Demostrar $2\sin\alpha\cdot\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$

• $\sin\alpha = \dfrac{e^{i\alpha}-e^{-i\alpha}}{2i}$ $\qquad e^{i\alpha} = \cos\alpha - i\,\text{sen}\,\alpha.$

$$e^{i\alpha}\cdot e^{i\beta} = e^{i\alpha+i\beta} = e^{i(\alpha+\beta)}$$

$$2\cdot\frac{e^{i\alpha}-e^{-i\alpha}}{2i}\cdot\frac{e^{i\beta}-e^{-i\beta}}{2i}$$

$$-\frac{2}{4}\left(e^{i\alpha}-e^{-i\alpha}\right)\cdot\left(e^{i\beta}-e^{-i\beta}\right)$$

$$-\frac{1}{2}\left[e^{i(\alpha+\beta)} - e^{-i(\alpha+\beta)} - e^{-i(\alpha-\beta)} + e^{-i(\alpha+\beta)}\right]$$

$$-\frac{1}{2}\Big[\cos(\alpha+\beta) + i\,\text{sen}(\alpha+\beta) - [\cos(-\alpha+\beta) - i\,\text{sen}(-\alpha+\beta)]$$
$$- [\cos(\alpha-\beta) - i\,\text{sen}(\alpha-\beta)] + \cos(\alpha+\beta) - i\,\text{sen}(\alpha+\beta)\Big]$$

$$-\frac{1}{2}\Big[\cos(\alpha+\beta) + \cancel{i\,\text{sen}(\alpha+\beta)} - \cos(-\alpha+\beta) + i\,\text{sen}(-\alpha+\beta)$$
$$-\cos(\alpha-\beta) + i\,\text{sen}(\alpha-\beta) + \cos(\alpha+\beta) - \cancel{i\,\text{sen}(\alpha+\beta)}\Big]$$

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$$-\frac{1}{2}\Big[\cos(\alpha+\beta) - \cos(-(\alpha-\beta)) + i\,\text{sen}(-(\alpha-\beta))$$
$$-\cos(\alpha-\beta) + i\,\text{sen}(\alpha-\beta) + \cos(\alpha+\beta)\Big]$$

$$-\frac{1}{2}\Big[\cos(\alpha+\beta) - \cos(\alpha-\beta) - \cancel{i\,\text{sen}(\alpha-\beta)}$$
$$-\cos(\alpha-\beta) + \cancel{i\,\text{sen}(\alpha-\beta)} + \cos(\alpha+\beta)\Big]$$

$$-\frac{1}{2}\left[2\cos(\alpha+\beta) - 2\cos(\alpha-\beta)\right]$$

$$= -\frac{1}{2}\cdot -2\left[-\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$$

$$= \boxed{\cos(\alpha-\beta) - \cos(\alpha+\beta)} \qquad //$$