The Holdout Problem in Sovereign Debt Markets

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Abstract

I develop a sovereign debt model with endogenous re-entry to international financial markets via debt renegotiation and a possibility for lenders to holdout and litigate. Thus, a haircut and a lenders' participation rate characterize the outcome of a renegotiation process. I use this model to show that the lenders' threat to litigate buys commitment to the sovereign. Precisely, to increase the lender's participation rate and hence reduce subsequent litigation, governments in default negotiate lower haircuts; as a result, lenders charge lower spreads ex-ante, during the periods in which the country has access to international financial markets. I use this model to evaluate the role of collective action clauses and find that the optimal threshold for the Argentine economy during the 1990s was 80%, which is only 5pp above the typical threshold currently used in sovereign debt contracts.

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1 Introduction

Sovereign debt markets feature a holdout problem: in any debt restructuring episode, each creditor has incentives to free-ride on the debt relief provided by the other creditors. Instead of accepting the deal as its peers, it may engage in a litigious process in an attempt to obtain a higher recovery rate from the then less financially-distressed government.

This process, which jeopardizes the post-restructuring recovery of debtor-countries, has become widespread in recent decades. Enderlein, Schumacher, and Trebesch (2018) document that the litigated claims in a US or UK court as a share of the debtor-countries' GDP has risen from 0.4% in the 1990s to 1.6% in the 2000s.

The escalation of litigation led the international community to look for ways to minimize this holdout problem, and the most prevalent solution involves a contractual approach: Collective Action Clauses (CACs) have been embedded in new bond issuances to prevent the emergence of holdouts[1]. In general, these clauses allow a majority to impose restructuring terms on a minority of bondholders. Bradley and Gulati (2014) document a shift towards CACs in 2003: 95% of sovereign bonds issued in New York required unanimity in the decade preceding this date, while virtually none in the subsequent one. Likewise, in response to the European sovereign debt crisis, all countries in the euro area are required to include CACs in their new sovereign bonds since 2013.

The purpose of this paper is to provide a default framework à la Eaton and Gersovitz (1981) to evaluate the effect of litigation in sovereign debt markets and the design of sovereign debt contracts. I introduce a theory of debt restructuring where identical risk-neutral foreign lenders make individual decisions on whether to accept the restructuring terms or to engage

[1] The international community has also looked for approaches other than CACs. In Belgium, a recently enacted law limits the creditors' ability, under certain circumstances, to recover through litigation more than the price they paid for the bonds. In 2018, the European Parliament incentivized member states to adopt similar regulations. The UK protects the Heavily Indebted Poor Countries (HIPC), as litigation cannot render more favorable terms than those agreed under the HIPC Initiative. Besides the "anti-vulture fund" legislation, the IMF proposed the Sovereign Debt Restructuring Mechanism (SDRM), which was rejected because of the lack of support from the US; see Krueger (2002).

in litigation. When litigation succeeds, the government is forced to either default on all bondholders or fully repay the holdouts. Thus, the model features an endogenous lenders' participation rate that helps discipline the debt relief through two channels. The first is very direct: high haircuts make the deal less attractive to lenders and induce low participation rates. The second one regards the value of a bond in legal disupute: when litigation succeeds, the government is more likely to fully repay holdouts and avoid a new default event if there is little holdout debt; thus, high participation rates require low haircuts to offset the intense free-riding incentives. Therefore, the threat to litigate enhances the government's commitment to repay its debt. As a consequence, litigation reduces sovereign spreads when the government is in good financial standing. Nevertheless, new borrowing becomes more expensive under the presence of holdouts, as they increase the default risk.

The introduction of CACs provide a balance between the ex-ante extra commitment for borrowing that stems from litigation and its associated post-restructuring (higher borrowing) costs. All agreements that lead to participation rates below the CAC threshold still benefit from the threat to litigate, ensuring lower haircuts for those lenders that participate in the deal. Yet, CACs prevent small shares of lenders from free-riding on the debt relief, thus minimizing the coordination problem between participating lenders, holdouts, and future lenders.

I calibrate my model using data from Argentina for the period preceding its 2001 default episode and find that the optimal CAC has an 80% threshold, which is only 5pp above the typical threshold currently used in sovereign debt contracts under NY law[2]. This 2001 event is one of the last default episodes before CACs became prevalent under NY law and illustrates how holdouts can disrupt the restructuring process when these clauses are not present. After the 2001 default and two rounds of restructuring in 2005 and 2010, Argentina modified the payment terms of 93% of its bonds with a 70% haircut. The holdouts, who represented 7%

[2]Bradley and Gulati (2014) report that CAC thresholds range from 18.75% to 85%. The 18.75%, though, usually applies only when an initial quorum requirement is not satisfied. And the most common threshold is 75%.

of the original stock of debt in default, got several favorable judgments during the 2000s that established that they were entitled to full face value rather than, for instance, the price for which they purchased the bonds or the value that other lenders settled in 2005 and 2010. Despite the barriers to seizing Argentine assets due to sovereign immunity, holdouts won an important injunction in 2012 that forced Argentina to either default on all lenders or restructure the bonds held by the holdouts[3].

The paper proceeds as follows. I briefly overview the related literature in the remainder of this introduction. In Section 2, I describe the model and, in Section 3, I inspect its mechanisms. Then, in Section 4, I calibrate the model and present the numerical results. Finally, I conclude the paper in section 5.

My paper is connected to the quantitative literature that follows Eaton and Gersovitz (1981) that had its early quantitative applications with Arellano (2008) and Aguiar and Gopinath (2006) in a setting with zero recovery rate on defaulted debt. Subsequently, Yue (2010) introduces renegotiation to standard sovereign default models using cooperative game theory solution concepts. Like Hatchondo, Martinez and Sosa-Padilla (2014), Gabriel Mihalache (2020), and Almeida et al. (2019), I follow Yue (2010) in the particular aspect of modeling debt restructuring as the outcome of a Nash bargaining problem between the government and the (participating) lenders. Nevertheless, the participation rate can be smaller than 100% in my model. This potential lack of cooperation, with non-participating lenders free-riding in the participating ones' debt relief, is exactly what gives rise to the holdout problem.

Benjamin and Wright (2009) introduce renegotiation using non-cooperative game theory solution-concepts to the sovereign default literature. My paper is mostly related to Pitchford and Wright's (2012) paper about the holdout problem. They use a non-cooperative approach to quantitatively analyze delays in debt restructurings. An essential difference between our

[3]The US courts had jurisdiction to issue an injunction relief because the contracts required payments to be made through a trustee. Thus, until Argentina settled an agreement with holdouts, the trustee could not realize the payments to those bondholders who had previously agreed to a haircut in 2005 and 2010. For further details on the Argentina negotiations, see Alfaro (2015).

papers regards the rounds of renegotiations and the sources of inefficiency. They assume that the government negotiates with one bondholder per time, and each defaulted bond guarantees its holder a veto power over the country's ability to reaccess international financial markets. Then, it creates incentives for each bondholder to be the last one to restructure the debt and, consequently, causes inefficiencies through delays. In contrast, in my paper, there is only one round of renegotiation: the restructuring offer is simultaneously available to all bondholders, who can reject it, hold out, and litigate. Here, the inefficiency stems from the higher borrowing costs the government faces while dealing with holdouts.

Closest to my paper is Anand and Gai (2019), who develop an analytical framework for sovereign debt negotiations with endogenous participation rates. In their setting, though, the government tailors the bankruptcy procedures by committing in advance to a haircut in the event it files for bankruptcy and seeks restructuring. In contrast, in my setting, the government lacks commitment regarding the haircut. Bi, Chamon, and Zettelmeyer (2016) also develop a simple and elegant framework to thoroughly discuss different aspects of sovereign debt contracts. Neither of these papers, though, provide a quantitative exercise.

My paper also complements the empirical literature on sovereign debt restructuring. Enderlein, Schumacher, and Trebesch (2018) provide many empirical regularities and a comprehensive discussion on relevant institutional changes that shaped sovereign debt markets and particularly litigation processes.

2 Model

I consider a small open economy à la Eaton and Gersovitz (1981) in which the government receives a stochastic endowment stream and issues non-state-contingent defaultable bonds to a large number of risk-neutral foreign lenders. Whenever the government defaults, it suffers a direct output cost and stays in financial autarky until the debt is restructured. A key feature is that each lender makes its individual decision on whether to accept or reject the restructuring terms, subject to the collective action clause, which renders the participation rate endogenously. Afterward, the holdouts immediately engage in litigation against the sovereign, which eventually forces the government to either fully repay them or default on the entire stock of debt.

2.1 Government

Time is discrete and indexed by $t \in \{0, 1, 2, \ldots\}$. In each period, households receive a stochastic endowment of a tradable good y_t that follows a finite-state Markov chain with transition probabilities $Prob\left(y_{t+1} = y' | y_t = y\right) = F\left(y'|y\right)$. The economy is populated by identical households, whose preferences are given by:

$$E_t \left[\sum_{j=t}^{\infty} \beta^{j-t} u\left(c_j\right) \right] \tag{1}$$

where $\beta \in (0, 1)$ is the subjective discount factor, c is consumption, and the utility function u is strictly increasing and strictly concave. The government is benevolent and can borrow from foreign lenders by issuing one-period non-contingent bonds.

Every period, the government observes the total stock of debt b, the stock of debt held by holdouts b_l, the income shock y, and whether it has access to international financial markets, where $z = 1$ indicates it does while $z = 0$ indicates it is in financial autarky. I assume $b \in \mathcal{B} = \left[0, \bar{b}\right]$ where $\bar{b} > 0$ is finite, so that the government cannot run a Ponzi scheme. Because the government savings are risk-free and the debt held by holdouts is, by construction, smaller than the total stock of debt, then $b_l \in [0, b]$. I also assume the government cannot come to any agreement with the holdouts through means other than litigation. In the periods in which the government does not inherit a previous default decision, litigation succeeds with an exogenous probability θ_L.

In case litigation fails, the government chooses between default and repayment. In this case, the value function of the government is:

$$V\left(b, b_l, y\right) = \max_{d \in \{0,1\}} \left\{ dV^D\left(b, y\right) + (1-d)\, V^P\left(b, b_d, y, 1\right) \right\} \tag{2}$$

where V^D is the default value, V^P is the repayment value, and default and repayment decisions are represented by $d = 1$ and $d = 0$, respectively.

In the case in which litigation succeeds, the government must fully repay the holdouts if it chooses to avoid a default episode. Then, the value function is:

$$V_L(b, b_l, y) = \max_{d_L \in \{0,1\}} \left\{ d_L V^D(b, y) + (1 - d_L) V^P(b, 0, y, 1) \right\} \tag{3}$$

where default and repayment decisions are represented by $d_L = 1$ and $d_L = 0$, respectively.

Finally, if the government inherits a previous default decision, renegotiation opportunities arise with probability θ_R. When these opportunities arise, the outcome that follows the bargaining process is characterized by a participation rate and a haircut. The government can choose to accept the newly restructured debt level or remain in default. The restructuring immediately ceases the direct output cost $\phi(y)$, but the government remains in financial autarky in the current period, i.e., $z = 0$, and only regains access to financial markets in the subsequent one[4]. The associated value function is:

$$V_R(b_d, y) = \max_{d_R \in \{0,1\}} \left\{ d_R V^D(b_d, y) + (1 - d_R) V^P\left(b^R(b_d, y), b_l^R(b_d, y), y, 0\right) \right\} \tag{4}$$

where $b^R(b_d, y) \equiv PR^R(b_d, y)\left[1 - h^R(b_d, y)\right] b_d + \left[1 - PR^R(b_d, y)\right] b_d$ is the new total stock of debt after the debt restructuring, $b_l^R(b_d, y) \equiv \left[1 - PR^R(b_d, y)\right] b_d$ is the part of it that is held by holdouts, $d_R = 0$ if the government takes the deal with lenders and $d_R = 0$ otherwise. In sections 2.2 and 2.3, I discuss in detail how $h^R(b_d, y)$ and $PR^R(b_d, y)$ are determined.

When the government chooses to default, the government is excluded from international financial markets, the debt service is suspended, the stock of debt is frozen, and the country

[4]This assumption of remaining in financial autarky in the period of the debt restructuring serves a computational purpose only. After the debt restructuing, the government does not make any immediate borrowing decisions, which simplifies the pricing equation 19. It has the benefit of reducing the jumps associated to the mapping of the prices that takes place in each iteration. For alternative solutions to this computational obstacle, see Gordon (2019) and Chatterjee and Eyigungor (2012).

suffers a direct output cost, $\phi(y)$, which is increasing in y. The associated value function is:

$$V^D(b_d, y) = u(y - \phi(y)) + \beta\mathbb{E}\left[\theta_R V_R(b_d, y') + (1-\theta_R) V^D(b_d, y')\right] \tag{5}$$

The value function when the government has access to international financial markets and chooses to repay (non-holdout) lenders is the following:

$$\begin{aligned} V^P(b, b_l, y, 1) &= \max_{b'} \left\{u(c) + \beta\mathbb{E}\left[(1-\theta_L) V(b', b_l, y') + \theta_L V_L(b', b_l, y')\right]\right\} \\ s.t. \quad c &= y - (b - b_l) + q^P(b', b_l, y)(b' - b_l) \end{aligned} \tag{6}$$

The government can finance its consumption with its income y and new debt issue $(b' - b_l)$ at price $q^P(b', b_l, y)$, net of the debt service $(b - b_l)$. Notice the country still faces litigation risk in the subsequent period.

The case in which the government repays its debt but has no access to international financial markets only takes place in the period with a debt restructuring episode. Its value function is slightly different than the previous one:

$$\begin{aligned} V^P(b, b_l, y, 0) &= u(c) + \beta\mathbb{E}\left[(1-\theta_L) V(b_l, b_l, y') + \theta_L V_L(b_l, b_l, y')\right] \\ s.t. \quad c &= y - (b - b_l) \end{aligned} \tag{7}$$

Notice that I assume the government faces litigation even if there is a measure zero of holdouts, $b_l = 0$. Besides avoiding a separate definition of another value function $V^P(b, 0, y, z)$, this approach is consistent with a price schedule in which a measure zero of holdouts still litigate and recover the full payment on their claims. Furthermore, since restructuring a measure zero of debt have no impact in the government's payoff, i.e., $V(b', b_l, y') = V_L(b', b_l, y')$ when $b_l = 0$, then the alternative value function $V^P(b, 0, y, z)$ would be isomorphic to the one I use.

The solution to the government's problem gives decision rules for consumption, $c^P(b, b_l, y)$, debt issuance, $\left[b^P(b,b_l,y)-b_l\right]$, default policies, $d^P(b,b_l,y)$ and $d^P_L(b,b_l,y)$, and restructuring policies, $d^P_R(b_d,y)$.

2.2 Renegotiation

Following a default episode, renegotiation opportunities arise with probability θ_R. In this case, the government and the lenders go through the following Nash bargaining problem, after observing the amount of debt in default b_d and the income shock y.

$$\begin{aligned} h^R(b_d,y) &= \arg\max_{\tilde{h}\in[0,1]} \left\{ S^{LEN}\left(\tilde{h},b_d,y\right)^{\alpha} S^{GOV}\left(\tilde{h},b_d,y\right)^{1-\alpha} \right\} \\ s.t. :& S^{LEN}\left(\tilde{h},b_d,y\right), S^{GOV}\left(\tilde{h},b_d,y\right) \geq 0 \end{aligned} \tag{8}$$

where α is the bargaining power of the participating foreign lenders, S^{LEN} is their surplus, and S^{GOV} is the government's. As usual, a constraint to this problem is that all parties need to be better off with the terms of renegotiation, otherwise it fails.

The participating lenders' surplus is the difference between resuming debt payments with a haircut $\tilde{h}$ and the market value of their bonds in case the government remains in default:

$$S^{LEN}\left(\tilde{b},b_d,y\right) \equiv \left(1-\tilde{h}\right)\tilde{PR}^R_{CAC}\left(\tilde{h},b_d,y\right)b_d - q^D(b_d,y)\,\tilde{PR}^R_{CAC}\left(\tilde{h},b_d,y\right)b_d \geq 0 \tag{9}$$

where $\tilde{PR}^R_{CAC}\left(\tilde{h},b_d,y\right)$ is the endogenous participation rate associated with a restructuing offer $\tilde{h}$ and $q^D(b_d,y)$ is the price schedule in secondary markets of a unit of a bond in default.

And I define the surplus of the govenment in an analogous way: it's the difference between the value of accepting the deal and remaining in default:

$$S^{GOV}\left(\tilde{h},b_d,y\right) \equiv V^P\left(b^R\left(\tilde{h},b_{d,}y\right), b^R_d\left(\tilde{h},b_d,y\right), y, 0\right) - V^D(b_d,y) \geq 0 \tag{10}$$

where $b^R\left(\tilde{h},b_{d,}y\right) \equiv \left(1-\tilde{h}\right)\tilde{PR}^R_{CAC}\left(\tilde{h},b_d,y\right)b_d + \left[1-\tilde{PR}^R_{CAC}\left(\tilde{h},b_d,y\right)\right]b_d$ is the new total stock of debt after the debt restructuring and $b_d^R\left(\tilde{h}, b_d, y\right) \equiv \left[1 - \tilde{PR}_{CAC}^R\left(\tilde{h}, b_d, y\right)\right] b_d$ is the part of it that is held by holdouts.

Finally, the outcome of this bargaining game is not only a haircut $h^R(b_d, y)$ but also a participation rate $PR^R(b_d, y) \equiv \tilde{PR}_{CAC}^R\left(h^R(b_d, y), b_d, y\right)$, i.e., the endogenous participation rate mentioned above, evaluated at the new restructured debt level. In the next section, I discuss how the participation rate for different restructuring offers is determined.

2.3 Lenders

There is a continuum of atomistic lenders indexed by i. Given their size, no lender can individually affect the participation rate. Thus, facing an offer $\tilde{h}$ and taking as given the participation rate $\tilde{PR}$, the problem of lender i in a restructuring episode is to choose whether to take the deal or to hold out:

$$a_i^P\left(\tilde{h}, \tilde{PR}, b_d, y\right) \in \arg\max_{a_i \in \{0,1\}} \left\{a_i RR\left(\tilde{h}\right) + (1 - a_i)\, HO\left(\tilde{h}, \tilde{PR}, b_d, y\right)\right\} \tag{11}$$

where $a_i = 1$ if the government takes the deal and receives as payoff a recovery rate RR on the unit of debt, and $a_i = 0$ if the government rejects the deal and receives the payoff HO associated with the expected future gains from litigation.

The recovery rate is defined as one minus the haircut:

$$RR\left(\tilde{h}\right) \equiv 1 - \tilde{h} \tag{12}$$

I define the value of the holdout strategy as:

$$HO\left(\tilde{h}, \tilde{PR}, b_d, y\right) \equiv (1 - \epsilon_{CAC})\, HO_T\left(\tilde{h}, \tilde{PR}, b_d, y\right) + \epsilon_{CAC} HO_1\left(\tilde{h}, \tilde{PR}, b_d, y\right) \tag{13}$$

I assume the CAC are unobserved with probability $\epsilon_{CAC} > 0$ close to zero. When it happens, the debt contract requires unanimity to implement changes in the payment terms. The terms

HO_1 and HO_T indicate the value of a contract when unanimity is required and when the CAC threshold T is observed, respectively. They are defined as follows:

$$HO_t\left(\tilde{h}, \tilde{PR}, b_d, y\right) \equiv \begin{cases} q^L\left(\left[1-\tilde{PR}\right]b_d, \left[1-\tilde{PR}\right]b_d, y\right) & \text{if } \tilde{PR} < t \\ RR\left(\tilde{h}\right) & \text{otherwise} \end{cases} \tag{14}$$

where $q^L\left(\left[1-\tilde{PR}\right]b_d, \left[1-\tilde{PR}\right]b_d, y\right)$ is the price schedule of a bond held by a holdout after the government restructures the debt of lenders who participate in the deal. The payoff when $\tilde{PR} \geq T$ captures the lenders' obedience to the CAC. The introduction of CAC failures eliminate an undesirable equilibrium outcome. In section 3, I explore in detail the equilibrium selection of this game.

For a consistency matter, the participation rate that lenders take as given should coincide with the aggregation of their individual decision rules:

$$\tilde{PR}^R\left(\tilde{h}, b_d, y\right) = \int a_i\left(\tilde{h}, \tilde{PR}^R\left(\tilde{h}, b_d, y\right), b_d, y\right) dF(i) \tag{15}$$

And, finally, the participation rate after the consideration of the CAC is:

$$\tilde{PR}^R_{CAC}\left(\tilde{h}, b_d, y\right) \equiv \begin{cases} \tilde{PR}^R\left(\tilde{h}, b_d, y\right) & \text{if } \tilde{PR}^R\left(\tilde{h}, b_d, y\right) \in [0, T) \\ 1 & \text{otherwise} \end{cases} \tag{16}$$

2.4 Equilibrium

An equilibrium is a set of:

- value functions V, V_L, V_R, V^P, and V^D,
- government policy functions c^P, b^P, d^P, d^P_L, and d^P_R,
- lenders' individual decision rules a^P_i,

- participation rate functions for given restructuring offers before and after consideration of the CAC, $\tilde{PR}^R$ and $\tilde{PR}^R_{CAC}$, respectively,
- bond price functions q^P, q^D, and q^L, and
- renegotiaton rules h^R and PR^R,

such that:

1. given the renegotiaton rules, h^R and PR^R, and the bond price function, q^P, the value and government policy functions solve the government's problem,
2. given the bond price functions, q^D, the value functions, V^P and V^D, and the participation rate function for given restructuring offers after consideration of the CAC, $\tilde{PR}^R$, the renegotiation rules, h^R and PR^R, solve the Nash bargaining problem,
3. given the price function, q^L, the lenders' individual decision rule, a_i^P, solve the lender's individual problem,
4. given the lenders' individual decision rule, a_i^P, the participation rate function for given restructuring offers before the consideration of the CAC, $\tilde{PR}^R$, solve the fixed point problem defined in equation 15,
5. given the participation rate functions for given restructuring offers before the consideration of the CAC, $\tilde{PR}^R$, the analogous function after the consideration of the CAC, $\tilde{PR}^R_{CAC}$, is defined by equation 16
6. and the bond prices are consistent with lenders making zero profits after adjusting for default risk.

Given the above definition, the price schedule needs to satisfy a few conditions. Next, I define the price of a bond held by non-holdouts when the government repaid its previous (non-holdout) lenders:

$$\begin{aligned} q^P\left(b', b_l, y\right) = & \frac{\left(1-\theta_L\right)}{1+r} \mathbb{E}\left[1-d^P\left(b', b_l, y'\right)\right] \\ & + \frac{\left(1-\theta_L\right)}{1+r} \mathbb{E}\left[d^P\left(b', b_l, y'\right) q^D\left(b', y'\right)\right] \\ & + \frac{\theta_L}{1+r} \mathbb{E}\left[1-d_L^P\left(b', b_l, y'\right)\right] \\ & + \frac{\theta_L}{1+r} \mathbb{E}\left[d_L^P\left(b', b_l, y'\right) q^D\left(b', y'\right)\right] \end{aligned} \tag{17}$$

The first two lines is standard for most quantitative sovereign debt models with short term bonds and renegotiation, as they refer to the states in which litigation did not succeed. In the first one, the government chooses to repay in full, $d^P\left(b', b_l, y\right) = 0$, while in the second one, the government chooses to default, $d^P\left(b', b_l, y\right) = 1$, in which case the lender holds a defaulted bond $q^D\left(b', y'\right)$. The third and fourth lines, on the other hand, refer to the states in which litigation succeeds. In the third one, bondholders are paid in full, while in the fourth one, they continue to hold a defaulted debt. Different than the early quantitative sovereign debt models, defaulted bonds generally feature positive market price $q^D\left(b', y'\right)$, because they are not forgiven and are eventually restructured.

The price of a bond held by holdouts (or any lender who purchased the bonds from a holdout in secondary markets) is the following:

$$\begin{aligned} q^L\left(b', b_l, y\right) = & \frac{\left(1-\theta_L\right)}{1+r} \mathbb{E}\left[\left[1-d^P\left(b', b_l, y'\right)\right] q^L\left(b^P\left(b', b_l, y'\right), b_l, y'\right)\right] \\ & + \frac{\left(1-\theta_L\right)}{1+r} \mathbb{E}\left[d^P\left(b', b_l, y'\right) q^D\left(b', y'\right)\right] \\ & + \frac{\theta_L}{1+r} \mathbb{E}\left[1-d_L^P\left(b', b_l, y'\right)\right] \\ & + \frac{\theta_L}{1+r} \mathbb{E}\left[d_L^P\left(b', b_l, y'\right) q^D\left(b', y'\right)\right] \end{aligned} \tag{18}$$

Its distinction from the bonds in good standing appears in the first line, where litigation fails and the holdouts continue to carry bonds priced at q^L. The third line is also worth mentioning: it indicates the holdouts successfully force the government to repay them in full.

The price of a debt in default is:

$$
\begin{aligned}
q^D(b_d, y) = {} & \frac{\theta_R}{1+r}\mathbb{E}\left[\left[1 - d_R^P(b_d, y')\right] X(b_d, y')\right] \\
& + \frac{\theta_R}{1+r}\mathbb{E}\left[d_R^P(b_d, y')\, q^D(b_d, y')\right] \\
& + \frac{(1-\theta_R)}{1+r}\mathbb{E}\left[q^D(b_d, y')\right]
\end{aligned} \tag{19}
$$

where

$$X(b_d, y') \equiv \max\left\{RR(b_d, y'), HO(b_d, y')\right\} \tag{20}$$

$$RR(b_d, y') \equiv 1 - h^R(b_d, y') \tag{21}$$

$$HO(b_d, y') \equiv \begin{cases} q^L\left(b_l^R(b_d, y), b_l^R(b_d, y), y'\right) & \text{if } PR^R(b_d, y') < T \\ RR(b_d, y') & \text{otherwise} \end{cases} \tag{22}$$

Lenders continue to hold defaulted debt if the government rejects the restructuring terms or if no restructuring opportunities even arise, as captured by the second and third line of equation 19, respectively. The first line of equation 19, though, represents the lenders' payoff X from a successful renegotiation. This payoff is the maximum between the recovery rate on the unit of debt or the market value of a bond in litigation, subject to the CAC. Notice that, since the government has to remain in financial autarky during the period of restructuring, then the total stock of debt in the end of the period coincides with the debt held by holdouts, $b_l^R(b_d, y) \equiv \left[1 - PR^R(b_d, y)\right] b_d$.

3 Inspecting the mechanism

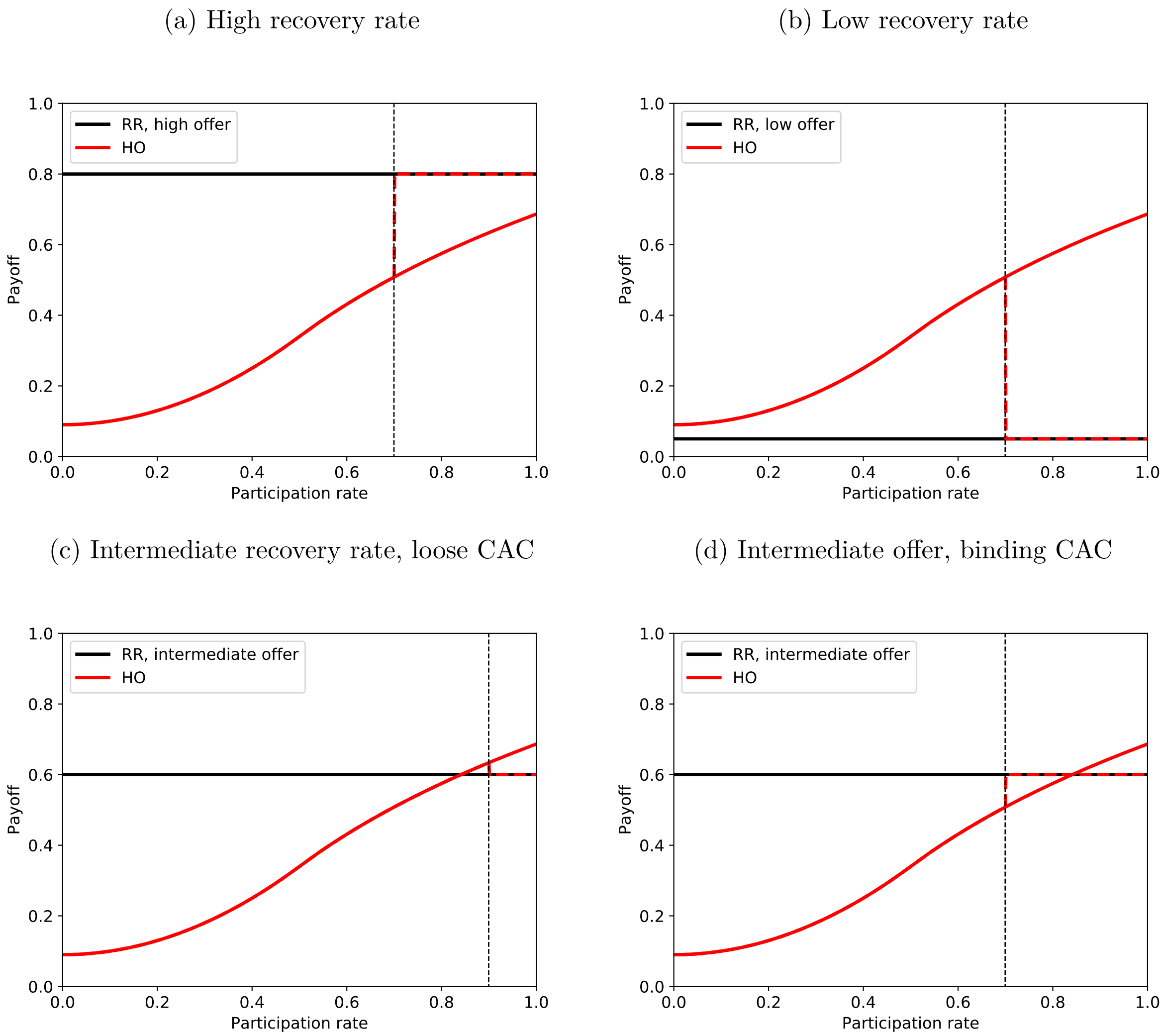
In this section, I discuss the endogeneity of the participation rate, which is a novel feature of my model. I leave for section 4 to discuss some parameter-dependent properties of my model.

The participation rate is the outcome of the aggregation of individual decisions of atomistic lenders. They take the deal when the recovery rate is higher than the value of being a holdout, and reject the deal otherwise. Figure 1 helps visualize this aggregation problem. In each plot of this figure, I keep fixed the government's stock of debt and income. The black vertical lines represent CAC thresholds. The black flat lines represent a recovery rate $RR\left(\tilde{h}\right) \equiv 1 - \tilde{h}$ associated with some haircut $\tilde{h}$. The dashed red line represents the lender's payoff when the CAC is active. Finally, the solid red lines represents the value of holding out of the restructuring when the CAC is off.

Notice from the solid red line that litigation becomes increasingly more valuable as the participation rate increases, ie $q^{D,P}\left(\left[1-\tilde{PR}\right]b, \left[1-\tilde{PR}\right]b, y\right)$ increases as $\tilde{PR}$ increases. The intuition behind this monotonicity is simple: holdouts free-ride on the debt relief provided by lenders who participate in debt restructurings. A successful litigation is more likely to trigger a new default episode if the amount of debt in dispute is higher; consequently, lenders have less incentives to become holdouts when a small pool of bondholders accept the restructuring terms. Since the recovery rate is independent of the participation rate $\tilde{PR}$ and only depends on the haircut $\tilde{h}$, then the curves can intersect each other in, at most, one point.

Figure 1a depicts the case in which the haircut $\tilde{h}$ offer is low enough to the point in which it's never advantageous to become a holdout. For any rate below 100%, holdouts have incentives to deviate from their strategy and take the deal. Thus, $\tilde{PR}^{R}_{CAC}\left(\tilde{h}, b, y\right) = 100\%$ is the only rate that satisfies the consistency condition of equation 16.

Figure 1: Lenders's payoff.



(a) High recovery rate

(b) Low recovery rate

(c) Intermediate recovery rate, loose CAC

(d) Intermediate offer, binding CAC

In the other extreme, figure 1b shows the case in which the haircut $\tilde{h}$ is high. Consider first an alternative framework in which we remove the risk of CAC failtures, i.e., the CAC applies whenever the participation rate is above the threshold T. Then, two participation rates $\{0\%, 100\%\}$ become consistent with equation 16. When a lender takes as given that all other lenders are accepting the deal, it becomes indifferent between accepting or rejecting the deal because both choices lead to the same payoff $1-\tilde{h}$. This equilibrium with 100% participartion rate for such a high haircut is undesirable, since there are no forces pushing any lender to accept the deal, except for the CAC itself. In my framework, though, lenders do not choose this weakly dominated strategy in equilibrium; only $\tilde{PR}^{R}_{CAC}\left(\tilde{h}, b, y\right) = 0\%$ is a possible equilibrium outcome. The introduction of CAC risk makes lenders reject the deal even if they believe that everybody else is taking the deal. Ultimately, as no lender takes the deal, an offer $\tilde{h}$ leads the renegotitation to failure.

Finally, figures 1c and 1d consider the same intermediate haicut offer, but different CAC thresholds. In figure 1c, the CAC is too high and hence does not bind. As a consequence, the only outcome consistent with equation 16 is $\tilde{PR}^{R}_{CAC}\left(\tilde{h}, b, y\right) \in (0, T)$, where $T \leq 1$[5]. On the other hand, in figure 1d, the CAC threshold is lower. In this case, the CAC binds and guarantees the government full participation rate, $\tilde{PR}^{R}_{CAC}\left(\tilde{h}, b, y\right) = 1$.

4 Quantitative analysis

In this section, I numerically solve my model using value function iteration and discuss some of its key features.

4.1 Computational algorithm

1. Start with a guess for value functions, V, V_L, V_R, V^P, V^D, and price functions, q^P, q^L, q^D.

2. Solve for V^P, V^D, and b^P using the guesses.

3. Solve for the renegotiation outcomes h^R and PR^R using the guesses and the solution from step 2 (V^P and V^D).

4. Solve for V, V_L, V_R and d^P, d^P_L, d^P_R using the solution to step 2 (V^P and V^D) and to step 3 (h^R and PR^R).

5. Solve for q^P, q^L, q^D using the guesses for prices, the solution to step 2 (b^P), to step 3 (h^R and PR^R), and to step 4 (d^P, d^P_L and d^P_R)

[5] A similar argument used when the haircut $\tilde{h}$ was high applies to this case of intermediate $\tilde{h}$ and loose CAC. The CAC risk eliminates the equilibrium with 100% participation rate.

6. Check for convergence of value functions and prices.

7. If no convergence, update guesses V, V_L, V_R, V^P, V^D, and price functions, q^P, q^L, q^D with the solution of the last iteration and repeat steps 2-6.

4.2 Calibration

Functional forms. I consider the case of the Argentine debt crisis in 2001 for the calibration of my model. The income shock follows a log-normal AR(1) process $\log(y_t) = \rho \log(y_{t-1}) + \epsilon_t$, with $|\rho| < 1$ and $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. And I assume the direct output cost of default has a quadratic functional form $\phi(y_t) = \max\{0, \phi_0 y_t + \phi_1 y_t^2\}$, with $\phi_0 < 0 < \phi_1$, which makes default more costly during high-endowment periods. In particular, the cost is zero when for $0 \leq y_t \leq -\frac{\phi_0}{\phi_1}$ and increases more than proportionally for $y_t > -\frac{\phi_0}{\phi_1}$. This asymmetry allows the model to match default episodes occurring during bad times and, more generally, to better match the dynamics of spreads observed in the data. I also assume that the utility function features a constant relative risk aversion (CRRA): $u(c_j) = \frac{c_j^{1-\eta}-1}{1-\eta}$ and that a period in the model corresponds to a quarter of a year.

Estimated parameters. I report in table 1 all the parameter values that can be directly calibrated from the data. The risk-free interest rate is set to 1.5%, the 1990-2001 average quarterly interest rate of a 5-year treasury bond[6]. The constant coefficient of relative risk aversion is set to a standard value, $\eta = 2$. Renegotiation opportunities arise with 2.7%, so that default episodes last 9 years, on average, while litigation succeeds with probability 5%, so that it is resolved in 5 years, on average[7]. As most debt contracts issued under the New York jurisdiction did not involve CACs before 2001, including those Argentine bonds, then

[6] I excluded the 1980s when computing the average of the US interest rate. Thus, I excluded the observations from the unusually high interest rates of the 1980s, when the then chairman of the Federal Reserve, Paul Volcker, raised interest rates to tame the exceptionally high US inflation.

[7] It took 4 years from the 2001 default episode until the first Argentine restructuring round, 9 years until the second round, and 14 years until the lawsuits succeeded.

I set the CAC threshold to 100% so that bondholders can provide debt relief only under unanimity. The AR(1) income process is estimated using HP-filtered logged Argentine GDP data from 1980 to 2001. This yields an auto-correlation parameter $\rho = 0.945$ and a standard deviation of innovations of $\sigma = 0.025$.

Table 1: Parameters directly calibrated from the data

Parameter		Value	Detail
Risk-free interest rate	r	0.015	1980-2001
Risk aversion	η	2	Standard
Prob(litigation)	θ_L	0.050	Duration of 5 years
Prob(renegotiation)	θ_R	0.027	Duration of 9 years
CAC threshold	T	1	No CAC
Income process	ρ_y	0.945	AR(1) estimation
	σ_y	0.025	

Calibrated parameters. In table 2, I report the internally calibrated parameters: the discount factor β, the direct output cost parameters ϕ_0 and ϕ_1, and the lenders' bargaining power α. I set them to match four moments of the Argentine economy: the default probability of 3.0%, the average debt service-to-GDP ratio of 5.5%, the trade balance volatility relative to the GDP volatility of 17.1% and the average spread of 8.1%.

Table 2: Internally calibrated parameters and moments

Parameter		Value	Moment	Data	Model
Discount factor	β	0.943	Default probability	3.0%	3.1%
Bargaining power	α	0.100	Debt service-to-GDP (mean)	5.5%	6.0%
Output cost	d_1	-0.191	$\frac{\text{Trade balance (volatility)}}{\text{GDP (volatility)}}$	17.1%	24.7%
	d_2	0.246	Spread (mean)	8.1%	4.2%

4.3 Renegotiation outcome

Before I discuss the role of litigation and collective action clauses, I explain how the consumption smoothing motive plays out during restructuring episodes.

Consumption smoothing motive. A poor and financially distressed government in default receives reasonable debt relief during restructuring episodes. As a consequence, the government does not guarantee a full participation rate, as the lenders have incentives to become holdouts and start a litigation process that may last for many periods. Thus, the govenment incurs in only part of the restructuring costs in the current period, leaving part of it to the next periods, when it is likely to be in better times, given the mean reversion of the income shock. Precisely, in the current period, the government services only part of the stock of debt, with a discount, while in the future, when litigation suceeds, it may service the debt held by holdouts in full and face higher borrowing costs until then.

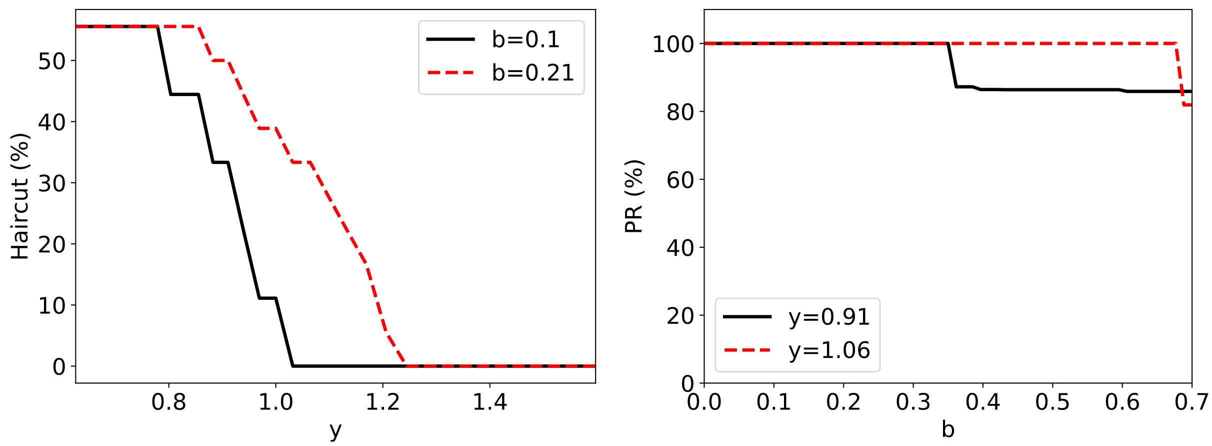
On the other hand, a rich country with low debt levels in default can afford a full participation rate by restructuring the debt with little or no haircut, which allows the country to prevent future costs, when the income of the country reverses downwards, towards the mean. Figure 2 depict this dynamics[8].

In figure 2a, the haircut is decreasing in the income level. For high enough income, the debt relief disappears and the government pays the full amount it owes; similarly, the debt relief also disappears if the governments had defaulted on a smaller amount of debt. In figure 2b, the participation rate is increasing in the income level and decreasing in the debt level.

[8]There are other forces working in the same direction as the consumption smoothing motive. The asymmetric default cost $\phi(y_t)$, which is increasing in the income level, reduces the outside option of the government during the Nash Bargaining problem disproportionally more during periods of high income; as long as the lenders' bargaining power is strictly greater than zero, it leads them to claim more favorable restructing terms. Also, since the income is persistent, this asymmetric cost also incentivizes government to provide generous restructuring terms during good times, as it prevents litigation from pushing the country towards a new costly default in the near future.

Figure 2: Consumption smoothing dynamics

(a) Haircut as a function of the income level. (b) Participation rate as a function of the debt level.

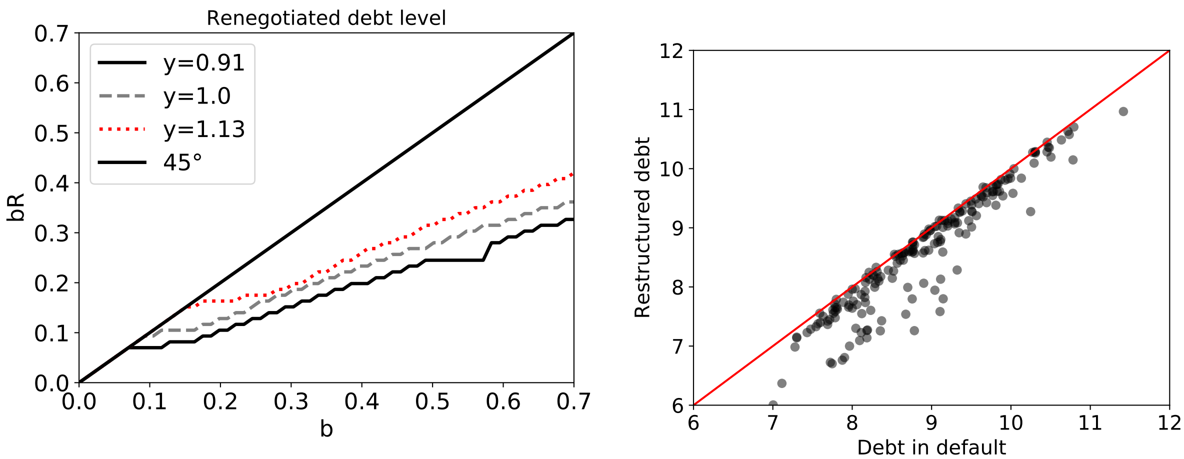


Bygones no longer. Another distinct feature of this model is that the default value $V^D(b, y)$ is decreasing in the amount of defaulted debt. Figure 3a highlights the reason: in the aftermath of sovereign default, the restructured debt is positively correlated with the defaulted debt. It is in sharp contrast with most of the literature that delivers the result that "bygones are bygones": in Arellano (2008), the country reaccess international financial markets after all the defaulted debt is forgiven; and models that follows Yue (2010), where renegotiation follows a Nash bargaining process, find that the restructured debt is independent of the amount of debt in default. Wang (2019) departs from the Nash bargaining process and employs a Kalai-Smorodinsky bargaining process, which captures an "equal sacrifice" condition, yielding the first quantitative sovereign debt model with the "bygones no longer" feature.

Based on the dataset of Cruces and Tresbesch (2013), figure 3b provides evidence that the default value indeed depends on the level of defaulted debt.

Figure 3: Restructured versus defaulted debt level.

(a) Model. (b) Data.



4.4 The role of litigation

What distinguishes my paper from Yue (2010)'s is the possibility to hold out of restructuring deals. Thus, I evaluate the role of litigation in sovereign debt markets by comparing our models. In this comparison, her model captures a legal framework that prevents lenders from holding out and, to some extent, can be interpreted as an extreme version of the "anti-vulture fund" legislation of Belgium and the UK.

My model nests Yue (2010)'s if I set the probability of litigation success to zero, $\theta_L = 0$. The remaining parameters that I use are the same ones described in the tables 1 and 2. Figure 4 illustrates two important effects of litigation: it lowers the borrowing costs and drives the government to increase borrowing.

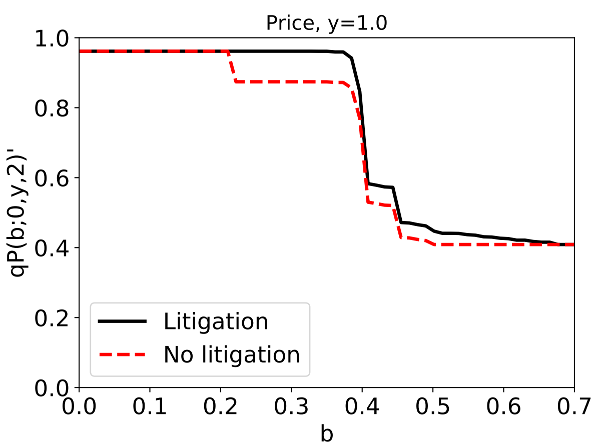
In figure 4a I plot the bond price schedule in two different environments: the red dashed line refers to the environment without litigation while the black solid line refers to the case where lenders can become holdouts and litigate. For comparability reasons, I consider states whith no debt held by holdouts in the environment that allows litigation. Except for low enough borrowing choices in which the default probability is zero, bond prices are higher when the threat of litigation is present.

Still with no debt held by holdouts, figure 4b depicts the government's policy function in these two different environments. It shows the government borrows more when lenders have the ability to become holdouts. Not coincidentally, the borrowing decisions differ the most in the states where the borrowing costs differ the most. A government with little debt does not borrow much and continue hence faces no default risk; in such case, as indicated in figure 4a, independent of litigation, the bond price pays only the risk-free rate, and consequently the borrowing decisions are similar in the two environments. But for more indebted governments, the presence of litigation can make further borrowing more accessible and hence lead to more debt accumulation.

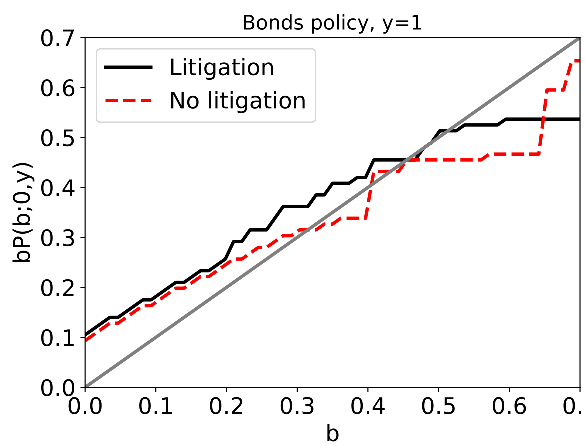
In a sense, these figures provide an intuitive illustration to the role of litigation in sovereign debt markets: it buys commitment to the government's borrowing decision.

Figure 4: The role of litigation in sovereign debt markets.

(a) Price schedule.



(b) Borrowing decisions.



I also perform the following exercise. I consider three different economies and, for each of them, simulate 5,000 periods and drop the first 500. First, I simulate the litigation-free and the litigation-prone economies and compute their average spreads and debt-to-GDP ratios. I find that the debt-to-GDP is 0.02pp higher in the litigation-prone and spreads are virtually the same. Of course, spreads are sensitive to the debt accumulation in these two different environments. Thus, I also consider an alternative setting. I simulate the litigation-free economy using the price schedule of the litigation-prone economy. In this alternative economy, spreads are 0.21pp lower than in the litigation-free economy, even though both share the same borrowing decision rule.

4.5 The role of collective action clauses

In this section, I consider the design of debt contracts for the Argentine economy. To quantify the welfare gains of transitioning from debt contracts that require unanimous decisions for changing payment terms to debt contracts with embedded CACs, I solve my model for many economies that share the same set of parameters described in the tables 1 and 2, except for the CAC threshold T. Then, I proceed as follows. First, I depart from the ergodic distribution of an economy that lacks CACs and simulate 2,000 periods of an economy with a CAC threshold T. Then, I compute its associated consumption path in these 2,000 periods and search for the optimal threshold T by varying it from 60% to 100% in jumps of 5pp. For each T, I repeat this procedure 200 times.

Table 3 summarizes the main findings. I report the welfare gain relative to an economy that continued to issue bonds with no CACs. Precisely, I compute the consumption compensation that would make the government indifferent to adding CACs to its debt contracts. The optimal clause has a threshold $T = 80\%$, and renders a welfare gain of 0.15%. This is not too far from the typical threshold present in sovereign debt contracts, $T = 75\%$, documented by Bradley and Gulati (2014), in which the welfare gain is only 0.03pp below the optimal contract.

Table 3: Welfare gains

Threshold T	Welfare gain
75%	0.12%
80%	0.15%
100%	0.00%

5 Conclusion

In this paper, I study the role of litigation and collective action clauses in sovereign debt markets. By introducing a holdout problem to an otherwise standard quantitative model, I show that litigation buys commitment to the government and hence facilitates borrowing. This framework rationalizes the empirical regularity on renegotiation outcomes that the restructured debt level is increasing in the amount of debt in default. In addition, it illustrates a consumption smoothing motive during restructuring episodes. Finally, the main finding of this paper is that a CAC with an 80% threshold is welfare improving for Argentina.

Further exploration of my framework may imply different countries can benefit from different regulations, especially the Heavily Indebted Poor Countries (HIPC). Countries with poor institutions, that are susceptible to more impatient governments with very short-term goals, may not benefit from litigation as countries like Argentina. Since their governments already borrow more than what their households prefer, litigation would drive them to further overborrow.

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