Logic and lambda calculus

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Model theoretic semantics

- linguistic expressions $\rightsquigarrow$ logic expressions
- logic expressions $\rightsquigarrow$ objects in a model
- the model provides
  - a set of basic objects
  - an assignment of objects to the basic expressions of the (logic) language
- for each rule combining expressions in the natural language there is a rule in the logic language for which there is a corresponding rule operating on the model-theoretic objects

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- for each rule combining expressions in the natural language there is a rule in the logic language for which there is a corresponding rule operating on the model-theoretic objects
- Formal languages:
  - Propositional logic
  - First order logic
  - Lambda calculus

Propositional logic

propositional-logic.ipynb

First-order logic

…predicate logic or predicate calculus

first-order-logic.ipynb

From linguistic structure to logic representations

- a turtle runs
- $\exists x[\mathrm{turtle}(x) \wedge \mathrm{run}(x)]$
- a: $\exists x[?(x) \wedge ?(x)]$

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From linguistic structure to logic representations

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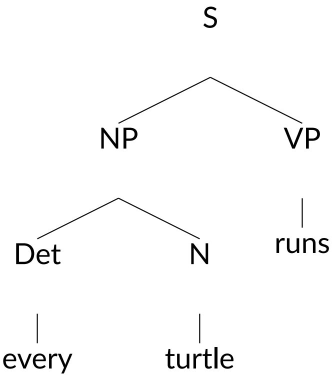
- $\exists x[\mathrm{turtle}(x) \land \mathrm{run}(x)]$
- a: $\exists x[?(x) \land ?(x)]$
- every turtle runs
- $\forall x[\mathrm{turtle}(x) \rightarrow \mathrm{run}(x)]$
- every: $\forall x[?(x) \rightarrow ?(x)]$

A mismatch in structure



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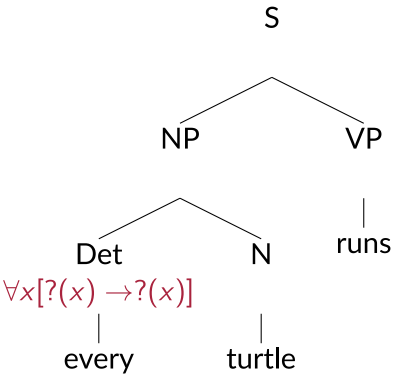

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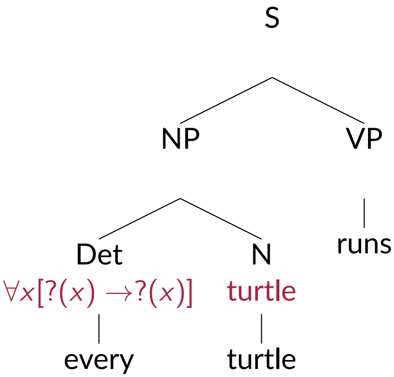

S

NP

VP

Det

$\forall x[?(x) \rightarrow ?(x)]$

every

N

turtle

turtle

runs

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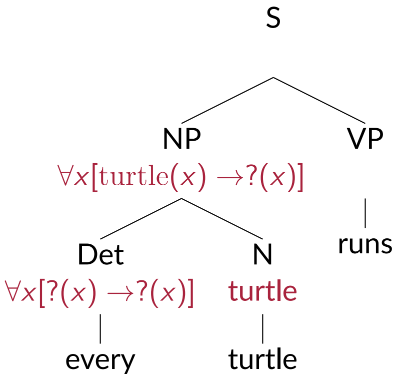

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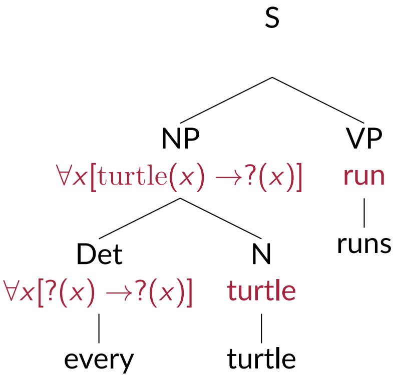
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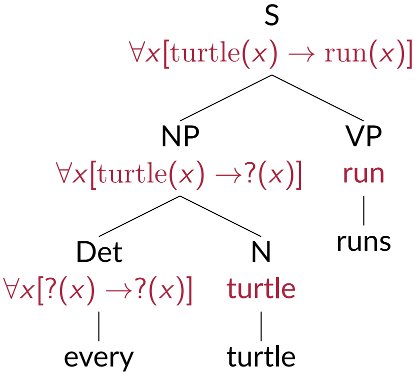

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Solution: typed functions and lambda calculus

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  $\lambda x[2x + 3](4)$
  $2 \times 4 + 3$
  11

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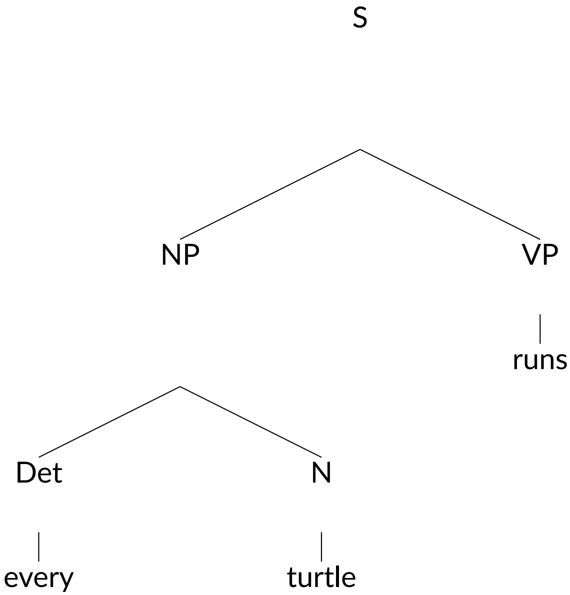
Solution for ? in the linguistic example



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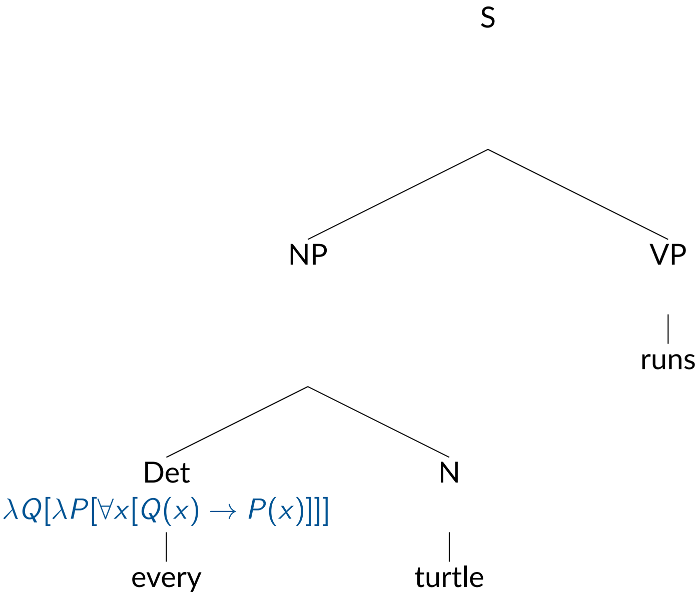

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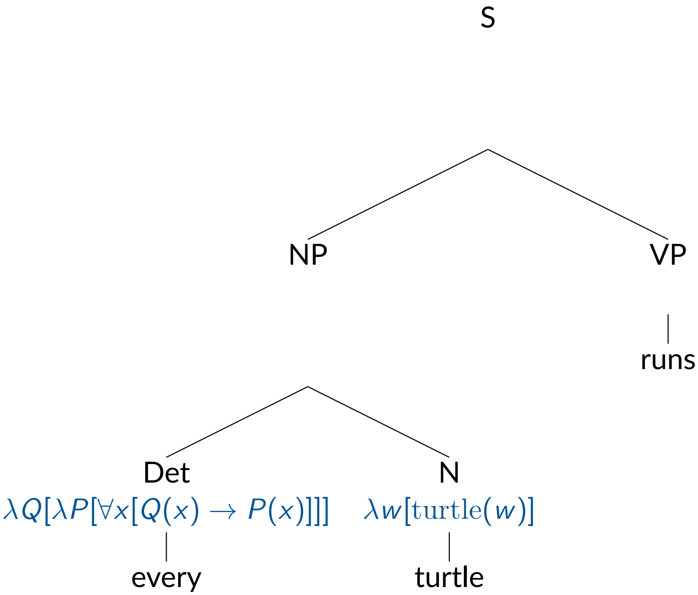

S

NP

VP

runs

Det

$\lambda Q[\lambda P[\forall x[Q(x) \rightarrow P(x)]]]$

every

N

$\lambda w[\mathrm{turtle}(w)]$

turtle

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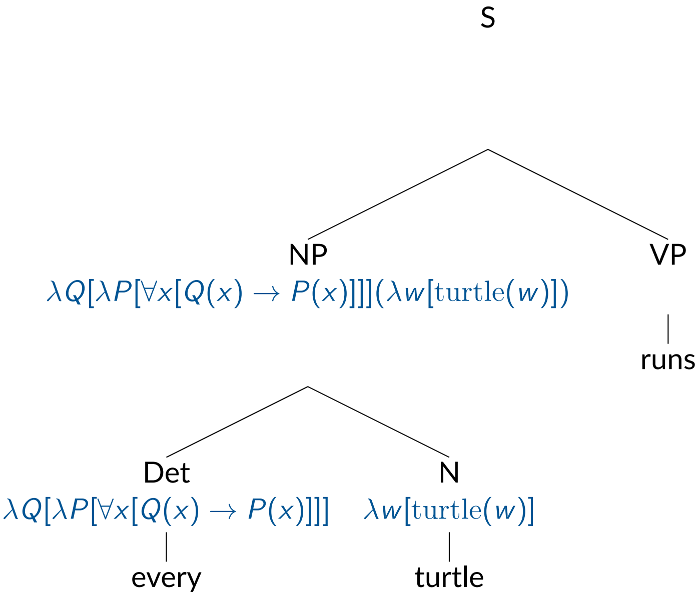

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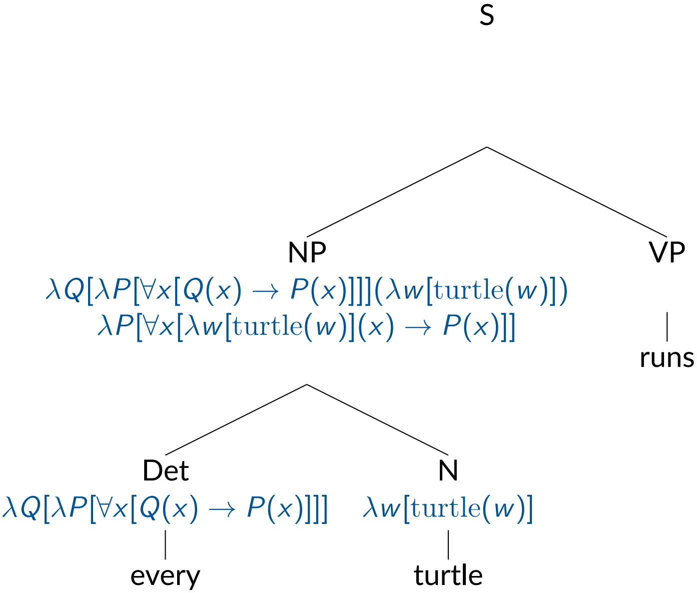
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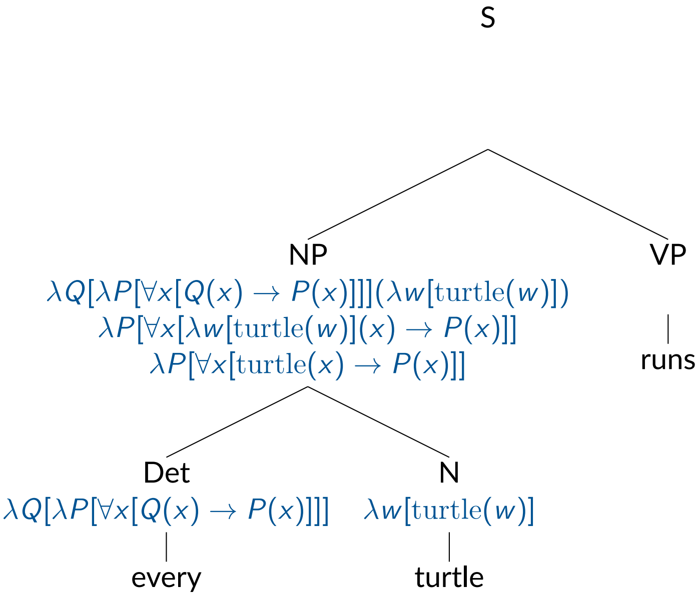

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$\lambda Q[\lambda P[\forall x[Q(x) \to P(x)]]](\lambda w[\mathrm{turtle}(w)])$

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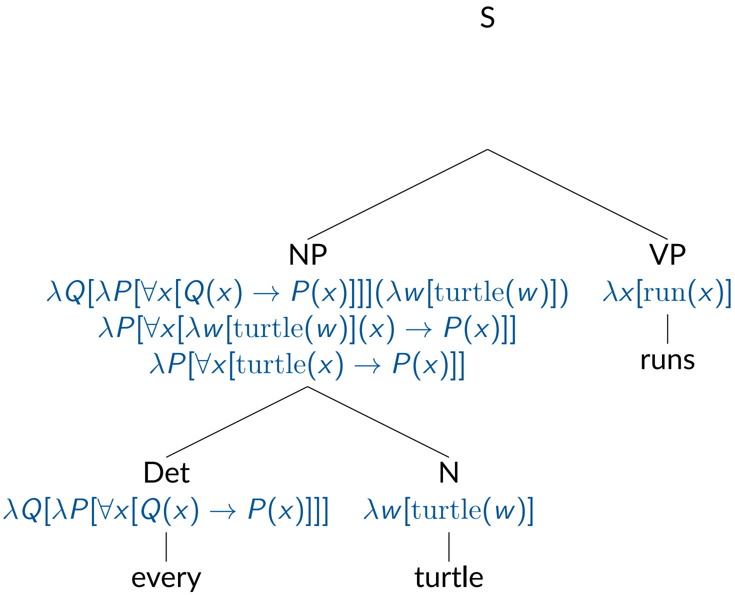

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$\lambda Q[\lambda P[\forall x[Q(x) \to P(x)]]](\lambda w[\mathrm{turtle}(w)])$

$\lambda P[\forall x[\lambda w[\mathrm{turtle}(w)](x) \to P(x)]]$

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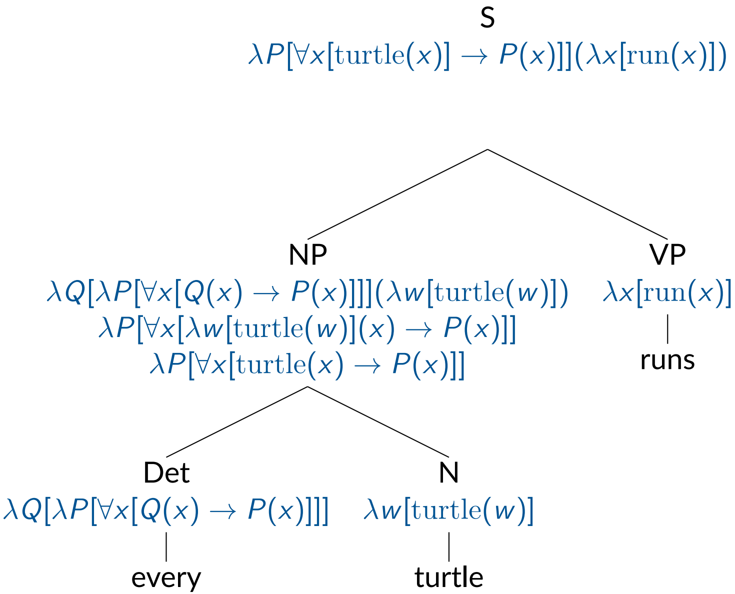

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$\lambda P[\forall x[\mathrm{turtle}(x)] \rightarrow P(x)]](\lambda x[\mathrm{run}(x)])$

NP

$\lambda Q[\lambda P[\forall x[Q(x) \rightarrow P(x)]]](\lambda w[\mathrm{turtle}(w)])$

$\lambda P[\forall x[\lambda w[\mathrm{turtle}(w)](x) \rightarrow P(x)]]$

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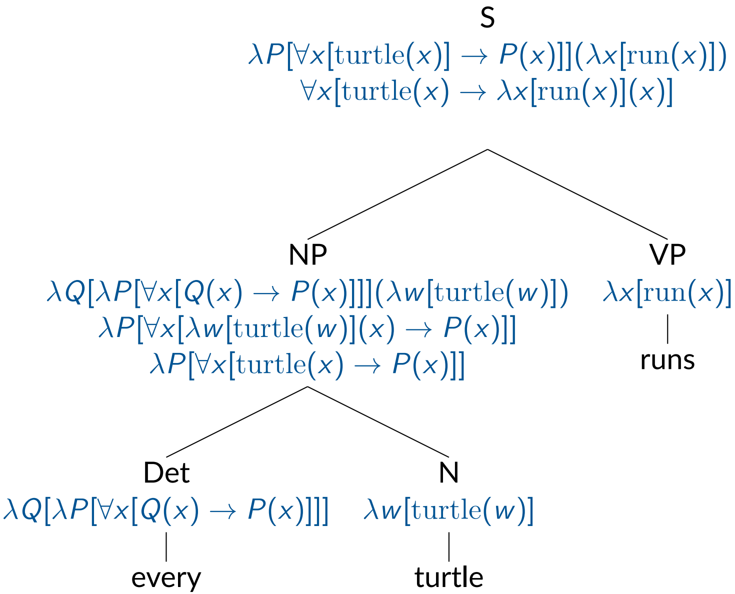

S

$\lambda P[\forall x[\mathrm{turtle}(x)] \rightarrow P(x)]](\lambda x[\mathrm{run}(x)])$

$\forall x[\mathrm{turtle}(x) \rightarrow \lambda x[\mathrm{run}(x)](x)]$

NP

$\lambda Q[\lambda P[\forall x[Q(x) \rightarrow P(x)]]](\lambda w[\mathrm{turtle}(w)])$

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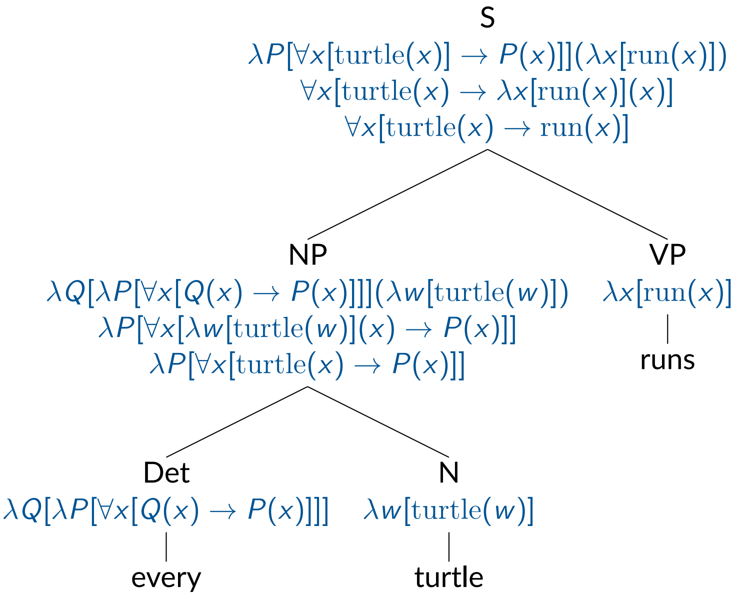

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$\lambda P[\forall x[\mathrm{turtle}(x)] \rightarrow P(x)]](\lambda x[\mathrm{run}(x)])$

$\forall x[\mathrm{turtle}(x) \rightarrow \lambda x[\mathrm{run}(x)](x)]$

$\forall x[\mathrm{turtle}(x) \rightarrow \mathrm{run}(x)]$

NP

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$\lambda P[\forall x[\lambda w[\mathrm{turtle}(w)](x) \rightarrow P(x)]]$

$\lambda P[\forall x[\mathrm{turtle}(x) \rightarrow P(x)]]$

VP

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runs

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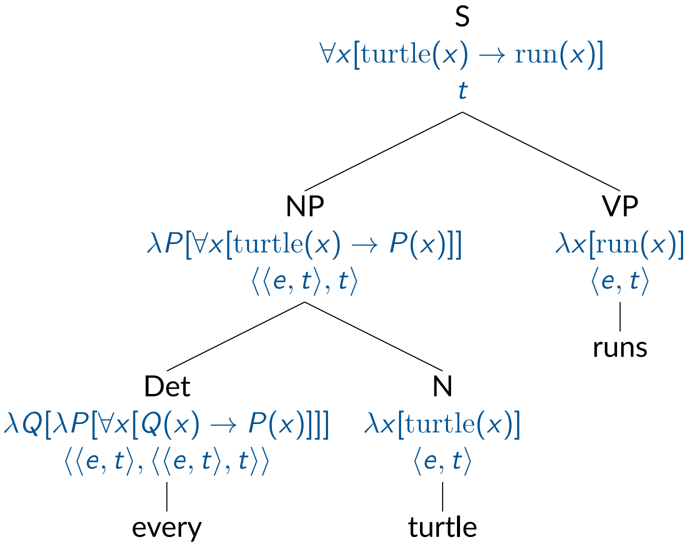
Balance correct composition AND semantic interpretation



S

$\forall x[\mathrm{turtle}(x) \to \mathrm{run}(x)]$

t

NP

$\lambda P[\forall x[\mathrm{turtle}(x) \to P(x)]]$

$\langle\langle e, t\rangle, t\rangle$

VP

$\lambda x[\mathrm{run}(x)]$

$\langle e, t\rangle$

runs

Det

$\lambda Q[\lambda P[\forall x[Q(x) \to P(x)]]]$

$\langle\langle e, t\rangle, \langle\langle e, t\rangle, t\rangle\rangle$

every

N

$\lambda x[\mathrm{turtle}(x)]$

$\langle e, t\rangle$

turtle

Grammar with first-order logic + lambda calculus

lambda-calculus.ipynb

simple-sem.fcfg

Summary

- Logic-based computational semantics can use model theory to evaluate natural language in a database

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Summary

- Logic-based computational semantics can use model theory to evaluate natural language in a database
- This is done by translating natural language into an extension of first order logic employing the λ-calculus
- These expressions can be then evaluated in the database
- Translations to logic and lambda calculus can be done for a significant part of natural language
- There are, however, important constructions of natural language which go beyond first order logic

Further reading

- Bird et al. (2009), Chapter 10: Analyzing the meaning of sentences
- Manning (2005)
- Eisenstein (2019), Chapter 13: Logical Semantics
- Jurafsky and Martin (2000): Chapter 17: Representing meaning and Chapter 18: Computational semantics

Advanced

- Blackburn and Bos (2005): Chapter 1: First-order logic and Chapter 2: Lambda calculus
- Eijck and Unger (2010)

Acknowledgement

Some slides based on the slides by Robin Cooper.

References I

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