Units and dB Conversion

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FIELD & POWER QUANTITIES

$\vec{E}$ is the electric field intensity [volt/meter or V/m] and $\vec{D}$ is the electric flux density [coulomb/meter^2 or C/m^2]:

$$\vec{D} = \varepsilon\vec{E} \qquad \text{with } \varepsilon = \varepsilon_r\varepsilon_0, \text{ with } \varepsilon_0 = 8.854\times10^{-12} \qquad (1)$$

$\vec{H}$ is the magnetic field intensity [ampere/meter or A/m] and $\vec{B}$ is the magnetic flux density [tesla or weber/meter^2 or T, Wb/m^2]:

$$\vec{B} = \mu\vec{H} \qquad \text{with } \mu = \mu_r\mu_0, \text{ with } \mu_0 = 4\pi10^{-7} \qquad (2)$$

and

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.7\Omega \approx 120\pi$$

In the **far-field** and **free-space**: $\vec{H} = \frac{1}{\eta}\hat{r} \times \vec{E} = \frac{1}{\eta_0}\hat{r} \times \vec{E}$, and the time-average radiated power **density** (based on the Poyinting vector for peak-valued vector phasors/sinusoidal excitations/**permanent response**) reduces to:

$$P_{radiated}\,[\text{w/m}^2] = \text{Re}\{\frac{1}{2}\vec{E} \times \vec{H}^*\} = \text{Re}\{\frac{1}{2\eta_0}\vec{E} \times \left(\hat{r} \times \vec{E}^*\right)\} =$$

$$P_{radiated} = \mathrm{Re}\{\frac{1}{2\eta_0}[\vec{E} \bullet \vec{E}^* \hat{r} - \vec{E} \bullet \hat{r}\,\hat{r}]\} = \mathrm{Re}\{\frac{1}{2\eta_0}\vec{E} \bullet \vec{E}^*\} = \frac{1}{2\eta_0}\left|\vec{E}\right|^2$$

OBS.: If working with time-varying fields, the *rms* power density does **not** contain the factor ½ ($\vec{S} = \vec{E} \times \vec{H}$, quantities are not phasors). This is also referred to as the *instantaneous* power density. Following a similar derivation

$$P_{radiated}\,[\mathrm{W/m^2}] = \vec{E} \times \vec{H} = \frac{1}{\eta_0}\left|\vec{E}\right|^2 \qquad (3)$$

where $\vec{E}$ contains explicit time-dependence (not a phasor) and is given in [V/m]. **Equation (3) is the one that is going to be used in class given that *rms* values are employed.** Note that the relationship between the peak-valued phasor and the *rms* quantity is $|\vec{E}_{\text{peak}}| = \sqrt{2}\ |\vec{E}_{\text{rms}}|$, and with that both equations yield the same result. **"In this class $|\vec{E}| = |\vec{E}_{\text{rms}}|$, unless explicitly indicated".**

Note that if an antenna with gain G is employed in the system (and remember that you may consider the receive case equal to the transmit due to reciprocity – isotropic materials), the radiated power density can be written as

$$P_{radiated}\,[\mathrm{W/m^2}] = \frac{P_{in}G}{4\pi r^2} \tag{4}$$

Where P_{in} is the input power to the antenna in [W] and r is the distance in [m]. Equaling (3) and (4):

$$\left|\vec{E}\right| = \sqrt{\frac{P_{in}G\eta_0}{4\pi r^2}} \tag{5}$$

which yields the electric field intensity in [V/m] at a distance r. Note that if you measure $\vec{E}$ with a field meter, you can use (5) to determine the antenna gain at that given direction (considering the probe truly isotropic). The ratio of two quantities in dB is given by

$$\text{POWER [dB]} = 10\log_{10}\left(\frac{P_2}{P_1}\right) \tag{6}$$

$$\text{VOLTAGE [dB]} = 20\log_{10}\left(\frac{V_2}{V_1}\right) \tag{7}$$

$$\text{CURRENT [dB]} = 20\log_{10}\left(\frac{I_2}{I_1}\right) \tag{8}$$

For example, a voltage of 1 V is 120 dBμV:

$$\text{VOLTAGE [dB}\mu V] = 20\log_{10}\left(\frac{1V}{1\mu V = 10^{-6}V}\right) = 20\log_{10} 10^{6} = 120\,\text{dB}\mu V$$