ITERATIVE MMSE PRECODING AND POWER ALLOCATION IN CELL-FREE MASSIVE MIMO SYSTEMS

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ABSTRACT

In this work, we consider the downlink channel of a Cell-Free Massive multiple-input multiple-output (MIMO) system where both users and access points (APs) are equipped with a single-antenna. We propose iterative linear minimum mean-square error (MMSE) precoding along with optimal and uniform power allocation. We then derive achievable rate expressions for the proposed iterative MMSE precoder and power allocation approaches. Simulations show that the proposed iterative MMSE precoder and power allocation techniques outperform existing conjugate beamforming (CB), zero-forcing (ZF), and MMSE schemes in terms of achievable sum-rate and bit error rate (BER), in the presence of perfect and imperfect channel state information (CSI).

***Index Terms*—Cell-Free Massive MIMO, MMSE precoding, power allocation, distributed antenna systems, communication systems.**

I. INTRODUCTION

Massive multiple-input multiple-output (MIMO) systems have been recognized as a promising approach to improve the efficiency, throughput and reliability of the fifth generation (5G) of wireless networks [1]. In these schemes, multiple antennas serve simultaneously multiple users in the same time-frequency resource and can be set in a collocated or distributed layout. Advances towards distributed architectures are promising to provide higher coverage probability, flexible resource management, higher power efficiency and larger capacities by the exploitation of smaller distances between base stations and users and diversity against shadow fading [2]–[4]. A large-scale distributed setup is considered, [5], where all access points (APs) are connected to a central processing unit (CPU) in order to process payload data and to perform precoding with power control. It should be noted that users, however, do not cooperate with each other. Among the many benefits of this system are its simple signal processing and exploitation of favorable propagation with channel hardening.

In the seminal work [6] that introduced the cell-free concept, capacity lower bounds for the downlink have been derived with finite number of APs and users using a conjugate beamforming (CB) precoder with power allocation. Then, in [7], CB and zero-forcing (ZF) precoders with power optimization are investigated. In [8], a minimum mean-square error (MMSE) processing was developed for the uplink channel of a cell-free system and in [9], an algorithm to maximize the minimum achievable rate, through resource allocation is proposed. Scalable cell-free schemes have been studied in [10], [11]. Several MMSE precoders have been developed for single-user and multiuser MIMO systems in [12]–[15] considering perfect and imperfect channel state information (CSI). A robust and a non-robust version of this precoder have been developed specifically for cell-free networks in [16], [17].

In this work, we propose iterative linear MMSE precoding with power allocation for cell-free networks that are suitable for 5G. In particular, an alternating precoding and power allocation scheme is presented, where precoding coefficients are calculated based on initial parameters, used in power allocation and recalculated based on power allocation coefficients. Optimal and uniform power allocation techniques are examined and compared, in terms of sum-rate and bit error rate (BER), with existing CB, ZF and MMSE precoding and power allocation approaches from [6], [7] and [12]. We also derive expressions to compute the achievable sum-rates of the proposed scheme. Perfect and imperfect CSI are taken into account when evaluating performance, similarly to other works in the area, where MMSE channel estimates are considered [7].

This paper is organized in the following sections: In Section II the Cell-Free Massive MIMO system model is shown and CSI setup is explained. In Section III, an iterative linear MMSE precoder with power allocation is presented. Numerical results are presented in Section IV. Finally, in Section V, conclusions and main points are highlighted.

Notation: Uppercase bold symbols denote matrices and lowercase bold symbols denote vectors. The superscripts $()^*$,$()^T$, $()^H$ stand for complex conjugate, transpose and Hermitian operations, respectively. The expectation, trace of a matrix, Euclidean norm and Frobenius norm are denoted by $\mathbb{E}[\cdot]$, $\operatorname{tr}(\cdot)$, $\|\cdot\|_2$ and $\|\cdot\|_F$, respectively. The operator $\operatorname{diag}\{\mathbf{A}\}$ stands for the column vector corresponding to the main diagonal elements of matrix $\mathbf{A}$. The $D \times D$ identity matrix is $\mathbf{I}_D$. The notation $x \sim \mathcal{N}(0, \sigma^2)$ refers to a Gaussian random variable (RV) x with zero mean and variance σ^2 and $x \sim \mathcal{CN}(0, \sigma^2)$ denotes a circularly symmetric complex Gaussian RV x with zero mean and variance σ^2.

II. SYSTEM MODEL

The downlink of a Cell-Free Massive MIMO System is assumed, with M randomly distributed single-antenna APs and K single-antenna users, where $M >> K$. In this system, all APs are connected to a CPU and serve simultaneously all users, as shown in Fig. 1. Each AP obtains CSI and sends them to the CPU where precoding and power allocation are executed and the coefficients are sent back to the APs.

The channel coefficient between the mth AP and the kth user is defined as [6],

$$g_{mk} = \sqrt{\beta_{mk}} h_{mk}, \tag{1}$$

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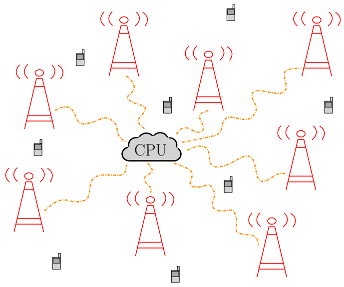

Fig. 1. Cell-Free Massive MIMO System Layout.

where β_{mk} is the large-scale fading coefficient (path loss and shadowing effects) and $h_{mk} \sim \mathcal{CN}(0,1)$ is the small-scale fading coefficient, defined as independent and identically distributed (i.i.d) RVs that remain constant during a coherence interval and are independent in different coherence intervals. On the other hand, the large-scale fading coefficients change less frequently, being constant for several coherence intervals. In this case we will assume that it changes at least 40 times slower than h_{mk} [18].

Since each user is served by all APs, in order to reduce interference between signals intended for different users, the APs should always take into account the channel coefficients when forming transmitted signals. Therefore, the time division duplex protocol is used in channel estimation. By considering an MMSE estimation of CSI computed in each AP, we define them as [7]

$$\hat{g}_{mk} \sim \mathcal{CN}(0, \alpha_{mk}),\ \tilde{g}_{mk} \sim \mathcal{CN}(0, \beta_{mk} - \alpha_{mk}), \qquad (2)$$

where $\hat{g}_{mk}$ is the CSI between the mth AP and the kth user and $\tilde{g}_{mk}$ is the CSI error between the mth AP and the kth user. In order to evaluate different levels of imperfect CSI in our numerical results, in a simpler way, we will consider α_{mk} as an adjustable percentage of β_{mk} $(0 \leq n \leq 1)$. Consequently, we have

$$\begin{aligned} \alpha_{mk} &= n\beta_{mk} \\ \tilde{g}_{mk} &= g_{mk} - \hat{g}_{mk},\ \text{and} \\ \mathbb{E}\left[|\tilde{g}_{mk}|^2\right] &= (1-n)\,\beta_{mk}. \end{aligned} \qquad (3)$$

III. PROPOSED ITERATIVE MMSE PRECODING AND POWER ALLOCATION

In this section, an iterative linear MMSE precoder for cell-free systems is presented and combined with power allocation techniques.

III-A. MMSE Precoder

Differently from previous linear techniques [7], [12], here, we take into account the power allocation matrix in the precoder and use the CSI matrix $\hat{\mathbf{G}}$, instead of the actual channel matrix $\mathbf{G}$, since the APs have no knowledge of the large-scale fading coefficients (β_{mk}). It is important to mention that similarly to multiuser MIMO schemes, the receivers of the users do not cooperate with each other. We use the normalization factor f^{-1} at the receivers, which can be interpreted as an automatic gain control [12]. Moreover, we consider an iterative MMSE precoder with power allocation in order to maximize the minimum signal-to-interference-plus-noise ratio (SINR). If the conventional MMSE precoder, [12], was applied to this type of system, performance would be degraded due to the lack of appropriate power allocation.

In the downlink channel of a cell-free system with precoding and power allocation, the signal received by the kth user is given by,

$$y_k = \sqrt{\rho_f}\ \mathbf{g}_k^T \mathbf{P}\mathbf{N}\ \mathbf{s} + w_k, \qquad (4)$$

where ρ_f is the maximum transmit power of each AP, $\mathbf{g}_k = [g_{1k}, \ldots, g_{Mk}]^T$ are the channel coefficients for user k, $\mathbf{P} \in \mathbb{C}^{M\times K}$ is the precoding matrix, $\mathbf{N} \in \mathbb{R}_+^{K\times K}$ is the power allocation diagonal matrix with $\sqrt{\eta_1}, \ldots, \sqrt{\eta_K}$ on its diagonal, η_k is the power coefficient of user k, $\mathbf{s} = [s_1, \ldots, s_K]^T$ is the zero mean symbol vector, with $\sigma_s^2 = \mathbb{E}(|s_k|^2)$, s_k is the data symbol intended for user k (uncorrelated between users), $w_k \sim \mathcal{CN}(0, \sigma_w^2)$ is the additive noise for user k and σ_w^2 is the noise variance. For all users combined, we have

$$\mathbf{y} = \sqrt{\rho_f}\ \mathbf{G}^T\mathbf{P}\mathbf{N}\mathbf{s} + \mathbf{w}, \qquad (5)$$

where $\mathbf{G} \in \mathbb{C}^{M\times K}$ is the channel matrix with elements $[\mathbf{G}]_{mk} = g_{mk}$ and $\mathbf{w} = [w_1, \ldots, w_K]^T$ is the noise vector. We remark that the proposed iterative MMSE precoder can be adapted to a scalable cell-free approach [10], [11].

In order to obtain the MMSE precoder, the following optimization is performed [12]:

$$\{\mathbf{P}_{\text{MMSE}}, \mathbf{N}, f_{\text{MMSE}}\} = \text{argmin}_{\{\mathbf{P},\mathbf{N},f\}}\mathbb{E}\left[\left\|\mathbf{s} - f^{-1}\mathbf{y}\right\|_2^2\right] \qquad (6a)$$

$$\text{s.t.: } \mathbb{E}\left[\|\mathbf{x}\|_2^2\right] = E_{tr}, \qquad (6b)$$

where the transmitted signal is given by

$$\mathbf{x} = \sqrt{\rho_f}\ \mathbf{P}\mathbf{N}\mathbf{s}. \qquad (7)$$

The average transmit power is described by

$$\mathbb{E}\left[\|\mathbf{x}\|_2^2\right] = \rho_f \text{tr}\left(\mathbf{P}\mathbf{N}\mathbf{C}_s\mathbf{N}^H\mathbf{P}^H\right) = E_{tr}, \qquad (8)$$

where $\mathbf{C}_s = \mathbb{E}\left[\mathbf{s}\mathbf{s}^H\right]$ is the symbol covariance matrix.

Then, the MMSE precoder that takes into account power allocation for cell-free networks is defined as

$$\mathbf{P}_{\text{MMSE}} = \frac{f_{\text{MMSE}}}{\sqrt{\rho_f}}\left(\hat{\mathbf{G}}^*\hat{\mathbf{G}}^T + \frac{\text{tr}\,(\mathbf{C}_w)}{E_{tr}}\mathbf{I}_M\right)^{-1}\hat{\mathbf{G}}^*\mathbf{N}^{-1}, \qquad (9)$$

where

$$f_{\text{MMSE}} = \sqrt{E_{tr}/\text{tr}\left(\tilde{\mathbf{P}}\mathbf{C}_s\tilde{\mathbf{P}}^H\right)}, \qquad (10)$$

$\tilde{\mathbf{P}} = \left(\hat{\mathbf{G}}^*\hat{\mathbf{G}}^T + \frac{\text{tr}(\mathbf{C}_w)}{E_{tr}}\mathbf{I}_M\right)^{-1}\hat{\mathbf{G}}^*$, $[\hat{\mathbf{G}}]_{mk} = \hat{g}_{mk}$ is the CSI matrix and we assumed that $\text{tr}\,(\mathbf{C}_w) = K\sigma_w^2$. We initialize the precoding technique considering a power allocation matrix $\mathbf{N} = \mathbf{I}_K$. After computing the precoding matrix, we perform power allocation based on the precoding vectors. With the new power allocation matrix $\mathbf{N}$, we substitute it in $\mathbf{P}_{\text{MMSE}}$. The last part of the iteration is to recalculate the matrix $\mathbf{N}$ by employing $\mathbf{P}_{\text{MMSE}}$ to perform power allocation. The final matrix $\mathbf{N}$, called $\mathbf{N}_{\text{MMSE}}$, guarantees that the power constraint is satisfied. The $\mathbf{N}$ present in the $\mathbf{P}_{\text{MMSE}}$ expression is different from the final $\mathbf{N}_{\text{MMSE}}$. Thus, $\mathbf{N}$ and $\mathbf{N}_{\text{MMSE}}$ will not cancel each other. The main point of this iterative process is to make sure that the power allocation is taken into account in the precoding matrix and that the power coefficients are calculated based on the precoding vectors. For this reason, only these few iterations are necessary.

By expanding (4) and (9), we will define the received signal by user k as

$$\begin{aligned} y_k &= \sqrt{\rho_f}\ \mathbf{g}_k^T\mathbf{P}_{\text{MMSE}}\ \mathbf{N}_{\text{MMSE}}\ \mathbf{s} + w_k \\ &= \sqrt{\rho_f}\ \left(\hat{\mathbf{g}}_k + \tilde{\mathbf{g}}_k\right)^T\frac{f_{\text{MMSE}}}{\sqrt{\rho_f}}\left(\hat{\mathbf{G}}^*\hat{\mathbf{G}}^T + \frac{K\sigma_w^2}{E_{tr}}\mathbf{I}_M\right)^{-1}\hat{\mathbf{G}}^*\mathbf{N}^{-1} \\ &\quad \mathbf{N}_{\text{MMSE}}\ \mathbf{s} + w_k \end{aligned} \qquad (11)$$

$$= \underbrace{\sqrt{\rho_f}\ \hat{\mathbf{g}}_k^T \frac{f_{\text{MMSE}}}{\sqrt{\rho_f}} \left(\hat{\mathbf{G}}^* \hat{\mathbf{G}}^T + \frac{K\sigma_w^2}{E_{tr}} \mathbf{I}_M\right)^{-1} \hat{\mathbf{G}}^* \mathbf{N}^{-1} \mathbf{N}_{\text{MMSE}}\ \mathbf{s} +}_{\text{desired signal + interference}}$$

$$\underbrace{\sqrt{\rho_f}\ \tilde{\mathbf{g}}_k^T \frac{f_{\text{MMSE}}}{\sqrt{\rho_f}} \left(\hat{\mathbf{G}}^* \hat{\mathbf{G}}^T + \frac{K\sigma_w^2}{E_{tr}} \mathbf{I}_M\right)^{-1} \hat{\mathbf{G}}^* \mathbf{N}^{-1} \mathbf{N}_{\text{MMSE}}\ \mathbf{s} + w_k,}_{\text{CSI error}}$$

where $\hat{\mathbf{g}}_k = [\hat{g}_{1k}, \dots, \hat{g}_{Mk}]^T$ is the CSI vector for user k and $\tilde{\mathbf{g}}_k = [\tilde{g}_{1k}, \dots, \tilde{g}_{Mk}]^T$ is the CSI error vector for user k.

Considering Gaussian signaling, the achievable rate of user k with the proposed iterative MMSE precoder is given by

$$R_{k,\text{MMSE}} = \log_2(1 + \text{SINR}_{k,\text{MMSE}}), \tag{12}$$

where $\text{SINR}_{k,\text{MMSE}}$ denotes the SINR of user k and can be expressed as $\text{SINR}_{k,\text{MMSE}} = \mathbb{E}[|A_1|^2]/(\sigma_w^2 + \sum_{i=1,i\neq k}^{K} \mathbb{E}[|A_{2,i}|^2] + \mathbb{E}[|A_3|^2])$. The term $A_1 = \sqrt{\rho_f}\hat{\mathbf{g}}_k^T \mathbf{p}_k \sqrt{\eta_k} s_k$ is the desired signal, $A_{2,i} = \sqrt{\rho_f}\hat{\mathbf{g}}_k^T \mathbf{p}_i \sqrt{\eta_i} s_i$, is the interference caused by user i, for $i \neq k, i = 1, \dots, K$ and $A_3 = \sqrt{\rho_f}\tilde{\mathbf{g}}_k^T \mathbf{P}_{\text{MMSE}} \mathbf{N}_{\text{MMSE}} \mathbf{s}$ refers to CSI error. The mean-square values of A_1, $A_{2,i}$ and A_3 are computed as follows:

$$\begin{aligned}
\mathbb{E}\left[|A_1|^2\right] &= \mathbb{E}\left[\left(\sqrt{\rho_f}\hat{\mathbf{g}}_k^T \mathbf{p}_k \sqrt{\eta_k} s_k\right)^* \left(\sqrt{\rho_f}\hat{\mathbf{g}}_k^T \mathbf{p}_k \sqrt{\eta_k} s_k\right)\right] \\
&= \rho_f \operatorname{tr}\left(\eta_k \mathbf{p}_k^H \hat{\mathbf{g}}_k^* \hat{\mathbf{g}}_k^T \mathbf{p}_k\right) \\
&= \rho_f \eta_k \psi_k \\
\mathbb{E}\left[|A_{2,i}|^2\right] &= \mathbb{E}\left[\left(\sqrt{\rho_f}\hat{\mathbf{g}}_k^T \mathbf{p}_i \sqrt{\eta_i} s_i\right)^* \left(\sqrt{\rho_f}\hat{\mathbf{g}}_k^T \mathbf{p}_i \sqrt{\eta_i} s_i\right)\right] \\
&= \rho_f \operatorname{tr}\left(\eta_i \mathbf{p}_i^H \hat{\mathbf{g}}_k^* \hat{\mathbf{g}}_k^T \mathbf{p}_i\right) \\
&= \rho_f \eta_i \phi_{ki} \\
\mathbb{E}\left[|A_3|^2\right] &= \mathbb{E}\left[\left|\sqrt{\rho_f}\tilde{\mathbf{g}}_k^T \mathbf{P}_{\text{MMSE}} \mathbf{N}_{\text{MMSE}} \mathbf{s}\right|^2\right] \\
&= \rho_f \operatorname{tr}\left(\mathbf{N}_{\text{MMSE}}^2 \mathbf{P}_{\text{MMSE}}^H \mathbb{E}\left[\tilde{\mathbf{g}}_k^* \tilde{\mathbf{g}}_k^T\right] \mathbf{P}_{\text{MMSE}}\right) \\
&= \rho_f \sum_{i=1}^{K} \eta_i \gamma_{ki}
\end{aligned} \tag{13}$$

In the expressions above, $\psi_k = \mathbf{p}_k^H \hat{\mathbf{g}}_k^* \hat{\mathbf{g}}_k^T \mathbf{p}_k$, $\boldsymbol{\psi} = [\psi_1, \dots, \psi_K]^T$, $\phi_{ki} = \mathbf{p}_i^H \hat{\mathbf{g}}_k^* \hat{\mathbf{g}}_k^T \mathbf{p}_i$, $i \neq k$, $i = 1, \dots, K$, $\mathbf{p}_k = [p_{1k}, \dots, p_{Mk}]^T$ is the column k of matrix $\mathbf{P}_{\text{MMSE}}$, $\boldsymbol{\gamma}_k = \operatorname{diag}\left\{\mathbf{P}_{\text{MMSE}}^H \mathbb{E}\left[\tilde{\mathbf{g}}_k^* \tilde{\mathbf{g}}_k^T\right] \mathbf{P}_{\text{MMSE}}\right\}$, γ_{ki} is the ith element of vector $\boldsymbol{\gamma}_k$, and $\mathbb{E}\left[\tilde{\mathbf{g}}_k^* \tilde{\mathbf{g}}_k^T\right]$ is a diagonal matrix with $((1-n)\beta_{mk})$ on its mth diagonal element.

By substituting (13) in the $\text{SINR}_{\text{k,MMSE}}$ expression we get

$$\text{SINR}_{\text{k,MMSE}} = \frac{\rho_f \eta_k \psi_k}{\sigma_w^2 + \rho_f \sum_{i=1,i\neq k}^{K} \eta_i \phi_{ki} + \rho_f \sum_{i=1}^{K} \eta_i \gamma_{ki}}. \tag{14}$$

Next, we will perform Optimal Power Allocation (OPA) and Uniform Power Allocation (UPA) in order to obtain $\mathbf{N}$, a diagonal matrix with $\sqrt{\eta_1}, \dots, \sqrt{\eta_K}$ on its diagonal, used to readjust the precoder $\mathbf{P}_{\text{MMSE}}$ and find the matrix $\mathbf{N}_{\text{MMSE}}$.

III-B. Optimal Power Allocation (OPA)

The max-min fairness power allocation problem considering AP constraint is given by

$$\max_{\boldsymbol{\eta}} \min_{k} \text{SINR}_{k,\text{MMSE}}(\boldsymbol{\eta}) \tag{15a}$$

$$\text{s.t.} \sum_{i=1}^{K} \eta_i \delta_{mi} \leq 1, m = 1, \dots, M, \tag{15b}$$

where

$$\boldsymbol{\delta}_m = \operatorname{diag}\left\{\mathbb{E}\left[\mathbf{p}_m^T \mathbf{p}_m^*\right]\right\}, \tag{16}$$

δ_{mi} is the ith element of vector $\boldsymbol{\delta}_m$ and $\mathbf{p}_m = [p_{m1}, \dots, p_{mK}]$ is the mth row of the precoding matrix $\mathbf{P}_{\text{MMSE}}$.

In $\text{SINR}_{\text{k,MMSE}}(\boldsymbol{\eta})$ (14), the numerator and denominator are linear functions of $\boldsymbol{\eta}$, which makes the total expression a quasilinear function, enabling us to use the bisection method presented in [19]. Then, the optimization problem in an epigraph form at each step of the bisection method is

$$\text{find } \boldsymbol{\eta} \tag{17a}$$

$$\text{s.t. } \text{SINR}_{\text{k,MMSE}}(\boldsymbol{\eta}) \geq t,\ k = 1, \dots, K, \tag{17b}$$

$$\sum_{i=1}^{K} \eta_i \delta_{mi} \leq 1,\ m = 1, \dots, M, \tag{17c}$$

where $t = (t_b + t_e)/2$ is the midpoint of a chosen interval (t_b, t_e) that contains the optimal value t^*.

III-C. Uniform Power Allocation (UPA)

A uniform solution is also proposed based on [7], to be compared with the OPA scheme. Suppose that we wish to obtain equal η_k and that a certain AP m transmits with full power. Then, considering η_k at its minimum possible value, we have

$$\eta_k = 1/\left(\max_{m} \sum_{i=1}^{K} \delta_{mi}\right),\ k = 1, \dots, K, \tag{18}$$

where δ_{mi} is the ith element of vector $\boldsymbol{\delta}_m$.

Although (18) is an approximation of the optimal solution, it is also a less complex and cost-effective alternative in order to show the benefits of the MMSE precoder.

IV. SIMULATIONS

Numerical results are presented for the proposed iterative technique where a linear MMSE precoder is calculated considering $\mathbf{N} = \mathbf{I}_K$, and then used to perform power allocation. With the corresponding power coefficients, a readjusted precoder is found and use to recompute the final power allocation, completing two iterations in total. In our simulations, users do not cooperate to decode the received signal, as often assumed in the study of precoders. Although the proposed iterative MMSE precoding and power allocation techniques do not consider scalability aspects for large systems, they can be extended according to the concepts in [10], [11]. In all experiments, we performed 120 channel realizations and assumed $\sigma_s^2 = 1$.

We considered M single-antenna APs and K single-antenna users uniformly distributed at random within an area of 1 km^2. The large-scale fading coefficients from (1) are modeled by

$$\beta_{mk} = \text{PL}_{mk} \cdot 10^{\frac{\sigma_{sh} z_{mk}}{10}} \tag{19}$$

where PL_{mk} is the path loss, and $10^{\frac{\sigma_{sh} z_{mk}}{10}}$ accounts for the shadow fading with standard deviation $\sigma_{sh} = 8$ dB and $z_{mk} \sim \mathcal{N}(0, 1)$. The path loss is based on a three slope model [20], in dB, defined as

$$\text{PL}_{mk} = \begin{cases} -L - 35\log_{10}(d_{mk}), & \text{if } d_{mk} > d_1 \\ -L - 15\log_{10}(d_1) - 20\log_{10}(d_{mk}), & \\ \qquad \text{if } d_0 < d_{mk} \leq d_1 & \\ -L - 15\log_{10}(d_1) - 20\log_{10}(d_0), & \text{if } d_{mk} \leq d_0 \end{cases} \tag{20}$$

where

$$\begin{aligned} L \triangleq{} & 46.3 + 33.9\log_{10}\left(f_{freq}\right) - 13.82\log_{10}\left(h_{\text{AP}}\right) \\ & - \left(1.1\log_{10}\left(f_{freq}\right) - 0.7\right) h_{\text{u}} + \left(1.56\log_{10}\left(f_{freq}\right) - 0.8\right), \end{aligned} \tag{21}$$

d_{mk} is the distance between the mth AP and the kth user, $d_1 = 50$ m, $d_0 = 10$ m, $f_{freq} = 1900$ MHz is the carrier frequency in MHz, $h_{\text{AP}} = 15$ m is the AP antenna height in meters and $h_{\text{u}} = 1.65$ m is the user antenna height in meters, as in [6]. When $d_{mk} \leq d_1$ there is no shadowing.

Considering strong path loss, which is characteristic of Cell-Free Massive MIMO systems, we will define ρ_f based on the signal-to-noise ratio (SNR) given by [21]

$$\rho_f = \frac{\text{SNR} \cdot \text{tr}\left(\mathbf{C}_w\right)}{\mathbb{E}[||\hat{\mathbf{G}}||_F^2]} = \frac{\text{SNR} \cdot K\sigma_w^2}{\text{tr}\left(\hat{\mathbf{G}}\hat{\mathbf{G}}^H\right)} \tag{22}$$

where,

$$\sigma_w^2 = T_0 \times k_B \times B \times NF(\text{W}), \tag{23}$$

$T_0 = 290$ (Kelvin) is the noise temperature, $k_B = 1.381 \times 10^{-23}$ (Joule per Kelvin) is the Boltzmann constant, $B = 20$ MHz is the bandwidth and $NF = 9$ dB is the noise figure.

In the first experiment, whose results are shown in Fig. 2 and 3, we compare the performance of the proposed iterative MMSE precoder considering UPA and OPA with that of the CB, ZF and MMSE precoders from [6], [7] and [12], in terms of sum-rate vs. SNR, taking into account perfect (Fig. 2) and imperfect CSI (Fig. 3).

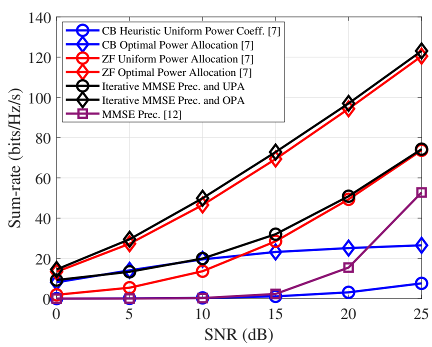

Fig. 2. Sum-rate vs. SNR with M = 128, K = 16, n = 1, 120 channel realizations and $E_{tr} = M\rho_f$.

First, we can observe that the iterative MMSE precoder with OPA has the best performance compared to other precoders, followed by the ZF precoder with Optimal Power Allocation. Second, the iterative MMSE precoder with UPA also outperforms both CB precoders, ZF with Uniform Power Allocation from [7] and the MMSE Precoder from [12]. We then notice that the optimal schemes, applied to all three precoders, are significantly better than the uniform techniques. As expected, when adding channel estimation error ($n < 1$), smaller rates are achieved, for all precoders.

Next, we investigate the performance of the proposed iterative MMSE precoder, with UPA and OPA, compared with the same CB, ZF and MMSE precoders, in terms of BER vs. SNR. In this case we consider quadrature phase shift keying (QPSK) modulation and both perfect (Fig. 4) and imperfect CSI (Fig. 5).

Fig. 4 and 5 provide almost the same insight given in the previous result, with the iterative MMSE precoder with OPA having the best performance. As expected, the proposed iterative MMSE precoder with UPA outperforms both CB precoders, the ZF precoder with Uniform Power Allocation from [7] and the MMSE precoder from [12]. The performance is deteriorated when adding channel estimation error ($n < 1$). Finally, the optimal techniques also outperform the uniform solutions in terms of BER for each precoding scheme.

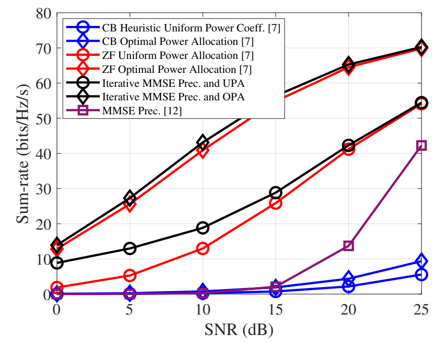

Fig. 3. Sum-rate vs. SNR with M = 128, K = 16, n = 0.9, 120 channel realizations and $E_{tr} = M\rho_f$.

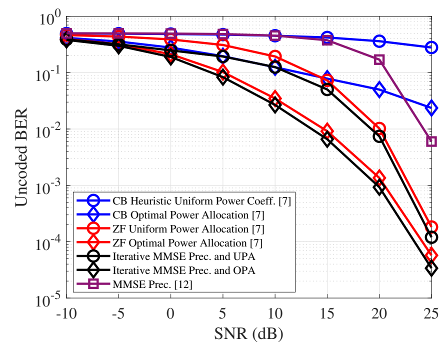

Fig. 4. BER vs. SNR with $M = 128$, $K = 16$, $n = 1$, 120 channel realizations, 100 symbols per packet and $E_{tr} = M\rho_f$.

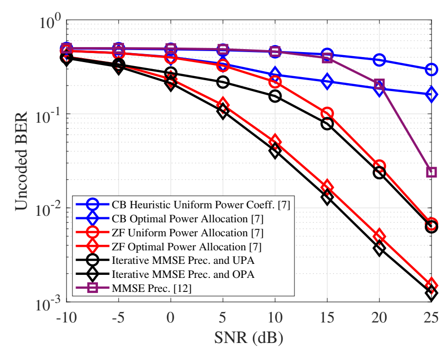

Fig. 5. BER vs. SNR with $M = 128$, $K = 16$, $n = 0.9$, 120 channel realizations, 100 symbols per packet and $E_{tr} = M\rho_f$.

V. CONCLUSIONS

We analyzed the downlink channel performance of a Cell-Free Massive MIMO system in terms of sum-rate and BER, in the presence of perfect and imperfect CSI. An iterative MMSE precoder is developed, together with UPA and OPA techniques. Numerical results indicate that the proposed iterative MMSE precoder with UPA and OPA, outperform existing CB, ZF and MMSE schemes. Furthermore, the performance is degraded when taking into account channel estimation errors, for all precoders. The scalability of the proposed system will be analyzed in future works.

VI. REFERENCES

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