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INF2178

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INF2178 Technical Assignment 3:

ANCOVAs

I. Introduction

Within liberal democratic systems of government like in Canada, the United States, and other wealthy, developed nations around the world, citizens participate in markets and state-run institutions which assume a certain level of independence and material security from citizens, and so work to maintain these minimum standards (Studebaker, 2023). However, independence and minimal material security, while necessary, are not sufficient to maintain liberal democracies (Rowe, 2017). Citizens need to understand enough about liberal democracy and the world more broadly to properly realize their political capabilities as citizens within the liberal democratic system (Studebaker, 2023). Further, liberal democratic states need to both keep existing citizens liberal and democratic while also creating new ones, so that the system can persist over time (Studebaker, 2013). To fulfill this fundamental need, as well as to self-correct in other ways (Carnegie, 2006), liberal democracies build and maintain public education systems; public education is therefore considered a bedrock of liberal democracy (Rowe, 2017).

However, the effectiveness of public education is considered sensitive to socioeconomic factors (Engle & Black, 2008). Specifically, education outcomes are considered worse for young students when they are relatively poorer (Engle & Black, 2008). Lack of success in public education is considered strongly related to lack of other kinds of success within liberal democracy's markets and institutions – success upon which the system depends (Studebaker, 2023). Disparities in success, known as inequality, are considered a fundamental threat to the larger democratic system when severe enough (Thomas & Ronald, 2021). All together, these findings outline the urgency of understanding inequality pouring into and arising out of public education systems.

Thus, this technical assignment investigates what presence inequality appears to have within the earliest days of participation in public education systems: primary education, or Kindergaten. To better understand how family income affects educational performance at this fundamental level, this assignment investigates three main research questions:

1. What effect does household income seem to have on school-based learning, represented by differences in reading test scores over one school year?

2. What effect does household income seem to have on school-based learning, represented by differences in math test scores over one school year?
3. What effect does household income seem to have on school-based learning, represented by differences in general knowledge test scores over one school year?

II. The Data and Data Cleaning

The data was retrieved from the INF2178 Quercus module as a Comma-Separated Value (CSV) file. The name of the file is "INF2178_A3_data.csv". The data has a length of 11,933 and has nine columns, including:

- Fallreadingscore, fallmathscore, and fallgeneralknowledgescore, each of which being a column of float64 type representing test scores in three main subject areas at the start of the school year
- Springreadingscore, springmathscore, and springgeneralknowledgescore, each of which the same as the fall columns, but at the end of the school year
- Totalhouseholdincome and incomeinthousands, also float64 columns representing family income per student
- Incomegroup, an int64-type column organizing the household income columns into quintiles

III. Testing Assumptions for ANCOVA

ANCOVA tests require that two assumptions be met: first, that residuals be normally distributed, and, second, heteroskedasticity. To test the first assumption for the three Research Questions presented above, we created an OLS (Ordinary Least Squares) model for each research question, and then performed Anderson-Darling tests for normality. The Anderson-Darling test was used instead of the Shapiro-Wilk because the data has a large length, and the Shapiro-Wilk test becomes less reliable at larger amounts of data points. The initial Anderson-Darling results were as follows:

Reading: Anderson-Darling Test Statistic: 154.48124076225577

Reading: Critical Values: [0.576 0.656 0.787 0.918 1.092]

Reading: Significance Levels: [15. 10. 5. 2.5 1.]

Math: Anderson-Darling Test Statistic: 74.66567741147264

Math: Critical Values: [0.576 0.656 0.787 0.918 1.092]

Math: Significance Levels: [15. 10. 5. 2.5 1.]

General Knowledge: Anderson-Darling Test Statistic: 1.7208135790970118

General Knowledge: Critical Values: [0.576 0.656 0.787 0.918 1.092]

General Knowledge: Significance Levels: [15. 10. 5. 2.5 1.]

As the test statistics for each of the three subjects are significantly above the corresponding critical values for each Anderson-Darling test, we conclude that the Null Hypotheses that the data are normally distributed are rejected. Therefore, these residuals do not appear to be normally distributed.

To proceed from here, we applied a Yeo-Johnson transformation to the residual data. (We originally applied a Box Cox transformation, but the Box Cox transformation did not succeed in altering the Anderson-Darling results). The Yeo-Johnson transformed residual data produced the following Anderson-Darling results:

Reading: Transformed Anderson-Darling Test Statistic: 112.14681634098633

Reading: Transformed Critical Values: [0.576 0.656 0.787 0.918 1.092]

Reading: Transformed Significance Levels: [15. 10. 5. 2.5 1.]

Math: Transformed Anderson-Darling Test Statistic: 33.082302250664725

Math: Transformed Critical Values: [0.576 0.656 0.787 0.918 1.092]

Math: Transformed Significance Levels: [15. 10. 5. 2.5 1.]

General Knowledge: Transformed Anderson-Darling Test Statistic: 1.0374357778891863

General Knowledge: Transformed Critical Values: [0.576 0.656 0.787 0.918 1.092]

General Knowledge: Transformed Significance Levels: [15. 10. 5. 2.5 1.]

Reviewing these results, we see that the transformed Anderson-Darling Test Statistics for the first reason question and second research question are still significantly above the corresponding critical values. So, we conclude that the reading and math scores are not normally distributed, and that the first assumption of ANCOVA tests is therefore not met. Therefore, frustratingly, we are unable to proceed with ANCOVAs for Research Questions 1 and 2. However, the third Research Question produced an Anderson-Darling Test Statistic within the tests' critical values. Therefore, we conclude that the general knowledge test scores are close enough to normally distributed, and can proceed with Research Question 3.

Next, we tested the second assumption for ANCOVA, which is heteroskedasticity of the relevant variable. To do this we used a White test. The White test produced the following results:

White Test Statistic: 26.79737470563559

p-value: 1.5171342676929652e-06

As the p-value here is quite small (<0.001), we reject the Null Hypothesis that the data is homoskedasticitous, and conclude that the data displays heteroskedasticity. Therefore, we fulfill both assumptions of ANCOVA and can proceed with the ANCOVA test for Research Question 3, on general knowledge scores.

IV. The one-way ANCOVA

Having confirmed that only the data underlying Research Question 3 aligns with the assumptions of a one-way ANCOVA test, we proceeded with the one-way ANCOVA test using the differences between spring and fall general knowledge test scores as the dependent variable, household income as the independent variable, and the early/Fall general knowledge test scores as the covariate. The resultant ANCOVA table is as follows:

	Sum_sq	df	F	PR(>F)
totalhouseholdincome	1863.445065	1.0	120.798022	<0.000001
fallgeneralknowledgescore	12151.352428	1.0	787.712698	<0.000001
Residual	184033.639329	11930.0		

In this table, we see that the F statistics are near to 120.8 and 787.7, whereas the p-values are both <0.000001. This indicates that both variables, household income and Fall scores, have large significant effects on the final scores attained by the students. Because the Fall scores have significantly higher F-statistic values than total household income, however, we see that the one-way ANCOVA suggests that the Fall scores have a higher impact on change in test scores over the course of the school year.

Thus, we conclude that, at least in the earlier stages of public education, household income inequality does not drive school performance. This is good news from the perspective of liberal democracy more broadly, as it provides evidence for the consensus on liberal democratic theory that public education has not been overtaken by inequality in its work to develop citizens at this level.

V. References

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