Understanding the Logit Function and Inverse Logit Function in Logistic Regression

Introduction:

Logistic regression is a statistical method used for modeling the probability of a binary outcome based on one or more predictor variables. Central to logistic regression are the logit function and its inverse, the inverse logit function. This document aims to elucidate these functions and their significance in logistic regression.

1. The Logit Function:

In logistic regression, the logit function is used to model the relationship between the independent variables and the binary dependent variable. It transforms the probability of the event occurring into the log-odds or logit, which is a linear function of the predictors.

The logit function is defined as:

$$\text{logit}(p) = \log\left(\frac{p}{1-p}\right)$$

Where:

- p is the probability of the event occurring.
- log denotes the natural logarithm.

2. Interpretation of Logit:

- The logit function transforms the probability p into a continuous variable that ranges from $-\infty$ to $+\infty$.
- It maps the probability scale (0 to 1) to the log-odds scale.
- Positive log-odds indicate that the event is more likely to occur, while negative log-odds suggest the event is less likely to occur.

3. The Inverse Logit Function:

The inverse logit function, also known as the logistic function or sigmoid function, is the inverse of the logit function. It converts the log-odds back into probabilities, allowing us to interpret the results of logistic regression models in terms of probabilities.

The inverse logit function is defined as:

$$p = \frac{e^{\beta_0 + \beta_1 x}}{1 + e^{\beta_0 + \beta_1 x}}$$

Where:

- p represents the probability of success ($P(y = 1)$).
- e is the base of the natural logarithm (Euler's number).

4. Interpretation of Inverse Logit:

- The inverse logit function transforms the log-odds $\beta_0 + \beta_1 x$ back into probabilities.
- It squashes the log-odds scale (from $-\infty$ to $+\infty$) into a probability scale (from 0 to 1).
- Probabilities obtained from the inverse logit function represent the likelihood of the event occurring.

5. Practical Application:

In logistic regression:

- We use the logit function to model the linear relationship between predictors and the log-odds of the binary outcome.
- After obtaining the model output (logit), we apply the inverse logit function to convert it into predicted probabilities.

Conclusion:

Understanding the logit and inverse logit functions is crucial for interpreting logistic regression results. They facilitate the transformation between probability and log-odds scales, enabling us to make predictions and infer the likelihood of binary outcomes based on predictor variables.

Practice Problems

1. Given a logistic regression model output of $\beta_0 + \beta_1 x = 0.75$, find the probability using the inverse logit function.

2. Calculate the probability for a logistic regression model output of $\beta_0 + \beta_1 x = 0.4$.

3. If the log-odds are 2.5, what is the corresponding probability using the inverse logit function?

4. Determine the probability when the log-odds are -1.8.

5. For a logistic regression model output of $\beta_0 + \beta_1 x = 0.6$, find the log-odds using the inverse logit function.

6. Find the log-odds for a probability of $p = 0.3$.

7. Given the log-odds of -1.2, calculate the probability using the inverse logit function.

8. Determine the probability when the log-odds are 0.8.

9. Find the log-odds for a probability of 0.2.

10. For a log-odds of 1.7, what is the probability, using the inverse logit function?

11. Calculate the logistic regression model output (i.e., probability) when the log-odds are -2.3.

12. Determine the log odds for a probability of 0.8.

13. For a probability of 0.7, what are the log-odds using the logit function?

14. Calculate the log-odds when the probability is 0.4.

15. If the log-odds are -0.5, what is the probability using the inverse logit function?

16. Determine the logistic regression model output for log-odds of 2.0.