ista_solve_hot

April 18, 2022

[54]:

```
import numpy as np
from scipy.io import loadmat
import matplotlib.pyplot as plt
from sklearn.svm import LinearSVC
import numpy.random
import math
```

[151]:

```
def ista_solve_hot( A, d, la_array ):
    # ista_solve_hot: Iterative soft-thresholding for multiple values of
    # lambda with hot start for each case - the converged value for the previous
    # value of lambda is used as an initial condition for the current lambda.
    # this function solves the minimization problem
    # Minimize |Ax-d|_2^2 + lambda*|x|_1 (Lasso regression)
    # using iterative soft-thresholding.
    max_iter = 10**4
    tol = 10**(-3)
    tau = 1/np.linalg.norm(A,2)**2
    n = A.shape[1]
    w = np.zeros((n,1))
    num_lam = len(la_array)
    X = np.zeros((n, num_lam))
    for i, each_lambda in enumerate(la_array):
        for j in range(max_iter):
            z = w - tau*(A.T@(A@w-d))
            w_old = w
            w = np.sign(z) * np.clip(np.abs(z)-tau*each_lambda/2, 0, np.inf)
            X[:, i:i+1] = w
            if np.linalg.norm(w - w_old) < tol:
                break
    return X
```

[242]:

```
in_data = loadmat('BreastCancer.mat')
X = in_data['X']
y = in_data['y']
```

[243]:

```
print(X.shape, y.shape)
```

```
(295, 8141) (295, 1)
```

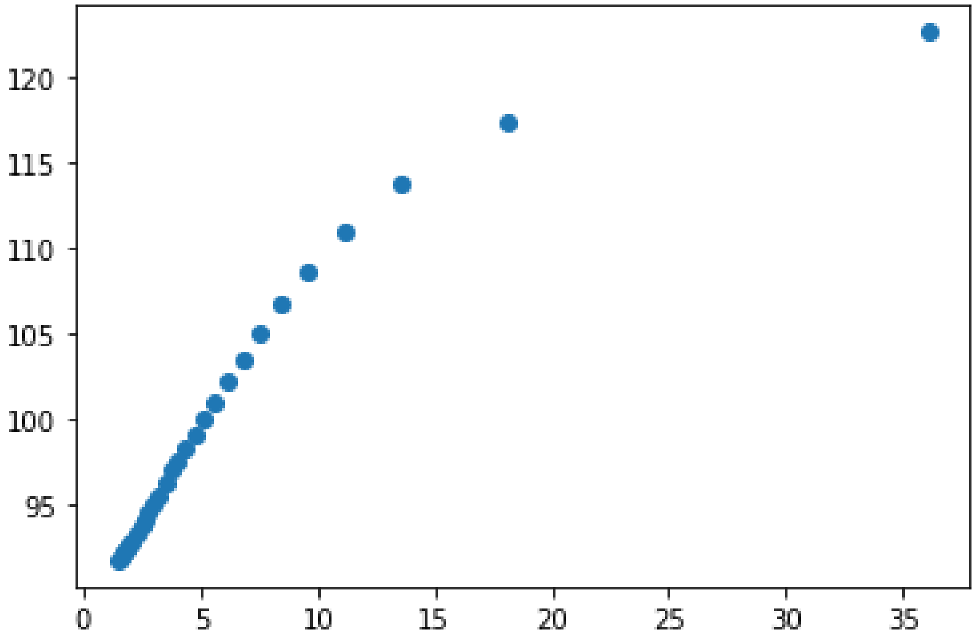
a)

[254]:
```
# We will use the first 100 patients
X_train = X[:100]
y_train = y[:100]
# Set lambda array with range (1e-6, 20) spaced logaritmically
lambda_array = []
i = 1e-6
while i < 19:
    lambda_array.append(i)
    i += math.log(2)
lambda_array.append(20)
w = ista_solve_hot(X_train, y_train, lambda_array)
```

[245]:
```
residual_error = []
w_norm = []
for i in range(len(lambda_array)):
    residual_error.append(np.linalg.norm(X_train@w[:,i] - y_train))
    w_norm.append(abs(w[:,i]).sum())
    print("w_norm: " , w_norm[i],"\tresidual_error: ", residual_error[i],␣
 ↪"\tlambda: ", lambda_array[i])
plt.scatter(x=w_norm, y=residual_error)
plt.show()
# As lambda increases the residual error and the norm of w decreases
```

```
w_norm:  36.109055952630406     residual_error:  122.62017023893092     lambda:
1e-06
w_norm:  18.1266980593891       residual_error:  117.3028693792493      lambda:
0.6931481805599453
w_norm:  13.51150995902426      residual_error:  113.75422580631017     lambda:
1.3862953611198905
w_norm:  11.207473857087024     residual_error:  110.96552155127395     lambda:
2.079442541679836
w_norm:  9.600042255143801      residual_error:  108.64335521508443     lambda:
2.7725897222397813
w_norm:  8.425137987735571      residual_error:  106.67098689091421     lambda:
3.4657369027997267
w_norm:  7.504537201776889      residual_error:  104.93988316904763     lambda:
4.158884083359672
w_norm:  6.785326085022339      residual_error:  103.44476518158844     lambda:
4.852031263919617
w_norm:  6.123578969469852      residual_error:  102.12035118812786     lambda:
5.545178444479562
w_norm:  5.579710963832548      residual_error:  100.97384444664706     lambda:
6.238325625039508
w_norm:  5.137386633432994      residual_error:  99.97513990804859      lambda:
6.931472805599453
w_norm:  4.7294810562842615     residual_error:  99.12557162414309      lambda:
7.624619986159399
```

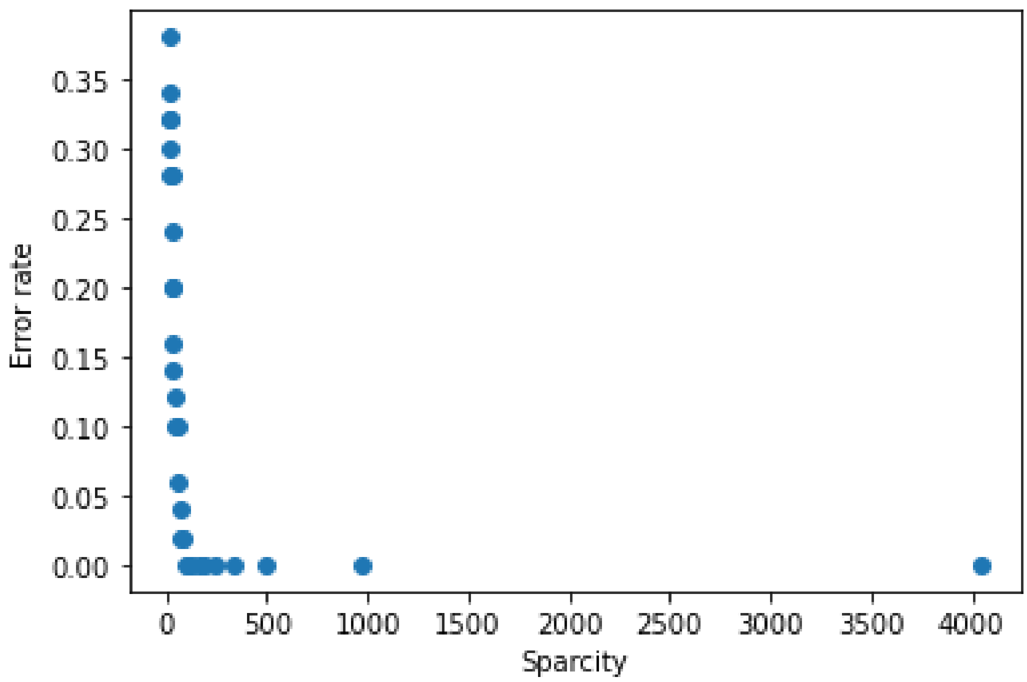
```
w_norm:  4.355999922114707      residual_error:  98.32365768967908      lambda:
8.317767166719344
w_norm:  3.9723493979938773     residual_error:  97.50426863953899      lambda:
9.01091434727929
w_norm:  3.75616629710407       residual_error:  97.0082963354693       lambda:
9.704061527839235
w_norm:  3.4778926816773117     residual_error:  96.27848306896577      lambda:
10.39720870839918
w_norm:  3.136669482594097      residual_error:  95.45134190969435      lambda:
11.090355888959126
w_norm:  2.9903158145972366     residual_error:  95.0538750806501       lambda:
11.783503069519071
w_norm:  2.764883558007083      residual_error:  94.46313591228582      lambda:
12.476650250079016
w_norm:  2.5863002243014237     residual_error:  93.99688909430621      lambda:
13.169797430638962
w_norm:  2.4504933690532065     residual_error:  93.6815802149507       lambda:
13.862944611198907
w_norm:  2.2633392534203525     residual_error:  93.24543582959832      lambda:
14.556091791758853
w_norm:  2.0540384197405395     residual_error:  92.78511251484086      lambda:
15.249238972318798
w_norm:  1.956318032917396      residual_error:  92.6079892263537       lambda:
15.942386152878743
w_norm:  1.8420240652914215     residual_error:  92.39630102542934      lambda:
16.635533333438687
w_norm:  1.7578344358851319     residual_error:  92.2403313247562       lambda:
17.32868051399863
w_norm:  1.6912694674229027     residual_error:  92.11599146101157      lambda:
18.021827694558574
w_norm:  1.5822541740103822     residual_error:  91.92717363184512      lambda:
18.714974875118518
w_norm:  1.426055638400412      residual_error:  91.72082256986741      lambda:
20
```



b)

[246]:

```
error_rate = []
sparcity = np.zeros(len(lambda_array))
for i in range(len(lambda_array)):
    diff = abs(np.sign(X_train@w[:,i]) - y_train[:,0])
    for j in w[:,i]:
        # Add to count the number of nonzero entries of w => if |w|_i > 1e-6
        if j > 1e-6:
            sparcity[i] += 1
    error_rate.append(diff.sum()/len(diff))
plt.scatter(x=sparcity, y=error_rate)
plt.xlabel("Sparcity")
plt.ylabel("Error rate")
plt.show()
# As the number of nonzero entries in w increases => the error rate decreases
```



c)

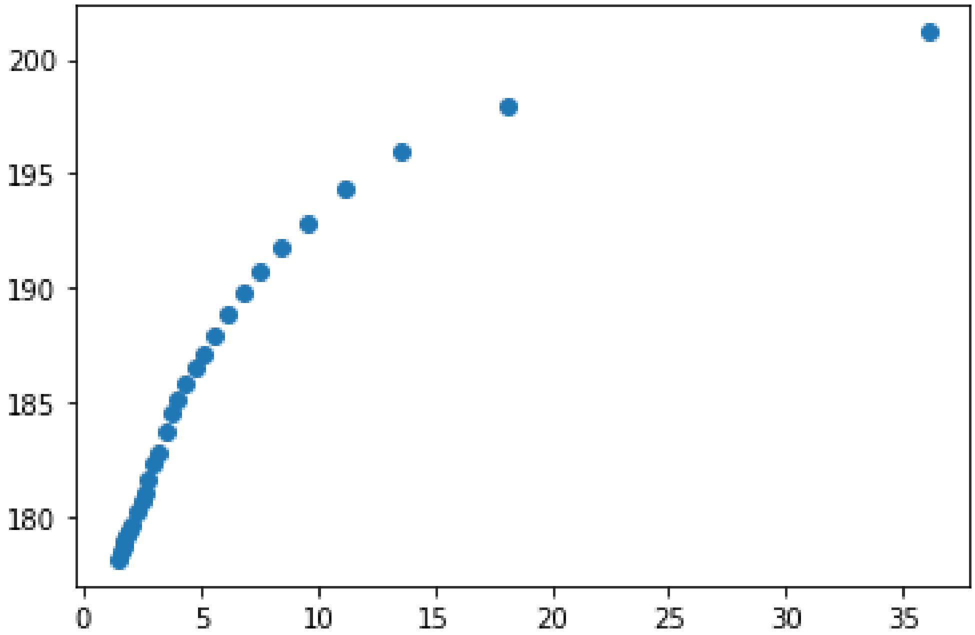
```
[247]: X_test = X[101:]
       y_test = y[101:]
```

```
[253]: # a)
       residual_error = []
       w_norm = []
       for i in range(len(lambda_array)):
           residual_error.append(np.linalg.norm(X_test@w[:,i] - y_test))
           w_norm.append(abs(w[:,i]).sum())
           print("w_norm: " , w_norm[i],"\tresidual_error: ", residual_error[i],␣
        ↪"\tlambda: ", lambda_array[i])
       plt.scatter(x=w_norm, y=residual_error)
       plt.show()
       # As with the train data, as lambda increases the residual error and the norm-1␣
        ↪of w decreases.
       # However, the plot show a more curved graph with it's point with a more␣
        ↪distance between them
```

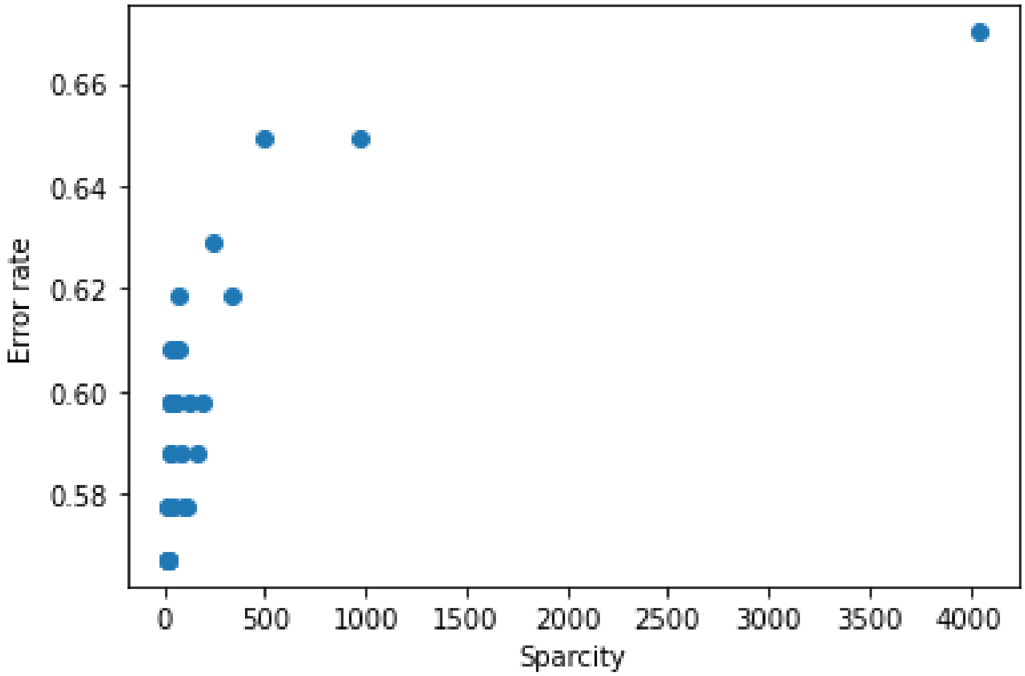
```
w_norm:  36.109055952630406     residual_error:  201.21304378897253     lambda:
1e-06
w_norm:  18.1266980593891       residual_error:  197.91570675411856     lambda:
0.6931481805599453
w_norm:  13.51150995902426      residual_error:  195.93637747867436     lambda:
1.3862953611198905
```

```
w_norm:  11.207473857087024     residual_error:  194.34688419042368     lambda:
2.079442541679836
w_norm:  9.600042255143801      residual_error:  192.82475715297167     lambda:
2.7725897222397813
w_norm:  8.425137987735571      residual_error:  191.78903343197146     lambda:
3.4657369027997267
w_norm:  7.504537201776889      residual_error:  190.782753642679       lambda:
4.158884083359672
w_norm:  6.785326085022339      residual_error:  189.77979622919315     lambda:
4.852031263919617
w_norm:  6.123578969469852      residual_error:  188.87130033785922     lambda:
5.545178444479562
w_norm:  5.579710963832548      residual_error:  187.91498643542022     lambda:
6.238325625039508
w_norm:  5.137386633432994      residual_error:  187.08197273897952     lambda:
6.931472805599453
w_norm:  4.7294810562842615     residual_error:  186.45962974159804     lambda:
7.624619986159399
w_norm:  4.355999922114707      residual_error:  185.78010100452067     lambda:
8.317767166719344
w_norm:  3.9723493979938773     residual_error:  185.0836551926895      lambda:
9.01091434727929
w_norm:  3.75616629710407       residual_error:  184.57120572253032     lambda:
9.704061527839235
w_norm:  3.4778926816773117     residual_error:  183.74603478027777     lambda:
10.39720870839918
w_norm:  3.136669482594097      residual_error:  182.7934822301498      lambda:
11.090355888959126
w_norm:  2.9903158145972366     residual_error:  182.30658498752715     lambda:
11.783503069519071
w_norm:  2.764883558007083      residual_error:  181.60219752866593     lambda:
12.476650250079016
w_norm:  2.5863002243014237     residual_error:  181.06824398874377     lambda:
13.169797430638962
w_norm:  2.4504933690532065     residual_error:  180.7230201718312      lambda:
13.862944611198907
w_norm:  2.2633392534203525     residual_error:  180.17953711686815     lambda:
14.556091791758853
w_norm:  2.0540384197405395     residual_error:  179.62115975839671     lambda:
15.249238972318798
w_norm:  1.956318032917396      residual_error:  179.41276239881375     lambda:
15.942386152878743
w_norm:  1.8420240652914215     residual_error:  179.11612841938015     lambda:
16.635533333438687
w_norm:  1.7578344358851319     residual_error:  178.87761119660146     lambda:
17.32868051399863
w_norm:  1.6912694674229027     residual_error:  178.69479609154843     lambda:
18.021827694558574
```

```
w_norm:  1.5822541740103822     residual_error:  178.4147295472156      lambda:
18.714974875118518
w_norm:  1.426055638400412      residual_error:  178.11348094047474     lambda:
20
```



[252]:
```
# b)
error_rate = []
sparcity = np.zeros(len(lambda_array))
for i in range(len(lambda_array)):
    diff = abs(np.sign(X_test@w[:,i]) - y_test[:,0])
    for j in w[:,i]:
        # Add to count the number of nonzero entries of w => if |w|_i > 1e-6
        if j > 1e-6:
            sparcity[i] += 1
    error_rate.append(diff.sum()/len(diff))
plt.scatter(x=sparcity, y=error_rate)
plt.xlabel("Sparcity")
plt.ylabel("Error rate")
plt.show()
# Now, the error rate vs sparcity behaves completely different as before. As␣
 ↪the sparcity increases, the error
# rate increases with it.
```


0.66
0.64
0.62
0.60
0.58
Error rate
0
500
1000
1500
2000
2500
3000
3500
4000
Sparcity