AMC12 Problems
2018

Problems

1. Kate bakes a 20-inch by 18-inch pan of cornbread. The cornbread is cut into pieces that measure 2 inches by 2 inches. How many pieces of cornbread does the pan contain?

   (A) 90 **(B)** 100 **(C)** 180 **(D)** 200 **(E)** 360

2. Sam drove 96 miles in 90 minutes. His average speed during the first 30 minutes was 60 mph (miles per hour), and his average speed during the second 30 minutes was 65 mph. What was his average speed, in mph, during the last 30 minutes?

   (A) 64 **(B)** 65 **(C)** 66 **(D)** 67 **(E)** 68

3. A line with slope 2 intersects a line with slope 6 at the point $(40, 30)$. What is the distance between the x-intercepts of these two lines?

   (A) 5 **(B)** 10 **(C)** 20 **(D)** 25 **(E)** 50

4. A circle has a chord of length 10, and the distance from the center of the circle to the chord is 5. What is the area of the circle?

   (A) 25π **(B)** 50π **(C)** 75π **(D)** 100π **(E)** 125π

5. How many subsets of $\{2, 3, 4, 5, 6, 7, 8, 9\}$ contain at least one prime number?

   (A) 128 **(B)** 192 **(C)** 224 **(D)** 240 **(E)** 256

6. Suppose S cans of soda can be purchased from a vending machine for Q quarters. Which of the following expressions describes the number of cans of soda that can be purchased for D dollars, where 1 dollar is worth 4 quarters?

   (A) $\frac{4DQ}{S}$ **(B)** $\frac{4DS}{Q}$ **(C)** $\frac{4Q}{DS}$ **(D)** $\frac{DQ}{4S}$ **(E)** $\frac{DS}{4Q}$

7. What is the value of
$$\log_3 7 \cdot \log_5 9 \cdot \log_7 11 \cdot \log_9 13 \cdots \log_{21} 25 \cdot \log_{23} 27?$$

   (A) 3 **(B)** $3\log_7 23$ **(C)** 6 **(D)** 9 **(E)** 10

8. Line segment $\overline{AB}$ is a diameter of a circle with $AB = 24$. Point C, not equal to A or B, lies on the circle. As point C moves around the circle, the centroid (center of mass) of $\triangle ABC$ traces out a closed curve missing two points. To the nearest positive integer, what is the area of the region bounded by this curve?

   (A) 25 **(B)** 38 **(C)** 50 **(D)** 63 **(E)** 75

9. What is
$$\sum_{i=1}^{100}\sum_{j=1}^{100}(i+j)?$$

   (A) 100,100 **(B)** 500,500 **(C)** 505,000 **(D)** 1,001,000 **(E)** 1,010,000

10. A list of 2018 positive integers has a unique mode, which occurs exactly 10 times. What is the least number of distinct values that can occur in the list?

    (A) 202 **(B)** 223 **(C)** 224 **(D)** 225 **(E)** 234

11. A closed box with a square base is to be wrapped with a square sheet of wrapping paper. The box is centered on the wrapping paper with the vertices of the base lying on the midlines of the square sheet of paper, as shown in the figure on the left. The four corners of the wrapping paper are to be folded up over the sides and brought together to meet at the center of the top of the box, point A in the figure on the right. The box has base length w and height h. What is the area of the sheet of wrapping paper?

    (A) $2(w+h)^2$ **(B)** $\frac{(w+h)^2}{2}$ **(C)** $2w^2 + 4wh$ **(D)** $2w^2$ **(E)** w^2h

12. Side $\overline{AB}$ of $\triangle ABC$ has length 10. The bisector of angle A meets $\overline{BC}$ at D, and $CD = 3$. The set of all possible values of AC is an open interval (m, n). What is $m + n$?

    (A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 20

13. Square $ABCD$ has side length 30. Point P lies inside the square so that $AP = 12$ and $BP = 26$. The centroids of $\triangle ABP$, $\triangle BCP$, $\triangle CDP$, and $\triangle DAP$ are the vertices of a convex quadrilateral. What is the area of that quadrilateral?

    (A) $100\sqrt{2}$ **(B)** $100\sqrt{3}$ **(C)** 200 **(D)** $200\sqrt{2}$ **(E)** $200\sqrt{3}$

14. Joey and Chloe and their daughter Zoe all have the same birthday. Joey is 1 year older than Chloe, and Zoe is exactly 1 year old today. Today is the first of the 9 birthdays on which Chloe's age will be an integral multiple of Zoe's age. What will be the sum of the two digits of Joey's age the next time his age is a multiple of Zoe's age?

    (A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

15. How many odd positive 3-digit integers are divisible by 3 but do not contain the digit 3?

    (A) 96 **(B)** 97 **(C)** 98 **(D)** 102 **(E)** 120

16. The solutions to the equation $(z+6)^8 = 81$ are connected in the complex plane to form a convex regular polygon, three of whose vertices are labeled A, B, and C. What is the least possible area of $\triangle ABC$?

    (A) $\frac{1}{6}\sqrt{6}$ **(B)** $\frac{3}{2}\sqrt{2} - \frac{3}{2}$ **(C)** $2\sqrt{3} - 3\sqrt{2}$ **(D)** $\frac{1}{2}\sqrt{2}$ **(E)** $\sqrt{3} - 1$

17. Let p and q be positive integers such that
$$\frac{5}{9} < \frac{p}{q} < \frac{4}{7}$$
    and q is as small as possible. What is $q - p$?

    (A) 7 **(B)** 11 **(C)** 13 **(D)** 17 **(E)** 19

18. A function f is defined recursively by $f(1) = f(2) = 1$ and
$$f(n) = f(n-1) - f(n-2) + n$$
    for all integers $n \geq 3$. What is $f(2018)$?

    (A) 2016 **(B)** 2017 **(C)** 2018 **(D)** 2019 **(E)** 2020

19. Mary chose an even 4-digit number n. She wrote down all the divisors of n in increasing order from left to right: $1, 2, \ldots, \dfrac{n}{2}, n$. At some moment Mary wrote 323 as a divisor of n. What is the smallest possible value of the next divisor written to the right of 323?

    (A) 324 **(B)** 330 **(C)** 340 **(D)** 361 **(E)** 646

20. Let $ABCDEF$ be a regular hexagon with side length 1. Denote by X, Y, and Z the midpoints of sides $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$, respectively. What is the area of the convex hexagon whose interior is the intersection of the interiors of $\triangle ACE$ and $\triangle XYZ$?

    (A) $\frac{3}{8}\sqrt{3}$ **(B)** $\frac{7}{16}\sqrt{3}$ **(C)** $\frac{15}{32}\sqrt{3}$ **(D)** $\frac{1}{2}\sqrt{3}$ **(E)** $\frac{9}{16}\sqrt{3}$

21. In $\triangle ABC$ with side lengths $AB = 13$, $AC = 12$, and $BC = 5$, let O and I denote the circumcenter and incenter, respectively. A circle with center M is tangent to the legs AC and BC and to the circumcircle of $\triangle ABC$. What is the area of $\triangle MOI$?

    (A) $\frac{5}{2}$ **(B)** $\frac{11}{4}$ **(C)** 3 **(D)** $\frac{13}{4}$ **(E)** $\frac{7}{2}$

22. Consider polynomials $P(x)$ of degree at most 3, each of whose coefficients is an element of $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. How many such polynomials satisfy $P(-1) = -9$?

    (A) 110 **(B)** 143 **(C)** 165 **(D)** 220 **(E)** 286

23. Ajay is standing at point A near Pontianak, Indonesia, $0°$ latitude and $110°$ E longitude. Billy is standing at point B near Big Baldy Mountain, Idaho, USA, $45°$ N latitude and $115°$ W longitude. Assume that Earth is a perfect sphere with center C. What is the degree measure of $\angle ACB$?

    (A) 105 **(B)** $112\frac{1}{2}$ **(C)** 120 **(D)** 135 **(E)** 150

24. Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. How many real numbers x satisfy the equation $x^2 + 10,000\lfloor x \rfloor = 10,000x$?

    (A) 197 **(B)** 198 **(C)** 199 **(D)** 200 **(E)** 201

25. Circles ω_1, ω_2, and ω_3 each have radius 4 and are placed in the plane so that each circle is externally tangent to the other two. Points P_1, P_2, and P_3 lie on ω_1, ω_2, and ω_3 respectively such that $P_1P_2 = P_2P_3 = P_3P_1$ and line P_iP_{i+1} is tangent to ω_i for each $i = 1, 2, 3$, where $P_4 = P_1$. See the figure below. The area of $\triangle P_1P_2P_3$ can be written in the form $\sqrt{a} + \sqrt{b}$ for positive integers a and b. What is $a + b$?

    (A) 546 **(B)** 548 **(C)** 550 **(D)** 552 **(E)** 554