1. Equation of the tangent line to curve $f(x) = \dfrac{3x+1}{(4x-3)^3}$ at (1,4):

$$f' = \frac{d}{dx}(3x+1) = 3$$

$$g' = \frac{d}{dx}\left((4x-3)^3\right) = (3)(4x-3)^2(4)$$

$$f'(x) = \frac{f'g - fg'}{g^2}$$

$$= \frac{(3)(4x-3)^3 - (3x+1)(3)(4x-3)^2(4)}{\left((4x-3)^6\right)}$$

$$= \frac{3\,(4x-3) - 12\,(3x+1)}{(4x-3)^4}$$

$$= f'(1) = \frac{3(1)-12(4)}{1} = -45\ slope$$

$$= y-4 = -45(x-1)$$

$$= y = -45x+49$$

2. Equation of Tangent and Normal line to curve $f(x) = \dfrac{x+8}{\sqrt{3x+1}}$ at (0,8). Answer in slope-int.

$$f' = x\,(1)$$

$$g' = \frac{d}{dx}\left[\sqrt{3x+1}\right] = \frac{1}{2}(3x+1)^{-\frac{1}{2}}(3)$$

$$= \frac{3(3x+1)^{-\frac{1}{2}}}{2}$$

$$Tangent\ line\ slope = f'(x) = \frac{f'g - fg'}{g^2}$$

$$= \frac{(1)\left(\sqrt{3x+1}\right) - (x+8)\left(\frac{3(3x+1)^{-\frac{1}{2}}}{2}\right)}{3x+1}$$

$$= \frac{\sqrt{3x+1} - (x+8)\left(3(3x+1)^{-\frac{1}{2}}\right)}{2(3x+1)}$$

$$= f'(0) = \frac{1 - (8)(3(1)^{-\frac{1}{2}}}{2(1)}$$
$$= \frac{1 - 24}{2}$$
$$Slope\ of\ tangent\ line = -\frac{23}{2}$$
$$Normal\ line = -\frac{1}{tangent\ m} = -\frac{1}{-\frac{23}{2}}$$
$$Slope\ of\ normal\ line = \frac{23}{2}$$

$$y - 8 = -\frac{23}{2}(x - 0)$$
$$Tangent\ line: y = -11.5x + 8$$
$$y - 8 = \frac{23}{2}(x - 0)$$
$$Normal\ line\ y = 11.5x + 8$$

3. At what points on curve $y = \sqrt{3}\cos x + \sin x$, $0 \leqslant x \leqslant 2\pi$ is the tangent line horizontal?

$$First,\ f'(x) = -sin\ x\ cos\ x$$

$$Zeros\ for\ f'\ have\ period = \frac{n\pi}{2}$$
$$There\ are\ 5\ points\ between\ 0\ and\ 2pi\ where\ y = 0\ and\ the\ tangent\ line\ would\ be\ horizontal:$$
$$Points: \left\{(0,0), \left(\frac{\pi}{2}, 0\right), (\pi, 0), \left(\frac{2\pi}{2},\ 0\right),\ (2\pi, 0)\right\}$$

4. Use implicit differentiation to find an equation of the tanget line to the graph at point (0,1) for
$ln\ (x + y) = x$

$$\frac{d}{dx}(ln(x + y) = \frac{d}{dx}x$$
$$= \frac{1 + \frac{dy}{dx}}{x + y} = 1$$

$$= \ 1 + \frac{dy}{dx} = x + y$$
$$= \ \frac{dy}{dx} = x + y - 1$$
$$\frac{dy}{dx} \ at \ (0, 1) \ = \ 0 \ + \ 1 \ - 1 \ = \ 0$$
$$y - 1 = 0(x - 0) \ = \ y = 1$$

The tangent line at point $0, 1$ *is horizonal with equation* $y = 1$

5. Find the third derivative of $f(x) \ = \ \sqrt[3]{5 - 3x}$. Simplify.

$$f' = \ \left(\frac{1}{3}\right)(5 - 3x)^{\frac{1}{3} - 1}$$
$$= \ \left(\frac{1}{3}\right)(5 - 3x)^{-\frac{2}{3}}$$
$$\frac{d}{dx} = \ \frac{1}{(5 - 3x)^{\frac{2}{3}}}$$
$$f'' \ = \ \frac{d^2}{dx^2} = f(x)g'(x) \ + \ f'(x)g(x)$$
$$f''(x) = \ f' \ of \left(\frac{1}{3}\right)(5 - 3x)^{-\frac{2}{3}} \ = \ \left(\frac{2}{3}\right)\left(\left(\frac{1}{3}\right)(5 - 3x)^{-\frac{2}{3} - 1}\right)(-3)$$
$$= \ \left(\frac{2}{3}\right)(5 - 3x)^{-\frac{5}{3}}(-3)$$
$$= \ \frac{-6}{3\ (5 - 3x)^{\frac{5}{3}}}$$
$$f''(x) = \ - \ \frac{2}{(5 - 3x)^{\frac{5}{3}}}$$

$$f''' \ = \ \frac{d^3}{dx^3}$$
$$= \ f' \ of \ \left(\frac{2}{3}\right)(5 - 3x)^{-\frac{5}{3}}(-3) \ = \ \left(\frac{5}{3}\right)\left(\left(\frac{2}{3}\right)(5 - 3x)^{-\frac{5}{3} - 1}\right)(-3)$$

$$\left(\frac{10}{3}\right)(5-3x)^{-\frac{8}{3}}(-3)$$

$$= -\frac{30}{3(5-3x)^{\frac{8}{3}}}$$

$$f'''(x) = \frac{10}{(5-3x)^{\frac{8}{3}}}$$

6. All edges of a cube are expanding at a rate of 6 centimeters per second. How fast is the volume changing when each edge is a.) 2cm b.) 10cm. Volume = a^3, surface area = $6a^2$

$$\frac{dx}{dt} = \frac{6cm}{1s}$$

$$\frac{dx}{dy} = 3a^2 * \frac{dx}{dy} = 3a^2(6)$$

$$f'(2) = 3(2)^2(6) = change\ in\ volume\ is\ 72cm/s$$

$$f'(10) = 3(10)^2(6) = change\ in\ volume\ is\ 1800cm/s$$

7. Find derivatives

A.) $f(x) = \left(1-x^{-7}\right)^{-11} - 7ex^{2e} + 2e\pi^{3e}$

$$A.)\ f'(x) = -11\left(1-x^{-7}\right)^{-12}\left(-7x^{-8}\right) - (2e)7ex^{2e-1} + 0$$

$$= \frac{-14e^2x^{2e-1}\left(1-x^{-7}\right)^{12} - 77}{x^8\left(1-x^{-7}\right)^{12}}$$