

INFO371: Causal Inference

Ott Toomet

April 12, 2018

1 Causal Inference

Motivating Example

2 What is Impact

3 Curse of Counterfactual

4 A Few Simple Estimators

Cross-Sectional Estimator

Before-After Estimator

Diff-in-Diff Estimator

1 Causal Inference

Motivating Example

2 What is Impact

3 Curse of Counterfactual

4 A Few Simple Estimators

- Cross-Sectional Estimator
- Before-After Estimator
- Diff-in-Diff Estimator

What is *Cause*?

- Link between two events, *cause* and *effect*
- Effect (partly) dependent on cause
- Cause precedes the effect in time
- 3 Cases
 - Sufficient and necessary
 - Sufficient but not necessary
 - Necessary but not sufficient

Does Flu Shot Help to Avoid Illness?

Ott Toomet

Causal
Inference

Example

Impact

Counterfactual

A Few
Simple
EstimatorsCross-
Sectional
Estimator
Before-After
Estimator
Diff-in-Diff
Estimator

- Collect data (get medical data):

gotFlu	fluShot
1	0
0	1
...	...

- Run a regression:

$$\text{gotFlu}_i = \alpha + \beta \cdot \text{fluShot}_i + \epsilon_i$$

- We want to know β
 - What does the regression tell?

1 Causal Inference

Motivating Example

2 What is Impact

3 Curse of Counterfactual

4 A Few Simple Estimators

Cross-Sectional Estimator

Before-After Estimator

Diff-in-Diff Estimator

A Causal Question

- Many important questions are causal
 - Does college pay off?
 - Which career path should I choose?
 - Does the drug cure illness?
 - Does the advertisement work?
- These models can be written as

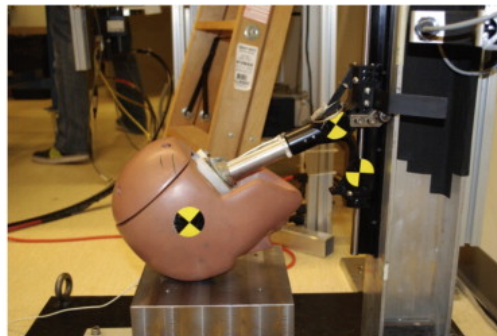
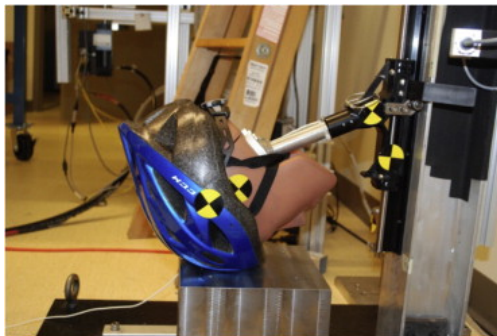
$$Y_i = \alpha + \beta \cdot T_i + \epsilon_i$$

What are *outcome* and *treatment*?

Ott Toomet

Example:

*Bicycle helmets are highly effective at preventing head injury [...]
(Cripton et al, 2014).*



Causal
Inference

Example

Impact

Counterfactual

A Few
Simple
Estimators

Cross-
Sectional
Estimator
Before-After
Estimator
Diff-in-Diff
Estimator

Should we make helmets mandatory?

Ott Toomet

Causal
Inference

Example

Impact

Counterfactual

A Few
Simple
Estimators

Cross-
Sectional
Estimator
Before-After
Estimator
Diff-in-Diff
Estimator

- In the paper, *treatment* is helmet and outcome is head injury.
- It is not *helmet law* and *public health*
- There are many additional factors:
 - Helmets may hurt in certain cases (rotational injury)
 - Helmeted cyclists may opt for less safe riding (risk compensation)
 - Motorists may opt for less safe techniques
 - When arguing for helmets, one argues that cycling is unsafe
 - Less cycling, more driving
 - Lower *safety in numbers*
 - Worse public health outcomes
 - Ditto with inconvenience of helmets

Want to earn *a lot* of money?

- ① Find stocks that are rising
- ② Buy!
- ③ Sell later
- ④ Enjoy life 😊

Problem:

- Everyone else is also going to buy the same stocks
- Stocks that were profitable a short while ago are not any more
- Have to re-assess the strategy all the time

Humans adapt to interventions and render these less effective

1 Causal Inference

Motivating Example

2 What is Impact

3 Curse of Counterfactual

4 A Few Simple Estimators

Cross-Sectional Estimator

Before-After Estimator

Diff-in-Diff Estimator

How to measure outcome?

Ott Toomet

Causal
Inference
Example

Impact

Counterfactual

A Few
Simple
Estimators

Cross-
Sectional
Estimator
Before-After
Estimator
Diff-in-Diff
Estimator

We want *numeric estimate* of the causal effect of our intervention

- $Y(1)$: outcome when treated ($T = 1$)
- $Y(0)$: when not treated ($T = 0$)
- The *treatment effect*:

$$Y(1) - Y(0) = \beta$$

- **Only observe** $Y(0)$ or $Y(1)$ but never both!

Estimate the regression model:

$$Y_i(T_i) = \alpha + \beta \cdot T_i + \epsilon_i$$

The regression coefficient is the average difference over $T = 1$ and $T = 0$ cases:

$$\begin{aligned}\mathbb{E}[Y(1) - Y(0)] &= \\ &= \mathbb{E} Y(1) - \mathbb{E} Y(0) = \\ &= \mathbb{E}[\alpha + \beta \cdot 1 + \epsilon | T = 1] - \mathbb{E}[\alpha + \beta \cdot 0 + \epsilon | T = 0] = \\ &= \beta + \mathbb{E}[\epsilon | T = 1] - \mathbb{E}[\epsilon | T = 0]\end{aligned}$$

This is not β

- unless $\mathbb{E}[\epsilon | T = 1] - \mathbb{E}[\epsilon | T = 0] = 0$

What Does Regression Do

Ott Toomet

Causal
Inference

Example

Impact

Counterfactual

A Few
Simple
Estimators

Cross-
Sectional
Estimator
Before-After
Estimator
Diff-in-Diff
Estimator

Regression compares Y for two groups: $T = 0$ and $T = 1$.

- And that's it.
- It is “just correlation”
- In order to interpret it causally, we have to ensure $T \perp\!\!\!\perp \epsilon$
 - i.e. treatment is unrelated to the disturbance term

There are always 3 stories:

- 1 T causes Y
“Because there are so many guns, there is so much violence”
- 2 Y causes T
“Because of the violence, people get guns”
- 3 something else causes both T and Y
“Because US history of being a frontier land, people want to have guns, and are used to only rely on themselves when seeking justice”

(sometimes some of the stories not convincing)

Amaros et al (2011)

- analyze French medical registries
- compare bicycle accident data with/without helmets
- compare 'similar' injuries outside of the head region
- control for road type, accident type, age, gender
- "...confirm the protective effect for head and facial injuries..."

Can you tell 3 stories?

- ① Helmet causes less head injuries
- ② Head injuries cause not wearing helmets
Does not sound convincing as we *know* the decision to (not to) wear helmet strictly preceded the crash. However, we also have to assume that riders did not want to get injured. These are pieces of extra information.
- ③ Those who care about their health both wear helmets and ride carefully.
Although the injuries outside of head region was controlled for, it may still be possible to take more care of your head than rest of the body

Note that this does not address the social psychology issues.

We have to know that $\mathbb{E}[\epsilon|T = 1] - \mathbb{E}[\epsilon|T = 0] = 0$

- This information is usually not called “data”
- Examples:
 - This was a randomized experiment
 - Treatment was caused by an external factor (natural experiment)
 - natural event
 - age-dependent rules
 - decision to change rules for some people
 - We know what was treatment into selection based on
 - We know functional form of ϵ .

Note: “big” data does not help here

1 Causal Inference

Motivating Example

2 What is Impact

3 Curse of Counterfactual

4 A Few Simple Estimators

Cross-Sectional Estimator

Before-After Estimator

Diff-in-Diff Estimator

Cross-Sectional Estimator

##	y	treatment
## 1	0.1575982	0
## 2	0.2978230	0
## 3	-0.3886009	0
## 4	1.1147280	1
## 5	1.3410852	1
## 6	0.7754202	1

Cross-Sectional Estimator

Ott Toomet

Causal
Inference

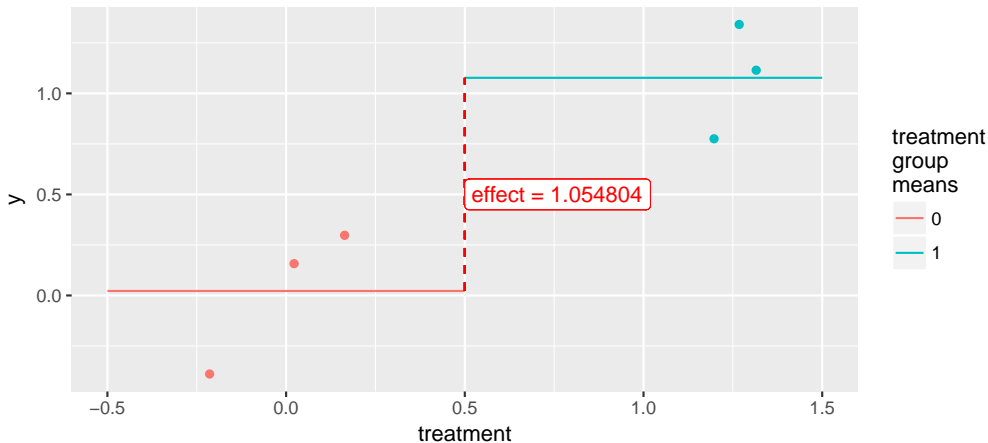
Example

Impact

Counterfactual

A Few
Simple
Estimators

**Cross-
Sectional
Estimator**
Before-After
Estimator
Diff-in-Diff
Estimator

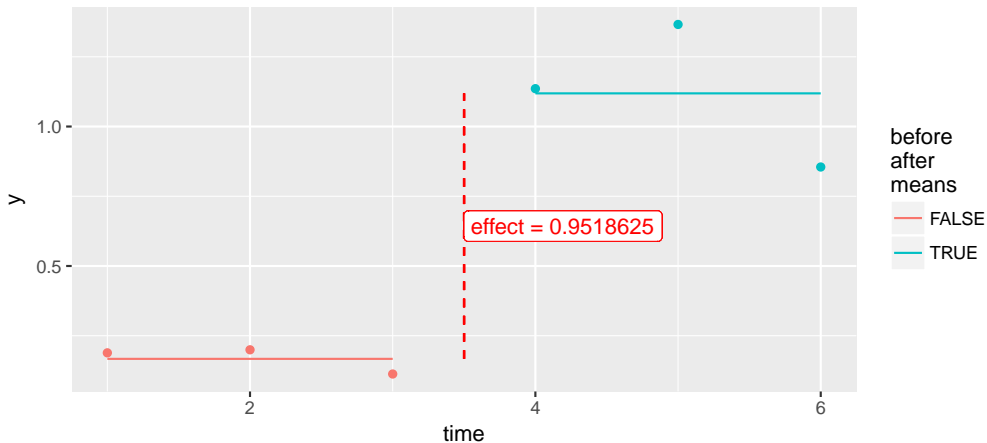


```
##  
## Call:  
## lm(formula = y ~ treatment, data = df)  
##  
## Residuals:  
##          1          2          3          4          5          6  
## 0.13532 0.27555 -0.41087 0.03765 0.26401 -0.30166  
##  
## Coefficients:  
##              Estimate Std. Error t value Pr(>|t|)  
## (Intercept) 0.02227    0.18823   0.118   0.9115  
## treatment   1.05480    0.26620   3.962   0.0166 *  
## ---  
## Signif. codes:  
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
##
```

Before-After Estimator

##		y	time
##	1	0.1887248	1
##	2	0.1996113	2
##	3	0.1127663	3
##	4	1.1357981	4
##	5	1.3659416	5
##	6	0.8549502	6

Cross-Sectional Estimator




```
##
## Call:
## lm(formula = y ~ time > 3, data = df)
##
## Residuals:
##          1          2          3          4          5          6
##  0.02169  0.03258 -0.05427  0.01690  0.24704 -0.26395
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    0.1670     0.1062   1.572  0.19102
## time > 3TRUE    0.9519     0.1503   6.335  0.00318 **
## ---
## Signif. codes:
##  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

Differences-in-differences

##	y	time	treatment
## 1	0.0	1	0
## 2	0.5	2	0
## 3	2.0	1	1
## 4	3.5	2	1

Ott Toomet

Causal
Inference

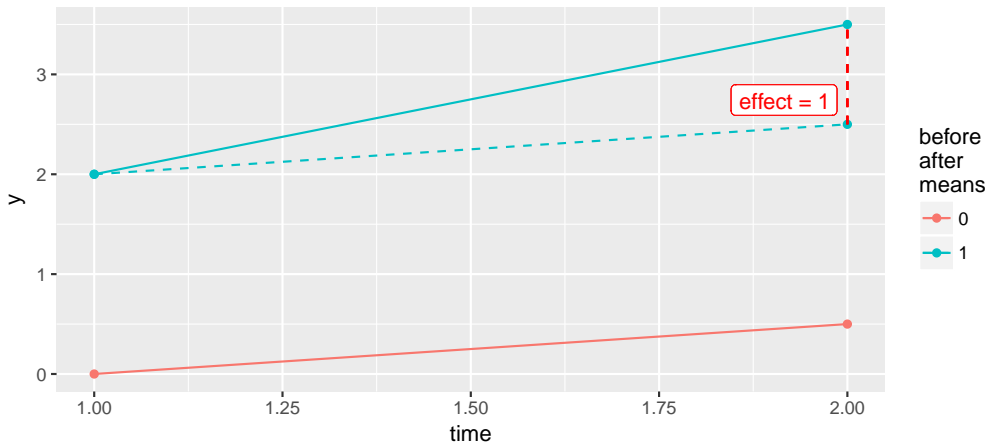
Example

Impact

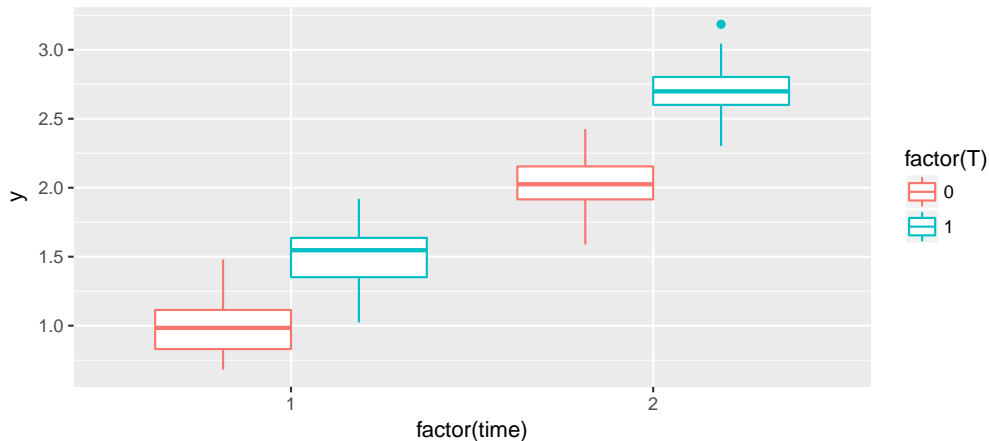
Counterfactual

A Few
Simple
Estimators

Cross-
Sectional
Estimator
Before-After
Estimator
**Diff-in-Diff
Estimator**



Random Data



```
##  
## Call:  
## lm(formula = y ~ time * T, data = df)  
##  
## Residuals:  
##      Min       1Q   Median       3Q      Max   
## -0.49302 -0.13115  0.00572  0.12300  0.48529   
##  
## Coefficients:  
##              Estimate Std. Error t value Pr(>|t|)      
## (Intercept) -0.03149    0.06317  -0.499   0.61868      
## time         1.02598    0.03995  25.680 < 2e-16 ***  
## T            0.36226    0.08518   4.253 3.27e-05 ***  
## time:T       0.16002    0.05387   2.970  0.00335 **   
## ---  
## Signif. codes:
```