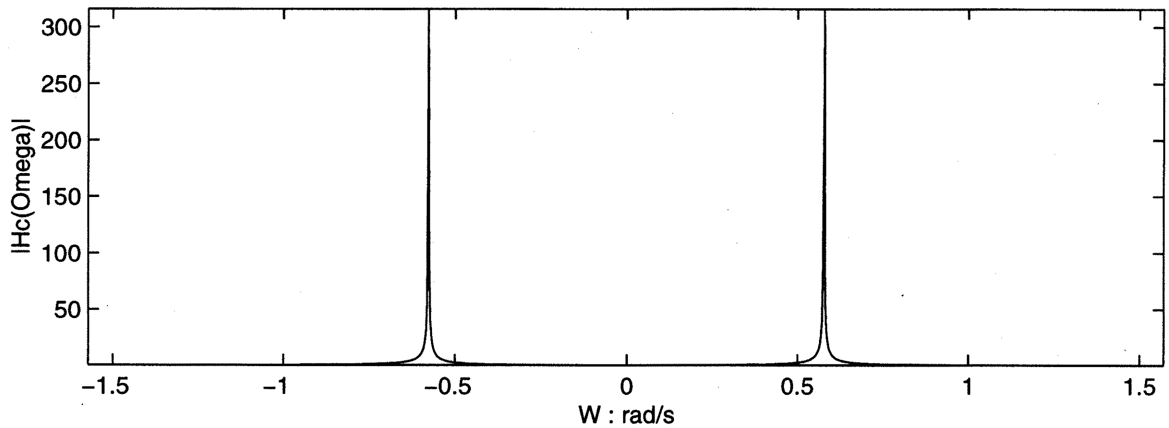




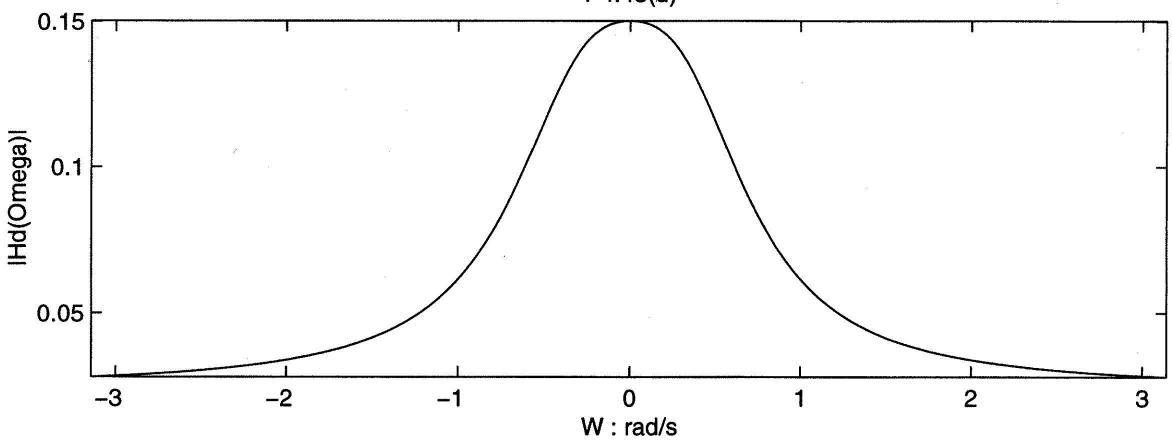
P 4.43

- Plot 2 of 2 -

P4.43(c)

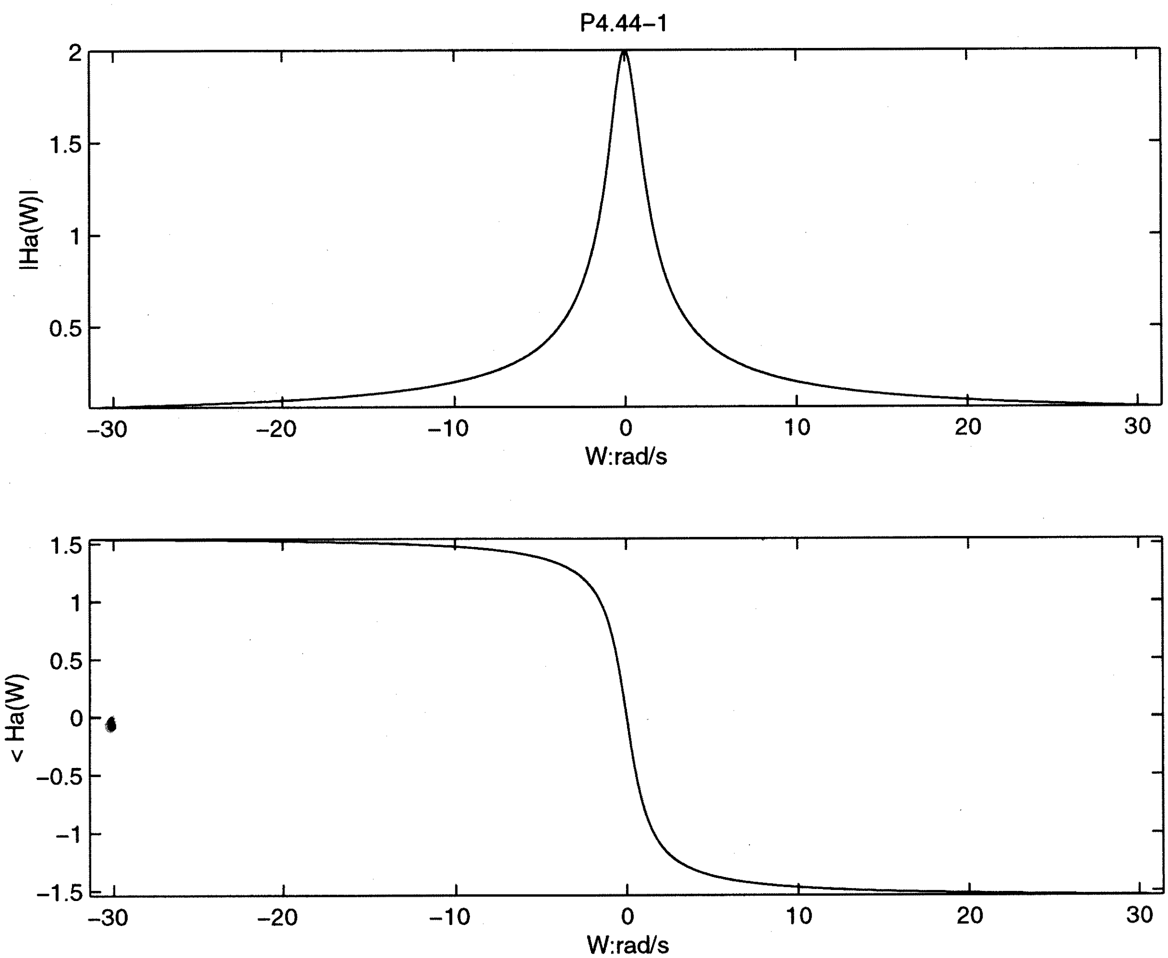


P4.43(d)



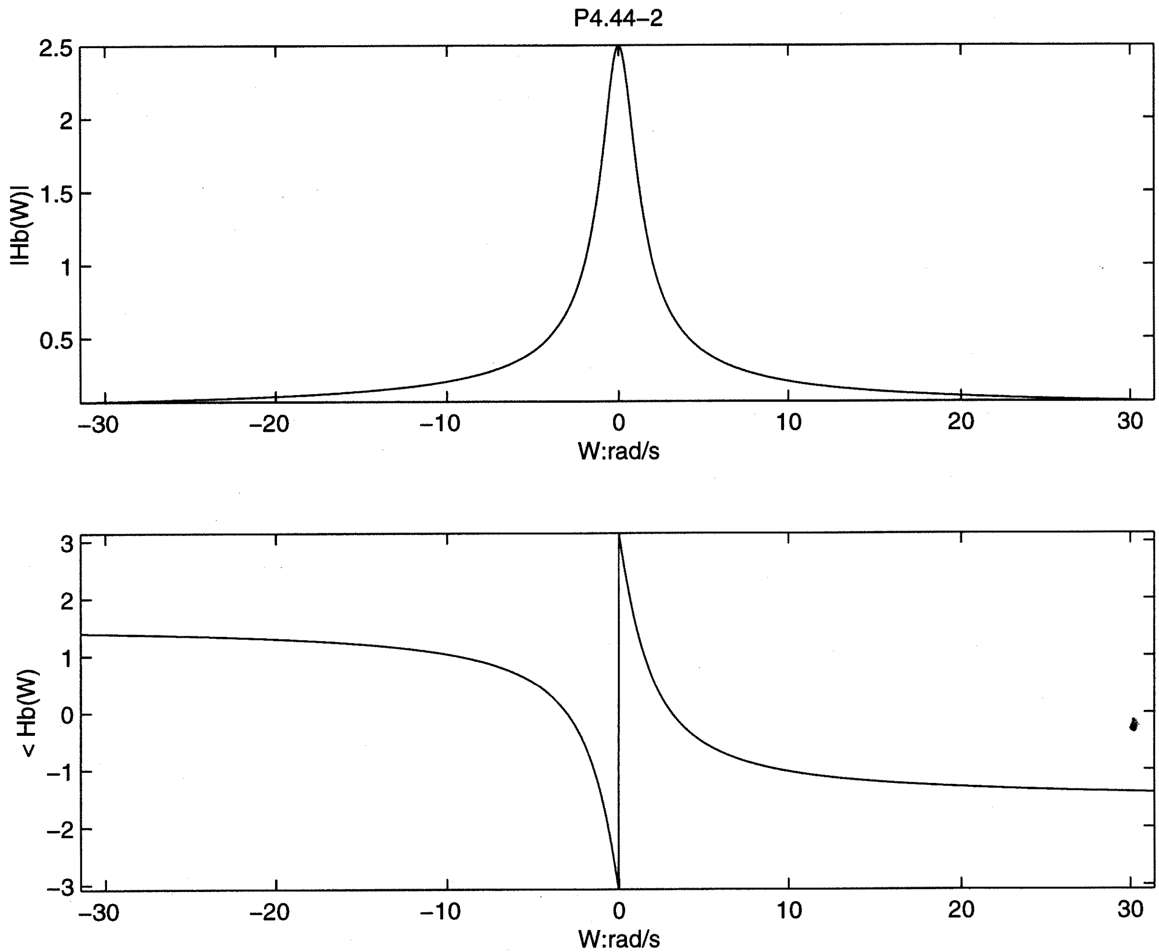
P. 4.44

– Plot 1 of 4 –



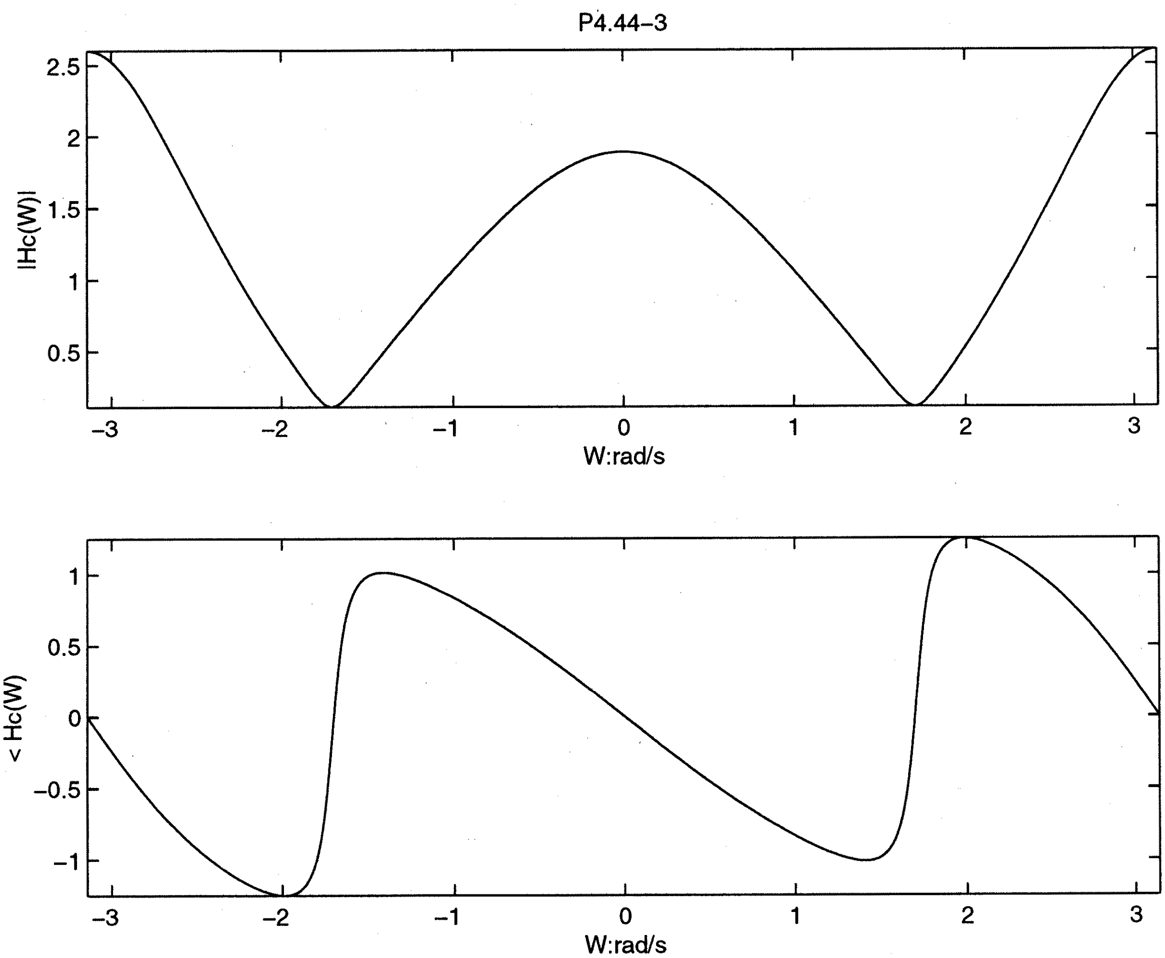
P 4.44

- Plot 2 of 4 -



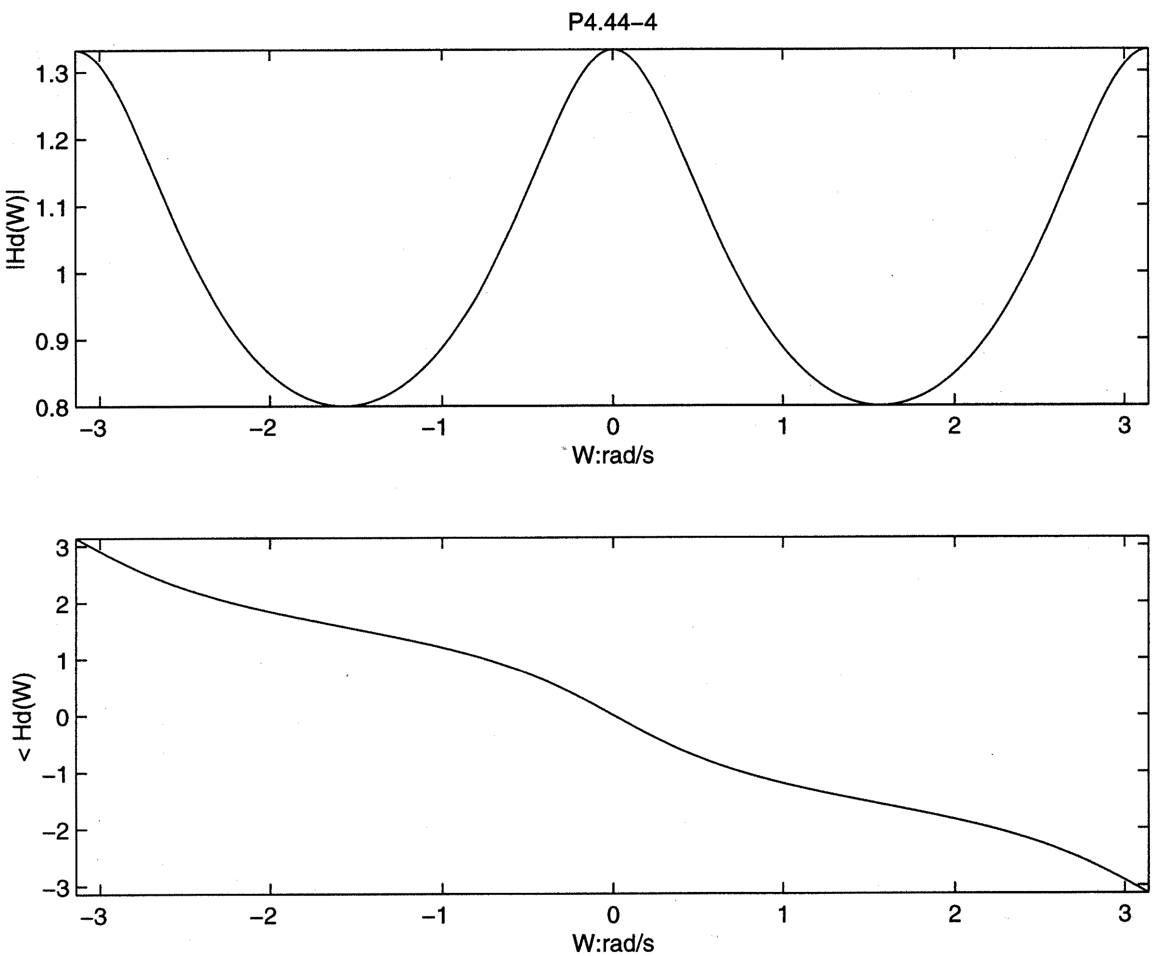
P 4.44

- Plot 3 of 4 -



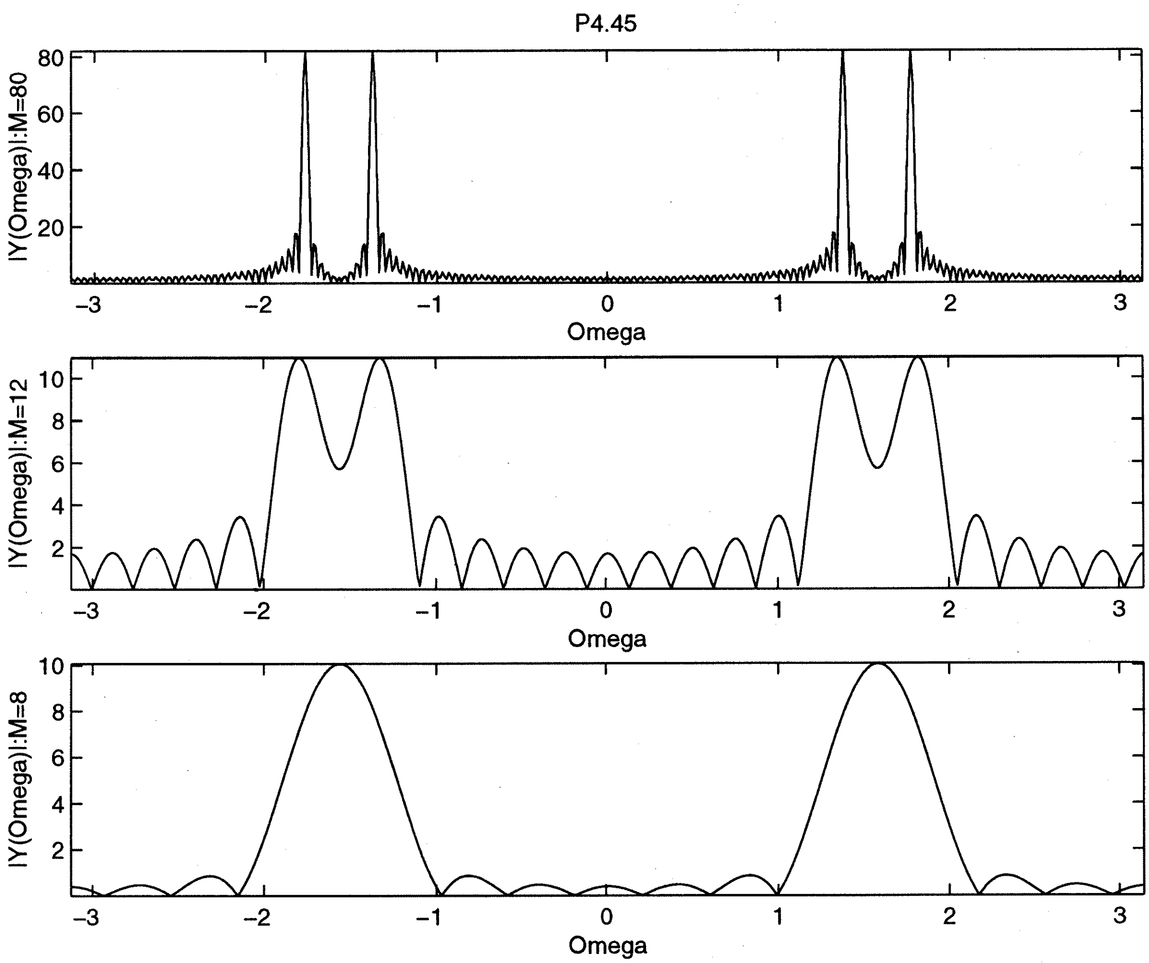
– P 4.44

– Plot 4 of 4 –



P 4.45

- Plot 1 of 1 -



$\boxed{4.46}$

(c) Using interval of $\frac{\pi}{200}$, the mainlobe width and peak sidelobe for each window can be estimated from the figure or finding the local minima and local nulls in the vicinity of the mainlobe

	Mainlobe width (Ω : rad)	Sidelobe height (dB)
Rectangular	0.25	-13.28
Hanning	0.50	-31.48

Note : sidelobe height is relative to the mainlobe

Hanning window has lower sidelobe, but wider mainlobe width compared to rectangular window

(d) Yes, because two sinusoids are very close to each other ($\frac{26\pi}{100}$ and $\frac{29\pi}{100}$).

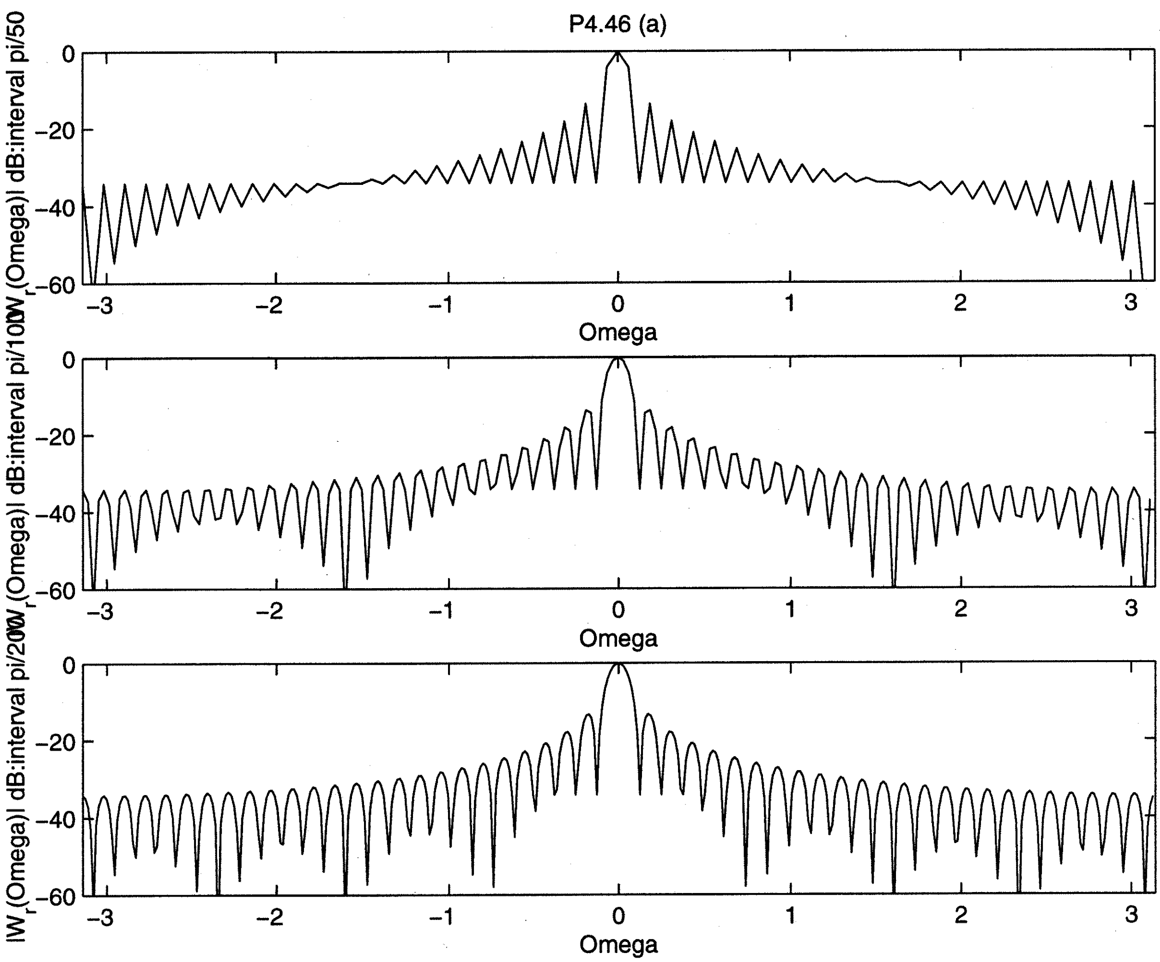
Since Hanning window has wider mainlobe, its resolution capability is inferior to rectangular window.
Notice from the plot that the existence of two sinusoids are indicated for the rectangular window, but not for Hanning window.

(e) Yes, here, two sinusoids are significantly far from each other (the separation is $\frac{\pi}{4}$ which is significantly bigger than the mainlobe width of both windows), hence resolution is not a problem for Hanning window.
Since the sidelobe magnitude is higher that 0.02 in rectangular window, the sinusoid at $\frac{51\pi}{100}$

is indistiguishable, unlike the case with Hanning window whose sidelobe is lower than 0.02

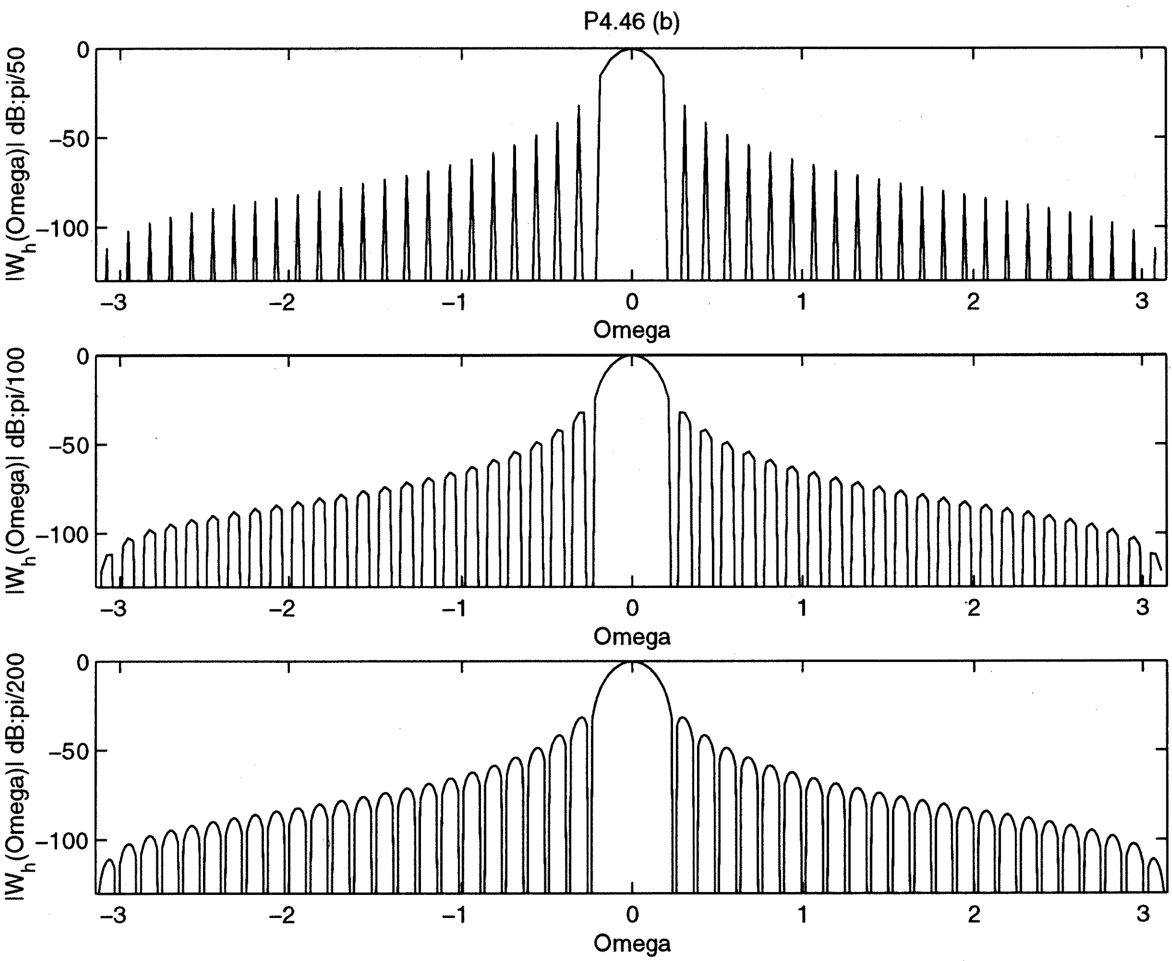
P 4.46

– Plot 1 of 5 –



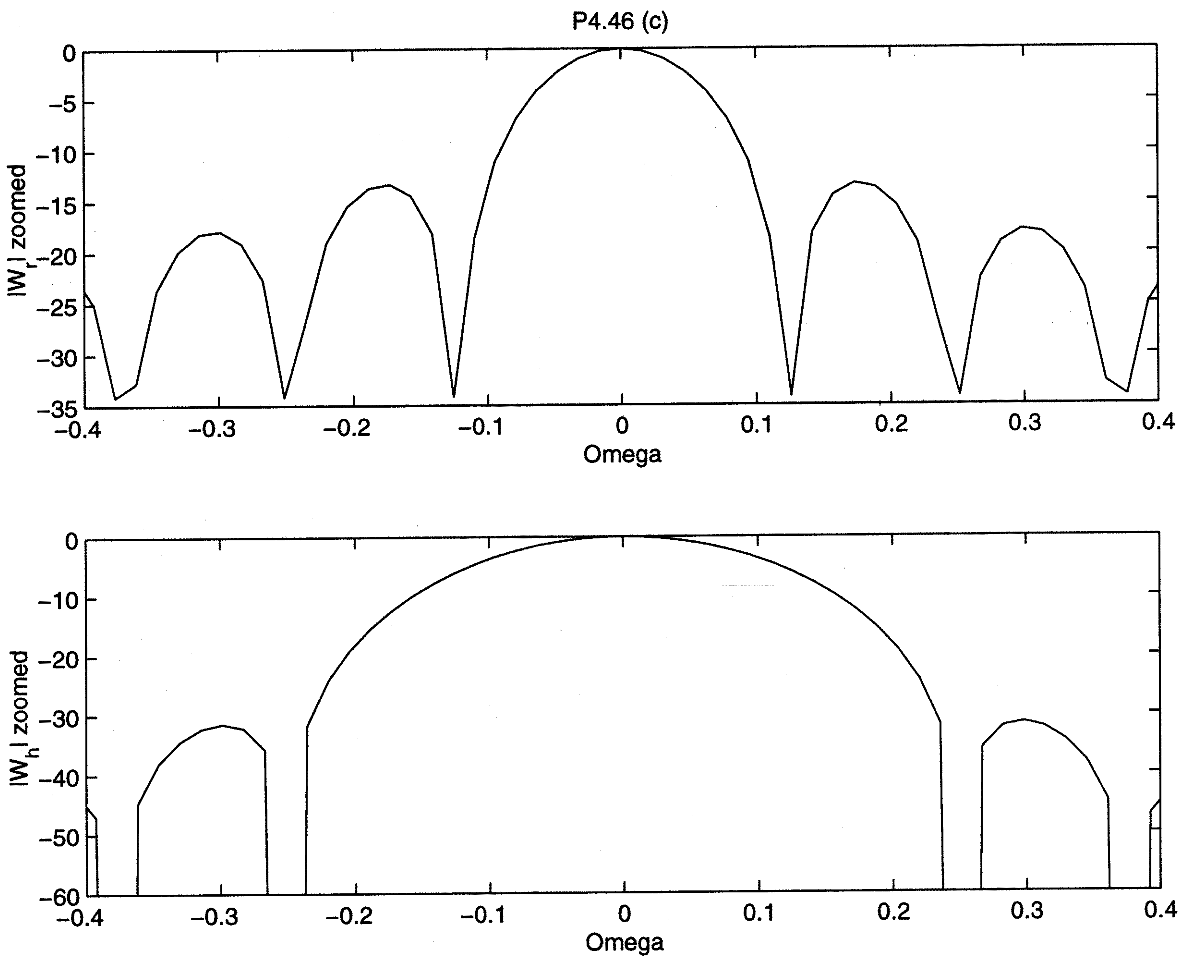
P 4.46

- Plot 2 of 5 -



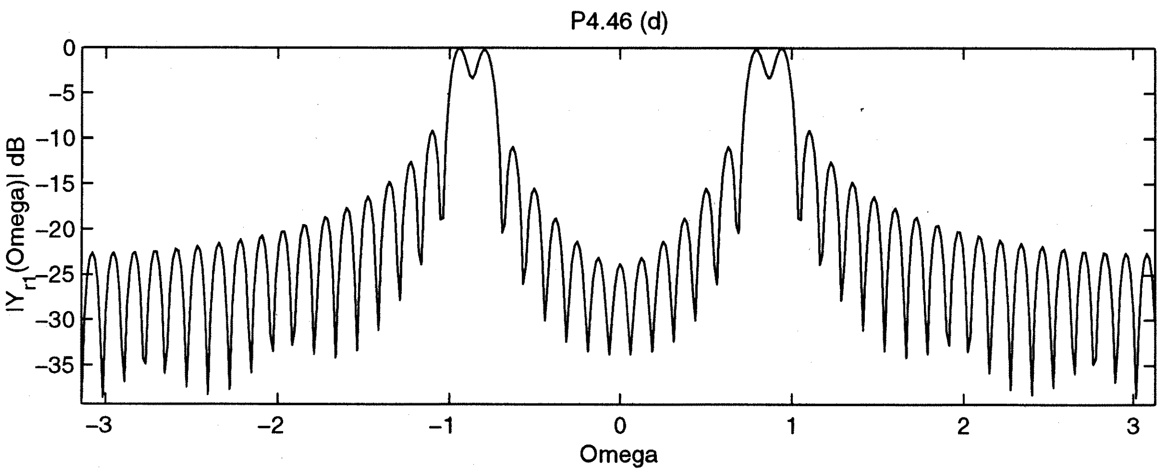
P 4.46

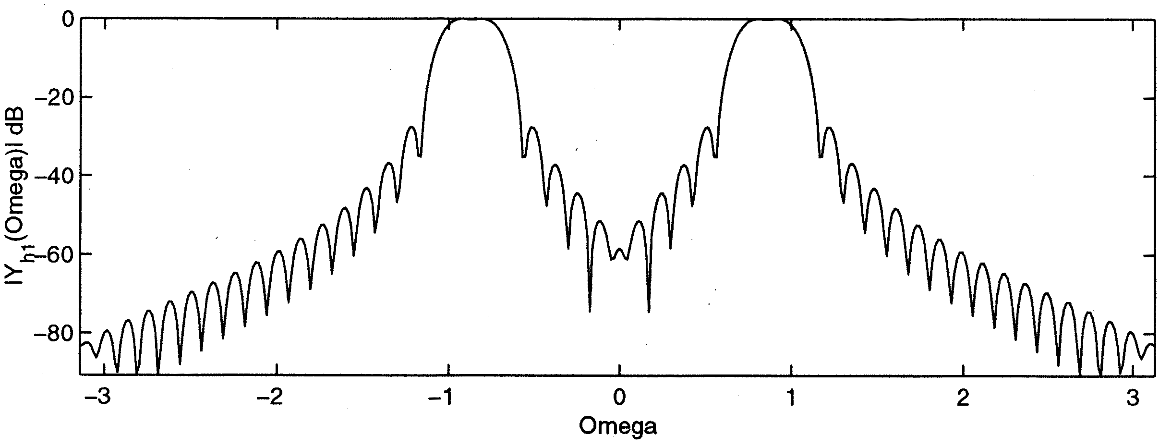
– Plot 3 of 5 –



P 4.46

- Plot 4 of 5 -

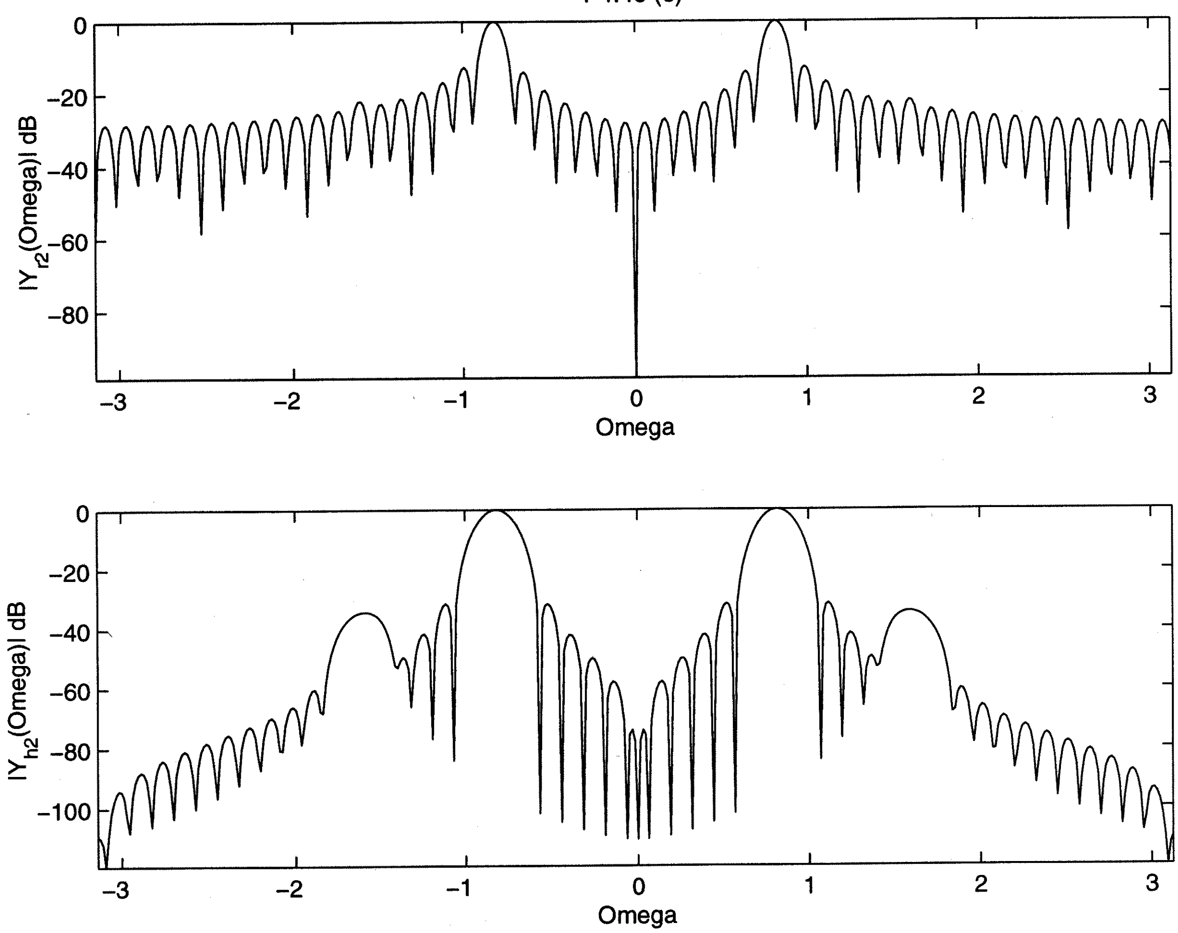




P 4.46

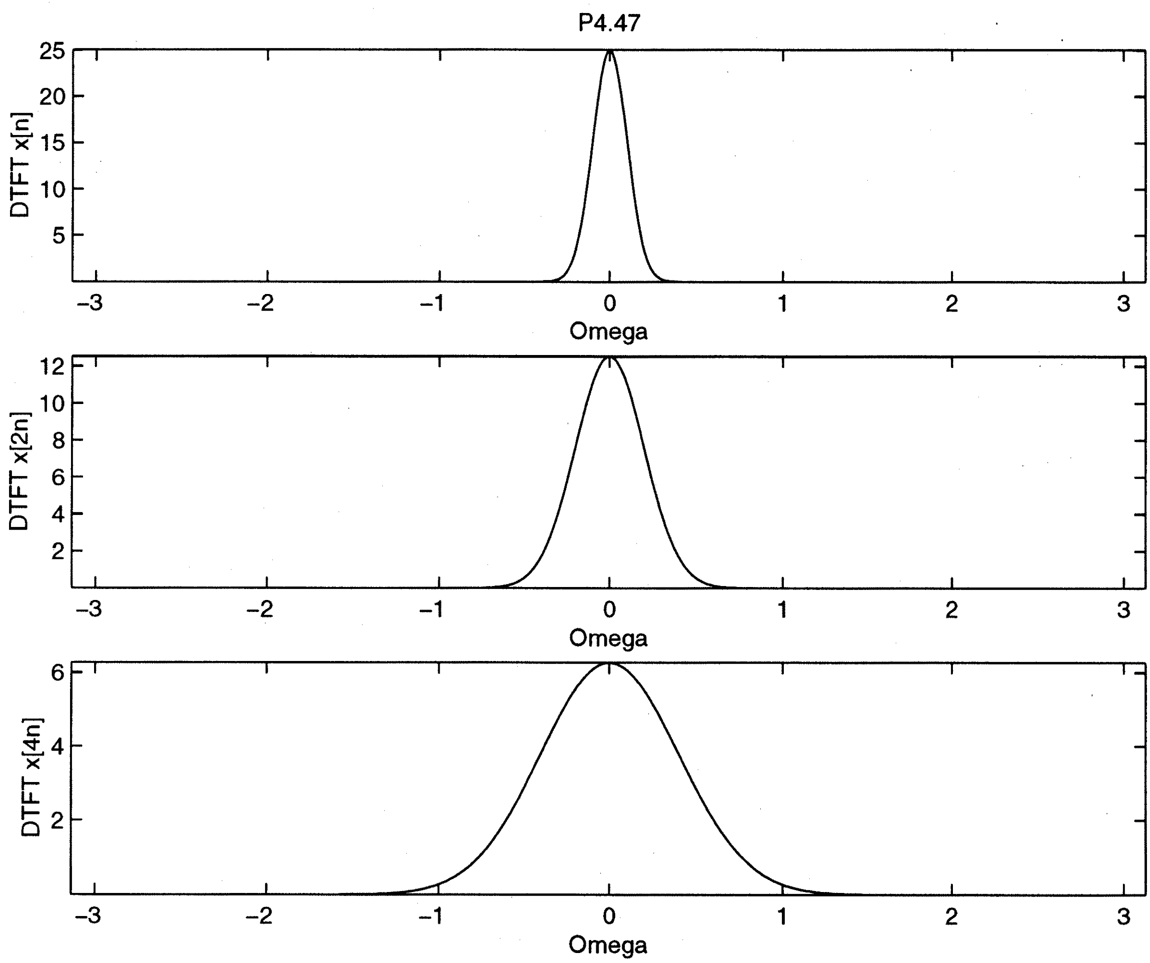
- Plot 5 of 5

P4.46 (e)



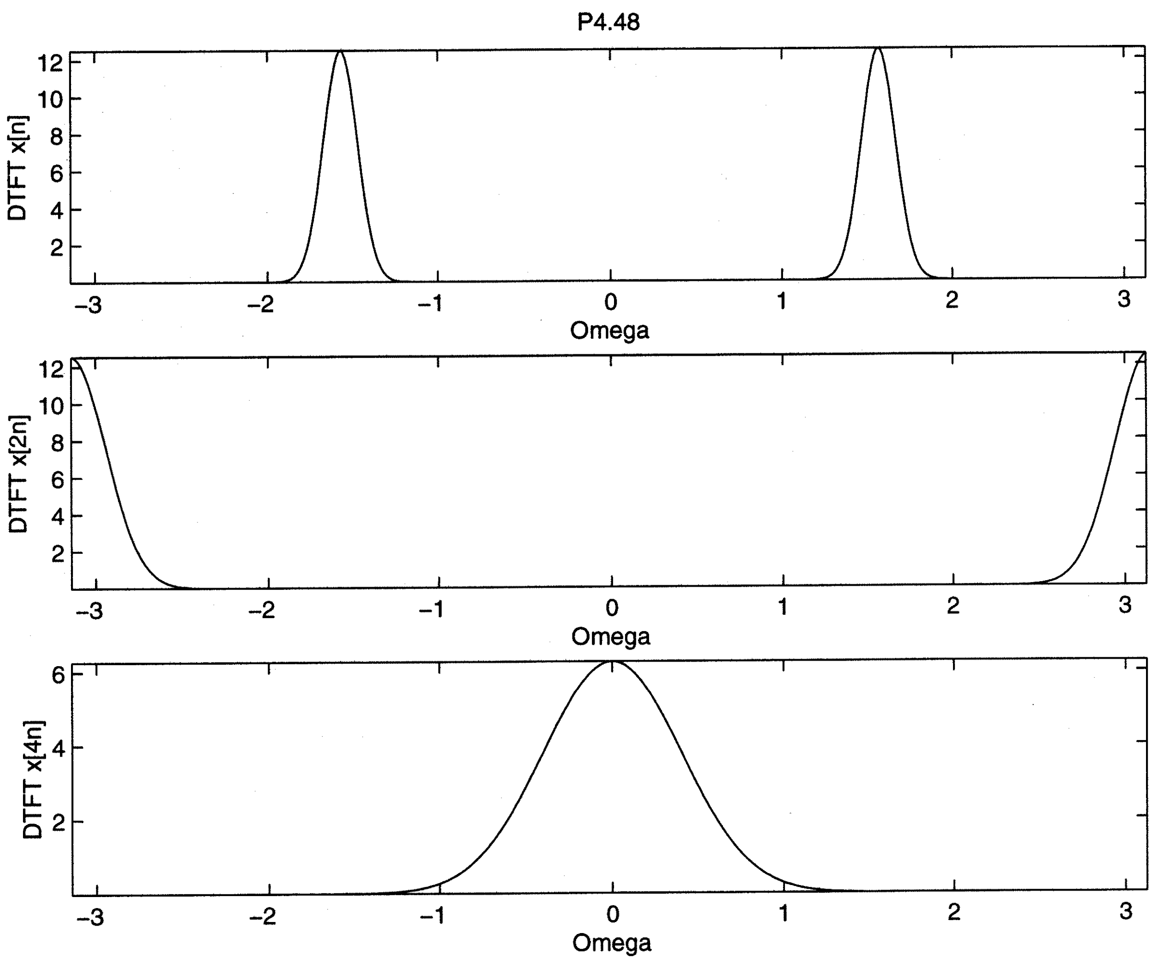
P 4.47

– Plot 1 of 1 –



P 4.48

– Plot 1 of 1 –



$\boxed{4.49}$

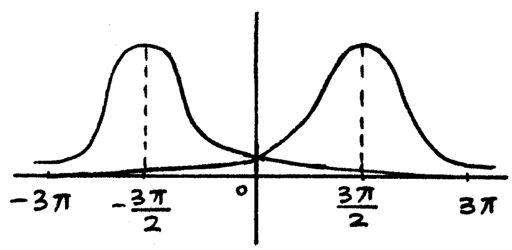
(a) $x(t) = \cos\left(\frac{3\pi}{2}t\right) e^{-t^2/2}$

$$X(j\omega) = \pi\left(\delta\left(\omega - \tfrac{3\pi}{2}\right) + \delta\left(\omega + \tfrac{3\pi}{2}\right)\right) * e^{-\omega^2/2}\sqrt{2\pi}$$

$$= \sqrt{2\pi^3}\left(e^{\frac{-(\omega - \frac{3\pi}{2})^2}{2}} + e^{-\frac{(\omega + \frac{3\pi}{2})^2}{2}}\right)$$

For $\underline{|\omega| > 3\pi}$, $|X(j\omega)| = \sqrt{2\pi^3}\left|e^{\frac{-(\omega - \frac{3\pi}{2})^2}{2}} + e^{-\frac{(\omega + \frac{3\pi}{2})^2}{2}}\right|$

$$|X(j\omega)| \leq \sqrt{2\pi^3}\left(e^{-\frac{(\frac{3\pi}{2})^2}{2}} + e^{-\frac{(\frac{9\pi}{2})^2}{2}}\right) \cong \underbrace{1.2 \times 10^{-4}}_{\text{negligible}}$$



(b), (c), (d) :

$$x[n] = \cos\left(\tfrac{3\pi}{2} nT\right) e^{-\frac{(nT)^2}{2}}$$

Notice that $x[n]$ is symmetric with respect to n, so

$$X_\delta(j\omega) = x[0] + 2\sum_{n=1}^{250} x[n]\cos(\omega . n . T)$$

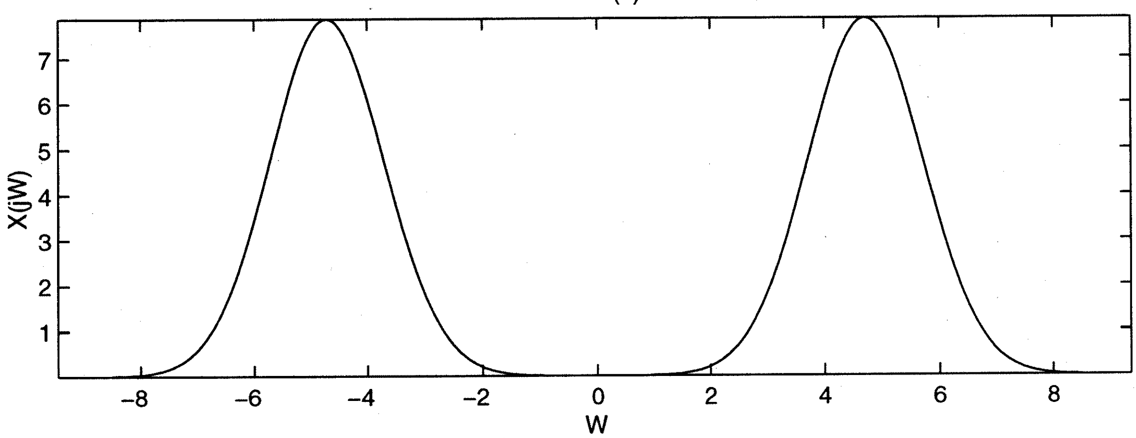
$$= 1 + 2\sum_{n=1}^{250} x[n]\cos(\omega n T)$$

Aside from the magnitude difference by $\frac{1}{T}$, $X(j\omega)$ and $X_\delta(j\omega)$ for each T are different within $[-3\pi, 3\pi]$ only when the sampling period T is big enough such that <u>significant</u> (noticeable) alliasing occurs. It is obvious that the worst alliasing occurs for $T = \frac{1}{2}$

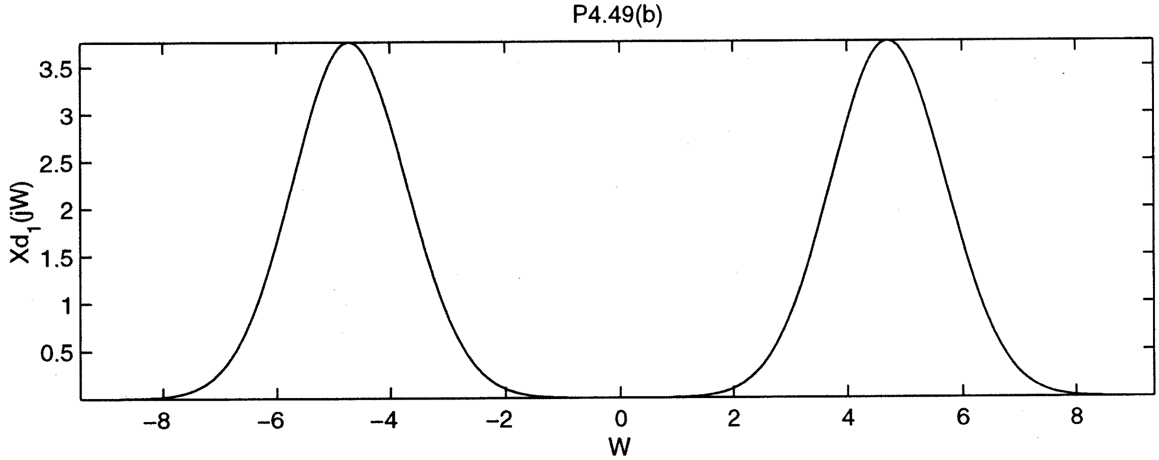
P 4.49

- Plot 1 of 2 -

P4.49(a)

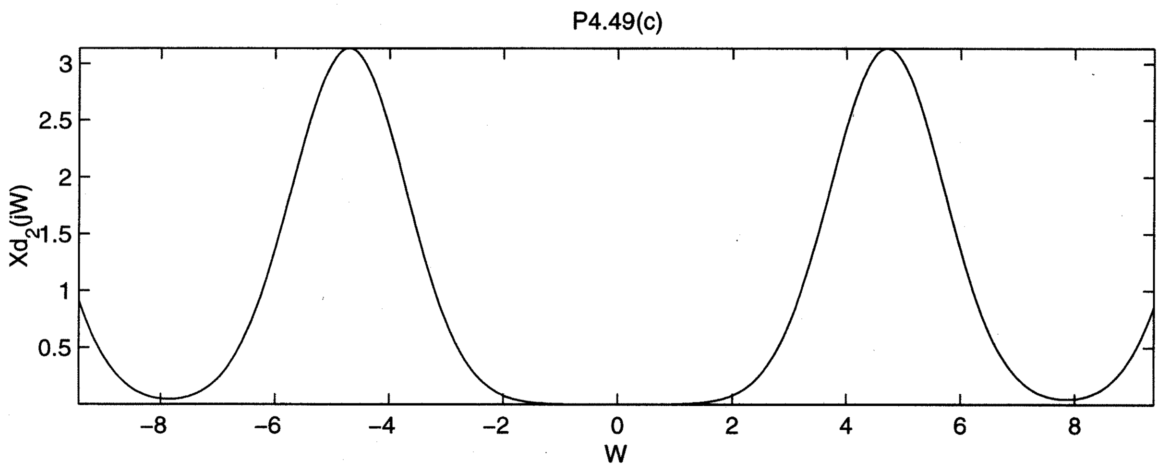


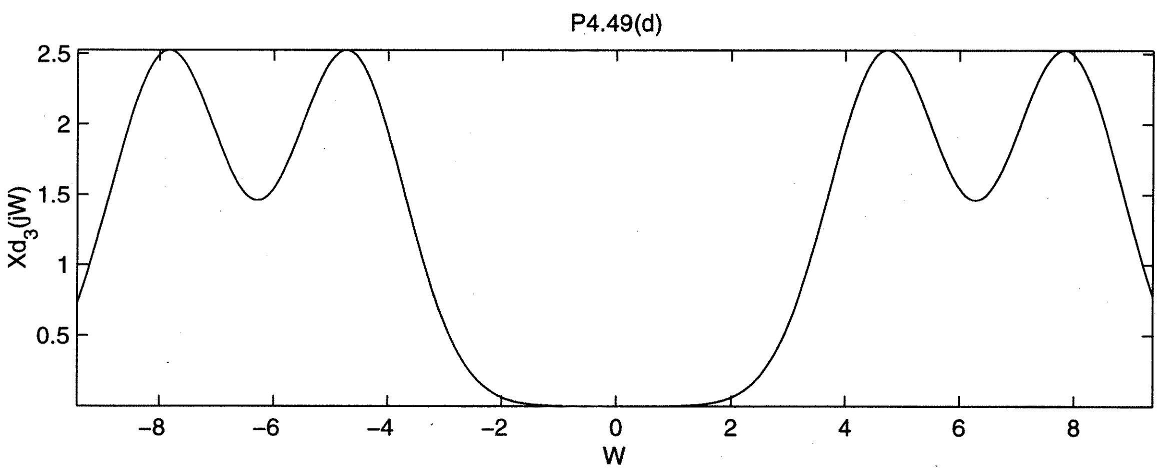
P4.49(b)



P 4.49

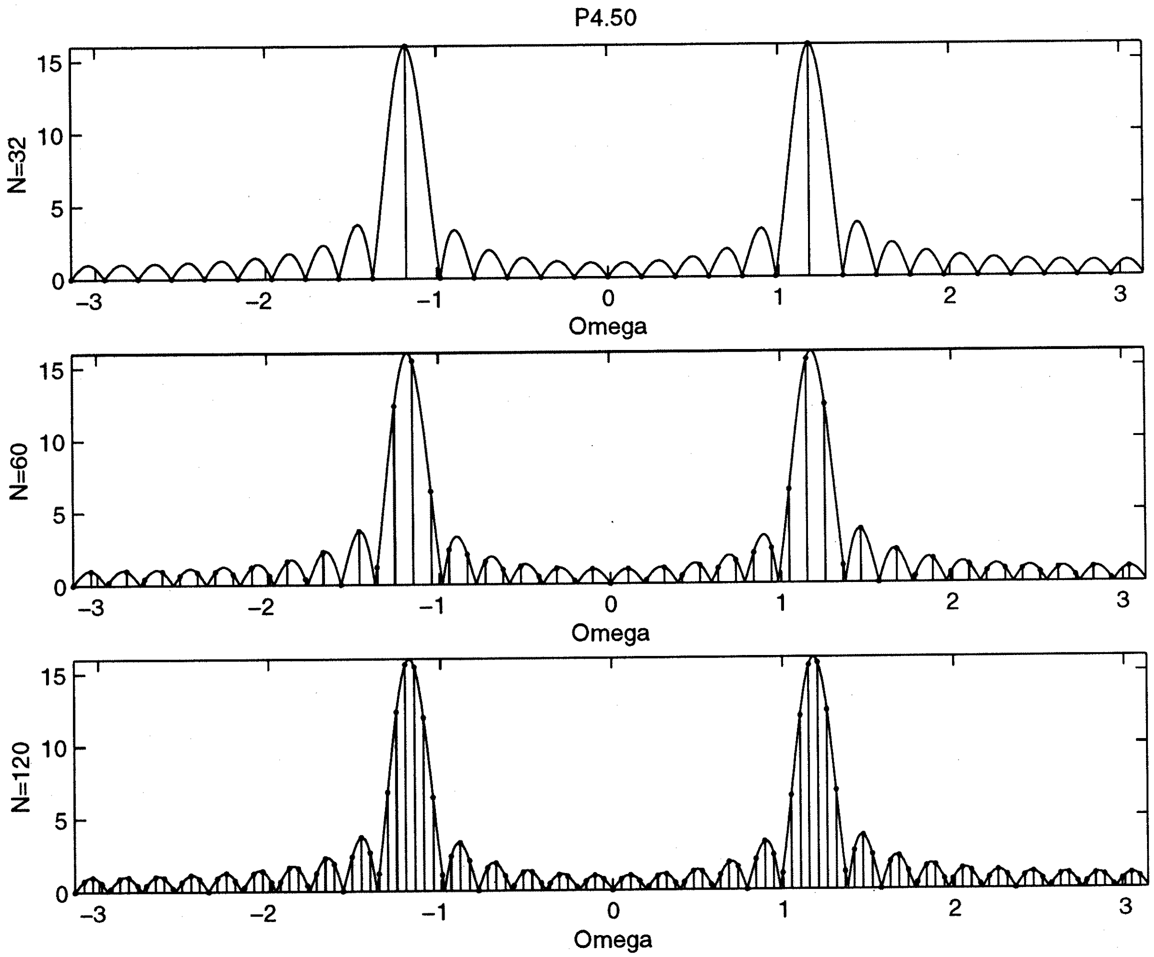
- Plot 2 of 2 -





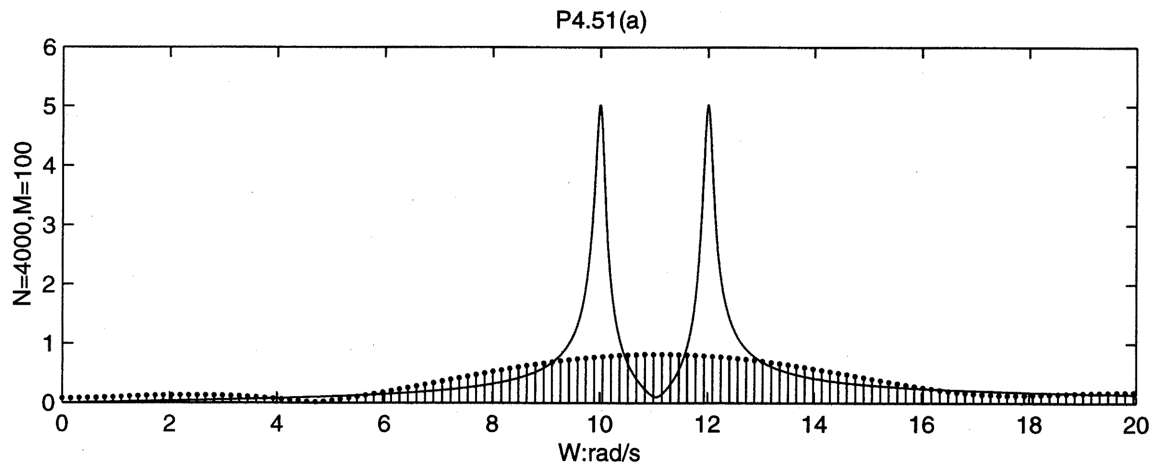
P 4.50

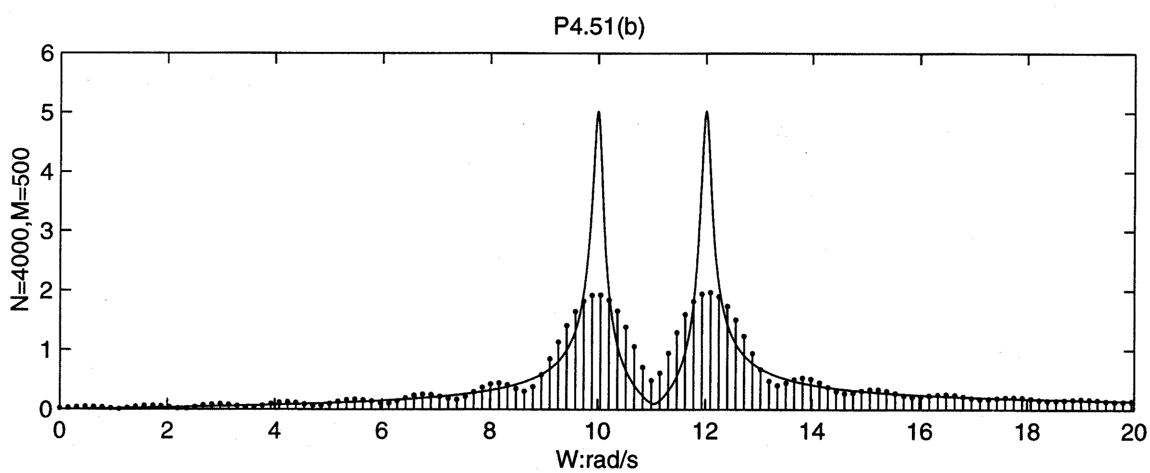
- Plot 1 of 1 -



P 4.51

- Plot 1 of 2 -

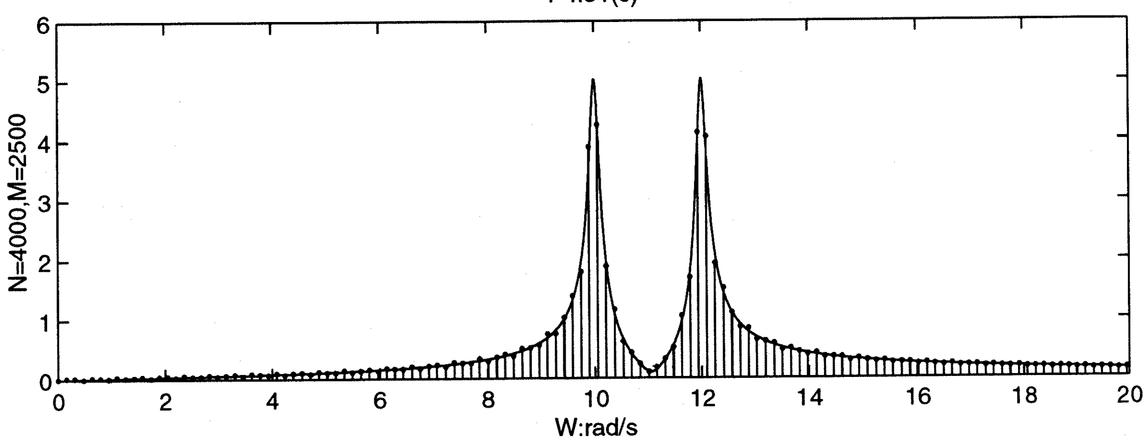




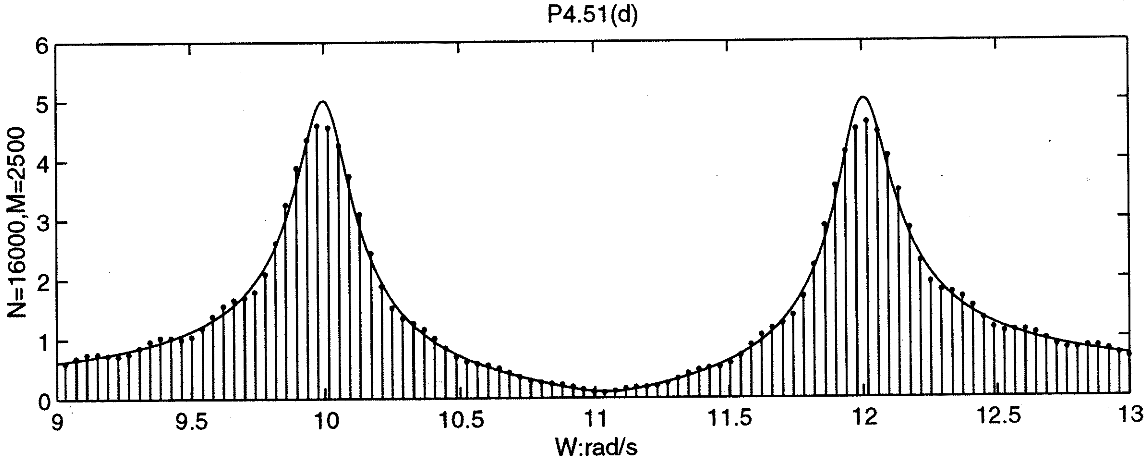
P 4.51

– Plot 2 of 2 –

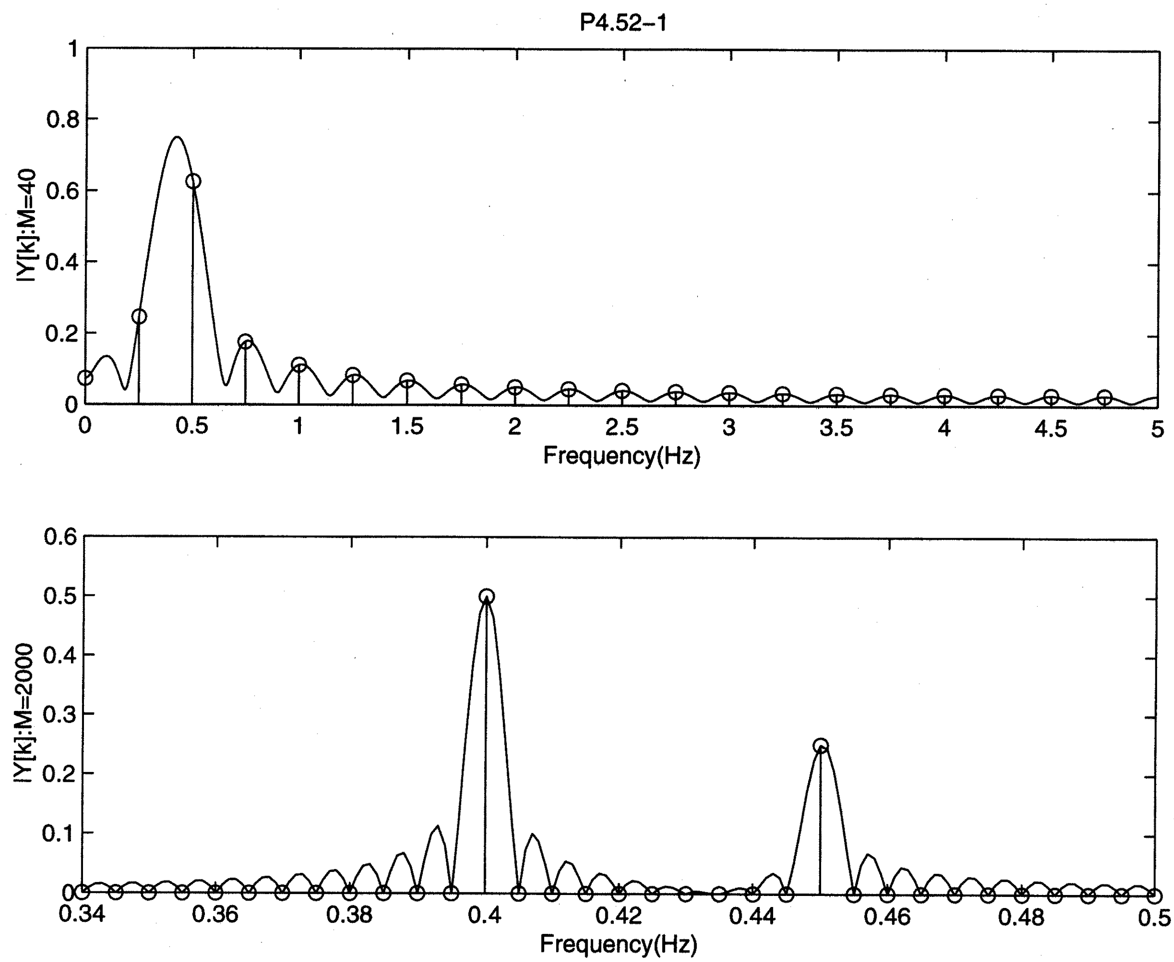
P4.51(c)



P4.51(d)

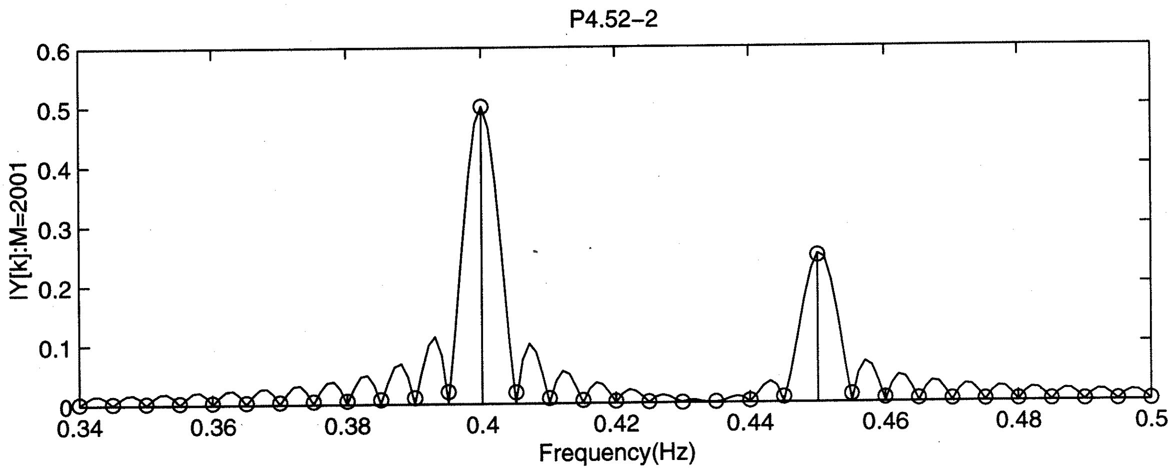


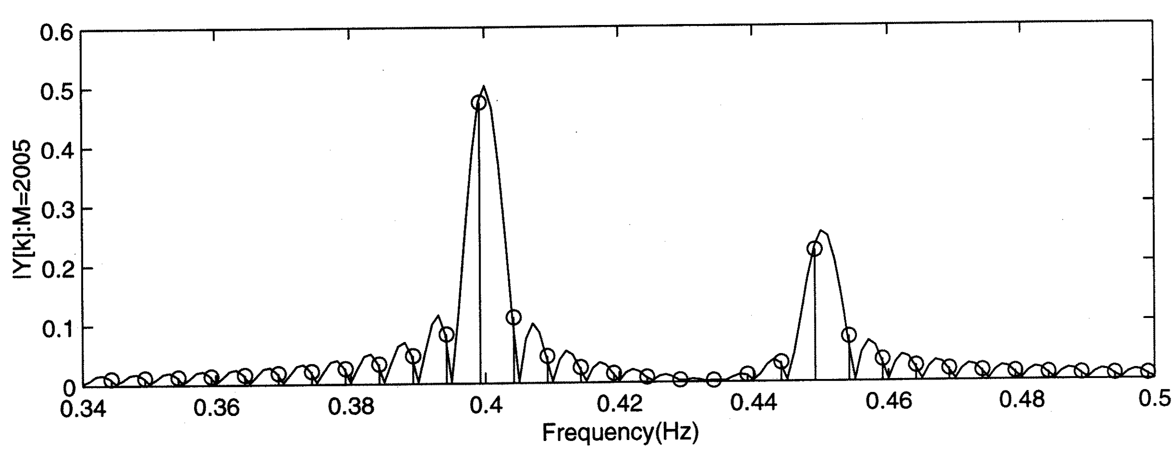
P 4.52
– Plot 1 of 3 –



P 4.52

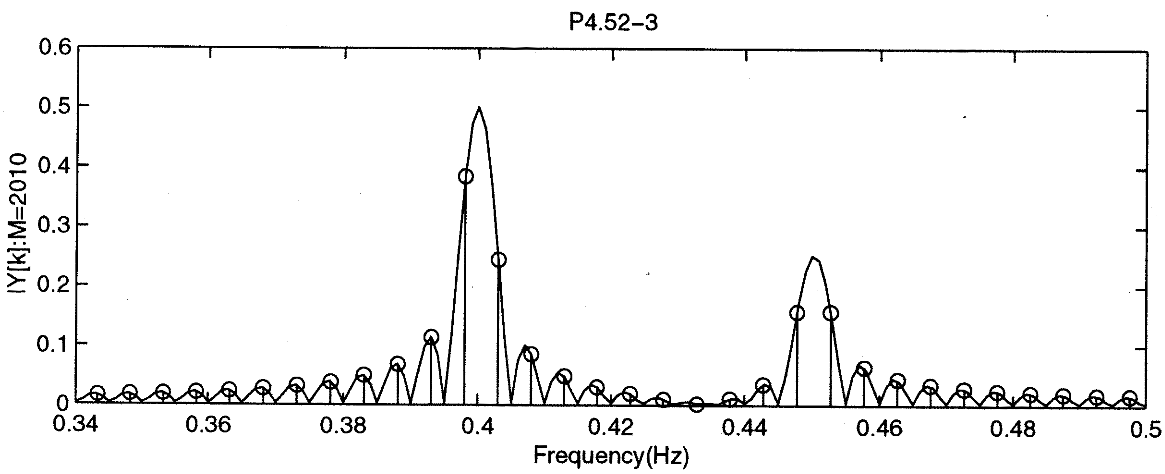
- Plot 2 of 3 -





P 4.52

- Plot 3 of 3 -



$\boxed{4.53}$

(a) $\omega_a = 5\pi \longrightarrow T < \dfrac{2\pi}{3\omega_a} = 0.133$, choose $\boxed{T = 0.1}$

(b) For a given M_0, the frequency interval for DFT $= \dfrac{2\pi}{M_0}$, we want

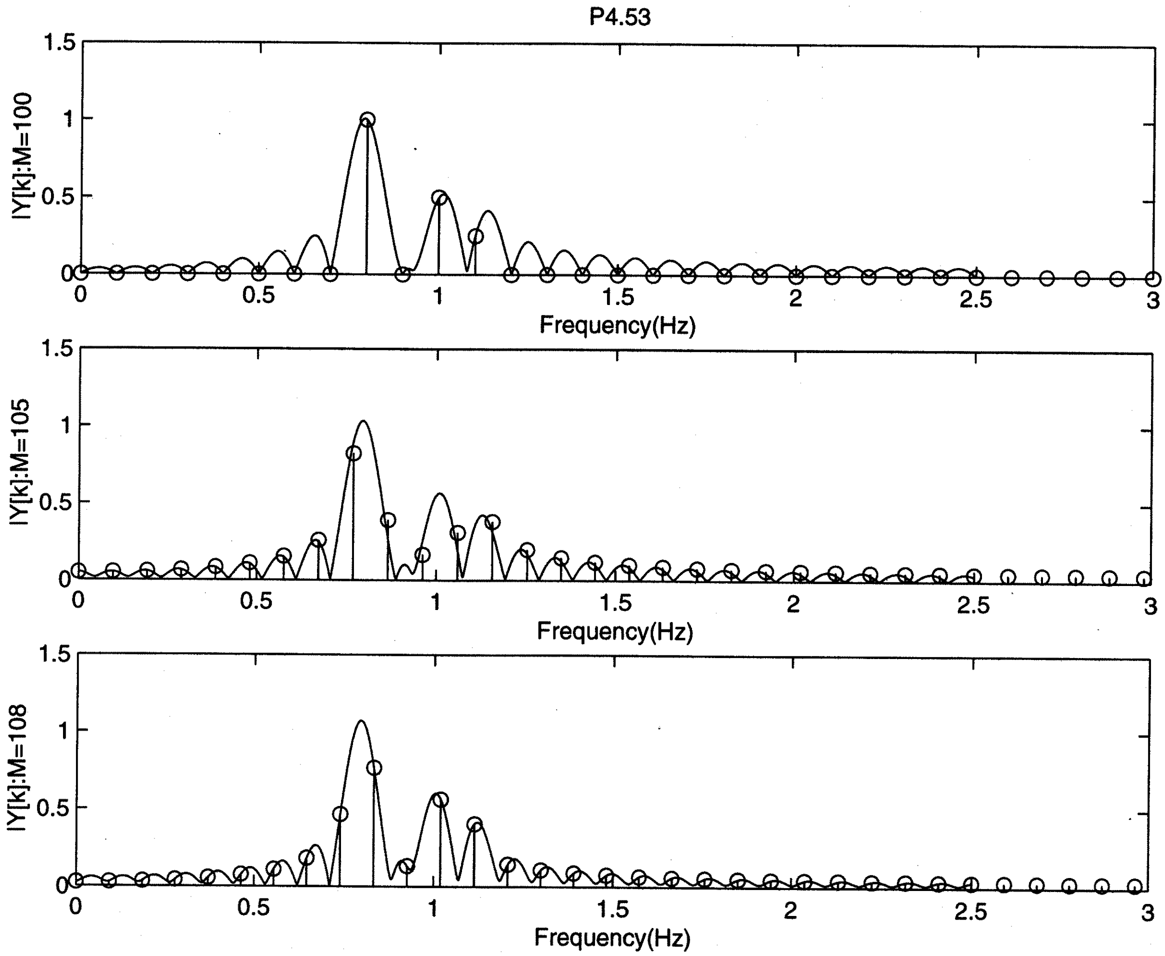
$$\left.\begin{aligned} \frac{2\pi}{M_0} \cdot k_1 &= 2\pi \cdot T \quad \text{and} \\ \frac{2\pi}{M_0} \cdot k_2 &= 2\pi (0.8) T \quad \text{and} \\ \frac{2\pi}{M_0} \cdot k_3 &= 2\pi (1.1) T \end{aligned}\right\} \quad \begin{aligned} M_0 &= \frac{k_1}{T} \quad \text{and} \\ M_0 &= \frac{k_2}{0.8T} \quad \text{and} \\ M_0 &= \frac{k_3}{1.1T} \end{aligned}$$

k_1, k_2, k_3 integers

By choosing $T = 0.1$, the minimum $\boxed{M_0 = 100}$ with $k_1 = 10$, $k_2 = 8$, $k_3 = 11$

P 4.53

- Plot 1 of 1 -



$\boxed{4.54}$ We use $T < \frac{2\pi}{\omega_m + \omega_a}$; $M \geqslant \frac{\omega_s}{\omega_r}$; $N > \frac{\omega_s}{\Delta\omega}$

to find the required T, M, N for part (a) and (b)

(a) $X(j\omega) = \frac{2\sin\omega}{\omega}$; want $\left|\frac{\sin\omega}{\omega}\right| \leqslant \frac{1}{15\pi}$ for $|\omega| \geqslant \omega_m$

, gives $\omega_m = 15\pi$

Hence, $T < \frac{2\pi}{15\pi + \frac{3\pi}{2}} \cong 0.12$, choose $T = 0.1$

$M \geqslant 26.67$, choose $M = 28$
$N \geqslant 160$, choose $N = 160$

(b) $X(j\omega) = \frac{1}{\sqrt{2\pi}} e^{-\frac{\omega^2}{2}}$; want $e^{-\frac{\omega^2}{2}} \leqslant \frac{1}{10} e^{-9/2}$,

gives $\omega_m \cong 3.69$
Hence, $T < 0.939$, choose $T = 0.9$
$M \geqslant 3.49$, choose $M = 4$
$N \geqslant 55.85$, choose $N = 56$

(c) $\omega_s = 2\omega_a = 80\pi$; $X(j\omega) = \pi(\delta(\omega - 20\pi) + \delta(\omega + 20\pi))$
Hence $T < \frac{2\pi}{3\omega_a} = 0.0167$, choose $T = 0.01$

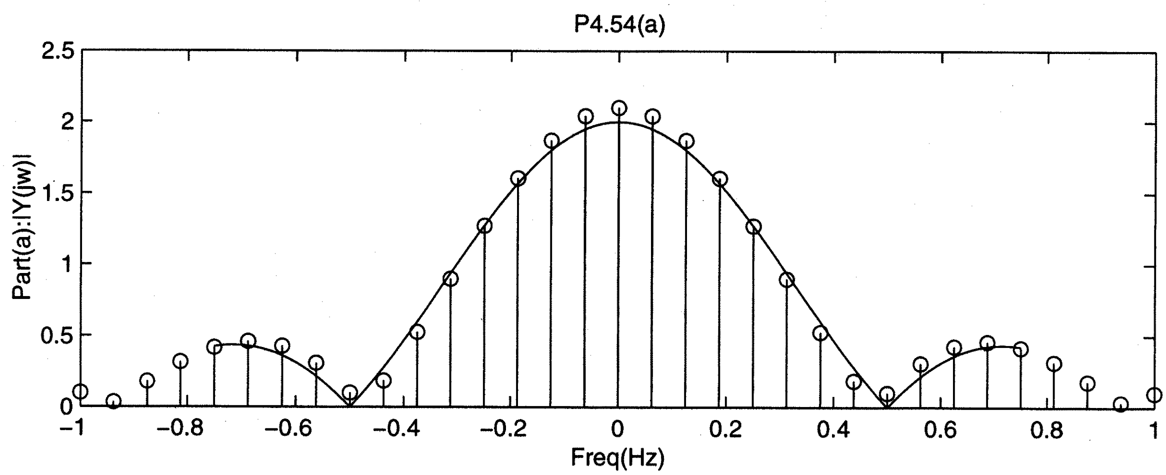
$M \geqslant 600$, choose $M = 600 = N$

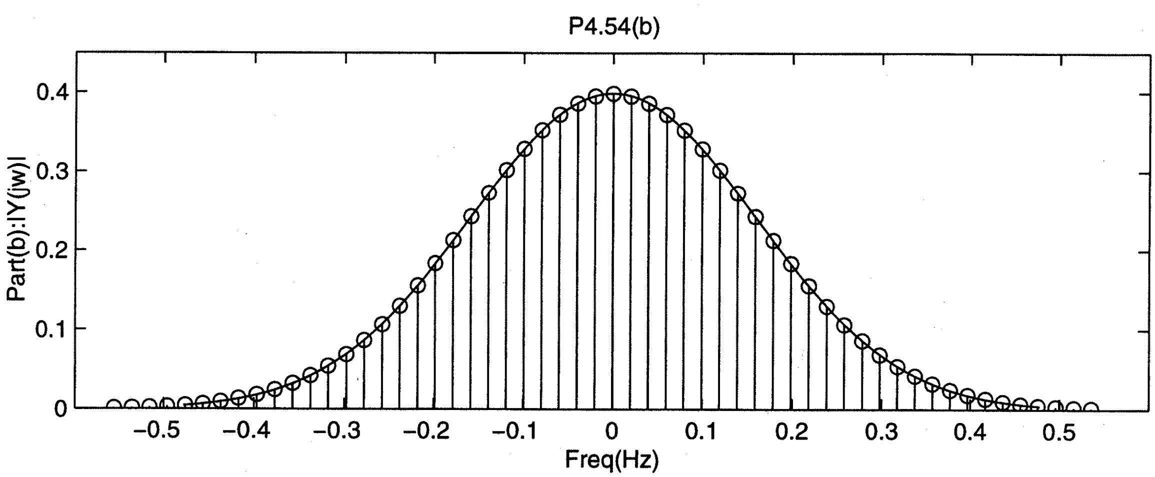
(d) $T = 0.01$, $M \geqslant 2000$, choose $M = N = 2000$
same as (c)

For (c) and (d), need to consider $X(j\omega) * W_\delta(j\omega)$
, where
$$W_\delta(j\omega) = e^{-j\omega T \frac{M-1}{2}} \frac{\sin\left(M\frac{\omega T}{2}\right)}{\sin\left(\omega\frac{T}{2}\right)}$$

P 4.54

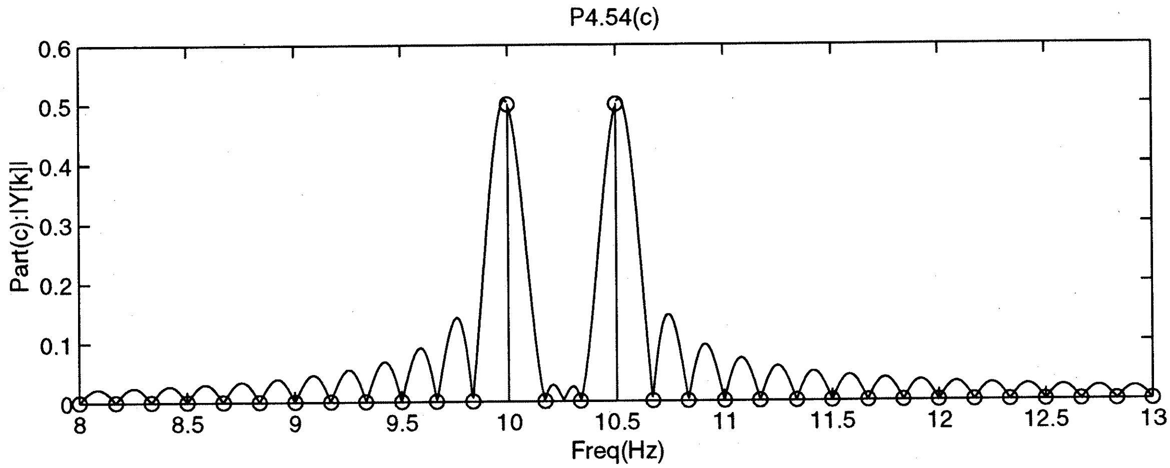
- Plot 1 of 2 -

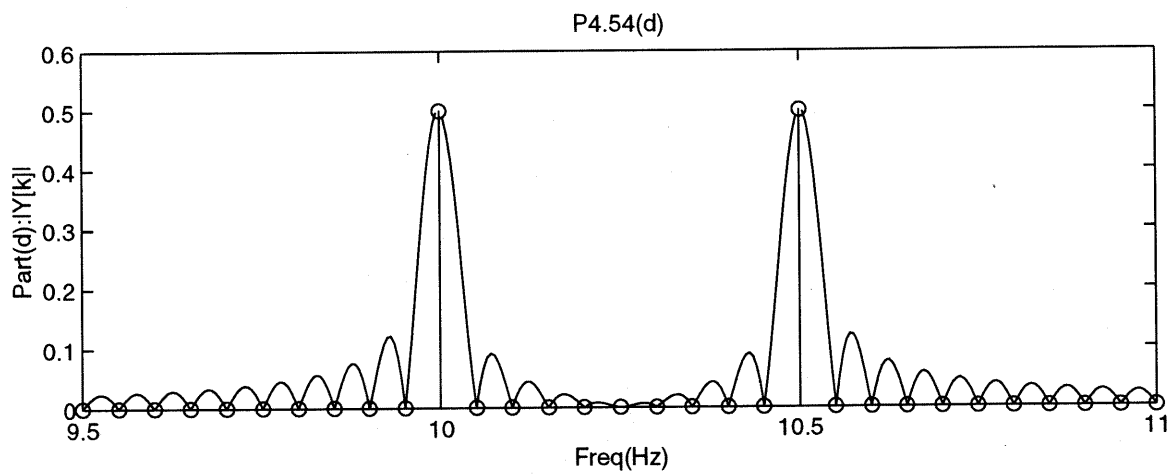




P 4.54

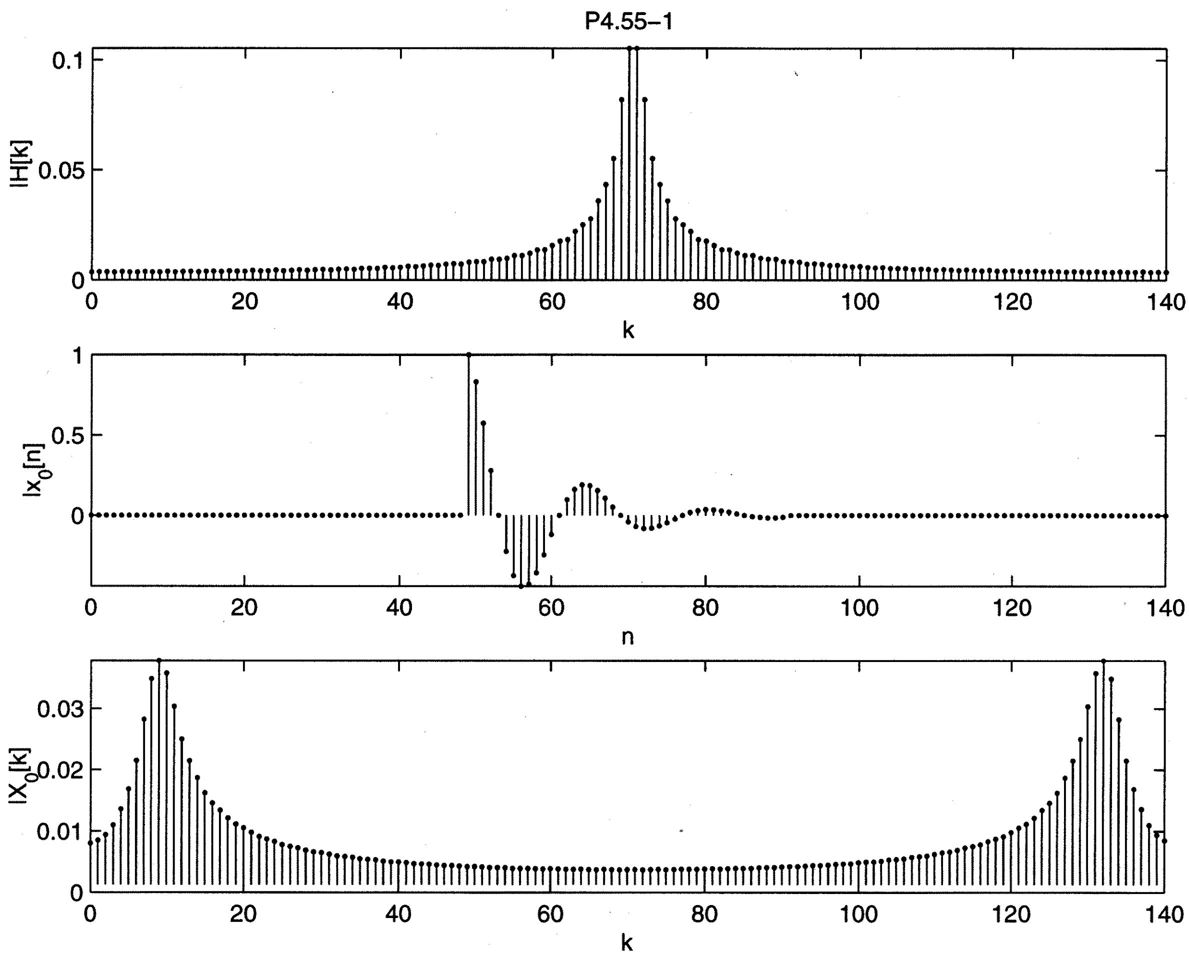
– Plot 2 of 2 –





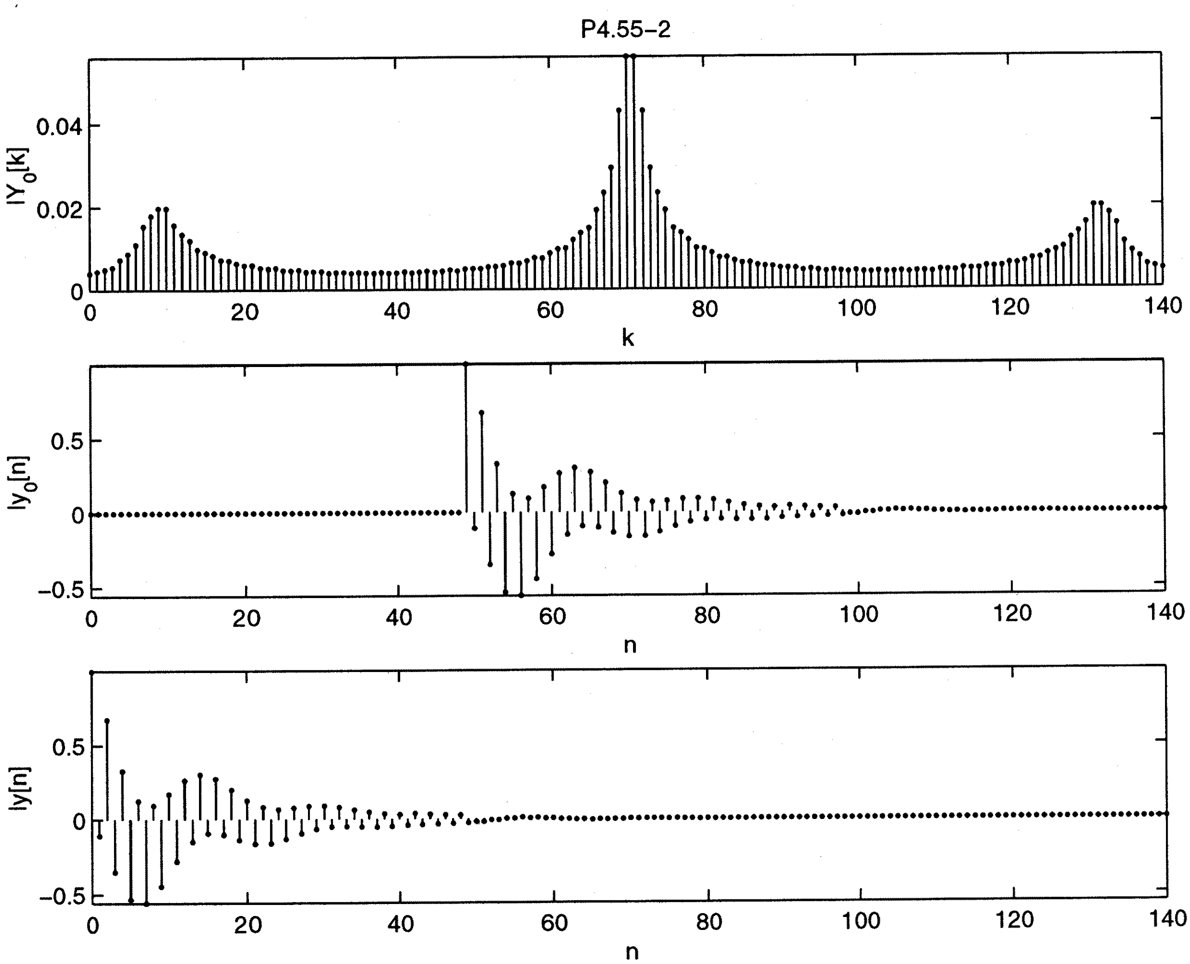
P 4.55

- Plot 1 of 2 -



P 4.55

- Plot 2 of 2 -

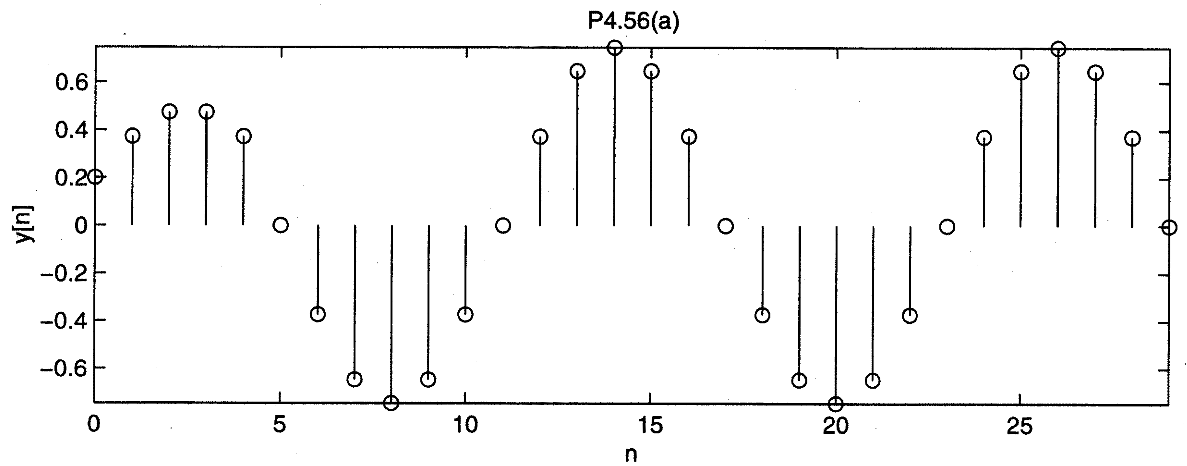


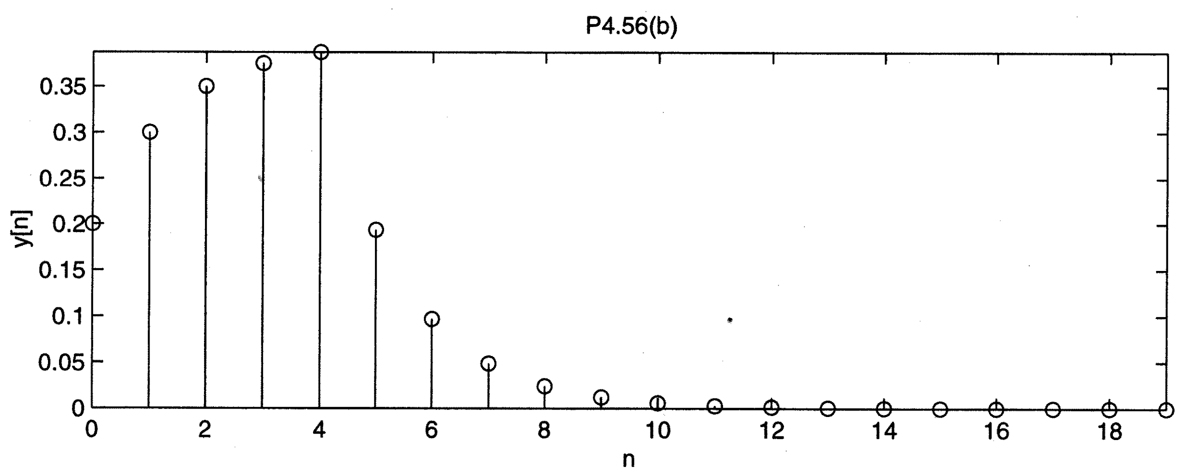
$\boxed{4.56}$

(a) $M = 5$, $L = 30$, $N = 34$

(b) $M = 5$, $L = 20$, $N = 24$

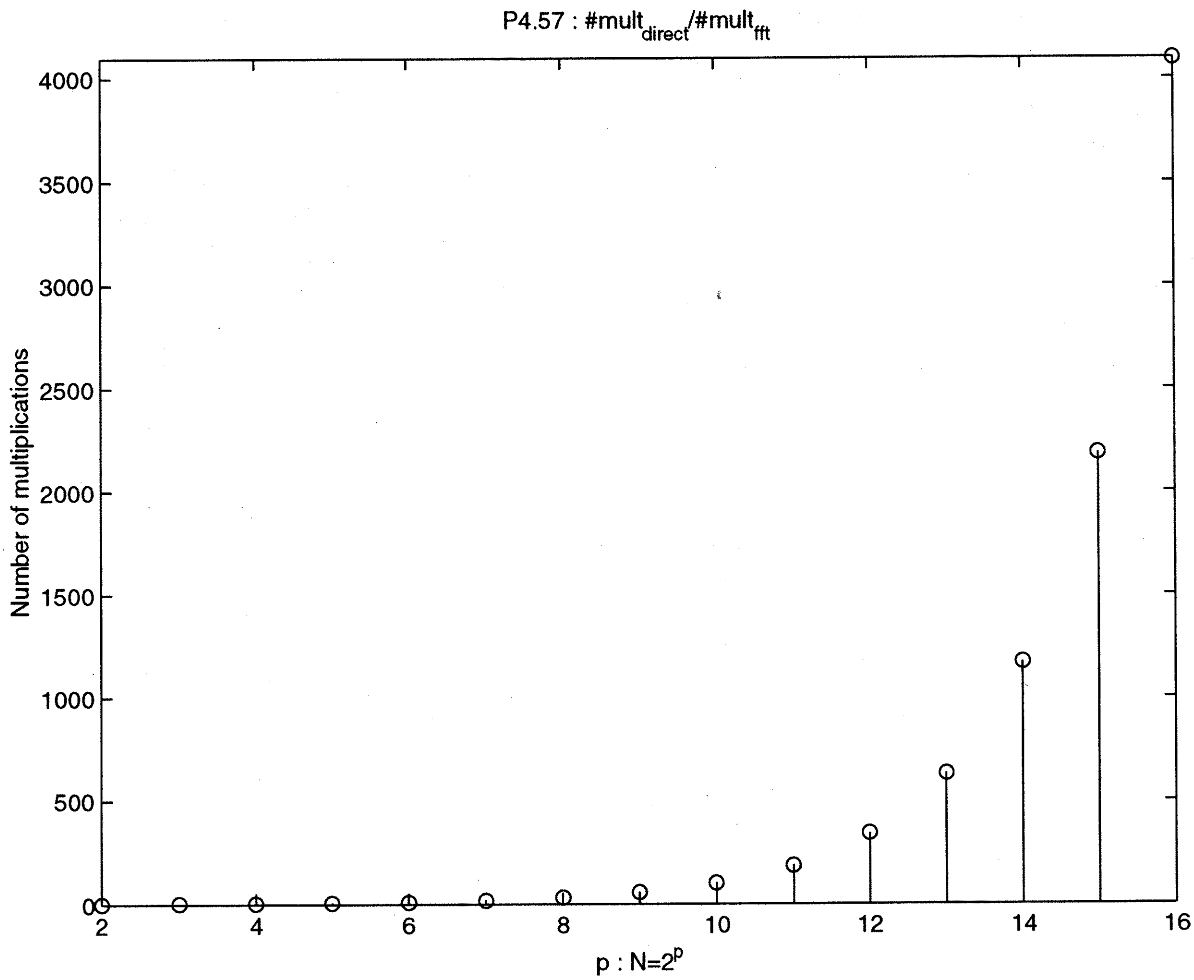
– Plot 1 of 1 –





P 4.57

- Plot 1 of 1 -



4.58

(a) direct method : $y[n] = \sum_{k=0}^{M-1} h[k] \cdot x[n-k]$,

obviously needs $(M-1)$ additions and M multiplications per output point

(b) overlap and save method : Assume $N = 2^k$

step 4 : need $N \log_2 N$ multiplications

5 : N multiplications

6 : $N \log_2 N$ multiplications

since each operation from step 1 - 7 computes the linear convolution of $(N-M+1)$ points of $y[n]$, the number of multiplication per output point needed $= \dfrac{2N\log_2 N + N}{N-M+1}$

(d) For $N \gg M$, $\dfrac{2N\log_2 N + N}{N-M+1} \approx \dfrac{N(2\log_2 N + 1)}{N}$

$= 2\log_2 N + 1$

If $M > 2\log_2 N + 1$, overlap and save algorithm requires fewer multiplications than direct method.

```
P4.58 :
=======

Part (i) :
-----------
M =
    10

N =
       16256        1024

Ratio =
    2.8993    2.1186


Part (ii) :
------------
M =
    20

Ratio =
    1.5486    1.0699

Part (iii) :
-------------
M =
   100

N =
       128256        1024

Ratio =
    0.3496    0.2325
```

$\boxed{4.59}$

(a) By setting $t_n = n\tau$, $dt \approx \Delta t = \tau$, so,

$$\int_{-\infty}^{\infty} t^2 |x(t)|^2 dt \approx \sum_{n=-M}^{M} (n\tau)^2 |x(n\tau)|^2 \tau$$

$$= \tau^3 \sum_{n=-M}^{M} n^2 |x[n]|^2$$

similarly $\int_{-\infty}^{\infty} |x(t)|^2 dt \approx \tau \sum_{n=-M}^{M} |x[n]|^2$

Hence, $T_d \approx \tau \left[\dfrac{\sum_{n=-M}^{M} n^2 |x[n]|^2}{\sum_{n=-M}^{M} |x[n]|^2} \right]^{1/2}$

(b) Using $(2M+1)$ - point DTFS approximation, we have

$$\omega_k = k \cdot \frac{\omega_s}{2M+1}, \text{ hence } d\omega \approx \Delta\omega = \frac{\omega_s}{2M+1}, \text{ and}$$

$$X[k] \approx \frac{1}{(2M+1)\tau} X\left(jk\frac{\omega_s}{2M+1}\right)$$

Hence, $\int_{-\infty}^{\infty} \omega^2 |X(j\omega)|^2 d\omega = \left(\dfrac{\omega_s}{2M+1}\right)^3 \sum_{k=-M}^{M} k^2 |X[k]|^2,$

and $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega \approx \left(\dfrac{\omega_s}{2M+1}\right) \sum_{k=-M}^{M} |X[k]|^2$

Hence, $B_w \approx \dfrac{\omega_s}{2M+1} \left[\dfrac{\sum_{k=-M}^{M} k^2 |X[k]|^2}{\sum_{k=-M}^{M} |X[k]|^2} \right]^{\frac{1}{2}}$

(c) Since $T_d - B_w \geq \frac{1}{2}$ and $\tau \cdot \dfrac{\omega_s}{2M+1} = \dfrac{2\pi}{2M+1}$, we have,

$$\left[\frac{\sum_{n=-M}^{M} n^2 |x[n]|^2}{\sum_{n=-M}^{M} |x[n]|^2}\right]^{\frac{1}{2}} \cdot \left[\frac{\sum_{k=-M}^{M} k^2 |X[k]|}{\sum_{k=-M}^{M} |X[k]|^2}\right]^{\frac{1}{2}} \geqslant \frac{2M+1}{4\pi}$$

P 4.59

- Plot 1 of 1 -

P4.59(d) : o = (2M+1)/(4pi)