Segment 2: Randomized Studies

Section 01: Completely Randomized Experiments

Operating Question: Segment 2

Why, exactly, are randomized studies so useful for estimating causal effects, and how can I thoughtfully design them?

Why Focus on Randomized Experiments?

- Learn some basic tenets of experimental design
  - Tailor prospective study design to questions of causal inference
- "Practice" the potential outcomes paradigm
- Use analysis of randomized studies as a template for the analysis of observational studies
  - That is, studies where the assignment mechanism is unknown and not under experimenter control
- Recurring theme: Observational studies framed as (approximate) randomized experiments

Different Randomized Designs

In general, the design of a randomized experiment can be captured by an expression of the **assignment mechanism**:

$$Pr(\mathbf{Z}|\mathbf{X}, \mathbf{Y}^c, \mathbf{Y}^t) = c \prod_i p_i(Z_i|X_i, Y_i^c, Y_i^t)$$

governing how units are assigned to different treatments

- Different designs will correspond to different assignment mechanisms
- Different assignment mechanisms will have implications for statistical estimation of causal effects
- We will assume throughout binary treatments, that is, designs with two arms: "treatment arm" and "control arm"
  - $Z \in \{c, t\}$ or $Z \in \{0, 1\}$

Simplest Example: Bernoulli Trial

Random coin flip for each of n units in the trial

```
Z<- rbinom(8, 1, q)
```

Assignment mechanism:

$$Pr(\mathbf{Z}|\mathbf{X}, \mathbf{Y}^c, \mathbf{Y}^t) = q^{n_t}(1-q)^{n-n_t}$$

where n_t is the number with $Z_i = 1$

- No restriction on the possible $\mathbf{Z}$, could be any n dimensional vector of 0/1s, i.e., $\mathbf{Z} \in [0, 1]^n$
- Probability of any particular $\mathbf{Z}$ depends only on the number of treated
- What could go wrong?

Completely Randomized Experiments

The probability of being assigned treatment is the same for each unit *and the number of treated units is fixed*

```
Z<-sample(c(0,0,0,0,1,1,1,1), length = 8)
```

Assignment mechanism:

$$Pr(\mathbf{Z}|\mathbf{X}, \mathbf{Y}^c, \mathbf{Y}^t) = \binom{n}{n_t}^{-1} \text{ if } \sum_{i=1}^{n} = n_t$$

and 0 otherwise, where n is the number of units in the sample and n_t is the desired number to receive the treatment

- Probability of any $\mathbf{Z}$ depends only on the number treated
- The set of possible $\mathbf{Z}$ is: $\{\mathbf{Z}; \sum_{i=1}^{n} Z_i = n_t\}$

Ignorability and Unconfoundedness

In both cases, the assignment mechanism:

$$Pr(\mathbf{Z}|\mathbf{X}, \mathbf{Y}^c, \mathbf{Y}^t) = \binom{n}{n_t}^{-1} \text{ if } \sum_{i=1}^{n} = n_t$$

or

$$Pr(\mathbf{Z}|\mathbf{X}, \mathbf{Y}^c, \mathbf{Y}^t) = q^{n_t}(1-q)^{n-n_t}$$

does not depend on:

- Potential outcomes
  - $\Rightarrow$ *Unconfounded* or *Ignorable*
- Unit characteristics

Can formalize this through the assumption of *ignorability*

$$Z \perp\!\!\!\perp Y^c, Y^t$$

Implications of Ignorability

$$Z \perp\!\!\!\perp Y^c, Y^t$$

- Units' potential outcomes do not provide any information about their treatment assignment
  - On average, potential outcomes will be *balanced* across treatment groups
- Comparison of mean outcomes in treated/control groups will provide an **unbiased estimate** of the average causal effect, τ_{PATE} (τ_{SATE})
- Randomization "solves" the fundamental problem by creating treatment groups that are, on average, *balanced* with respect to Y^t, Y^c
  - Average outcomes in $Z = c$ units are a "good substitute" for the unobserved $Y^c | Z = t$

Reminder: Sampling Distributions

Recall: An *estimand* is an underlying quantity we wish to estimate (e.g., unknown parameter, τ_{PATE}, etc.)

- We *estimate* estimands with *estimators*

Properties of a statistical estimate are reflected in the distribution of the estimate over repeated samples

- Typically formulated as a *sampling distribution* derived from calculating the estimate in the repeated samples from a population of units
- E.g., Estimand is the mean of Y in some population, μ
- (1) take a sample of units; (2) calculate the sample mean of Y ($\hat{\mu}$); (3) repeat

Sampling Distributions

Unbiasedness

An estimate is **unbiased** if the mean of the sampling distribution is equal to the true value of the estimand

- E.g., $E[\hat{\mu}] = \mu$

An esitmate is *efficient* if the sampling distribution has small variance

Unbiased Estimation of Causal Effects

In completely randomized experiments

Estimand: $\tau_{PATE} = \frac{1}{N}\sum_{i=1}^{N}(Y_i^t - Y_i^c) = E[Y^t - Y^c] = E[Y^t] - E[Y^c]$

Estimator: $\hat{\tau} = \left(\frac{1}{n_t}\sum Y_i - \frac{1}{n-n_t}\sum Y_i\right) = E[Y|Z=t] - E[Y|Z=c]$

Ignorability: $Z \perp\!\!\!\perp Y^c, Y^t$

Average *observed* outcomes for those assigned $Z = t$ is equal to the average of Y^t in the sample/population

Implications of Ignorability

In completely randomized designs, Z is a random variable that is independent of the potential outcomes

- Under *repeated sampling/randomization* there will be no differences, *on average*, in the potential outcomes between treated and control groups
  - Comparison groups should be balanced, *on average* across repeated randomizations
  - Imbalance may exist in any *single* sample/randomization
- Independence between Z and X too
- Motivates "checking" that variables are balanced to see whether the realized randomization was any good

Example: Hypothetical Dietary Experiment

Observed Data

Table: Observed Data from the Hypothetical Dietary Experiment, **Idealized Assignment**

Unit, i	Female, x_{1i}	Age, x_{2i}	Treatment Z_i	Potential Y_i^c	Potential Y_i^t
Audrey	1	40	0	140	135
Anna	1	40	1	140	135
Bob	0	50	0	150	140
Bill	0	50	1	150	140
Caitlin	1	60	0	160	155
Cara	1	60	1	160	155
Dave	0	70	0	170	160
Doug	0	70	1	170	160

Example: Hypothetical Dietary Experiment
Observed Data

Table: Observed Data from the Hypothetical Dietary Experiment, **Unlucky Assignment**

Unit, i	Female, x_{1i}	Age, x_{2i}	Treatment Z_i	Potential Y_i^c	Potential Y_i^t
Audrey	1	40	1	140	135
Anna	1	40	1	140	135
Bob	0	50	1	150	140
Bill	0	50	0	150	140
Caitlin	1	60	0	160	155
Cara	1	60	0	160	155
Dave	0	70	0	170	160
Doug	0	70	1	170	160