Default and Interest Rate Shocks: Renegotiation Matters

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Abstract

In this paper we develop a sovereign default model with endogenous re-entry to financial markets via debt renegotiation. We use this model to evaluate how shocks to risk-free interest rates trigger default episodes through two channels: borrowing costs and expected renegotiation terms after default. The first channel makes repayment less attractive when risk-free interest rates are high due to higher borrowing costs. The second channel works through the expected subsequent renegotiation process: when risk-free rates are high, lenders are willing to accept a higher haircut in exchange for resuming payments. Thus, high risk-free rates imply better renegotiation terms for a borrower, making default more attractive ex-ante. We calibrate the model to study the 1982 Mexican default, which was preceded by a drastic increase in federal funds rates in the US. We find that the renegotiation process is key for reconciling the model to the widespread narrative that the increase in US interest rates triggered the 1982 default episode.

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1 Introduction

Why and when do countries default? The last decade had witnessed remarkable progress in the development of quantitative models that offer answers to that question. Taking as a primitive assumption that governments cannot issue state contingent securities, these models first emphasized the role played by the state of the economy in the borrowing country, as well as the existing level of the debt. A narrative along this line explains default as the resulty of a series of negative domestic macroeconomic outcomes, that lead the country first to borrow and eventually to default if the bad macroeconomic outcomes persist over time.

Besides these domestic causes of defaults, versions of the models also emphasize factors that are external to the country. For instance, under certain conditions, the models exhibit multiplicity of equilibria, so pessimistic expectations in international markets can coordinate in high interest rate equilibria that may accelerate the defaul. In addition, a drop in the terms of trade can also accelerate a default decision, particularly in small open economies that have their exports concentrated in a single primary commodity.

The purpose of this paper is to quantitatively evaluate the role of a third foreign factor that affects the probability of defaults: shocks to the foreign interest rate. There is an obvious reason why increases in the foreign interest rate may accelerate a default decision: as the cost of servicing the debt goes up, the value of repaying goes down, which may induce a country to stop debt payments. Any individual that held a variable interest rate mortgage understands this channel.

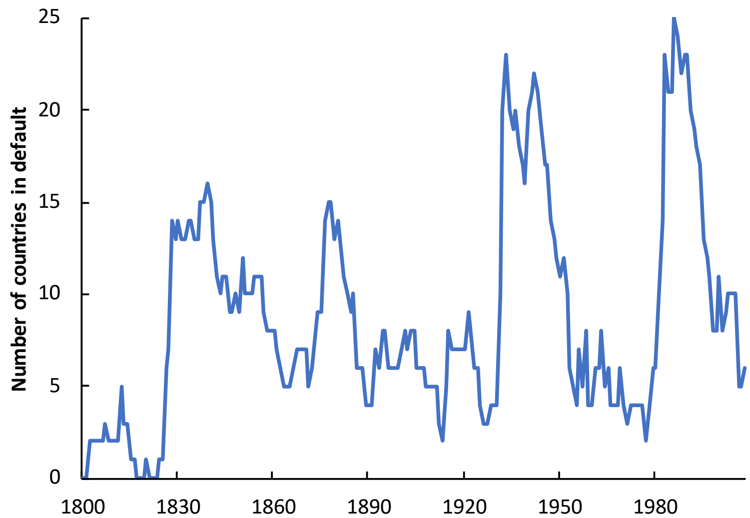
Evaluating this channel is particularly interesting, since what became the worst sovereign debt crisis in the world occured precisely at the time in which the real interest rate in the US reached its historical maximum[1]. In figure 1, we plot the number of countries in default in each year between 1800 and 2008[2]. It's striking that the 1980s is comparable to periods

[1] According to the database of Reinhart and Rogoff (2009), there were on average 18.9 countries in default in each year throughout the 1980s; 11.4 of them from Latin American.

[2] The data comes from Reinhart and Rogoff (2009). It consists of 66 countries that together amount to more than 90% of global GDP.

like the Napoleonic Wars, the World Wars I and II, and the Great Depression.[3]

Figure 1: Default status over time

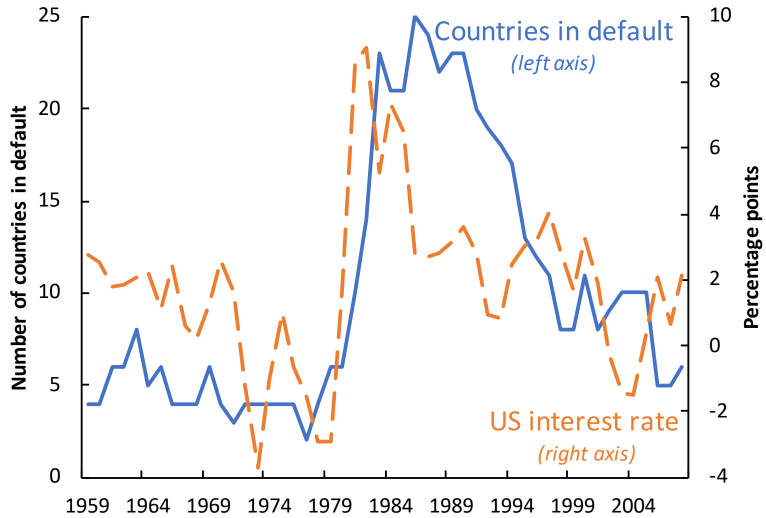


The series of defaults that followed the Mexican case in 1982, unravelled during the sucessful program to tame inflation carried on by the Federal Reserve under Chairman Paul Volcker. At the onset of the stabilization plan, the real interest rate increased at very high levels, reaching 6% at the begining of 1980, and remained high for over two years. Existing narratives of the debt crisis that started with the default in Mexico in the summer of 1982 and spread over many other countries put the "Volcker shock" high in the list of culprits.

In figure 2, we plot both the default status and the annual interest rate on 1-year US treasury bonds in each year between 1959 and 2008. The figure features a positive correlation (46.9%), consistent with the widespread narrative about the 1980s.

[3]Similar comparison applies if we look at the share of independent countries in default as well as if we weight countries by their respective GDP.

Figure 2: Default status and US interest rate over time



In this paper, we evaluate the effect of a Volcker shock in a quantitative model of sovereign default. To calibrate the model, we use data from Mexico, but as it will become obvious from our robustness exercises, the results are very general. We first show that the obvious direct effect of the increase in the servicing cost is quantitatively negligible: according to the model, it cannot explain the default in Mexico, even if we make the shock unrealistically large.[4] But we also show that changes in the foreign interest rate have a more subtle effect once renegotiation is explicitly modeled. The reason is that a higher interest rate makes foreign lenders more impatient, and are therefore more willing to take haricuts on the value of the debt upon renegotiation. Thus, a high interest rate implies better future expected deals for the goverment, increasing the value of default. Our numerical exploration shows that this effect is several order of magnitudes more important that the direct effect on the cost of servicing the debt. In discussing the specific case of Mexico, we show that while the model without renegotiation is unable to explain why Mexico defaulted in 1982 for any reasonable parameter values, the model with renegotiation does, for many plausible parameter values. This is the main contribution of our paper.

[4] We are not the first ones to make this point. In their models without any explicit renegotiation process, Toure (2017) and Singh (2018) find the Volcker shock had virtually no role in triggering the Mexican default decision.

In order to make the point, we use a simple off-the-shelf model of renegotiation, in which, after default, the country faces an exogenous probability to enter into a renegotiation process. The simplicity of the model implies that, once a renegotiation opportunity arises, an agreement is reached. As it will become clear in the paper, this is not an important ingredient in the result, since the key is the effect of a higher interest rate in the outside option value for the lenders. Therefore, this effect will still be present in more complicated bargaining scenarios. We believe this is the rigth choice, since the purpose of the paper is to highlight the way interest rate shocks and renegotiation interact in a quantitative model of sovereign default.

As it turns out, however, in order to understand the experience of Mexico and of several other Latin-American countries that defaulted in those same years that feature is undesirable. The reason is that in most of those cases, it took around a whole decade for the countries to reach agreements with their lenders. During that decade there were several rounds of renegotiations in many of these countries that were unsuccessful. It was only through the the Brady plan of the early 90s that those countries could renegotiate their defaulted debts. Thus, while the model can potentially explain why Mexico defaulted, it does not explain why it took so long for a successful renegotiation to occur.

The model in this paper provides a step forward in understanding the role of shocks to the international interest rate, by analyzing the renegotiation game that follows all default eposides. But fully understanding the series of defaults of the early 80's and its posterior renegotiation processes will require more sophisticated theories of renegotiation. We collected interesting anecdotal evidence that suggest interesting hypothesis for future work, that involve US regulators. We discuss and document them in our concluding Section.

The paper proceeds as follows. In the remains of this introduction we briefly overview related papers. Then, in Section 2 we describe the model and in Section 3 we explain in detail the new mechanism. In Section 4 we calibrate the model and in Section 5 we present the numerical results. Section 6 concludes.

The role of debt restructuring has recently be analyzed by Yue (2010), Hatchondo, Martinez and Sosa-Padilla (2014) in sovereign default models pioneered by Aguiar and Gopinath (2006) and Arellano (2008). Our model of renegotiation is similar to the one in Yue (2010).

Recent literature has studied the role of interest rate shocks in models without renegotiation, such as Johri, Khan, and Sosa-Padilla (2018)(2018), Toure (2017), and Singh (2018). None of them consider the effect that interest rate shocks have on the renegotiation game that follows default.

Closest to ours is Guimarães (2011), who shows that shocks to world interest rates affect the incentive compatible level of debt more than output shocks do. We complement this work by highlighting an important channel through which, given a certain debt level, shocks to the risk free interest rate can trigger a default episode.

2 Model

Time is discrete and indexed by $t \in \{0, 1, 2, \ldots\}$. We model a small open economy with a stochastic endowment and a benevolent government without commitment. There is a large number of risk neutral foreign lenders that have access to a risk-free asset. Each period, the return to the risk free asset can take one of two values $r_t \in \{r_L, r_H\}$, with $r_L < r_H$. Given r_t, the probability of $r_{t+1} = r_H$ is $Prob\,(r_{t+1} = r_H | r_t) = \lambda\,(r_t)$.

A crucial feature of the model is that after default: (i) debt is frozen and not forgiven, (ii) opportunities to renegotiate the debt level arise stochastically, and (iii) upon an renegotiation opportunity, the country and the lenders enter a Nash bargaining process.

2.1 The government

Each period, the economy receives a stochastic endowment of a tradable good y_t that follows a finite-state Markov chain with transition law $Prob\,(y_{t+1} = y' | y_t = y) = F\,(y, y')$.[5] The

[5]The numerical solution in this paper relies on a similar algorithm as that in Chatterjee and Eyigungor (2012). Accordingly, we introduce some additional disturbance to the income process for dealing with

government has preferences for consumption of the tradable good in each period given by

$$E_t \left[\sum_{j=t}^{\infty} \beta^{j-t} u\left(c_j\right) \right]$$

where c is consumption, $\beta \in (0, 1)$ is the subjective discount factor, and the utility function u is strictly increasing and strictly concave.

The government can issue non-contingent long-term debt in international financial markets. Similar to Hatchondo and Martinez (2009), a bond consists of a perpetuity with geometrically declining payments: a bond issued in period t promises to pay $\gamma\left(1-\gamma\right)^{j-1}$ units of the tradable good in period $t+j$, $\forall j \geq 1$. The law of motion for bonds is given by

$$b_{t+1} = \left(1-\gamma\right) b_t + x_t$$

where b_t is the amount of bonds due at the beginning of period t, γ is the fraction of bonds that matures each period, and x_t is the issuance of new bonds in period t. We assume $b_t \in \mathcal{B} = \left[\underline{b}, \bar{b}\right]$ where $\underline{b} \leq 0$ and $\bar{b} > 0$. In our environment positive b is debt owed to international lenders.

If the government is in good standing with international lenders, then, at the beginning of the period, it observes the current realization of all shocks in the economy and decides whether to default or repay, its value in this case is:

$$V\left(b, y, r, 0\right) = \max_{d \in \{0,1\}} \left\{ \left(1-d\right) V^P\left(b, y, r\right) + dV^D\left(b, y, r\right) \right\} \tag{1}$$

where $d = 1$ if the government decides to default on its debt and $d = 0$ if it decides to repay. If the government decides to repay it makes coupon payments γb and gets to issue

the computational issues that arise from the long term structure of bonds. Different than Chatterjee and Eyigungor (2012), we also make changes to the timing of the model in order to deal with issues that arise from the inclusion of a renegotiation process. See the online Appendix for a description of the model we numerically solve.

new bonds. The value of the government in repayment is then:

$$V^P(b, y, r) = \max_{c,b'} \{u(c) + \beta \mathbb{E}[V(b', y', r', 0)]\} \tag{2}$$
$$s.t. \quad c + \gamma b \leq y + q^P(b', y, r)[b' - (1-\gamma) b]$$

where $q^P(b', y, r)$ is the market price of bonds when the government is in good financial standing. If the government chooses to default, then debt payments are suspended, the government is excluded from credit markets, debt is frozen so $b_{t+1} = b_t$, and income is $h(y_t) = y_t - \phi(y_t)$, where $\phi(y_t) \geq 0$ and $h(y_t)$ is increasing in y_t. The value of the government in default is:

$$V^D(b, y, r) = u(h(y)) + \beta \left\{\theta \mathbb{E}[V(b, y', r', 1)] + (1-\theta)\mathbb{E}\left[V^D(b, y', r')\right]\right\}. \tag{3}$$

where $V(b, y', r', 1)$ is the value of the government when an opportunity to renegotiate arises, which happens with probability θ.[6] If an opportunity to renegotiate arises, the government can choose to accept to start repaying a new debt level b^R or to remain in default:

$$V(b, y, r, 1) = \max_{a \in \{0,1\}} \left\{aV^P\left(b^R(b, y, r), y, r\right) + (1-a) V^D(b, y, r)\right\} \tag{4}$$

where $a = 1$ if the government accepts the deal and $a = 0$ if the government decides to remain in default. In section 2.2 below we discuss in detail how b^R is determined. If a new debt level b^R is agreed upon, the government regains access to financial markets and there is no longer an output cost of being in default.

The solution to the government's problem gives decision rules for consumption, $c^P(b, y, r)$, debt issuance, $b^P(b, y, r)$, default policy, $d(b, y, r)$, and the decision to accept a renegotiation offer, $a(b, y, r)$.

[6] Note that, in order for renegotiation to happen, the government has to be in default at least for one period. Hatchondo, Martinez and Sosa-Padilla (2014) consider the option to renegotiate as an alternative to default. Our results would not change if we considered this option, which we exclude for simplicity.

2.2 Renegotiation

When the government is in bad standing with international lenders, an opportunity to renegotiate its debt arises with probability θ. In this case, at the beginning of the period, the lenders and the government observe the endowment y and the risk-free interest rate r. Then, the new debt level b^R offered to the government is determined in the following Nash bargaining game:

$$\begin{aligned} b^R(b,y,r) &= \arg\max_{\tilde{b}} \left[S^{LEN}\left(\tilde{b},b,y,r\right)\right]^{\alpha}\left[S^{GOV}\left(\tilde{b},b,y,r\right)\right]^{1-\alpha} \\ s.t.: \quad & S^{LEN}\left(\tilde{b},b,y,r\right) \geq 0,\ S^{GOV}\left(\tilde{b},b,y,r\right) \geq 0 \end{aligned} \tag{5}$$

where α is the bargaining power of international lenders and S^{LEN} and S^{GOV} are the surpluses of the lenders and the government, respectively. We define the surplus of the government as the difference between accepting the new deal and remaining in default:

$$S^{GOV}\left(\tilde{b},b,y,r\right) = V^P\left(\tilde{b},y,r\right) - V^D(b,y,r) \tag{6}$$

Similarly, the surplus of the lenders is the difference between resuming debt payments with the new debt level $\tilde{b}$ and the market value of the previously defaulted bonds:

$$S^{LEN}\left(\tilde{b},b,y,r\right) = \gamma\tilde{b} + q^P\left(b^P\left(\tilde{b},y,r\right),y,r\right)(1-\gamma)\,\tilde{b} - q^D(b,y,r)\,b \tag{7}$$

where $q^D(b,y,r)$ is the market price schedule of a unit of defaulted bonds.

We assume b^R is determined after the realization of y and r to capture the fact that current economic conditions affect the outcome of a renegotiation.

2.3 Equilibrium

An equilibrium is value functions V, V^P, and V^D, policy functions c^P, b^P, d, and a, bond prices q^P and q^D, and a best feasible renegotiated debt level b^R such that: (i) given b^R and the bond prices, the value and policy functions solve (1) through (4), (ii) given prices and the value and policy functions, b^R is defined by (5), (iii) bond prices are consistent with lenders making zero profits after adjusting for default risk.

Given the above definition, the price of bonds when the government is in good standing with international lenders is:

$$\begin{aligned} q^P(b',y,r) = & \frac{1}{1+r}\mathbb{E}\left[\{1-d(b',y',r')\}\left\{\gamma+(1-\gamma)\,q^P\left(b^P(b',y',r'),y',r'\right)\right\}\right] \\ & +\frac{1}{1+r}\mathbb{E}\left[d(b',y',r')\,q^D(b',y',r')\right] \end{aligned} \tag{8}$$

The first line of the RHS is standard for models with this long term debt structure. If the government does not default in the next period ($d=0$) lenders get the fraction γ of debt that matures and the market value of the fraction $(1-\gamma)$ of the remaining debt. The second line takes into account the fact that debt is not forgiven after default. Defaulted bonds have, in general, a positive market price q^D, that captures the proceeds of future debt renegotiation:[7]

$$\begin{aligned} q^D(b,y,r) = & \frac{\theta}{1+r}\mathbb{E}\left[a(b,y',r')\,\frac{b^R(b,y',r')}{b}\,\{\gamma\right. \\ & \left. +(1-\gamma)\,q^P\left(b^P\left(b^R(b,y',r'),y',r'\right),y',r'\right)\}\right] \\ & +\frac{\theta}{1+r}\mathbb{E}\left[\{1-a(b,y',r')\}\,q^D(b,y',r')\right]+\frac{1-\theta}{1+r}\mathbb{E}\left[q^D(b,y',r')\right] \end{aligned} \tag{9}$$

Conditional on having an opportunity to renegotiate in the next period, if the government accepts the new amount b^R, then lenders get the fraction γ of the new debt that matures in that period and the market value of the remaining fraction $(1-\gamma)$. If the government rejects b^R, or if there is no renegotiation opportunity at all, then lenders get the market

[7] As we discuss in section 3, the market price of defaulted bonds q^0 is positive and approaches zero in the limit case when $\alpha \to 0$.

value of defaulted bonds $q^D(b, y', r')$.

3 Characterization of an equilibrium

The main purpose of this section is to provide a theory regarding how renegotiation outcomes change with regard to variations in the risk-free interest rate. This result is especially important as it makes default decisions in quantitative sovereign debt models more sensitive to interest rate shocks.

We look at a particular class of economies. We restrict our analysis to models with one period bonds and assume debt and endowment are both bounded, ie, $y \in \left[\underline{y}, \overline{y}\right]$ and $b \in [0, B]$, where $\underline{y} > 0$ as well as $\overline{y}$ and $\overline{b} > 0$ are both finite.

Proposition 1: In equilibrium, the bargaining problem always have a solution and, accordingly, $a(b, y, r) = 1$ and $V(b, y, r, 1) = V^P\left(b^R(b, y, r), y, r\right)$, for all (b, y, r).

Proof: See appendix.

This proposition implies that, whenever a renegotiation opportunity arises, there is always a new debt level that makes both government and lenders better off. The intuition is that there is always some output cost from default that can be avoided and split among the parties. This is true even for periods in which there is no direct output cost, as long as the endowment process features strictly positive probability that before the next renegotiation opportunity arises some direct output cost reduces resources in the defaulted economy.

Denote $\overline{\mathcal{B}}(b, y, r)$ and $\underline{\mathcal{B}}(b, y, r)$, the sets of debt levels that make the government and the lenders, respectively, indifferent between accepting or rejecting the offer. Formally, $\overline{b} \in \overline{\mathcal{B}}(b, y, r)$ and $\underline{b} \in \underline{\mathcal{B}}(b, y, r)$ are such that $S^{GOV}\left(\overline{b}, b, y, r\right) = 0$ and $S^{LEN}\left(\underline{b}, b, y, r\right) = 0$. Next proposition states that the debt level that makes lenders indifferent between accepting or rejecting a renegotiation deal is smaller than the level that makes government indifferent.

Proposition 2: The sets $\overline{\mathcal{B}}(b, y, r)$ and $\underline{\mathcal{B}}(b, y, r)$ are singleton. Furthermore, if $\bar{b} \in \overline{\mathcal{B}}(b, y, r)$ and $\underline{b} \in \underline{\mathcal{B}}(b, y, r)$, then it must be that $\bar{b} \geq \underline{b}$.

Proof: See appendix.

Given this result, denote $\bar{b}(b, y, r)$ and $\underline{b}(b, y, r)$ the unique elements in $\overline{\mathcal{B}}(b, y, r)$ and $\underline{\mathcal{B}}(b, y, r)$, respectively.

From hereafter, we focus on a particular equilibrium in this economy, namely one in which the amount of debt defaulted on is irrelevant for the renegotiation process. In this case, the price of defaulted bonds held by lenders as well as the government's value in default are also independent on the amount of defaulted debt.

Proposition 3: The market value of defaulted bonds and the value in default are independent of the stock of debt if and only if renegotion outcomes does not depend on the stock of debt, ie, $q^D(b_1, y, r) b_1 = q^D(b_2, y, r) b_2$ and $V^D(b_1, y, r) = V^D(b_2, y, r) \iff b^R(b_1, y, r) = b^R(b_2, y, r)$ for all (y, r) and $b_1, b_2 > 0$.

Proof: See appendix.

The next theorem demonstrates the irrelevance of the risk-free interest rate for the conditions in which the government returns from financial autarky when it enjoys full bargaining power. In this setting, the unit of a defaulted bond has no value as the government reaccess international financial markets after a default episode with no debt, regardless of the risk-free interest rate. This 100% haircut indicates that this economy is equivalent to one in which renegotiation is exogenously shut down.

Theorem 1: If $\alpha = 0$, then $q^D(b, y, r) = 0$ and $b^R(b, y, r) = 0$, for all (b, y, r).

Proof: See appendix.

As we depart from the standard models with no renegotiation by increasing the bargaining power of lenders, haircuts become a function of the risk-free interest rate. For a wide range set of functional forms and parameters, the model features high risk-free interest rates inducing relatively high haircuts in equilibrium. Nevertheless, the model may also feature opposite or even ambiguous relationship between risk-free interest rates and haircuts.

As an empirical matter, under the functional forms and parameters used for the calibration of the Mexican economy up to the 1982 default episode, the model displays high haircuts when the risk-free interest rate is high. Next section shows how this feature provides an interesting role for risk-free interest rate spikes regarding sovereign debt matters.

4 Discussion

In this section, we assume some persistence on the risk-free interest rate shock so that the probability of high next-period risk-free interest rate increases with the current interest rate, ie, $\lambda(r_H) > \lambda(r_L)$ and focus on an equilibrium in which higher risk-free interest rates induce higher haircuts The main contribution of this paper is to show how renegotiation dynamics may introduce a new mechanism through which interest rates have a significant impact on equilibrium outcomes, in particular on default decisions and spreads.

4.1 Default incentives

There are two channels through which shocks to risk-free interest rates affect default decisions. The first channel is through the borrowing cost. Risk-free interest rate spikes make the price of bonds lower, which tightens the budget constraint and hence decreases the value of repayment. This effect is already present in simpler settings, but is enlarged in the presence of renegotiation.

$$V^P(b, y, r) = \max_{c, b'} \{u(c) + \beta \mathbb{E}[V(b', y', r', 0)]\}$$
$$s.t. \quad c + \gamma b \leq y + \underbrace{q^P(b', y, r)}_{\text{First channel}} [b' - (1-\gamma) b]$$

The second channel is exclusive to models with renegotiation and works through the expectations of future haircut. Once the risk-free shock hits the economy, the risk-free interest rate becomes more likely to be high in the future, as $\lambda(r_H) > \lambda(r_L)$. In case the government defaults, this implies a lower future expected haircut, which causes the continuation value in default to increase and hence makes the default decision more attractive.

$$V^D(b, y, r) = u(h(y)) + \beta \left\{ \theta \underbrace{\mathbb{E}[V(b, y', r', 1)]}_{\text{Second channel}} + (1-\theta) \mathbb{E}\left[V^D(b, y', r')\right] \right\}$$

We label renegotiation-mechanism and standard-mechanism those present in models with and without renegotiation, respectively, that drive world interest rate to alter default incentives. In section 6, we show quantitatively the renegotiation mechanism has a large effect in triggering default decisions, while the effect of the standard-mechanism by itself is neglibible.

$$\text{First channel} \uparrow r \Longrightarrow \downarrow q^P \Longrightarrow \downarrow V^P$$
$$\text{Second channel} \uparrow r \Longrightarrow \downarrow b^R \Longrightarrow \uparrow V(., 1) \Longrightarrow \uparrow V^D$$

4.2 Effect on spreads

Equation (8), rewritten below, highlights how the standard and the renegotiation-mechanisms work through the borrowing costs. The standard one follows through the higher discount that lenders give to future payments: as r increases, q^P decreases:

$$q^P(b,y,r) = \mathbb{E}\left[\{1 - D(b,y',r',0)\}\left\{\frac{\gamma}{1+r} + \frac{1-\gamma}{1+r}q^P\left(b^P(b,y',r'),y',r'\right)\right\}\right.$$
$$\underbrace{\left.+\underbrace{D(b,y',r',0)}_{\text{Part 1}}\underbrace{q^D(b,y',r')}_{\text{Part 2}}\right]}_{\text{Renegotiation mechanism}}\underbrace{\frac{1}{1+r}}_{\text{Standard mechanism}}$$

The renegotiation-mechanism can be split in two parts. Both originate from the anticipation of what a rise in risk free interest rate causes to renegotiation terms: all else constant, as r increases, the expected renegotiated debt b^R decreases. The first part regards the anticipation by a government with access to international financial markets. As explained before, default becomes relatively more attractive when r is high. Therefore, as lenders are more likely to be holding defaulted bonds in the subsequent period, increases in r drives the price of a bond in good standing q^1 to be more heavily determined by the price of a defaulted bond q^0.

The second part regards the anticipation by lenders. As the expected haircut increases, the price of defaulted bonds decreases. Therefore, increases in r reduce the price of a bond in good standing as its payoff is lower in subsequent period's default states.

$$\text{Standard-mechanism: } \uparrow r \Longrightarrow \downarrow \frac{1}{1+r} \Longrightarrow \downarrow q^P$$

$$\text{Renegotiation-mechanism: } \uparrow r \Longrightarrow \downarrow b^R \Longrightarrow \begin{cases} \Longrightarrow \uparrow V^D \Longrightarrow \uparrow \mathbb{E}\left[D(.,0)\right] \\ \downarrow q^D \end{cases} \Longrightarrow \downarrow q^P$$

In section (6), we show the quantitative importance of this new mechanism. We show its effect on renegotiation terms, how it reinforces the effect of risk free interest rate shocks on spreads, and more importantly, we show the standard mechanism alone is not enough for making risk free rate shocks a relevant factor for government's default decisions.

5 Calibration

For our calibration exercise we will consider the case of the Mexican debt crisis in 1982, since it was preceded by a sizable increase in US risk-free rates and this increase has been considered a determinant cause of the crisis. Given data limitations on spread dynamics and GDP (only yearly GDP data is available), more than matching business cycle properties of the data we will use our model to assess the extent to which the increase in risk-free interest rates triggered the default decision and whether renegotiation dynamics played an essential role. We assume the utility function displays a constant coefficient of relative risk aversion η:

$$E_t \left[\sum_{j=t}^{\infty} \beta^{j-t} \frac{c_j^{1-\eta} - 1}{1 - \eta} \right]$$

The income shock y_t follows a log-normal AR(1) process $\log(y_t) = \rho \log(y_{t-1}) + \epsilon_t$, with $|\rho| < 1$ and $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ and the output cost is $\phi(y_t) = \max\{0, \phi_0 y_t + \phi_1 y_t^2\}$, with $\phi_0 < 0 < \phi_1$. Introducing this quadratic functional form makes the output cost be zero for $0 \leq y_t \leq -\frac{\phi_0}{\phi_1}$ and more than proportionally increasing for $y_t > -\frac{\phi_0}{\phi_1}$. This direct output cost, coupled with income shock and long-term debt, has become standard in the literature as it allows the model to match the spread dynamics observed in the data (see Chatterjee and Eyigungor (2012) and Hatchondo, Martinez and Sosa-Padilla (2014)). We do preserve them, even though our renegotiation process, coupled with shocks to interest rates, also help at matching these dynamics.

Table 1 presents all parameter values that can be directly calibrated from the data. Each period in the model corresponds to 1 year. The discount factor and the risk aversion parameters are set to standard values, $\beta = 0.94$ and $\eta = 2$. The AR(1) income process is estimated using HP-filtered logged Mexican GDP data from 1921 to 1983. This yields an auto-correlation parameter $\rho = 0.705$ and a standard deviation of innovations of $\sigma = 0.040$.

Table 1: Parameters

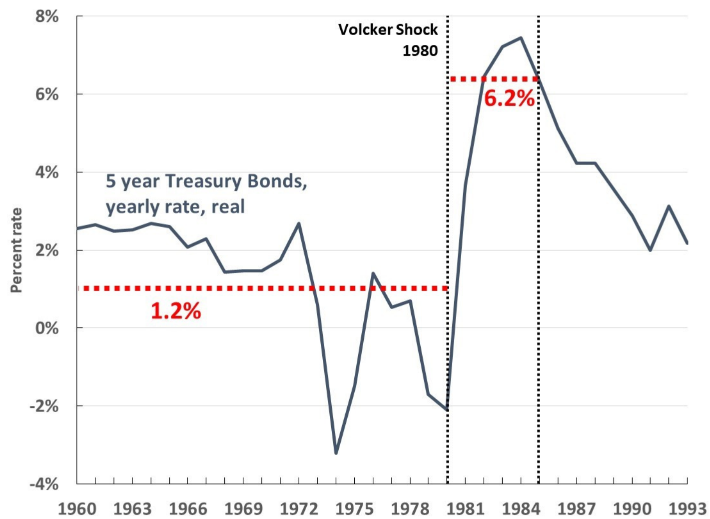
Parameter		Value	Details
Low r	r_L	1.2%	1955 - 1980
High r	r_H	6.2%	1981 - 1985
Pr(low to high r)	$\lambda(r_L)$	1%	Duration of 100 years
Pr(high to low r)	$1-\lambda(r_H)$	20%	Duration of 5 years
Pr(renegotiation)	θ	28%	Arellano (2008)
Maturity rate	γ	0.75	16 month bonds
Discount factor	β	0.94	LR interest rate of 6%
Risk aversion	η	2	Standard
Income process	ρ	0.705	AR(1) estimation
	σ_ϵ	0.040	

The probability of switching from the high risk-free interest rate regime to the low one is set to $1-\lambda(r_H) = 20\%$ so that it generates an expected duration of 5 years for the high regime. This is the time the Volcker shock lasted in the US economy, as can be seen in Figure 3. Hence, implicit in our analysis is the assumption the Mexican government had the correct expectation for the duration of high world interest rates. We set the probability of switching from the low to the high risk-free interest rate regime to $\lambda(r_L) = 1\%$ so that shocks like the one we are studying are very infrequent events. Figure 3 also displays the average interest rate during the Volcker shock (1980-1985) and the average interest rate before that (1955-1980).[8] Therefore, the risk-free interest rate is set to $r_L = 1.2\%$ in the low regime and

[8]For simplicity, we assume only two possible states for the risk-free interest rate. This highlights the mechanics of the model as well as the role of the Volcker shock in triggering default episodes.

$r_H = 6.2\%$ in the high regime.

Figure 3: Annual real interest rate of 5-year treasury constant maturity rate



We set the output cost parameters ϕ_0 and ϕ_1 and the lenders' bargaining power parameter α to match three moments of the Mexican economy: a default probability of 3%[9], an average debt-to-GDP ratio of 19.3%, and the haircut of 30.5% following the Brady plan. Table 2 summarizes the calibration of these parameters.

Table 2: Moments

Parameter		Value	Moment	Data	Model
Bargaining power	α	0.40	Default probability	3.0%	3.5%
Quadratic income cost function	ϕ_0	-.025	Debt-to-GDP ratio	19.3%	18.6%
	ϕ_1	0.26	Haircut in 1990	30.5%	23.1%

[9]In the 100 years prior to and including 1982 the Mexican government defaulted on its debt three times: 1914, 1928 and 1982. In 1899 there was a debt restructuring, however, we don't consider this a default episode in the sense of this version of the model since there was no suspension of payments. Moreover, the restructuring in that period was in part promoted by lenders themselves, as Lill 1919 points out: "Due to the increasing prosperity of the nation and the good results which we are beginning to obtain in the finances of the Federal Government, the value of the securities of the public debt has gradually risen, and the question has presented itself in financial circles and those interested in Mexican securities as to the possibility of reducing the annual charge against the nation for debt service".

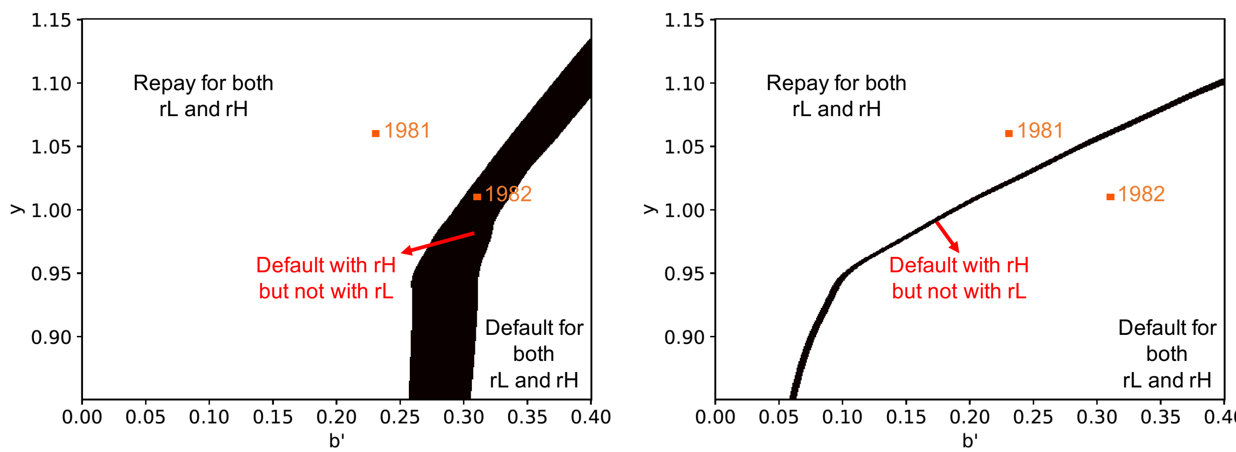
6 Results

Figure 4 exhibits the main result of this paper. When $\alpha = 0$, that is, in the model that is isomorphic to a model without renegotiation, interest rate shocks have a negligible effect on default incentives. The default set when $r = r_L$ is almost identical to the default set when $r = r_H$. Panels (a) and (b) overlap the default and repayment regions in two versions of the model: with and without renegotiation. In both figures 4(a) and 4(b) we identify three regions: (i) one in which the government defaults for any risk-free interest rate (the bottom right white area), (ii) one in which it repays for any risk-free interest rate (top left white area), and the region in which the government defaults only when the risk-free interest rate is high (the black area).

Figure 4: States in which the Volcker shock explains the crisis

(a) With renegotiation

(b) Without renegotiation

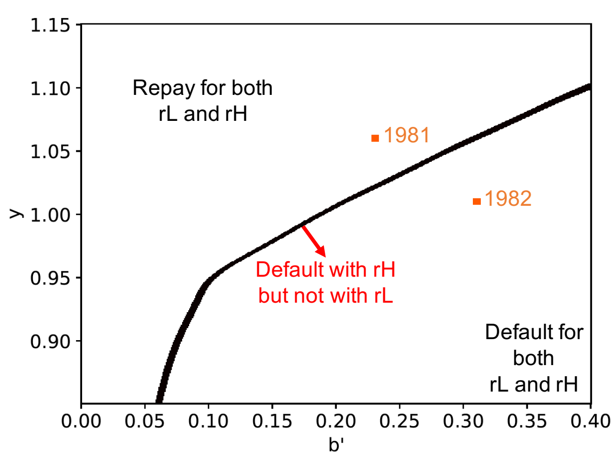


The black area is considerably thicker in the model with renegotiation, figure 4(a). This highlights the importance of the renegotiation-mechanism for interest rate shocks to trigger default episodes. On the other hand, in the model without renegotiation, figure 4(b), the only motive for the government to react to interest rate regime switches is the standard channel through the cost of borrowing described in section (4).

The absence of the renegotiation-mechanisms that originates from government and lenders anticipating movements to haircuts cause spreads to be little sensitive to the risk-free rate. Moreover, there is no role for a default decision in which the government is strategically motivated to take advantage of improved renegotiation terms during periods of high interest rate regime. Therefore, the less sensitive spreads and lack of default decisions motivated by favorable future haircuts cause the simplified model to be almost silent about risk-free interest rate spikes triggering default episodes. This is exactly what figure 4 illustrates: if the risk-free interest rate is to be of any quantitative relevance for default decisions, then it is crucial to model the renegotiation process.

Notice from figure 5 that the area in which the Volcker shock explains the crisis is insignificant in the simplified model even if we exaggeratedly deviate from our benchmark parameters. In particular, even if we consider a much stronger risk-free rate spike in which it increases to 20% instead of 6.2%, the model still predicts virtually no role for the shock in explaining default episodes. The same applies for the duration of the shock: even if the government expected the shock to last 20 years instead of 5, it plays no relevant role. This exercise highlights once again that the standard mechanism is not enough for having the Volcker shock to be the trigger of the 1982 Mexican default episode.

Figure 5: States in which the Volcker shock explains the crisis, no renegotiation, high risk free interest rate $r_H = 21.2\%$, and expected duration of high interest rate regime $\frac{1}{(1-\lambda(r_H))} =$ 20 years

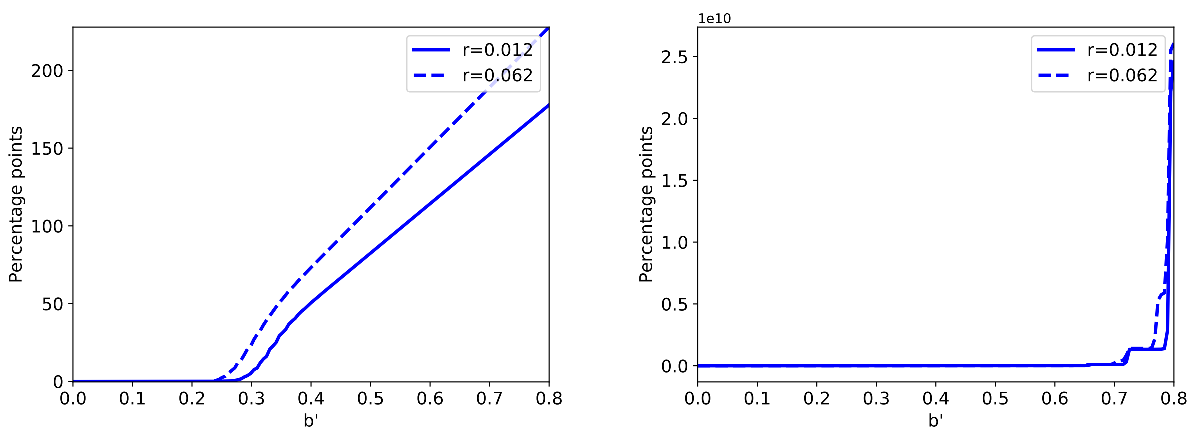


6.1 Renegotiation-mechanism in action

Spreads for a debt in good standing, $spreads\,(b,y,r)$, is computed as the difference between the interest rate on the sovereign bond, $i\,(b,y,r)=\frac{\gamma}{q^P(b,y,r)}-\gamma$, minus the risk-free interest rate r, ie, $spreads\,(b,y,r)=i\,(b,y,r)-r$. Notice the spreads become distinct from one another around the same debt level where the risk-free interest rate shock explains the crisis in figure 6 when endowment $y=1$. Notice how bond prices and spreads are more sensitive in a model with renegotiation, where the standard mechanism is not the only one in place. This figure also illustrates why our model does not need to rely as much on income shocks in order to generate high volatility of spreads.

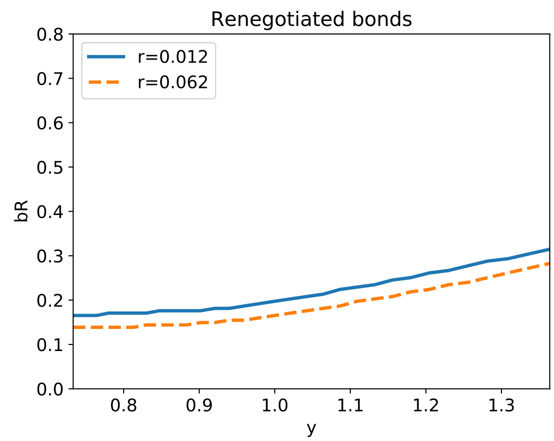
Figure 6: Spreads for different risk-free interest rates

(a) With renegotiation (b) Without renegotiation



One feature of our bargaining process is that its outcome does not depend on the amount of debt in default. Nevertheless, Figure 7 illustrates how the renegotiated debt level b^R varies with the interest rate regime as well as the persistent income shock y. Notice the terms are more favorable to the government when it is in the worst states of the world: haircuts are higher when the income shock is lower and risk-free rate is higher. Furthermore, this figure is consistent with Guimarães 2011 in the sense that the regime switching is a lot more relevant to renegotiation outcomes than a significant output drop. Specially important is the reaction of renegotiation terms to the interest rate regime, which is at the heart of our new mechanism.

Figure 7: Renegotiated debt level

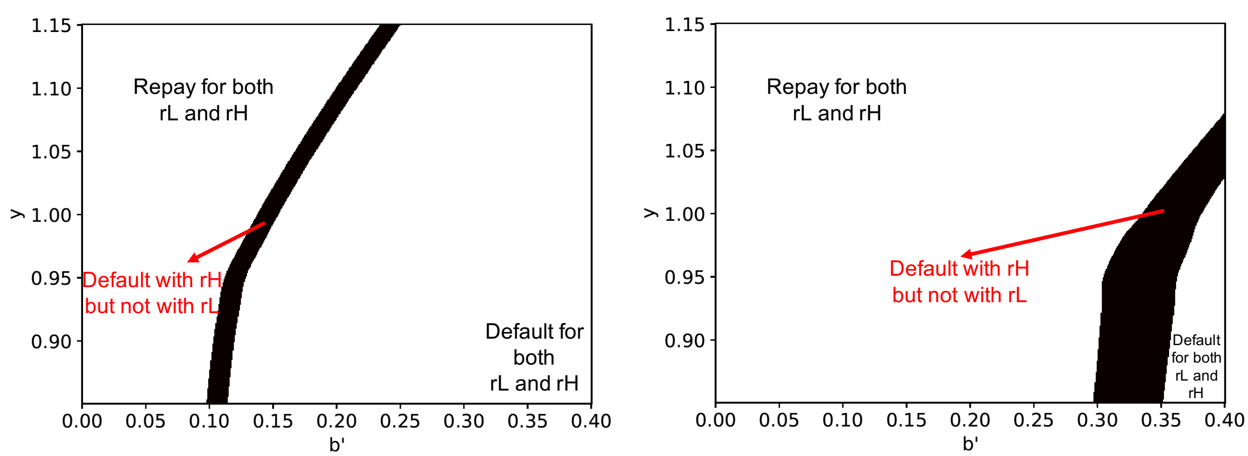


6.2 Lenders' bargaining power

Figure 8 shows the same regions as figure 4 for different values of α. Notice the area where only the high and not the low interest rate triggers default becomes thicker as α increases. This is consistent with the discussion in section (4): the higher the bargaining power of the lenders, the higher the role that the interest rate plays.

Figure 8: Variations of the lenders' bargaining power α

(a) $\alpha = 0.10$ (b) $\alpha = 0.50$



Additionally, note that with high α higher levels of debt can be sustained. This is because higher levels of lenders' bargaining power makes default more painful since renegotiation will be more stringent. This future threat is akin to giving more commitment for the government to borrow ex-ante, as this parameters governs renegotiation outcomes.

7 Conclusion

We provide a framework in which external shocks may have a relevant role in triggereing debt crisis in emerging economies. For this, we provide a new mechanism through which risk-free interest rates affect borrowing costs and ultimately default incentives.

This feature has not been present in many quantitative sovereign default models that studies exactly the role of such shocks. We show that out model is able to reconcile sovereign default models to the widespread narrative about the 1980's.

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Appendix: Proofs

Proposition 1: In equilibrium, the bargaining problem always have a solution and, accordingly, $a(b,y,r)=1$ and $V(b,y,r,1)=V^P\left(b^R(b,y,r),y,r\right)$, for all (b,y,r).

Proof: [...]

Proposition 2: The sets $\overline{\mathcal{B}}(b,y,r)$ and $\underline{\mathcal{B}}(b,y,r)$ are singleton. Furthermore, if $\bar{b}\in\overline{\mathcal{B}}(b,y,r)$ and $\underline{b}\in\underline{\mathcal{B}}(b,y,r)$, then it must be that $\bar{b}\geq\underline{b}$.

Proof: As the Nash bargaining problem in our economy always has a solution, then the sets $\overline{\mathcal{B}}(b,y,r)$ and $\underline{\mathcal{B}}(b,y,r)$ are non-empty. Also, notice that the lenders' surplus $S^{LEN}\left(\tilde{b},b,y,r\right)$ is linearly increasing in the renegotiated debt level $\tilde{b}$ and that the government's surplus S^{GOV} is strictly decreasing in $\tilde{b}$, as it makes its budget tighter. . Therefore, the strict monotonicity implies that there must be only one debt level that makes lenders indifferent and one that makes the government. Therefore, the sets $\overline{\mathcal{B}}(b,y,r)$ and $\underline{\mathcal{B}}(b,y,r)$ are singleton

Consider $\bar{b}\in\overline{\mathcal{B}}(b,y,r)$ and $\underline{b}\in\underline{\mathcal{B}}(b,y,r)$. Suppose for contradiction that $\bar{b}<\underline{b}$. As the lenders' surplus is strictly increasing in the renegotiated debt level $\tilde{b}$, they reject any $\tilde{b}<\underline{b}$. Similarly, as the government's surplus is strictly decreasing in $\tilde{b}$, they reject any $\tilde{b}\geq\underline{b}>\bar{b}$. Therefore, there is no feasible renegotiated debt level for this barganining problem, which contradicts proposition 1.

Proposition 3: The market value of defaulted bonds and the value in default are independent of the stock of debt if and only if renegotion outcomes does not depend on the stock of debt, ie, $q^D(b_1,y,r)\,b_1 = q^D(b_2,y,r)\,b_2$ and $V^D(b_1,y,r) = V^D(b_2,y,r) \iff$

$b^R(b_1, y, r) = b^R(b_2, y, r)$, for all (y, r) and $b_1, b_2 > 0$.

Proof: First, consider the if part of the statement. Suppose $b^R(b_1, y, r) = b^R(b_2, y, r)$, for all (y, r) and $b_1, b_2 > 0$. From proposition 1, $V(b, y, r, 1) = V^P\left(b^R(y, r), y, r\right)$. Thus, the value in default is

$$\begin{aligned} V^D(b, y, r) &= u(h(y)) + \beta\theta\mathbb{E}\left[V^P\left(b^R(y', r'), y', r'\right)\right] \\ &\quad + \beta(1-\theta)\mathbb{E}\left[V^D(b, y', r')\right] \end{aligned}$$

and, by iterating with time subscripts, it can be rewritten as

$$\begin{aligned} V^D(b_t, y_t, r_t) &= u(h(y_t)) + \sum_{h=1}^{\infty}\left\{\beta^h(1-\theta)^h\mathbb{E}_t\left[u(h(y_{t+h}))\right]\right\} \\ &\quad + \sum_{h=1}^{\infty}\left\{\beta^h(1-\theta)^{h-1}\theta\mathbb{E}_t\left[V^P\left(b^R(y_{t+h}, r_{t+h}), y_{t+h}, r_{t+h}\right)\right]\right\} \\ &\quad + \lim_{\Delta\to\infty}\beta^{t+\Delta}(1-\theta)^{t+\Delta}\mathbb{E}_t\left[V^D(b_t, y_{t+\Delta}, r_{t+\Delta})\right] \end{aligned}$$

As $|\beta(1-\theta)| < 1$ and V^D is bounded, then the last term of this expression converges to zero and hence the RHS does not depend on b. Therefore, $V^D(b_1, y, r) = V^D(b_2, y, r)$, for all (y, r) and $b_1, b_2 > 0$.

Similarly, proposition 1 implies that $q^D(b, y, r)\, b = \frac{\theta}{1+r}\mathbb{E}\left[b^R(y', r')\right] + \frac{(1-\theta)}{1+r}\mathbb{E}\left[q^D(b, y', r')\, b\right]$. By iterating with time subscripts, we can rewrite the price of defaulted debt as

$$\begin{aligned} q^D(b_t, y_t, r_t)\, b_t &= \sum_{h=1}^{\infty}(1-\theta)^{h-1}\theta\mathbb{E}_t\left[\prod_{\tau=1}^{h}\left(\frac{1}{1+r_{t+\tau-1}}\right)b^R(y_{t+h}, r_{t+h})\right] \\ &\quad + \lim_{\Delta\to\infty}(1-\theta)^{\Delta}\mathbb{E}_t\left[\prod_{\tau=1}^{\Delta}\left(\frac{1}{1+r_{t+\tau-1}}\right)q^D(b_t, y_{t+\Delta}, r_{t+\Delta})\, b_t\right] \end{aligned}$$

Notice the price of defaulted bonds is bounded above by the highest possible income realization . Accordingly, the last term of this expression converges to zero and hence the

RHS does not depend on b. Therefore, $q^D(b_1,y,r)\,b_1 = q^D(b_2,y,r)\,b_2$, for all (y,r) and $b_1,b_2>0$.

Next, consider the only if part of the proposition. Suppose $q^D(b_1,y,r)\,b_1 = q^D(b_2,y,r)\,b_2$ and $V^D(b_1,y,r) = V^D(b_2,y,r)$, for all (y,r) and $b_1,b_2>0$, and denote $MV^D(y,r) \equiv q^D(b,y,r)\,b$, for any b. This implies that the government's surplus $S^{GOV}\left(\tilde{b},b,y,r\right) = V^P\left(\tilde{b},y,r\right) - V^D(y,r)$ and the lenders' surplus $S^{LEN}\left(\tilde{b},b,y,r\right) = \tilde{b} - MV^D(y,r)$ in the renegotiation game do not depend on the debt level b. It follows then that the game can be rewritten as

$$\begin{aligned} b^R(b,y,r) = &\arg\max_{\tilde{b}} S^{LEN}\left(\tilde{b},y,r\right)^{\alpha} S^{GOV}\left(\tilde{b},y,r\right)^{(1-\alpha)} \\ &\text{s.t.: } S^{LEN}\left(\tilde{b},y,r\right) \geq 0,\ S^{GOV}\left(\tilde{b},y,r\right) \geq 0 \end{aligned}$$

where the RHS clearly does not depend on b. Therefore, $b^R(b_1,y,r) = b^R(b_2,y,r)$, for all (y,r) and $b_1,b_2>0$.

Theorem 1: If $\alpha = 0$, then $q^D(b,y,r) = 0$ and $b^R(b,y,r) = 0$, for all (b,y,r).

Proof: In an equilibrium where the amount of debt defaulted on is irrelevant for the renegotiation process, proposition 3 gives that the Nash bargaining problem can be rewritten as

$$\begin{aligned} b^R(y,r) = &\arg\max_{\tilde{b}} S^{LEN}\left(\tilde{b},y,r\right) \\ &\text{s.t.: } S^{LEN}\left(\tilde{b},y,r\right) \geq 0,\ S^{GOV}\left(\tilde{b},y,r\right) \geq 0 \end{aligned}$$

As $S^{LEN}\left(\tilde{b},y,r\right) = \tilde{b} - q^D(b,y,r)\,b$ is linearly increasing in $\tilde{b}$ while $S^{GOV} = V^P\left(\tilde{b},y,r\right) - V^D(y,r)$ is decreasing in $\tilde{b}$, then the renegotiated debt level is as low as feasible. Specifically, by proposition 2, the outcome of this game is such that lenders are indifferent between renegotiating or remaining in default: $b^R(y,r) = \underline{b}(y,r)$. From the definition of $\underline{b}(y,r)$, this result implies $b^R(y,r) = q^D(b,y,r)\,b$.

As proposition 1 states that renegotiation is always succesful in equilibrium, then a unit of a defaulted bond can be rewritten as $q^D(b,y,r) = \frac{\theta}{1+r}\mathbb{E}\left[\frac{b^R(y',r')}{b}\right] + \frac{(1-\theta)}{1+r}\mathbb{E}\left[q^D(b,y',r')\right]$. Evaluated at renegotiation outcomes in which the government is always indifferent between accepting the terms or not, it becomes $q^D(b,y,r) = \frac{1}{1+r}\mathbb{E}\left[q^D(b,y',r')\right]$. As $q^D(b,y,r) \geq 0$ for any state, it must be that $q^D(b,y,r) = 0$, for all (b,y,r).

Then, it follows from the definition of $\underline{b}(y,r)$ that $b^R(y,r) = 0$ for all (b,y,r) with $b > 0$.

Online appendix. Computation

The numerical solution of models in the style of Eaton and Gersovitz (1981) with long term debt can be quite challenging. Nevertheless, Chatterjee and Eyigungor (2012) explain how this can be overcome with the introduction to the endowment process of an independent and identically distributed shock m that has a continuous cumulative distribution function. While these features guarantee good convergence properties, setting zero mean and low variance prevent this shock to alter the model's outcome in any meaningful way.

As renegotiation provides a new layer of complication for the numerical solution, we explain in this online appendix the changes we impose to the model for computing equilibrium.

Model

The following basic structure remains the same. Time is discrete. Government is benevolent, borrows from risk neutral foreign lenders by issuing long term non-contingent debt, and lacks commitment. Risk free interest rate follows a Markov process.

Nevertheless, the income process now is composed of two components $w_t = y_t + m_t$. As before, there is a persistent component y_t that follows a finite-state Markov chain with transition law $Prob\left(y_{t+1}=y'|y_t=y\right) = F\left(y,y'\right)$. Additionally, there is a transitory shock m_t, assumed to be *iid* with continuous CDF $G\left(m_t\right)$ over $\left[-\bar{m},\bar{m}\right]$ for some small $\bar{m}>0$.

Preferences for consumption in each period are given by $E_t\left[\sum_{j=t}^{\infty}\beta^{j-t}u\left(c_j\right)\right]$ and a bond consists of the same perpetuity as in our original model. Thus, a bond issued in period t promises to pay $\gamma\left(1-\gamma\right)^{j-1}$ units of the tradable good in period $t+j$, $\forall j\geq 1$, and the law of motion for bonds is given by $b_{t+1}=\left(1-\gamma\right)b_t+x_t$.

If the government is in good standing with international lenders, then, at the beginning of the period, it observes the current realization of all shocks in the economy, which includes the new transitory income shock, and decides whether to default or repay. Its value in this case is:

$$V\left(b,y,m,r,0\right)=\max_{d\in\{0,1\}}\left\{\left(1-d\right)V^P\left(b,y,m,r\right)+dV^D\left(b,y,r\right)\right\} \tag{10}$$

If the government decides to repay it makes coupon payments γb and gets to issue new bonds. The value of the government in repayment is then:

$$V^P\left(b,y,m,r\right)=\max_{c,b'}\left\{u(c)+\beta\mathbb{E}\left[V\left(b',y',m',r',0\right)\right]\right\} \tag{11}$$

$$s.t.\quad c+\gamma b\leq y+m+q^P\left(b',y,r\right)\left[b'-\left(1-\gamma\right)b\right]$$

As in Chatterjee and Eyigungor (2012), notice the price of bonds does not depend on m. The reason is that, given the shock's *iid* nature, its current value does not affect the probability of defaulting in the following period.

If the government chooses to default, then debt payments are suspended, the government is excluded from credit markets, debt is frozen so $b_{t+1}=b_t$, and income is $h\left(y_t\right)=y_t-\phi\left(y_t\right)-\bar{m}$, where $\phi\left(y_t\right)\geq 0$ and $y_t-\phi\left(y_t\right)$ is increasing in y_t. The value of the government in default is:

$$V^D\left(b,y,r\right)=u(h\left(y\right))+\beta\left\{\theta\mathbb{E}\left[V\left(b,y',m',r',1\right)\right]+\left(1-\theta\right)\mathbb{E}\left[V^D\left(b,y',r'\right)\right]\right\}. \tag{12}$$

If an opportunity to renegotiate arises, the government can choose to accept to start repaying a new debt level b^R or to remain in default. If a new debt level b^R is agreed upon, the government regains access to financial markets and there is no longer an output cost of being in default:

$$V(b, y, m, r, 1) = \max_{a} \left\{ aV^P \left(b^R(b, y, r), y, m, r \right) + (1-a) V^D(b, y, r) \right\}. \tag{13}$$

Note that b^R does not depend on m. This is a result of a timing assumption we make in the bargaining problem between lenders and the government, when the government is in bad financial standing and an opportunity to renegotiate its debt arises. In particular, we assume that at the beginning of the period the lenders and the government observe the persistent component of income y and the risk-free interest rate r. Then, before observing the transitory component m, the new debt level b^R offered to the government is determined in the following Nash bargaining game:

$$\begin{aligned} b^R(b, y, r) &= \arg\max_{\tilde{b}} \left[S^{LEN}\left(\tilde{b}, b, y, r\right) \right]^{\alpha} \left[S^{GOV}\left(\tilde{b}, b, y, r\right) \right]^{1-\alpha} \\ s.t.: \quad & S^{LEN}\left(\tilde{b}, b, y, r\right) \geq 0,\ S^{GOV}\left(\tilde{b}, b, y, r\right) \geq 0 \end{aligned} \tag{14}$$

Once m is realized, the offer b^R cannot be changed. Given this timing, we define the surplus of the government as the expectation over m of the difference between accepting the new deal and remaining in default:

$$S^{GOV}\left(\tilde{b}, b, y, r\right) = \int_{-\bar{m}}^{\bar{m}} \left[V^P\left(\tilde{b}, y, m, r\right) - V^D(b, y, r) \right] dG(m) \tag{15}$$

Similarly, the surplus of the lenders is the expectation over m of the difference between resuming debt payments with the new debt level $\tilde{b}$ and the market value of the previously defaulted bonds:

$$S^{LEN}\left(\tilde{b},b,y,r\right)=\int_{-\bar{m}}^{\bar{m}}\left[\gamma\tilde{b}+q^{P}\left(b^{P}\left(\tilde{b},y,m,r\right),y,r\right)(1-\gamma)\,\tilde{b}-q^{D}\left(b,y,r\right)b\right]dG\left(m\right) \quad (16)$$

In our quantitative exercises we assume the bound $\bar{m}$ to the support of the shock m is small enough so that its effect on the results is negligible. Our timing assumption regarding the shock m is akin to randomizing over decisions, which is crucial for the convergence of the solution method we implement. Dvorkin, Sánchez, Sapriza, and Yurdagul 2018 implement a similar method to randomize over renegotiation decisions in an environment where bargaining is over both debt levels and maturity extensions.

The definition of equilibrium is analogous to that of our original model. An equilibrium is value functions V, V^P, and V^D, policy functions c^P, b^P, d, and a, bond prices q^P and q^D, and a best feasible renegotiated debt level b^R such that: (i) given b^R and the bond prices, the value and policy functions solve (10) through (13), (ii) given prices and the value and policy functions, b^R is defined by (14) , (iii) bond prices are consistent with lenders making zero profits after adjusting for default risk.

Thus, the price of bonds when the government is in good standing with international lenders is:

$$\begin{aligned} q^{P}\left(b',y,r\right)=&\frac{1}{1+r}\mathbb{E}\left[\left\{1-d\left(b',y',m',r'\right)\right\}\left\{\gamma+(1-\gamma)\,q^{P}\left(b^{P}\left(b',y',m',r'\right),y',r'\right)\right\}\right] \quad (17)\\ &+\frac{1}{1+r}\mathbb{E}\left[d\left(b',y',m',r'\right)q^{D}\left(b',y',r'\right)\right] \end{aligned}$$

And the price of bonds when the government has in bad standing is:

$$
\begin{aligned}
q^{D}(b,y,r) = {} & \frac{\theta}{1+r}\mathbb{E}\left[a\left(b,y',m',r'\right)\frac{b^{R}\left(b,y',r'\right)}{b}\left\{\gamma\right.\right. \\
& \left.\left. +\left(1-\gamma\right)q^{P}\left(b^{P}\left(b^{R}\left(b,y',r'\right),y',m',r'\right),y',r'\right)\right\}\right] \\
& +\frac{\theta}{1+r}\mathbb{E}\left[\left\{1-a\left(b,y',m',r'\right)\right\}q^{D}\left(b,y',r'\right)\right]+\frac{1-\theta}{1+r}\mathbb{E}\left[q^{D}\left(b,y',r'\right)\right]
\end{aligned} \tag{18}
$$

Calibration

As before, the utility function displays a constant coefficient of relative risk aversion η, $E_t\left[\sum_{j=t}^{\infty}\beta^{j-t}\frac{c_j^{1-\eta}-1}{1-\eta}\right]$, the persistent component of income y_t follows a log-normal AR(1) process $\log\left(y_t\right)=\rho\log\left(y_{t-1}\right)+\epsilon_t$, with $|\rho|<1$ and $\epsilon_t\sim N\left(0,\sigma_\epsilon^2\right)$, and the output cost is $\phi\left(y_t\right)=\max\left\{0,\phi_0 y_t+\phi_1 y_t^2\right\}$, with $\phi_0<0<\phi_1$.

We assume the transitory component m_t follows a truncated normal distribution, $m_t\sim$ trunc$N\left(0,\sigma_m^2\right)$, with points of truncation $-\bar{m}$ and $\bar{m}$. In all figures and simulations in this paper, we set $m=0$.