Segment 2: Randomized Studies

Section 03: Blocked and Paired Designs

Motivating Idea: Reducing "Unlucky" Assignments

- Randomized assignment will give us the "right" answer *on average*
  - Because potential outcomes will be *balanced* between groups, on average
  - Covariates, too
- Does not guarantee the estimate for any particular random assignment will be close to the truth
  - "Unlucky" assignment
- Reducing the chances of "unlucky" randomizations $\rightarrow$ decrease the variance of effect estimates
- The comparison between Bernoulli randomization and the Completely Randomized Design is one example

Example: Hypothetical Dietary Experiment

Observed Data

Table: Observed Data from the Hypothetical Dietary Experiment, **Idealized Assignment**

Unit, i	Female, x_{1i}	Age, x_{2i}	Treatment Z_i	Potential Y_i^c	Potential Y_i^t
Audrey	1	40	0	140	135
Anna	1	40	1	140	135
Bob	0	50	0	150	140
Bill	0	50	1	150	140
Caitlin	1	60	0	160	155
Cara	1	60	1	160	155
Dave	0	70	0	170	160
Doug	0	70	1	170	160

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Table: Observed Data from the Hypothetical Dietary Experiment, **Unlucky Assignment**

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Question: Is there anything we can do in our randomized study design to reduce the chance of this type of unlucky imbalance?

Blocking in Design

Key Idea: If there are certain types of units with the same potential outcomes, randomize (possibly differently) within these types of units (i.e., within *blocks*)

Blocking in Design

Key Idea: If there are certain types of units with the same potential outcomes, randomize (possibly differently) within these types of units (i.e., within *blocks*)

- We will never know the potential outcomes

$\Rightarrow$ determine *randomization blocks* based on observed covariates, $\mathbf{X}$

  - E.g., if we expect sex and age to be important in determining blood pressure, consider sex/age strata as *blocking factors* to ensure equal numbers in each treatment group
  - Guarantee that all randomizations will be *balanced* with respect to these factors

- Result will be a randomization distribution that *only* considers randomizations that are balanced with respect to these factors

$\Rightarrow$ Fewer "unlucky" randomizations, less uncertainty in the inference

Block Randomized Experiment

- Divide units into J blocks based on covariates
  - $B_i = B(\mathbf{X}_i) \in \{1, 2, \ldots, J\}$
- Conduct a completely randomized experiment in each block:

$$Pr(\mathbf{Z}|\mathbf{X}, \mathbf{Y}^c, \mathbf{Y}^t) = \prod_{j=1}^{J} \binom{n(j)}{n_t(j)}^{-1}$$

- Set of possible $\mathbf{Z}$ is: $\{\mathbf{Z}; \sum_{i:B_i=j}^{n} Z_i = n_t(j)$ for $j = 1, 2, \ldots, J\}$
- Randomization depends on $\mathbf{X}$ through definition of blocks

Ignorability and Unconfoundedness

For a block-randomized experiment, the assignment mechanism:

$$Pr(\mathbf{Z}|\mathbf{X}, \mathbf{Y}^c, \mathbf{Y}^t) = \prod_{j=1}^{J} \binom{n(j)}{n_t(j)}^{-1}$$

Depends (overall) on:

- Unit characteristics, $\mathbf{X}$
- Potential outcomes

$\Rightarrow$ Not unconfounded!

Block-randomized design is only unconfounded *within* blocks
(just a completely-randomized design within each bock)

Formalized with the assumption of *conditional ignorability*

Implications of (Conditional) Ignorability

$$Z \perp\!\!\!\perp Y^c, Y^t | \mathbf{X}$$

Within blocks (or strata) defined by $\mathbf{X}$,

- Observed covariates provide information about treatment assignment
- Within values of $\mathbf{X}$, potential outcomes do not
- Within values of $\mathbf{X}$, comparison of observed outcomes in treated/control groups will provide an **unbiased estimate** of the conditional average causal effect, $\tau_{CATE|X}$
- An unbaised estimate of τ_{SATE} can be obtained by averaging block-specific estimates

Key Implication: The analysis must "adjust" for X in order to provide unbiased estimation of τ_{PATE}

- A weighted average of block-specific estimates
- A regression that adjusts for indicator of block

Example: Hypothetical Dietary Experiment

Now with more units!

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Unit, i	Female, x_{1i}	Age, x_{2i}	Treatment Z_i	Potential Y_i^c	Potential Y_i^t
Audrey	1	40	0	140	135
Abigail	1	40	0	140	135
Arielle	1	40	0	140	135
Anna	1	40	1	140	135
Bob	0	50	0	150	140
Bill	0	50	0	150	140
Burt	0	50	0	150	140
Brad	0	50	1	150	140
Caitlin	1	60	0	160	155
Cara	1	60	1	160	155
Cassie	1	60	1	160	155
Cindy	1	60	1	160	155
Dave	0	70	0	170	160
Doug	0	70	1	170	160
Dylan	0	70	1	170	160
Derik	0	70	1	170	160

Example: Hypothetical Dietary Experiment

Now with more units! And blocks!

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Need to Account for Design in the Analysis

If observations were randomized within blocks, *we need to take this into account when we analyze the results of the trial*

- Calculate treatment effects within each block
  - Randomized within block $\Rightarrow$ unbiased for treatment effect in that block
- Average block-specific estimates to estimates
  - Represent the effect in the sample (or population)
  - τ_{SATE} or τ_{PATE}

Examples of Blocking Factors

Blocking factors work best when they arise "naturally"

- Randomize students within blocks defined by school
  - Avoid problems where school membership is *imbalanced* across treated/control students
- Randomize products within blocks defined by product class
  - Avoid problems where a treatment group of products is dominated by products in a certain class
- Randomize surgery recovery programs within blocks defined by the type of surgery
  - Make sure there are equal numbers of pancreatic, liver, stomach cancer patients in the two treatment groups

Bottom Line: Define blocks based on covariates believed to be predictive of the outcome, but balance against practical constraints

Matched Pairs Design

Basic Idea: push the idea of blocking to the extreme $\Rightarrow$ have as many blocks as units in the treatment group

- Special case of randomized block design
- Units are arranged into pairs based on having closely-matched characteristics, $\mathbf{X}$
  - 2-unit blocks
- Randomize exactly one unit per pair to receive treatment
- **Practical limitation** is the ability to find units that are well matched based on the entire $\mathbf{X}$
- Can arise naturally when pairs present themselves "naturally"
  - E.g., twins, family members

Paired Randomized Experiment

Same as a blocked experiment where there are 2 units within each stratum and one is randomized to treatment:

$$Pr(\mathbf{Z}|\mathbf{X}, \mathbf{Y}^c, \mathbf{Y}^t) = 2^{n/2}$$

- Set of possible $\mathbf{Z}$ is: $\{\mathbf{Z}; \sum_{i:B_i=j}^{n} Z_i = 1$ for $j = 1, 2, \ldots, n/2\}$
- Extreme stratified/block experiment
- Randomization depends on $\mathbf{X}$ through definition of pairs

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Analysis of Matched Pair Designs

- Ignorability: $Z \perp\!\!\!\perp Y^c, Y^t | \mathbf{X}$
- Equal probability of treatment in each block $\Rightarrow$ adjustment for pair not required for unbiasedness
- For efficiency gains, analysis must account for pairs
- Analysis:
  - Simple difference in means:
  - Average of within-pair differences: