Earmarked Loans and Economic Performance in Brazil

Victor Almeida
University of Minnesota

Pedro Tanure Veloso
University of Minnesota

March 5, 2019

Abstract

This paper develops a general equilibrium model with sectoral linkages in which firms face borrowing constraints that can be alleviated by government subsidies. We use this model to evaluate how the Brazilian government's policy to direct subsidized credit to specific sectors, called *earmarked* loans, impacts output per worker through two channels. The first one is the general equilibrium effect of alleviating the borrowing constraint of a sector, increasing output. The second channel works in the other direction. In order to raise funds to subsidize loans, the government needs to tax labor and hence distorts households' consumption–labor supply decisions. Whether the first effect dominates the second depends on how relevant the subsidized sector is in the economy's production network structure. We calibrate the model using Brazilian data to study the federal government's decision to increase subsidies for specific sectors in the credit market, perform optimal policy analysis, and investigate how the economy would have performed had the policy not changed. We find that the behavior of sectoral productivity was more important to explain sluggish performance of the Brazilian economy after the Great Recession than changes in government intervention in the credit markets. In addition, we find that the optimal subsidy policy would require higher sectoral subsidies than the ones observed in the data.

1 Introduction

This paper develops a multi-sector model to understand the impact of government credit subsidy policies directed towards specific sectors on output per worker in the presence of a distortionary labor tax, using the Brazilian economy's recent experience with such policy as its quantitative experiment. The model allows for sectoral linkages in production and imposes working capital constraints in the firms' maximization problem. We allow for heterogeneous input demands in the network structure, which implies that some sectors impact others, and thus aggregate output, differently. Therefore, marginally relaxing different sectors' borrowing constraint can generate different impacts on aggregate output due to general equilibrium effects, which implies that any subsidy should take these effects into account.

Another consequence of the credit subsidy policy is the distortion in consumption-labor decisions by households. Because the government needs to tax labor income to subsidize interest rates on loans, any decision to increase subsidies to a specific sector involves a tradeoff.

Since 2010, Brazil has implemented a series of policies to foster economic growth. To achieve this goal, the government heavily used three policy instruments: i) price freezes of "strategic" inputs whose prices are regulated, such as the price of electricity and gasoline; ii) tax exemptions for some sectors, in particular manufacturing industries; and iii) directed, subsidized lending to firms and households. However, as Figure 1 shows, the drop in TFP since 2010 has been sharp and consistent. This suggests that these policies may have contributed negatively to the country's economic performance. For instance, after growing 7.6% in 2010, output growth slowed down to an average rate of 2.2% from 2011 to 2014.

In this paper, we narrow down our analysis to the third instrument. Directed, subsidized lending to specific sectors or household demographics is formally called earmarked lending. One of the main sources of earmarked credit to firms is the Brazilian Social and

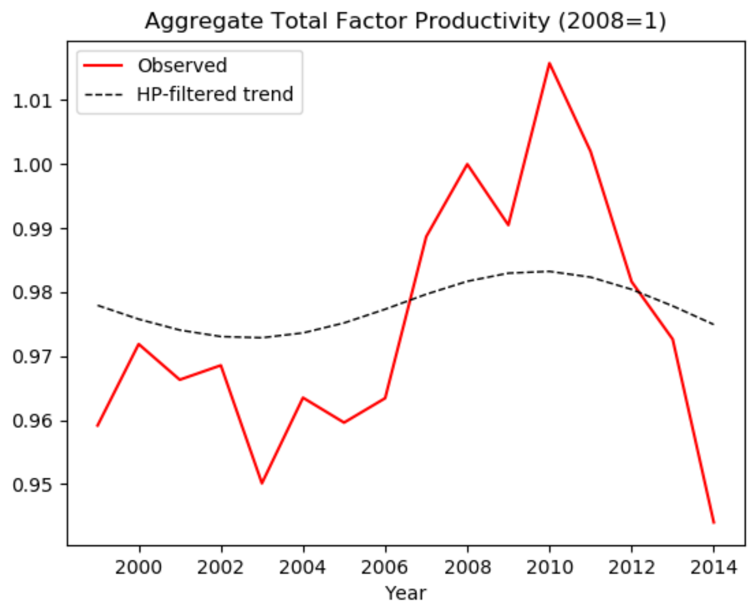


Figure 1: Total Factor Productivity–Observed and Trend (2008=1)

Economic Development Bank (BNDES hereafter). By earmarking credit, the government generates substantial differences in borrowing costs across firms and sectors. Therefore, this credit policy change in Brazil potentially had non-negligible effect on the economy. By varying how much credit to subsidize and which sectors to focus the policy on, financial frictions were exacerbated in some sectors and remedied in others.

To implement such a large-scale subsidy policy, the government has to generate revenues. In Brazil, this is done mainly through a combination of labor income taxes and government borrowing. This means that even if alleviating firms' financial constraints has the beneficial outcome of increasing output, the distortions arising from taxation can potentially counterbalance this effect, making such policy undesirable.

The way we chose to think about this policy is by looking at how financial constraints affect an economy with sectoral linkages. In such environment, the effect of lowering borrowing costs in one sector propagates throughout the economy. In our model, therefore, subsidies to a sector have effects that go beyond simply increasing production of such sector. By increasing its production and reducing its prices, the working capital constraints of its downstream sectors are alleviated. Indeed, since their input costs decreases, their demand increases and production gets closer to its frictionless counterpart. This also decreases their prices, which consequently also alleviates their own downstream sectors' working capital constraints.

In general, sectors downstream of the targeted ones experience positive spillovers from the earmarked credit policy program through general equilibrium effects. Therefore, subsidies feature heterogeneous effects on the economy, since sectors face different financial constrains and are not equally relevant in the production network.

There are two forces in the model associated to earmarked credit policy that could justify aggregate TFP movements: labor market distortions and internalizing pecuniary externalities. The first one comes from the need to finance subsidies with distortionary taxes, which consequently causes misallocation of resources at the firm-level and hence a fall in aggregate TFP. The second one comes from the positive spillover that are only (either partially or fully) internalized by sectors via interest rate subsidies. Therefore, a government could lead the economy to higher or lower aggregate TFP levels depending on how much and how well distributed across sectors subsidies are.

Using Brazilian input-output tables and data on sectoral interest rates, we use the model to understand the effect of sectoral productivity and financial frictions on aggregate productivity. Our functional form assumptions for the production sector and the financial frictions introduced in the model allow us to write aggregate productivity that is multiplicatively separable on sectoral productivities and borrowing costs (market interest rates and credit subsidies). We find that the change in earmarked credit policy that occurred from 2008 to 2014 played a modest positive role in increasing GDP per worker and aggregate productivity. The observed policy was effective in raising the economy's efficiency and hence output per worker when compared to a policy that kept sectoral interest-rate subsidies in the levels observed in 2008, and especially in a counterfactual economy with no government subsidies whatsoever. Additionally, the model suggests that credit subsidies should have been even higher than the ones observed in the data. Welfare analysis indicates that implementing such a policy would have increased output per worker and welfare significantly.

1.1 Related Literature

The intervention on the credit market by the federal government was mainly to provide firms with cheaper credit than they would in the free credit market. The policy is also partly motivated by the idea that firms should receive subsidized credit due to financial constraints: these firms are either very young or are part of a sector in which the social benefit of their economic activity are bigger than the private one. As Bonomo et al. (2015) find out, however, this credit seems to be directed towards older and larger firms, which would likely be able to finance themselves on the private and free credit market more easily. Carvalho (2014) also finds that these loans occur more often to companies in regions where local elections are coming, and the incumbent government is less likely to win by the time the loan is approved.

Both papers cited above hint that the use of earmarked credit as a government policy may have generated misallocation of productive resources in Brazil. The magnitude in which such policy affected resource allocation is an open question. Following the methodology in Hsieh and Klenow (2009), Vasconcelos (2017) finds evidence that resource misallocation increased in Brazil from 2005 to 2011. On the other hand, Cavalcanti and Vaz (2017) find that when a firm has access to earmarked credit in Brazil, it increases its investment and productivity if the access is permanent, not temporary.

The literature on the link between financial frictions and misallocation is also extensive. Gilchrist et al. (2013) look at the effect of heterogeneous borrowing cost for firms in the U.S. financial market and find little negative aggregate effects of such heterogeneity. Midrigan and Xu (2014) use a model in which financial frictions affect both technology adoption between firms and dispersion in returns to capital. They find little losses in misallocation due to such frictions. Gopinath et al. (2017) provide evidence of capital misallocation effects of a decline in the real interest rate in Spain in the presence of financial frictions. They find that capital inflows are misallocated toward firms with higher net worth, which aren't necessarily more productive.

Recently, papers such as Acemoglu et al. (2012) emphasize the role of sectoral shocks in driving the aggregate economy. The role of small sectoral distortions in generating substantial macroeconomic effects has also been recently explored in the literature. Baqaee and Farhi (2017) show that the aggregate impact of a sectoral shock can be summarized into two components: a technology shock and its effect on allocative efficiency arising from the reallocation of resources. They show that the latter effect can be quantitatively substantial.

Justification for industrial policies are usually related to the presence of externalities (Atkinson and Stiglitz, 2015). Liu (2017) finds that effects of market imperfections accumulate through *backward demand linkages*, that is, distortions accumulate upstream in a production network. Therefore, a benevolent government should focus on subsidizing sectors which are upstream from highly distorted ones to improve aggregate welfare.

Network externalities can arise from production networks in which firms face financial constraints (Altinoglu, 2018; Bigio and La'O, 2016; Luo, 2018). Under such circumstances, financial constraints in one sector have effects in the rest of the economy via the intermediate demand. A distorted firm will demand less than optimal inputs from its suppliers, which will therefore have depressed sales and will themselves demand less than optimal from their own suppliers.

2 The Role of Earmarked Credit in Brazil

In Brazil, a segment of the credit market consists of credit directed towards predetermined sectors or activities, using resources regulated by law or discretionarily allocated by the government. Three main entities provide funds for the lending operations. The BNDES provides credit to private firms, either directly or through commercial banks, for their investment and working capital decisions. Housing financing for households is mostly financed through the public bank Caixa Econômica Federal, while credit for the agricultural sector is mainly provided by the public bank Banco do Brasil.

Since 2008, earmarked lending has steadily increased its share in total credit. Initially implemented as a policy response to the liquidity shortage in the global economy due to the credit crunch of 2008, it later became a government policy tool to bring cheaper credit to firms. In particular, the creation of the Program to Sustain Investment (*Programa de Sustentação do Investimento*, in Portuguese) in mid-2009 made the use of earmarked credit to stimulate private sector's investment a key component for the government's long-term economic development project. As Figure 2 shows, the increase in the earmarked credit participation in the total stock of credit in Brazilian's financial system has been steady since 2008 until 2016, when the persistent and ever-increasing budget deficits forced the government to wane the policy. The stock of earmarked credit rose from 11.52 percent of GDP in January 2008 to 26.37 percent in December 2015. By the end of 2015, the amount of earmarked credit in the Brazilian economy had reached about half the stock of credit. As Figure 2 also shows, the stock of earmarked credit coming directly from BNDES, not counting its loans made through commercial banks, also substantially increased.

The objective of the earmarked credit is to provide credit at a lower rate than the one available in the private market. For instance, the Long Term Interest Rate (TJLP), the benchmark rate for credit supplied by the BNDES, is set below the monetary policy rate, and frequently reaches negative values in real terms. As Figure 3 shows, earmarked rates, from the BNDES or other sources, are on average substantially lower than the ones in the free credit market, and less volatile as well.

This difference is less striking, but still present, when we compare interest rates charged in the earmarked and free credit market for loans with similar characteristics. We investi-

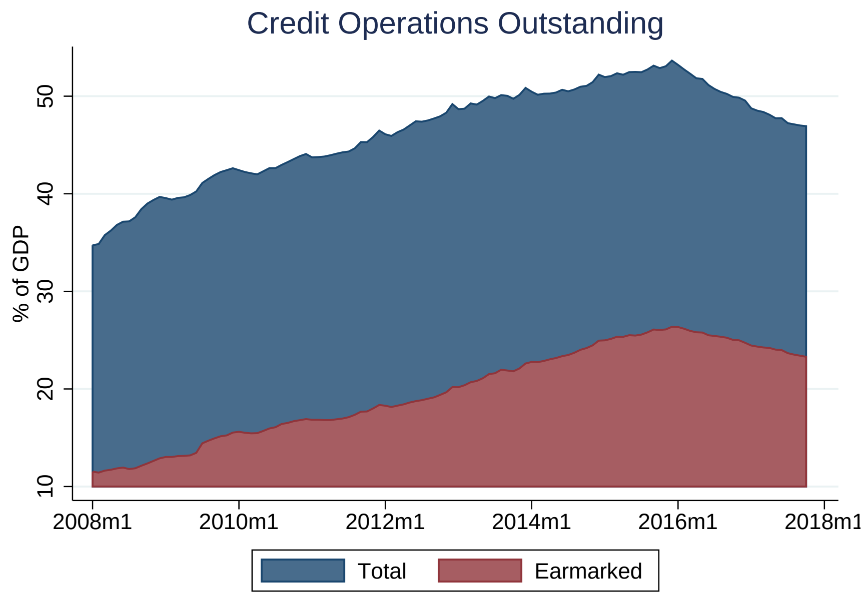


Figure 2: Credit Operations Outstanding

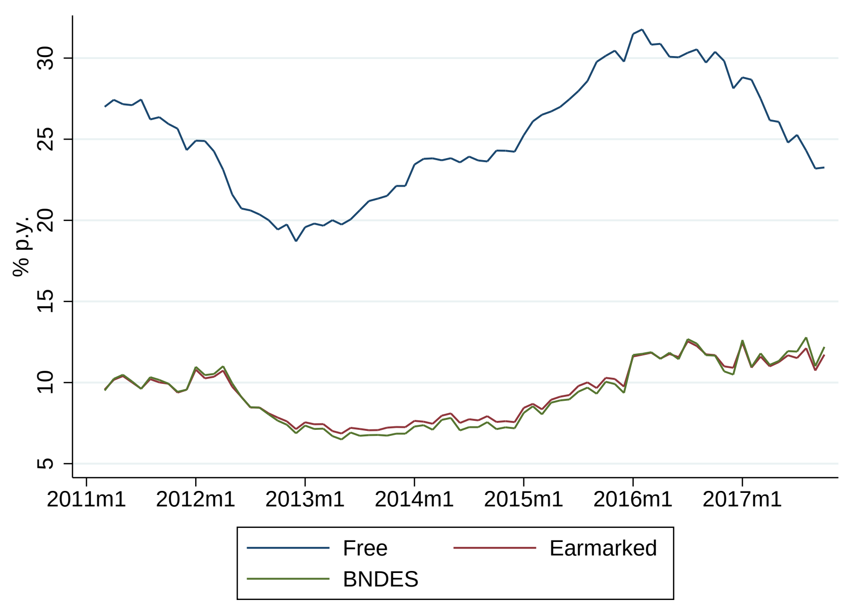


Figure 3: Average Interest Rate of New Credit Operations

Non-Financial Corporations

gate this by using a monthly dataset provided by the Central Bank of Brazil that contains data on loans for firms, aggregated by sector, using the 6-digit National Classification of Economic Activities (CNAE) code.[1] Our dataset contains information on the average amount lent, average interest rate, maturity, average value of guarantees, type of loan (earmarked or nonearmarked), and the category of the loan (working capital, fixed investments, export credits and so on). A regression of interest rates on a dummy for whether the credit is earmarked, controlling for maturity, guarantees, category, year and sector dummies, indicates that interest rates charged on earmarked credit are, on average, ten percentage points lower than the ones in the nonearmarked segment of the credit market.

Using financial data aggregated to the 12-sector division in the Brazilian national accounts,[2] Figures 5 and 6 show that, even though the policy has increased its role in the Brazilian economy, it did not increase symmetrically across sectors. In particular, Figure 5 shows that the share of total earmarked credit in 2008 that was directed to each sector was fairly different than in 2013.

Naturally, these movements could had been simply caused by changes in productivity across sectors over time and, hence, be unrelated to the government's policy. Nevertheless, Figure 6 indicates they were, to some extent, a consequence of a new targeting of earmarked credit policy. The earmarked-to-credit-outstanding ratio in each sector has also changed from 2008 to 2013, evidencing that the dynamics in Figure 5 should not only be attributed to private sector factors.

Moreover, in order to finance the loans subsidies, the government needs to raise funds. According to Pazarbasioglu-Dutz (2017), demand deposits, special funds, and direct lending from the fiscal sector are the main funding sources of earmarked credit. The funding of earmarked credit in 2015 comes from the following sources:

1. Savers fund about 40 percent of the earmarked credit. Funding comes from demand

[1] The Brazilian National Institute of Geography and Statistics (IBGE) has its own industry classification system. It can be mapped into the common ISIC industry classifications.

[2] A description of the sectors used can be found in Section 5.

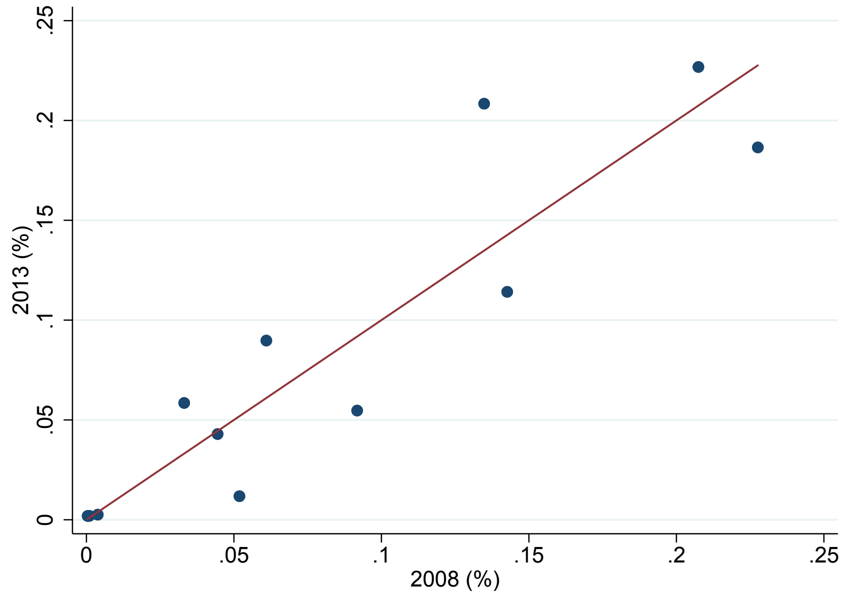


Figure 5: Inter-sectoral share of earmarked credit–2008-2013

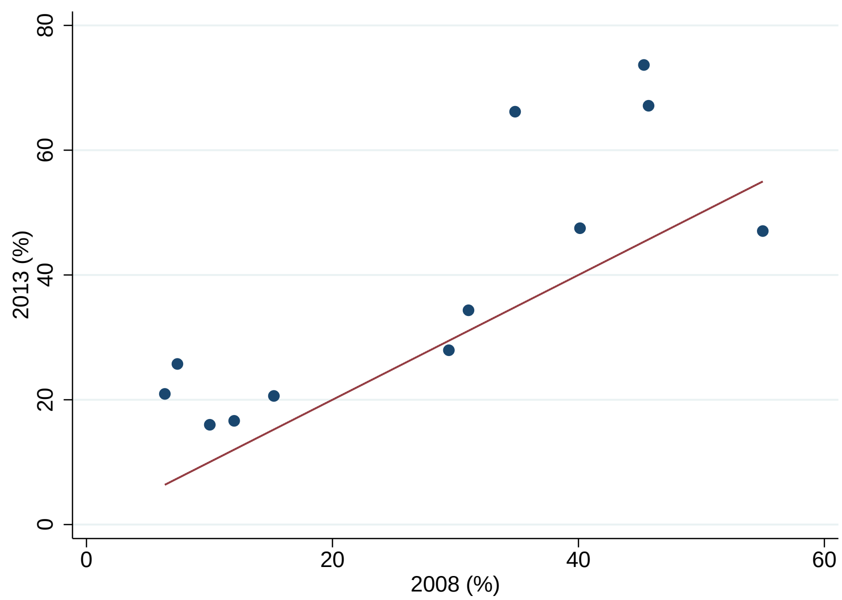


Figure 6: Intra-sector share of earmarked credit–2008-2013

and savings deposits as well as tax-exempt financial instruments from specific sectors, such as real estate and agriculture;

2. Employees fund about 12 percent through monthly salary deductions to the Severance Indemnity Fund (FGTS);

3. The fiscal sector funds about 48 percent through direct lending to BNDES and through various special and constitutional funds, such as the Support Fund to the Employee (FAT).

3 A Model of Sectoral Linkages with Labor Taxes

We study the impact of earmarked credit in the Brazilian economy using a simple static general equilibrium model with sectoral linkages. The model closely related to the one presented in Bigio and La'O (2016), and especially in Luo (2018), which includes working capital constraints in a model with sectoral linkages and trade credit, where part of the firms' financing costs can be paid after production without borrowing costs. The difference between our model and the one in Luo (2018) is the absence of trade credit, the inclusion of government subsidies, and the introduction of labor taxes, which distorts household's consumption–labor supply decisions.

Even though, as Section 2 above discussed, the majority of resources used to finance the earmarked credit policy in Brazil does not come from labor taxes, most of the sources of revenue, if not all, involve some sort of distortion in the private agents' decision-making. For instance, the existence of tax-exempt financial instrument generate distortion in portfolio allocation in the financial markets. The FAT is financed mainly by taxing firms' gross revenues, as well as payroll taxes from non-profit organizations. Increased government borrowing can also be thought of as a potential for distorting consumption-labor supply decisions, either through the use of other distortive taxes or through higher inflation. To keep the model simple, we choose to abstract from most of these details, leave out government debt, and make use of a tax on wages as a way to discipline it.

The economy is composed of N sectors, indexed by $i = 1, ..., N$. Each sector consists of a continuum of competitive firms. Goods are identical across firms within a sector but differentiated across sectors. We index goods by $j = 1, ..., N$, with the understanding that there is a one-to-one mapping between each sector and the good that it produces.

3.1 Firms

Firms in sector i produces a specialized product y_i using a Cobb-Douglas production function:

$$y_i = z_i \ell_i^{\alpha_i} \left(\prod_{j=1}^{N} x_{ij}^{\omega_{ij}} \right)^{1-\alpha_i}$$

where x_{ij} denotes the intermediate input used by firm in sector i from sector j; ℓ_i represents the amount of labor input used by the firm i, and z_i is a sector-specific productivity measure. We assume $\sum_{j=1}^{N} \omega_{ij} = 1$ for all i.

There exists a working capital requirement on labor and intermediate inputs. Firms need to borrow in order to pay for their input cost at the beginning of each production period. We assume a small open economy in which firms obtain credit from the rest of the world with a fixed interest rate R, plus an idiosyncratic risk e_i. Policymakers can subsidize the loan on each unit of the credit obtained by the firm in the amount s_i. In this case, the borrowing fee of the firm becomes $R_i = R + e_i - s_i$.

The output of sector i can be used as an intermediate input for other sectors, x_{ji}, as well as for final consumption, which we denote c_i. Due to the financial consraint, the price the firm sells its goods to other firms, p_{ji}, is not the same as the price it sells to households, p_i. Each firm solves the following problem:

$$\max_{\ell_i, \{x_{ij}\}_{j=1}^{N}} p_i c_i + \sum_{j=1}^{N} R_i p_{ji} x_{ji} - \sum_{j=1}^{N} R_i p_{ij} x_{ij} - R_i w \ell_i$$

$$\text{subject to } z_i \ell_i^{\alpha_i} \left(\prod_{j=1}^{N} x_{ij}^{\omega_{ij}} \right)^{1-\alpha_i} \geq y_i = c_i + \sum_{j=1}^{N} x_{ji}$$

3.2 Households

The preferences of the representative household are given by:

$$U(c, \ell) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{\ell^{1+\varphi}}{1+\varphi}$$

where ℓ denotes labor and c the household's final consumption basket. The final consumption basket of the household is a composite of the differentiated goods in the economy:

$$c = \prod_{j=1}^{N} \left(\frac{c_j}{\nu_j} \right)^{\nu_j}$$

where the parameter $\nu_j \in [0, 1]$ is the household's expenditure share on good j. If the household does not consume good j, then $\nu_j = 0$. Without loss of generality we set $\sum_{j=1}^{N} \nu_j = 1$.

The budget constraint of the household is given by:

$$Pc \leq (1-\tau) w\ell + \sum_{i=1}^{N} \pi_i$$

where the left hand side is total expenditure and the right hand side includes the after-tax wage income of the household and dividends from owning all of the firms. P is the price level, given by $P \equiv \Pi_i p_i^{\nu_i}$, which comes from the expenditure minimization problem.

3.3 Government

The government taxes labor in order to finance their sectoral loan subsidy policy. We assume it can not borrow in order to finance its expenditures. Therefore, the government chooses a tax rate in order to satisfy its budget constraint.

$$\sum_{i=1}^{N} s_i \left[\sum_{j=1}^{N} \left(p_{ij} x_{ij} - p_{ji} x_{ji} \right) + w \ell_i \right] = \tau w \ell$$

3.4 Market clearing

The output of any given sector may be either consumed by the household or used by other sectors as an input to production. Commodity market clearing for each good i is thus given by:

$$y_i = c_i + \sum_{j=1}^{N} x_{ji}$$

Similarly, labor market clearing satisfies $\sum_{i=1}^{N} \ell_i = \ell$.

3.5 Equilibrium

Given the environment described above, the equilibrium in this model is defined as follows.

Definition 1. *Given the sector-specific interest rate and subsidy, $(R_i, s_i)_{i=1}^{N}$, a competitive equilibrium consists of a vector of commodity prices, $(p_i)_{i=1}^{N}$, a consumption bundle, $(c_i)_{i=1}^{N}$, and sectoral output, intermediate goods, and labor allocations $\left(y_i, (x_{ij})_{j=1}^{N}, \ell_i\right)_{i=1}^{N}$, such that:*

1. *the household and firms are at their respective optima;*

2. *prices and wages clear the commodity and labor markets.*

4 Financial Frictions and Aggregate Distortions

Given the amount of labor supplied by the household ℓ, Proposition 1 below characterizes the equilibrium aggregate value added, tax rate, wages and sectoral prices.

Proposition 1. *Let $\circ$ and $\oslash$ denote the Hadamard (entrywise) product and division, respectively. Define the vectors $\mathbf{s} \equiv [s_1 \ldots s_N]'$, $\mathbf{R} \equiv [R_1 \ldots R_N]'$, $\mathbf{z} \equiv [z_1 \ldots z_N]'$, $\boldsymbol{\alpha} \equiv [\alpha_1 \ldots \alpha_N]'$, and $\boldsymbol{\nu} \equiv [\nu_1 \ldots \nu_N]'$. Let $\Theta_i \equiv \alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i}\prod_{j=1}^N \omega_{ij}^{(1-\alpha_i)\omega_{ij}}$ and Ω and Λ be matrices with entries $\Omega_{ij} = (1-\alpha_i)\omega_{ij}$ and $\Lambda_{ij} = R_i/R_j$, respectively. In addition, define $\beta \equiv (I-\Omega')^{-1}\boldsymbol{\nu}$.*

The aggregate production function of this economy is linear in labor, and given by

$$GDP(\mathbf{z}, \mathbf{R}, \mathbf{s}) = A(\mathbf{z})\mu(\mathbf{R}, \mathbf{s})\ell, \tag{1}$$

where the functions $A(\mathbf{z})$ and $\mu(\mathbf{R}, \mathbf{s})$ are given by

$$A(\mathbf{z}) \equiv \prod_i \Theta_i^{\beta_i} z_i^{\beta_i}$$

$$\mu(\mathbf{R}, \mathbf{s}) \equiv \frac{\prod_i R_i^{-\alpha_i\beta_i}\prod_{j=1}^N \left(\frac{R_i}{R_j}\right)^{-\omega_{ij}(1-\alpha_i)\beta_i}}{1 + \left((\mathbf{s}\oslash\mathbf{R})' - \mathbf{s}'(\Omega\oslash(\mathbf{R}\mathbf{1}'))'\right)\left(I-\Omega'\circ\Lambda\right)^{-1}\boldsymbol{\nu}}.$$

The endogenous tax rate that is consistent with the government policy $\mathbf{s}$ is given by

$$\tau = \frac{\left((\mathbf{s}\oslash\mathbf{R})' - \mathbf{s}'(\Omega\oslash(\mathbf{R}\mathbf{1}'))'\right)\left(I-\Omega'\circ\Lambda\right)^{-1}\boldsymbol{\nu}}{1 + \left((\mathbf{s}\oslash\mathbf{R})' - \mathbf{s}'(\Omega\oslash(\mathbf{R}\mathbf{1}'))'\right)\left(I-\Omega'\circ\Lambda\right)^{-1}\boldsymbol{\nu}}. \tag{2}$$

The log of the wage rate is given by

$$\log w = \beta'\left[\log\Theta + \log\mathbf{z} - (I-\Omega)\log\mathbf{R}\right]. \tag{3}$$

Aggregate price level is normalized to one ($P=1$). The log of the sectoral price vector $\mathbf{p}$ is *given by*

$$\log \mathbf{p} = [I - (I-\Omega)^{-1}\boldsymbol{\alpha}\beta'(I-\Omega)]\log \mathbf{R} - (I-\Omega)^{-1}(I-\boldsymbol{\alpha}\beta')(\log\Theta + \log \mathbf{z}). \quad (4)$$

4.1 The Efficiency Wedge

Proposition 1 shows that the production function aggregates into one that is linear on labor. This aggregate output function is similar from the one in Luo (2018) and in Bigio and La'O (2016). The difference between ours and the latter is our assumption of constant returns to scale. The aggregate production function contains an aggregate TFP that can be decomposed into two terms–one that is a function of sectoral productivity, $A(\mathbf{z})$, and one that depends on the distribution of interest rate spreads and government subsidies, $\mu(\mathbf{R}, \mathbf{s})$. The former is the common aggregation of sectoral productivity, while the latter is the efficiency wedge, i.e., the effect of sectoral distortions on aggregate productivity.

4.1.1 The Impact of a Sectoral Productivity Shock on Aggregate TFP

It's easier to see how aggregate TFP moves due to a sectoral productivity shock by taking logs on $A(\mathbf{z})$ and using the definition of β:

$$\log A(\mathbf{z}) = (I-\Omega')^{-1}\boldsymbol{\nu}(\log\Theta + \log \mathbf{z}).$$

The term $(I-\Omega')^{-1}$ is called the Leontief Inverse matrix. It encapsulates the infinite impact a sectoral productivity shock has in the economic network: a positive productivity shock in sector i simultaneously decreases the sector's price and increases its output. This in turns induces sectors that demand i's intermediate good to purchase more from this sector, increasing their production as well as decreasing their prices. These effects propagate through the economic network, and the magnitude of this effects depends on the network structure in Ω.

4.1.2 The Impact of Government Subsidies on Aggregate TFP

To build intuition understand the impact of government subsidies on aggregate productivity, we split the discussion in three parts.

The impact of borrowing costs on TFP. Assume no subsidies and that borrowing costs are equal across sectors and $R > 1$. It can be shown that $\boldsymbol{\alpha}'\beta = 1$, so therefore the efficiency wedge can be expressed as follows:

$$\log \mu(\mathbf{R}, \mathbf{s}) = -\log R.$$

In this case, the effect of borrowing costs on aggregate productivity is represented by a downward shift in the aggregate production function.

The impact of dispersions in borrowing costs on TFP. Assume no subsidies, but now borrowing costs are allowed to vary across sectors. The efficiency wedge can be expressed as follows:

$$\begin{aligned}
\log \mu(\mathbf{R}, \mathbf{s}) &= -\sum_i \beta_i \log R_i + \sum_i \left[\sum_j (1-\alpha_j)\omega_{ji}\beta_j\right] \log R_i \\
&= -\sum_i \left[\beta_i + \sum_j (1-\alpha_j)\omega_{ji}\beta_j\right] \log R_i \\
&= -(\beta + \Omega'\beta)' \log \mathbf{R} \\
&= -\beta'(I + \Omega) \log \mathbf{R}.
\end{aligned}$$

In this setting, the network structure of the economy plays a role in shifting the aggregate TFP downwards–a higher borrowing cost in sector i distorts the first-order condition for intermediate goods in all sectors that demand i's output. This creates resource misallocation, which is amplified through the network structure of the economy.

The impact of borrowing costs subsidies on TFP. When we add government subsidies to borrowing costs, the efficiency wedge can be expressed as follows:

$$
\begin{aligned}
\log \mu(\mathbf{R}, \mathbf{s}) = & -\beta'(I+\Omega)\log \mathbf{R} \\
& -\log\left(1 + ((\mathbf{s}\oslash\mathbf{R})' - \mathbf{s}'(\Omega\oslash(\mathbf{R}\mathbf{1}'))')\left(I - \Omega'\circ[(\mathbf{R}\mathbf{1}')\circ(\mathbf{1}\mathbf{R}')]\right)^{-1}\boldsymbol{\nu}\right),
\end{aligned}
$$

where $\mathbf{1}$ is a vector of ones and the fact that Λ can be written as $(\mathbf{R}\mathbf{1}')\circ(\mathbf{1}\mathbf{R}')$.

Note that, in our definition, $R_i = R + e_i - s_i$. Thus, it's hard to make comparative statics using the expression above. Nevertheless, we can see from above that if the term $((\mathbf{s}\oslash\mathbf{R})' - \mathbf{s}'(\Omega\oslash(\mathbf{R}\mathbf{1}'))')\left(I - \Omega'\circ[(\mathbf{R}\mathbf{1}')\circ(\mathbf{1}\mathbf{R}')]\right)^{-1}\boldsymbol{\nu}$ is positive, government subsidies have deleterious effects on aggregate productivity.

To think about how government subsidies affect the economy's aggregate productivity, start by assuming initially that there are no subsidies to borrowing costs, so that $s_i = 0$ for all $i \in \{1, \dots, N\}$ and thus $\tau = 0$. Denote the vector of the log of the sectors' borrowing costs as $\log \mathbf{R}^0$. A necessary and sufficient condition for a given vector of sectoral subsidies $\mathbf{s}$ to improve aggregate TFP is that $\log \mu(\mathbf{R}, \mathbf{s}) - \log \mu(\mathbf{R}^0, \mathbf{0}) > 0$, or

$$
\begin{aligned}
& -\beta'(I+\Omega)(\log \mathbf{R} - \log \mathbf{R}^0) \\
& -\log\left(1 + ((\mathbf{s}\oslash\mathbf{R})' - \mathbf{s}'(\Omega\oslash(\mathbf{R}\mathbf{1}'))')\left(I - \Omega'\circ[(\mathbf{R}\mathbf{1}')\circ(\mathbf{1}\mathbf{R}')]\right)^{-1}\boldsymbol{\nu}\right) > 0.
\end{aligned}
$$

4.2 The Absence of the Labor Wedge

Now, we close the model using the household's optimality condition to derive the labor wedge. The optimality condition for the household is given by

$$
\frac{\ell^{\varphi}}{c^{-\sigma}} = (1-\tau)w.
$$

The labor wedge, $1-\tau_\ell$, is implicitly defined by the relation

$$\frac{\ell^{\varphi}}{c^{-\sigma}} = (1-\tau_\ell(\mathbf{s},\mathbf{R}))\frac{c}{\ell}.$$

Combining both, we get

$$(1-\tau_\ell(\mathbf{s},\mathbf{R}))A(\mathbf{z})\mu(\mathbf{R},\mathbf{s}) = (1-\tau)w.$$

Plugging in the functional forms for the tax rate, wages, sectoral productivity and the efficiency wedge, we get the following result.

Proposition 2. *In equilibrium, the economy's aggregate labor wedge is always equal one. That is,* $\tau_\ell(\mathbf{s},\mathbf{R}) = 0$.

The reason why our model does not contain the labor wedge rests solely on the fact that we are not allowing the interest-rate spread e_i to return to the household as a lump-sum transfer. Here, the revenue generated from those spreads are simply thrown into the ocean. In this setting, because the sectoral production function is constant returns to scale and all the distortions are incorporated in the after-tax wage rate, the household has no other source of income to finance consumption. In Appendix B, we derive the labor wedge for the case in which the revenue generated from the interest-rate spreads are rebated back to the consumer. In such setting, both the efficiency wedge and the tax rate change compared to our baseline model.

4.3 Equilibrium path

Starting from the labor wedge definition, we get

$$\ell^{1+\varphi} = (1-\tau_\ell(\mathbf{s},\mathbf{R}))c^{1-\sigma}.$$

Given that $c(\mathbf{z}, \mathbf{R}, \mathbf{s}) = A(\mathbf{z})\mu(\mathbf{R}, \mathbf{s})\ell$, we take logs and arrive at the following lemma:

Lemma 1. *Given sectoral productivity, efficiency and labor wedges, equilibrium aggregate consumption and labor supply are given by*

$$\log \ell(\mathbf{R}, \mathbf{s}) = \frac{1-\sigma}{\sigma+\varphi}(\log A(\mathbf{z}) + \log \mu(\mathbf{R}, \mathbf{s})) + \frac{1}{\sigma+\varphi}\log(1-\tau_\ell(\mathbf{s}, \mathbf{R})) \tag{5}$$

$$\log c(\mathbf{R}, \mathbf{s}) = \frac{1+\varphi}{\sigma+\varphi}(\log A(\mathbf{z}) + \log \mu(\mathbf{R}, \mathbf{s})) + \frac{1}{\sigma+\varphi}\log(1-\tau_\ell(\mathbf{s}, \mathbf{R})). \tag{6}$$

4.4 Intuition Behind the Model

In the model, the government needs to raise funds in order to subsidize firms via distortionary labor taxes. Whenever a firm's borrowing costs are subsidized, it increases its input demand and, consequently, its output. By supplying more goods, this sector's price is reduced. Therefore, the subsidy works as if it was alleviating the borrowing constraint of firms in all the downstream sectors from the original firm: all its customers purchases their intermediate goods at lower prices. Consequently, all downstream sectors produce more and sell their output at lower prices, allowing their own downstream sectors to purchase more intermediate goods, produce more, and sell their output at lower prices. This effect goes on forever, which implies that total output in this economy increases due to lower interest rates.

Therefore, this multi-sector economy with borrowing constraints features externalities that are only internalized with government subsidizes. Subsidy will only be beneficial to the economy when the spillover of the targeted sector towards its downstream sectors compensates the labor market distortion.

5 Calibration

We use annual data on National Accounts provided by the Brazilian Institute of Geography and Statistics (IBGE), in particular their Use tables, disaggregated at 12 industries for the years 2000-2015. The industries are: Agriculture, Extraction Industries, Manufacturing, Utilities, Construction, Retail, Transportation and Storage, ICT, Financial Services and Insurance, Real Estate, Other Services and Public Administration. Because we assume sectors borrow from the international financial markets, we exclude Financial Services and Insurance from our quantitative exercise. For each year, this table identifies sector i's uses (expenditures) of goods produced by sector j, the empirical analogue of $p_{ij}x_{ij}$. Final use of sector i net of gross fixed capital formation and inventories is the empirical analogue of $p_i c_i$. For the empirical analogue of $w\ell_i$, we add to sector i's labor compensation half of the sector's gross mixed income. This is motivated by the fact that this item in the national accounts includes components of both labor and capital income. We take the agnostic stance of assigning assigning half of such income to labor.

5.1 Input shares

Due to the Cobb-Douglas production function of intermediate goods and the Cobb-Douglas final goods aggregator, this data is sufficient for computing the labor shares, α_i, the intermediate inputs shares, ω_{ij}, and the consumption shares, ν_i. Therefore, we compute these parameters for each year in our sample using the following model's first order conditions:

$$\alpha_i = \frac{w\ell_i}{w\ell_i + \sum_{k=1}^{N}(p_{ik}x_{ik})}$$

$$\omega_{ij} = \frac{p_{ij}x_{ij}}{\sum_{k=1}^{N}(p_{ik}x_{ik})}$$

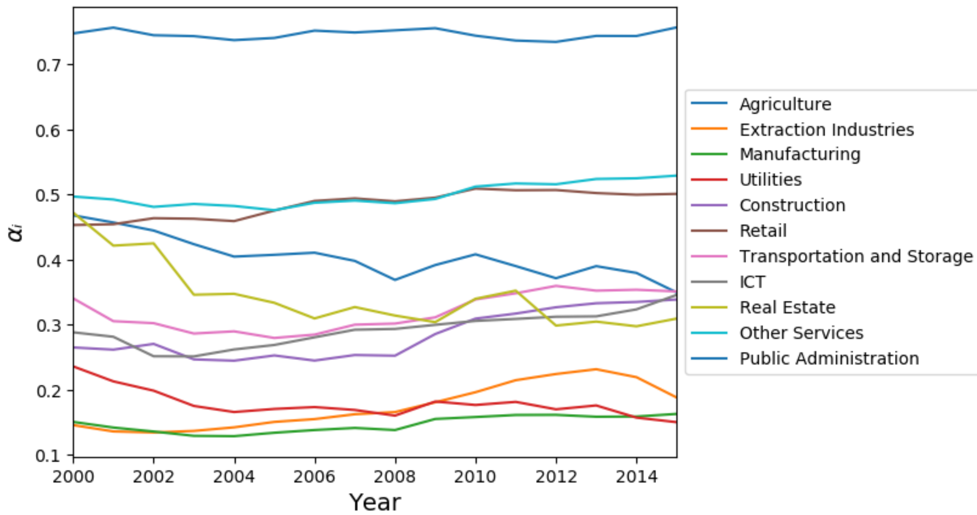


Figure 7: Labor share (α_i) by sector

$$\nu_i = \frac{p_i c_i}{\sum_{k=1}^{N}(p_k c_k)}.$$

We also retrieve a sector-specific price index from the Make tables, since they are available in current prices as well as previous-year prices. We observe, therefore, the empirical analogue of (p_{ij}), which allows us to recover each sectors' intermediate input demand (x_{ij}) and output (y_i). IBGE also reports the number of employers in each sector, which is the empirical analogue of ℓ_i. This suffices to compute sector-specific productivities:

$$z_i = \frac{y_i}{\ell_i^{\alpha_i}\left(\prod_{j=1}^{N} x_{ij}^{\omega_{ij}}\right)^{1-\alpha_i}}.$$

We chose to let parameters vary by year instead of taking the yearly average due to the nature of the Cobb-Douglas function. The value of intermediate demand from some sectors is zero in some years. If the parameters ω_{ij} are constant, in the years where x_{ij} takes the value of zero, the entire expression $\ell_i^{\alpha_i}\left(\prod_{j=1}^{N} x_{ij}^{\omega_{ij}}\right)^{1-\alpha_i}$ would take the value of zero as well. Nevertheless, they are fairly stable. Figure 7 depicts the sectoral labor share across the years.

5.2 Sectoral subsidies

The Central Bank of Brazil manages a loan-level database called Credit Information System (SIC) that consists of all loans above R$5,000 (during the 2008-2015 period, its dollar-equivalent nominal value was always in the $1,250–3,000 range). It provides information on loan amount, maturity, collateral value, interest rate, industry, and category (project financing, vehicle financing, bill discounting, etc). Our data set is monthly and spans from the years 2004 to 2018. Markedly, it also discriminates earmarked from non-earmarked loans. Therefore, our sample offers a good representation of the entire Brazilian economy, suitable for analyzing earmarked credit policy.

We compute the sum of world interest rate plus idiosyncratic risk, $R + e_i$, as the sector i's average real interest rate of non-earmarked loans from this database. Analogously, we use the sector i's average real interest rate of earmarked loans as the sum of world interest rate plus idiosyncratic risk minus subsidy rate, $R + e_i - \varsigma_i$. Sector i's subsidy rate, ς_i, is then calculated as the difference between these two objects. We also extract, for each sector i, the share of total loans that are earmarked, ϕ_i, directly from the data. We use these two objects to compute the subsidy by each unit of credit to sector i as $s_i = \phi_i \varsigma_i$.

In our quantitative exercise, we choose to focus on the transition between the years 2008 and 2013. We chose 2008 since it's the last year before the government started using earmarked lending as a countercyclical measure to deal with the 2008 financial crisis. In 2014, the presidential election took place, and the economy started to experience a slowdown. The latter may have been affected by other factors outside our model, which is why we decided to use 2013 instead of 2014. Table 1 presents summary statistics for the interest rate spread and share of earmarked credit in the eleven sectors. The table shows that, on average, the reduction in interest rate per sector from the earmarked lending decreases from 2008 to 2013, while the share of earmarked lending in each sector significantly increased. A reason why the interest rate spread is smaller in 2013 than in 2008 on average is potentially explained by the decrease in the average lending rate in Brazil due

Table 1: Productivity and Credit Data by Sector

Sector	Interest rate spread (p.p.)		Share of earmarked credit (%)	
	2008	2013	2008	2013
Agriculture	5.27	4.68	41.64	48.27
Extraction Industries	3.26	5.22	57.25	47.42
Manufacturing	4.48	3.76	15.22	20.56
Utilities	-1.03	2.02	46.09	72.95
Construction	11.03	10.59	30.36	34.08
Retail	4.78	4.95	9.24	15.87
Transportation & Storage	11.7	13.09	45.56	67.39
ICT	13.85	-6.14	3.96	21.14
Real Estate	6.22	9.05	11.57	16.58
Other Services	13.77	4.25	27.57	28.39
Public Administration	16.08	13.03	4.67	25.77

Note: The interest rate spread, ς_i, represents the difference, in percentage points, between the interest rates charged for nonearmarked and earmarked loans in sector i. The share of earmarked credit, ϕ_i, represents the share or earmarked loans in sector i in the given year.

to monetary easing in Brazil and in the world: the rate was on average 47.25% in 2008, and 27.39% in 2013.

6 Quantitative Exercises

This section presents the main quantitative findings of the paper. We first investigate the role of the dynamics of sectoral productivity to aggregate productivity. Next, we use the model to assess the optimal vector of sector subsidies given exogenous sector-specific interest rates observed in the data, as well as considering the counterfactual policy of not intervening in the credit markets through subsidy, consequently not taxing labor income.

6.1 Decomposing aggregate productivity

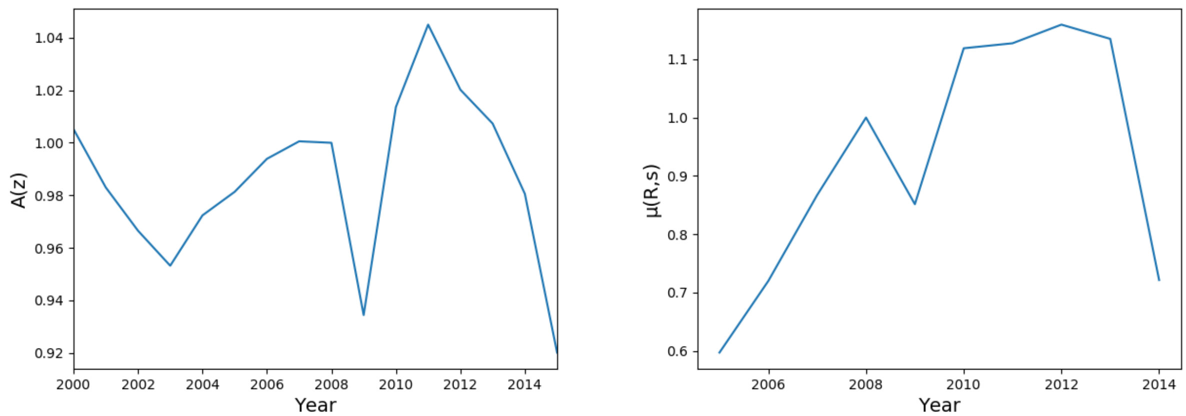
Using equation (1), we can infer the contribution of the sectoral productivity component of TFP to aggregate productivity. The left panel of Figure 8 plots the function $A(\mathbf{z}_t)$ for years 2000 to 2015. The values were normalized so that $A(\mathbf{z}_{2008}) = 1$. The figure shows a productivity slowdown between the years 2000 and 2003, followed by a recovery from 2004 to 2008. After the Great Recession in the United States in 2008, there was a sharp drop of more than six percent in the sectoral productivity component of TFP in 2009, followed by a quick recovery in 2010 and 2011. By the latter year, the sectoral productivity component of TFP was about four percent higher than in 2008. From 2012 onwards, this component of aggregate productivity starts to fall considerably. In 2015, it was about eight percent lower than in 2008.

We can use equation (1) to decompose the dynamics of the efficiency wedge as well. The right panel of Figure 8 plots the function $\mu(\mathbf{R}_t, \mathbf{s}_t)$ for years 2004 to 2015. we observe that it contributed positively to aggregate TFP growth between 2004 and 2008. In 2009, it contributed negatively, just as the sectoral productivity component. From 2010 to 2012, it again contributed to productivity growth, contributing negatively afterwards.

If we apply a comparative statics exercise, analyzing the contribution of each component to GDP per worker growth from 2008 to 2013, the year before the presidential elections, the model indicates that aggregate TFP grew 0.73% due to sectoral productivity growth, and 13.48% due to the efficiency wedge. This implies that factors affecting the behavior of each sector's productivity were much more important to explain why Brazil did not grow as fast after recovering from the spillovers of the Great Recession in the United States.

From 2013 onwards, both the sectoral productivity component and the efficiency wedge were responsible for a sharp drop in aggregate productivity. In particular, the efficiency wedge dropped as much as forty percent from 2013 to 2014. This can be partly explained by the increase in world interest rates.[3] In the next section, we will investigate whether the government policy was responsible for such a sharp drop in this component of TFP.

[3] For instance, five-year U.S. Treasury bonds increased half a percentage point.



(a) Sectoral productivity

(b) Efficiency wedge

Figure 8: Contributions to aggregate productivity (2008=1)

6.2 Optimal interest rate subsidy policy

In this section, we use the optimal consumption and labor derived from the household optimality conditions, (5) and (6), to choose the vector of government subsidies that maximizes household utility. For this exercise, we choose $\sigma = 2$ and $\varphi = 0.5$, common values in the business cycle literature.

Figure 9 compares the observed subsidy policy to the optimal one for the years 2008, 2010 and 2013 and is the most important finding in this paper. Each dot represents the observed and optimal subsidy for a single sector in one given year We find that, through the lens of our model, the optimal subsidy policy should generally be much higher than the one observed in the data. In general, lower subsidies were required for 2013 when compared to 2008, even though they should have been substantially higher than the actual policy in both years. This is in line with the discussion in Section 5.2. The decrease in the average lending rate in Brazil due to monetary easing in Brazil and in the world required a less aggressive subsidy policy from the government.

Next, we analyze overall welfare gains of implementing the optimal policy from 2008 to 2013. Figure 10 plots the results. We find that applying the optimal policy improves welfare substantially, especially between 2008 and 2009. In particular, would have been

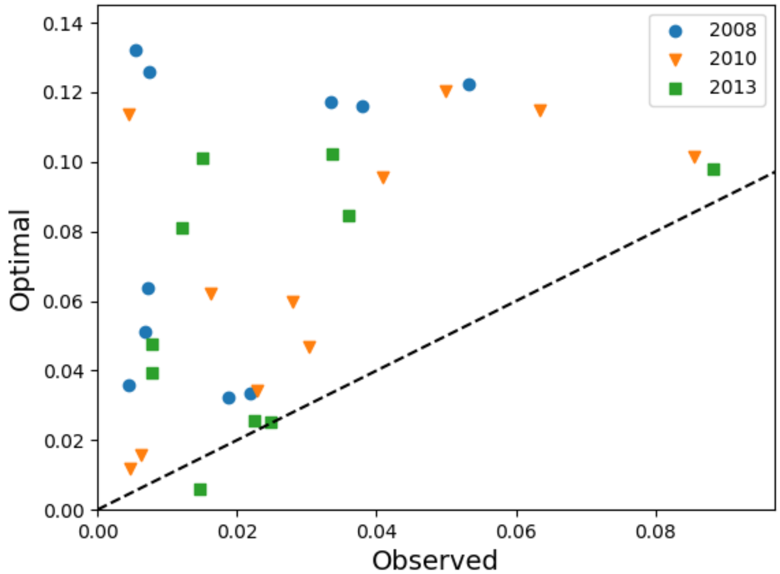


Figure 9: Optimal vs. observed policies

about twenty five percent higher in 2009 if the optimal policy was implemented compared to the actual policy.

Figures 11 and 12 plot the behavior of the efficiency wedge and GDP per worker. Both measures would be at least twenty percent higher than the actual observed value in 2008 in every year of the sample. The optimal policy would have offset the negative contribution of the sectoral productivity and the efficiency wedge components of TFP more substantially.

We also perform two counterfactuals: fixing the government policy to the one observed in 2008, as well as removing all sectoral subsidies. Figure 10 presents these results. In terms of welfare gains, both policies would have been welfare-reducing, especially the one where the government removes all sectoral subsidies. Focusing on freezing the policy to 2008 levels, Figures 11 and 12 indicate that the behavior of the efficiency wedge and output per worker would have been roughly similar to the data from 2004 to 2015. The changes in credit policy from 2008 onwards were not as relevant as the behavior of sectoral productivities to explain the behavior of aggregate TFP. Nevertheless, the policy

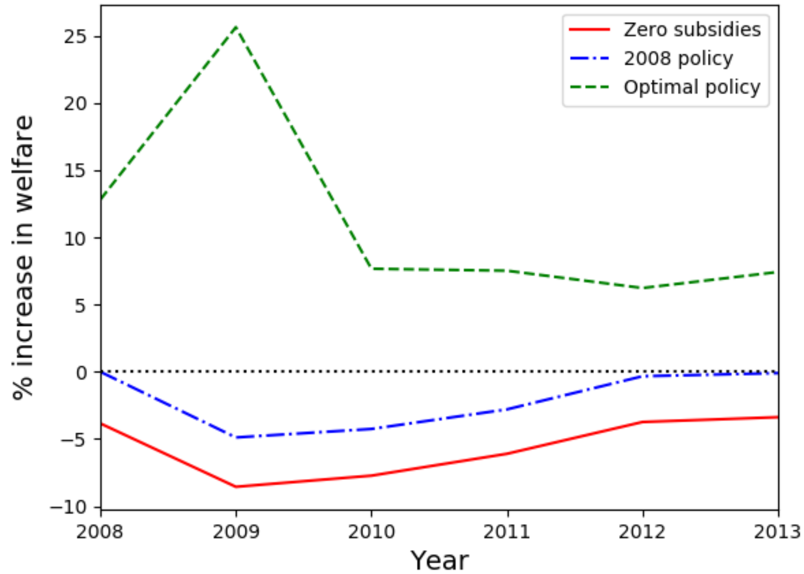


Figure 10: Welfare relative to actual policy

implemented by the government contributed to offset the negative effects of interest rate shocks on the efficiency wedge.

7 Conclusion and Directions for Further Research

Under the lens of our model, the change in earmarked credit policy that occurred from 2008 to 2014 played a positive, albeit modest, role in the economy in terms of GDP per worker. The observed policy was effective in raising the economy's efficiency and hence output per worker when compared to a policy that kept sectoral interest-rate subsidies in the levels observed in 2008, and especially in a counterfactual economy with no government subsidies whatsoever. More importantly, the model suggests that the government should have provided sectors with even more subsidies than the ones observed in the data. Implementing such a policy would have increased output per worker and welfare significantly.

A potential model is one in which the government can finance its credit policy not only

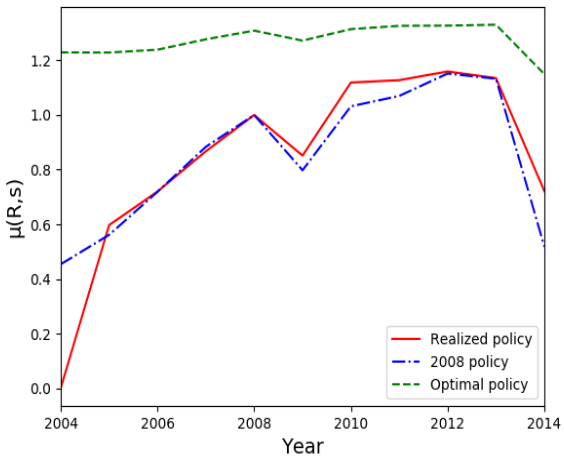


Figure 11: Efficiency wedge

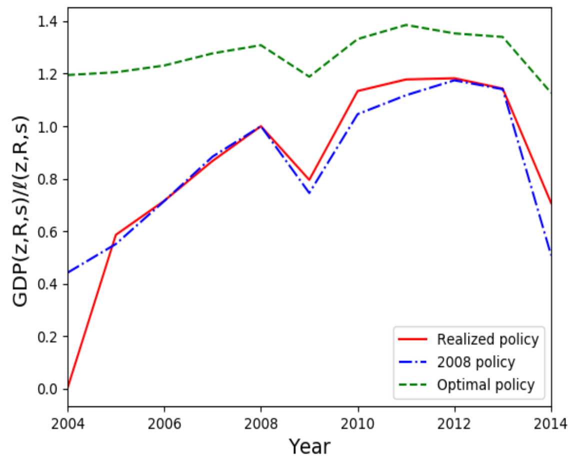


Figure 12: GDP per worker

by taxing labor, but also by borrowing. In this setting, expanding earmarked loans may increase the government's default risk. If the idiosyncratic borrowing cost faced by each sector is correlated with the government's, sharp increases in earmarked credit policy force the government to increase both labor taxes and debt, which imposes not only labor distortions but also higher borrowing costs for non-earmarked loans.

Even though our current model is not appropriate for addressing such channel, it does illustrate one of our next steps. To our knowledge, sovereign default risk was never used in a general equilibrium model for modeling the costs of industrial policies. Or, similarly, earmarked loans in an economy with sectoral linkages were never used to explicitly model rolling over debt costs.

Another challenge to our theoretical framework and therefore our conclusions in this paper is the absence of capital as a factor of production. Because we do not have sectoral data on fixed assets from the national accounts, it's not feasible to introduce capital accumulation to the current setting. Since a substantial share of the earmarked loans were directed towards long-term investments, omitting the investment channel in our analysis may have significantly affected our conclusions. One potential solution is to use the Annual Survey of Industry (PIA). Conducted by the IBGE, it is a yearly census of the

Brazilian manufacturing sector. This data set covers all firms in the manufacturing and extractive industries, including firm-level data on labor compensation, capital expenditures, fixed assets and purchases of intermediate goods. Even though it would restrict the analysis to a subset of sectors of the Brazilian economy, the potential gains of providing more detailed analysis of the role of earmarked credit in the firms' investment decisions could therefore shed some light in the role of the government policy in capital (mis)allocation across these sectors.

Additionally, our current analysis is silent about the effect of this policy on industry concentration. In Section 6.1, we provided evidence that the decline in sectoral productivity was a relevant driver of the slowdown in aggregate TFP. It is possible that assuming a representative firm in each sector may be hiding interesting intra-sector firm dynamics. As shown by Bonomo et al. (2015), inside each industrial sector, earmarked loans were directed towards older and bigger firms. The government policy may be leading to deleterious market concentration by allocating cheaper capital to bigger, unproductive firms, while forcing younger and productive firms to exit the market, thus lowering average productivity by sector. Again, using the PIA could answer this question for the manufacturing and extractive industries.

References

Acemoglu, D., Carvalho, V. M., Ozdaglar, A., and Tahbaz-Salehi, A. (2012). The Network Origins of Aggregate Fluctuations. *Econometrica*, 80(5):1977–2016.

Altinoglu, E. L. (2018). The Origins of Aggregate Fluctuations in a Credit Network Economy. Finance and Economics Discussion Series 2018-031, Board of Governors of the Federal Reserve System (U.S.).

Atkinson, A. B. and Stiglitz, J. E. (2015). *Lectures on Public Economics Updated edition*. Number 10493 in Economics Books. Princeton University Press.

Baqaee, D. R. and Farhi, E. (2017). Productivity and Misallocation in General Equilibrium. NBER Working Papers 24007, National Bureau of Economic Research, Inc.

Bigio, S. and La'O, J. (2016). Financial Frictions in Production Networks. NBER Working Papers 22212, National Bureau of Economic Research, Inc.

Bonomo, M., Brito, R. D., and Martins, B. (2015). The after crisis government-driven credit expansion in brazil: A firm level analysis. *Journal of International Money and Finance*, 55:111 – 134. Macroeconomic and financial challenges facing Latin America and the Caribbean after the crisis.

Carvalho, D. (2014). The Real Effects of Government-Owned Banks: Evidence from an Emerging Market. *Journal of Finance*, 69(2):577–609.

Cavalcanti, T. and Vaz, P. H. (2017). Access to Long-Term Credit and Productivity of Small and Medium Firms: A Causal Evidence. Reap working papers, Rede de Economia Aplicada.

Chari, V. V., Kehoe, P. J., and McGrattan, E. R. (2007). Business Cycle Accounting. *Econometrica*, 75(3):781–836.

Gilchrist, S., Sim, J. W., and Zakrajsek, E. (2013). Misallocation and Financial Market Frictions: Some Direct Evidence from the Dispersion in Borrowing Costs. *Review of Economic Dynamics*, 16(1):159–176.

Gopinath, G., Şebnem Kalemli-Özcan, Karabarbounis, L., and Villegas-Sanchez, C. (2017). Capital Allocation and Productivity in South Europe. *The Quarterly Journal of Economics*, 132(4):1915–1967.

Hsieh, C.-T. and Klenow, P. J. (2009). Misallocation and Manufacturing TFP in China and India. *The Quarterly Journal of Economics*, 124(4):1403–1448.

Liu, E. (2017). Industrial policies in production networks.

Luo, S. (2018). Propagation of financial shocks in an input-output economy with trade and financial linkages of firms.

Midrigan, V. and Xu, D. Y. (2014). Finance and misallocation: Evidence from plant-level data. *American Economic Review*, 104(2):422–58.

Pazarbasioglu-Dutz, Ceyla; Byskov, S. B. M. . C. I. . M. B. A. H. P. A. (2017). Brazil - Financial intermediation costs and credit allocation : discussion paper for workshop (English). Working paper, World Bank.

Vasconcelos, R. d. S. (2017). Misallocation in the Brazilian Manufacturing Sector. *Brazilian Review of Econometrics*, 37(2).

Appendix

A Proofs

Proof of Proposition 1

We begin by first proving the following Lemma:

Lemma 2. *Given sectoral prices and wages, the firm i's optimality condition for output, intermediate goods and labor satisfies*

$$(1-\alpha_i)\omega_{ij}p_i y_i = \frac{R_i}{R_j} p_j x_{ij} \quad \textit{for all } j\textit{, and} \tag{7}$$

$$\alpha_i p_i y_i = R_i w \ell_i. \tag{8}$$

In addition, given sectoral prices and wages, the firm i's cost function for a given output level is given by

$$C(y_i; w, \{p_{ij}\}) = \Theta_i^{-1} \frac{y_i}{z_i} w^{\alpha_i} R_i^{\alpha_i} \prod_{j=1}^{N} \left(p_j \frac{R_i}{R_j} \right)^{\omega_{ij}(1-\alpha_i)}, \tag{9}$$

where $\Theta_i \equiv \alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i} \prod_{j=1}^{N} \omega_{ij}^{\omega_{ij}(1-\alpha_i)}$.

Proof. Because the production constraint binds, we can rewrite the maximization problem as

$$\max_{\ell_i, \{x_{ij}\}_{j=1}^N} \quad p_i z_i \ell_i^{\alpha_i} \left(\prod_{j=1}^{N} x_{ij}^{\omega_{ij}} \right)^{1-\alpha_i} - R_i \left\{ \sum_j p_{ij} x_{ij} + w\ell_i \right\}$$

Optimality conditions are

$$(1-\alpha_i)\omega_{ij} p_i y_i = R_i p_{ij} x_{ij}$$

$$\alpha_i p_i y_i = R_i w \ell_i.$$

In order for it not to be profitable for the firm to focus on selling their goods either to firms or to households, the marginal revenues have to be equal: $p_i = R_i p_{ji}$. Thus, we can rewrite the optimality condition for intermediate goods as

$$(1-\alpha_i)\omega_{ij} p_i y_i = \frac{R_i}{R_j} p_j x_{ij}.$$

The cost minimization problem for the firm is

$$\min_{\ell_i, \{x_{ij}\}} R_i \left[\sum_j p_{ij} x_{ij} + w\ell_i \right] \quad \text{s.t.} \quad y_i = z_i \ell_i^{\alpha_i} \left(\prod_{j=1}^N x_{ij}^{\omega_{ij}} \right)^{1-\alpha_i}.$$

First-order conditions imply the following ratios:

$$\frac{p_{ij}}{w} = \frac{(1-\alpha_i)\omega_{ij}}{\alpha_i} \frac{\ell_i}{x_{ij}} \quad \text{and} \quad \frac{p_{ij}}{p_{ik}} = \frac{\omega_{ij}}{\omega_{ik}} \frac{x_{ik}}{x_{ij}},\ j \neq k.$$

Therefore, we can write

$$\prod_j x_{ij}^{\omega_{ij}} = w\ell_i \left(\frac{1-\alpha_i}{\alpha_i} \right) \prod_j \left(\frac{\omega_{ij}}{p_{ij}} \right)^{\omega_{ij}},$$
$$\prod_{k\neq j} x_{ik}^{\omega_{ik}} = \left(\frac{p_{ij}}{\omega_{ij}} x_{ij} \right)^{1-\omega_{ij}} \prod_{k \neq j} \left(\frac{\omega_{ik}}{p_{ik}} \right)^{\omega_{ik}}$$

Plugging these results back in the production function, we get

$$y_i = z_i \ell_i \left[w \left(\frac{1-\alpha_i}{\alpha_i} \right) \right]^{1-\alpha_i} \prod_j \left(\frac{\omega_{ij}}{p_{ij}} \right)^{\omega_{ij}(1-\alpha_i)} \tag{ℓ_i}$$

$$y_i = z_i \left[\left(\frac{p_{ij}}{w} \right) \frac{\alpha_i}{\omega_{ij}(1-\alpha_i)} x_{ij} \right]^{\alpha_i} \left[x_{ij} \left(\frac{p_{ij}}{\omega_{ij}} \right)^{1-\omega_{ij}} \prod_{k \neq j} \left(\frac{\omega_{ik}}{p_{ik}} \right)^{\omega_{ik}} \right]^{1-\alpha_i} \tag{x_{ij}}$$
$$= z_i x_{ij} \left(\frac{p_{ij}}{\omega_{ij}} \right) \left[\frac{\alpha_i}{w(1-\alpha_i)} \right]^{\alpha_i} \prod_{k=1}^N \left(\frac{\omega_{ik}}{p_{ik}} \right)^{\omega_{ik}(1-\alpha_i)}$$

We then get the following conditional input demands:

$$
\begin{aligned}
\ell_i &= \frac{y_i}{z_i}\left[w\left(\frac{1-\alpha_i}{\alpha_i}\right)\right]^{\alpha_i-1}\prod_j\left(\frac{\omega_{ij}}{p_{ij}}\right)^{-\omega_{ij}(1-\alpha_i)}, \\
x_{ij} &= \frac{y_i}{z_i}\left(\frac{\omega_{ij}}{p_{ij}}\right)\left[\frac{\alpha_i}{w(1-\alpha_i)}\right]^{-\alpha_i}\prod_{k=1}^{N}\left(\frac{\omega_{ik}}{p_{ik}}\right)^{-\omega_{ik}(1-\alpha_i)}.
\end{aligned}
$$

Therefore, the cost function is given by

$$
\begin{aligned}
R_i\left[\sum_j p_{ij}x_{ij}+w\ell_i\right] &= R_i\frac{y_i}{z_i}\prod_{j=1}^{N}\left(\frac{\omega_{ij}}{p_{ij}}\right)^{-\omega_{ij}(1-\alpha_i)}\left[\sum_j\omega_{ij}\left[\frac{\alpha_i}{w(1-\alpha_i)}\right]^{-\alpha_i}+w\left[w\left(\frac{1-\alpha_i}{\alpha_i}\right)\right]^{\alpha_i-1}\right] \\
&= R_i\frac{y_i}{z_i}\prod_{j=1}^{N}\left(\frac{\omega_{ij}}{p_{ij}}\right)^{-\omega_{ij}(1-\alpha_i)}\left\{\left[\frac{\alpha_i}{w(1-\alpha_i)}\right]^{-\alpha_i}+w\left[w\left(\frac{1-\alpha_i}{\alpha_i}\right)\right]^{\alpha_i-1}\right\} \\
&= R_i\frac{y_i}{z_i}w^{\alpha_i}\prod_{j=1}^{N}\left(\frac{\omega_{ij}}{p_{ij}}\right)^{-\omega_{ij}(1-\alpha_i)}\left[\left(\frac{1-\alpha_i}{\alpha_i}\right)^{\alpha_i}+\left(\frac{\alpha_i}{1-\alpha_i}\right)^{1-\alpha_i}\right] \\
&= R_i\frac{y_i}{z_i}w^{\alpha_i}\prod_{j=1}^{N}\left(\frac{\omega_{ij}}{p_j}R_j\right)^{-\omega_{ij}(1-\alpha_i)}\left[\alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i}\right]^{-1} \\
&= \frac{y_i}{z_i}w^{\alpha_i}R_i^{\alpha_i+\sum_j\omega_{ij}(1-\alpha_i)}\prod_{j=1}^{N}\omega_{ij}^{-\omega_{ij}(1-\alpha_i)}\prod_{j=1}^{N}\left(\frac{p_j}{R_j}\right)^{\omega_{ij}(1-\alpha_i)}\left[\alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i}\right]^{-1} \\
&= \Theta_i^{-1}\frac{y_i}{z_i}w^{\alpha_i}R_i^{\alpha_i}\prod_{j=1}^{N}\left(p_j\frac{R_i}{R_j}\right)^{\omega_{ij}(1-\alpha_i)},
\end{aligned}
$$

where $\Theta_i \equiv \alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i}\prod_{j=1}^{N}\omega_{ij}^{\omega_{ij}(1-\alpha_i)}$. □

Normalize $P \equiv \Pi_i p_i^{\nu_i} = 1$. Let $g_i \equiv p_i y_i$. Then, using (7),

$$
\begin{aligned}
p_i y_i &= p_i c_i + \sum_j p_i x_{ji} \\
&= p_i c_i + \sum_j (1-\alpha_j)\omega_{ji}p_j y_j\frac{R_i}{R_j}.
\end{aligned}
$$

Stacking the equations, we have

$$\begin{bmatrix} g_1 \\ \vdots \\ g_N \end{bmatrix} = \begin{bmatrix} \nu_1 \\ \vdots \\ \nu_N \end{bmatrix} c + \left(\begin{bmatrix} (1-\alpha_1)\omega_{11} & \dots & (1-\alpha_1)\omega_{N1} \\ \vdots & \ddots & \vdots \\ (1-\alpha_N)\omega_{1N} & \dots & (1-\alpha_N)\omega_{NN} \end{bmatrix} \circ \begin{bmatrix} \frac{R_1}{R_1} & \dots & \frac{R_1}{R_N} \\ \vdots & \ddots & \vdots \\ \frac{R_N}{R_1} & \dots & \frac{R_N}{R_N} \end{bmatrix} \right) \begin{bmatrix} g_1 \\ \vdots \\ g_N \end{bmatrix}$$

where $\circ$ denote the Hadamard (entrywise) product. Normalizing $P = 1$ implies $p_i c_i = \nu_i c$. Define

$$\Omega \equiv \begin{bmatrix} (1-\alpha_1)\omega_{11} & (1-\alpha_1)\omega_{12} & \dots & (1-\alpha_1)\omega_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ (1-\alpha_N)\omega_{N1} & (1-\alpha_N)\omega_{N2} & \dots & (1-\alpha_N)\omega_{NN} \end{bmatrix}, \quad \Lambda \equiv \begin{bmatrix} \frac{R_1}{R_1} & \frac{R_1}{R_2} & \dots & \frac{R_1}{R_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{R_N}{R_1} & \frac{R_N}{R_2} & \dots & \frac{R_N}{R_N} \end{bmatrix}.$$

Let $\mathbf{g} \equiv [g_1 \, \dots \, g_N]'$ and $\boldsymbol{\nu} \equiv [\nu_1 \, \dots \, \nu_N]'$. Then, from the algebra above:

$$\mathbf{g} = \boldsymbol{\nu} c + (\Omega' \circ \Lambda)\mathbf{g}$$
$$\Rightarrow \mathbf{g} = \left(I - \Omega' \circ \Lambda\right)^{-1} \boldsymbol{\nu} c.$$

Using (9), perfect competition implies that sectoral price equals marginal cost:

$$p_i = \Theta_i^{-1} \frac{1}{z_i} w^{\alpha_i} R_i^{\alpha_i} \prod_{j=1}^{N} \left(p_j \frac{R_i}{R_j} \right)^{\omega_{ij}(1-\alpha_i)}$$

and thus

$$w^{\alpha_i} = \Theta_i p_i \prod_{j=1}^{N} \left(p_j\right)^{-\omega_{ij}(1-\alpha_i)} z_i R_i^{-\alpha_i} \prod_{j=1}^{N} \left(\frac{R_i}{R_j} \right)^{-\omega_{ij}(1-\alpha_i)} .$$

Define $\beta \equiv (I - \Omega')^{-1}\boldsymbol{\nu}$, which is a centrality measure. It can be shown that $\sum_i \alpha_i \beta_i = 1$. From the definition of β, $-\Omega'\beta = \boldsymbol{\nu} - \beta$. Therefore,

$$\prod_i p_i^{\beta_i} \prod_j \left(p_j^{-\omega_{ij}(1-\alpha_i)\beta_i} \right) = \prod_i p_i^{\beta_i - \sum_j \omega_{ji}(1-\alpha_j)\beta_j} = \prod_i p_i^{\beta_i + \nu_i - \beta_i} = \prod_i p_i^{\nu_i} = P = 1.$$

We can write the wage in the economy as a function of labor shares, $\{\alpha_i\}$, intermediate good shares, $\{\omega_{ij}\}$, and the interest rates faced by the firms, $\{R_i\}$:

$$
\begin{aligned}
\prod_i w^{\alpha_i\beta_i} &= \prod_i \Theta_i^{\beta_i} p_i^{\beta_i} \prod_{j=1}^{N} (p_j)^{-\omega_{ij}(1-\alpha_i)\beta_i} z_i^{\beta_i} R_i^{-\alpha_i\beta_i} \prod_{j=1}^{N} \left(\frac{R_i}{R_j}\right)^{-\omega_{ij}(1-\alpha_i)\beta_i} \\
\Rightarrow w &= \prod_i \Theta_i^{\beta_i} z_i^{\beta_i} R_i^{-\alpha_i\beta_i} \prod_{j=1}^{N} \left(\frac{R_i}{R_j}\right)^{-\omega_{ij}(1-\alpha_i)\beta_i}.
\end{aligned}
$$

Let $\mathbf{s} \equiv [s_1 \ldots s_N]'$ and $\mathbf{R} \equiv [R_1 \ldots R_N]'$. Denote $\oslash$ as the Hadamard division and $\mathbf{1}$ as a vector of ones. Using (7) and (8) on the government's budget constraint:

$$
\begin{aligned}
\tau w\ell &= \sum_i^N s_i \left[\sum_j (p_{ij}x_{ij} - p_{ji}x_{ji}) + w\ell_i\right] \\
&= \sum_i^N s_i \left(\sum_j p_{ij}x_{ij} + w\ell_i\right) - \sum_i^N s_i \left(\sum_j p_{ji}x_{ji}\right) \\
&= \sum_i^N s_i \left(\sum_j \frac{(1-\alpha_i)\omega_{ij}p_iy_i}{R_i} + \frac{\alpha_i p_i y_i}{R_i}\right) - \sum_i^N s_i \left(\sum_j \frac{(1-\alpha_j)\omega_{ji}p_jy_j}{R_j}\right) \\
&= \sum_i^N s_i \left((1-\alpha_i)\sum_j \omega_{ij} + \alpha_i\right)\frac{g_i}{R_i} - \sum_i^N s_i \left(\sum_j \frac{(1-\alpha_j)\omega_{ji}g_j}{R_j}\right) \\
&= \sum_i^N s_i \frac{g_i}{R_i} - \sum_i^N s_i \left(\sum_j \frac{(1-\alpha_j)\omega_{ji}g_j}{R_j}\right) \\
&= ((\mathbf{s} \oslash \mathbf{R})' - \mathbf{s}'(\Omega \oslash (\mathbf{R}\mathbf{1}'))')\mathbf{g}.
\end{aligned}
$$

Using it in the household's budget constraint,

$$
\begin{aligned}
c &= (1-\tau)w\ell \\
&= w\ell - ((\mathbf{s} \oslash \mathbf{R})' - \mathbf{s}'(\Omega \oslash (\mathbf{R}\mathbf{1}'))')\mathbf{g} \\
&= w\ell - ((\mathbf{s} \oslash \mathbf{R})' - \mathbf{s}'(\Omega \oslash (\mathbf{R}\mathbf{1}'))')\left(I - \Omega' \circ \Lambda\right)^{-1} \boldsymbol{\nu} c.
\end{aligned}
$$

Let $\mathbf{z} \equiv [z_1 \ldots z_N]'$. Through market clearing, we can express GDP as a function of total labor supply, ℓ, sectoral productivities $\{z_i\}$, sectoral interest rates $\{R_i\}$, sectoral subsidies $\{s_i\}$, and exogenous parameters:

$$GDP(\mathbf{z},\mathbf{R},\mathbf{s}) = \frac{\prod_i \Theta_i^{\beta_i} z_i^{\beta_i} R_i^{-\alpha_i\beta_i} \prod_{j=1}^{N} \left(\frac{R_i}{R_j}\right)^{-\omega_{ij}(1-\alpha_i)\beta_i}}{1 + \left((\mathbf{s}\oslash\mathbf{R})' - \mathbf{s}'(\Omega\oslash(\mathbf{R}\mathbf{1}'))'\right)\left(I - \Omega'\circ\Lambda\right)^{-1}\boldsymbol{\nu}}\ell.$$

Using this result in the household's budget constraint, we can write the tax rate τ as

$$\tau = \frac{\left((\mathbf{s}\oslash\mathbf{R})' - \mathbf{s}'(\Omega\oslash(\mathbf{R}\mathbf{1}'))'\right)\left(I - \Omega'\circ\Lambda\right)^{-1}\boldsymbol{\nu}}{1 + \left((\mathbf{s}\oslash\mathbf{R})' - \mathbf{s}'(\Omega\oslash(\mathbf{R}\mathbf{1}'))'\right)\left(I - \Omega'\circ\Lambda\right)^{-1}\boldsymbol{\nu}}.$$

From the sector's zero-profit condition, we take logs:

$$\begin{aligned}
\log p_i &= -(\log\Theta_i + \log z_i) + \alpha_i(\log w + \log R_i) + \sum_{j=1}^{N}(1-\alpha_i)\omega_{ij}\left(\log p_j + \log R_i - \log R_j\right) \\
&= -(\log\Theta_i + \log z_i) + \log R_i + \alpha_i \log w + \sum_{j=1}^{N}(1-\alpha_i)\omega_{ij}\left(\log p_j - \log R_j\right)
\end{aligned}$$

Let $\mathbf{p} \equiv [p_1 \ldots p_N]'$, $\Theta \equiv [\Theta_1 \ldots \Theta_N]'$ and $\boldsymbol{\alpha} \equiv [\alpha_1 \ldots \alpha_N]'$. Then, we can write the system as

$$\log\mathbf{p} = -(\log\Theta + \log\mathbf{z}) + \log\mathbf{R} + \boldsymbol{\alpha}\log w + \Omega(\log\mathbf{p} - \log\mathbf{R}).$$

Which yields

$$\log\mathbf{p} = \log\mathbf{R} + (I-\Omega)^{-1}\left[\boldsymbol{\alpha}\log w - (\log\Theta + \log\mathbf{z})\right].$$

Taking logs on the equation for wages

$$\log w = \sum_i \beta_i \left[\log \Theta_i + \log z_i - \alpha_i \log R_i - (1-\alpha_i) \sum_{j=1}^N \omega_{ij} \left(\log R_i - \log R_j \right) \right]$$
$$= \sum_i \beta_i \left[\log \Theta_i + \log z_i - \log R_i + (1-\alpha_i) \sum_{j=1}^N \omega_{ij} \log R_j \right]$$
$$= \beta' \left[\log \Theta + \log \mathbf{z} - (I-\Omega) \log \mathbf{R} \right]$$

We can then solve for the price vector:

$$\log \mathbf{p} = \log \mathbf{R} + (I-\Omega)^{-1} \left\{ \boldsymbol{\alpha} \beta' \left[\log \Theta + \log \mathbf{z} - (I-\Omega)\mathbf{R} \right] - (\log \Theta + \log \mathbf{z}) \right\}$$
$$= \log \mathbf{R} - (I-\Omega)^{-1}(I - \boldsymbol{\alpha}\beta')(\log \Theta + \log \mathbf{z}) - (I-\Omega)^{-1} \boldsymbol{\alpha} \beta' (I-\Omega)\mathbf{R}$$
$$= \left[I - (I-\Omega)^{-1} \boldsymbol{\alpha} \beta' (I-\Omega) \right] \log \mathbf{R} - (I-\Omega)^{-1}(I - \boldsymbol{\alpha}\beta')(\log \Theta + \log \mathbf{z})$$

Proof of Proposition 2

Household's optimality condition is given by

$$\frac{\ell^\epsilon}{c^\sigma} = (1-\tau)w.$$

The labor wedge is implicitly defined by the relation

$$\frac{\ell^\epsilon}{c^\sigma} = (1-\tau_\ell)\frac{c}{\ell}.$$

Combining both, we get

$$(1-\tau_\ell)\frac{c}{\ell} = (1-\tau)w.$$

The expression $1-\tau$ can be written as

$$1-\tau = \frac{1}{1+\left((\mathbf{s} \oslash \mathbf{R})' - \mathbf{s}'(\Omega \oslash (\mathbf{R}\mathbf{1}'))'\right)\left(I-\Omega' \circ \Lambda\right)^{-1} \boldsymbol{\nu}}.$$

Using the results from Proposition 1, the equivalence results becomes

$$\frac{1-\tau_\ell}{1+\left((\mathbf{s} \oslash \mathbf{R})' - \mathbf{s}'(\Omega \oslash (\mathbf{R}\mathbf{1}'))'\right)\left(I-\Omega' \circ \Lambda\right)^{-1} \boldsymbol{\nu}} = \frac{1}{1+\left((\mathbf{s} \oslash \mathbf{R})' - \mathbf{s}'(\Omega \oslash (\mathbf{R}\mathbf{1}'))'\right)\left(I-\Omega' \circ \Lambda\right)^{-1} \boldsymbol{\nu}}.$$

which implies that $1-\tau_\ell = 1$ and thus $\tau_\ell = 0$.

Proof of Lemma 1

Starting from the labor wedge definition, we get

$$\ell^{1+\varphi} = (1-\tau_\ell(\mathbf{s},\mathbf{R}))c^{1-\sigma}.$$

Given that $c(\mathbf{z},\mathbf{R},\mathbf{s}) = A(\mathbf{z})\mu(\mathbf{R},\mathbf{s})\ell$, we take logs

$$(1+\varphi)\log \ell = (1-\sigma)(\log A(\mathbf{z}) + \log \mu(\mathbf{R},\mathbf{s}) + \log \ell) + \log(1-\tau_\ell(\mathbf{s},\mathbf{R})),$$

which gives us

$$\log \ell = \frac{1-\sigma}{\sigma+\varphi}(\log A(\mathbf{z}) + \log \mu(\mathbf{R},\mathbf{s})) + \frac{1}{\sigma+\varphi}\log(1-\tau_\ell(\mathbf{s},\mathbf{R})),$$

and thus

$$\begin{aligned}\log c &= \log A(\mathbf{z}) + \log \mu(\mathbf{R},\mathbf{s})\frac{1-\sigma}{\sigma+\varphi}(\log A(\mathbf{z}) + \log \mu(\mathbf{R},\mathbf{s})) + \frac{1}{\sigma+\varphi}\log(1-\tau_\ell(\mathbf{s},\mathbf{R})) \\ &= \frac{1+\varphi}{\sigma+\varphi}(\log A(\mathbf{z}) + \log \mu(\mathbf{R},\mathbf{s})) + \frac{1}{\sigma+\varphi}\log(1-\tau_\ell(\mathbf{s},\mathbf{R})).\end{aligned}$$

B Deriving the lump-sum transfers to consumers

If the idiosyncratic borrowing cost e_i is transfered lump-sum to the consumer instead of being thrown into the ocean, the transfer, Ψ, is given by

$$\Psi = \sum_{i=1}^{N} e_i \left[\sum_{j=1}^{N} \left(p_{ij} x_{ij} - p_{ji} x_{ji} \right) + w\ell_i \right].$$

Remember that $e_i = R_i + s_i - 1$, since we set $R = 1$. Thus

$$\begin{aligned}
\Psi &= \sum_{i}^{N} (R_i + s_i - 1) \left[\sum_{j} (p_{ij} x_{ij} - p_{ji} x_{ji}) + w\ell_i \right] \\
&= \sum_{i}^{N} (R_i + s_i - 1) \left(\sum_{j} p_{ij} x_{ij} + w\ell_i \right) - \sum_{i}^{N} (R_i + s_i - 1) \left(\sum_{j} p_{ji} x_{ji} \right) \\
&= \sum_{i}^{N} (R_i + s_i - 1) \left(\sum_{j} \frac{(1-\alpha_i)\omega_{ij} p_i y_i}{R_i} + \frac{\alpha_i p_i y_i}{R_i} \right) - \sum_{i}^{N} (R_i + s_i - 1) \left(\sum_{j} \frac{(1-\alpha_j)\omega_{ji} p_j y_j}{R_j} \right) \\
&= \sum_{i}^{N} (R_i + s_i - 1) \left((1-\alpha_i) \sum_{j} \omega_{ij} + \alpha_i \right) \frac{g_i}{R_i} - \sum_{i}^{N} (R_i + s_i - 1) \left(\sum_{j} \frac{(1-\alpha_j)\omega_{ji} g_j}{R_j} \right) \\
&= \sum_{i}^{N} (R_i + s_i - 1) \frac{g_i}{R_i} - \sum_{i}^{N} (R_i + s_i - 1) \left(\sum_{j} \frac{(1-\alpha_j)\omega_{ji} g_j}{R_j} \right) \\
&= [((\mathbf{R} + \mathbf{s} - \mathbf{1}) \oslash \mathbf{R})' - (\mathbf{R} + \mathbf{s} - \mathbf{1})'(\Omega \oslash (\mathbf{R}\mathbf{1}'))']\mathbf{g},
\end{aligned}$$

where $\mathbf{1}$ is a vector where all entries equal one. Using it in the household's budget constraint,

$$\begin{aligned}
c &= (1-\tau) w\ell + \Psi \\
&= w\ell - ((\mathbf{s} \oslash \mathbf{R})' - \mathbf{s}'(\Omega \oslash (\mathbf{R}\mathbf{1}'))')\mathbf{g} + [((\mathbf{R} + \mathbf{s} - \mathbf{1}) \oslash \mathbf{R})' - (\mathbf{R} + \mathbf{s} - \mathbf{1})'(\Omega \oslash (\mathbf{R}\mathbf{1}'))']\mathbf{g} \\
&= w\ell + [((\mathbf{R} - \mathbf{1}) \oslash \mathbf{R})' - (\mathbf{R} - \mathbf{1})'(\Omega \oslash (\mathbf{R}\mathbf{1}'))']\mathbf{g} \\
&= w\ell + [((\mathbf{R} - \mathbf{1}) \oslash \mathbf{R})' - (\mathbf{R} - \mathbf{1})'(\Omega \oslash (\mathbf{R}\mathbf{1}'))'] \left(I - \Omega' \circ \Lambda \right)^{-1} \nu c.
\end{aligned}$$

The expression for GDP is as follows

$$GDP(\mathbf{z}, \mathbf{R}, \mathbf{s}) = \frac{\prod_i \Theta_i^{\beta_i} z_i^{\beta_i} R_i^{-\alpha_i \beta_i} \prod_{j=1}^N \left(\frac{R_i}{R_j}\right)^{-\omega_{ij}(1-\alpha_i)\beta_i}}{1 - \left[((\mathbf{R}-\mathbf{1}) \oslash \mathbf{R})' - (\mathbf{R}-\mathbf{1})'(\Omega \oslash (\mathbf{R}\mathbf{1}'))'\right] (I - \Omega' \circ \Lambda)^{-1} \boldsymbol{\nu}} \ell.$$

Using this result in the household's budget constraint, we can write the tax rate τ as

$$\tau = \frac{((\mathbf{s} \oslash \mathbf{R})' - \mathbf{s}'(\Omega \oslash (\mathbf{R}\mathbf{1}'))') (I - \Omega' \circ \Lambda)^{-1} \boldsymbol{\nu}}{1 - \left[((\mathbf{R}-\mathbf{1}) \oslash \mathbf{R})' - (\mathbf{R}-\mathbf{1})'(\Omega \oslash (\mathbf{R}\mathbf{1}'))'\right] (I - \Omega' \circ \Lambda)^{-1} \boldsymbol{\nu}}.$$

In this case, the efficiency wedge is given by

$$\mu(\mathbf{R}, \mathbf{s}) \equiv \frac{\prod_i R_i^{-\alpha_i \beta_i} \prod_{j=1}^N \left(\frac{R_i}{R_j}\right)^{-\omega_{ij}(1-\alpha_i)\beta_i}}{1 - \left[((\mathbf{R}-\mathbf{1}) \oslash \mathbf{R})' - (\mathbf{R}-\mathbf{1})'(\Omega \oslash (\mathbf{R}\mathbf{1}'))'\right] (I - \Omega' \circ \Lambda)^{-1} \boldsymbol{\nu}}.$$

The labor wedge can be derived from

$$\begin{aligned}
(1-\tau_\ell)\frac{c}{\ell} &= (1-\tau)w \\
(1-\tau_\ell) &= 1 - \left[((\mathbf{R}+\mathbf{s}-\mathbf{1}) \oslash \mathbf{R})' - (\mathbf{R}+\mathbf{s}-\mathbf{1})'(\Omega \oslash (\mathbf{R}\mathbf{1}'))'\right] \left(I - \Omega' \circ \Lambda\right)^{-1} \boldsymbol{\nu}.
\end{aligned}$$