Métodos Numéricos LE2.2

④ $P(D)y = e^{ct}$

$y_p = \dfrac{e^{ct}}{P(c)}$

a) $y_p'' + 3y_p' + 5y_p = e^t$

Solução Particular:

$y_p = A \cdot e^t$

$y_p' = A \cdot e^t$

$y_p'' = A \cdot e^t$

$A \cdot e^t + 3A \cdot e^t + 5Ae^t = e^t$

$\cancel{e^t}(A + 3A + 5A) = \cancel{e^t}$

$9A = 1$

$A = 1/9$

$\boxed{y_p = \dfrac{e^t}{9}}$

b) $2y_p'' + 4y_p = e^{it}$

Solução Particular

$y_p = A \cdot e^{it}$

$y_p' = A \cdot i \cdot e^{it}$

$y_p'' = Ai^2 \cdot e^{it} = -A \cdot e^{it} = -y_p$

$2(-A e^{it}) + 4A \cdot e^{it} = e^{it}$

$\cancel{e^{it}}(-2A + 4A) = \cancel{e^{it}}$

$2A = 1$

$A = 1/2$

$\boxed{y_p = \dfrac{e^{it}}{2}}$

c) $y'''' = e^t$

$\boxed{y = e^t}$ pois

$y' = e^t,$

$y'' = e^t$

$y''' = e^t$

$y'''' = e^t$

⑤ $y_p(t) = \dfrac{e^{cT}}{P(c)}$

$y'' + y = e^T \cdot e^{iT}$

$y'' + y = e^{T(1+i)}$

$y_p = A \cdot e^{T(1+i)}$

$y_p' = A(1+i) \cdot e^{T(1+i)}$

$y_p'' = A(1+i)^2 \cdot e^{T(1+i)}$

$Ae^{T(1+i)} + A(1+i)^2 e^{T(1+i)} = e^{T(1+i)}$

$\cancel{e^{T(1+i)}}\left[A + A(\cancel{1} + 2i + \cancel{i^2})\right] = \cancel{e^{T(1+i)}}$

$A + A \cdot 2i = 1$

$A(1 + 2i) = 1$

$A = \dfrac{1}{1+2i} \cdot \dfrac{(1-2i)}{(1-2i)} = \dfrac{1-2i}{\underbrace{1+4i^2}_{1-4}} = \dfrac{2i-1}{3}$

$\boxed{y_p = \dfrac{(2i-1)}{3} \cdot e^{T(1+i)}}$

$y'' + y = e^t \cos(t)$ produto de funções

$y_p = A \cdot e^t \cdot [B\,\text{sen}(t) + C\cos(t)]$

$y_p = \underset{A}{\cancel{AB}}\,\text{sen}(t) \cdot e^t + \underset{B}{\cancel{AC}} \cdot \cos(t) \cdot e^t$

$y_p = e^t[A\,\text{sen}(t) + B \cdot \cos(t)]$

$y_p' = e^t[A\,\text{sen}(t) + B\cos(t)] + e^t \cdot [A\cos(t) - B\,\text{sen}(t)]$

$y_p' = e^t[\text{sen}(t) \cdot (A-B) + \cos(t) \cdot (A+B)]$

$y_p'' = e^t[\text{sen}(t) \cdot (A-B) + \cos(t) \cdot (A+B)] + e^t[(A-B)\cos(t) + \text{sen}(-A-B)]$

$y_p'' = e^t[-2B\,\text{sen}(t) + 2A\cos(t)]$

$\cancel{e^t}[-2B\,\text{sen}(t) + 2A\cos(t)] + \cancel{e^t}[A\,\text{sen}(t) + B \cdot \cos(t)] = \cancel{e^t}\cos(t)$

$\text{sen}(t) \cdot (A - 2B) + \cos(t) \cdot (2A + B) = \cos(t) + 0 \cdot \text{sen}(t)$

$2A + B = 1$

$A - 2B = 0 \longrightarrow A = 2B$

$4B + B = 1$

$B = 1/5 \quad A = 2/5$

$\boxed{y_p = \dfrac{e^t}{5}[2\,\text{sen}(t) + \cos(t)]}$

⑥ $y'' + Cy = e^{i\omega t}$

$y_p = Ae^{i\omega t}$

$y_p' = Ai\omega e^{i\omega t}$

$y_p'' = Ai^2\omega^2 e^{i\omega t} = -A\omega^2 e^{i\omega t}$

$-A\omega^2 \cancel{e^{i\omega t}} + CA\cancel{e^{i\omega t}} = \cancel{e^{i\omega t}}$

$C\cancel{A} = \cancel{A}\omega^2$

$\boxed{C = \omega^2}$

$y_p'' + 5y_p' + Cy_p = e^{i\omega T}$

$y_p = A \cdot e^{i\omega T}$

$y_p' = Ai\omega e^{i\omega t}$

$y_p'' = -A\omega^2 e^{i\omega t}$

$-A\omega^2 e^{i\omega t} + 5Ai\omega e^{i\omega t} + C \cdot A \cdot e^{i\omega T} = e^{i\omega t}$

$A \cdot \cancel{e^{i\omega T}}(-\omega^2 + 5i\omega + C) = \cancel{e^{i\omega t}}$

$A = \dfrac{1}{5i\omega + C - \omega^2}$

$\boxed{y_p = \dfrac{e^{i\omega T}}{5i\omega + C - \omega^2}}$

$P(C) = 5i\omega + C - \omega^2$

Para ocorrer ressonância, $P(C) = 0$, o que nunca vai ocorrer já que $P(C)$ nunca vai zerar

⑦ a) $y'' + 4y' + 3y = 5\cos(\omega t)$

$y_p = A\,\text{sen}(\omega t) + B\cos(\omega t)$

$y_p' = A\omega\cos(\omega t) - B\omega\,\text{sen}(\omega t)$

$y_p'' = -A\omega^2\text{sen}(\omega t) - B\omega^2\cos(\omega t)$

$-A\omega^2\text{sen}(\omega t) - B\omega^2\cos(\omega t) + 4[A\omega\cos(\omega t) - B\omega\,\text{sen}(\omega t)] + 3[A\,\text{sen}(\omega t) + B\cos(\omega t)] = 5\cos(\omega t)$

$\text{sen}(\omega t) \cdot [-A\omega^2 - 4B\omega + 3A] + \cos(\omega t) \cdot [-B\omega^2 + 4A\omega + 3B] = 5\cos(\omega t) + 0 \cdot \text{sen}(\omega T)$

$-B\omega^2 + 4A\omega + 3B = 5$

$-A\omega^2 - 4B\omega + 3A = 0$

$3A = A\omega^2 + 4B\omega$

$3A - A\omega^2 = 4B\omega$

$A(3 - \omega^2) = 4B\omega$

$\rightarrow A = \dfrac{4B\omega}{3 - \omega^2}$

$-B\omega^2 + 4\omega\left(\dfrac{4B\omega}{3 - \omega^2}\right) + 3B = 5$

$-B\omega^2 + \dfrac{16B\omega^2}{3 - \omega^2} + 3B = 5$

$B\left[-\omega^2 + \dfrac{16\omega^2}{3 - \omega^2} + 3\right] = 5$

$\rightarrow B = 5 \cdot \left[-\omega^2 + \dfrac{16\omega^2}{3 - \omega^2} + 3\right]^{-1}$

$\rightarrow A = \dfrac{20\,\omega}{3 - \omega^2} \cdot \left[-\omega^2 + \dfrac{16\omega^2}{3 - \omega^2} + 3\right]^{-1}$

Portanto...

$y_p = \dfrac{20\,\omega}{3 - \omega^2} \cdot \left[-\omega^2 + \dfrac{16\omega^2}{3 - \omega^2} + 3\right]^{-1} \text{sen}(\omega T) + 5 \cdot \left[-\omega^2 + \dfrac{16\omega^2}{3 - \omega^2} + 3\right]^{-1} \cdot \cos(\omega T)$

CONTINUA →

7) a) $y_p = \frac{20w}{3-w^2} \cdot \left[-w^2 + \frac{16w^2}{3-w^2} + 3 \right]^{-1} \operatorname{sen}(wT) + 5 \cdot \left[-w^2 + \frac{16w^2}{3-w^2} + 3 \right]^{-1} \cdot \cos(wT)$

simplificando y_p:

$$\frac{20w}{3-w^2} \cdot \frac{1}{-w^2 + \frac{16w^2}{3-w^2} + 3} = \qquad \frac{5}{-w^2 + \frac{16w^2}{3-w^2} + 3} =$$

$$\frac{20w}{\cancel{3-w^2}} \quad \frac{\cancel{3-w^2}}{-w^2(3-w^2) + 16w^2 + 3(3-w^2)} = \qquad \frac{5(3-w^2)}{-w^2(3-w^2) + 16w^2 + 3(3-w^2)} =$$

$$\frac{20w}{-3w^2 + w^4 + 16w^2 + 9 - 3w^2} = \qquad \frac{15 - 5w^2}{-3w^2 + w^4 + 16w^2 + 9 - 3w^2} = \frac{15 - 5w^2}{w^4 + 10w^2 + 9}$$

$$\frac{20w}{w^4 + 10w^2 + 9}$$

finalmente... $\boxed{y_p = \frac{1}{w^4 + 10w^2 + 9} \cdot \left[20w \cdot \operatorname{sen}(wT) + (15 - 5w^2) \cdot \cos(wT) \right]}$

b) $A = 1$, $B = 4$, $C = 3$ $\quad \operatorname{tg} \alpha = \frac{Bw}{C - Aw^2} \rightarrow$ Phase Lag $\boxed{\operatorname{tg} \alpha = \frac{4w}{3 - w^2}}$

Amplitude

$$G = \frac{1}{(\underbrace{16w^2 + 9 - 6w^2 + w^4}_{D})^{1/2}} = \boxed{\frac{1}{(w^4 + 10w^2 + 9)^{1/2}}}$$

c) $G_{\text{MÁXIMO}} \rightarrow \frac{\partial G}{\partial w} = 0$

$$G' = (w^4 + 10w^2 + 9)^{-1/2} = -\frac{1}{2}(w^4 + 10w^2 + 9)^{-3/2} \cdot (4w^3 + 20w)$$

$$-\cancel{\frac{1}{2}} \cancel{(w^4 + 10w^2 + 9)^{-3/2}} \cdot (4w^3 + 20w) = 0 \rightarrow w^3 = -20w \rightarrow w^2 = -5$$

$\boxed{w_1 = 0 \quad w_2 = i\sqrt{5} \quad w_3 = -i\sqrt{5}}$

$2(3 - w^2) \cdot -2w + 2 \cdot 4w \cdot 4$

$-(6 - 2w^2) \cdot 2w + 32w =$

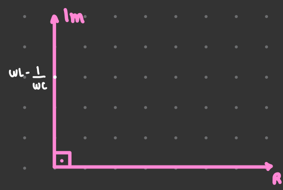
$-12w + 4w^3 + 32w = 0$

$4w^3 + 20w = 0$

8 Tem-se que o ângulo de fase $tg\,\alpha = \frac{\text{imaginário}}{\text{real}} = \frac{wL - 1/wc}{R}$

se $r = 0$, $tg\,\alpha = i(wL - 1/wc)$, que só possui parte imaginária

Dessa forma, $\theta = \pm\frac{\pi}{2}$



9 a) $y'' + 9y = e^{2T}$

$y_p = Ae^{2T}$

$y_p' = 2Ae^{2T}$

$y_p'' = 4Ae^{2T}$

$4Ae^{2T} + 9Ae^{2T} = e^{2T}$

$\cancel{e^{2T}}(4A + 9A) = \cancel{e^{2T}}$

$13A = 1$

$A = \frac{1}{13}$

$$\boxed{y_p = \frac{e^{2T}}{13}}$$

b) $y'' + 9y = Te^{2T}$

$y_1 = At + B$

$y_2 = C \cdot e^{2T}$

$y_p = e^{2T}(\cancel{C}\overset{A}{A}t + \cancel{C}\overset{B}{B})$

$y_p = e^{2T}(At + B)$

$y_p' = Ae^{2T} + 2e^{t}(At + B)$

$y_p'' = 2Ae^{2T} + [4e^{2T}(At + B) + 2e^{2T} \cdot A]$

$y''_p = 4e^{2T}(A + B + At)$

$4\cancel{e^{2T}}(A + B + A \cdot t) + 9\cancel{e^{2T}}(B + At) = t \cdot \cancel{e^{2t}}$

$4A + 4B + 4A \cdot t + 9B + 9At = t$

$\underbrace{4A + 13B}_{0} + 13A\cancel{t} = \cancel{t}$

$13A = 1$

$A = \frac{1}{13}$, $\frac{4}{13} = -13B$, $B = \frac{-4}{169}$

$$\boxed{y_p = e^{2T}\left(\frac{t}{13} - \frac{4}{169}\right)}$$

(10)

$$y'' + y = e^{it}$$

$y_p = Ae^{it}$
$y_p' = Aie^{it}$
$y_p'' = Ai^2e^{it} = -Ae^{it}$

$-Ae^{it} + Ae^{it} = e^{it}$
$e^{it}(-A+A) = e^{it}$
$0 = 1$
$\hookrightarrow P(c) = 0 \to$ Ressonância

Portanto, y_p é a solução homogênea

Logo: $y_p = tAe^{it}$
$y_p' = Ae^{it} + t \cdot iAe^{it}$
$y_p'' = iAe^{it} + iAe^{it} + t \cdot i^2Ae^{it}$
$y_p'' = 2iAe^{it} + ti^2Ae^{it}$

$tAe^{it} + 2iAe^{it} + ti^2Ae^{it} = e^{it}$
$\cancel{e^{it}}(\cancel{At} + 2iA + \cancel{ti^2A}) = \cancel{e^{it}}$
$2iA = 1$
$A = \frac{1}{2i} \to \boxed{y_p = \frac{te^{it}}{2i}}$

$$y'' + y = \cos(t)$$

$y_p = A\,\text{sen}(t) + B\cos(t)$

$y_p' = A\cos(t) - B\,\text{sen}(t)$

$y_p'' =$

$-\cancel{A\,\text{sen}(t)} - \cancel{B\cos(t)} + \cancel{A\,\text{sen}(t)} + \cancel{B\cos(t)} = \cos(t)$

$\cos(t) = 0$
$\hookrightarrow P(c) = 0 \to$ Ressonância
y_p é a solução homogênea

$y_p = t(A\,\text{sen}(t) + B\cos(t))$
$y_p' = A\,\text{sen}(t) + B\cos(t) + t(A\cos(t) - B\,\text{sen}(t))$
$y_p'' = A\cos(t) - B\,\text{sen}(t) + A\cos(t) - B\,\text{sen}(t) + t(-A\,\text{sen}(t) - B\cos(t))$
$y_p'' = 2A\cos(t) - 2B\,\text{sen}(t) - At \cdot \text{sen}(t) - Bt\cos(t)$

$\cancel{At\,\text{sen}(t)} + \cancel{Bt\cos(t)} + 2A\cos(t) - 2B\,\text{sen}(t) - \cancel{At \cdot \text{sen}(t)} - \cancel{Bt\cos(t)} = \cos(t)$

$2A\cos(t) - 2B\,\text{sen}(t) = 1 \cdot \cos(t) + 0 \cdot \text{sen}(t)$

$2A = 1 \to A = 1/2$
$-2B = 0 \to B = 0$
$\to \boxed{y_p = \frac{t\,\text{sen}(t)}{2}}$

(11) $y'' + 4y = e^t \cdot \text{sen}(t)$

a) $y_p = Me^t \cdot \cos(t) + Ne^t \cdot \text{sen}(t)$

$y_p' = Me^t \cdot \cos(t) - Me^t \cdot \text{sen}(t) + Ne^t \cdot \text{sen}(t) + Ne^t \cdot \cos(t)$

$y_p'' = \cancel{Me^t\cos(t)} - Me^t \cdot \text{sen}(t) - Me^t \cdot \text{sen}(t) - \cancel{Me^t\cos(t)} + \cancel{Ne^t\,\text{sen}(t)} + Ne^t \cdot \cos(t) + Ne^t \cdot \cos(t) - \cancel{Ne^t\,\text{sen}(t)}$

$y_p'' = -2Me^t\,\text{sen}(t) + 2Ne^t\cos(t)$

$-2Me^t\,\text{sen}(t) + 2Ne^t\cos(t) + 4[Me^t \cdot \cos(t) + Ne^t \cdot \text{sen}(t)] = e^t \cdot \text{sen}(t)$

$-2M\cancel{e^t}\,\text{sen}(t) + 2N\cancel{e^t}\cos(t) + 4M\cancel{e^t} \cdot \cos(t) + 4N\cancel{e^t}\,\text{sen}(t) = \cancel{e^t} \cdot \text{sen}(t)$
$(4N - 2M)\,\text{sen}(t) + (4M + 2N)\cos(t) = \text{sen}(t)$

$+\ 4N - 2M = 1 \to 4(-2M) - 2M = 1$
$4M + 2N = 0 \qquad -8M - 2M = 1$
$\hookrightarrow 4M = -2N \to M = -\frac{1}{10}, \to N = \frac{1}{5}$
$-2M = N$

$$\boxed{y_p = e^t\left(-\frac{\cos(t)}{10} + \frac{\text{sen}(t)}{5}\right)}$$

11 b) $z'' + 4z = e^{t(1+i)}$

O segundo método é mais simples. Portanto, prefiro ele.

$z_p = Ae^{t(1+i)}$

$z_p' = (1+i)Ae^{t(1+i)}$

$z_p'' = (1+i)^2 Ae^{t(1+i)}$

$(1+i)^2 A\cancel{e^{t(1+i)}} + 4A\cancel{e^{t(1+i)}} = \cancel{e^{t(1+i)}}$

$A(1+i)^2 + 4A = 1$

$A\left[(1+i)^2 + 4\right] = 1$

$A[\cancel{1} + 2i \cancel{-1} + 4] = 1$

$A = \frac{1}{2i+4} \cdot \frac{(2i-4)}{(2i-4)} = \frac{2i-4}{\underbrace{4i^2-16}_{-4-16=-20}} = \frac{2i-4}{-20} = \frac{2-i}{10}$

$z_p = \frac{(2-i)\, e^{t(1+i)}}{10} = I\{z\} = y$

12

a) $y'' + 3y' + 2y = e^t$

método da variação de parâmetros:

$y'' + 3y' + 2y = e^t$

Equação característica:

$r^2 + 3r + 2 = (r+1)(r+2) = 0$

Por soma e produto, $r_1 = -1$ e $r_2 = -2$

$y_n = C_1 \cdot \underbrace{e^{-t}}_{y_1} + C_2 \cdot \underbrace{e^{-2t}}_{y_2}$

$y_1' = -e^{-t}, \quad y_2' = -2 \cdot e^{-2t}$

$y_p = u(t)\, y_1 + v(t)\, y_2$

$M = \begin{vmatrix} y_1' & y_2' \\ y_1 & y_2 \end{vmatrix} = y_1' \cdot y_2' - y_1 \cdot y_2$

$M = (-e^{-t} \cdot -2e^{-2t}) - e^{-t} \cdot e^{-2t}$

$M = 2e^{-3t} - e^{-3t} = e^{-3t}$

Encontrando u(t) e v(t)

$\frac{e^{-t}}{e^{-3t}} = e^{2t}$

$u(t) = \int \frac{y_2 \cdot G(t)}{M} dt = \int \frac{e^{-2t} \cdot e^{t}}{e^{-3t}} dt = \int e^{2t} dt = \frac{1}{2}\int e^{x} dx = \frac{e^{2t}}{2}$

$x = 2t$

$dx = 2\,dt$

$v(t) = -\int \frac{y_1 \cdot G(t)}{M} dt = \int \frac{e^{-t} \cdot e^{t}}{e^{-3t}} dt = \int e^{3t} dt = \frac{-e^{3t}}{3}$

$u(t) = \frac{e^{2t}}{2} \quad v(t) = \frac{-e^{3t}}{3}$

Logo, $y_p = \frac{-e^{2t}}{2}(-e^{-t}) - \frac{e^{3t}}{3} \cdot (e^{-2t})$

$y_p = \frac{e^t}{2} - \frac{e^t}{3} = \frac{3e^t - 2e^t}{6} = \frac{e^t}{6}$

$$\boxed{y = y_n + y_p = C_1 e^{-t} + C_2 \cdot e^{-2t} + \frac{e^t}{6}}$$

12 b) $y'' + 3y' + 2y = e^{-t}$

y_n vai ser o mesmo da letra A pois a equação característica é a mesma.

$$y_n = C_1 \cdot \underbrace{e^{-t}}_{y_1} + C_2 \cdot \underbrace{e^{-2t}}_{y_2}$$

$M = e^{-3t}$, também reaproveitado da letra A

$$y_p = u(t)\, y_1 + v(t)\, y_2$$

Encontrando $u(t)$ e $v(t)$

$$u(t) = \int \frac{y_2 \cdot G(t)\, dt}{M} = \int \frac{e^{-2t} \cdot e^{-t}}{e^{-3t}}\, dt = \int dt = t$$

ressonância

$$v(t) = \int \frac{-y_1 \cdot G(t)\, dt}{M} = -\int \frac{e^{-t} \cdot e^{-t}}{e^{-3t}}\, dt = -\int e^{t}\, dt = -e^{t}$$

$\hookrightarrow \frac{e^{-2t}}{e^{-3t}} = e^{-2t+3t}$

$u(t) = t \quad v(t) = -e^{t}$

Logo, $y_p = t \cdot e^{-t} - e^{t} \cdot e^{-2t} = t e^{-t} - e^{-t} = e^{-t}(t-1)$

Como $y = y_p + y_n$, $y = C_1 \cdot e^{-t} + C_2 \cdot e^{-2t} + e^{-t}(t-1)$

$$y = C_1 \cdot e^{-t} + C_2 \cdot [e^{-t} \cdot e^{-t}] + e^{-t} \cdot T - e^{-t}$$

$$C_1 \cdot e^{-t} - e^{-t} = e^{-t} \cdot \underbrace{(C_1 - 1)}_{CTE}$$

$$\boxed{\text{Simplificando, } \quad y = C_1\, e^{-t} + C_2 \cdot e^{-2t} + t \cdot e^{-t}}$$

13 a) $y'' = e^{at} \quad y(0) = A \quad y'(0) = B$

$$\mathcal{L}\{y''\} = \mathcal{L}\{e^{at}\}$$

$$s^2 \cdot Y(s) - s\, y(0) - s\, y'(0)$$

$$s^2 \cdot Y(s) - s \cdot A - B = \frac{1}{s-a}$$

$$Y(s) = \frac{1}{(s-a)s^2} + \frac{s \cdot A}{s^2} + \frac{B}{s^2}$$

$$\boxed{Y(s) = \frac{1}{s^2(s-a)} + \frac{A}{s} + \frac{B}{s^2}}$$

b)

$$\frac{C_1}{s-a} + \frac{C_2}{s} + \frac{C_3}{s^2} = \frac{1}{s^2(s-a)}$$

$$\frac{C_1 \cdot s^2 + C_2 \cdot s(s-a) + C_3(s-a)}{s^2(s-a)} = \frac{1}{s^2(s-a)}$$

continua →

13 b) $\frac{C_1}{s-a} + \frac{C_2}{s} + \frac{C_3}{s^2} \cdot s^2(s-a) = 1$

$$\frac{C_1 \cdot s^2 \cancel{(s-a)}}{\cancel{s-a}} + \frac{C_2 \cdot s^{\cancel{2}}(s-a)}{\cancel{s}} + \frac{C_3 \cdot \cancel{s^2}(s-a)}{\cancel{s^2}} = 1$$

$$C_1 \cdot s^2 + C_2 \cdot (s-a) \cdot s + C_3 \cdot (s-a) = 1$$

supondo $s=0$:

$1 = -C_3 \cdot a$

$\rightarrow C_3 = \frac{-1}{a}$

supondo $s-a=0 \rightarrow s=a$.

$C_1 \cdot s^2 = 1$

$C_1 \cdot a^2 = 1$

$\rightarrow C_1 = 1/a^2$

Encontrando C_2 com $s=1$:

$\frac{s^2}{a^2} + C_2 \cdot s(s-a) \quad \frac{-(s-a)}{a} = 1$

$\frac{1}{a^2} + C_2(1-a) - \frac{1+a}{a} = 1$

$1 + a^2 \cdot C_2(1-a) + a(-1+a) = a^2$

$a^2 \cdot C_2(1-a) - a + \cancel{a^2} = \cancel{a^2}$

$a^2 \cdot C_2(1-a) = a - 1$

$a^2 \cdot C_2 \cdot \cancel{-(a-1)} = \cancel{a-1}$

$-a^2 \cdot C_2 = 1$

$\rightarrow C_2 = \frac{-1}{a^2}$

Achando C_1, C_2 e C_3, temos que:

$$Y(s) = \frac{1/a^2}{s-a} + \frac{-1/a^2}{s} + \frac{-1/a}{s^2} + \frac{A}{s} + \frac{B}{s^2}$$

$$\boxed{Y(s) = \frac{1/a^2}{s-a} + \frac{A - 1/a^2}{s} + \frac{B - 1/a}{s^2}}$$

transformando:

c) $y(t) = \frac{1}{a^2} \cdot e^{at} - \frac{1}{a^2} \cdot 1 - \frac{1}{a} \cdot t + A + Bt$

$\rightarrow y(0) = \frac{1}{a^2} - \frac{1}{a^2} - 0 + A + 0 = A$ ok!

$y = C_1 e^{at} - C_2 - C_3 t + A + Bt$

$y' = C_1 \cdot a \cdot e^{at} - C_3 + B$

$y' = \frac{1}{a^2} \cdot a \cdot e^{at} - \frac{1}{a} + B$

$\rightarrow y'(0) = \frac{1}{a} - \frac{1}{a} + B = B$ ok!

$y'' = C_1 \cdot a^2 \cdot e^{at}$

$\rightarrow y'' = \frac{a^2}{a^2} \cdot e^{at} = e^{at}$ ok!

14) $$\frac{Hs+K}{(s-a)(s-b)} = \frac{Ha+K}{(s-a)(a-b)} + \frac{Hb+K}{(b-a)(s-b)}$$

$$Hs+K = \frac{[Ha+K]\cdot(s-b)}{(a-b)} + \frac{[Hb+K](s-a)}{(b-a)}$$

$$Hs+K = \frac{[Ha+K]\cdot(s-b)}{(a-b)} - \frac{[Hb+K](s-a)}{(a-b)}$$

$$[Hs+K] = \frac{Ha\cdot(s-b) + K(s-b) - [Hb(s-a) + K(s-a)]}{a-b}$$

$$Hs+K = \frac{Has - Ha\cdot b + Ks - Kb - [Hb\cdot s - Hb\cdot a + Ks - Ka]}{a-b}$$

$$Hs+K = \frac{Ha\cdot s - H\cancel{a\cdot b} + \cancel{Ks} - Kb - Hbs + \cancel{Hba} \cancel{-Ks} + Ka}{a-b}$$

$$Hs+K = \frac{Has - Kb - Hbs + Ka}{a-b}$$

$$(a-b)(Hs+K) = Has - Kb - Hbs + Ka$$

$$Hs\cdot a - Hsb + Ka - Kb = Has - Kb - Hbs + Ka$$

ok!