Solutions to Practice Problems: Using the Inverse Logit Function in Logistic Regression

Solutions

1. **Given a logistic regression model output of $\beta_0 + \beta_1 x = 0.75$, find the probability using the inverse logit function.** We apply the inverse logit function to the given logistic regression model output:

$$p = \frac{e^{0.75}}{1 + e^{0.75}}$$

$$p \approx \frac{e^{0.75}}{1 + e^{0.75}} \approx \frac{2.117}{1 + 2.117} \approx \frac{2.117}{3.117} \approx 0.6805$$

So, the probability is approximately 0.6805.

2. **Calculate the probability for a logistic regression model output of $\beta_0 + \beta_1 x = 0.4$.**

Using the inverse logit function:

$$p = \frac{e^{0.4}}{1 + e^{0.4}}$$

$$p \approx \frac{e^{0.4}}{1 + e^{0.4}} \approx \frac{1.491}{1 + 1.491} \approx \frac{1.491}{2.491} \approx 0.599$$

So, the probability is approximately 0.599.

3. **If the log-odds are 2.5, what is the corresponding probability using the inverse logit function?**

Applying the inverse logit function:

$$p = \frac{e^{2.5}}{1 + e^{2.5}}$$

$$p \approx \frac{e^{2.5}}{1 + e^{2.5}} \approx \frac{12.182}{1 + 12.182} \approx \frac{12.182}{13.182} \approx 0.924$$

So, the corresponding probability is approximately 0.924.

4. **Determine the probability when the log-odds are -1.8.**

   Using the inverse logit function:

$$p = \frac{e^{-1.8}}{1 + e^{-1.8}}$$

$$p \approx \frac{e^{-1.8}}{1 + e^{-1.8}} \approx \frac{0.165}{1 + 0.165} \approx \frac{0.165}{1.165} \approx 0.142$$

   So, the probability is approximately 0.142.

5. **For a logistic regression model output of $\beta_0 + \beta_1 x = 0.6$, find the probability using the inverse logit function.**

   Applying the inverse logit function:

$$p = \frac{e^{0.6}}{1 + e^{0.6}}$$

$$p \approx \frac{e^{0.6}}{1 + e^{0.6}} \approx \frac{1.822}{1 + 1.822} \approx \frac{1.822}{2.822} \approx 0.646$$

   So, the probability is approximately 0.646.

6. **Find the log-odds for a probability of $p = 0.3$.**

   Using the logit function:

$$\text{logit}(p) = \log\left(\frac{p}{1-p}\right)$$

$$\text{logit}(0.3) = \log\left(\frac{0.3}{1-0.3}\right)$$

$$\text{logit}(0.3) = \log\left(\frac{0.3}{0.7}\right) \approx \log(0.429) \approx -0.845$$

   So, the log-odds for $p = 0.3$ is approximately -0.845.

7. **Given the log-odds of -1.2, calculate the probability using the inverse logit function.**

   Applying the inverse logit function:

$$p = \frac{e^{-1.2}}{1 + e^{-1.2}}$$

$$p \approx \frac{e^{-1.2}}{1 + e^{-1.2}} \approx \frac{0.301}{1 + 0.301} \approx \frac{0.301}{1.301} \approx 0.231$$

   So, the probability is approximately 0.231.

8. **Determine the probability when the log-odds are 0.8.**

   Using the inverse logit function:

$$p = \frac{e^{0.8}}{1 + e^{0.8}}$$

$$p \approx \frac{e^{0.8}}{1 + e^{0.8}} \approx \frac{2.225}{1 + 2.225} \approx \frac{2.225}{3.225} \approx 0.690$$

   So, the probability is approximately 0.690.

9. **Find the log-odds for a probability of 0.2.**

   Using the logit function:

$$\text{logit}(p) = \log\left(\frac{p}{1-p}\right)$$

$$\text{logit}(0.2) = \log\left(\frac{0.2}{1-0.2}\right)$$

$$\text{logit}(0.2) = \log\left(\frac{0.2}{0.8}\right) \approx \log(0.25) \approx -1.386$$

   So, the log-odds for $p = 0.2$ is approximately -1.386.

10. **For a log-odds of 1.7, what is the probability, using the inverse logit function?**

   Applying the inverse logit function:

$$p = \frac{e^{1.7}}{1 + e^{1.7}}$$

$$p \approx \frac{e^{1.7}}{1 + e^{1.7}} \approx \frac{5.473}{1 + 5.473} \approx \frac{5.473}{6.473} \approx 0.845$$

   So, the probability is approximately 0.845.

11. **Calculate the logistic regression model output (i.e., probability) when the log-odds are -2.3.**

   Applying the inverse logit function:

$$p = \frac{e^{-2.3}}{1 + e^{-2.3}}$$

$$p \approx \frac{e^{-2.3}}{1 + e^{-2.3}} \approx \frac{0.100}{1 + 0.100} \approx \frac{0.100}{1.100} \approx 0.476$$

   So, the probability is approximately 0.476.

12. **Determine the log odds for a probability of 0.8.**

    Using the logit function:

$$\text{logit}(p) = \log\left(\frac{p}{1-p}\right)$$

$$\text{logit}(0.8) = \log\left(\frac{0.8}{1-0.8}\right)$$

$$\text{logit}(0.8) = \log\left(\frac{0.8}{0.2}\right) \approx \log(4) \approx 1.386$$

    So, the log odds for $p = 0.8$ is approximately 1.386.

13. **For a probability of 0.7, what are the log-odds using the logit function?**

    Using the logit function:

$$\text{logit}(p) = \log\left(\frac{p}{1-p}\right)$$

$$\text{logit}(0.7) = \log\left(\frac{0.7}{1-0.7}\right)$$

$$\text{logit}(0.7) = \log\left(\frac{0.7}{0.3}\right) \approx \log(2.333) \approx 0.847$$

    So, the log-odds for $p = 0.7$ is approximately 0.847.

14. **Calculate the log-odds when the probability is 0.4.**

    Using the logit function:

$$\text{logit}(p) = \log\left(\frac{p}{1-p}\right)$$

$$\text{logit}(0.4) = \log\left(\frac{0.4}{1-0.4}\right)$$

$$\text{logit}(0.4) = \log\left(\frac{0.4}{0.6}\right) \approx \log(0.667) \approx -0.405$$

    So, the log-odds for $p = 0.4$ is approximately -0.405.

15. **If the log-odds are -0.5, what is the probability using the inverse logit function?**

    Applying the inverse logit function:

$$p = \frac{e^{-0.5}}{1+e^{-0.5}}$$

$$p \approx \frac{e^{-0.5}}{1+e^{-0.5}} \approx \frac{0.606}{1+0.606} \approx \frac{0.606}{1.606} \approx 0.274$$

    So, the probability is approximately 0.274.

16. **Determine the probability for log-odds of 2.0.**

    Applying the inverse logit function:

$$p = \frac{e^{2.0}}{1 + e^{2.0}}$$

$$p \approx \frac{e^{2.0}}{1 + e^{2.0}} \approx \frac{7.389}{1 + 7.389} \approx \frac{7.389}{8.389} \approx 0.880$$

    So, the probability is approximately 0.880.