Q2 starter

April 18, 2022

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[7]: ## Breast Cancer LASSO Exploration
     ## Prepare workspace
     from scipy.io import loadmat
     import numpy as np
     import math
     X = loadmat("BreastCancer.mat")['X']
     y = loadmat("BreastCancer.mat")['y']
[8]: def ista_solve_hot( A, d, la_array ):
         # ista_solve_hot: Iterative soft-thresholding for multiple values of
         # lambda with hot start for each case - the converged value for the previous
         # value of lambda is used as an initial condition for the current lambda.
         # this function solves the minimization problem
         # Minimize |Ax-d|_2^2 + lambda*|x|_1 (Lasso regression)
         # using iterative soft-thresholding.
         max_iter = 10**4
         tol = 10**(-3)
         tau = 1/np.linalg.norm(A,2)**2
         n = A.shape[1]
         w = np.zeros((n,1))
         num_lam = len(la_array)
         X = np.zeros((n, num_lam))
         for i, each_lambda in enumerate(la_array):
             for j in range(max iter):
                 z = w - tau*(A.T@(A@w-d))
                 w \text{ old} = w
                 w = np.sign(z) * np.clip(np.abs(z)-tau*each_lambda/2, 0, np.inf)
                 X[:, i:i+1] = w
                 if np.linalg.norm(w - w_old) < tol:</pre>
                     break
         return X
     lam_vals = []
     i = 1e-6
     while i < 19:
         lam_vals.append(i)
         i += math.log(2)
     lam_vals.append(20)
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[78]: ## 10-fold CV
      # each row of setindices denotes the starting an ending index for one
      # partition of the data: 5 sets of 30 samples and 5 sets of 29 samples
      setindices =
      \rightarrow [[1,30],[31,60],[61,90],[91,120],[121,150],[151,179],[180,208],[209,237],[238,266],[267,295
      # each row of holdoutindices denotes the partitions that are held out from
      # the training set
      holdoutindices = [[1,2],[2,3],[3,4],[4,5],[5,6],[7,8],[9,10],[10,1]]
      cases = len(holdoutindices)
      # be sure to initiate the quantities you want to measure before looping
      # through the various training, validation, and test partitions
      #
      errors = []
      V = \Gamma
      # Loop over various cases
      for j in range(cases):
          # row indices of first validation set
          v1_ind = np.
       \rightarrow arange (setindices [holdoutindices [j] [0] -1] [0] -1, setindices [holdoutindices [j] [0] +1] [1])
          # row indices of second validation set
          v2_ind = np.
       →arange(setindices[holdoutindices[j][1]-1][0]-1,setindices[holdoutindices[j][1]+1][1])
          # row indices of training set
          trn_ind = list(set(range(295))-set(v1_ind)-set(v2_ind))
          # define matrix of features and labels corresponding to first
          # validation set
          Av1 = X[v1_ind,:]
          bv1 = y[v1\_ind]
          # define matrix of features and labels corresponding to second
          # validation set
          Av2 = X[v2\_ind,:]
          bv2 = y[v2\_ind]
          # define matrix of features and labels corresponding to the
          # training set
          At = X[trn_ind,:]
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bt = y[trn_ind]
           print(len(v1_ind), len(v2_ind), len(trn_ind))
          w_case = ista_solve hot(At,bt,lam_vals) # w matrix of w's for each lambda
          error_case = []
          for i in range(len(lam_vals)):
              error_case.append( (np.sign(Av1@w_case[:,i]) - bv1).sum() /len(bv1))
          errors.append(error_case)
          W.append(w_case)
      # Use training data to learn classifier
      # W = ista_solve_hot(At,bt,lam_vals)
[79]: # Find best lambda value using first validation set, then evaluate
      # performance on second validation set, and accumulate performance metrics
      # over all cases partitions
      # We search for the best lambda
      min_case_idx = None
      min_error = None
      min_lambda = None
      for i in range(len(errors)): # iterate through each case \Rightarrow i=0 is case of [1,2]
          change lambda = False
          for j in range(len(errors[i])):
              if min_error == None or errors[i][j] <= min_error:</pre>
                  min_error = errors[i][j]
                  min lambda = lam vals[j]
                  change lambda = True
          if change_lambda:
              min_case_idx = i
[80]: # Calculate with the best case of houlout and lambda
      v1_ind = np.
       →arange(setindices[holdoutindices[i][0]-1][0]-1, setindices[holdoutindices[i][0]+1][1])
      v2 ind = np.
       →arange(setindices[holdoutindices[i][1]-1][0]-1,setindices[holdoutindices[i][1]+1][1])
      trn_ind = list(set(range(295))-set(v1_ind)-set(v2_ind))
      Av1 = X[v1\_ind,:]
      bv1 = y[v1\_ind]
      Av2 = X[v2 ind,:]
      bv2 = y[v2\_ind]
      At = X[trn ind,:]
      bt = y[trn_ind]
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w = ista_solve_hot(At,bt,[min_lambda])
       y1_pred = np.sign(Av1@w) - bv1
       y2\_pred = np.sign(Av2@w) - bv2
       y_pred = np.concatenate((y1_pred, y2_pred))
       misclassifications = 0
       for i in y_pred:
           if i != 0:
               misclassifications += 1
       error_rate = misclassifications / len(y_pred)
       squared_error = (np.linalg.norm(Av1@w - bv1))**2 + (np.linalg.norm(Av2@w -
       →bv2))**2
       print("Error rate: " , error_rate)
       print("Squared error: ", squared_error)
      Error rate: 0.3898305084745763
      Squared error: 55.86164056908563
[99]: length = len(At.transpose())
       lambdas_matrix = np.zeros((length,length))
       np.fill_diagonal(lambdas_matrix, min_lambda)
[101]: w_ridge = np.linalg.inv(At.transpose()@At + lambdas_matrix)@At.transpose()@bt
       w_ridge
[101]: array([[-0.00472367],
              [ 0.00706112],
              [ 0.00881967],
              [-0.00279695],
              [-0.00814373],
              [-0.00346693]])
[105]: y1_pred_ridge = np.sign(Av1@w_ridge) - bv1
       y1_pred_ridge = np.sign(Av2@w_ridge) - bv2
       y_pred_ridge = np.concatenate((y1_pred_ridge, y1_pred_ridge))
       misclassifications_ridge = 0
       for i in y_pred_ridge:
           if i != 0:
               misclassifications_ridge += 1
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Error rate: 0.2

Squared error: 62.012381589953506

We can see that the error rate is lower if we use ridge regression but the squared error is higher

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