University of Nevada, Reno

Lecture 8: Likelihood

Perry Williams, PhD

NRES 779

Bayesian Hierarchical Modeling in Natural Resources

Why Likelihood

- Likelihood is a component of all Bayesian models.
- Maximum likelihood is an important statistical approach in its own right

Learning Objectives (Lecture and Lab)

- Understand the concept of likelihood and its relationship to the probability of the data conditional on the parameters
- Describe a likelihood profile and how it differs from the plot of a probability density function.
- Be able to use single and multiple observations to obtain maximum likelihood estimates of single and multiple parameters

Two Perspectives

1. $[y|\boldsymbol{\theta}]$

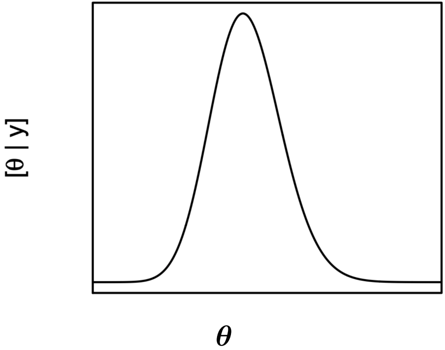
Prevalence is a term used in disease ecology to indicate the proportion of a population that is infected. Prevalence of chronic wasting disease in male mule deer on winter range in Georgetown, CA average 12%. A sample of 24 male mule deer includes 4 infected individuals. **What is the probability of obtaining these data if the estimate of prevalence is true?**

Two Perspectives

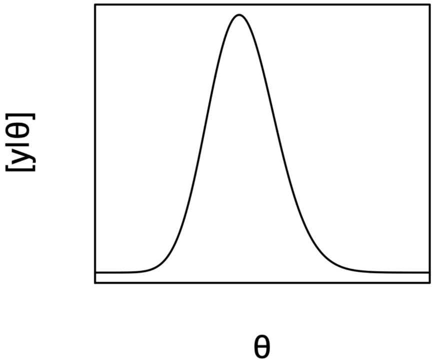
2. $[\boldsymbol{\theta}|y]$

We obtain a sample of 24 male mule deer that include 4 infected individuals from the mule deer winter range in Georgetown, CA. In light of these data, **what is the probability that the true value of prevalence, θ, is found in $q_L \leq \theta \leq q_U$?**

Bayesian inference is based on $[\theta|y]$



Likelihood inference is based on $[y|\theta]$



Likelihood allows us to compare alternative parameter values and models by calculating the probability of the data, conditional on the parameters $[y|\theta]$. As you will see, all evidence based on likelihood is relative.

The Key Idea in Likelihood

- In a probability mass or probability density function, the parameter $\boldsymbol{\theta}$ is constant (known) and the data y are random variables. The function sums or integrates to 1 over its support (normal pdf example; $[y|\theta]$).
- In a likelihood function, the data are constant (known) and the parameter is unknown but fixed. We use $[y|\boldsymbol{\theta}]$ to assess the likelihood of different values of $\boldsymbol{\theta}$ in light of the data. In this case, the function does not sum or integrate to one over all possible values of the parameter.

$$\underbrace{L(\boldsymbol{\theta}|y)}_{\text{likelihood function}} \propto \underbrace{[y|\boldsymbol{\theta}]}_{\text{PDF or PMF}} \tag{1}$$

Likelihood is *proportional* to probability or probability density.

Discuss Notation

$$L(\boldsymbol{\theta}|y) \propto [y|\boldsymbol{\theta}]$$
$$L(\boldsymbol{\theta}|y) = c[y|\boldsymbol{\theta}]$$
$$L(\boldsymbol{\theta}|y) = [y|\boldsymbol{\theta}]$$

Intuition for Likelihood

The parameter is fixed

We *know* the parameter value, $\theta = 0.5$. We make three draws.

- What are the possible outcomes for the number of white beans?
- What probability mass function would you use to model these data?
- What is the probability of each outcome?
- What is the sum of the individual probabilities?

The Parameter is Fixed and the Data Vary

We make three draws

θ	number of white beans, y_i	$[y_i \mid \theta = .5]$
.5	0	.125
.5	1	.375
.5	2	.375
.5	3	.125
	$\sum_{i=1}^{4}[y_i \mid \theta = .5]$	1

The Data are Fixed and the Parameter Varies

We make a single draw of three beans and obtain two whites. We seek evidence in the data for three hypothesized parameter values,

- $\boldsymbol{\theta} = \frac{1}{6}$
- $\boldsymbol{\theta} = \frac{1}{2}$
- $\boldsymbol{\theta} = \frac{5}{6}$

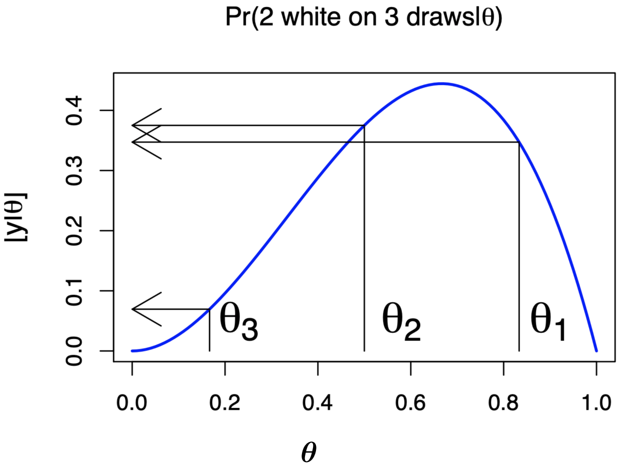
Compute the likelihoods for these parameter values and sum them.

The Data are Fixed and the Parameter Varies

We have three hypothesized parameter values. Data in hand are 2 whites on three draws. What is the likelihood of each parameter value?

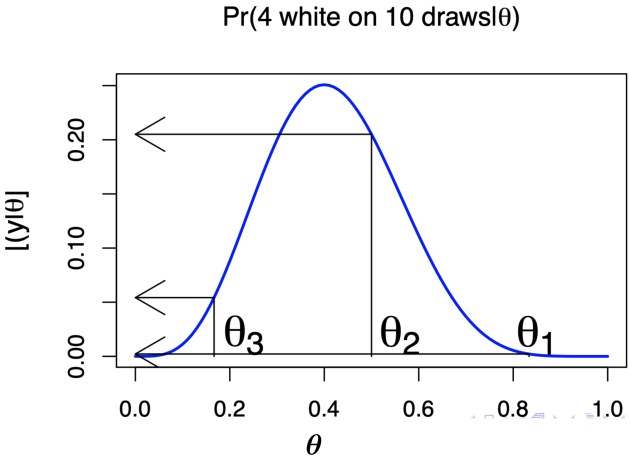
Hypothesis, θ_i	$[y \mid \theta_i]$
$\theta_1 = 5/6$	.347
$\theta_2 = 1/2$	.375
$\theta_3 = 1/6$	.069
$\sum_{i=1}^{3}[y \mid \theta_i]$	.791

A Likelihood Profile

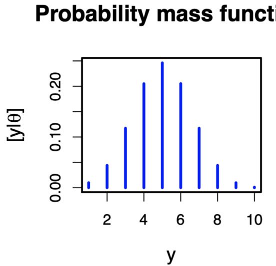


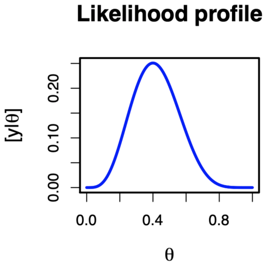
A Likelihood Profile

We now draw 10 beans and obtain 4 whites



Likelihood vs Probability





The *points* on each graph are probabilities. The graph on the left is a true PMF. The graph on the right is not.

Exercise

We fix $\boldsymbol{\theta} = 0.5$ and vary the data in the probability mass function. We fix the data at two whites on three draws and vary $\boldsymbol{\theta}$ in the likelihood profile. There is a single point on each graph with the exact same value. What is that point?