Lab 1

Victoria Peechatt & Khushi

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1 Introduction

This project deals with exploring chaos with the discrete logistic equation.

$$x_{t+1} = \lambda x_t(1 - x_t) \tag{1}$$

To explore chaos with the discrete logistic equation, we built an R function to run simulations with varying parameters. We wanted to understand the dynamics of population change as a function of λ. By producing a bifurcation diagram, we can visualize the chaotic trends in population growth as λ increases. Finally, we used a brute force approach to fit a logistic equation to a dataset measuring elk population over 35 years.

2 Discussion

With this simulation, we show population dynamics with a changing λ. When λ ε 0.25-0.75, the population is decreasing, similar to a negative exponential curve. When λ reaches one, population decay is linear. When λ increases from 1.25-2, population growth is becoming a logistic growth curve and population growth is stable after a certain time. When λ hits 2.25, fluctuations start at around $t = 6$. The fluctuations are subtle at first, and intensify when λ ε 2.5-3.5. When $\lambda = 3.75$ the intensity of fluctuations is dramatic and by $\lambda = 4$, population growth is so chaotic that the population size is close to zero.

When per capita growth is less than 1, the population is declining, which means that per individual in a population, less than one is replacing it in the next time period. When per capita growth is between 1-2.25, the population is growing, which means that per individual in a population, one or more exists in the next time period. But when there is a drastic increase in per capita growth, the population increases very fast, hits a threshold, and decreases. This is likely due to competition, limited resources, finite space, etc.

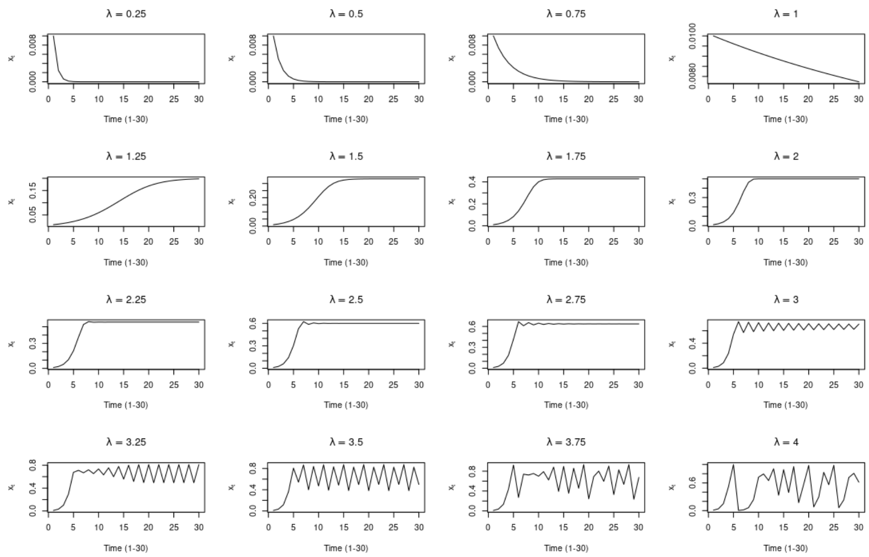
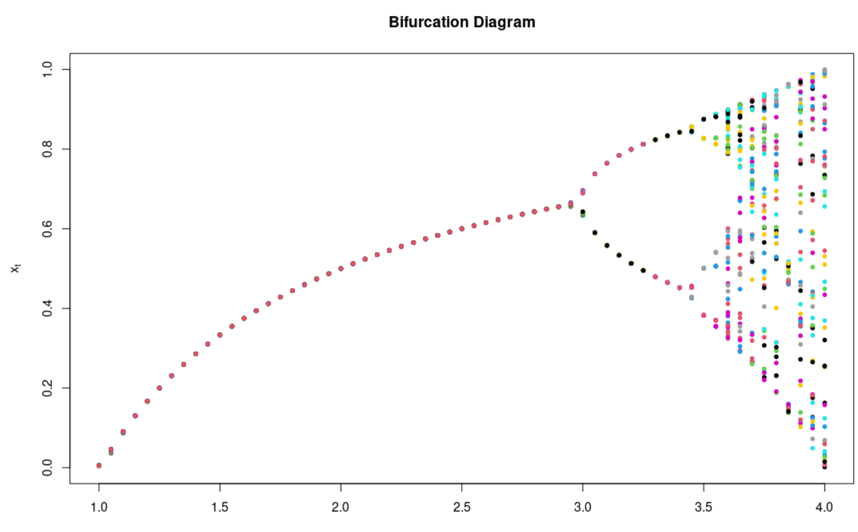


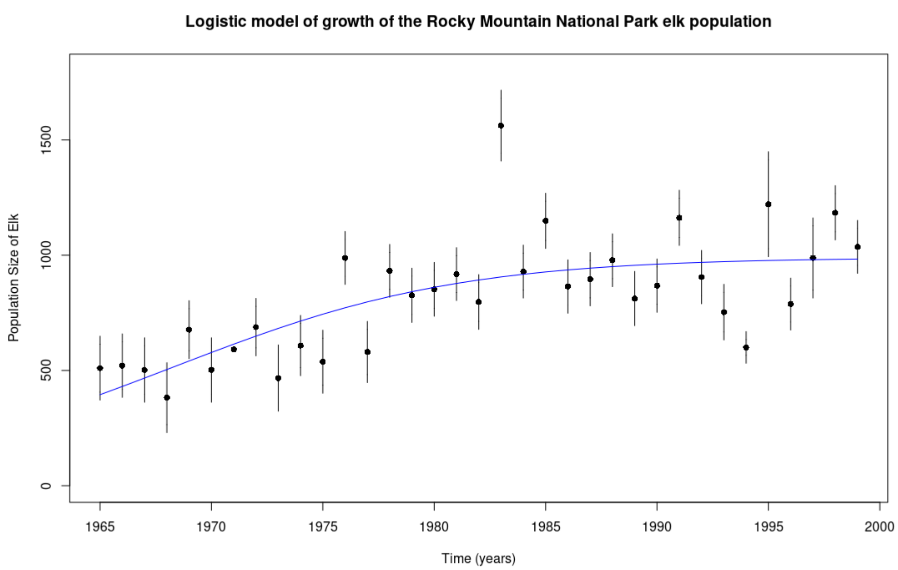
Figure 1: Population dynamics over time, with varying λ

The bifurcation diagram is showing that as λ increases from 1-3, the population is increasing at a stable rate. But when λ reaches a threshold 3, population growth is chaotic. Population size decreases and increases until $\lambda = 3.5$, at which point population growth is extremely chaotic.



Our previous 16 plots showed population dynamics for each λ over time, while the bifurcation diagram shows population dynamics over a change in λ . As per capita growth increases and reaches a certain threshold, population dynamics become chaotic.

Out of all of the problems with this brute force approach, the incapability to reproduce these results with this exact code is the greatest drawback. If we take the time and effort to write code, we might as well make it useful for future iterations of the same project/experiment, even if it takes a little bit of extra time. We're also unable to determine the limitations of our approach. Will this same equation work for more variable parameter values? Will this work for modeling data for different time periods? Instead of hard coding, we can make code more flexible to reuse and reproduce consistent results.



3 Conclusion

Using R to visualize, analyze, and understand data is one of the most important things we can learn as graduate students. This lab helped us refresh our memories and learn some new things. Even though hard-coding and using a brute force approach to determine the best fit model is not the best way to analyze our own data, it's an important exercise to learn the fundamentals of how code works with a deterministic model and an existing dataset. We learned how to apply code to the most specific scenarios and to answer the most specific questions, which is what we will try to do as scientists in our respective study systems.