University of Nevada, Reno

Lecture 02: What sets Bayes apart?

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NRES 779

Bayesian Hierarchical Modeling in Natural Resources

Today

- A high elevation view of approaches for statistical inference
- Some motivation for learning
- The basic ideas of Bayesian inference

Learning Objectives

- Understand basic notation for data, parameters, and conditional distributions
- Understand fundemental difference between frequentist inference and Bayesian inference
- Understand and interpret the Bayesian credible interval vs. frequentist confidence interval

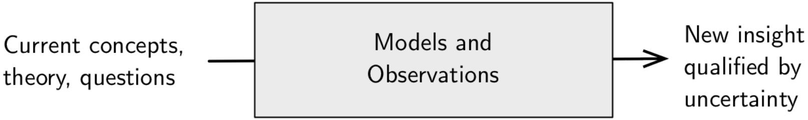
Exercise

Write the definition of a frequentist, 95% confidence interval on a parameter of interest, θ.

Exercise

In frequentist statistics, a 95% confidence interval represents an interval of a specified width such that if the experiment or sample were repeated many times, 95% of the intervals would contain the true parameter value.

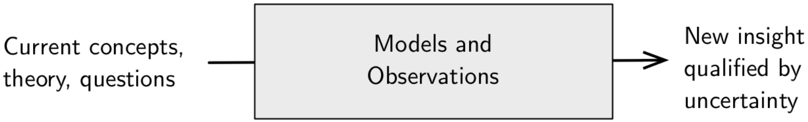
A line of inference



Some notation

- y is data
- $\boldsymbol{\theta}$ a parameter or other unknown quantity of interest
- $[\boldsymbol{\theta}|y]$ The probability distribution of $\boldsymbol{\theta}$ conditional on y
- $[y|\boldsymbol{\theta}]$ The probability distribution of y conditional on $\boldsymbol{\theta}$
- $[y|\boldsymbol{\theta}] = P(y|\boldsymbol{\theta}) = p(y|\boldsymbol{\theta}) = f(y|\boldsymbol{\theta}) f(y, \boldsymbol{\theta})$, different notation that means the same thing.

Bayesian Credible Interval



Exercise

Describe how Bayesian analysis differs from other types of statistical analyses.

What sets Bayes apart?

- Bayesians divide the world into things that are observed (y) and unobserved ($\boldsymbol{\theta}$).
- All unobserved quantities are treated as *random variables*.
- A random variable is a quantity whose behavior is governed by chance.
- Probability distributions are mathematical abstractions of "governed by chance."
- We seek to understand the characteristics of these probability distributions, particularly $[\boldsymbol{\theta}|y]$.

What sets Bayes apart?

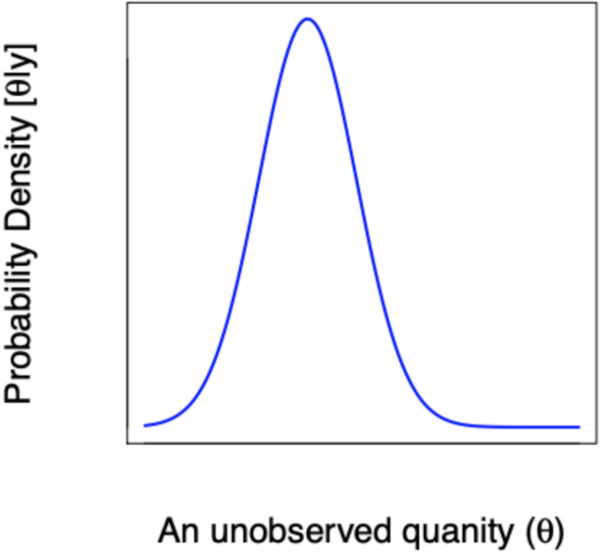
Treating unobserved quantities as random variables is profound!

What sets Bayes apart?

All unobserved quantities are treated in exactly the same way.

- Parameters
- Latent states
- Missing data
- Censored data
- Predictions in space (interpolations)
- Predictions in time (forecasts)

Bayesian Credible Interval



You Can Understand It

KEY TO STATISTICAL METHODS

	Design or Purpose	Measurement Variables	Ranked Variables	Attributes
1 variable 1 sample	Examination of a single sample	Procedure for grouping a frequency distribution, Box 2.1; stem-and-leaf display, Section 2.5; testing for outliers, Section 13.4 Computing median of frequency distribution, Box 4.1 Computing arithmetic mean: unordered sample, Box 4.2; frequency distribution, Box 4.3 Computing standard deviation: unordered sample, Box 4.2; frequency distribution, Box 4.3 Setting confidence limits: mean, Box 7.2; variance, Box 7.3 Computing g_1 and g_2, Box 6.2		Confidence limits for a percentage, Section 17.1 Runs test for randomness in dichotomized data, Box 18.3
	Comparison of a single sample with an expected frequency distribution	Normal expected frequencies, Box 6.1 Goodness of fit tests: parameters from an extrinsic hypothesis, Box 17.1; from an intrinsic hypothesis, Box 17.2 Kolmogorov–Smirnov test of goodness of fit, Box 17.3 Graphic "tests" for normality: large sample sizes, Box 6.3; small sample sizes (rankit test), Box 6.4 Test of sample statistic against expected value, Box 7.4		Binomial expected frequencies, Box 5.1 Poisson expected frequencies, Box 5.2 Goodness of fit tests: parameters from an extrinsic hypothesis, Box 17.1; from an intrinsic hypothesis, Box 17.2
1 ariable ≥2 amples	Single classification	Single classification anova: unequal sample sizes, Box 9.1; equal sample sizes, Box 9.4 Planned comparison of means in anova, Box 9.8; single degree of freedom comparisons of means, Box 14.10 Unplanned comparison of means: T-method, equal sample sizes, Box 9.9; T′, GT2, and Tukey–Kramer, unequal sample sizes, Box 9.10; Welsch step-up, Box 9.11; STP test, Section 9.7; contrasts using Scheffé, T, and GT2, Box 9.12; multiple confidence limits, Section 14.10 Estimate variance components: unequal sample sizes, Box 9.2; equal sample sizes, Box 9.3 Setting confidence limits to a variance component, Box 9.3 Tests of homogeneity of variances, Box 13.1 Tests of equality of means when variances are heterogeneous, Box 13.2	Kruskal–Wallis test, Box 13.5 Unplanned comparison of means by a nonparametric STP, Box 17.5	*G*-test for homogeneity of percentages, Boxes 17.5 and 17.8 Comparison of several samples with an expected frequency distribution, Box 17.4; unplanned analysis of replicated tests of goodness of fit, Box 17.5
	Nested classification	Two-level nested anova: equal sample sizes, Box 10.1; unequal sample sizes, Box 10.4 Three-level nested anova: equal sample sizes, Box 10.3; unequal sample sizes, Box 10.5		
	Two-way or multi-way classification	Two-way anova: with replication, Box 11.1; without replication, Box 11.2; unequal but proportional subclass sizes, Box 11.4; with a single missing observation, Box 11.5 Three-way anova, Box 12.1 More-than-three-way classification, Section 12.3 and Box 12.2 Test for nonadditivity in a two-way anova, Box 13.4	Friedman's method for randomized blocks, Box 13.9	Three-way log-linear model, Box 17.9 Randomized blocks for frequency data (repeated testing of the same individuals), Box 17.11

You Can Understand It

P value:

A number that shows the likelyhood that a value is the same as another

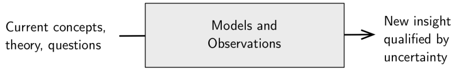
Confidence Interval- ~~Shows~~ A range of values that we have a certain level of confidence our value of interest falls in.

1) Definition of Pvalue
The probability of the significant difference between measured (observed) value & other measured values

2) What is confidence Interval?
The range of measured (observed) true population mean value can occur within it

You Can Understand It

- Rules of probability
  - Conditioning and independence
  - Law of total probability
  - Factoring joint probabilities
- Distribution theory
- Markov chain Monte Carlo



One Approach Tailored to Many Problems

- An unobservable state of interest, z
- A deterministic model of a process, $f(\boldsymbol{\theta}, x)$ controlling the state
- A model of the data
- Models of parameters (priors)