Métodos Numéricos - LE 2

Victória Xavier Queiroz

1) 1. **(3p)** Encontre uma solução específica **yp(t) = R cos(wt-a) para y" + 100y = cos(wt) - sin(wt).**

$$\boxed{\begin{array}{l} y'' + 100y = \cos(wt) - \sin(wt) \\ y_p = R\cos(wt - a) \\ y = y_p + y_h \end{array}}$$

$A = 1 \quad B = -1$

$\boxed{R^2 = A^2 + B^2}$

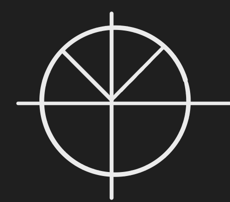
$R^2 = (1)^2 + (-1)^2$

$R^2 = 1 + 1$

$R^2 = 2 \rightarrow R = \pm\sqrt{2}$

$\boxed{tg\, a = \frac{B}{A}} = -\frac{1}{1} = -1$

$a = arc\, tg(-1) = -\frac{\pi}{4}$



$y_p = R\cos(\omega t - a) = A\cos(\omega t) + B\,sen(\omega t)$

$$\frac{d[y_p]}{dt} = -WR\,sen(\omega t - a)$$

$$\frac{d^2[y_p]}{dt^2} = -\omega^2 R\cos(\omega t - a)$$

substituindo

$-\omega^2[R\cos(\omega t - a)] + 100\cdot[R\cos(\omega t - a)] = \cos(\omega t) - sen(\omega t)$

$[100 - \omega^2]\cdot[R\cos(\omega t - a)] = \sqrt{2}\cdot\cos(\omega t + \pi/4)$

$[100 - \omega^2]\cdot[R\,\cancel{\cos(\omega t + \pi/4)}] = \sqrt{2}\cdot\cancel{\cos(\omega t + \pi/4)}$

$R(100 - \omega^2) = \sqrt{2}$

$R = \sqrt{2}/(100 - \omega^2) \rightarrow$

$y_p = R\cos(\omega t - a)$

$$\boxed{y_p = \frac{\sqrt{2}\cdot\cos(\omega t + \pi/4)}{100 - \omega^2}}$$

2) **(3p)** (a) Se você conhece **exp(iθ)** e **exp(-iθ)**, como pode encontrar **sin(θ)**? (b) Encontre todos os ângulos **θ** com **exp(iθ)** = -1, e (c) todos os ângulos ϕ com **exp(iϕ)** = i.

a) $e^{i\theta} = \cos(\theta) + i\,sen(\theta)$

$e^{-i\theta} = \cos(\theta) + (-i)\cdot sen(\theta)$

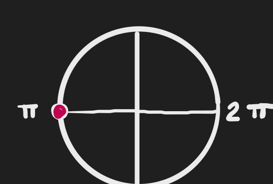
$e^{i\theta} - e^{-i\theta} = \cos(\theta) + i\,sen(\theta) - \cos(\theta) + i\,sen(\theta)$

$e^{i\theta} - e^{-i\theta} = 2i\cdot sen(\theta)$

$$\boxed{sen(\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2i}}$$

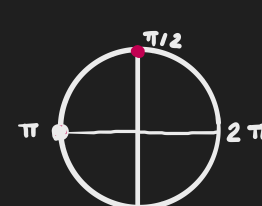
b) $e^{i\theta} = -1 \longrightarrow \cos(\theta) + i\,\cancel{sen(\theta)}^{0} = -1$

$\boxed{\theta = \pi(2k+1);\ k \in \mathbb{N}}$



c) $e^{i\phi} = i \longrightarrow \cancel{\cos(\phi)}^{0} + i\,sen(\phi) = i$

$\boxed{\phi = 2\pi\cdot k + \frac{\pi}{2};\ k \in \mathbb{N}}$



3) **(4p)** Qual equação de segunda ordem é resolvida por **y(t) = c1 exp(-2t) + c2 exp(-4t)**? Ou **y(t) = t exp (5t)**?

a) $y(t) = c_1\cdot e^{-2t} + c_2\cdot e^{-4t}$

-2 e -4 são raízes! $(\Delta > 0)$

$f(t) = (t+4)(t+2)$

$f(t) = t^2 + 6t + 8$

$\boxed{y'' + 6y' + 8y = 0}$

b) $y(t) = t\cdot e^{5t}$

5 é a única raiz! $(\Delta = 0)$

$f(t) = (t-5)(t-5)$

$f(t) = t^2 - 10t + 25$

$\boxed{y'' - 10y' + 25y = 0}$