AMC12 Problems
2010-2024

Contents

2010 AMC1212A

1. What is $(20-(2010-201))+(2010-(201-20))$?

   (A) -4020 **(B)** 0 **(C)** 40 **(D)** 401 **(E)** 4020

2. A ferry boat shuttles tourists to an island every hour starting at 10 AM until its last trip, which starts at 3 PM. One day the boat captain notes that on the 10 AM trip there were 100 tourists on the ferry boat, and that on each successive trip, the number of tourists was 1 fewer than on the previous trip. How many tourists did the ferry take to the island that day?

   (A) 585 **(B)** 594 **(C)** 672 **(D)** 679 **(E)** 694

3. Rectangle $ABCD$, pictured below, shares 50% of its area with square $EFGH$. Square $EFGH$ shares 20% of its area with rectangle $ABCD$. What is $\frac{AB}{AD}$?

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   (A) 4 **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 10

4. If $x<0$, then which of the following must be positive?

   (A) $\frac{x}{|x|}$ **(B)** $-x^2$ **(C)** -2^x **(D)** $-x^{-1}$ **(E)** $\sqrt[3]{x}$

5. Halfway through a 100-shot archery tournament, Chelsea leads by 50 points. For each shot a bullseye scores 10 points, with other possible scores being 8, 4, 2, and 0 points. Chelsea always scores at least 4 points on each shot. If Chelsea's next n shots are bullseyes she will be guaranteed victory. What is the minimum value for n?

   (A) 38 **(B)** 40 **(C)** 42 **(D)** 44 **(E)** 46

6. A *palindrome*, such as 83438, is a number that remains the same when its digits are reversed. The numbers x and $x+32$ are three-digit and four-digit palindromes, respectively. What is the sum of the digits of x?

   (A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

7. Logan is constructing a scaled model of his town. The city's water tower stands 40 meters high, and the top portion is a sphere that holds 100,000 liters of water. Logan's miniature water tower holds 0.1 liters. How tall, in meters, should Logan make his tower?

   (A) 0.04 **(B)** $\frac{0.4}{\pi}$ **(C)** 0.4 **(D)** $\frac{4}{\pi}$ **(E)** 4

8. Triangle ABC has $AB=2\cdot AC$. Let D and E be on $\overline{AB}$ and $\overline{BC}$, respectively, such that $\angle BAE=\angle ACD$. Let F be the intersection of segments AE and CD, and suppose that $\triangle CFE$ is equilateral. What is $\angle ACB$?

   (A) 60° **(B)** 75° **(C)** 90° **(D)** 105° **(E)** 120°

9. A solid cube has side length 3 inches. A 2-inch by 2-inch square hole is cut into the center of each face. The edges of each cut are parallel to the edges of the cube, and each hole goes all the way through the cube. What is the volume, in cubic inches, of the remaining solid?

   (A) 7 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 15

10. The first four terms of an arithmetic sequence are p, 9, $3p-q$, and $3p+q$. What is the 2010^{th} term of this sequence?

    (A) 8041 **(B)** 8043 **(C)** 8045 **(D)** 8047 **(E)** 8049

11. The solution of the equation $7^{x+7}=8^x$ can be expressed in the form $x=\log_b 7^7$. What is b?

    (A) $\frac{7}{15}$ **(B)** $\frac{7}{8}$ **(C)** $\frac{8}{7}$ **(D)** $\frac{15}{8}$ **(E)** $\frac{15}{7}$

12. In a magical swamp there are two species of talking amphibians: toads, whose statements are always true, and frogs, whose statements are always false. Four amphibians, Brian, Chris, LeRoy, and Mike live together in this swamp, and they make the following statements.

    Brian: "Mike and I are different species."

    Chris: "LeRoy is a frog."

    LeRoy: "Chris is a frog."

    Mike: "Of the four of us, at least two are toads."

    How many of these amphibians are frogs?

    (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

13. For how many integer values of k do the graphs of $x^2 + y^2 = k^2$ and $xy = k$ not intersect?

    (A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 8

14. Nondegenerate $\triangle ABC$ has integer side lengths, $\overline{BD}$ is an angle bisector, $AD = 3$, and $DC = 8$. What is the smallest possible value of the perimeter?

    (A) 30 **(B)** 33 **(C)** 35 **(D)** 36 **(E)** 37

15. A coin is altered so that the probability that it lands on heads is less than $\frac{1}{2}$ and when the coin is flipped four times, the probability of an equal number of heads and tails is $\frac{1}{6}$. What is the probability that the coin lands on heads?

    (A) $\frac{\sqrt{15}-3}{6}$ **(B)** $\frac{6-\sqrt{6\sqrt{6}+2}}{12}$ **(C)** $\frac{\sqrt{2}-1}{2}$ **(D)** $\frac{3-\sqrt{3}}{6}$ **(E)** $\frac{\sqrt{3}-1}{2}$

16. Bernardo randomly picks 3 distinct numbers from the set $\{1, 2, 3, ..., 7, 8, 9\}$ and arranges them in descending order to form a 3-digit number. Silvia randomly picks 3 distinct numbers from the set $\{1, 2, 3, ..., 6, 7, 8\}$ and also arranges them in descending order to form a 3-digit number. What is the probability that Bernardo's number is larger than Silvia's number?

    (A) $\frac{47}{72}$ **(B)** $\frac{37}{56}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{49}{72}$ **(E)** $\frac{39}{56}$

17. Equiangular hexagon $ABCDEF$ has side lengths $AB = CD = EF = 1$ and $BC = DE = FA = r$. The area of $\triangle ACE$ is 70% of the area of the hexagon. What is the sum of all possible values of r?

    (A) $\frac{4\sqrt{3}}{3}$ **(B)** $\frac{10}{3}$ **(C)** 4 **(D)** $\frac{17}{4}$ **(E)** 6

18. A 16-step path is to go from $(-4, -4)$ to $(4, 4)$ with each step increasing either the x-coordinate or the y-coordinate by 1. How many such paths stay outside or on the boundary of the square $-2 \le x \le 2$, $-2 \le y \le 2$ at each step?

    (A) 92 **(B)** 144 **(C)** 1568 **(D)** 1698 **(E)** $12,800$

19. Each of 2010 boxes in a line contains a single red marble, and for $1 \le k \le 2010$, the box in the kth position also contains k white marbles. Isabella begins at the first box and successively draws a single marble at random from each box, in order. She stops when she first draws a red marble. Let $P(n)$ be the probability that Isabella stops after drawing exactly n marbles. What is the smallest value of n for which $P(n) < \frac{1}{2010}$?

    (A) 45 **(B)** 63 **(C)** 64 **(D)** 201 **(E)** 1005

20. Arithmetic sequences (a_n) and (b_n) have integer terms with $a_1 = b_1 = 1 < a_2 \le b_2$ and $a_n b_n = 2010$ for some n. What is the largest possible value of n?

    (A) 2 **(B)** 3 **(C)** 8 **(D)** 288 **(E)** 2009

21. The graph of $y = x^6 - 10x^5 + 29x^4 - 4x^3 + ax^2$ lies above the line $y = bx + c$ except at three values of x, where the graph and the line intersect. What is the largest of these values?

    (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

22. What is the minimum value of $|x - 1| + |2x - 1| + |3x - 1| + \cdots + |119x - 1|$?

    (A) 49 **(B)** 50 **(C)** 51 **(D)** 52 **(E)** 53

23. The number obtained from the last two nonzero digits of $90!$ is equal to n. What is n?

    (A) 12 **(B)** 32 **(C)** 48 **(D)** 52 **(E)** 68

24. Let $f(x) = \log_{10}\left(\sin(\pi x) \cdot \sin(2\pi x) \cdot \sin(3\pi x) \cdots \sin(8\pi x)\right)$. The intersection of the domain of $f(x)$ with the interval $[0, 1]$ is a union of n disjoint open intervals. What is n?

    (A) 2 **(B)** 12 **(C)** 18 **(D)** 22 **(E)** 36

25. Two quadrilaterals are considered the same if one can be obtained from the other by a rotation and a translation. How many different convex cyclic quadrilaterals are there with integer sides and perimeter equal to 32?

    (A) 560 **(B)** 564 **(C)** 568 **(D)** 1498 **(E)** 2255

2010 AMC1212B

1. Makarla attended two meetings during her 9-hour work day. The first meeting took 45 minutes and the second meeting took twice as long. What percent of her work day was spent attending meetings?

   (A) 15 **(B)** 20 **(C)** 25 **(D)** 30 **(E)** 35

2. A big L is formed as shown. What is its area?

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   (A) 22 **(B)** 24 **(C)** 26 **(D)** 28 **(E)** 30

3. A ticket to a school play cost x dollars, where x is a whole number. A group of 9¡sup¿th¡/sup¿ graders buys tickets costing a total of \$48, and a group of 10¡sup¿th¡/sup¿ graders buys tickets costing a total of \$64. How many values for x are possible?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

4. A month with 31 days has the same number of Mondays and Wednesdays. How many of the seven days of the week could be the first day of this month?

   (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

5. Lucky Larry's teacher asked him to substitute numbers for a, b, c, d, and e in the expression $a-(b-(c-(d+e)))$ and evaluate the result. Larry ignored the parentheses but added and subtracted correctly and obtained the correct result by coincidence. The numbers Larry substituted for a, b, c, and d were 1, 2, 3, and 4, respectively. What number did Larry substitute for e?

   (A) -5 **(B)** -3 **(C)** 0 **(D)** 3 **(E)** 5

6. At the beginning of the school year, 50% of all students in Mr. Wells' math class answered "Yes" to the question "Do you love math", and 50% answered "No." At the end of the school year, 70% answered "Yes" and 30% answered "No." Altogether, $x\%$ of the students gave a different answer at the beginning and end of the school year. What is the difference between the maximum and the minimum possible values of x?

   (A) 0 **(B)** 20 **(C)** 40 **(D)** 60 **(E)** 80

7. Shelby drives her scooter at a speed of 30 miles per hour if it is not raining, and 20 miles per hour if it is raining. Today she drove in the sun in the morning and in the rain in the evening, for a total of 16 miles in 40 minutes. How many minutes did she drive in the rain?

   (A) 18 **(B)** 21 **(C)** 24 **(D)** 27 **(E)** 30

8. Every high school in the city of Euclid sent a team of 3 students to a math contest. Each participant in the contest received a different score. Andrea's score was the median among all students, and hers was the highest score on her team. Andrea's teammates Beth and Carla placed 37¡sup¿th¡/sup¿ and 64¡sup¿th¡/sup¿, respectively. How many schools are in the city?

   (A) 22 **(B)** 23 **(C)** 24 **(D)** 25 **(E)** 26

9. Let n be the smallest positive integer such that n is divisible by 20, n^2 is a perfect cube, and n^3 is a perfect square. What is the number of digits of n?

   (A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

10. The average of the numbers $1, 2, 3, \cdots, 98, 99,$ and x is $100x$. What is x?

    (A) $\frac{49}{101}$ **(B)** $\frac{50}{101}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{51}{101}$ **(E)** $\frac{50}{99}$

11. A palindrome between 1000 and $10,000$ is chosen at random. What is the probability that it is divisible by 7?

    (A) $\frac{1}{10}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{1}{7}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{5}$

12. For what value of x does

$$\log_{\sqrt{2}} \sqrt{x} + \log_2 x + \log_4 x^2 + \log_8 x^3 + \log_{16} x^4 = 40?$$

(A) 8 **(B)** 16 **(C)** 32 **(D)** 256 **(E)** 1024

13. In $\triangle ABC$, $\cos(2A - B) + \sin(A + B) = 2$ and $AB = 4$. What is BC?

(A) $\sqrt{2}$ **(B)** $\sqrt{3}$ **(C)** 2 **(D)** $2\sqrt{2}$ **(E)** $2\sqrt{3}$

14. Let a, b, c, d, and e be positive integers with $a+b+c+d+e = 2010$ and let M be the largest of the sums $a+b$, $b+c$, $c+d$ and $d+e$. What is the smallest possible value of M?

(A) 670 **(B)** 671 **(C)** 802 **(D)** 803 **(E)** 804

15. For how many ordered triples (x, y, z) of nonnegative integers less than 20 are there exactly two distinct elements in the set $\{i^x, (1+i)^y, z\}$, where $i = \sqrt{-1}$?

(A) 149 **(B)** 205 **(C)** 215 **(D)** 225 **(E)** 235

16. Positive integers a, b, and c are randomly and independently selected with replacement from the set $\{1, 2, 3, \dots, 2010\}$. What is the probability that $abc + ab + a$ is divisible by 3?

(A) $\frac{1}{3}$ **(B)** $\frac{29}{81}$ **(C)** $\frac{31}{81}$ **(D)** $\frac{11}{27}$ **(E)** $\frac{13}{27}$

17. The entries in a 3×3 array include all the digits from 1 through 9, arranged so that the entries in every row and column are in increasing order. How many such arrays are there?

(A) 18 **(B)** 24 **(C)** 36 **(D)** 42 **(E)** 60

18. A frog makes 3 jumps, each exactly 1 meter long. The directions of the jumps are chosen independently at random. What is the probability that the frog's final position is no more than 1 meter from its starting position?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

19. A high school basketball game between the Raiders and Wildcats was tied at the end of the first quarter. The number of points scored by the Raiders in each of the four quarters formed an increasing geometric sequence, and the number of points scored by the Wildcats in each of the four quarters formed an increasing arithmetic sequence. At the end of the fourth quarter, the Raiders had won by one point. Neither team scored more than 100 points. What was the total number of points scored by the two teams in the first half?

(A) 30 **(B)** 31 **(C)** 32 **(D)** 33 **(E)** 34

20. A geometric sequence (a_n) has $a_1 = \sin x$, $a_2 = \cos x$, and $a_3 = \tan x$ for some real number x. For what value of n does $a_n = 1 + \cos x$?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

21. Let $a > 0$, and let $P(x)$ be a polynomial with integer coefficients such that

¡center¿ $P(1) = P(3) = P(5) = P(7) = a$, and¡br/¿ $P(2) = P(4) = P(6) = P(8) = -a$. ¡/center¿

What is the smallest possible value of a?

(A) 105 **(B)** 315 **(C)** 945 **(D)** $7!$ **(E)** $8!$

22. Let $ABCD$ be a cyclic quadrilateral. The side lengths of $ABCD$ are distinct integers less than 15 such that $BC \cdot CD = AB \cdot DA$. What is the largest possible value of BD?

(A) $\sqrt{\frac{325}{2}}$ **(B)** $\sqrt{185}$ **(C)** $\sqrt{\frac{389}{2}}$ **(D)** $\sqrt{\frac{425}{2}}$ **(E)** $\sqrt{\frac{533}{2}}$

23. Monic quadratic polynomials $P(x)$ and $Q(x)$ have the property that $P(Q(x))$ has zeros at $x = -23, -21, -17$, and -15, and $Q(P(x))$ has zeros at $x = -59, -57, -51$ and -49. What is the sum of the minimum values of $P(x)$ and $Q(x)$?

    (A) -100 **(B)** -82 **(C)** -73 **(D)** -64 **(E)** 0

24. The set of real numbers x for which

$$\frac{1}{x-2009} + \frac{1}{x-2010} + \frac{1}{x-2011} \geq 1$$

is the union of intervals of the form $a < x \leq b$. What is the sum of the lengths of these intervals?

(A) $\frac{1003}{335}$ **(B)** $\frac{1004}{335}$ **(C)** 3 **(D)** $\frac{403}{134}$ **(E)** $\frac{202}{67}$

25. For every integer $n \geq 2$, let $\text{pow}(n)$ be the largest power of the largest prime that divides n. For example $\text{pow}(144) = \text{pow}(2^4 \cdot 3^2) = 3^2$. What is the largest integer m such that 2010^m divides

¡center¿ $\prod_{n=2}^{5300} \text{pow}(n)$? ¡/center¿

(A) 74 **(B)** 75 **(C)** 76 **(D)** 77 **(E)** 78

2011 AMC1212A

1. A cell phone plan costs \$20 dollars each month, plus 5 cents per text message sent, plus 10 cents for each minute used over 30 hours. In January Michelle sent 100 text messages and talked for 30.5 hours. How much did she have to pay? **(A)** 24.00 **(B)** 24.50 **(C)** 25.50 **(D)** 28.00 **(E)** 30.00

2. There are 5 coins placed flat on a table according to the figure. What is the order of the coins from top to bottom?

   (A) (C, A, E, D, B) **(B)** (C, A, D, E, B) **(C)** (C, D, E, A, B) **(D)** (C, E, A, D, B) **(E)** (C, E, D, A, B)

3. A small bottle of shampoo can hold 35 milliliters of shampoo, whereas a large bottle can hold 500 milliliters of shampoo. Jasmine wants to buy the minimum number of small bottles necessary to completely fill a large bottle. How many bottles must she buy?

   (A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

4. At an elementary school, the students in third grade, fourth grade, and fifth grade run an average of 12, 15, and 10 minutes per day, respectively. There are twice as many third graders as fourth graders, and twice as many fourth graders as fifth graders. What is the average number of minutes run per day by these students?

   (A) 12 **(B)** $\frac{37}{3}$ **(C)** $\frac{88}{7}$ **(D)** 13 **(E)** 14

5. Last summer 30% of the birds living on Town Lake were geese, 25% were swans, 10% were herons, and 35% were ducks. What percent of the birds that were not swans were geese?

   (A) 20 **(B)** 30 **(C)** 40 **(D)** 50 **(E)** 60

6. The players on a basketball team made some three-point shots, some two-point shots, and some one-point free throws. They scored as many points with two-point shots as with three-point shots. Their number of successful free throws was one more than their number of successful two-point shots. The team's total score was 61 points. How many free throws did they make?

   (A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

7. A majority of the 30 students in Ms. Demeanor's class bought pencils at the school bookstore. Each of these students bought the same number of pencils, and this number was greater than 1. The cost of a pencil in cents was greater than the number of pencils each student bought, and the total cost of all the pencils was 17.71. What was the cost of a pencil in cents?

   (A) 7 **(B)** 11 **(C)** 17 **(D)** 23 **(E)** 77

8. In the eight term sequence A, B, C, D, E, F, G, H, the value of C is 5 and the sum of any three consecutive terms is 30. What is $A + H$?

   (A) 17 **(B)** 18 **(C)** 25 **(D)** 26 **(E)** 43

9. At a twins and triplets convention, there were 9 sets of twins and 6 sets of triplets, all from different families. Each twin shook hands with all the twins except his/her siblings and with half the triplets. Each triplet shook hands with all the triplets except his/her siblings and with half the twins. How many handshakes took place?

   (A) 324 **(B)** 441 **(C)** 630 **(D)** 648 **(E)** 882

10. A pair of standard 6-sided dice is rolled once. The sum of the numbers rolled determines the diameter of a circle. What is the probability that the numerical value of the area of the circle is less than the numerical value of the circle's circumference?

    (A) $\frac{1}{36}$ **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{5}{18}$

11. Circles A, B, and C each have radius 1. Circles A and B share one point of tangency. Circle C has a point of tangency with the midpoint of $\overline{AB}$. What is the area inside circle C but outside circle A and circle B?

(A) $3 - \frac{\pi}{2}$ **(B)** $\frac{\pi}{2}$ **(C)** 2 **(D)** $\frac{3\pi}{4}$ **(E)** $1 + \frac{\pi}{2}$

12. A power boat and a raft both left dock A on a river and headed downstream. The raft drifted at the speed of the river current. The power boat maintained a constant speed with respect to the river. The power boat reached dock B downriver, then immediately turned and traveled back upriver. It eventually met the raft on the river 9 hours after leaving dock A. How many hours did it take the power boat to go from A to B?

(A) 3 **(B)** 3.5 **(C)** 4 **(D)** 4.5 **(E)** 5

13. Triangle ABC has side-lengths $AB = 12, BC = 24$, and $AC = 18$. The line through the incenter of $\triangle ABC$ parallel to $\overline{BC}$ intersects $\overline{AB}$ at M and $\overline{AC}$ at N. What is the perimeter of $\triangle AMN$?

(A) 27 **(B)** 30 **(C)** 33 **(D)** 36 **(E)** 42

14. Suppose a and b are single-digit positive integers chosen independently and at random. What is the probability that the point (a, b) lies above the parabola $y = ax^2 - bx$?

(A) $\frac{11}{81}$ **(B)** $\frac{13}{81}$ **(C)** $\frac{5}{27}$ **(D)** $\frac{17}{81}$ **(E)** $\frac{19}{81}$

15. The circular base of a hemisphere of radius 2 rests on the base of a square pyramid of height 6. The hemisphere is tangent to the other four faces of the pyramid. What is the edge-length of the base of the pyramid?

(A) $3\sqrt{2}$ **(B)** $\frac{13}{3}$ **(C)** $4\sqrt{2}$ **(D)** 6 **(E)** $\frac{13}{2}$

16. Each vertex of convex polygon $ABCDE$ is to be assigned a color. There are 6 colors to choose from, and the ends of each diagonal must have different colors. How many different colorings are possible?

(A) 2520 **(B)** 2880 **(C)** 3120 **(D)** 3250 **(E)** 3750

17. Circles with radii 1, 2, and 3 are mutually externally tangent. What is the area of the triangle determined by the points of tangency?

(A) $\frac{3}{5}$ **(B)** $\frac{4}{5}$ **(C)** 1 **(D)** $\frac{6}{5}$ **(E)** $\frac{4}{3}$

18. Suppose that $|x + y| + |x - y| = 2$. What is the maximum possible value of $x^2 - 6x + y^2$?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

19. At a competition with N players, the number of players given elite status is equal to $2^{1+\lfloor \log_2(N-1) \rfloor} - N$. Suppose that 19 players are given elite status. What is the sum of the two smallest possible values of N?

(A) 38 **(B)** 90 **(C)** 154 **(D)** 406 **(E)** 1024

20. Let $f(x) = ax^2 + bx + c$, where a, b, and c are integers. Suppose that $f(1) = 0$, $50 < f(7) < 60$, $70 < f(8) < 80$, $5000k < f(100) < 5000(k+1)$ for some integer k. What is k?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

21. Let $f_1(x) = \sqrt{1-x}$, and for integers $n \geq 2$, let $f_n(x) = f_{n-1}(\sqrt{n^2 - x})$. If N is the largest value of n for which the domain of f_n is nonempty, the domain of f_N is $\{c\}$. What is $N + c$?

(A) -226 **(B)** -144 **(C)** -20 **(D)** 20 **(E)** 144

22. Let R be a unit square region and $n \geq 4$ an integer. A point X in the interior of R is called "n-ray partitional" if there are n rays emanating from X that divide R into n triangles of equal area. How many points are 100-ray partitional but not 60-ray partitional?

(A) 1500 **(B)** 1560 **(C)** 2320 **(D)** 2480 **(E)** 2500

23. Let $f(z) = \frac{z+a}{z+b}$ and $g(z) = f(f(z))$, where a and b are complex numbers. Suppose that $|a| = 1$ and $g(g(z)) = z$ for all z for which $g(g(z))$ is defined. What is the difference between the largest and smallest possible values of $|b|$?

(A) 0 **(B)** $\sqrt{2} - 1$ **(C)** $\sqrt{3} - 1$ **(D)** 1 **(E)** 2

24. Consider all quadrilaterals $ABCD$ such that $AB = 14$, $BC = 9$, $CD = 7$, and $DA = 12$. What is the radius of the largest possible circle that fits inside or on the boundary of such a quadrilateral?

    (A) $\sqrt{15}$ **(B)** $\sqrt{21}$ **(C)** $2\sqrt{6}$ **(D)** 5 **(E)** $2\sqrt{7}$

25. Triangle ABC has $\angle BAC = 60^\circ$, $\angle CBA \leq 90^\circ$, $BC = 1$, and $AC \geq AB$. Let H, I, and O be the orthocenter, incenter, and circumcenter of $\triangle ABC$, respectively. Assume that the area of pentagon $BCOIH$ is the maximum possible. What is $\angle CBA$?

    (A) 60° **(B)** 72° **(C)** 75° **(D)** 80° **(E)** 90°

2011 AMC1212B

1. What is ¡center¿$\frac{2+4+6}{1+3+5} - \frac{1+3+5}{2+4+6}$?¡/center¿

   (A) -1 **(B)** $\frac{5}{36}$ **(C)** $\frac{7}{12}$ **(D)** $\frac{147}{60}$ **(E)** $\frac{43}{3}$

2. Josanna's test scores to date are 90, 80, 70, 60, and 85. Her goal is to raise her test average at least 3 points with her next test. What is the minimum test score she would need to accomplish this goal?

   (A) 80 **(B)** 82 **(C)** 85 **(D)** 90 **(E)** 95

3. LeRoy and Bernardo went on a week-long trip together and agreed to share the costs equally. Over the week, each of them paid for various joint expenses such as gasoline and car rental. At the end of the trip it turned out that LeRoy had paid A dollars and Bernardo had paid B dollars, where $A < B$. How many dollars must LeRoy give to Bernardo so that they share the costs equally?

   (A) $\frac{A+B}{2}$ **(B)** $\frac{A-B}{2}$ **(C)** $\frac{B-A}{2}$ **(D)** $B - A$ **(E)** $A + B$

4. In multiplying two positive integers a and b, Ron reversed the digits of the two-digit number a. His erroneous product was 161. What is the correct value of the product of a and b?

   (A) 116 **(B)** 161 **(C)** 204 **(D)** 214 **(E)** 224

5. Let N be the second smallest positive integer that is divisible by every positive integer less than 7. What is the sum of the digits of N?

   (A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 9

6. Two tangents to a circle are drawn from a point A. The points of contact B and C divide the circle into arcs with lengths in the ratio $2 : 3$. What is the degree measure of $\angle BAC$?

   (A) 24 **(B)** 30 **(C)** 36 **(D)** 48 **(E)** 60

7. Let x and y be two-digit positive integers with mean 60. What is the maximum value of the ratio $\frac{x}{y}$?

   (A) 3 **(B)** $\frac{33}{7}$ **(C)** $\frac{39}{7}$ **(D)** 9 **(E)** $\frac{99}{10}$

8. Keiko walks once around a track at exactly the same constant speed every day. The sides of the track are straight, and the ends are semicircles. The track has width 6 meters, and it takes her 36 seconds longer to walk around the outside edge of the track than around the inside edge. What is Keiko's speed in meters per second?

   (A) $\frac{\pi}{3}$ **(B)** $\frac{2\pi}{3}$ **(C)** π **(D)** $\frac{4\pi}{3}$ **(E)** $\frac{5\pi}{3}$

9. Two real numbers are selected independently and at random from the interval $[-20, 10]$. What is the probability that the product of those numbers is greater than zero?

   (A) $\frac{1}{9}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{4}{9}$ **(D)** $\frac{5}{9}$ **(E)** $\frac{2}{3}$

10. Rectangle $ABCD$ has $AB = 6$ and $BC = 3$. Point M is chosen on side AB so that $\angle AMD = \angle CMD$. What is the degree measure of $\angle AMD$?

    (A) 15 **(B)** 30 **(C)** 45 **(D)** 60 **(E)** 75

11. A frog located at (x, y), with both x and y integers, makes successive jumps of length 5 and always lands on points with integer coordinates. Suppose that the frog starts at $(0, 0)$ and ends at $(1, 0)$. What is the smallest possible number of jumps the frog makes?

    (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

12. A dart board is a regular octagon divided into regions as shown below. Suppose that a dart thrown at the board is equally likely to land anywhere on the board. What is the probability that the dart lands within the center square?

    (A) $\frac{\sqrt{2}-1}{2}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{2-\sqrt{2}}{2}$ **(D)** $\frac{\sqrt{2}}{4}$ **(E)** $2 - \sqrt{2}$

13. Brian writes down four integers $w > x > y > z$ whose sum is 44. The pairwise positive differences of these numbers are $1, 3, 4, 5, 6$ and 9. What is the sum of the possible values of w?

(A) 16 **(B)** 31 **(C)** 48 **(D)** 62 **(E)** 93

14. A segment through the focus F of a parabola with vertex V is perpendicular to $\overline{FV}$ and intersects the parabola in points A and B. What is $\cos(\angle AVB)$?

(A) $-\frac{3\sqrt{5}}{7}$ **(B)** $-\frac{2\sqrt{5}}{5}$ **(C)** $-\frac{4}{5}$ **(D)** $-\frac{3}{5}$ **(E)** $-\frac{1}{2}$

15. How many positive two-digit integers are factors of $2^{24} - 1$?

(A) 4 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 14

16. Rhombus $ABCD$ has side length 2 and $\angle B = 120°$. Region R consists of all points inside of the rhombus that are closer to vertex B than any of the other three vertices. What is the area of R?

(A) $\frac{\sqrt{3}}{3}$ **(B)** $\frac{\sqrt{3}}{2}$ **(C)** $\frac{2\sqrt{3}}{3}$ **(D)** $1 + \frac{\sqrt{3}}{3}$ **(E)** 2

17. Let $f(x) = 10^{10x}, g(x) = \log_{10}\left(\frac{x}{10}\right), h_1(x) = g(f(x))$, and $h_n(x) = h_1(h_{n-1}(x))$ for integers $n \geq 2$. What is the sum of the digits of $h_{2011}(1)$?

(A) 16081 **(B)** 16089 **(C)** 18089 **(D)** 18098 **(E)** 18099

18. A pyramid has a square base with side of length 1 and has lateral faces that are equilateral triangles. A cube is placed within the pyramid so that one face is on the base of the pyramid and its opposite face has all its edges on the lateral faces of the pyramid. What is the volume of this cube?

(A) $5\sqrt{2} - 7$ **(B)** $7 - 4\sqrt{3}$ **(C)** $\frac{2\sqrt{2}}{27}$ **(D)** $\frac{\sqrt{2}}{9}$ **(E)** $\frac{\sqrt{3}}{9}$

19. A lattice point in an xy-coordinate system is any point (x, y) where both x and y are integers. The graph of $y = mx + 2$ passes through no lattice point with $0 < x \leq 100$ for all m such that $\frac{1}{2} < m < a$. What is the maximum possible value of a?

(A) $\frac{51}{101}$ **(B)** $\frac{50}{99}$ **(C)** $\frac{51}{100}$ **(D)** $\frac{52}{101}$ **(E)** $\frac{13}{25}$

20. Triangle ABC has $AB = 13, BC = 14$, and $AC = 15$. The points D, E, and F are the midpoints of $\overline{AB}, \overline{BC}$, and $\overline{AC}$ respectively. Let $X \neq E$ be the intersection of the circumcircles of ΔBDE and ΔCEF. What is $XA + XB + XC$?

(A) 24 **(B)** $14\sqrt{3}$ **(C)** $\frac{195}{8}$ **(D)** $\frac{129\sqrt{7}}{14}$ **(E)** $\frac{69\sqrt{2}}{4}$

21. The arithmetic mean of two distinct positive integers x and y is a two-digit integer. The geometric mean of x and y is obtained by reversing the digits of the arithmetic mean. What is $|x - y|$?

(A) 24 **(B)** 48 **(C)** 54 **(D)** 66 **(E)** 70

22. Let T_1 be a triangle with side lengths $2011, 2012$, and 2013. For $n \geq 1$, if $T_n = \Delta ABC$ and D, E, and F are the points of tangency of the incircle of ΔABC to the sides AB, BC, and AC, respectively, then T_{n+1} is a triangle with side lengths AD, BE, and CF, if it exists. What is the perimeter of the last triangle in the sequence (T_n)?

(A) $\frac{1509}{8}$ **(B)** $\frac{1509}{32}$ **(C)** $\frac{1509}{64}$ **(D)** $\frac{1509}{128}$ **(E)** $\frac{1509}{256}$

23. A bug travels in the coordinate plane, moving only along the lines that are parallel to the x-axis or y-axis. Let $A = (-3, 2)$ and $B = (3, -2)$. Consider all possible paths of the bug from A to B of length at most 20. How many points with integer coordinates lie on at least one of these paths?

(A) 161 **(B)** 185 **(C)** 195 **(D)** 227 **(E)** 255

24. Let $P(z) = z^8 + \left(4\sqrt{3} + 6\right)z^4 - \left(4\sqrt{3} + 7\right)$. What is the minimum perimeter among all the 8-sided polygons in the complex plane whose vertices are precisely the zeros of $P(z)$?

(A) $4\sqrt{3} + 4$ **(B)** $8\sqrt{2}$ **(C)** $3\sqrt{2} + 3\sqrt{6}$ **(D)** $4\sqrt{2} + 4\sqrt{3}$ **(E)** $4\sqrt{3} + 6$

25. For every m and k integers with k odd, denote by $\left[\frac{m}{k}\right]$ the integer closest to $\frac{m}{k}$. For every odd integer k, let $P(k)$ be the probability that

$$\left[\frac{n}{k}\right] + \left[\frac{100-n}{k}\right] = \left[\frac{100}{k}\right]$$

for an integer n randomly chosen from the interval $1 \leq n \leq 99!$. What is the minimum possible value of $P(k)$ over the odd integers k in the interval $1 \leq k \leq 99$?

(A) $\frac{1}{2}$ **(B)** $\frac{50}{99}$ **(C)** $\frac{44}{87}$ **(D)** $\frac{34}{67}$ **(E)** $\frac{7}{13}$

2012 AMC1212A

1. A bug crawls along a number line, starting at -2. It crawls to -6, then turns around and crawls to 5. How many units does the bug crawl altogether?

   (A) 9 **(B)** 11 **(C)** 13 **(D)** 14 **(E)** 15

2. Cagney can frost a cupcake every 20 seconds and Lacey can frost a cupcake every 30 seconds. Working together, how many cupcakes can they frost in 5 minutes?

   (A) 10 **(B)** 15 **(C)** 20 **(D)** 25 **(E)** 30

3. A box 2 centimeters high, 3 centimeters wide, and 5 centimeters long can hold 40 grams of clay. A second box with twice the height, three times the width, and the same length as the first box can hold n grams of clay. What is n?

   (A) 120 **(B)** 160 **(C)** 200 **(D)** 240 **(E)** 280

4. In a bag of marbles, $\frac{3}{5}$ of the marbles are blue and the rest are red. If the number of red marbles is doubled and the number of blue marbles stays the same, what fraction of the marbles will be red?

   (A) $\dfrac{2}{5}$ **(B)** $\dfrac{3}{7}$ **(C)** $\dfrac{4}{7}$ **(D)** $\dfrac{3}{5}$ **(E)** $\dfrac{4}{5}$

5. A fruit salad consists of blueberries, raspberries, grapes, and cherries. The fruit salad has a total of 280 pieces of fruit. There are twice as many raspberries as blueberries, three times as many grapes as cherries, and four times as many cherries as raspberries. How many cherries are there in the fruit salad?

   (A) 8 **(B)** 16 **(C)** 25 **(D)** 64 **(E)** 96

6. The sums of three whole numbers taken in pairs are 12, 17, and 19. What is the middle number?

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

7. Mary divides a circle into 12 sectors. The central angles of these sectors, measured in degrees, are all integers and they form an arithmetic sequence. What is the degree measure of the smallest possible sector angle?

   (A) 5 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

8. An "iterative average" of the numbers 1, 2, 3, 4, and 5 is computed in the following way. Arrange the five numbers in some order. Find the mean of the first two numbers, then find the mean of that with the third number, then the mean of that with the fourth number, and finally the mean of that with the fifth number. What is the difference between the largest and smallest possible values that can be obtained using this procedure?

   (A) $\frac{31}{16}$ **(B)** 2 **(C)** $\frac{17}{8}$ **(D)** 3 **(E)** $\frac{65}{16}$

9. A year is a leap year if and only if the year number is divisible by 400 (such as 2000) or is divisible by 4 but not by 100 (such as 2012). The 200th anniversary of the birth of novelist Charles Dickens was celebrated on February 7, 2012, a Tuesday. On what day of the week was Dickens born?

   (A) Friday **(B)** Saturday **(C)** Sunday **(D)** Monday **(E)** Tuesday

10. A triangle has area 30, one side of length 10, and the median to that side of length 9. Let θ be the acute angle formed by that side and the median. What is $\sin\theta$?

    (A) $\frac{3}{10}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{9}{20}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{9}{10}$

11. Alex, Mel, and Chelsea play a game that has 6 rounds. In each round there is a single winner, and the outcomes of the rounds are independent. For each round the probability that Alex wins is $\frac{1}{2}$, and Mel is twice as likely to win as Chelsea. What is the probability that Alex wins three rounds, Mel wins two rounds, and Chelsea wins one round?

    (A) $\frac{5}{72}$ **(B)** $\frac{5}{36}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{3}$ **(E)** 1

12. A square region $ABCD$ is externally tangent to the circle with equation $x^2+y^2=1$ at the point $(0,1)$ on the side CD. Vertices A and B are on the circle with equation $x^2+y^2=4$. What is the side length of this square?

(A) $\frac{\sqrt{10}+5}{10}$ **(B)** $\frac{2\sqrt{5}}{5}$ **(C)** $\frac{2\sqrt{2}}{3}$ **(D)** $\frac{2\sqrt{19}-4}{5}$ **(E)** $\frac{9-\sqrt{17}}{5}$

13. Paula the painter and her two helpers each paint at constant, but different, rates. They always start at 8:00 AM, and all three always take the same amount of time to eat lunch. On Monday the three of them painted 50% of a house, quitting at 4:00 PM. On Tuesday, when Paula wasn't there, the two helpers painted only 24% of the house and quit at 2:12 PM. On Wednesday Paula worked by herself and finished the house by working until 7:12 PM. How long, in minutes, was each day's lunch break?

(A) 30 **(B)** 36 **(C)** 42 **(D)** 48 **(E)** 60

14. The closed curve in the figure is made up of 9 congruent circular arcs each of length $\frac{2\pi}{3}$, where each of the centers of the corresponding circles is among the vertices of a regular hexagon of side 2. What is the area enclosed by the curve?

(A) $2\pi+6$ **(B)** $2\pi+4\sqrt{3}$ **(C)** $3\pi+4$ **(D)** $2\pi+3\sqrt{3}+2$ **(E)** $\pi+6\sqrt{3}$

15. A 3×3 square is partitioned into 9 unit squares. Each unit square is painted either white or black with each color being equally likely, chosen independently and at random. The square is then rotated 90° clockwise about its center, and every white square in a position formerly occupied by a black square is painted black. The colors of all other squares are left unchanged. What is the probability that the grid is now entirely black?

(A) $\dfrac{49}{512}$ **(B)** $\dfrac{7}{64}$ **(C)** $\dfrac{121}{1024}$ **(D)** $\dfrac{81}{512}$ **(E)** $\dfrac{9}{32}$

16. Circle C_1 has its center O lying on circle C_2. The two circles meet at X and Y. Point Z in the exterior of C_1 lies on circle C_2 and $XZ=13$, $OZ=11$, and $YZ=7$. What is the radius of circle C_1?

(A) 5 **(B)** $\sqrt{26}$ **(C)** $3\sqrt{3}$ **(D)** $2\sqrt{7}$ **(E)** $\sqrt{30}$

17. Let S be a subset of $\{1,2,3,\ldots,30\}$ with the property that no pair of distinct elements in S has a sum divisible by 5. What is the largest possible size of S?

(A) 10 **(B)** 13 **(C)** 15 **(D)** 16 **(E)** 18

18. Triangle ABC has $AB=27$, $BC=25$, and $CA=26$. Let I denote the intersection of the internal angle bisectors of $\triangle ABC$. What is BI?

(A) 15 **(B)** $5+\sqrt{26}+3\sqrt{3}$ **(C)** $3\sqrt{26}$ **(D)** $\frac{2}{3}\sqrt{546}$ **(E)** $9\sqrt{3}$

19. Adam, Benin, Chiang, Deshawn, Esther, and Fiona have internet accounts. Some, but not all, of them are internet friends with each other, and none of them has an internet friend outside this group. Each of them has the same number of internet friends. In how many different ways can this happen?

(A) 60 **(B)** 170 **(C)** 290 **(D)** 320 **(E)** 660

20. Consider the polynomial

$$P(x)=\prod_{k=0}^{10}(x^{2^k}+2^k)=(x+1)(x^2+2)(x^4+4)\cdots(x^{1024}+1024)$$

The coefficient of x^{2012} is equal to 2^a. What is a?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 10 **(E)** 24

21. Let a, b, and c be positive integers with $a\ge b\ge c$ such that

$$a^2-b^2-c^2+ab=2011 \text{ and}$$
$$a^2+3b^2+3c^2-3ab-2ac-2bc=-1997$$

What is a?

(A) 249 **(B)** 250 **(C)** 251 **(D)** 252 **(E)** 253

22. Distinct planes $p_1, p_2, ..., p_k$ intersect the interior of a cube Q. Let S be the union of the faces of Q and let $P = \bigcup_{j=1}^{k} p_j$. The intersection of P and S consists of the union of all segments joining the midpoints of every pair of edges belonging to the same face of Q. What is the difference between the maximum and minimum possible values of k?

(A) 8 **(B)** 12 **(C)** 20 **(D)** 23 **(E)** 24

23. Let S be the square one of whose diagonals has endpoints $(0.1, 0.7)$ and $(-0.1, -0.7)$. A point $v = (x, y)$ is chosen uniformly at random over all pairs of real numbers x and y such that $0 \leq x \leq 2012$ and $0 \leq y \leq 2012$. Let $T(v)$ be a translated copy of S centered at v. What is the probability that the square region determined by $T(v)$ contains exactly two points with integer coordinates in its interior?

(A) $\frac{1}{8}$ **(B)** $\frac{7}{50}$ **(C)** $\frac{4}{25}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{8}{25}$

24. Let $\{a_k\}_{k=1}^{2011}$ be the sequence of real numbers defined by $a_1 = 0.201$, $a_2 = (0.2011)^{a_1}$, $a_3 = (0.20101)^{a_2}$, $a_4 = (0.201011)^{a_3}$, and in general,

$$a_k = \begin{cases} (0.\underbrace{20101\cdots0101}_{k+2 \text{ digits}})^{a_{k-1}} & \text{if } k \text{ is odd,} \\ (0.\underbrace{20101\cdots01011}_{k+2 \text{ digits}})^{a_{k-1}} & \text{if } k \text{ is even.} \end{cases}$$

Rearranging the numbers in the sequence $\{a_k\}_{k=1}^{2011}$ in decreasing order produces a new sequence $\{b_k\}_{k=1}^{2011}$. What is the sum of all integers k, $1 \leq k \leq 2011$, such that $a_k = b_k$?

(A) 671 **(B)** 1006 **(C)** 1341 **(D)** 2011 **(E)** 2012

25. Let $f(x) = |2\{x\} - 1|$ where $\{x\}$ denotes the fractional part of x. The number n is the smallest positive integer such that the equation

$$nf(xf(x)) = x$$

has at least 2012 real solutions. What is n? "'Note:"' the fractional part of x is a real number $y = \{x\}$ such that $0 \leq y < 1$ and $x - y$ is an integer.

(A) 30 **(B)** 31 **(C)** 32 **(D)** 62 **(E)** 64

2012 AMC1212B

1. Each third-grade classroom at Pearl Creek Elementary has 18 students and 2 pet rabbits. How many more students than rabbits are there in all 4 of the third-grade classrooms?

   (A) 48 **(B)** 56 **(C)** 64 **(D)** 72 **(E)** 80

2. A circle of radius 5 is inscribed in a rectangle as shown. The ratio of the length of the rectangle to its width is 2:1. What is the area of the rectangle?

   (A) 50 **(B)** 100 **(C)** 125 **(D)** 150 **(E)** 200

3. For a science project, Sammy observed a chipmunk and squirrel stashing acorns in holes. The chipmunk hid 3 acorns in each of the holes it dug. The squirrel hid 4 acorns in each of the holes it dug. They each hid the same number of acorns, although the squirrel needed 4 fewer holes. How many acorns did the chipmunk hide?

   (A) 30 **(B)** 36 **(C)** 42 **(D)** 48 **(E)** 54

4. Suppose that the euro is worth 1.30 dollars. If Diana has 500 dollars and Etienne has 400 euros, by what percent is the value of Etienne's money greater than the value of Diana's money?

   (A) 2 **(B)** 4 **(C)** 6.5 **(D)** 8 **(E)** 13

5. Two integers have a sum of 26. When two more integers are added to the first two, the sum is 41. Finally, when two more integers are added to the sum of the previous 4 integers, the sum is 57. What is the minimum number of even integers among the 6 integers?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

6. In order to estimate the value of $x - y$ where x and y are real numbers with $x > y > 0$, Xiaoli rounded x up by a small amount, rounded y down by the same amount, and then subtracted her rounded values. Which of the following statements is necessarily correct?

   (A) Her estimate is larger than $x - y$

   (B) Her estimate is smaller than $x - y$

   (C) Her estimate equals $x - y$

   (D) Her estimate equals $y - x$

   (E) Her estimate is 0

7. Small lights are hung on a string 6 inches apart in the order red, red, green, green, green, red, red, green, green, green, and so on continuing this pattern of 2 red lights followed by 3 green lights. How many feet separate the 3rd red light and the 21st red light?

   ”’Note:”’ 1 foot is equal to 12 inches.

   (A) 18 **(B)** 18.5 **(C)** 20 **(D)** 20.5 **(E)** 22.5

8. A dessert chef prepares the dessert for every day of a week starting with Sunday. The dessert each day is either cake, pie, ice cream, or pudding. The same dessert may not be served two days in a row. There must be cake on Friday because of a birthday. How many different dessert menus for the week are possible?

   (A) 729 **(B)** 972 **(C)** 1024 **(D)** 2187 **(E)** 2304

9. It takes Clea 60 seconds to walk down an escalator when it is not moving, and 24 seconds when it is moving. How many seconds would it take Clea to ride the escalator down when she is not walking?

   (A) 36 **(B)** 40 **(C)** 42 **(D)** 48 **(E)** 52

10. What is the area of the polygon whose vertices are the points of intersection of the curves $x^2 + y^2 = 25$ and $(x - 4)^2 + 9y^2 = 81$?

    (A) 24 **(B)** 27 **(C)** 36 **(D)** 37.5 **(E)** 42

11. In the equation below, A and B are consecutive positive integers, and A, B, and $A+B$ represent number bases:
$$132_A + 43_B = 69_{A+B}.$$
What is $A+B$?
(A) 9 **(B)** 11 **(C)** 13 **(D)** 15 **(E)** 17

12. How many sequences of zeros and ones of length 20 have all the zeros consecutive, or all the ones consecutive, or both?
(A) 190 **(B)** 192 **(C)** 211 **(D)** 380 **(E)** 382

13. Two parabolas have equations $y = x^2 + ax + b$ and $y = x^2 + cx + d$, where a, b, c, and d are integers, each chosen independently by rolling a fair six-sided die. What is the probability that the parabolas will have at least one point in common?
(A) $\frac{1}{2}$ **(B)** $\frac{25}{36}$ **(C)** $\frac{5}{6}$ **(D)** $\frac{31}{36}$ **(E)** 1

14. Bernardo and Silvia play the following game. An integer between 0 and 999 inclusive is selected and given to Bernardo. Whenever Bernardo receives a number, he doubles it and passes the result to Silvia. Whenever Silvia receives a number, she adds 50 to it and passes the result to Bernardo. The winner is the last person who produces a number less than 1000. Let N be the smallest initial number that results in a win for Bernardo. What is the sum of the digits of N?
(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

15. Jesse cuts a circular paper disk of radius 12 along two radii to form two sectors, the smaller having a central angle of 120 degrees. He makes two circular cones, using each sector to form the lateral surface of a cone. What is the ratio of the volume of the smaller cone to that of the larger one?
(A) $\frac{1}{8}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{\sqrt{10}}{10}$ **(D)** $\frac{\sqrt{5}}{6}$ **(E)** $\frac{\sqrt{5}}{5}$

16. Amy, Beth, and Jo listen to four different songs and discuss which ones they like. No song is liked by all three. Furthermore, for each of the three pairs of the girls, there is at least one song liked by those girls but disliked by the third. In how many different ways is this possible?
(A) 108 **(B)** 132 **(C)** 671 **(D)** 846 **(E)** 1105

17. Square $PQRS$ lies in the first quadrant. Points $(3,0), (5,0), (7,0)$, and $(13,0)$ lie on lines SP, RQ, PQ, and SR, respectively. What is the sum of the coordinates of the center of the square $PQRS$?
(A) 6 **(B)** $\frac{31}{5}$ **(C)** $\frac{32}{5}$ **(D)** $\frac{33}{5}$ **(E)** $\frac{34}{5}$

18. Let $(a_1, a_2, \ldots, a_{10})$ be a list of the first 10 positive integers such that for each $2 \le i \le 10$ either $a_i + 1$ or $a_i - 1$ or both appear somewhere before a_i in the list. How many such lists are there?
(A) 120 **(B)** 512 **(C)** 1024 **(D)** $181,440$ **(E)** $362,880$
label("P_1", P[1], dir(P[1])); label("P_2", P[2], dir(P[2])); label("P_3", P[3], dir(-45)); label("P_4", P[4], dir(P[4])); label("P_1'", Pp[1], dir(Pp[1])); label("P_2'", Pp[2], dir(Pp[2])); label("P_3'", Pp[3], dir(-100)); label("P_4'", Pp[4], dir(Pp[4]));

19. A trapezoid has side lengths 3, 5, 7, and 11. The sums of all the possible areas of the trapezoid can be written in the form of $r_1\sqrt{n_1} + r_2\sqrt{n_2} + r_3$, where r_1, r_2, and r_3 are rational numbers and n_1 and n_2 are positive integers not divisible by the square of any prime. What is the greatest integer less than or equal to $r_1 + r_2 + r_3 + n_1 + n_2$?
(A) 57 **(B)** 59 **(C)** 61 **(D)** 63 **(E)** 65

20. Square $AXYZ$ is inscribed in equiangular hexagon $ABCDEF$ with X on $\overline{BC}$, Y on $\overline{DE}$, and Z on $\overline{EF}$. Suppose that $AB = 40$, and $EF = 41(\sqrt{3} - 1)$. What is the side-length of the square?
(A) $29\sqrt{3}$ **(B)** $\frac{21}{2}\sqrt{2} + \frac{41}{2}\sqrt{3}$ **(C)** $20\sqrt{3} + 16$
(D) $20\sqrt{2} + 13\sqrt{3}$ **(E)** $21\sqrt{6}$

21. A bug travels from A to B along the segments in the hexagonal lattice pictured below. The segments marked with an arrow can be traveled only in the direction of the arrow, and the bug never travels the same segment more than once. How many different paths are there?

(A) 2112 **(B)** 2304 **(C)** 2368 **(D)** 2384 **(E)** 2400

22. Consider all polynomials of a complex variable, $P(z) = 4z^4 + az^3 + bz^2 + cz + d$, where a, b, c, and d are integers, $0 \le d \le c \le b \le a \le 4$, and the polynomial has a zero z_0 with $|z_0| = 1$. What is the sum of all values $P(1)$ over all the polynomials with these properties?

(A) 84 **(B)** 92 **(C)** 100 **(D)** 108 **(E)** 120

23. Define the function f_1 on the positive integers by setting $f_1(1) = 1$ and if $n = p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$ is the prime factorization of $n > 1$, then

$$f_1(n) = (p_1 + 1)^{e_1 - 1}(p_2 + 1)^{e_2 - 1} \cdots (p_k + 1)^{e_k - 1}.$$

For every $m \ge 2$, let $f_m(n) = f_1(f_{m-1}(n))$. For how many Ns in the range $1 \le N \le 400$ is the sequence $(f_1(N), f_2(N), f_3(N), \dots)$ unbounded?

”’Note:”’ A sequence of positive numbers is unbounded if for every integer B, there is a member of the sequence greater than B.

(A) 15 **(B)** 16 **(C)** 17 **(D)** 18 **(E)** 19

24. Let $S = \{(x, y) : x \in \{0, 1, 2, 3, 4\}, y \in \{0, 1, 2, 3, 4, 5\}, \text{ and } (x, y) \ne (0, 0)\}$. Let T be the set of all right triangles whose vertices are in S. For every right triangle $t = \triangle ABC$ with vertices A, B, and C in counter-clockwise order and right angle at A, let $f(t) = \tan(\angle CBA)$. What is

$$\prod_{t \in T} f(t)?$$

(A) 1 **(B)** $\frac{625}{144}$ **(C)** $\frac{125}{24}$ **(D)** 6 **(E)** $\frac{625}{24}$

2013 AMC1212A

1. Square $ABCD$ has side length 10. Point E is on $\overline{BC}$, and the area of $\triangle ABE$ is 40. What is BE?

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

2. A softball team played ten games, scoring $1, 2, 3, 4, 5, 6, 7, 8, 9$, and 10 runs. They lost by one run in exactly five games. In each of the other games, they scored twice as many runs as their opponent. How many total runs did their opponents score?

   (A) 35 **(B)** 40 **(C)** 45 **(D)** 50 **(E)** 55

3. A flower bouquet contains pink roses, red roses, pink carnations, and red carnations. One third of the pink flowers are roses, three fourths of the red flowers are carnations, and six tenths of the flowers are pink. What percent of the flowers are carnations?

   (A) 15 **(B)** 30 **(C)** 40 **(D)** 60 **(E)** 70

4. What is the value of
$$\frac{2^{2014}+2^{2012}}{2^{2014}-2^{2012}}?$$

   (A) -1 **(B)** 1 **(C)** $\frac{5}{3}$ **(D)** 2013 **(E)** 2^{4024}

5. Tom, Dorothy, and Sammy went on a vacation and agreed to split the costs evenly. During their trip Tom paid \$105, Dorothy paid \$125, and Sammy paid \$175. In order to share the costs equally, Tom gave Sammy t dollars, and Dorothy gave Sammy d dollars. What is $t - d$?

   (A) 15 **(B)** 20 **(C)** 25 **(D)** 30 **(E)** 35

6. In a recent basketball game, Shenille attempted only three-point shots and two-point shots. She was successful on 20% of her three-point shots and 30% of her two-point shots. Shenille attempted 30 shots. How many points did she score?

   (A) 12 **(B)** 18 **(C)** 24 **(D)** 30 **(E)** 36

7. The sequence $S_1, S_2, S_3, \cdots, S_{10}$ has the property that every term beginning with the third is the sum of the previous two. That is,
$$S_n = S_{n-2} + S_{n-1} \text{ for } n \geq 3.$$
   Suppose that $S_9 = 110$ and $S_7 = 42$. What is S_4?

   (A) 4 **(B)** 6 **(C)** 10 **(D)** 12 **(E)** 16

8. Given that x and y are distinct nonzero real numbers such that $x + \frac{2}{x} = y + \frac{2}{y}$, what is xy?

   (A) $\frac{1}{4}$ **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** 2 **(E)** 4

9. In $\triangle ABC$, $AB = AC = 28$ and $BC = 20$. Points D, E, and F are on sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, respectively, such that $\overline{DE}$ and $\overline{EF}$ are parallel to $\overline{AC}$ and $\overline{AB}$, respectively. What is the perimeter of parallelogram $ADEF$?

   (A) 48 **(B)** 52 **(C)** 56 **(D)** 60 **(E)** 72

10. Let S be the set of positive integers n for which $\frac{1}{n}$ has the repeating decimal representation $0.\overline{ab} = 0.ababab\cdots$, with a and b different digits. What is the sum of the elements of S?

    (A) 11 **(B)** 44 **(C)** 110 **(D)** 143 **(E)** 155

11. Triangle ABC is equilateral with $AB = 1$. Points E and G are on $\overline{AC}$ and points D and F are on $\overline{AB}$ such that both $\overline{DE}$ and $\overline{FG}$ are parallel to $\overline{BC}$. Furthermore, triangle ADE and trapezoids $DFGE$ and $FBCG$ all have the same perimeter. What is $DE + FG$?

(A) 1 **(B)** $\frac{3}{2}$ **(C)** $\frac{21}{13}$ **(D)** $\frac{13}{8}$ **(E)** $\frac{5}{3}$

12. The angles in a particular triangle are in arithmetic progression, and the side lengths are $4, 5, x$. The sum of the possible values of x equals $a + \sqrt{b} + \sqrt{c}$ where a, b, and c are positive integers. What is $a + b + c$?

(A) 36 **(B)** 38 **(C)** 40 **(D)** 42 **(E)** 44

13. Let points $A = (0, 0)$, $B = (1, 2)$, $C = (3, 3)$, and $D = (4, 0)$. Quadrilateral $ABCD$ is cut into equal area pieces by a line passing through A. This line intersects $\overline{CD}$ at point $\left(\frac{p}{q}, \frac{r}{s}\right)$, where these fractions are in lowest terms. What is $p + q + r + s$?

(A) 54 **(B)** 58 **(C)** 62 **(D)** 70 **(E)** 75

14. The sequence

$\log_{12} 162$, $\log_{12} x$, $\log_{12} y$, $\log_{12} z$, $\log_{12} 1250$

is an arithmetic progression. What is x?

(A) $125\sqrt{3}$ **(B)** 270 **(C)** $162\sqrt{5}$ **(D)** 434 **(E)** $225\sqrt{6}$

15. Rabbits Peter and Pauline have three offspringFlopsie, Mopsie, and Cotton-tail. These five rabbits are to be distributed to four different pet stores so that no store gets both a parent and a child. It is not required that every store gets a rabbit. In how many different ways can this be done?

(A) 96 **(B)** 108 **(C)** 156 **(D)** 204 **(E)** 372

16. A, B, C are three piles of rocks. The mean weight of the rocks in A is 40 pounds, the mean weight of the rocks in B is 50 pounds, the mean weight of the rocks in the combined piles A and B is 43 pounds, and the mean weight of the rocks in the combined piles A and C is 44 pounds. What is the greatest possible integer value for the mean in pounds of the rocks in the combined piles B and C?

(A) 55 **(B)** 56 **(C)** 57 **(D)** 58 **(E)** 59

17. A group of 12 pirates agree to divide a treasure chest of gold coins among themselves as follows. The k^{th} pirate to take a share takes $\frac{k}{12}$ of the coins that remain in the chest. The number of coins initially in the chest is the smallest number for which this arrangement will allow each pirate to receive a positive whole number of coins. How many coins does the 12^{th} pirate receive?

(A) 720 **(B)** 1296 **(C)** 1728 **(D)** 1925 **(E)** 3850

18. Six spheres of radius 1 are positioned so that their centers are at the vertices of a regular hexagon of side length 2. The six spheres are internally tangent to a larger sphere whose center is the center of the hexagon. An eighth sphere is externally tangent to the six smaller spheres and internally tangent to the larger sphere. What is the radius of this eighth sphere?

(A) $\sqrt{2}$ **(B)** $\frac{3}{2}$ **(C)** $\frac{5}{3}$ **(D)** $\sqrt{3}$ **(E)** 2

19. In $\triangle ABC$, $AB = 86$, and $AC = 97$. A circle with center A and radius AB intersects $\overline{BC}$ at points B and X. Moreover $\overline{BX}$ and $\overline{CX}$ have integer lengths. What is BC?

(A) 11 **(B)** 28 **(C)** 33 **(D)** 61 **(E)** 72

20. Let S be the set $\{1, 2, 3, ..., 19\}$. For $a, b \in S$, define $a \succ b$ to mean that either $0 < a - b \leq 9$ or $b - a > 9$. How many ordered triples (x, y, z) of elements of S have the property that $x \succ y$, $y \succ z$, and $z \succ x$?

(A) 810 **(B)** 855 **(C)** 900 **(D)** 950 **(E)** 988

21. Consider $A = \log(2013 + \log(2012 + \log(2011 + \log(\cdots + \log(3 + \log 2) \cdots))))$. Which of the following intervals contains A?

(A) $(\log 2016, \log 2017)$ **(B)** $(\log 2017, \log 2018)$ **(C)** $(\log 2018, \log 2019)$ **(D)** $(\log 2019, \log 2020)$ **(E)** $(\log 2020, \log 2021)$

22. A palindrome is a nonnegative integer number that reads the same forwards and backwards when written in base 10 with no leading zeros. A 6-digit palindrome n is chosen uniformly at random. What is the probability that $\frac{n}{11}$ is also a palindrome?

(A) $\frac{8}{25}$ **(B)** $\frac{33}{100}$ **(C)** $\frac{7}{20}$ **(D)** $\frac{9}{25}$ **(E)** $\frac{11}{30}$

23. $ABCD$ is a square of side length $\sqrt{3}+1$. Point P is on $\overline{AC}$ such that $AP = \sqrt{2}$. The square region bounded by $ABCD$ is rotated 90° counterclockwise with center P, sweeping out a region whose area is $\frac{1}{c}(a\pi + b)$, where a, b, and c are positive integers and $\gcd(a, b, c) = 1$. What is $a + b + c$?

(A) 15 **(B)** 17 **(C)** 19 **(D)** 21 **(E)** 23

24. Three distinct segments are chosen at random among the segments whose end-points are the vertices of a regular 12-gon. What is the probability that the lengths of these three segments are the three side lengths of a triangle with positive area?

(A) $\frac{553}{715}$ **(B)** $\frac{443}{572}$ **(C)** $\frac{111}{143}$ **(D)** $\frac{81}{104}$ **(E)** $\frac{223}{286}$

25. Let $f : \mathbb{C} \to \mathbb{C}$ be defined by $f(z) = z^2 + iz + 1$. How many complex numbers z are there such that $\text{Im}(z) > 0$ and both the real and the imaginary parts of $f(z)$ are integers with absolute value at most 10?

(A) 399 **(B)** 401 **(C)** 413 **(D)** 431 **(E)** 441

2013 AMC1212B

1. On a particular January day, the high temperature in Lincoln, Nebraska, was 16 degrees higher than the low temperature, and the average of the high and low temperatures was 3. In degrees, what was the low temperature in Lincoln that day?

   (A) -13 **(B)** -8 **(C)** -5 **(D)** -3 **(E)** 11

2. Mr. Green measures his rectangular garden by walking two of the sides and finds that it is 15 steps by 20 steps. Each of Mr. Greens steps is 2 feet long. Mr. Green expects a half a pound of potatoes per square foot from his garden. How many pounds of potatoes does Mr. Green expect from his garden?

   (A) 600 **(B)** 800 **(C)** 1000 **(D)** 1200 **(E)** 1400

3. When counting from 3 to 201, 53 is the 51^{st} number counted. When counting backwards from 201 to 3, 53 is the n^{th} number counted. What is n?

   (A) 146 **(B)** 147 **(C)** 148 **(D)** 149 **(E)** 150

4. Ray's car averages 40 miles per gallon of gasoline, and Tom's car averages 10 miles per gallon of gasoline. Ray and Tom each drive the same number of miles. What is the cars' combined rate of miles per gallon of gasoline?¡br

   (A) 10 **(B)** 16 **(C)** 25 **(D)** 30 **(E)** 40

5. The average age of 33 fifth-graders is 11. The average age of 55 of their parents is 33. What is the average age of all of these parents and fifth-graders?

   (A) 22 **(B)** 23.25 **(C)** 24.75 **(D)** 26.25 **(E)** 28

6. Real numbers x and y satisfy the equation $x^2 + y^2 = 10x - 6y - 34$. What is $x + y$?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 8

7. Jo and Blair take turns counting from 1 to one more than the last number said by the other person. Jo starts by saying "1", so Blair follows by saying "$1, 2$". Jo then says "$1, 2, 3$", and so on. What is the 53^{rd} number said?

   (A) 2 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 8

8. Line l_1 has equation $3x - 2y = 1$ and goes through $A = (-1, -2)$. Line l_2 has equation $y = 1$ and meets line l_1 at point B. Line l_3 has positive slope, goes through point A, and meets l_2 at point C. The area of $\triangle ABC$ is 3. What is the slope of l_3?

   (A) $\frac{2}{3}$ **(B)** $\frac{3}{4}$ **(C)** 1 **(D)** $\frac{4}{3}$ **(E)** $\frac{3}{2}$

9. What is the sum of the exponents of the prime factors of the square root of the largest perfect square that divides 12! ?

   (A) 5 **(B)** 7 **(C)** 8 **(D)** 10 **(E)** 12

10. Alex has 75 red tokens and 75 blue tokens. There is a booth where Alex can give two red tokens and receive in return a silver token and a blue token, and another booth where Alex can give three blue tokens and receive in return a silver token and a red token. Alex continues to exchange tokens until no more exchanges are possible. How many silver tokens will Alex have at the end?

    (A) 62 **(B)** 82 **(C)** 83 **(D)** 102 **(E)** 103

11. Two bees start at the same spot and fly at the same rate in the following directions. Bee A travels 1 foot north, then 1 foot east, then 1 foot upwards, and then continues to repeat this pattern. Bee B travels 1 foot south, then 1 foot west, and then continues to repeat this pattern. In what directions are the bees traveling when they are exactly 10 feet away from each other?

    (A) A east, B west¡br **(B)** A north, B south¡br **(C)** A north, B west¡br **(D)** A up, B south¡br **(E)** A up, B west¡br

12. Cities A, B, C, D, and E are connected by roads $\widetilde{AB}$, $\widetilde{AD}$, $\widetilde{AE}$, $\widetilde{BC}$, $\widetilde{BD}$, $\widetilde{CD}$, and $\widetilde{DE}$. How many different routes are there from A to B that use each road exactly once? (Such a route will necessarily visit some cities more than once.)

(A) 7 **(B)** 9 **(C)** 12 **(D)** 16 **(E)** 18

13. The internal angles of quadrilateral $ABCD$ form an arithmetic progression. Triangles ABD and DCB are similar with $\angle DBA = \angle DCB$ and $\angle ADB = \angle CBD$. Moreover, the angles in each of these two triangles also form an arithmetic progression. In degrees, what is the largest possible sum of the two largest angles of $ABCD$?

(A) 210 **(B)** 220 **(C)** 230 **(D)** 240 **(E)** 250

14. Two non-decreasing sequences of nonnegative integers have different first terms. Each sequence has the property that each term beginning with the third is the sum of the previous two terms, and the seventh term of each sequence is N. What is the smallest possible value of N ?

(A) 55 **(B)** 89 **(C)** 104 **(D)** 144 **(E)** 273

15. The number 2013 is expressed in the form ¡br ¡center¿ $2013 = \frac{a_1!a_2!...a_m!}{b_1!b_2!...b_n!}$,¡/center¿¡br /¿where $a_1 \geq a_2 \geq ... \geq a_m$ and $b_1 \geq b_2 \geq ... \geq b_n$ are positive integers and $a_1 + b_1$ is as small as possible. What is $|a_1 - b_1|$?
(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

16. Let $ABCDE$ be an equiangular convex pentagon of perimeter 1. The pairwise intersections of the lines that extend the sides of the pentagon determine a five-pointed star polygon. Let s be the perimeter of this star. What is the difference between the maximum and the minimum possible values of s?

(A) 0 **(B)** $\frac{1}{2}$ **(C)** $\frac{\sqrt{5}-1}{2}$ **(D)** $\frac{\sqrt{5}+1}{2}$ **(E)** $\sqrt{5}$

17. Let a, b, and c be real numbers such that

$$a + b + c = 2, \text{ and}$$

$$a^2 + b^2 + c^2 = 12$$

What is the difference between the maximum and minimum possible values of c?

(A) 2 **(B)** $\frac{10}{3}$ **(C)** 4 **(D)** $\frac{16}{3}$ **(E)** $\frac{20}{3}$

18. Barbara and Jenna play the following game, in which they take turns. A number of coins lie on a table. When it is Barbaras turn, she must remove 2 or 4 coins, unless only one coin remains, in which case she loses her turn. When it is Jennas turn, she must remove 1 or 3 coins. A coin flip determines who goes first. Whoever removes the last coin wins the game. Assume both players use their best strategy. Who will win when the game starts with 2013 coins and when the game starts with 2014 coins?

(A) Barbara will win with 2013 coins and Jenna will win with 2014 coins.

(B) Jenna will win with 2013 coins, and whoever goes first will win with 2014 coins.

(C) Barbara will win with 2013 coins, and whoever goes second will win with 2014 coins.

(D) Jenna will win with 2013 coins, and Barbara will win with 2014 coins.

(E) Whoever goes first will win with 2013 coins, and whoever goes second will win with 2014 coins.

19. In triangle ABC, $AB = 13$, $BC = 14$, and $CA = 15$. Distinct points D, E, and F lie on segments $\overline{BC}$, $\overline{CA}$, and $\overline{DE}$, respectively, such that $\overline{AD} \perp \overline{BC}$, $\overline{DE} \perp \overline{AC}$, and $\overline{AF} \perp \overline{BF}$. The length of segment $\overline{DF}$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

(A) 18 **(B)** 21 **(C)** 24 **(D)** 27 **(E)** 30

20. For $135^\circ < x < 180^\circ$, points $P = (\cos x, \cos^2 x), Q = (\cot x, \cot^2 x), R = (\sin x, \sin^2 x)$ and $S = (\tan x, \tan^2 x)$ are the vertices of a trapezoid. What is $\sin(2x)$?

(A) $2 - 2\sqrt{2}$ **(B)**$3\sqrt{3} - 6$ **(C)** $3\sqrt{2} - 5$ **(D)** $-\frac{3}{4}$ **(E)** $1 - \sqrt{3}$

21. Consider the set of 30 parabolas defined as follows: all parabolas have as focus the point $(0, 0)$ and the directrix lines have the form $y = ax + b$ with a and b integers such that $a \in \{-2, -1, 0, 1, 2\}$ and $b \in \{-3, -2, -1, 1, 2, 3\}$. No three of these parabolas have a common point. How many points in the plane are on two of these parabolas?

    (A) 720 **(B)** 760 **(C)** 810 **(D)** 840 **(E)** 870

22. Let $m > 1$ and $n > 1$ be integers. Suppose that the product of the solutions for x of the equation

$$8(\log_n x)(\log_m x) - 7\log_n x - 6\log_m x - 2013 = 0$$

    is the smallest possible integer. What is $m + n$?

    (A) 12 **(B)** 20 **(C)** 24 **(D)** 48 **(E)** 272

23. Bernardo chooses a three-digit positive integer N and writes both its base-5 and base-6 representations on a blackboard. Later LeRoy sees the two numbers Bernardo has written. Treating the two numbers as base-10 integers, he adds them to obtain an integer S. For example, if $N = 749$, Bernardo writes the numbers 10444 and 3245, and LeRoy obtains the sum $S = 13,689$. For how many choices of N are the two rightmost digits of S, in order, the same as those of $2N$?

    (A) 5 **(B)** 10 **(C)** 15 **(D)** 20 **(E)** 25

24. Let ABC be a triangle where M is the midpoint of $\overline{AC}$, and $\overline{CN}$ is the angle bisector of $\angle ACB$ with N on $\overline{AB}$. Let X be the intersection of the median $\overline{BM}$ and the bisector $\overline{CN}$. In addition $\triangle BXN$ is equilateral with $AC = 2$. What is BX^2?

    (A) $\frac{10-6\sqrt{2}}{7}$ **(B)** $\frac{2}{9}$ **(C)** $\frac{5\sqrt{2}-3\sqrt{3}}{8}$ **(D)** $\frac{\sqrt{2}}{6}$ **(E)** $\frac{3\sqrt{3}-4}{5}$

25. Let G be the set of polynomials of the form

$$P(z) = z^n + c_{n-1}z^{n-1} + \cdots + c_2z^2 + c_1z + 50,$$

    where $c_1, c_2, \cdots, c_{n-1}$ are integers and $P(z)$ has distinct roots of the form $a + ib$ with a and b integers. How many polynomials are in G?

    (A) 288 **(B)** 528 **(C)** 576 **(D)** 992 **(E)** 1056

2014 AMC1212A

1. What is $10 \cdot \left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right)^{-1}$?

   (A) 3 **(B)** 8 **(C)** $\frac{25}{2}$ **(D)** $\frac{170}{3}$ **(E)** 170

2. At the theater children get in for half price. The price for 5 adult tickets and 4 child tickets is \$24.50. How much would 8 adult tickets and 6 child tickets cost?

   (A) \$35 **(B)** \$38.50 **(C)** \$40 **(D)** \$42 **(E)** \$42.50

3. Walking down Jane Street, Ralph passed four houses in a row, each painted a different color. He passed the orange house before the red house, and he passed the blue house before the yellow house. The blue house was not next to the yellow house. How many orderings of the colored houses are possible?

   (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

4. Suppose that a cows give b gallons of milk in c days. At this rate, how many gallons of milk will d cows give in e days?

   (A) $\frac{bde}{ac}$ **(B)** $\frac{ac}{bde}$ **(C)** $\frac{abde}{c}$ **(D)** $\frac{bcde}{a}$ **(E)** $\frac{abc}{de}$

5. On an algebra quiz, 10% of the students scored 70 points, 35% scored 80 points, 30% scored 90 points, and the rest scored 100 points. What is the difference between the mean and median score of the students' scores on this quiz?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

6. The difference between a two-digit number and the number obtained by reversing its digits is 5 times the sum of the digits of either number. What is the sum of the two digit number and its reverse?

   (A) 44 **(B)** 55 **(C)** 77 **(D)** 99 **(E)** 110

7. The first three terms of a geometric progression are $\sqrt{3}$, $\sqrt[3]{3}$, and $\sqrt[6]{3}$. What is the fourth term?

   (A) 1 **(B)** $\sqrt[7]{3}$ **(C)** $\sqrt[8]{3}$ **(D)** $\sqrt[9]{3}$ **(E)** $\sqrt[10]{3}$

8. A customer who intends to purchase an appliance has three coupons, only one of which may be used:

   Coupon 1: 10% off the listed price if the listed price is at least \$50

   Coupon 2: \$20 off the listed price if the listed price is at least \$100

   Coupon 3: 18% off the amount by which the listed price exceeds \$100

   For which of the following listed prices will coupon 1 offer a greater price reduction than either coupon 2 or coupon 3?

   (A) \$179.95 **(B)** \$199.95 **(C)** \$219.95 **(D)** \$239.95 **(E)** \$259.95

9. Five positive consecutive integers starting with a have average b. What is the average of 5 consecutive integers that start with b?

   (A) $a+3$ **(B)** $a+4$ **(C)** $a+5$ **(D)** $a+6$ **(E)** $a+7$

10. Three congruent isosceles triangles are constructed with their bases on the sides of an equilateral triangle of side length 1. The sum of the areas of the three isosceles triangles is the same as the area of the equilateral triangle. What is the length of one of the two congruent sides of one of the isosceles triangles?

    (A) $\frac{\sqrt{3}}{4}$ **(B)** $\frac{\sqrt{3}}{3}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{\sqrt{2}}{2}$ **(E)** $\frac{\sqrt{3}}{2}$

11. David drives from his home to the airport to catch a flight. He drives 35 miles in the first hour, but realizes that he will be 1 hour late if he continues at this speed. He increases his speed by 15 miles per hour for the rest of the way to the airport and arrives 30 minutes early. How many miles is the airport from his home?

    (A) 140 **(B)** 175 **(C)** 210 **(D)** 245 **(E)** 280

12. Two circles intersect at points A and B. The minor arcs AB measure 30° on one circle and 60° on the other circle. What is the ratio of the area of the larger circle to the area of the smaller circle?

(A) 2 **(B)** $1+\sqrt{3}$ **(C)** 3 **(D)** $2+\sqrt{3}$ **(E)** 4

13. A fancy bed and breakfast inn has 5 rooms, each with a distinctive color-coded decor. One day 5 friends arrive to spend the night. There are no other guests that night. The friends can room in any combination they wish, but with no more than 2 friends per room. In how many ways can the innkeeper assign the guests to the rooms?

(A) 2100 **(B)** 2220 **(C)** 3000 **(D)** 3120 **(E)** 3125

14. Let $a < b < c$ be three integers such that a, b, c is an arithmetic progression and a, c, b is a geometric progression. What is the smallest possible value of c?

(A) -2 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 6

15. A five-digit palindrome is a positive integer with respective digits $abcba$, where a is non-zero. Let S be the sum of all five-digit palindromes. What is the sum of the digits of S?

(A) 9 **(B)** 18 **(C)** 27 **(D)** 36 **(E)** 45

16. The product $(8)(888\dots8)$, where the second factor has k digits, is an integer whose digits have a sum of 1000. What is k?

(A) 901 **(B)** 911 **(C)** 919 **(D)** 991 **(E)** 999

17. A $4 \times 4 \times h$ rectangular box contains a sphere of radius 2 and eight smaller spheres of radius 1. The smaller spheres are each tangent to three sides of the box, and the larger sphere is tangent to each of the smaller spheres. What is h?

¡center¿

¡/center¿

(A) $2+2\sqrt{7}$ **(B)** $3+2\sqrt{5}$ **(C)** $4+2\sqrt{7}$ **(D)** $4\sqrt{5}$ **(E)** $4\sqrt{7}$

18. The domain of the function $f(x) = \log_{\frac{1}{2}}(\log_4(\log_{\frac{1}{4}}(\log_{16}(\log_{\frac{1}{16}} x))))$ is an interval of length $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

(A) 19 **(B)** 31 **(C)** 271 **(D)** 319 **(E)** 511

19. There are exactly N distinct rational numbers k such that $|k| < 200$ and

$$5x^2 + kx + 12 = 0$$

has at least one integer solution for x. What is N?

(A) 6 **(B)** 12 **(C)** 24 **(D)** 48 **(E)** 78

20. In $\triangle BAC$, $\angle BAC = 40^\circ$, $AB = 10$, and $AC = 6$. Points D and E lie on $\overline{AB}$ and $\overline{AC}$ respectively. What is the minimum possible value of $BE + DE + CD$?

(A) $6\sqrt{3}+3$ **(B)** $\dfrac{27}{2}$ **(C)** $8\sqrt{3}$ **(D)** 14 **(E)** $3\sqrt{3}+9$

21. For every real number x, let $\lfloor x \rfloor$ denote the greatest integer not exceeding x, and let

$$f(x) = \lfloor x \rfloor (2014^{x - \lfloor x \rfloor} - 1).$$

The set of all numbers x such that $1 \le x < 2014$ and $f(x) \le 1$ is a union of disjoint intervals. What is the sum of the lengths of those intervals?

(A) 1 **(B)** $\dfrac{\log 2015}{\log 2014}$ **(C)** $\dfrac{\log 2014}{\log 2013}$ **(D)** $\dfrac{2014}{2013}$ **(E)** $2014^{\frac{1}{2014}}$

22. The number 5^{867} is between 2^{2013} and 2^{2014}. How many pairs of integers (m, n) are there such that $1 \leq m \leq 2012$ and
$$5^n < 2^m < 2^{m+2} < 5^{n+1}?$$
(A) 278 **(B)** 279 **(C)** 280 **(D)** 281 **(E)** 282

23. The fraction
$$\frac{1}{99^2} = 0.\overline{b_{n-1}b_{n-2}\ldots b_2b_1b_0},$$
where n is the length of the period of the repeating decimal expansion. What is the sum $b_0+b_1+\cdots+b_{n-1}$?
(A) 874 **(B)** 883 **(C)** 887 **(D)** 891 **(E)** 892

24. Let $f_0(x) = x + |x - 100| - |x + 100|$, and for $n \geq 1$, let $f_n(x) = |f_{n-1}(x)| - 1$. For how many values of x is $f_{100}(x) = 0$?
(A) 299 **(B)** 300 **(C)** 301 **(D)** 302 **(E)** 303

25. The parabola P has focus $(0, 0)$ and goes through the points $(4, 3)$ and $(-4, -3)$. For how many points $(x, y) \in P$ with integer coordinates is it true that $|4x + 3y| \leq 1000$?
(A) 38 **(B)** 40 **(C)** 42 **(D)** 44 **(E)** 46

2014 AMC1212B

1. Leah has 13 coins, all of which are pennies and nickels. If she had one more nickel than she has now, then she would have the same number of pennies and nickels. In cents, how much are Leah's coins worth?

   (A) 33 **(B)** 35 **(C)** 37 **(D)** 39 **(E)** 41

2. Orvin went to the store with just enough money to buy 30 balloons. When he arrived he discovered that the store had a special sale on balloons: buy 1 balloon at the regular price and get a second at $\frac{1}{3}$ off the regular price. What is the greatest number of balloons Orvin could buy?

   (A) 33 **(B)** 34 **(C)** 36 **(D)** 38 **(E)** 39

3. Randy drove the first third of his trip on a gravel road, the next 20 miles on pavement, and the remaining one-fifth on a dirt road. In miles, how long was Randy's trip?

   (A) 30 **(B)** $\frac{400}{11}$ **(C)** $\frac{75}{2}$ **(D)** 40 **(E)** $\frac{300}{7}$

4. Susie pays for 4 muffins and 3 bananas. Calvin spends twice as much paying for 2 muffins and 16 bananas. A muffin is how many times as expensive as a banana?

   (A) $\frac{3}{2}$ **(B)** $\frac{5}{3}$ **(C)** $\frac{7}{4}$ **(D)** 2 **(E)** $\frac{13}{4}$

5. Doug constructs a square window using 8 equal-size panes of glass, as shown. The ratio of the height to width for each pane is $5:2$, and the borders around and between the panes are 2 inches wide. In inches, what is the side length of the square window?

   (A) 26 **(B)** 28 **(C)** 30 **(D)** 32 **(E)** 34

6. Ed and Ann both have lemonade with their lunch. Ed orders the regular size. Ann gets the large lemonade, which is 50

   (A) 30 **(B)** 32 **(C)** 36 **(D)** 40 **(E)** 50

7. For how many positive integers n is $\frac{n}{30-n}$ also a positive integer?

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

8. In the addition shown below A, B, C, and D are distinct digits. How many different values are possible for D?

$$\begin{array}{cccccc} & \mathrm{A} & \mathrm{B} & \mathrm{B} & \mathrm{C} & \mathrm{B} \\ + & \mathrm{B} & \mathrm{C} & \mathrm{A} & \mathrm{D} & \mathrm{A} \\ \hline & \mathrm{D} & \mathrm{B} & \mathrm{D} & \mathrm{D} & \mathrm{D} \end{array}$$

   (A) 2 **(B)** 4 **(C)** 7 **(D)** 8 **(E)** 9

9. Convex quadrilateral $ABCD$ has $AB = 3$, $BC = 4$, $CD = 13$, $AD = 12$, and $\angle ABC = 90°$, as shown. What is the area of the quadrilateral?

   (A) 30 **(B)** 36 **(C)** 40 **(D)** 48 **(E)** 58.5

10. Danica drove her new car on a trip for a whole number of hours, averaging 55 miles per hour. At the beginning of the trip, abc miles was displayed on the odometer, where abc is a 3-digit number with $a \geq 1$ and $a + b + c \leq 7$. At the end of the trip, the odometer showed cba miles. What is $a^2 + b^2 + c^2$?.

    (A) 26 **(B)** 27 **(C)** 36 **(D)** 37 **(E)** 41

11. A list of 11 positive integers has a mean of 10, a median of 9, and a unique mode of 8. What is the largest possible value of an integer in the list?

    (A) 24 **(B)** 30 **(C)** 31 **(D)** 33 **(E)** 35

12. A set S consists of triangles whose sides have integer lengths less than 5, and no two elements of S are congruent or similar. What is the largest number of elements that S can have?

    (A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

13. Real numbers a and b are chosen with $1 < a < b$ such that no triangles with positive area has side lengths $1, a,$ and b or $\frac{1}{b}, \frac{1}{a}$, and 1. What is the smallest possible value of b?

    (A) $\frac{3+\sqrt{3}}{2}$ **(B)** $\frac{5}{2}$ **(C)** $\frac{3+\sqrt{5}}{2}$ **(D)** $\frac{3+\sqrt{6}}{2}$ **(E)** 3

14. A rectangular box has a total surface area of 94 square inches. The sum of the lengths of all its edges is 48 inches. What is the sum of the lengths in inches of all of its interior diagonals?

    (A) $8\sqrt{3}$ **(B)** $10\sqrt{2}$ **(C)** $16\sqrt{3}$ **(D)** $20\sqrt{2}$ **(E)** $40\sqrt{2}$

15. When $p = \sum\limits_{k=1}^{6} k \ln k$, the number e^p is an integer. What is the largest power of 2 that is a factor of e^p ?

    (A) 2^{12} **(B)** 2^{14} **(C)** 2^{16} **(D)** 2^{18} **(E)** 2^{20}

16. Let P be a cubic polynomial with $P(0) = k$, $P(1) = 2k$, and $P(-1) = 3k$. What is $P(2) + P(-2)$?

    (A) 0 **(B)** k **(C)** $6k$ **(D)** $7k$ **(E)** $14k$

17. Let P be the parabola with equation $y = x^2$ and let $Q = (20, 14)$. There are real numbers r and s such that the line through Q with slope m does not intersect P if and only if $r < m < s$. What is $r + s$?

    (A) 1 **(B)** 26 **(C)** 40 **(D)** 52 **(E)** 80

18. The numbers 1, 2, 3, 4, 5, are to be arranged in a circle. An arrangement is *bad* if it is not true that for every n from 1 to 15 one can find a subset of the numbers that appear consecutively on the circle that sum to n. Arrangements that differ only by a rotation or a reflection are considered the same. How many different bad arrangements are there?

    (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

19. A sphere is inscribed in a truncated right circular cone as shown. The volume of the truncated cone is twice that of the sphere. What is the ratio of the radius of the bottom base of the truncated cone to the radius of the top base of the truncated cone?

    (A) $\dfrac{3}{2}$ (B) $\dfrac{1+\sqrt{5}}{2}$ (C) $\sqrt{3}$ (D) 2 (E) $\dfrac{3+\sqrt{5}}{2}$

20. For how many positive integers x is $\log_{10}(x-40) + \log_{10}(60-x) < 2$?

    (A) 10 **(B)** 18 **(C)** 19 **(D)** 20 **(E)** infinitely many

21. In the figure, $ABCD$ is a square of side length 1. The rectangles $JKHG$ and $EBCF$ are congruent. What is BE?

    (A) $\frac{1}{2}(\sqrt{6}-2)$ **(B)** $\frac{1}{4}$ **(C)** $2-\sqrt{3}$ **(D)** $\frac{\sqrt{3}}{6}$ **(E)** $1-\frac{\sqrt{2}}{2}$

22. In a small pond there are eleven lily pads in a row labeled 0 through 10. A frog is sitting on pad 1. When the frog is on pad N, $0 < N < 10$, it will jump to pad $N-1$ with probability $\frac{N}{10}$ and to pad $N+1$ with probability $1-\frac{N}{10}$. Each jump is independent of the previous jumps. If the frog reaches pad 0 it will be eaten by a patiently waiting snake. If the frog reaches pad 10 it will exit the pond, never to return. What is the probability that the frog will escape without being eaten by the snake?

    (A) $\frac{32}{79}$ **(B)** $\frac{161}{384}$ **(C)** $\frac{63}{146}$ **(D)** $\frac{7}{16}$ **(E)** $\frac{1}{2}$

23. The number 2017 is prime. Let $S = \sum_{k=0}^{62} \binom{2014}{k}$. What is the remainder when S is divided by 2017?

    (A) 32 **(B)** 684 **(C)** 1024 **(D)** 1576 **(E)** 2016

24. Let $ABCDE$ be a pentagon inscribed in a circle such that $AB = CD = 3$, $BC = DE = 10$, and $AE = 14$. The sum of the lengths of all diagonals of $ABCDE$ is equal to $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

    (A) 129 **(B)** 247 **(C)** 353 **(D)** 391 **(E)** 421

25. Find the sum of all the positive solutions of

$$2\cos(2x)\left(\cos(2x) - \cos\left(\frac{2014\pi^2}{x}\right)\right) = \cos(4x) - 1$$

    (A) π **(B)** 810π **(C)** 1008π **(D)** 1080π **(E)** 1800π

2015 AMC1212A

1. What is the value of $(2^0 - 1 + 5^2 - 0)^{-1} \times 5$?

   (A) -125 **(B)** -120 **(C)** $\frac{1}{5}$ **(D)** $\frac{5}{24}$ **(E)** 25

2. Two of the three sides of a triangle are 20 and 15. Which of the following numbers is not a possible perimeter of the triangle?

   (A) 52 **(B)** 57 **(C)** 62 **(D)** 67 **(E)** 72

3. Mr. Patrick teaches math to 15 students. He was grading tests and found that when he graded everyone's test except Payton's, the average grade for the class was 80. After he graded Payton's test, the class average became 81. What was Payton's score on the test?

   (A) 81 **(B)** 85 **(C)** 91 **(D)** 94 **(E)** 95

4. The sum of two positive numbers is 5 times their difference. What is the ratio of the larger number to the smaller?

   (A) $\frac{5}{4}$ **(B)** $\frac{3}{2}$ **(C)** $\frac{9}{5}$ **(D)** 2 **(E)** $\frac{5}{2}$

5. Amelia needs to estimate the quantity $\frac{a}{b} - c$, where a, b, and c are large positive integers. She rounds each of the integers so that the calculation will be easier to do mentally. In which of these situations will her answer necessarily be greater than the exact value of $\frac{a}{b} - c$?

   (A) She rounds all three numbers up.
   (B) She rounds a and b up, and she rounds c down.
   (C) She rounds a and c up, and she rounds b down.
   (D) She rounds a up, and she rounds b and c down.
   (E) She rounds c up, and she rounds a and b down.

6. Two years ago Pete was three times as old as his cousin Claire. Two years before that, Pete was four times as old as Claire. In how many years will the ratio of their ages be $2 : 1$?

   (A) 2 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 8

7. Two right circular cylinders have the same volume. The radius of the second cylinder is 10% more than the radius of the first. What is the relationship between the heights of the two cylinders?

   (A) The second height is 10% less than the first.
   (B) The first height is 10% more than the second.
   (C) The second height is 21% less than the first.
   (D) The first height is 21% more than the second.
   (E) The second height is 80% of the first.

8. The ratio of the length to the width of a rectangle is $4 : 3$. If the rectangle has diagonal of length d, then the area may be expressed as kd^2 for some constant k. What is k?

   (A) $\frac{2}{7}$ **(B)** $\frac{3}{7}$ **(C)** $\frac{12}{25}$ **(D)** $\frac{16}{25}$ **(E)** $\frac{3}{4}$

9. A box contains 2 red marbles, 2 green marbles, and 2 yellow marbles. Carol takes 2 marbles from the box at random; then Claudia takes 2 of the remaining marbles at random; and then Cheryl takes the last 2 marbles. What is the probability that Cheryl gets 2 marbles of the same color?

   (A) $\frac{1}{10}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

10. Integers x and y with $x > y > 0$ satisfy $x + y + xy = 80$. What is x?

    (A) 8 **(B)** 10 **(C)** 15 **(D)** 18 **(E)** 26

11. On a sheet of paper, Isabella draws a circle of radius 2, a circle of radius 3, and all possible lines simultaneously tangent to both circles. Isabella notices that she has drawn exactly $k \geq 0$ lines. How many different values of k are possible?

    (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

12. The parabolas $y = ax^2 - 2$ and $y = 4 - bx^2$ intersect the coordinate axes in exactly four points, and these four points are the vertices of a kite of area 12. What is $a + b$?

    (A) 1 **(B)** 1.5 **(C)** 2 **(D)** 2.5 **(E)** 3

13. A league with 12 teams holds a round-robin tournament, with each team playing every other team exactly once. Games either end with one team victorious or else end in a draw. A team scores 2 points for every game it wins and 1 point for every game it draws. Which of the following is NOT a true statement about the list of 12 scores?

    (A) There must be an even number of odd scores.
    (B) There must be an even number of even scores.
    (C) There cannot be two scores of 0.
    (D) The sum of the scores must be at least 100.
    (E) The highest score must be at least 12.

14. What is the value of a for which $\frac{1}{\log_2 a} + \frac{1}{\log_3 a} + \frac{1}{\log_4 a} = 1$?

    (A) 9 **(B)** 12 **(C)** 18 **(D)** 24 **(E)** 36

15. What is the minimum number of digits to the right of the decimal point needed to express the fraction $\frac{123456789}{2^{26} \cdot 5^4}$ as a decimal?

    (A) 4 **(B)** 22 **(C)** 26 **(D)** 30 **(E)** 104

16. Tetrahedron $ABCD$ has $AB = 5, AC = 3, BC = 4, BD = 4, AD = 3$, and $CD = \frac{12}{5}\sqrt{2}$. What is the volume of the tetrahedron?

    (A) $3\sqrt{2}$ **(B)** $2\sqrt{5}$ **(C)** $\frac{24}{5}$ **(D)** $3\sqrt{3}$ **(E)** $\frac{24}{5}\sqrt{2}$

17. Eight people are sitting around a circular table, each holding a fair coin. All eight people flip their coins and those who flip heads stand while those who flip tails remain seated. What is the probability that no two adjacent people will stand?

    (A) $\frac{47}{256}$ **(B)** $\frac{3}{16}$ **(C)** $\frac{49}{256}$ **(D)** $\frac{25}{128}$ **(E)** $\frac{51}{256}$

18. The zeros of the function $f(x) = x^2 - ax + 2a$ are integers. What is the sum of the possible values of a?

    (A) 7 **(B)** 8 **(C)** 16 **(D)** 17 **(E)** 18

19. For some positive integers p, there is a quadrilateral $ABCD$ with positive integer side lengths, perimeter p, right angles at B and C, $AB = 2$, and $CD = AD$. How many different values of $p < 2015$ are possible?

    (A) 30 **(B)** 31 **(C)** 61 **(D)** 62 **(E)** 63

20. Isosceles triangles T and T' are not congruent but have the same area and the same perimeter. The sides of T have lengths of $5, 5$, and 8, while those of T' have lengths of a, a, and b. Which of the following numbers is closest to b?

    (A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 8

21. A circle of radius r passes through both foci of, and exactly four points on, the ellipse with equation $x^2 + 16y^2 = 16$. The set of all possible values of r is an interval $[a, b)$. What is $a + b$?

    (A) $5\sqrt{2} + 4$ **(B)** $\sqrt{17} + 7$ **(C)** $6\sqrt{2} + 3$ **(D)** $\sqrt{15} + 8$ **(E)** 12

22. For each positive integer n, let $S(n)$ be the number of sequences of length n consisting solely of the letters A and B, with no more than three As in a row and no more than three Bs in a row. What is the remainder when $S(2015)$ is divided by 12?

    (A) 0 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 10

23. Let S be a square of side length 1. Two points are chosen independently at random on the sides of S. The probability that the straight-line distance between the points is at least $\frac{1}{2}$ is $\frac{a-b\pi}{c}$, where a, b, and c are positive integers and $\gcd(a, b, c) = 1$. What is $a + b + c$?

    (A) 59 **(B)** 60 **(C)** 61 **(D)** 62 **(E)** 63

24. Rational numbers a and b are chosen at random among all rational numbers in the interval $[0, 2)$ that can be written as fractions $\frac{n}{d}$ where n and d are integers with $1 \le d \le 5$. What is the probability that

$$(\cos(a\pi) + i\sin(b\pi))^4$$

is a real number?

(A) $\frac{3}{50}$ **(B)** $\frac{4}{25}$ **(C)** $\frac{41}{200}$ **(D)** $\frac{6}{25}$ **(E)** $\frac{13}{50}$

25. A collection of circles in the upper half-plane, all tangent to the x-axis, is constructed in layers as follows. Layer L_0 consists of two circles of radii 70^2 and 73^2 that are externally tangent. For $k \ge 1$, the circles in $\bigcup_{j=0}^{k-1} L_j$ are ordered according to their points of tangency with the x-axis. For every pair of consecutive circles in this order, a new circle is constructed externally tangent to each of the two circles in the pair. Layer L_k consists of the 2^{k-1} circles constructed in this way. Let $S = \bigcup_{j=0}^{6} L_j$, and for every circle C denote by $r(C)$ its radius. What is

$$\sum_{C \in S} \frac{1}{\sqrt{r(C)}}?$$

(A) $\frac{286}{35}$ **(B)** $\frac{583}{70}$ **(C)** $\frac{715}{73}$ **(D)** $\frac{143}{14}$ **(E)** $\frac{1573}{146}$

2015 AMC1212B

1. What is the value of $2 - (-2)^{-2}$?

   (A) -2 **(B)** $\frac{1}{16}$ **(C)** $\frac{7}{4}$ **(D)** $\frac{9}{4}$ **(E)** 6

2. Marie does three equally time-consuming tasks in a row without taking breaks. She begins the first task at 1:00 PM and finishes the second task at 2:40 PM. When does she finish the third task?

   (A) 3:10 PM **(B)** 3:30 PM **(C)** 4:00 PM **(D)** 4:10 PM **(E)** 4:30 PM

3. Isaac has written down one integer two times and another integer three times. The sum of the five numbers is 100, and one of the numbers is 28. What is the other number?

   (A) 8 **(B)** 11 **(C)** 14 **(D)** 15 **(E)** 18

4. David, Hikmet, Jack, Marta, Rand, and Todd were in a 12-person race with 6 other people. Rand finished 6 places ahead of Hikmet. Marta finished 1 place behind Jack. David finished 2 places behind Hikmet. Jack finished 2 places behind Todd. Todd finished 1 place behind Rand. Marta finished in 6th place. Who finished in 8th place?

   (A) David **(B)** Hikmet **(C)** Jack **(D)** Rand **(E)** Todd

5. The Tigers beat the Sharks 2 out of the 3 times they played. They then played N more times, and the Sharks ended up winning at least 95

   (A) 35 **(B)** 37 **(C)** 39 **(D)** 41 **(E)** 43

6. Back in 1930, Tillie had to memorize her multiplication facts from 0×0 to 12×12. The multiplication table she was given had rows and columns labeled with the factors, and the products formed the body of the table. To the nearest hundredth, what fraction of the numbers in the body of the table are odd?

   (A) 0.21 **(B)** 0.25 **(C)** 0.46 **(D)** 0.50 **(E)** 0.75

7. A regular 15-gon has L lines of symmetry, and the smallest positive angle for which it has rotational symmetry is R degrees. What is $L + R$?

   (A) 24 **(B)** 27 **(C)** 32 **(D)** 39 **(E)** 54

8. What is the value of $(625^{\log_5 2015})^{\frac{1}{4}}$?

   (A) 5 **(B)** $\sqrt[4]{2015}$ **(C)** 625 **(D)** 2015 **(E)** $\sqrt[4]{5^{2015}}$

9. Larry and Julius are playing a game, taking turns throwing a ball at a bottle sitting on a ledge. Larry throws first. The winner is the first person to knock the bottle off the ledge. At each turn the probability that a player knocks the bottle off the ledge is $\frac{1}{2}$, independently of what has happened before. What is the probability that Larry wins the game?

   (A) $\frac{1}{2}$ **(B)** $\frac{3}{5}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{3}{4}$ **(E)** $\frac{4}{5}$

10. How many noncongruent integer-sided triangles with positive area and perimeter less than 15 are neither equilateral, isosceles, nor right triangles?

    (A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

11. The line $12x + 5y = 60$ forms a triangle with the coordinate axes. What is the sum of the lengths of the altitudes of this triangle?

    (A) 20 **(B)** $\frac{360}{17}$ **(C)** $\frac{107}{5}$ **(D)** $\frac{43}{2}$ **(E)** $\frac{281}{13}$

12. Let a, b, and c be three distinct one-digit numbers. What is the maximum value of the sum of the roots of the equation $(x - a)(x - b) + (x - b)(x - c) = 0$?

    (A) 15 **(B)** 15.5 **(C)** 16 **(D)** 16.5 **(E)** 17

13. Quadrilateral $ABCD$ is inscribed in a circle with $\angle BAC = 70^\circ, \angle ADB = 40^\circ, AD = 4$, and $BC = 6$. What is AC?

(A) $3+\sqrt{5}$ **(B)** 6 **(C)** $\frac{9}{2}\sqrt{2}$ **(D)** $8-\sqrt{2}$ **(E)** 7

14. A circle of radius 2 is centered at A. An equilateral triangle with side 4 has a vertex at A. What is the difference between the area of the region that lies inside the circle but outside the triangle and the area of the region that lies inside the triangle but outside the circle?

(A) $8-\pi$ **(B)** $\pi+2$ **(C)** $2\pi-\frac{\sqrt{2}}{2}$ **(D)** $4(\pi-\sqrt{3})$ **(E)** $2\pi-\frac{\sqrt{3}}{2}$

15. At Rachelle's school an A counts 4 points, a B 3 points, a C 2 points, and a D 1 point. Her GPA on the four classes she is taking is computed as the total sum of points divided by 4. She is certain that she will get As in both Mathematics and Science, and at least a C in each of English and History. She thinks she has a $\frac{1}{6}$ chance of getting an A in English, and a $\frac{1}{4}$ chance of getting a B. In History, she has a $\frac{1}{4}$ chance of getting an A, and a $\frac{1}{3}$ chance of getting a B, independently of what she gets in English. What is the probability that Rachelle will get a GPA of at least 3.5?

(A) $\frac{11}{72}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{3}{16}$ **(D)** $\frac{11}{24}$ **(E)** $\frac{1}{2}$

16. A regular hexagon with sides of length 6 has an isosceles triangle attached to each side. Each of these triangles has two sides of length 8. The isosceles triangles are folded to make a pyramid with the hexagon as the base of the pyramid. What is the volume of the pyramid?

(A) 18 **(B)** 162 **(C)** $36\sqrt{21}$ **(D)** $18\sqrt{138}$ **(E)** $54\sqrt{21}$

17. An unfair coin lands on heads with a probability of $\frac{1}{4}$. When tossed n times, the probability of exactly two heads is the same as the probability of exactly three heads. What is the value of n ?

(A) 5 **(B)** 8 **(C)** 10 **(D)** 11 **(E)** 13

18. For every composite positive integer n, define $r(n)$ to be the sum of the factors in the prime factorization of n. For example, $r(50) = 12$ because the prime factorization of 50 is 2×5^2, and $2+5+5 = 12$. What is the range of the function r, $\{r(n) : n \text{ is a composite positive integer}\}$?

(A) the set of positive integers
(B) the set of composite positive integers
(C) the set of even positive integers
(D) the set of integers greater than 3
(E) the set of integers greater than 4

19. In $\triangle ABC$, $\angle C = 90^\circ$ and $AB = 12$. Squares $ABXY$ and $CBWZ$ are constructed outside of the triangle. The points X, Y, Z, and W lie on a circle. What is the perimeter of the triangle?

(A) $12+9\sqrt{3}$ **(B)** $18+6\sqrt{3}$ **(C)** $12+12\sqrt{2}$ **(D)** 30 **(E)** 32

20. For every positive integer n, let $\text{mod}_5(n)$ be the remainder obtained when n is divided by 5. Define a function $f : \{0,1,2,3,\dots\} \times \{0,1,2,3,4\} \to \{0,1,2,3,4\}$ recursively as follows:

$$f(i,j) = \begin{cases} \text{mod}_5(j+1) & \text{if } i=0 \text{ and } 0 \le j \le 4, \\ f(i-1,1) & \text{if } i \ge 1 \text{ and } j=0, \text{ and} \\ f(i-1, f(i,j-1)) & \text{if } i \ge 1 \text{ and } 1 \le j \le 4. \end{cases}$$

What is $f(2015,2)$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

21. Cozy the Cat and Dash the Dog are going up a staircase with a certain number of steps. However, instead of walking up the steps one at a time, both Cozy and Dash jump. Cozy goes two steps up with each jump (though if necessary, he will just jump the last step). Dash goes five steps up with each jump (though if necessary, he will just jump the last steps if there are fewer than 5 steps left). Suppose that Dash takes 19 fewer jumps than Cozy to reach the top of the staircase. Let s denote the sum of all possible numbers of steps this staircase can have. What is the sum of the digits of s?

(A) 9 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 15

22. Six chairs are evenly spaced around a circular table. One person is seated in each chair. Each person gets up and sits down in a chair that is not the same chair and is not adjacent to the chair he or she originally occupied, so that again one person is seated in each chair. In how many ways can this be done?

    (A) 14 **(B)** 16 **(C)** 18 **(D)** 20 **(E)** 24

23. A rectangular box measures $a \times b \times c$, where a, b, and c are integers and $1 \leq a \leq b \leq c$. The volume and the surface area of the box are numerically equal. How many ordered triples (a, b, c) are possible?

    (A) 4 **(B)** 10 **(C)** 12 **(D)** 21 **(E)** 26

24. Four circles, no two of which are congruent, have centers at A, B, C, and D, and points P and Q lie on all four circles. The radius of circle A is $\frac{5}{8}$ times the radius of circle B, and the radius of circle C is $\frac{5}{8}$ times the radius of circle D. Furthermore, $AB = CD = 39$ and $PQ = 48$. Let R be the midpoint of $\overline{PQ}$. What is $AR + BR + CR + DR$?

    (A) 180 **(B)** 184 **(C)** 188 **(D)** 192 **(E)** 196

25. A bee starts flying from point P_0. She flies 1 inch due east to point P_1. For $j \geq 1$, once the bee reaches point P_j, she turns 30° counterclockwise and then flies $j + 1$ inches straight to point P_{j+1}. When the bee reaches P_{2015} she is exactly $a\sqrt{b} + c\sqrt{d}$ inches away from P_0, where a, b, c and d are positive integers and b and d are not divisible by the square of any prime. What is $a + b + c + d$?

    (A) 2016 **(B)** 2024 **(C)** 2032 **(D)** 2040 **(E)** 2048

2016 AMC1212A

1. What is the value of $\frac{11!-10!}{9!}$?

   (A) 99 **(B)** 100 **(C)** 110 **(D)** 121 **(E)** 132

2. For what value of x does $10^x \cdot 100^{2x} = 1000^5$?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

3. The remainder can be defined for all real numbers x and y with $y \neq 0$ by

   $$\text{rem}(x, y) = x - y \left\lfloor \frac{x}{y} \right\rfloor$$

   where $\left\lfloor \frac{x}{y} \right\rfloor$ denotes the greatest integer less than or equal to $\frac{x}{y}$. What is the value of $\text{rem}(\frac{3}{8}, -\frac{2}{5})$?

   (A) $-\frac{3}{8}$ **(B)** $-\frac{1}{40}$ **(C)** 0 **(D)** $\frac{3}{8}$ **(E)** $\frac{31}{40}$

4. The mean, median, and mode of the 7 data values $60, 100, x, 40, 50, 200, 90$ are all equal to x. What is the value of x?

   (A) 50 **(B)** 60 **(C)** 75 **(D)** 90 **(E)** 100

5. Goldbach's conjecture states that every even integer greater than 2 can be written as the sum of two prime numbers (for example, $2016 = 13 + 2003$). So far, no one has been able to prove that the conjecture is true, and no one has found a counterexample to show that the conjecture is false. What would a counterexample consist of?

   (A) an odd integer greater than 2 that can be written as the sum of two prime numbers
   (B) an odd integer greater than 2 that cannot be written as the sum of two prime numbers
   (C) an even integer greater than 2 that can be written as the sum of two numbers that are not prime
   (D) an even integer greater than 2 that can be written as the sum of two prime numbers
   (E) an even integer greater than 2 that cannot be written as the sum of two prime numbers

6. A triangular array of 2016 coins has 1 coin in the first row, 2 coins in the second row, 3 coins in the third row, and so on up to N coins in the Nth row. What is the sum of the digits of N ?

   (A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

7. Which of these describes the graph of $x^2(x + y + 1) = y^2(x + y + 1)$?

   (A) two parallel lines
   (B) two intersecting lines
   (C) three lines that all pass through a common point
   (D) three lines that do not all pass through a common point
   (E) a line and a parabola

8. What is the area of the shaded region of the given 8×5 rectangle?

   (A) 4.75 **(B)** 5 **(C)** 5.25 **(D)** 6.5 **(E)** 8

9. The five small shaded squares inside this unit square are congruent and have disjoint interiors. The midpoint of each side of the middle square coincides with one of the vertices of the other four small squares as shown. The common side length is $\frac{a-\sqrt{2}}{b}$, where a and b are positive integers. What is $a + b$?

   (A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

10. Five friends sat in a movie theater in a row containing 5 seats, numbered 1 to 5 from left to right. (The directions "left" and "right" are from the point of view of the people as they sit in the seats.) During the movie Ada went to the lobby to get some popcorn. When she returned, she found that Bea had moved two seats to the right, Ceci had moved one seat to the left, and Dee and Edie had switched seats, leaving an end seat for Ada. In which seat had Ada been sitting before she got up?

    (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

11. Each of the 100 students in a certain summer camp can either sing, dance, or act. Some students have more than one talent, but no student has all three talents. There are 42 students who cannot sing, 65 students who cannot dance, and 29 students who cannot act. How many students have two of these talents?

    (A) 16 **(B)** 25 **(C)** 36 **(D)** 49 **(E)** 64

12. In $\triangle ABC$, $AB = 6$, $BC = 7$, and $CA = 8$. Point D lies on $\overline{BC}$, and $\overline{AD}$ bisects $\angle BAC$. Point E lies on $\overline{AC}$, and $\overline{BE}$ bisects $\angle ABC$. The bisectors intersect at F. What is the ratio $AF : FD$?

    (A) $3:2$ **(B)** $5:3$ **(C)** $2:1$ **(D)** $7:3$ **(E)** $5:2$

13. Let N be a positive multiple of 5. One red ball and N green balls are arranged in a line in random order. Let $P(N)$ be the probability that at least $\frac{3}{5}$ of the green balls are on the same side of the red ball. Observe that $P(5) = 1$ and that $P(N)$ approaches $\frac{4}{5}$ as N grows large. What is the sum of the digits of the least value of N such that $P(N) < \frac{321}{400}$?

    (A) 12 **(B)** 14 **(C)** 16 **(D)** 18 **(E)** 20

14. Each vertex of a cube is to be labeled with an integer from 1 through 8, with each integer being used once, in such a way that the sum of the four numbers on the vertices of a face is the same for each face. Arrangements that can be obtained from each other through rotations of the cube are considered to be the same. How many different arrangements are possible?

    (A) 1 **(B)** 3 **(C)** 6 **(D)** 12 **(E)** 24

15. Circles with centers P, Q and R, having radii $1, 2$ and 3, respectively, lie on the same side of line l and are tangent to l at P', Q' and R', respectively, with Q' between P' and R'. The circle with center Q is externally tangent to each of the other two circles. What is the area of triangle PQR?

    (A) 0 **(B)** $\frac{\sqrt{6}}{3}$ **(C)** 1 **(D)** $\sqrt{6}-\sqrt{2}$ **(E)** $\frac{\sqrt{6}}{2}$

16. The graphs of $y = \log_3 x, y = \log_x 3, y = \log_{\frac{1}{3}} x$, and $y = \log_x \dfrac{1}{3}$ are plotted on the same set of axes. How many points in the plane with positive x-coordinates lie on two or more of the graphs?

    (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

17. Let $ABCD$ be a square. Let E, F, G and H be the centers, respectively, of equilateral triangles with bases $\overline{AB}, \overline{BC}, \overline{CD}$, and $\overline{DA}$, each exterior to the square. What is the ratio of the area of square $EFGH$ to the area of square $ABCD$?

    (A) 1 **(B)** $\frac{2+\sqrt{3}}{3}$ **(C)** $\sqrt{2}$ **(D)** $\frac{\sqrt{2}+\sqrt{3}}{2}$ **(E)** $\sqrt{3}$

18. For some positive integer n, the number $110n^3$ has 110 positive integer divisors, including 1 and the number $110n^3$. How many positive integer divisors does the number $81n^4$ have?

    (A) 110 **(B)** 191 **(C)** 261 **(D)** 325 **(E)** 425

19. Jerry starts at 0 on the real number line. He tosses a fair coin 8 times. When he gets heads, he moves 1 unit in the positive direction; when he gets tails, he moves 1 unit in the negative direction. The probability that he reaches 4 at some time during this process is $\frac{a}{b}$, where a and b are relatively prime positive integers. What is $a + b$? (For example, he succeeds if his sequence of tosses is $HTHHHHHH$.)

    (A) 69 **(B)** 151 **(C)** 257 **(D)** 293 **(E)** 313

20. A binary operation $\diamondsuit$ has the properties that $a \diamondsuit (b \diamondsuit c) = (a \diamondsuit b) \cdot c$ and that $a \diamondsuit a = 1$ for all nonzero real numbers a, b and c. (Here the dot $\cdot$ represents the usual multiplication operation.) The solution to the equation $2016 \diamondsuit (6 \diamondsuit x) = 100$ can be written as $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $p + q$?

    (A) 109 **(B)** 201 **(C)** 301 **(D)** 3049 **(E)** $33,601$

21. A quadrilateral is inscribed in a circle of radius $200\sqrt{2}$. Three of the sides of this quadrilateral have length 200. What is the length of its fourth side?

    (A) 200 **(B)** $200\sqrt{2}$ **(C)** $200\sqrt{3}$ **(D)** $300\sqrt{2}$ **(E)** 500

22. How many ordered triples (x, y, z) of positive integers satisfy $\text{lcm}(x, y) = 72, \text{lcm}(x, z) = 600$ and $\text{lcm}(y, z) = 900$?

    (A) 15 **(B)** 16 **(C)** 24 **(D)** 27 **(E)** 64

23. Three numbers in the interval $[0, 1]$ are chosen independently and at random. What is the probability that the chosen numbers are the side lengths of a triangle with positive area?

    (A) $\dfrac{1}{6}$ **(B)** $\dfrac{1}{3}$ **(C)** $\dfrac{1}{2}$ **(D)** $\dfrac{2}{3}$ **(E)** $\dfrac{5}{6}$

24. There is a smallest positive real number a such that there exists a positive real number b such that all the roots of the polynomial $x^3 - ax^2 + bx - a$ are real. In fact, for this value of a the value of b is unique. What is the value of b?

    (A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

25. Let k be a positive integer. Bernardo and Silvia take turns writing and erasing numbers on a blackboard as follows: Bernardo starts by writing the smallest perfect square with $k+1$ digits. Every time Bernardo writes a number, Silvia erases the last k digits of it. Bernardo then writes the next perfect square, Silvia erases the last k digits of it, and this process continues until the last two numbers that remain on the board differ by at least 2. Let $f(k)$ be the smallest positive integer not written on the board. For example, if $k = 1$, then the numbers that Bernardo writes are $16, 25, 36, 49, 64$, and the numbers showing on the board after Silvia erases are $1, 2, 3, 4$, and 6, and thus $f(1) = 5$. What is the sum of the digits of $f(2) + f(4) + f(6) + ... + f(2016)$?

    (A) 7986 **(B)** 8002 **(C)** 8030 **(D)** 8048 **(E)** 8064

2016 AMC1212B

1. What is the value of $\frac{2a^{-1}+\frac{a^{-1}}{2}}{a}$ when $a=\frac{1}{2}$?
   (A) 1 **(B)** 2 **(C)** $\frac{5}{2}$ **(D)** 10 **(E)** 20

2. The harmonic mean of two numbers can be calculated as twice their product divided by their sum. The harmonic mean of 1 and 2016 is closest to which integer?
   (A) 2 **(B)** 45 **(C)** 504 **(D)** 1008 **(E)** 2015

3. Let $x=-2016$. What is the value of $\Bigg\vert \Big\vert |x|-x\Big\vert - |x| \Bigg\vert - x$?
   (A) -2016 **(B)** 0 **(C)** 2016 **(D)** 4032 **(E)** 6048

4. The ratio of the measures of two acute angles is $5:4$, and the complement of one of these two angles is twice as large as the complement of the other. What is the sum of the degree measures of the two angles?
   (A) 75 **(B)** 90 **(C)** 135 **(D)** 150 **(E)** 270

5. The War of 1812 started with a declaration of war on Thursday, June 18, 1812. The peace treaty to end the war was signed 919 days later, on December 24, 1814. On what day of the week was the treaty signed?
   (A) Friday **(B)** Saturday **(C)** Sunday **(D)** Monday **(E)** Tuesday

6. All three vertices of $\triangle ABC$ lie on the parabola defined by $y=x^2$, with A at the origin and $\overline{BC}$ parallel to the x-axis. The area of the triangle is 64. What is the length of BC?
   (A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 16

7. Josh writes the numbers $1,2,3,\dots,99,100$. He marks out 1, skips the next number (2), marks out 3, and continues skipping and marking out the next number to the end of the list. Then he goes back to the start of his list, marks out the first remaining number (2), skips the next number (4), marks out 6, skips 8, marks out 10, and so on to the end. Josh continues in this manner until only one number remains. What is that number?
   (A) 13 **(B)** 32 **(C)** 56 **(D)** 64 **(E)** 96

8. A thin piece of wood of uniform density in the shape of an equilateral triangle with side length 3 inches weighs 12 ounces. A second piece of the same type of wood, with the same thickness, also in the shape of an equilateral triangle, has side length of 5 inches. Which of the following is closest to the weight, in ounces, of the second piece?
   (A) 14.0 **(B)** 16.0 **(C)** 20.0 **(D)** 33.3 **(E)** 55.6

9. Carl decided to fence in his rectangular garden. He bought 20 fence posts, placed one on each of the four corners, and spaced out the rest evenly along the edges of the garden, leaving exactly 4 yards between neighboring posts. The longer side of his garden, including the corners, has twice as many posts as the shorter side, including the corners. What is the area, in square yards, of Carls garden?
   (A) 256 **(B)** 336 **(C)** 384 **(D)** 448 **(E)** 512

10. A quadrilateral has vertices $P(a,b)$, $Q(b,a)$, $R(-a,-b)$, and $S(-b,-a)$, where a and b are integers with $a>b>0$. The area of $PQRS$ is 16. What is $a+b$?
    (A) 4 **(B)** 5 **(C)** 6 **(D)** 12 **(E)** 13

11. How many squares whose sides are parallel to the axes and whose vertices have coordinates that are integers lie entirely within the region bounded by the line $y=\pi x$, the line $y=-0.1$ and the line $x=5.1$?
    (A) 30 **(B)** 41 **(C)** 45 **(D)** 50 **(E)** 57

12. All the numbers $1,2,3,4,5,6,7,8,9$ are written in a 3×3 array of squares, one number in each square, in such a way that if two numbers are consecutive then they occupy squares that share an edge. The numbers in the four corners add up to 18. What is the number in the center?
    (A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

13. Alice and Bob live 10 miles apart. One day Alice looks due north from her house and sees an airplane. At the same time Bob looks due west from his house and sees the same airplane. The angle of elevation of the airplane is 30° from Alice's position and 60° from Bob's position. Which of the following is closest to the airplane's altitude, in miles?

    (A) 3.5 **(B)** 4 **(C)** 4.5 **(D)** 5 **(E)** 5.5

14. The sum of an infinite geometric series is a positive number S, and the second term in the series is 1. What is the smallest possible value of S?

    (A) $\frac{1+\sqrt{5}}{2}$ **(B)** 2 **(C)** $\sqrt{5}$ **(D)** 3 **(E)** 4

15. All the numbers $2,3,4,5,6,7$ are assigned to the six faces of a cube, one number to each face. For each of the eight vertices of the cube, a product of three numbers is computed, where the three numbers are the numbers assigned to the three faces that include that vertex. What is the greatest possible value of the sum of these eight products?

    (A) 312 **(B)** 343 **(C)** 625 **(D)** 729 **(E)** 1680

16. In how many ways can 345 be written as the sum of an increasing sequence of two or more consecutive positive integers?

    (A) 1 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 7

17. In $\triangle ABC$ shown in the figure, $AB = 7$, $BC = 8$, $CA = 9$, and $\overline{AH}$ is an altitude. Points D and E lie on sides $\overline{AC}$ and $\overline{AB}$, respectively, so that $\overline{BD}$ and $\overline{CE}$ are angle bisectors, intersecting $\overline{AH}$ at Q and P, respectively. What is PQ?

    (A) 1 **(B)** $\frac{5}{8}\sqrt{3}$ **(C)** $\frac{4}{5}\sqrt{2}$ **(D)** $\frac{8}{15}\sqrt{5}$ **(E)** $\frac{6}{5}$

18. What is the area of the region enclosed by the graph of the equation $x^2 + y^2 = |x| + |y|$?

    (A) $\pi + \sqrt{2}$ **(B)** $\pi + 2$ **(C)** $\pi + 2\sqrt{2}$ **(D)** $2\pi + \sqrt{2}$ **(E)** $2\pi + 2\sqrt{2}$

19. Tom, Dick, and Harry are playing a game. Starting at the same time, each of them flips a fair coin repeatedly until he gets his first head, at which point he stops. What is the probability that all three flip their coins the same number of times?

    (A) $\frac{1}{8}$ **(B)** $\frac{1}{7}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{1}{3}$

20. A set of teams held a round-robin tournament in which every team played every other team exactly once. Every team won 10 games and lost 10 games; there were no ties. How many sets of three teams $\{A, B, C\}$ were there in which A beat B, B beat C, and C beat A?

    (A) 385 **(B)** 665 **(C)** 945 **(D)** 1140 **(E)** 1330

21. Let $ABCD$ be a unit square. Let Q_1 be the midpoint of $\overline{CD}$. For $i = 1, 2, \ldots$, let P_i be the intersection of $\overline{AQ_i}$ and $\overline{BD}$, and let Q_{i+1} be the foot of the perpendicular from P_i to $\overline{CD}$. What is

    $$\sum_{i=1}^{\infty} \text{Area of } \triangle DQ_iP_i\,?$$

    (A) $\frac{1}{6}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{2}$ **(E)** 1

22. For a certain positive integer n less than 1000, the decimal equivalent of $\frac{1}{n}$ is $0.\overline{abcdef}$, a repeating decimal of period of 6, and the decimal equivalent of $\frac{1}{n+6}$ is $0.\overline{wxyz}$, a repeating decimal of period 4. In which interval does n lie?

    (A) $[1, 200]$ **(B)** $[201, 400]$ **(C)** $[401, 600]$ **(D)** $[601, 800]$ **(E)** $[801, 999]$

23. What is the volume of the region in three-dimensional space defined by the inequalities $|x|+|y|+|z| \le 1$ and $|x|+|y|+|z-1| \le 1$?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{2}$ **(E)** 1

24. There are exactly $77,000$ ordered quadruplets (a, b, c, d) such that $\gcd(a, b, c, d) = 77$ and $\operatorname{lcm}(a, b, c, d) = n$. What is the smallest possible value for n?

(A) $13,860$ **(B)** $20,790$ **(C)** $21,560$ **(D)** $27,720$ **(E)** $41,580$

25. The sequence (a_n) is defined recursively by $a_0 = 1$, $a_1 = \sqrt[19]{2}$, and $a_n = a_{n-1}a_{n-2}^2$ for $n \ge 2$. What is the smallest positive integer k such that the product $a_1 a_2 \cdots a_k$ is an integer?

(A) 17 **(B)** 18 **(C)** 19 **(D)** 20 **(E)** 21

2017 AMC1212A

1. Pablo buys popsicles for his friends. The store sells single popsicles for \$1 each, 3-popsicle boxes for \$2, and 5-popsicle boxes for \$3. What is the greatest number of popsicles that Pablo can buy with \$8?

   (A) 8 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 15

2. The sum of two nonzero real numbers is 4 times their product. What is the sum of the reciprocals of the two numbers?

   (A) 1 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 12

3. Ms. Carroll promised that anyone who got all the multiple choice questions right on the upcoming exam would receive an A on the exam. Which one of these statements necessarily follows logically?

   (A) If Lewis did not receive an A, then he got all of the multiple choice questions wrong.
   (B) If Lewis did not receive an A, then he got at least one of the multiple choice questions wrong.
   (C) If Lewis got at least one of the multiple choice questions wrong, then he did not receive an A.
   (D) If Lewis received an A, then he got all of the multiple choice questions right.
   (E) If Lewis received an A, then he got at least one of the multiple choice questions right.

4. Jerry and Silvia wanted to go from the southwest corner of a square field to the northeast corner. Jerry walked due east and then due north to reach the goal, but Silvia headed northeast and reached the goal walking in a straight line. Which of the following is closest to how much shorter Silvia's trip was, compared to Jerry's trip?

   (A) 30% **(B)** 40% **(C)** 50% **(D)** 60% **(E)** 70%

5. At a gathering of 30 people, there are 20 people who all know each other and 10 people who know no one. People who know each other hug, and people who do not know each other shake hands. How many handshakes occur?

   (A) 240 **(B)** 245 **(C)** 290 **(D)** 480 **(E)** 490

6. Joy has 30 thin rods, one each of every integer length from 1 cm through 30 cm. She places the rods with lengths 3 cm, 7 cm, and 15cm on a table. She then wants to choose a fourth rod that she can put with these three to form a quadrilateral with positive area. How many of the remaining rods can she choose as the fourth rod?

   (A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 20

7. Define a function on the positive integers recursively by $f(1) = 2$, $f(n) = f(n-1) + 1$ if n is even, and $f(n) = f(n-2) + 2$ if n is odd and greater than 1. What is $f(2017)$?

   (A) 2017 **(B)** 2018 **(C)** 4034 **(D)** 4035 **(E)** 4036

8. The region consisting of all points in three-dimensional space within 3 units of line segment $\overline{AB}$ has volume 216π. What is the length AB?

   (A) 6 **(B)** 12 **(C)** 18 **(D)** 20 **(E)** 24

9. Let S be the set of points (x, y) in the coordinate plane such that two of the three quantities 3, $x+2$, and $y-4$ are equal and the third of the three quantities is no greater than the common value. Which of the following is a correct description of S?

   (A) a single point **(B)** two intersecting lines
   (C) three lines whose pairwise intersections are three distinct points
   (D) a triangle **(E)** three rays with a common point

10. Chlo chooses a real number uniformly at random from the interval $[0, 2017]$. Independently, Laurent chooses a real number uniformly at random from the interval $[0, 4034]$. What is the probability that Laurent's number is greater than Chloe's number?

    (A) $\frac{1}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{5}{6}$ **(E)** $\frac{7}{8}$

11. Claire adds the degree measures of the interior angles of a convex polygon and arrives at a sum of 2017. She then discovers that she forgot to include one angle. What is the degree measure of the forgotten angle?

(A) 37 **(B)** 63 **(C)** 117 **(D)** 143 **(E)** 163

12. There are 10 horses, named Horse 1, Horse 2, ..., Horse 10. They get their names from how many minutes it takes them to run one lap around a circular race track: Horse k runs one lap in exactly k minutes. At time 0 all the horses are together at the starting point on the track. The horses start running in the same direction, and they keep running around the circular track at their constant speeds. The least time $S > 0$, in minutes, at which all 10 horses will again simultaneously be at the starting point is $S = 2520$. Let $T > 0$ be the least time, in minutes, such that at least 5 of the horses are again at the starting point. What is the sum of the digits of T?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

13. Driving at a constant speed, Sharon usually takes 180 minutes to drive from her house to her mother's house. One day Sharon begins the drive at her usual speed, but after driving $\frac{1}{3}$ of the way, she hits a bad snowstorm and reduces her speed by 20 miles per hour. This time the trip takes her a total of 276 minutes. How many miles is the drive from Sharon's house to her mother's house?

(A) 132 **(B)** 135 **(C)** 138 **(D)** 141 **(E)** 144

14. Alice refuses to sit next to either Bob or Carla. Derek refuses to sit next to Eric. How many ways are there for the five of them to sit in a row of 5 chairs under these conditions?

(A) 12 **(B)** 16 **(C)** 28 **(D)** 32 **(E)** 40

15. Let $f(x) = \sin x + 2\cos x + 3\tan x$, using radian measure for the variable x. In what interval does the smallest positive value of x for which $f(x) = 0$ lie?

(A) $(0,1)$ **(B)** $(1,2)$ **(C)** $(2,3)$ **(D)** $(3,4)$ **(E)** $(4,5)$

16. In the figure below, semicircles with centers at A and B and with radii 2 and 1, respectively, are drawn in the interior of, and sharing bases with, a semicircle with diameter JK. The two smaller semicircles are externally tangent to each other and internally tangent to the largest semicircle. A circle centered at P is drawn externally tangent to the two smaller semicircles and internally tangent to the largest semicircle. What is the radius of the circle centered at P?

(A) $\frac{3}{4}$ **(B)** $\frac{6}{7}$ **(C)** $\frac{\sqrt{3}}{2}$ **(D)** $\frac{5}{8}\sqrt{2}$ **(E)** $\frac{11}{12}$

17. There are 24 different complex numbers z such that $z^{24} = 1$. For how many of these is z^6 a real number?

(A) 0 **(B)** 4 **(C)** 6 **(D)** 12 **(E)** 24

18. Let $S(n)$ equal the sum of the digits of positive integer n. For example, $S(1507) = 13$. For a particular positive integer n, $S(n) = 1274$. Which of the following could be the value of $S(n+1)$?

(A) 1 **(B)** 3 **(C)** 12 **(D)** 1239 **(E)** 1265

19. A square with side length x is inscribed in a right triangle with sides of length 3, 4, and 5 so that one vertex of the square coincides with the right-angle vertex of the triangle. A square with side length y is inscribed in another right triangle with sides of length 3, 4, and 5 so that one side of the square lies on the hypotenuse of the triangle. What is $\frac{x}{y}$?

(A) $\frac{12}{13}$ **(B)** $\frac{35}{37}$ **(C)** 1 **(D)** $\frac{37}{35}$ **(E)** $\frac{13}{12}$

20. How many ordered pairs (a, b) such that a is a positive real number and b is an integer between 2 and 200, inclusive, satisfy the equation $(\log_b a)^{2017} = \log_b(a^{2017})$?

(A) 198 **(B)** 199 **(C)** 398 **(D)** 399 **(E)** 597

21. A set S is constructed as follows. To begin, $S = \{0, 10\}$. Repeatedly, as long as possible, if x is an integer root of some polynomial $a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$ for some $n \geq 1$, all of whose coefficients a_i are elements of S, then x is put into S. When no more elements can be added to S, how many elements does S have?

(A) 4 **(B)** 5 **(C)** 7 **(D)** 9 **(E)** 11

22. A square is drawn in the Cartesian coordinate plane with vertices at $(2, 2)$, $(-2, 2)$, $(-2, -2)$, $(2, -2)$. A particle starts at $(0, 0)$. Every second it moves with equal probability to one of the eight lattice points (points with integer coordinates) closest to its current position, independently of its previous moves. In other words, the probability is $1/8$ that the particle will move from (x, y) to each of $(x, y+1)$, $(x+1, y+1)$, $(x+1, y)$, $(x+1, y-1)$, $(x, y-1)$, $(x-1, y-1)$, $(x-1, y)$, or $(x-1, y+1)$. The particle will eventually hit the square for the first time, either at one of the 4 corners of the square or at one of the 12 lattice points in the interior of one of the sides of the square. The probability that it will hit at a corner rather than at an interior point of a side is m/n, where m and n are relatively prime positive integers. What is $m + n$?

(A) 4 **(B)** 5 **(C)** 7 **(D)** 15 **(E)** 39

23. For certain real numbers a, b, and c, the polynomial

$$g(x) = x^3 + ax^2 + x + 10$$

has three distinct roots, and each root of $g(x)$ is also a root of the polynomial

$$f(x) = x^4 + x^3 + bx^2 + 100x + c.$$

What is $f(1)$?

(A) -9009 **(B)** -8008 **(C)** -7007 **(D)** -6006 **(E)** -5005

24. Quadrilateral $ABCD$ is inscribed in circle O and has side lengths $AB = 3, BC = 2, CD = 6$, and $DA = 8$. Let X and Y be points on $\overline{BD}$ such that $\frac{DX}{BD} = \frac{1}{4}$ and $\frac{BY}{BD} = \frac{11}{36}$. Let E be the intersection of line AX and the line through Y parallel to $\overline{AD}$. Let F be the intersection of line CX and the line through E parallel to $\overline{AC}$. Let G be the point on circle O other than C that lies on line CX. What is $XF \cdot XG$?

(A) 17 **(B)** $\frac{59-5\sqrt{2}}{3}$ **(C)** $\frac{91-12\sqrt{3}}{4}$ **(D)** $\frac{67-10\sqrt{2}}{3}$ **(E)** 18

25. The vertices V of a centrally symmetric hexagon in the complex plane are given by

$$V = \left\{ \sqrt{2}i, -\sqrt{2}i, \frac{1}{\sqrt{8}}(1+i), \frac{1}{\sqrt{8}}(-1+i), \frac{1}{\sqrt{8}}(1-i), \frac{1}{\sqrt{8}}(-1-i) \right\}.$$

For each j, $1 \leq j \leq 12$, an element z_j is chosen from V at random, independently of the other choices. Let $P = \prod_{j=1}^{12} z_j$ be the product of the 12 numbers selected. What is the probability that $P = -1$?

(A) $\dfrac{5 \cdot 11}{3^{10}}$ **(B)** $\dfrac{5^2 \cdot 11}{2 \cdot 3^{10}}$ **(C)** $\dfrac{5 \cdot 11}{3^9}$ **(D)** $\dfrac{5 \cdot 7 \cdot 11}{2 \cdot 3^{10}}$ **(E)** $\dfrac{2^2 \cdot 5 \cdot 11}{3^{10}}$

2017 AMC1212B

1. Kymbrea's comic book collection currently has 30 comic books in it, and she is adding to her collection at the rate of 2 comic books per month. LaShawn's collection currently has 10 comic books in it, and he is adding to his collection at the rate of 6 comic books per month. After how many months will LaShawn's collection have twice as many comic books as Kymbrea's?

   (A) 1 **(B)** 4 **(C)** 5 **(D)** 20 **(E)** 25

2. Real numbers x, y, and z satisfy the inequalities $0 < x < 1$, $-1 < y < 0$, and $1 < z < 2$. Which of the following numbers is necessarily positive?

   (A) $y + x^2$ **(B)** $y + xz$ **(C)** $y + y^2$ **(D)** $y + 2y^2$ **(E)** $y + z$

3. Supposed that x and y are nonzero real numbers such that $\frac{3x+y}{x-3y} = -2$. What is the value of $\frac{x+3y}{3x-y}$?

   (A) -3 **(B)** -1 **(C)** 1 **(D)** 2 **(E)** 3

4. Samia set off on her bicycle to visit her friend, traveling at an average speed of 17 kilometers per hour. When she had gone half the distance to her friend's house, a tire went flat, and she walked the rest of the way at 5 kilometers per hour. In all it took her 44 minutes to reach her friend's house. In kilometers rounded to the nearest tenth, how far did Samia walk?

   (A) 2.0 **(B)** 2.2 **(C)** 2.8 **(D)** 3.4 **(E)** 4.4

5. The data set $[6, 19, 33, 33, 39, 41, 41, 43, 51, 57]$ has median $Q_2 = 40$, first quartile $Q_1 = 33$, and third quartile $Q_3 = 43$. An outlier in a data set is a value that is more than 1.5 times the interquartile range below the first quartile (Q_1) or more than 1.5 times the interquartile range above the third quartile (Q_3), where the interquartile range is defined as $Q_3 - Q_1$. How many outliers does this data set have?

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

6. The circle having $(0, 0)$ and $(8, 6)$ as the endpoints of a diameter intersects the x-axis at a second point. What is the x-coordinate of this point?

   (A) $4\sqrt{2}$ **(B)** 6 **(C)** $5\sqrt{2}$ **(D)** 8 **(E)** $6\sqrt{2}$

7. The functions $\sin(x)$ and $\cos(x)$ are periodic with least period 2π. What is the least period of the function $\cos(\sin(x))$?

   (A) $\frac{\pi}{2}$ **(B)** π **(C)** 2π **(D)** 4π **(E)** It's not periodic.

8. The ratio of the short side of a certain rectangle to the long side is equal to the ratio of the long side to the diagonal. What is the square of the ratio of the short side to the long side of this rectangle?

   (A) $\frac{\sqrt{3}-1}{2}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{\sqrt{5}-1}{2}$ **(D)** $\frac{\sqrt{2}}{2}$ **(E)** $\frac{\sqrt{6}-1}{2}$

9. A circle has center $(-10, -4)$ and radius 13. Another circle has center $(3, 9)$ and radius $\sqrt{65}$. The line passing through the two points of intersection of the two circles has equation $x + y = c$. What is c?

   (A) 3 **(B)** $3\sqrt{3}$ **(C)** $4\sqrt{2}$ **(D)** 6 **(E)** $\frac{13}{2}$

10. At Typico High School, 60% of the students like dancing, and the rest dislike it. Of those who like dancing, 80% say that they like it, and the rest say that they dislike it. Of those who dislike dancing, 90% say that they dislike it, and the rest say that they like it. What fraction of students who say they dislike dancing actually like it?

    (A) 10% **(B)** 12% **(C)** 20% **(D)** 25% **(E)** $33\frac{1}{3}$%

11. Call a positive integer *monotonous* if it is a one-digit number or its digits, when read from left to right, form either a strictly increasing or a strictly decreasing sequence. For example, 3, 23578, and 987620 are monotonous, but 88, 7434, and 23557 are not. How many monotonous positive integers are there?

    (A) 1024 **(B)** 1524 **(C)** 1533 **(D)** 1536 **(E)** 2048

12. What is the sum of the roots of $z^{12} = 64$ that have a positive real part?

    (A) 2 **(B)** 4 **(C)** $\sqrt{2} + 2\sqrt{3}$ **(D)** $2\sqrt{2} + \sqrt{6}$ **(E)** $(1 + \sqrt{3}) + (1 + \sqrt{3})i$

13. In the figure below, 3 of the 6 disks are to be painted blue, 2 are to be painted red, and 1 is to be painted green. Two paintings that can be obtained from one another by a rotation or a reflection of the entire figure are considered the same. How many different paintings are possible?

    (A) 6 **(B)** 8 **(C)** 9 **(D)** 12 **(E)** 15

14. An ice-cream novelty item consists of a cup in the shape of a 4-inch-tall frustum of a right circular cone, with a 2-inch-diameter base at the bottom and a 4-inch-diameter base at the top, packed solid with ice cream, together with a solid cone of ice cream of height 4 inches, whose base, at the bottom, is the top base of the frustum. What is the total volume of the ice cream, in cubic inches?

    (A) 8π **(B)** $\frac{28\pi}{3}$ **(C)** 12π **(D)** 14π **(E)** $\frac{44\pi}{3}$

15. Let ABC be an equilateral triangle. Extend side $\overline{AB}$ beyond B to a point B' so that $BB' = 3 \cdot AB$. Similarly, extend side $\overline{BC}$ beyond C to a point C' so that $CC' = 3 \cdot BC$, and extend side $\overline{CA}$ beyond A to a point A' so that $AA' = 3 \cdot CA$. What is the ratio of the area of $\triangle A'B'C'$ to the area of $\triangle ABC$?

    (A) 9 **(B)** 16 **(C)** 25 **(D)** 36 **(E)** 37

16. The number $21! = 51{,}090{,}942{,}171{,}709{,}440{,}000$ has over $60{,}000$ positive integer divisors. One of them is chosen at random. What is the probability that it is odd?

    (A) $\frac{1}{21}$ **(B)** $\frac{1}{19}$ **(C)** $\frac{1}{18}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{11}{21}$

17. A coin is biased in such a way that on each toss the probability of heads is $\frac{2}{3}$ and the probability of tails is $\frac{1}{3}$. The outcomes of the tosses are independent. A player has the choice of playing Game A or Game B. In Game A she tosses the coin three times and wins if all three outcomes are the same. In Game B she tosses the coin four times and wins if both the outcomes of the first and second tosses are the same and the outcomes of the third and fourth tosses are the same. How do the chances of winning Game A compare to the chances of winning Game B?

    (A) The probability of winning Game A is $\frac{4}{81}$ less than the probability of winning Game B.

    (B) The probability of winning Game A is $\frac{2}{81}$ less than the probability of winning Game B.

    (C) The probabilities are the same.

    (D) The probability of winning Game A is $\frac{2}{81}$ greater than the probability of winning Game B.

    (E) The probability of winning Game A is $\frac{4}{81}$ greater than the probability of winning Game B.

18. The diameter AB of a circle of radius 2 is extended to a point D outside the circle so that $BD = 3$. Point E is chosen so that $ED = 5$ and line ED is perpendicular to line AD. Segment AE intersects the circle at a point C between A and E. What is the area of $\triangle ABC$?

    (A) $\frac{120}{37}$ **(B)** $\frac{140}{39}$ **(C)** $\frac{145}{39}$ **(D)** $\frac{140}{37}$ **(E)** $\frac{120}{31}$

19. Let $N = 123456789101112\dots4344$ be the 79-digit number that is formed by writing the integers from 1 to 44 in order, one after the other. What is the remainder when N is divided by 45?

    (A) 1 **(B)** 4 **(C)** 9 **(D)** 18 **(E)** 44

20. Real numbers x and y are chosen independently and uniformly at random from the interval $(0, 1)$. What is the probability that $\lfloor \log_2 x \rfloor = \lfloor \log_2 y \rfloor$?

    (A) $\frac{1}{8}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

21. Last year Isabella took 7 math tests and received 7 different scores, each an integer between 91 and 100, inclusive. After each test she noticed that the average of her test scores was an integer. Her score on the seventh test was 95. What was her score on the sixth test?

    (A) 92 **(B)** 94 **(C)** 96 **(D)** 98 **(E)** 100

22. Abby, Bernardo, Carl, and Debra play a game in which each of them starts with four coins. The game consists of four rounds. In each round, four balls are placed in an urn—one green, one red, and two white. The players each draw a ball at random without replacement. Whoever gets the green ball gives one coin to whoever gets the red ball. What is the probability that, at the end of the fourth round, each of the players has four coins?

    (A) $\frac{7}{576}$ **(B)** $\frac{5}{192}$ **(C)** $\frac{1}{36}$ **(D)** $\frac{5}{144}$ **(E)** $\frac{7}{48}$

23. The graph of $y = f(x)$, where $f(x)$ is a polynomial of degree 3, contains points $A(2,4)$, $B(3,9)$, and $C(4,16)$. Lines AB, AC, and BC intersect the graph again at points D, E, and F, respectively, and the sum of the x-coordinates of D, E, and F is 24. What is $f(0)$? **(A)** -2 **(B)** 0 **(C)** 2 **(D)** $\frac{24}{5}$ **(E)** 8

24. Quadrilateral $ABCD$ has right angles at B and C, $\triangle ABC \sim \triangle BCD$, and $AB > BC$. There is a point E in the interior of $ABCD$ such that $\triangle ABC \sim \triangle CEB$ and the area of $\triangle AED$ is 17 times the area of $\triangle CEB$. What is $\frac{AB}{BC}$?

    (A) $1+\sqrt{2}$ **(B)** $2+\sqrt{2}$ **(C)** $\sqrt{17}$ **(D)** $2+\sqrt{5}$ **(E)** $1+2\sqrt{3}$

25. A set of n people participate in an online video basketball tournament. Each person may be a member of any number of 5-player teams, but no teams may have exactly the same 5 members. The site statistics show a curious fact: The average, over all subsets of size 9 of the set of n participants, of the number of complete teams whose members are among those 9 people is equal to the reciprocal of the average, over all subsets of size 8 of the set of n participants, of the number of complete teams whose members are among those 8 people. How many values n, $9 \le n \le 2017$, can be the number of participants?

    (A) 477 **(B)** 482 **(C)** 487 **(D)** 557 **(E)** 562

2018 AMC1212A

1. A large urn contains 100 balls, of which 36% are red and the rest are blue. How many of the blue balls must be removed so that the percentage of red balls in the urn will be 72%? (No red balls are to be removed.)

   (A) 28 **(B)** 32 **(C)** 36 **(D)** 50 **(E)** 64

2. While exploring a cave, Carl comes across a collection of 5-pound rocks worth \$14 each, 4-pound rocks worth \$11 each, and 1-pound rocks worth \$2 each. There are at least 20 of each size. He can carry at most 18 pounds. What is the maximum value, in dollars, of the rocks he can carry out of the cave?

   (A) 48 **(B)** 49 **(C)** 50 **(D)** 51 **(E)** 52

3. How many ways can a student schedule 3 mathematics courses – algebra, geometry, and number theory – in a 6-period day if no two mathematics courses can be taken in consecutive periods? (What courses the student takes during the other 3 periods is of no concern here.)

   (A) 3 **(B)** 6 **(C)** 12 **(D)** 18 **(E)** 24

4. Alice, Bob, and Charlie were on a hike and were wondering how far away the nearest town was. When Alice said, "We are at least 6 miles away," Bob replied, "We are at most 5 miles away." Charlie then remarked, "Actually the nearest town is at most 4 miles away." It turned out that none of the three statements were true. Let d be the distance in miles to the nearest town. Which of the following intervals is the set of all possible values of d?

   (A) $(0, 4)$ **(B)** $(4, 5)$ **(C)** $(4, 6)$ **(D)** $(5, 6)$ **(E)** $(5, \infty)$

5. What is the sum of all possible values of k for which the polynomials $x^2 - 3x + 2$ and $x^2 - 5x + k$ have a root in common?

   (A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 10

6. For positive integers m and n such that $m + 10 < n + 1$, both the mean and the median of the set $\{m, m + 4, m + 10, n + 1, n + 2, 2n\}$ are equal to n. What is $m + n$?

   (A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

7. For how many (not necessarily positive) integer values of n is the value of $4000 \cdot \left(\frac{2}{5}\right)^n$ an integer?

   (A) 3 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 9

8. All of the triangles in the diagram below are similar to isosceles triangle ABC, in which $AB = AC$. Each of the 7 smallest triangles has area 1, and $\triangle ABC$ has area 40. What is the area of trapezoid $DBCE$?

   (A) 16 **(B)** 18 **(C)** 20 **(D)** 22 **(E)** 24

9. Which of the following describes the largest subset of values of y within the closed interval $[0, \pi]$ for which

   $$\sin(x + y) \leq \sin(x) + \sin(y)$$

   for every x between 0 and π, inclusive?

   (A) $y = 0$ **(B)** $0 \leq y \leq \frac{\pi}{4}$ **(C)** $0 \leq y \leq \frac{\pi}{2}$ **(D)** $0 \leq y \leq \frac{3\pi}{4}$ **(E)** $0 \leq y \leq \pi$

10. How many ordered pairs of real numbers (x, y) satisfy the following system of equations?

    $$x + 3y = 3$$
    $$\big||x| - |y|\big| = 1$$

    (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 8

11. A paper triangle with sides of lengths $3, 4$, and 5 inches, as shown, is folded so that point A falls on point B. What is the length in inches of the crease?

(A) $1 + \frac{1}{2}\sqrt{2}$ **(B)** $\sqrt{3}$ **(C)** $\frac{7}{4}$ **(D)** $\frac{15}{8}$ **(E)** 2

12. Let S be a set of 6 integers taken from $\{1, 2, \ldots, 12\}$ with the property that if a and b are elements of S with $a < b$, then b is not a multiple of a. What is the least possible value of an element in S?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 7

13. How many nonnegative integers can be written in the form

$$a_7 \cdot 3^7 + a_6 \cdot 3^6 + a_5 \cdot 3^5 + a_4 \cdot 3^4 + a_3 \cdot 3^3 + a_2 \cdot 3^2 + a_1 \cdot 3^1 + a_0 \cdot 3^0,$$

where $a_i \in \{-1, 0, 1\}$ for $0 \le i \le 7$?

(A) 512 **(B)** 729 **(C)** 1094 **(D)** 3281 **(E)** $59,048$

14. The solution to the equation $\log_{3x} 4 = \log_{2x} 8$, where x is a positive real number other than $\frac{1}{3}$ or $\frac{1}{2}$, can be written as $\frac{p}{q}$ where p and q are relatively prime positive integers. What is $p + q$?

(A) 5 **(B)** 13 **(C)** 17 **(D)** 31 **(E)** 35

15. A scanning code consists of a 7×7 grid of squares, with some of its squares colored black and the rest colored white. There must be at least one square of each color in this grid of 49 squares. A scanning code is called *symmetric* if its look does not change when the entire square is rotated by a multiple of 90° counterclockwise around its center, nor when it is reflected across a line joining opposite corners or a line joining midpoints of opposite sides. What is the total number of possible symmetric scanning codes?

(A) 510 **(B)** 1022 **(C)** 8190 **(D)** 8192 **(E)** 65,534

16. Which of the following describes the set of values of a for which the curves $x^2 + y^2 = a^2$ and $y = x^2 - a$ in the real xy-plane intersect at exactly 3 points?

(A) $a = \frac{1}{4}$ **(B)** $\frac{1}{4} < a < \frac{1}{2}$ **(C)** $a > \frac{1}{4}$ **(D)** $a = \frac{1}{2}$ **(E)** $a > \frac{1}{2}$

17. Farmer Pythagoras has a field in the shape of a right triangle. The right triangle's legs have lengths 3 and 4 units. In the corner where those sides meet at a right angle, he leaves a small unplanted square S so that from the air it looks like the right angle symbol. The rest of the field is planted. The shortest distance from S to the hypotenuse is 2 units. What fraction of the field is planted?

(A) $\frac{25}{27}$ **(B)** $\frac{26}{27}$ **(C)** $\frac{73}{75}$ **(D)** $\frac{145}{147}$ **(E)** $\frac{74}{75}$

18. Triangle ABC with $AB = 50$ and $AC = 10$ has area 120. Let D be the midpoint of $\overline{AB}$, and let E be the midpoint of $\overline{AC}$. The angle bisector of $\angle BAC$ intersects $\overline{DE}$ and $\overline{BC}$ at F and G, respectively. What is the area of quadrilateral $FDBG$?

(A) 60 **(B)** 65 **(C)** 70 **(D)** 75 **(E)** 80

19. Let A be the set of positive integers that have no prime factors other than 2, 3, or 5. The infinite sum

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{12} + \frac{1}{15} + \frac{1}{16} + \frac{1}{18} + \frac{1}{20} + \cdots$$

of the reciprocals of the elements of A can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

(A) 16 **(B)** 17 **(C)** 19 **(D)** 23 **(E)** 36

20. Triangle ABC is an isosceles right triangle with $AB = AC = 3$. Let M be the midpoint of hypotenuse $\overline{BC}$. Points I and E lie on sides $\overline{AC}$ and $\overline{AB}$, respectively, so that $AI > AE$ and $AIME$ is a cyclic quadrilateral. Given that triangle EMI has area 2, the length CI can be written as $\frac{a-\sqrt{b}}{c}$, where a, b, and c are positive integers and b is not divisible by the square of any prime. What is the value of $a+b+c$?

    (A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

21. Which of the following polynomials has the greatest real root?

    (A) $x^{19} + 2018x^{11} + 1$ **(B)** $x^{17} + 2018x^{11} + 1$ **(C)** $x^{19} + 2018x^{13} + 1$ **(D)** $x^{17} + 2018x^{13} + 1$ **(E)** $2019x + 2018$

22. The solutions to the equations $z^2 = 4 + 4\sqrt{15}i$ and $z^2 = 2 + 2\sqrt{3}i$, where $i = \sqrt{-1}$, form the vertices of a parallelogram in the complex plane. The area of this parallelogram can be written in the form $p\sqrt{q} - r\sqrt{s}$, where p, q, r, and s are positive integers and neither q nor s is divisible by the square of any prime number. What is $p+q+r+s$?

    (A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

23. In $\triangle PAT$, $\angle P = 36^\circ$, $\angle A = 56^\circ$, and $PA = 10$. Points U and G lie on sides $\overline{TP}$ and $\overline{TA}$, respectively, so that $PU = AG = 1$. Let M and N be the midpoints of segments $\overline{PA}$ and $\overline{UG}$, respectively. What is the degree measure of the acute angle formed by lines MN and PA?

    (A) 76 **(B)** 77 **(C)** 78 **(D)** 79 **(E)** 80

24. Alice, Bob, and Carol play a game in which each of them chooses a real number between 0 and 1. The winner of the game is the one whose number is between the numbers chosen by the other two players. Alice announces that she will choose her number uniformly at random from all the numbers between 0 and 1, and Bob announces that he will choose his number uniformly at random from all the numbers between $\frac{1}{2}$ and $\frac{2}{3}$. Armed with this information, what number should Carol choose to maximize her chance of winning?

    (A) $\frac{1}{2}$ **(B)** $\frac{13}{24}$ **(C)** $\frac{7}{12}$ **(D)** $\frac{5}{8}$ **(E)** $\frac{2}{3}$

25. For a positive integer n and nonzero digits a, b, and c, let A_n be the n-digit integer each of whose digits is equal to a; let B_n be the n-digit integer each of whose digits is equal to b, and let C_n be the $2n$-digit (not n-digit) integer each of whose digits is equal to c. What is the greatest possible value of $a+b+c$ for which there are at least two values of n such that $C_n - B_n = A_n^2$?

    (A) 12 **(B)** 14 **(C)** 16 **(D)** 18 **(E)** 20

2018 AMC1212B

1. Kate bakes a 20-inch by 18-inch pan of cornbread. The cornbread is cut into pieces that measure 2 inches by 2 inches. How many pieces of cornbread does the pan contain?

   (A) 90 **(B)** 100 **(C)** 180 **(D)** 200 **(E)** 360

2. Sam drove 96 miles in 90 minutes. His average speed during the first 30 minutes was 60 mph (miles per hour), and his average speed during the second 30 minutes was 65 mph. What was his average speed, in mph, during the last 30 minutes?

   (A) 64 **(B)** 65 **(C)** 66 **(D)** 67 **(E)** 68

3. A line with slope 2 intersects a line with slope 6 at the point $(40, 30)$. What is the distance between the x-intercepts of these two lines?

   (A) 5 **(B)** 10 **(C)** 20 **(D)** 25 **(E)** 50

4. A circle has a chord of length 10, and the distance from the center of the circle to the chord is 5. What is the area of the circle?

   (A) 25π **(B)** 50π **(C)** 75π **(D)** 100π **(E)** 125π

5. How many subsets of $\{2, 3, 4, 5, 6, 7, 8, 9\}$ contain at least one prime number?

   (A) 128 **(B)** 192 **(C)** 224 **(D)** 240 **(E)** 256

6. Suppose S cans of soda can be purchased from a vending machine for Q quarters. Which of the following expressions describes the number of cans of soda that can be purchased for D dollars, where 1 dollar is worth 4 quarters?

   (A) $\frac{4DQ}{S}$ **(B)** $\frac{4DS}{Q}$ **(C)** $\frac{4Q}{DS}$ **(D)** $\frac{DQ}{4S}$ **(E)** $\frac{DS}{4Q}$

7. What is the value of

   $$\log_3 7 \cdot \log_5 9 \cdot \log_7 11 \cdot \log_9 13 \cdots \log_{21} 25 \cdot \log_{23} 27?$$

   (A) 3 **(B)** $3\log_7 23$ **(C)** 6 **(D)** 9 **(E)** 10

8. Line segment $\overline{AB}$ is a diameter of a circle with $AB = 24$. Point C, not equal to A or B, lies on the circle. As point C moves around the circle, the centroid (center of mass) of $\triangle ABC$ traces out a closed curve missing two points. To the nearest positive integer, what is the area of the region bounded by this curve?

   (A) 25 **(B)** 38 **(C)** 50 **(D)** 63 **(E)** 75

9. What is

   $$\sum_{i=1}^{100} \sum_{j=1}^{100} (i + j)?$$

   (A) 100,100 **(B)** 500,500 **(C)** 505,000 **(D)** 1,001,000 **(E)** 1,010,000

10. A list of 2018 positive integers has a unique mode, which occurs exactly 10 times. What is the least number of distinct values that can occur in the list?

    (A) 202 **(B)** 223 **(C)** 224 **(D)** 225 **(E)** 234

11. A closed box with a square base is to be wrapped with a square sheet of wrapping paper. The box is centered on the wrapping paper with the vertices of the base lying on the midlines of the square sheet of paper, as shown in the figure on the left. The four corners of the wrapping paper are to be folded up over the sides and brought together to meet at the center of the top of the box, point A in the figure on the right. The box has base length w and height h. What is the area of the sheet of wrapping paper?

    (A) $2(w + h)^2$ **(B)** $\frac{(w+h)^2}{2}$ **(C)** $2w^2 + 4wh$ **(D)** $2w^2$ **(E)** w^2h

12. Side $\overline{AB}$ of $\triangle ABC$ has length 10. The bisector of angle A meets $\overline{BC}$ at D, and $CD = 3$. The set of all possible values of AC is an open interval (m, n). What is $m + n$?

(A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 20

13. Square $ABCD$ has side length 30. Point P lies inside the square so that $AP = 12$ and $BP = 26$. The centroids of $\triangle ABP$, $\triangle BCP$, $\triangle CDP$, and $\triangle DAP$ are the vertices of a convex quadrilateral. What is the area of that quadrilateral?

(A) $100\sqrt{2}$ **(B)** $100\sqrt{3}$ **(C)** 200 **(D)** $200\sqrt{2}$ **(E)** $200\sqrt{3}$

14. Joey and Chloe and their daughter Zoe all have the same birthday. Joey is 1 year older than Chloe, and Zoe is exactly 1 year old today. Today is the first of the 9 birthdays on which Chloe's age will be an integral multiple of Zoe's age. What will be the sum of the two digits of Joey's age the next time his age is a multiple of Zoe's age?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

15. How many odd positive 3-digit integers are divisible by 3 but do not contain the digit 3?

(A) 96 **(B)** 97 **(C)** 98 **(D)** 102 **(E)** 120

16. The solutions to the equation $(z + 6)^8 = 81$ are connected in the complex plane to form a convex regular polygon, three of whose vertices are labeled $A, B,$ and C. What is the least possible area of $\triangle ABC$?

(A) $\frac{1}{6}\sqrt{6}$ **(B)** $\frac{3}{2}\sqrt{2} - \frac{3}{2}$ **(C)** $2\sqrt{3} - 3\sqrt{2}$ **(D)** $\frac{1}{2}\sqrt{2}$ **(E)** $\sqrt{3} - 1$

17. Let p and q be positive integers such that

$$\frac{5}{9} < \frac{p}{q} < \frac{4}{7}$$

and q is as small as possible. What is $q - p$?

(A) 7 **(B)** 11 **(C)** 13 **(D)** 17 **(E)** 19

18. A function f is defined recursively by $f(1) = f(2) = 1$ and

$$f(n) = f(n - 1) - f(n - 2) + n$$

for all integers $n \geq 3$. What is $f(2018)$?

(A) 2016 **(B)** 2017 **(C)** 2018 **(D)** 2019 **(E)** 2020

19. Mary chose an even 4-digit number n. She wrote down all the divisors of n in increasing order from left to right: $1, 2, \ldots, \dfrac{n}{2}, n$. At some moment Mary wrote 323 as a divisor of n. What is the smallest possible value of the next divisor written to the right of 323?

(A) 324 **(B)** 330 **(C)** 340 **(D)** 361 **(E)** 646

20. Let $ABCDEF$ be a regular hexagon with side length 1. Denote by X, Y, and Z the midpoints of sides $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$, respectively. What is the area of the convex hexagon whose interior is the intersection of the interiors of $\triangle ACE$ and $\triangle XYZ$?

(A) $\frac{3}{8}\sqrt{3}$ **(B)** $\frac{7}{16}\sqrt{3}$ **(C)** $\frac{15}{32}\sqrt{3}$ **(D)** $\frac{1}{2}\sqrt{3}$ **(E)** $\frac{9}{16}\sqrt{3}$

21. In $\triangle ABC$ with side lengths $AB = 13$, $AC = 12$, and $BC = 5$, let O and I denote the circumcenter and incenter, respectively. A circle with center M is tangent to the legs AC and BC and to the circumcircle of $\triangle ABC$. What is the area of $\triangle MOI$?

(A) $\frac{5}{2}$ **(B)** $\frac{11}{4}$ **(C)** 3 **(D)** $\frac{13}{4}$ **(E)** $\frac{7}{2}$

22. Consider polynomials $P(x)$ of degree at most 3, each of whose coefficients is an element of $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. How many such polynomials satisfy $P(-1) = -9$?

(A) 110 **(B)** 143 **(C)** 165 **(D)** 220 **(E)** 286

23. Ajay is standing at point A near Pontianak, Indonesia, $0°$ latitude and $110°$ E longitude. Billy is standing at point B near Big Baldy Mountain, Idaho, USA, $45°$ N latitude and $115°$ W longitude. Assume that Earth is a perfect sphere with center C. What is the degree measure of $\angle ACB$?

(A) 105 **(B)** $112\frac{1}{2}$ **(C)** 120 **(D)** 135 **(E)** 150

24. Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. How many real numbers x satisfy the equation $x^2 + 10,000\lfloor x \rfloor = 10,000x$?

(A) 197 **(B)** 198 **(C)** 199 **(D)** 200 **(E)** 201

25. Circles ω_1, ω_2, and ω_3 each have radius 4 and are placed in the plane so that each circle is externally tangent to the other two. Points P_1, P_2, and P_3 lie on ω_1, ω_2, and ω_3 respectively such that $P_1P_2 = P_2P_3 = P_3P_1$ and line P_iP_{i+1} is tangent to ω_i for each $i = 1, 2, 3$, where $P_4 = P_1$. See the figure below. The area of $\triangle P_1P_2P_3$ can be written in the form $\sqrt{a} + \sqrt{b}$ for positive integers a and b. What is $a + b$?

(A) 546 **(B)** 548 **(C)** 550 **(D)** 552 **(E)** 554

2019 AMC1212A

1. The area of a pizza with radius 4 inches is N percent larger than the area of a pizza with radius 3 inches. What is the integer closest to N?

   (A) 25 **(B)** 33 **(C)** 44 **(D)** 66 **(E)** 78

2. Suppose a is 150% of b. What percent of a is $3b$?

   (A) 50 **(B)** $66 + \frac{2}{3}$ **(C)** 150 **(D)** 200 **(E)** 450

3. A box contains 28 red balls, 20 green balls, 19 yellow balls, 13 blue balls, 11 white balls, and 9 black balls. What is the minimum number of balls that must be drawn from the box without replacement to guarantee that at least 15 balls of a single color will be drawn?

   (A) 75 **(B)** 76 **(C)** 79 **(D)** 84 **(E)** 91

4. What is the greatest number of consecutive integers whose sum is 45?

   (A) 9 **(B)** 25 **(C)** 45 **(D)** 90 **(E)** 120

5. Two lines with slopes $\dfrac{1}{2}$ and 2 intersect at $(2, 2)$. What is the area of the triangle enclosed by these two lines and the line $x + y = 10$?

   (A) 4 **(B)** $4\sqrt{2}$ **(C)** 6 **(D)** 8 **(E)** $6\sqrt{2}$

6. The figure below shows line ℓ with a regular, infinite, recurring pattern of squares and line segments.

   How many of the following four kinds of rigid motion transformations of the plane in which this figure is drawn, other than the identity transformation, will transform this figure into itself? *some rotation around a point of line ℓ *some translation in the direction parallel to line ℓ *the reflection across line ℓ *some reflection across a line perpendicular to line ℓ **(A)** 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

7. Melanie computes the mean μ, the median M, and the modes of the 365 values that are the dates in the months of 2019. Thus her data consist of 12 1s, 12 2s, . . . , 12 28s, 11 29s, 11 30s, and 7 31s. Let d be the median of the modes. Which of the following statements is true?

   (A) $\mu < d < M$ **(B)** $M < d < \mu$ **(C)** $d = M = \mu$ **(D)** $d < M < \mu$ **(E)** $d < \mu < M$

8. For a set of four distinct lines in a plane, there are exactly N distinct points that lie on two or more of the lines. What is the sum of all possible values of N?

   (A) 14 **(B)** 16 **(C)** 18 **(D)** 19 **(E)** 21

9. A sequence of numbers is defined recursively by $a_1 = 1$, $a_2 = \frac{3}{7}$, and

   $$a_n = \frac{a_{n-2} \cdot a_{n-1}}{2a_{n-2} - a_{n-1}}$$

   for all $n \geq 3$. Then a_{2019} can be written as $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $p + q$?

   (A) 2020 **(B)** 4039 **(C)** 6057 **(D)** 6061 **(E)** 8078

10. The figure below shows 13 circles of radius 1 within a larger circle. All the intersections occur at points of tangency. What is the area of the region, shaded in the figure, inside the larger circle but outside all the circles of radius 1?

    (A) $4\pi\sqrt{3}$ **(B)** 7π **(C)** $\pi\left(3\sqrt{3} + 2\right)$ **(D)** $10\pi\left(\sqrt{3} - 1\right)$ **(E)** $\pi\left(\sqrt{3} + 6\right)$

11. For some positive integer k, the repeating base-k representation of the (base-ten) fraction $\frac{7}{51}$ is $0.\overline{23}_k = 0.232323..._k$. What is k?

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

12. Positive real numbers $x \neq 1$ and $y \neq 1$ satisfy $\log_2 x = \log_y 16$ and $xy = 64$. What is $(\log_2 \frac{x}{y})^2$?

(A) $\frac{25}{2}$ **(B)** 20 **(C)** $\frac{45}{2}$ **(D)** 25 **(E)** 32

13. How many ways are there to paint each of the integers $2, 3, \ldots, 9$ either red, green, or blue so that each number has a different color from each of its proper divisors?

(A) 144 **(B)** 216 **(C)** 256 **(D)** 384 **(E)** 432

14. For a certain complex number c, the polynomial

$$P(x) = (x^2 - 2x + 2)(x^2 - cx + 4)(x^2 - 4x + 8)$$

has exactly 4 distinct roots. What is $|c|$?

(A) 2 **(B)** $\sqrt{6}$ **(C)** $2\sqrt{2}$ **(D)** 3 **(E)** $\sqrt{10}$

15. Positive real numbers a and b have the property that

$$\sqrt{\log a} + \sqrt{\log b} + \log \sqrt{a} + \log \sqrt{b} = 100$$

and all four terms on the left are positive integers, where log denotes the base-10 logarithm. What is ab?

(A) 10^{52} **(B)** 10^{100} **(C)** 10^{144} **(D)** 10^{164} **(E)** 10^{200}

16. The numbers $1, 2, \ldots, 9$ are randomly placed into the 9 squares of a 3×3 grid. Each square gets one number, and each of the numbers is used once. What is the probability that the sum of the numbers in each row and each column is odd?

(A) $\frac{1}{21}$ **(B)** $\frac{1}{14}$ **(C)** $\frac{5}{63}$ **(D)** $\frac{2}{21}$ **(E)** $\frac{1}{7}$

17. Let s_k denote the sum of the kth powers of the roots of the polynomial $x^3 - 5x^2 + 8x - 13$. In particular, $s_0 = 3$, $s_1 = 5$, and $s_2 = 9$. Let a, b, and c be real numbers such that $s_{k+1} = a\, s_k + b\, s_{k-1} + c\, s_{k-2}$ for $k = 2$, 3, ... What is $a + b + c$?

(A) -6 **(B)** 0 **(C)** 6 **(D)** 10 **(E)** 26

18. A sphere with center O has radius 6. A triangle with sides of length $15, 15$, and 24 is situated in space so that each of its sides is tangent to the sphere. What is the distance between O and the plane determined by the triangle?

(A) $2\sqrt{3}$ **(B)** 4 **(C)** $3\sqrt{2}$ **(D)** $2\sqrt{5}$ **(E)** 5

19. In $\triangle ABC$ with integer side lengths,

$$\cos A = \frac{11}{16}, \qquad \cos B = \frac{7}{8}, \qquad \text{and} \qquad \cos C = -\frac{1}{4}.$$

What is the least possible perimeter for $\triangle ABC$?

(A) 9 **(B)** 12 **(C)** 23 **(D)** 27 **(E)** 44

20. Real numbers between 0 and 1, inclusive, are chosen in the following manner. A fair coin is flipped. If it lands heads, then it is flipped again and the chosen number is 0 if the second flip is heads and 1 if the second flip is tails. On the other hand, if the first coin flip is tails, then the number is chosen uniformly at random from the closed interval $[0, 1]$. Two random numbers x and y are chosen independently in this manner. What is the probability that $|x - y| > \frac{1}{2}$?

(A) $\frac{1}{3}$ **(B)** $\frac{7}{16}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{9}{16}$ **(E)** $\frac{2}{3}$

21. Let
$$z = \frac{1+i}{\sqrt{2}}.$$
What is
$$\left(z^{1^2} + z^{2^2} + z^{3^2} + \cdots + z^{12^2}\right) \cdot \left(\frac{1}{z^{1^2}} + \frac{1}{z^{2^2}} + \frac{1}{z^{3^2}} + \cdots + \frac{1}{z^{12^2}}\right)?$$
(A) 18 **(B)** $72 - 36\sqrt{2}$ **(C)** 36 **(D)** 72 **(E)** $72 + 36\sqrt{2}$

22. Circles ω and γ, both centered at O, have radii 20 and 17, respectively. Equilateral triangle ABC, whose interior lies in the interior of ω but in the exterior of γ, has vertex A on ω, and the line containing side $\overline{BC}$ is tangent to γ. Segments $\overline{AO}$ and $\overline{BC}$ intersect at P, and $\dfrac{BP}{CP} = 3$. Then AB can be written in the form $\dfrac{m}{\sqrt{n}} - \dfrac{p}{\sqrt{q}}$ for positive integers m, n, p, q with $\gcd(m,n) = \gcd(p,q) = 1$. What is $m+n+p+q$?
(A) 42 **(B)** 86 **(C)** 92 **(D)** 114 **(E)** 130

23. Define binary operations $\diamondsuit$ and $\heartsuit$ by
$$a \diamondsuit b = a^{\log_7(b)} \qquad \text{and} \qquad a \heartsuit b = a^{\frac{1}{\log_7(b)}}$$
for all real numbers a and b for which these expressions are defined. The sequence (a_n) is defined recursively by $a_3 = 3 \heartsuit 2$ and
$$a_n = (n \heartsuit (n-1)) \diamondsuit a_{n-1}$$
for all integers $n \geq 4$. To the nearest integer, what is $\log_7(a_{2019})$?
(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

24. For how many integers n between 1 and 50, inclusive, is
$$\frac{(n^2-1)!}{(n!)^n}$$
an integer? (Recall that $0! = 1$.)
(A) 31 **(B)** 32 **(C)** 33 **(D)** 34 **(E)** 35

25. Let $\triangle A_0B_0C_0$ be a triangle whose angle measures are exactly 59.999°, 60°, and 60.001°. For each positive integer n, define A_n to be the foot of the altitude from A_{n-1} to line $B_{n-1}C_{n-1}$. Likewise, define B_n to be the foot of the altitude from B_{n-1} to line $A_{n-1}C_{n-1}$, and C_n to be the foot of the altitude from C_{n-1} to line $A_{n-1}B_{n-1}$. What is the least positive integer n for which $\triangle A_nB_nC_n$ is obtuse?
(A) 10 **(B)** 11 **(C)** 13 **(D)** 14 **(E)** 15

2019 AMC1212B

1. Alicia had two containers. The first was $\frac{5}{6}$ full of water and the second was empty. She poured all the water from the first container into the second container, at which point the second container was $\frac{3}{4}$ full of water. What is the ratio of the volume of the first container to the volume of the second container?

   (A) $\frac{5}{8}$ **(B)** $\frac{4}{5}$ **(C)** $\frac{7}{8}$ **(D)** $\frac{9}{10}$ **(E)** $\frac{11}{12}$

2. Consider the statement, "If n is not prime, then $n-2$ is prime." Which of the following values of n is a counterexample to this statement?

   (A) 11 **(B)** 15 **(C)** 19 **(D)** 21 **(E)** 27

3. Which one of the following rigid transformations (isometries) maps the line segment $\overline{AB}$ onto the line segment $\overline{A'B'}$ so that the image of $A(-2,1)$ is $A'(2,-1)$ and the image of $B(-1,4)$ is $B'(1,-4)$?

   (A) reflection in the y-axis

   (B) counterclockwise rotation around the origin by 90°

   (C) translation by 3 units to the right and 5 units down

   (D) reflection in the x-axis

   (E) clockwise rotation about the origin by 180°

4. A positive integer n satisfies the equation $(n+1)! + (n+2)! = 440 \cdot n!$. What is the sum of the digits of n?

   (A) 2 **(B)** 5 **(C)** 10 **(D)** 12 **(E)** 15

5. Each piece of candy in a store costs a whole number of cents. Casper has exactly enough money to buy either 12 pieces of red candy, 14 pieces of green candy, 15 pieces of blue candy, or n pieces of purple candy. A piece of purple candy costs 20 cents. What is the smallest possible value of n?

   (A) 18 **(B)** 21 **(C)** 24 **(D)** 25 **(E)** 28

6. In a given plane, points A and B are 10 units apart. How many points C are there in the plane such that the perimeter of $\triangle ABC$ is 50 units and the area of $\triangle ABC$ is 100 square units?

   (A) 0 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** infinitely many

7. What is the sum of all real numbers x for which the median of the numbers $4, 6, 8, 17,$ and x is equal to the mean of those five numbers?

   (A) -5 **(B)** 0 **(C)** 5 **(D)** $\frac{15}{4}$ **(E)** $\frac{35}{4}$

8. Let $f(x) = x^2(1-x)^2$. What is the value of the sum

$$f\left(\frac{1}{2019}\right) - f\left(\frac{2}{2019}\right) + f\left(\frac{3}{2019}\right) - f\left(\frac{4}{2019}\right) + \cdots + f\left(\frac{2017}{2019}\right) - f\left(\frac{2018}{2019}\right)?$$

   (A) 0 **(B)** $\frac{1}{2019^4}$ **(C)** $\frac{2018^2}{2019^4}$ **(D)** $\frac{2020^2}{2019^4}$ **(E)** 1

9. For how many integral values of x can a triangle of positive area be formed having side lengths $\log_2 x, \log_4 x, 3$?

   (A) 57 **(B)** 59 **(C)** 61 **(D)** 62 **(E)** 63

10. The figure below is a map showing 12 cities and 17 roads connecting certain pairs of cities. Paula wishes to travel along exactly 13 of those roads, starting at city A and ending at city L, without traveling along any portion of a road more than once. (Paula is allowed to visit a city more than once.)

    How many different routes can Paula take?

    (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

11. How many unordered pairs of edges of a given cube determine a plane?

(A) 12 **(B)** 28 **(C)** 36 **(D)** 42 **(E)** 66

12. Right triangle ACD with right angle at C is constructed outwards on the hypotenuse $\overline{AC}$ of isosceles right triangle ABC with leg length 1, as shown, so that the two triangles have equal perimeters. What is $\sin(2\angle BAD)$?

(A) $\frac{1}{3}$ **(B)** $\frac{\sqrt{2}}{2}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{7}{9}$ **(E)** $\frac{\sqrt{3}}{2}$

13. A red ball and a green ball are randomly and independently tossed into bins numbered with the positive integers so that for each ball, the probability that it is tossed into bin k is 2^{-k} for $k = 1, 2, 3....$ What is the probability that the red ball is tossed into a higher-numbered bin than the green ball?¡br¿

(A) $\frac{1}{4}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{3}{8}$ **(E)** $\frac{3}{7}$

14. Let S be the set of all positive integer divisors of $100,000$. How many numbers are the product of two distinct elements of S?

(A) 98 **(B)** 100 **(C)** 117 **(D)** 119 **(E)** 121

15. As shown in the figure, line segment $\overline{AD}$ is trisected by points B and C so that $AB = BC = CD = 2$. Three semicircles of radius 1, 5.0pt24.88pt$\widehat{\mathrm{AEB}}$, 5.0pt24.88pt$\widehat{\mathrm{BFC}}$, and 5.0pt24.88pt$\widehat{\mathrm{CGD}}$, have their diameters on $\overline{AD}$, and are tangent to line EG at $E, F,$ and G, respectively. A circle of radius 2 has its center on F. The area of the region inside the circle but outside the three semicircles, shaded in the figure, can be expressed in the form

$$\frac{a}{b}\cdot\pi-\sqrt{c}+d,$$

where $a, b, c,$ and d are positive integers and a and b are relatively prime. What is $a + b + c + d$?

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

16. There are lily pads in a row numbered 0 to 11, in that order. There are predators on lily pads 3 and 6, and a morsel of food on lily pad 10. Fiona the frog starts on pad 0, and from any given lily pad, has a $\frac{1}{2}$ chance to hop to the next pad, and an equal chance to jump 2 pads. What is the probability that Fiona reaches pad 10 without landing on either pad 3 or pad 6?

(A) $\frac{15}{256}$ **(B)** $\frac{1}{16}$ **(C)** $\frac{15}{128}$ **(D)** $\frac{1}{8}$ **(E)** $\frac{1}{4}$

17. How many nonzero complex numbers z have the property that $0, z,$ and z^3, when represented by points in the complex plane, are the three distinct vertices of an equilateral triangle?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** infinitely many

18. Square pyramid $ABCDE$ has base $ABCD$, which measures 3 cm on a side, and altitude $\overline{AE}$ perpendicular to the base, which measures 6 cm. Point P lies on $\overline{BE}$, one third of the way from B to E; point Q lies on $\overline{DE}$, one third of the way from D to E; and point R lies on $\overline{CE}$, two thirds of the way from C to E. What is the area, in square centimeters, of $\triangle PQR$?

(A) $\frac{3\sqrt{2}}{2}$ **(B)** $\frac{3\sqrt{3}}{2}$ **(C)** $2\sqrt{2}$ **(D)** $2\sqrt{3}$ **(E)** $3\sqrt{2}$

19. Raashan, Sylvia, and Ted play the following game. Each person starts with \$1. A bell rings every 15 seconds, at which time each of the players who currently have money simultaneously chooses one of the other two players independently and at random and gives \$1 to that player. What is the probability that after the bell has rung 2019 times, each player will have \$1? (For example, Raashan and Ted may each decide to give \$1 to Sylvia, and Sylvia may decide to give her dollar to Ted, at which point Raashan will have \$0, Sylvia will have \$2, and Ted will have \$1, and that is the end of the first round of play. In the

second round Rashaan has no money to give, but Sylvia and Ted might choose each other to give their \$1 to, and the holdings will be the same at the end of the second round.)

(A) $\frac{1}{7}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{2}{3}$

20. Points $A(6,13)$ and $B(12,11)$ lie on circle ω in the plane. Suppose that the tangent lines to ω at A and B intersect at a point on the x-axis. What is the area of ω?

(A) $\frac{83\pi}{8}$ **(B)** $\frac{21\pi}{2}$ **(C)** $\frac{85\pi}{8}$ **(D)** $\frac{43\pi}{4}$ **(E)** $\frac{87\pi}{8}$

21. How many quadratic polynomials with real coefficients are there such that the set of roots equals the set of coefficients? (For clarification: If the polynomial is $ax^2+bx+c, a \neq 0$, and the roots are r and s, then the requirement is that $\{a,b,c\} = \{r,s\}$.)

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** infinitely many

22. Define a sequence recursively by $x_0 = 5$ and

$$x_{n+1} = \frac{x_n^2 + 5x_n + 4}{x_n + 6}$$

for all nonnegative integers n. Let m be the least positive integer such that

$$x_m \leq 4 + \frac{1}{2^{20}}.$$

In which of the following intervals does m lie?

(A) $[9,26]$ **(B)** $[27,80]$ **(C)** $[81,242]$ **(D)** $[243,728]$ **(E)** $[729,\infty]$

23. How many sequences of 0s and 1s of length 19 are there that begin with a 0, end with a 0, contain no two consecutive 0s, and contain no three consecutive 1s?

(A) 55 **(B)** 60 **(C)** 65 **(D)** 70 **(E)** 75

24. Let $\omega = -\frac{1}{2} + \frac{1}{2}i\sqrt{3}$. Let S denote all points in the complex plane of the form $a + b\omega + c\omega^2$, where $0 \leq a \leq 1, 0 \leq b \leq 1$, and $0 \leq c \leq 1$. What is the area of S?

(A) $\frac{1}{2}\sqrt{3}$ **(B)** $\frac{3}{4}\sqrt{3}$ **(C)** $\frac{3}{2}\sqrt{3}$ **(D)** $\frac{1}{2}\pi\sqrt{3}$ **(E)** π

25. Let $ABCD$ be a convex quadrilateral with $BC = 2$ and $CD = 6$. Suppose that the centroids of $\triangle ABC, \triangle BCD$, and $\triangle ACD$ form the vertices of an equilateral triangle. What is the maximum possible value of the area of $ABCD$?

(A) 27 **(B)** $16\sqrt{3}$ **(C)** $12 + 10\sqrt{3}$ **(D)** $9 + 12\sqrt{3}$ **(E)** 30

2020 AMC1212A

1. Carlos took 70% of a whole pie. Maria took one third of the remainder. What portion of the whole pie was left?

   (A) 10% **(B)** 15% **(C)** 20% **(D)** 30% **(E)** 35%

2. The acronym AMC is shown in the rectangular grid below with grid lines spaced 1 unit apart. In units, what is the sum of the lengths of the line segments that form the acronym AMC?

   (A) 17 **(B)** $15 + 2\sqrt{2}$ **(C)** $13 + 4\sqrt{2}$ **(D)** $11 + 6\sqrt{2}$ **(E)** 21

3. A driver travels for 2 hours at 60 miles per hour, during which her car gets 30 miles per gallon of gasoline. She is paid \$0.50 per mile, and her only expense is gasoline at \$2.00 per gallon. What is her net rate of pay, in dollars per hour, after this expense?

   (A) 20 **(B)** 22 **(C)** 24 **(D)** 25 **(E)** 26

4. How many 4-digit positive integers (that is, integers between 1000 and 9999, inclusive) having only even digits are divisible by 5?

   (A) 80 **(B)** 100 **(C)** 125 **(D)** 200 **(E)** 500

5. The 25 integers from -10 to 14, inclusive, can be arranged to form a 5-by-5 square in which the sum of the numbers in each row, the sum of the numbers in each column, and the sum of the numbers along each of the main diagonals are all the same. What is the value of this common sum?

   (A) 2 **(B)** 5 **(C)** 10 **(D)** 25 **(E)** 50

6. In the plane figure shown below, 3 of the unit squares have been shaded. What is the least number of additional unit squares that must be shaded so that the resulting figure has two lines of symmetry?

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

7. Seven cubes, whose volumes are 1, 8, 27, 64, 125, 216, and 343 cubic units, are stacked vertically to form a tower in which the volumes of the cubes decrease from bottom to top. Except for the bottom cube, the bottom face of each cube lies completely on top of the cube below it. What is the total surface area of the tower (including the bottom) in square units?

   (A) 644 **(B)** 658 **(C)** 664 **(D)** 720 **(E)** 749

8. What is the median of the following list of 4040 numbers?

   $$1, 2, 3, \ldots, 2020, 1^2, 2^2, 3^2, \ldots, 2020^2$$

   (A) 1974.5 **(B)** 1975.5 **(C)** 1976.5 **(D)** 1977.5 **(E)** 1978.5

9. How many solutions does the equation $\tan(2x) = \cos\left(\frac{x}{2}\right)$ have on the interval $[0, 2\pi]$?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

10. There is a unique positive integer n such that

    $$\log_2(\log_{16} n) = \log_4(\log_4 n).$$

    What is the sum of the digits of n?

    (A) 4 **(B)** 7 **(C)** 8 **(D)** 11 **(E)** 13

11. A frog sitting at the point $(1, 2)$ begins a sequence of jumps, where each jump is parallel to one of the coordinate axes and has length 1, and the direction of each jump (up, down, right, or left) is chosen independently at random. The sequence ends when the frog reaches a side of the square with vertices $(0,0), (0,4), (4,4)$, and $(4,0)$. What is the probability that the sequence of jumps ends on a vertical side of the square?

    (A) $\frac{1}{2}$ **(B)** $\frac{5}{8}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{3}{4}$ **(E)** $\frac{7}{8}$

12. Line ℓ in the coordinate plane has the equation $3x - 5y + 40 = 0$. This line is rotated $45°$ counterclockwise about the point $(20, 20)$ to obtain line k. What is the x-coordinate of the x-intercept of line k?

    (A) 10 **(B)** 15 **(C)** 20 **(D)** 25 **(E)** 30

13. There are integers a, b, and c, each greater than 1, such that

    $$\sqrt[a]{N\sqrt[b]{N\sqrt[c]{N}}} = \sqrt[36]{N^{25}}$$

    for all $N > 1$. What is b?

    (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

14. Regular octagon $ABCDEFGH$ has area n. Let m be the area of quadrilateral $ACEG$. What is $\frac{m}{n}$?

    (A) $\frac{\sqrt{2}}{4}$ **(B)** $\frac{\sqrt{2}}{2}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{3\sqrt{2}}{5}$ **(E)** $\frac{2\sqrt{2}}{3}$

15. In the complex plane, let A be the set of solutions to $z^3 - 8 = 0$ and let B be the set of solutions to $z^3 - 8z^2 - 8z + 64 = 0$. What is the greatest distance between a point of A and a point of B?

    (A) $2\sqrt{3}$ **(B)** 6 **(C)** 9 **(D)** $2\sqrt{21}$ **(E)** $9 + \sqrt{3}$

16. A point is chosen at random within the square in the coordinate plane whose vertices are $(0,0), (2020,0), (2020,2020)$, and $(0,2020)$. The probability that the point is within d units of a lattice point is $\frac{1}{2}$. (A point (x, y) is a lattice point if x and y are both integers.) What is d to the nearest tenth?

    (A) 0.3 **(B)** 0.4 **(C)** 0.5 **(D)** 0.6 **(E)** 0.7

17. The vertices of a quadrilateral lie on the graph of $y = \ln x$, and the x-coordinates of these vertices are consecutive positive integers. The area of the quadrilateral is $\ln \frac{91}{90}$. What is the x-coordinate of the leftmost vertex?

    (A) 6 **(B)** 7 **(C)** 10 **(D)** 12 **(E)** 13

18. Quadrilateral $ABCD$ satisfies $\angle ABC = \angle ACD = 90°, AC = 20$, and $CD = 30$. Diagonals $\overline{AC}$ and $\overline{BD}$ intersect at point E, and $AE = 5$. What is the area of quadrilateral $ABCD$?

    (A) 330 **(B)** 340 **(C)** 350 **(D)** 360 **(E)** 370

19. There exists a unique strictly increasing sequence of nonnegative integers $a_1 < a_2 << a_k$ such that

    $$\frac{2^{289} + 1}{2^{17} + 1} = 2^{a_1} + 2^{a_2} + +2^{a_k}.$$

    What is k?

    (A) 117 **(B)** 136 **(C)** 137 **(D)** 273 **(E)** 306

20. Let T be the triangle in the coordinate plane with vertices $(0,0)$, $(4,0)$, and $(0,3)$. Consider the following five isometries (rigid transformations) of the plane: rotations of $90°$, $180°$, and $270°$ counterclockwise around the origin, reflection across the x-axis, and reflection across the y-axis. How many of the 125 sequences of three of these transformations (not necessarily distinct) will return T to its original position? (For example, a $180°$ rotation, followed by a reflection across the x-axis, followed by a reflection across the y-axis will return T to its original position, but a $90°$ rotation, followed by a reflection across the x-axis, followed by another reflection across the x-axis will not return T to its original position.)

    (A) 12 **(B)** 15 **(C)** 17 **(D)** 20 **(E)** 25

21. How many positive integers n are there such that n is a multiple of 5, and the least common multiple of $5!$ and n equals 5 times the greatest common divisor of $10!$ and n?

    (A) 12 **(B)** 24 **(C)** 36 **(D)** 48 **(E)** 72

22. Let (a_n) and (b_n) be the sequences of real numbers such that

$$(2+i)^n = a_n + b_n i$$

for all integers $n \geq 0$, where $i = \sqrt{-1}$. What is

$$\sum_{n=0}^{\infty} \frac{a_n b_n}{7^n}\ ?$$

    (A) $\frac{3}{8}$ **(B)** $\frac{7}{16}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{9}{16}$ **(E)** $\frac{4}{7}$

23. Jason rolls three fair standard six-sided dice. Then he looks at the rolls and chooses a subset of the dice (possibly empty, possibly all three dice) to reroll. After rerolling, he wins if and only if the sum of the numbers face up on the three dice is exactly 7. Jason always plays to optimize his chances of winning. What is the probability that he chooses to reroll exactly two of the dice?

    (A) $\frac{7}{36}$ **(B)** $\frac{5}{24}$ **(C)** $\frac{2}{9}$ **(D)** $\frac{17}{72}$ **(E)** $\frac{1}{4}$

24. Suppose that $\triangle ABC$ is an equilateral triangle of side length s, with the property that there is a unique point P inside the triangle such that $AP = 1$, $BP = \sqrt{3}$, and $CP = 2$. What is s?

    (A) $1+\sqrt{2}$ **(B)** $\sqrt{7}$ **(C)** $\frac{8}{3}$ **(D)** $\sqrt{5+\sqrt{5}}$ **(E)** $2\sqrt{2}$

25. The number $a = \frac{p}{q}$, where p and q are relatively prime positive integers, has the property that the sum of all real numbers x satisfying

$$\lfloor x \rfloor \cdot \{x\} = a \cdot x^2$$

is 420, where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x and $\{x\} = x - \lfloor x \rfloor$ denotes the fractional part of x. What is $p+q$?

    (A) 245 **(B)** 593 **(C)** 929 **(D)** 1331 **(E)** 1332

2020 AMC1212B

1. What is the value in simplest form of the following expression?

$$\sqrt{1}+\sqrt{1+3}+\sqrt{1+3+5}+\sqrt{1+3+5+7}$$

(A) 5 **(B)** $4+\sqrt{7}+\sqrt{10}$ **(C)** 10 **(D)** 15 **(E)** $4+3\sqrt{3}+2\sqrt{5}+\sqrt{7}$

2. What is the value of the following expression?

$$\frac{100^2-7^2}{70^2-11^2}\cdot\frac{(70-11)(70+11)}{(100-7)(100+7)}$$

(A) 1 **(B)** $\frac{9951}{9950}$ **(C)** $\frac{4780}{4779}$ **(D)** $\frac{108}{107}$ **(E)** $\frac{81}{80}$

3. The ratio of w to x is $4:3$, the ratio of y to z is $3:2$, and the ratio of z to x is $1:6$. What is the ratio of w to y?

(A) $4:3$ **(B)** $3:2$ **(C)** $8:3$ **(D)** $4:1$ **(E)** $16:3$

4. The acute angles of a right triangle are a° and b°, where $a>b$ and both a and b are prime numbers. What is the least possible value of b?

(A) 2 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 11

5. Teams A and B are playing in a basketball league where each game results in a win for one team and a loss for the other team. Team A has won $\frac{2}{3}$ of its games and team B has won $\frac{5}{8}$ of its games. Also, team B has won 7 more games and lost 7 more games than team A. How many games has team A played?

(A) 21 **(B)** 27 **(C)** 42 **(D)** 48 **(E)** 63

6. For all integers $n\geq 9$, the value of

$$\frac{(n+2)!-(n+1)!}{n!}$$

is always which of the following?

(A) a multiple of 4 **(B)** a multiple of 10 **(C)** a prime number **(D)** a perfect square **(E)** a perfect cube

7. Two nonhorizontal, non vertical lines in the xy-coordinate plane intersect to form a 45° angle. One line has slope equal to 6 times the slope of the other line. What is the greatest possible value of the product of the slopes of the two lines?

(A) $\frac{1}{6}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{2}$ **(D)** 3 **(E)** 6

8. How many ordered pairs of integers (x,y) satisfy the equation

$$x^{2020}+y^2=2y?$$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** infinitely many

9. A three-quarter sector of a circle of radius 4 inches together with its interior can be rolled up to form the lateral surface of a right circular cone by taping together along the two radii shown. What is the volume of the cone in cubic inches?

(A) $3\pi\sqrt{5}$ **(B)** $4\pi\sqrt{3}$ **(C)** $3\pi\sqrt{7}$ **(D)** $6\pi\sqrt{3}$ **(E)** $6\pi\sqrt{7}$

10. In unit square $ABCD$, the inscribed circle ω intersects $\overline{CD}$ at M, and $\overline{AM}$ intersects ω at a point P different from M. What is AP?

(A) $\frac{\sqrt{5}}{12}$ **(B)** $\frac{\sqrt{5}}{10}$ **(C)** $\frac{\sqrt{5}}{9}$ **(D)** $\frac{\sqrt{5}}{8}$ **(E)** $\frac{2\sqrt{5}}{15}$

11. As shown in the figure below, six semicircles lie in the interior of a regular hexagon with side length 2 so that the diameters of the semicircles coincide with the sides of the hexagon. What is the area of the shaded regioninside the hexagon but outside all of the semicircles?

(A) $6\sqrt{3} - 3\pi$ **(B)** $\frac{9\sqrt{3}}{2} - 2\pi$ **(C)** $\frac{3\sqrt{3}}{2} - \frac{\pi}{3}$ **(D)** $3\sqrt{3} - \pi$ **(E)** $\frac{9\sqrt{3}}{2} - \pi$

12. Let $\overline{AB}$ be a diameter in a circle of radius $5\sqrt{2}$. Let $\overline{CD}$ be a chord in the circle that intersects $\overline{AB}$ at a point E such that $BE = 2\sqrt{5}$ and $\angle AEC = 45^\circ$. What is $CE^2 + DE^2$?

(A) 96 **(B)** 98 **(C)** $44\sqrt{5}$ **(D)** $70\sqrt{2}$ **(E)** 100

13. Which of the following is the value of $\sqrt{\log_2 6 + \log_3 6}$?

(A) 1 **(B)** $\sqrt{\log_5 6}$ **(C)** 2 **(D)** $\sqrt{\log_2 3} + \sqrt{\log_3 2}$ **(E)** $\sqrt{\log_2 6} + \sqrt{\log_3 6}$

14. Bela and Jenn play the following game on the closed interval $[0, n]$ of the real number line, where n is a fixed integer greater than 4. They take turns playing, with Bela going first. At his first turn, Bela chooses any real number in the interval $[0, n]$. Thereafter, the player whose turn it is chooses a real number that is more than one unit away from all numbers previously chosen by either player. A player unable to choose such a number loses. Using optimal strategy, which player will win the game?

(A) Bela will always win. **(B)** Jenn will always win. **(C)** Bela will win if and only if n is odd. **(D)** Jenn will w
8.

15. There are 10 people standing equally spaced around a circle. Each person knows exactly 3 of the other 9 people: the 2 people standing next to her or him, as well as the person directly across the circle. How many ways are there for the 10 people to split up into 5 pairs so that the members of each pair know each other?

(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

16. An urn contains one red ball and one blue ball. A box of extra red and blue balls lie nearby. George performs the following operation four times: he draws a ball from the urn at random and then takes a ball of the same color from the box and returns those two matching balls to the urn. After the four iterations the urn contains six balls. What is the probability that the urn contains three balls of each color?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

17. How many polynomials of the form $x^5 + ax^4 + bx^3 + cx^2 + dx + 2020$, where a, b, c, and d are real numbers, have the property that whenever r is a root, so is $\frac{-1+i\sqrt{3}}{2} \cdot r$? (Note that $i = \sqrt{-1}$)

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

18. In square $ABCD$, points E and H lie on $\overline{AB}$ and $\overline{DA}$, respectively, so that $AE = AH$. Points F and G lie on $\overline{BC}$ and $\overline{CD}$, respectively, and points I and J lie on $\overline{EH}$ so that $\overline{FI} \perp \overline{EH}$ and $\overline{GJ} \perp \overline{EH}$. See the figure below. Triangle AEH, quadrilateral $BFIE$, quadrilateral $DHJG$, and pentagon $FCGJI$ each has area 1. What is FI^2?

(A) $\frac{7}{3}$ **(B)** $8 - 4\sqrt{2}$ **(C)** $1 + \sqrt{2}$ **(D)** $\frac{7}{4}\sqrt{2}$ **(E)** $2\sqrt{2}$

19. Square $ABCD$ in the coordinate plane has vertices at the points $A(1, 1), B(-1, 1), C(-1, -1)$, and $D(1, -1)$. Consider the following four transformations:

- L, a rotation of 90° counterclockwise around the origin;
- R, a rotation of 90° clockwise around the origin;
- H, a reflection across the x-axis; and
- V, a reflection across the y-axis.

Each of these transformations maps the squares onto itself, but the positions of the labeled vertices will change. For example, applying R and then V would send the vertex A at $(1, 1)$ to $(-1, -1)$ and would send the vertex B at $(-1, 1)$ to itself. How many sequences of 20 transformations chosen from $\{L, R, H, V\}$ will send all of the labeled vertices back to their original positions? (For example, R, R, V, H is one sequence of 4 transformations that will send the vertices back to their original positions.)

(A) 2^{37} **(B)** $3 \cdot 2^{36}$ **(C)** 2^{38} **(D)** $3 \cdot 2^{37}$ **(E)** 2^{39}

20. Two different cubes of the same size are to be painted, with the color of each face being chosen independently and at random to be either black or white. What is the probability that after they are painted, the cubes can be rotated to be identical in appearance?

(A) $\frac{9}{64}$ **(B)** $\frac{289}{2048}$ **(C)** $\frac{73}{512}$ **(D)** $\frac{147}{1024}$ **(E)** $\frac{589}{4096}$

21. How many positive integers n satisfy

$$\frac{n + 1000}{70} = \lfloor \sqrt{n} \rfloor?$$

(Recall that $\lfloor x \rfloor$ is the greatest integer not exceeding x.)

(A) 2 **(B)** 4 **(C)** 6 **(D)** 30 **(E)** 32

22. What is the maximum value of $\frac{(2^t - 3t)t}{4^t}$ for real values of t?

(A) $\frac{1}{16}$ **(B)** $\frac{1}{15}$ **(C)** $\frac{1}{12}$ **(D)** $\frac{1}{10}$ **(E)** $\frac{1}{9}$

23. How many integers $n \geq 2$ are there such that whenever $z_1, z_2, ..., z_n$ are complex numbers such that

$$|z_1| = |z_2| = ... = |z_n| = 1 \text{ and } z_1 + z_2 + ... + z_n = 0,$$

then the numbers $z_1, z_2, ..., z_n$ are equally spaced on the unit circle in the complex plane?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

24. Let $D(n)$ denote the number of ways of writing the positive integer n as a product

$$n = f_1 \cdot f_2 \cdots f_k,$$

where $k \geq 1$, the f_i are integers strictly greater than 1, and the order in which the factors are listed matters (that is, two representations that differ only in the order of the factors are counted as distinct). For example, the number 6 can be written as 6, $2 \cdot 3$, and $3 \cdot 2$, so $D(6) = 3$. What is $D(96)$?

(A) 112 **(B)** 128 **(C)** 144 **(D)** 172 **(E)** 184

25. For each real number a with $0 \leq a \leq 1$, let numbers x and y be chosen independently at random from the intervals $[0, a]$ and $[0, 1]$, respectively, and let $P(a)$ be the probability that

$$\sin^2(\pi x) + \sin^2(\pi y) > 1$$

What is the maximum value of $P(a)$?

(A) $\frac{7}{12}$ **(B)** $2 - \sqrt{2}$ **(C)** $\frac{1+\sqrt{2}}{4}$ **(D)** $\frac{\sqrt{5}-1}{2}$ **(E)** $\frac{5}{8}$

2021 AMC1212A

1. What is the value of $\frac{(2112-2021)^2}{169}$?

(A) 7 **(B)** 21 **(C)** 49 **(D)** 64 **(E)** 91

2. Menkara has a 4×6 index card. If she shortens the length of one side of this card by 1 inch, the card would have area 18 square inches. What would the area of the card be in square inches if instead she shortens the length of the other side by 1 inch?

(A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 20

3. Mr. Lopez has a choice of two routes to get to work. Route A is 6 miles long, and his average speed along this route is 30 miles per hour. Route B is 5 miles long, and his average speed along this route is 40 miles per hour, except for a $\frac{1}{2}$-mile stretch in a school zone where his average speed is 20 miles per hour. By how many minutes is Route B quicker than Route A?

(A) $2\frac{3}{4}$ **(B)** $3\frac{3}{4}$ **(C)** $4\frac{1}{2}$ **(D)** $5\frac{1}{2}$ **(E)** $6\frac{3}{4}$

4. The six-digit number $\underline{2}\,\underline{0}\,\underline{2}\,\underline{1}\,\underline{0}\,\underline{A}$ is prime for only one digit A. What is A?

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

5. Elmer the emu takes 44 equal strides to walk between consecutive telephone poles on a rural road. Oscar the ostrich can cover the same distance in 12 equal leaps. The telephone poles are evenly spaced, and the 41st pole along this road is exactly one mile (5280 feet) from the first pole. How much longer, in feet, is Oscar's leap than Elmer's stride?

(A) 6 **(B)** 8 **(C)** 10 **(D)** 11 **(E)** 15

6. As shown in the figure below, point E lies on the opposite half-plane determined by line CD from point A so that $\angle CDE = 110^\circ$. Point F lies on $\overline{AD}$ so that $DE = DF$, and $ABCD$ is a square. What is the degree measure of $\angle AFE$?

(A) 160 **(B)** 164 **(C)** 166 **(D)** 170 **(E)** 174

7. A school has 100 students and 5 teachers. In the first period, each student is taking one class, and each teacher is teaching one class. The enrollments in the classes are $50, 20, 20, 5,$ and 5. Let t be the average value obtained if a teacher is picked at random and the number of students in their class is noted. Let s be the average value obtained if a student was picked at random and the number of students in their class, including the student, is noted. What is $t - s$?

(A) -18.5 **(B)** -13.5 **(C)** 0 **(D)** 13.5 **(E)** 18.5

8. Let M be the least common multiple of all the integers 10 through 30, inclusive. Let N be the least common multiple of $M, 32, 33, 34, 35, 36, 37, 38, 39,$ and 40. What is the value of $\frac{N}{M}$?

(A) 1 **(B)** 2 **(C)** 37 **(D)** 74 **(E)** 2886

9. A right rectangular prism whose surface area and volume are numerically equal has edge lengths $\log_2 x, \log_3 x,$ and $\log_4 x$. What is x?

(A) $2\sqrt{6}$ **(B)** $6\sqrt{6}$ **(C)** 24 **(D)** 48 **(E)** 576

10. The base-nine representation of the number N is $27{,}006{,}000{,}052_{\text{nine}}$. What is the remainder when N is divided by 5?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

11. Consider two concentric circles of radius 17 and 19. The larger circle has a chord, half of which lies inside the smaller circle. What is the length of the chord in the larger circle?

(A) $12\sqrt{2}$ **(B)** $10\sqrt{3}$ **(C)** $\sqrt{17 \cdot 19}$ **(D)** 18 **(E)** $8\sqrt{6}$

12. What is the number of terms with rational coefficients among the 1001 terms in the expansion of $\left(x\sqrt[3]{2}+y\sqrt{3}\right)^{1000}$?

(A) 0 **(B)** 166 **(C)** 167 **(D)** 500 **(E)** 501

13. The angle bisector of the acute angle formed at the origin by the graphs of the lines $y = x$ and $y = 3x$ has equation $y = kx$. What is k?

(A) $\frac{1+\sqrt{5}}{2}$ **(B)** $\frac{1+\sqrt{7}}{2}$ **(C)** $\frac{2+\sqrt{3}}{2}$ **(D)** 2 **(E)** $\frac{2+\sqrt{5}}{2}$

14. In the figure, equilateral hexagon $ABCDEF$ has three nonadjacent acute interior angles that each measure $30°$. The enclosed area of the hexagon is $6\sqrt{3}$. What is the perimeter of the hexagon?

(A) 4 **(B)** $4\sqrt{3}$ **(C)** 12 **(D)** 18 **(E)** $12\sqrt{3}$

15. Recall that the conjugate of the complex number $w = a + bi$, where a and b are real numbers and $i = \sqrt{-1}$, is the complex number $\overline{w} = a - bi$. For any complex number z, let $f(z) = 4i\overline{z}$. The polynomial

$$P(z) = z^4 + 4z^3 + 3z^2 + 2z + 1$$

has four complex roots: z_1, z_2, z_3, and z_4. Let

$$Q(z) = z^4 + Az^3 + Bz^2 + Cz + D$$

be the polynomial whose roots are $f(z_1)$, $f(z_2)$, $f(z_3)$, and $f(z_4)$, where the coefficients A, B, C, and D are complex numbers. What is $B + D$?

(A) -304 **(B)** -208 **(C)** $12i$ **(D)** 208 **(E)** 304

16. An organization has 30 employees, 20 of whom have a brand A computer while the other 10 have a brand B computer. For security, the computers can only be connected to each other and only by cables. The cables can only connect a brand A computer to a brand B computer. Employees can communicate with each other if their computers are directly connected by a cable or by relaying messages through a series of connected computers. Initially, no computer is connected to any other. A technician arbitrarily selects one computer of each brand and installs a cable between them, provided there is not already a cable between that pair. The technician stops once every employee can communicate with each other. What is the maximum possible number of cables used?

(A) 190 **(B)** 191 **(C)** 192 **(D)** 195 **(E)** 196

17. For how many ordered pairs (b, c) of positive integers does neither $x^2 + bx + c = 0$ nor $x^2 + cx + b = 0$ have two distinct real solutions?

(A) 4 **(B)** 6 **(C)** 8 **(D)** 12 **(E)** 16

18. Each of 20 balls is tossed independently and at random into one of 5 bins. Let p be the probability that some bin ends up with 3 balls, another with 5 balls, and the other three with 4 balls each. Let q be the probability that every bin ends up with 4 balls. What is $\frac{p}{q}$?

(A) 1 **(B)** 4 **(C)** 8 **(D)** 12 **(E)** 16

19. Let x be the least real number greater than 1 such that $\sin(x) = \sin(x^2)$, where the arguments are in degrees. What is x rounded up to the closest integer?

(A) 10 **(B)** 13 **(C)** 14 **(D)** 19 **(E)** 20

20. For each positive integer n, let $f_1(n)$ be twice the number of positive integer divisors of n, and for $j \geq 2$, let $f_j(n) = f_1(f_{j-1}(n))$. For how many values of $n \leq 50$ is $f_{50}(n) = 12$?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

21. Let $ABCD$ be an isosceles trapezoid with $\overline{BC} \parallel \overline{AD}$ and $AB = CD$. Points X and Y lie on diagonal $\overline{AC}$ with X between A and Y, as shown in the figure. Suppose $\angle AXD = \angle BYC = 90°$, $AX = 3$, $XY = 1$, and $YC = 2$. What is the area of $ABCD$?

(A) 15 **(B)** $5\sqrt{11}$ **(C)** $3\sqrt{35}$ **(D)** 18 **(E)** $7\sqrt{7}$

22. Azar and Carl play a game of tic-tac-toe. Azar places an X in one of the boxes in a 3-by-3 array of boxes, then Carl places an O in one of the remaining boxes. After that, Azar places an X in one of the remaining boxes, and so on until all boxes are filled or one of the players has of their symbols in a rowhorizontal, vertical, or diagonalwhichever comes first, in which case that player wins the game. Suppose the players make their moves at random, rather than trying to follow a rational strategy, and that Carl wins the game when he places his third O. How many ways can the board look after the game is over?

(A) 36 **(B)** 112 **(C)** 120 **(D)** 148 **(E)** 160

23. A quadratic polynomial with real coefficients and leading coefficient 1 is called *disrespectful* if the equation $p(p(x)) = 0$ is satisfied by exactly three real numbers. Among all the disrespectful quadratic polynomials, there is a unique such polynomial $\tilde{p}(x)$ for which the sum of the roots is maximized. What is $\tilde{p}(1)$?

(A) $\frac{5}{16}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{5}{8}$ **(D)** 1 **(E)** $\frac{9}{8}$

24. Convex quadrilateral $ABCD$ has $AB = 18, \angle A = 60^\circ$, and $\overline{AB} \parallel \overline{CD}$. In some order, the lengths of the four sides form an arithmetic progression, and side $\overline{AB}$ is a side of maximum length. The length of another side is a. What is the sum of all possible values of a?

(A) 24 **(B)** 42 **(C)** 60 **(D)** 66 **(E)** 84

25. Let $m \geq 5$ be an odd integer, and let $D(m)$ denote the number of quadruples (a_1, a_2, a_3, a_4) of distinct integers with $1 \leq a_i \leq m$ for all i such that m divides $a_1 + a_2 + a_3 + a_4$. There is a polynomial

$$q(x) = c_3x^3 + c_2x^2 + c_1x + c_0$$

such that $D(m) = q(m)$ for all odd integers $m \geq 5$. What is c_1?

(A) -6 **(B)** -1 **(C)** 4 **(D)** 6 **(E)** 11

2021 AMC1212B

1. What is the value of $1234 + 2341 + 3412 + 4123$?

   (A) 10,000 **(B)** 10,010 **(C)** 10,110 **(D)** 11,000 **(E)** 11,110

2. What is the area of the shaded figure shown below?

   (A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

3. At noon on a certain day, Minneapolis is N degrees warmer than St. Louis. At 4:00 the temperature in Minneapolis has fallen by 5 degrees while the temperature in St. Louis has risen by 3 degrees, at which time the temperatures in the two cities differ by 2 degrees. What is the product of all possible values of N?

   (A) 10 **(B)** 30 **(C)** 60 **(D)** 100 **(E)** 120

4. Let $n = 8^{2022}$. Which of the following is equal to $\frac{n}{4}$?

   (A) 4^{1010} **(B)** 2^{2022} **(C)** 8^{2018} **(D)** 4^{3031} **(E)** 4^{3032}

5. Call a fraction $\frac{a}{b}$, not necessarily in the simplest form, "special" if a and b are positive integers whose sum is 15. How many distinct integers can be written as the sum of two, not necessarily different, special fractions?

   (A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

6. The greatest prime number that is a divisor of 16,384 is 2 because $16{,}384 = 2^{14}$. What is the sum of the digits of the greatest prime number that is a divisor of 16,383?

   (A) 3 **(B)** 7 **(C)** 10 **(D)** 16 **(E)** 22

7. Which of the following conditions is sufficient to guarantee that integers x, y, and z satisfy the equation

   $$x(x-y) + y(y-z) + z(z-x) = 1?$$

   (A) $x > y$ and $y = z$

   (B) $x = y - 1$ and $y = z - 1$

   (C) $x = z + 1$ and $y = x + 1$

   (D) $x = z$ and $y - 1 = x$

   (E) $x + y + z = 1$

8. The product of the lengths of the two congruent sides of an obtuse isosceles triangle is equal to the product of the base and twice the triangle's height to the base. What is the measure, in degrees, of the vertex angle of this triangle?

   (A) 105 **(B)** 120 **(C)** 135 **(D)** 150 **(E)** 165

9. Triangle ABC is equilateral with side length 6. Suppose that O is the center of the inscribed circle of this triangle. What is the area of the circle passing through A, O, and C?

   (A) 9π **(B)** 12π **(C)** 18π **(D)** 24π **(E)** 27π

10. What is the sum of all possible values of t between 0 and 360 such that the triangle in the coordinate plane whose vertices are

    $$(\cos 40^\circ, \sin 40^\circ), (\cos 60^\circ, \sin 60^\circ), \text{ and } (\cos t^\circ, \sin t^\circ)$$

    is isosceles?

    (A) 100 **(B)** 150 **(C)** 330 **(D)** 360 **(E)** 380

11. Una rolls 6 standard 6-sided dice simultaneously and calculates the product of the 6 numbers obtained. What is the probability that the product is divisible by 4?

(A) $\frac{3}{4}$ **(B)** $\frac{57}{64}$ **(C)** $\frac{59}{64}$ **(D)** $\frac{187}{192}$ **(E)** $\frac{63}{64}$

12. For n a positive integer, let $f(n)$ be the quotient obtained when the sum of all positive divisors of n is divided by n. For example,

$$f(14) = (1+2+7+14) \div 14 = \frac{12}{7}$$

What is $f(768) - f(384)$?

(A) $\frac{1}{768}$ **(B)** $\frac{1}{192}$ **(C)** 1 **(D)** $\frac{4}{3}$ **(E)** $\frac{8}{3}$

13. Let $c = \frac{2\pi}{11}$. What is the value of

$$\frac{\sin 3c \cdot \sin 6c \cdot \sin 9c \cdot \sin 12c \cdot \sin 15c}{\sin c \cdot \sin 2c \cdot \sin 3c \cdot \sin 4c \cdot \sin 5c}?$$

(A) -1 **(B)** $-\frac{\sqrt{11}}{5}$ **(C)** $\frac{\sqrt{11}}{5}$ **(D)** $\frac{10}{11}$ **(E)** 1

14. Suppose that $P(z), Q(z)$, and $R(z)$ are polynomials with real coefficients, having degrees 2, 3, and 6, respectively, and constant terms 1, 2, and 3, respectively. Let N be the number of distinct complex numbers z that satisfy the equation $P(z) \cdot Q(z) = R(z)$. What is the minimum possible value of N?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 5

15. Three identical square sheets of paper each with side length 6 are stacked on top of each other. The middle sheet is rotated clockwise 30° about its center and the top sheet is rotated clockwise 60° about its center, resulting in the 24-sided polygon shown in the figure below. The area of this polygon can be expressed in the form $a - b\sqrt{c}$, where a, b, and c are positive integers, and c is not divisible by the square of any prime. What is $a + b + c$? ¡center¿

¡/center¿ **(A)** 75 **(B)** 93 **(C)** 96 **(D)** 129 **(E)** 147

16. Suppose a, b, c are positive integers such that

$$a + b + c = 23$$

and

$$\gcd(a, b) + \gcd(b, c) + \gcd(c, a) = 9.$$

What is the sum of all possible distinct values of $a^2 + b^2 + c^2$?

(A) 259 **(B)** 438 **(C)** 516 **(D)** 625 **(E)** 687

17. A bug starts at a vertex of a grid made of equilateral triangles of side length 1. At each step the bug moves in one of the 6 possible directions along the grid lines randomly and independently with equal probability. What is the probability that after 5 moves the bug never will have been more than 1 unit away from the starting position?

(A) $\frac{13}{108}$ **(B)** $\frac{7}{54}$ **(C)** $\frac{29}{216}$ **(D)** $\frac{4}{27}$ **(E)** $\frac{1}{16}$

18. Set $u_0 = \frac{1}{4}$, and for $k \geq 0$ let u_{k+1} be determined by the recurrence

$$u_{k+1} = 2u_k - 2u_k^2.$$

This sequence tends to a limit; call it L. What is the least value of k such that

$$|u_k - L| \leq \frac{1}{2^{1000}}?$$

(A) 10 **(B)** 87 **(C)** 123 **(D)** 329 **(E)** 401

19. Regular polygons with $5, 6, 7,$ and 8 sides are inscribed in the same circle. No two of the polygons share a vertex, and no three of their sides intersect at a common point. At how many points inside the circle do two of their sides intersect?

$\textbf{(A)}\ 52 \qquad \textbf{(B)}\ 56 \qquad \textbf{(C)}\ 60 \qquad \textbf{(D)}\ 64 \qquad \textbf{(E)}\ 68$

20. A cube is constructed from 4 white unit cubes and 4 blue unit cubes. How many different ways are there to construct the $2 \times 2 \times 2$ cube using these smaller cubes? (Two constructions are considered the same if one can be rotated to match the other.)

$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 11$

21. For real numbers x, let

$$P(x) = 1 + \cos(x) + i\sin(x) - \cos(2x) - i\sin(2x) + \cos(3x) + i\sin(3x)$$

where $i = \sqrt{-1}$. For how many values of x with $0 \le x < 2\pi$ does

$$P(x) = 0?$$

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

22. Right triangle ABC has side lengths $BC = 6$, $AC = 8$, and $AB = 10$. A circle centered at O is tangent to line BC at B and passes through A. A circle centered at P is tangent to line AC at A and passes through B. What is OP?

$\textbf{(A)}\ \frac{23}{8} \qquad \textbf{(B)}\ \frac{29}{10} \qquad \textbf{(C)}\ \frac{35}{12} \qquad \textbf{(D)}\ \frac{73}{25} \qquad \textbf{(E)}\ 3$

23. What is the average number of pairs of consecutive integers in a randomly selected subset of 5 distinct integers chosen from the set $\{1, 2, 3, , 30\}$? (For example the set $\{1, 17, 18, 19, 30\}$ has 2 pairs of consecutive integers.)

$\textbf{(A)}\ \frac{2}{3} \qquad \textbf{(B)}\ \frac{29}{36} \qquad \textbf{(C)}\ \frac{5}{6} \qquad \textbf{(D)}\ \frac{29}{30} \qquad \textbf{(E)}\ 1$

24. Triangle ABC has side lengths $AB = 11, BC = 24$, and $CA = 20$. The bisector of $\angle BAC$ intersects $\overline{BC}$ in point D, and intersects the circumcircle of $\triangle ABC$ in point $E \ne A$. The circumcircle of $\triangle BED$ intersects the line AB in points B and $F \ne B$. What is CF?

$\textbf{(A)}\ 28 \qquad \textbf{(B)}\ 20\sqrt{2} \qquad \textbf{(C)}\ 30 \qquad \textbf{(D)}\ 32 \qquad \textbf{(E)}\ 20\sqrt{3}$

25. For n a positive integer, let $R(n)$ be the sum of the remainders when n is divided by 2, 3, 4, 5, 6, 7, 8, 9, and 10. For example, $R(15) = 1 + 0 + 3 + 0 + 3 + 1 + 7 + 6 + 5 = 26$. How many two-digit positive integers n satisfy $R(n) = R(n + 1)$?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

2022 AMC1212A

1. What is the value of
$$3+\frac{1}{3+\frac{1}{3+\frac{1}{3}}}?$$
(A) $\frac{31}{10}$ **(B)** $\frac{49}{15}$ **(C)** $\frac{33}{10}$ **(D)** $\frac{109}{33}$ **(E)** $\frac{15}{4}$

2. The sum of three numbers is 96. The first number is 6 times the third number, and the third number is 40 less than the second number. What is the absolute value of the difference between the first and second numbers?
(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

3. Five rectangles, A, B, C, D, and E, are arranged in a square as shown below. These rectangles have dimensions 1×6, 2×4, 5×6, 2×7, and 2×3, respectively. (The figure is not drawn to scale.) Which of the five rectangles is the shaded one in the middle?
(A) A **(B)** B **(C)** C **(D)** D **(E)** E

4. The least common multiple of a positive integer n and 18 is 180, and the greatest common divisor of n and 45 is 15. What is the sum of the digits of n?
(A) 3 **(B)** 6 **(C)** 8 **(D)** 9 **(E)** 12

5. The ¡em¿taxicab distance¡/em¿ between points (x_1, y_1) and (x_2, y_2) in the coordinate plane is given by
$$|x_1-x_2|+|y_1-y_2|.$$
For how many points P with integer coordinates is the taxicab distance between P and the origin less than or equal to 20?
(A) 441 **(B)** 761 **(C)** 841 **(D)** 921 **(E)** 924

6. A data set consists of 6 (not distinct) positive integers: 1, 7, 5, 2, 5, and X. The average (arithmetic mean) of the 6 numbers equals a value in the data set. What is the sum of all possible values of X?
(A) 10 **(B)** 26 **(C)** 32 **(D)** 36 **(E)** 40

7. A rectangle is partitioned into 5 regions as shown. Each region is to be painted a solid color - red, orange, yellow, blue, or green - so that regions that touch are painted different colors, and colors can be used more than once. How many different colorings are possible?
(A) 120 **(B)** 270 **(C)** 360 **(D)** 540 **(E)** 720

8. The infinite product
$$\sqrt[3]{10}\cdot\sqrt[3]{\sqrt[3]{10}}\cdot\sqrt[3]{\sqrt[3]{\sqrt[3]{10}}}\cdots$$
evaluates to a real number. What is that number?
(A) $\sqrt{10}$ **(B)** $\sqrt[3]{100}$ **(C)** $\sqrt[4]{1000}$ **(D)** 10 **(E)** $10\sqrt[3]{10}$

9. On Halloween 31 children walked into the principal's office asking for candy. They can be classified into three types: Some always lie; some always tell the truth; and some alternately lie and tell the truth. The alternaters arbitrarily choose their first response, either a lie or the truth, but each subsequent statement has the opposite truth value from its predecessor. The principal asked everyone the same three questions in this order.

"Are you a truth-teller?" The principal gave a piece of candy to each of the 22 children who answered yes.

"Are you an alternater?" The principal gave a piece of candy to each of the 15 children who answered yes.

"Are you a liar?" The principal gave a piece of candy to each of the 9 children who answered yes.

How many pieces of candy in all did the principal give to the children who always tell the truth?

(A) 7 **(B)** 12 **(C)** 21 **(D)** 27 **(E)** 31

10. How many ways are there to split the integers 1 through 14 into 7 pairs such that in each pair, the greater number is at least 2 times the lesser number?

(A) 108 **(B)** 120 **(C)** 126 **(D)** 132 **(E)** 144

11. What is the product of all real numbers x such that the distance on the number line between $\log_6 x$ and $\log_6 9$ is twice the distance on the number line between $\log_6 10$ and 1?

(A) 10 **(B)** 18 **(C)** 25 **(D)** 36 **(E)** 81

12. Let M be the midpoint of $\overline{AB}$ in regular tetrahedron $ABCD$. What is $\cos(\angle CMD)$?

(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{2}{5}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{\sqrt{3}}{2}$

13. Let $\mathcal{R}$ be the region in the complex plane consisting of all complex numbers z that can be written as the sum of complex numbers z_1 and z_2, where z_1 lies on the segment with endpoints 3 and $4i$, and z_2 has magnitude at most 1. What integer is closest to the area of $\mathcal{R}$?

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

14. What is the value of

$$(\log 5)^3 + (\log 20)^3 + (\log 8)(\log 0.25)$$

where log denotes the base-ten logarithm?

(A) $\frac{3}{2}$ **(B)** $\frac{7}{4}$ **(C)** 2 **(D)** $\frac{9}{4}$ **(E)** 3

15. The roots of the polynomial $10x^3 - 39x^2 + 29x - 6$ are the height, length, and width of a rectangular box (right rectangular prism). A new rectangular box is formed by lengthening each edge of the original box by 2 units. What is the volume of the new box?

(A) $\frac{24}{5}$ **(B)** $\frac{42}{5}$ **(C)** $\frac{81}{5}$ **(D)** 30 **(E)** 48

16. A triangular number is a positive integer that can be expressed in the form $t_n = 1 + 2 + 3 + \cdots + n$, for some positive integer n. The three smallest triangular numbers that are also perfect squares are $t_1 = 1 = 1^2, t_8 = 36 = 6^2$, and $t_{49} = 1225 = 35^2$. What is the sum of the digits of the fourth smallest triangular number that is also a perfect square?

(A) 6 **(B)** 9 **(C)** 12 **(D)** 18 **(E)** 27

17. Suppose a is a real number such that the equation

$$a \cdot (\sin x + \sin(2x)) = \sin(3x)$$

has more than one solution in the interval $(0, \pi)$. The set of all such a that can be written in the form

$$(p, q) \cup (q, r),$$

where p, q, and r are real numbers with $p < q < r$. What is $p + q + r$?

(A) -4 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 4

18. Let T_k be the transformation of the coordinate plane that first rotates the plane k degrees counterclockwise around the origin and then reflects the plane across the y-axis. What is the least positive integer n such that performing the sequence of transformations $T_1, T_2, T_3, \ldots, T_n$ returns the point $(1, 0)$ back to itself?

(A) 359 **(B)** 360 **(C)** 719 **(D)** 720 **(E)** 721

19. Suppose that 13 cards numbered $1, 2, 3, \ldots, 13$ are arranged in a row. The task is to pick them up in numerically increasing order, working repeatedly from left to right. In the example below, cards $1, 2, 3$ are picked up on the first pass, 4 and 5 on the second pass, 6 on the third pass, $7, 8, 9, 10$ on the fourth pass, and $11, 12, 13$ on the fifth pass. For how many of the 13! possible orderings of the cards will the 13 cards be picked up in exactly two passes?

    (A) 4082 **(B)** 4095 **(C)** 4096 **(D)** 8178 **(E)** 8191

20. Isosceles trapezoid $ABCD$ has parallel sides $\overline{AD}$ and $\overline{BC}$, with $BC < AD$ and $AB = CD$. There is a point P in the plane such that $PA = 1, PB = 2, PC = 3$, and $PD = 4$. What is $\frac{BC}{AD}$?

    (A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

21. Let
$$P(x) = x^{2022} + x^{1011} + 1.$$
Which of the following polynomials is a factor of $P(x)$?

    (A) $x^2 - x + 1$ **(B)** $x^2 + x + 1$ **(C)** $x^4 + 1$ **(D)** $x^6 - x^3 + 1$ **(E)** $x^6 + x^3 + 1$

22. Let c be a real number, and let z_1 and z_2 be the two complex numbers satisfying the equation $z^2 - cz + 10 = 0$. Points z_1, z_2, $\frac{1}{z_1}$, and $\frac{1}{z_2}$ are the vertices of (convex) quadrilateral $\mathcal{Q}$ in the complex plane. When the area of $\mathcal{Q}$ obtains its maximum possible value, c is closest to which of the following?

    (A) 4.5 **(B)** 5 **(C)** 5.5 **(D)** 6 **(E)** 6.5

23. Let h_n and k_n be the unique relatively prime positive integers such that
$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} = \frac{h_n}{k_n}.$$
Let L_n denote the least common multiple of the numbers $1, 2, 3, \ldots, n$. For how many integers with $1 \le n \le 22$ is $k_n < L_n$?

    (A) 0 **(B)** 3 **(C)** 7 **(D)** 8 **(E)** 10

24. How many strings of length 5 formed from the digits 0, 1, 2, 3, 4 are there such that for each $j \in \{1, 2, 3, 4\}$, at least j of the digits are less than j? (For example, 02214 satisfies this condition because it contains at least 1 digit less than 1, at least 2 digits less than 2, at least 3 digits less than 3, and at least 4 digits less than 4. The string 23404 does not satisfy the condition because it does not contain at least 2 digits less than 2.)

    (A) 500 **(B)** 625 **(C)** 1089 **(D)** 1199 **(E)** 1296

25. A circle with integer radius r is centered at (r, r). Distinct line segments of length c_i connect points $(0, a_i)$ to $(b_i, 0)$ for $1 \le i \le 14$ and are tangent to the circle, where a_i, b_i, and c_i are all positive integers and $c_1 \le c_2 \le \cdots \le c_{14}$. What is the ratio $\frac{c_{14}}{c_1}$ for the least possible value of r?

    (A) $\frac{21}{5}$ **(B)** $\frac{85}{13}$ **(C)** 7 **(D)** $\frac{39}{5}$ **(E)** 17

2022 AMC1212B

1. Define $x \diamond y$ to be $|x-y|$ for all real numbers x and y. What is the value of

$$(1 \diamond (2 \diamond 3)) - ((1 \diamond 2) \diamond 3)?$$

(A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

2. In rhombus $ABCD$, point P lies on segment $\overline{AD}$ so that $\overline{BP} \perp \overline{AD}$, $AP = 3$, and $PD = 2$. What is the area of $ABCD$? (Note: The figure is not drawn to scale.)

(A) $3\sqrt{5}$ **(B)** 10 **(C)** $6\sqrt{5}$ **(D)** 20 **(E)** 25

3. How many of the first ten numbers of the sequence $121, 11211, 1112111, \ldots$ are prime numbers?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

4. For how many values of the constant k will the polynomial $x^2 + kx + 36$ have two distinct integer roots?

(A) 6 **(B)** 8 **(C)** 9 **(D)** 14 **(E)** 16

5. The point $(-1, -2)$ is rotated $270°$ counterclockwise about the point $(3, 1)$. What are the coordinates of its new position?

(A) $(-3, -4)$ **(B)** $(0, 5)$ **(C)** $(2, -1)$ **(D)** $(4, 3)$ **(E)** $(6, -3)$

6. Consider the following 100 sets of 10 elements each:

$$\begin{aligned}&\{1, 2, 3, \ldots, 10\},\\&\{11, 12, 13, \ldots, 20\},\\&\{21, 22, 23, \ldots, 30\},\\&\vdots\\&\{991, 992, 993, \ldots, 1000\}.\end{aligned}$$

How many of these sets contain exactly two multiples of 7?

(A) 40 **(B)** 42 **(C)** 43 **(D)** 49 **(E)** 50

7. Camila writes down five positive integers. The unique mode of these integers is 2 greater than their median, and the median is 2 greater than their arithmetic mean. What is the least possible value for the mode?

(A) 5 **(B)** 7 **(C)** 9 **(D)** 11 **(E)** 13

8. What is the graph of $y^4 + 1 = x^4 + 2y^2$ in the coordinate plane?

(A) two intersecting parabolas **(B)** two nonintersecting parabolas **(C)** two intersecting circles
(D) a circle and a hyperbola **(E)** a circle and two parabolas

9. The sequence $a_0, a_1, a_2, \cdots$ is a strictly increasing arithmetic sequence of positive integers such that

$$2^{a_7} = 2^{27} \cdot a_7.$$

What is the minimum possible value of a_2?

(A) 8 **(B)** 12 **(C)** 16 **(D)** 17 **(E)** 22

10. Regular hexagon $ABCDEF$ has side length 2. Let G be the midpoint of $\overline{AB}$, and let H be the midpoint of $\overline{DE}$. What is the perimeter of $GCHF$?

(A) $4\sqrt{3}$ **(B)** 8 **(C)** $4\sqrt{5}$ **(D)** $4\sqrt{7}$ **(E)** 12

11. Let $f(n) = \left(\frac{-1+i\sqrt{3}}{2}\right)^n + \left(\frac{-1-i\sqrt{3}}{2}\right)^n$, where $i = \sqrt{-1}$. What is $f(2022)$?

(A) -2 **(B)** -1 **(C)** 0 **(D)** $\sqrt{3}$ **(E)** 2

12. Kayla rolls four fair 6-sided dice. What is the probability that at least one of the numbers Kayla rolls is greater than 4 and at least two of the numbers she rolls are greater than 2?

(A) $\frac{2}{3}$ **(B)** $\frac{19}{27}$ **(C)** $\frac{59}{81}$ **(D)** $\frac{61}{81}$ **(E)** $\frac{7}{9}$

13. The diagram below shows a rectangle with side lengths 4 and 8 and a square with side length 5. Three vertices of the square lie on three different sides of the rectangle, as shown. What is the area of the region inside both the square and the rectangle?

(A) $15\frac{1}{8}$ **(B)** $15\frac{3}{8}$ **(C)** $15\frac{1}{2}$ **(D)** $15\frac{5}{8}$ **(E)** $15\frac{7}{8}$

14. The graph of $y = x^2 + 2x - 15$ intersects the x-axis at points A and C and the y-axis at point B. What is $\tan(\angle ABC)$?

(A) $\frac{1}{7}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{3}{7}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{4}{7}$

15. One of the following numbers is not divisible by any prime number less than 10. Which is it?

(A) $2^{606} - 1$ **(B)** $2^{606} + 1$ **(C)** $2^{607} - 1$ **(D)** $2^{607} + 1$ **(E)** $2^{607} + 3^{607}$

16. Suppose x and y are positive real numbers such that

$$x^y = 2^{64} \text{ and } (\log_2 x)^{\log_2 y} = 2^7.$$

What is the greatest possible value of $\log_2 y$?

(A) 3 **(B)** 4 **(C)** $3 + \sqrt{2}$ **(D)** $4 + \sqrt{3}$ **(E)** 7

17. How many 4×4 arrays whose entries are 0s and 1s are there such that the row sums (the sum of the entries in each row) are $1, 2, 3,$ and 4, in some order, and the column sums (the sum of the entries in each column) are also $1, 2, 3,$ and 4, in some order? For example, the array

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

satisfies the condition.

(A) 144 **(B)** 240 **(C)** 336 **(D)** 576 **(E)** 624

18. Each square in a 5×5 grid is either filled or empty, and has up to eight adjacent neighboring squares, where neighboring squares share either a side or a corner. The grid is transformed by the following rules:

* Any filled square with two or three filled neighbors remains filled.

* Any empty square with exactly three filled neighbors becomes a filled square.

* All other squares remain empty or become empty.

A sample transformation is shown in the figure below.

Suppose the 5×5 grid has a border of empty squares surrounding a 3×3 subgrid. How many initial configurations will lead to a transformed grid consisting of a single filled square in the center after a single transformation? (Rotations and reflections of the same configuration are considered different.)

(A) 14 **(B)** 18 **(C)** 22 **(D)** 26 **(E)** 30

19. In $\triangle ABC$ medians $\overline{AD}$ and $\overline{BE}$ intersect at G and $\triangle AGE$ is equilateral. Then $\cos(C)$ can be written as $\frac{m\sqrt{p}}{n}$, where m and n are relatively prime positive integers and p is a positive integer not divisible by the square of any prime. What is $m+n+p$?

(A) 44 **(B)** 48 **(C)** 52 **(D)** 56 **(E)** 60

20. Let $P(x)$ be a polynomial with rational coefficients such that when $P(x)$ is divided by the polynomial x^2+x+1, the remainder is $x+2$, and when $P(x)$ is divided by the polynomial x^2+1, the remainder is $2x+1$. There is a unique polynomial of least degree with these two properties. What is the sum of the squares of the coefficients of that polynomial?

(A) 10 **(B)** 13 **(C)** 19 **(D)** 20 **(E)** 23

21. Let S be the set of circles in the coordinate plane that are tangent to each of the three circles with equations $x^2+y^2=4$, $x^2+y^2=64$, and $(x-5)^2+y^2=3$. What is the sum of the areas of all circles in S?

(A) 48π **(B)** 68π **(C)** 96π **(D)** 102π **(E)** 136π

22. Ant Amelia starts on the number line at 0 and crawls in the following manner. For $n=1,2,3,$ Amelia chooses a time duration t_n and an increment x_n independently and uniformly at random from the interval $(0,1)$. During the nth step of the process, Amelia moves x_n units in the positive direction, using up t_n minutes. If the total elapsed time has exceeded 1 minute during the nth step, she stops at the end of that step; otherwise, she continues with the next step, taking at most 3 steps in all. What is the probability that Amelias position when she stops will be greater than 1?

(A) $\frac{1}{3}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{3}{4}$ **(E)** $\frac{5}{6}$

23. Let $x_0, x_1, x_2, \ldots$ be a sequence of numbers, where each x_k is either 0 or 1. For each positive integer n, define

$$S_n = \sum_{k=0}^{n-1} x_k 2^k$$

Suppose $7S_n \equiv 1 \pmod{2^n}$ for all $n \geq 1$. What is the value of the sum

$$x_{2019} + 2x_{2020} + 4x_{2021} + 8x_{2022}?$$

(A) 6 **(B)** 7 **(C)** 12 **(D)** 14 **(E)** 15

24. The figure below depicts a regular 7-gon inscribed in a unit circle.

What is the sum of the 4th powers of the lengths of all 21 of its edges and diagonals?

(A) 49 **(B)** 98 **(C)** 147 **(D)** 168 **(E)** 196

25. Four regular hexagons surround a square with a side length 1, each one sharing an edge with the square, as shown in the figure below. The area of the resulting 12-sided outer nonconvex polygon can be written as $m\sqrt{n}+p$, where m, n, and p are integers and n is not divisible by the square of any prime. What is $m+n+p$?

(A) -12 **(B)** -4 **(C)** 4 **(D)** 24 **(E)** 32

2023 AMC1212A

1. Cities A and B are 45 miles apart. Alicia lives in A and Beth lives in B. Alicia bikes towards B at 18 miles per hour. Leaving at the same time, Beth bikes toward A at 12 miles per hour. How many miles from City A will they be when they meet?

   (A) 20 **(B)** 24 **(C)** 25 **(D)** 26 **(E)** 27

2. The weight of $\frac{1}{3}$ of a large pizza together with $3\frac{1}{2}$ cups of orange slices is the same weight of $\frac{3}{4}$ of a large pizza together with $\frac{1}{2}$ cups of orange slices. A cup of orange slices weigh $\frac{1}{4}$ of a pound. What is the weight, in pounds, of a large pizza?

   (A) $1\frac{4}{5}$ **(B)** 2 **(C)** $2\frac{2}{5}$ **(D)** 3 **(E)** $3\frac{3}{5}$

3. How many positive perfect squares less than 2023 are divisible by 5?

   (A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

4. How many digits are in the base-ten representation of $8^5 \cdot 5^{10} \cdot 15^5$?

   (A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

5. Janet rolls a standard 6-sided die 4 times and keeps a running total of the numbers she rolls. What is the probability that at some point, her running total will equal 3?

   (A) $\frac{2}{9}$ **(B)** $\frac{49}{216}$ **(C)** $\frac{25}{108}$ **(D)** $\frac{17}{72}$ **(E)** $\frac{13}{54}$

6. Points A and B lie on the graph of $y = \log_2 x$. The midpoint of $\overline{AB}$ is $(6, 2)$. What is the positive difference between the x-coordinates of A and B?

   (A) $2\sqrt{11}$ **(B)** $4\sqrt{3}$ **(C)** 8 **(D)** $4\sqrt{5}$ **(E)** 9

7. A digital display shows the current date as an 8-digit integer consisting of a 4-digit year, followed by a 2-digit month, followed by a 2-digit date within the month. For example, Arbor Day this year is displayed as 20230428. For how many dates in 2023 will each digit appear an even number of times in the 8-digital display for that date?

   (A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

8. Maureen is keeping track of the mean of her quiz scores this semester. If Maureen scores an 11 on the next quiz, her mean will increase by 1. If she scores an 11 on each of the next three quizzes, her mean will increase by 2. What is the mean of her quiz scores currently?

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

9. A square of area 2 is inscribed in a square of area 3, creating four congruent triangles, as shown below. What is the ratio of the shorter leg to the longer leg in the shaded right triangle?

   (A) $\frac{1}{5}$ **(B)** $\frac{1}{4}$ **(C)** $2-\sqrt{3}$ **(D)** $\sqrt{3}-\sqrt{2}$ **(E)** $\sqrt{2}-1$

10. Positive real numbers x and y satisfy $y^3 = x^2$ and $(y-x)^2 = 4y^2$. What is $x+y$?

    (A) 12 **(B)** 18 **(C)** 24 **(D)** 36 **(E)** 42

11. What is the degree measure of the acute angle formed by lines with slopes 2 and $\frac{1}{3}$?

    (A) 30 **(B)** 37.5 **(C)** 45 **(D)** 52.5 **(E)** 60

12. What is the value of

    $$2^3 - 1^3 + 4^3 - 3^3 + 6^3 - 5^3 + \cdots + 18^3 - 17^3?$$

    (A) 2023 **(B)** 2679 **(C)** 2941 **(D)** 3159 **(E)** 3235

13. In a table tennis tournament every participant played every other participant exactly once. Although there were twice as many right-handed players as left-handed players, the number of games won by left-handed players was 40% more than the number of games won by right-handed players. (There were no ties and no ambidextrous players.) What is the total number of games played?

(A) 15 **(B)** 36 **(C)** 45 **(D)** 48 **(E)** 66

14. How many complex numbers satisfy the equation $z^5 = \overline{z}$, where $\overline{z}$ is the conjugate of the complex number z?

(A) 2 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 7

15. Usain is walking for exercise by zigzagging across a 100-meter by 30-meter rectangular field, beginning at point A and ending on the segment $\overline{BC}$. He wants to increase the distance walked by zigzagging as shown in the figure below ($APQRS$). What angle $\theta \angle PAB = \angle QPC = \angle RQB = \cdots$ will produce in a length that is 120 meters? (This figure is not drawn to scale. Do not assume that the zigzag path has exactly four segments as shown; there could be more or fewer.)

(A) $\arccos \frac{5}{6}$ **(B)** $\arccos \frac{4}{5}$ **(C)** $\arccos \frac{3}{10}$ **(D)** $\arcsin \frac{4}{5}$ **(E)** $\arcsin \frac{5}{6}$

16. Consider the set of complex numbers z satisfying $|1 + z + z^2| = 4$. The maximum value of the imaginary part of z can be written in the form $\frac{\sqrt{m}}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

(A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

17. Flora the frog starts at 0 on the number line and makes a sequence of jumps to the right. In any one jump, independent of previous jumps, Flora leaps a positive integer distance m with probability $\frac{1}{2^m}$. What is the probability that Flora will eventually land at 10?

(A) $\frac{5}{512}$ **(B)** $\frac{45}{1024}$ **(C)** $\frac{127}{1024}$ **(D)** $\frac{511}{1024}$ **(E)** $\frac{1}{2}$

18. Circle C_1 and C_2 each have radius 1, and the distance between their centers is $\frac{1}{2}$. Circle C_3 is the largest circle internally tangent to both C_1 and C_2. Circle C_4 is internally tangent to both C_1 and C_2 and externally tangent to C_3. What is the radius of C_4?

(A) $\frac{1}{14}$ **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{10}$ **(D)** $\frac{3}{28}$ **(E)** $\frac{1}{9}$

19. What is the product of all the solutions to the equation

$$\log_{7x} 2023 \cdot \log_{289x} 2023 = \log_{2023x} 2023?$$

(A) $(\log_{2023} 7 \cdot \log_{2023} 289)^2$ **(B)** $\log_{2023} 7 \cdot \log_{2023} 289$ **(C)** 1

(D) $\log_7 2023 \cdot \log_{289} 2023$ **(E)** $(\log_7 2023 \cdot \log_{289} 2023)^2$

20. Rows 1, 2, 3, 4, and 5 of a triangular array of integers are shown below:

Each row after the first row is formed by placing a 1 at each end of the row, and each interior entry is 1 greater than the sum of the two numbers diagonally above it in the previous row. What is the units digit of the sum of the 2023 numbers in the 2023rd row?

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

21. If A and B are vertices of a polyhedron, define the distance $d(A, B)$ to be the minimum number of edges of the polyhedron one must traverse in order to connect A and B. For example, if $\overline{AB}$ is an edge of the polyhedron, then $d(A, B) = 1$, but if $\overline{AC}$ and $\overline{CB}$ are edges and $\overline{AB}$ is not an edge, then $d(A, B) = 2$. Let Q, R, and S be randomly chosen distinct vertices of a regular icosahedron (regular polyhedron made up of 20 equilateral triangles). What is the probability that $d(Q, R) > d(R, S)$?

    (A) $\frac{7}{22}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{3}{8}$ **(D)** $\frac{5}{12}$ **(E)** $\frac{1}{2}$

22. Let f be the unique function defined on the positive integers such that

$$\sum_{d|n} d \cdot f\left(\frac{n}{d}\right) = 1$$

    for all positive integers n, where the sum is taken over all positive divisors of n. What is $f(2023)$?

    (A) -1536 **(B)** 96 **(C)** 108 **(D)** 116 **(E)** 144

23. How many ordered pairs of positive real numbers (a, b) satisfy the equation

$$(1 + 2a)(2 + 2b)(2a + b) = 32ab?$$

    (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** an infinite number

24. Let K be the number of sequences A_1, A_2, $\ldots$, A_n such that n is a positive integer less than or equal to 10, each A_i is a subset of $\{1, 2, 3, \ldots, 10\}$, and A_{i-1} is a subset of A_i for each i between 2 and n, inclusive. For example, $\{\}$, $\{5, 7\}$, $\{2, 5, 7\}$, $\{2, 5, 7\}$, $\{2, 5, 6, 7, 9\}$ is one such sequence, with $n = 5$.What is the remainder when K is divided by 10?

    (A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

25. There is a unique sequence of integers $a_1, a_2, \cdots a_{2023}$ such that

$$\tan 2023x = \frac{a_1 \tan x + a_3 \tan^3 x + a_5 \tan^5 x + \cdots + a_{2023} \tan^{2023} x}{1 + a_2 \tan^2 x + a_4 \tan^4 x \cdots + a_{2022} \tan^{2022} x}$$

    whenever $\tan 2023x$ is defined. What is a_{2023}?

    (A) -2023 **(B)** -2022 **(C)** -1 **(D)** 1 **(E)** 2023

2023 AMC1212B

1. Mrs. Jones is pouring orange juice into four identical glasses for her four sons. She fills the first three glasses completely but runs out of juice when the fourth glass is only $\frac{1}{3}$ full. What fraction of a glass must Mrs. Jones pour from each of the first three glasses into the fourth glass so that all four glasses will have the same amount of juice?

   (A) $\frac{1}{12}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{8}$ **(E)** $\frac{2}{9}$

2. Carlos went to a sports store to buy running shoes. Running shoes were on sale, with prices reduced by 20%on every pair of shoes. Carlos also knew that he had to pay a 7.5% sales tax on the discounted price. He had 43 dollars. What is the original (before discount) price of the most expensive shoes he could afford to buy?

   (A) 46 **(B)** 50 **(C)** 48 **(D)** 47 **(E)** 49

3. A $3-4-5$ right triangle is inscribed in circle A, and a $5-12-13$ right triangle is inscribed in circle B. What is the ratio of the area of circle A to the area of circle B?

   (A) $\frac{9}{25}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{25}{169}$ **(E)** $\frac{4}{25}$

4. Jackson's paintbrush makes a narrow strip with a width of 6.5 millimeters. Jackson has enough paint to make a strip 25 meters long. How many square centimeters of paper could Jackson cover with paint?

   (A) $162,500$ **(B)** 162.5 **(C)** $1,625$ **(D)** $1,625,000$ **(E)** $16,250$

5. You are playing a game. A 2×1 rectangle covers two adjacent squares (oriented either horizontally or vertically) of a 3×3 grid of squares, but you are not told which two squares are covered. Your goal is to find at least one square that is covered by the rectangle. A "turn" consists of you guessing a square, after which you are told whether that square is covered by the hidden rectangle. What is the minimum number of turns you need to ensure that at least one of your guessed squares is covered by the rectangle?

   (A) 3 **(B)** 5 **(C)** 4 **(D)** 8 **(E)** 6

6. When the roots of the polynomial

   $$P(x) = (x-1)^1(x-2)^2(x-3)^3 \cdots (x-10)^{10}$$

   are removed from the number line, what remains is the union of 11 disjoint open intervals. On how many of these intervals is $P(x)$ positive?

   (A) 3 **(B)** 7 **(C)** 6 **(D)** 4 **(E)** 5

7. For how many integers n does the expression

   $$\sqrt{\frac{\log(n^2) - (\log n)^2}{\log n - 3}}$$

   represent a real number, where log denotes the base 10 logarithm?

   (A) 900 **(B)** 3 **(C)** 902 **(D)** 2 **(E)** 901

8. How many nonempty subsets B of $\{0, 1, 2, 3, \dots, 12\}$ have the property that the number of elements in B is equal to the least element of B? For example, $B = \{4, 6, 8, 11\}$ satisfies the condition.

   (A) 256 **(B)** 136 **(C)** 108 **(D)** 144 **(E)** 156

9. What is the area of the region in the coordinate plane defined by
   $||x|-1| + ||y|-1| \leq 1$?

   (A) 2 **(B)** 8 **(C)** 4 **(D)** 15 **(E)** 12

10. In the xy-plane, a circle of radius 4 with center on the positive x-axis is tangent to the y-axis at the origin, and a circle with radius 10 with center on the positive y-axis is tangent to the x-axis at the origin. What is the slope of the line passing through the two points at which these circles intersect?

    (A) $\dfrac{2}{7}$ **(B)** $\dfrac{3}{7}$ **(C)** $\dfrac{2}{\sqrt{29}}$ **(D)** $\dfrac{1}{\sqrt{29}}$ **(E)** $\dfrac{2}{5}$

11. What is the maximum area of an isosceles trapezoid that has legs of length 1 and one base twice as long as the other?

(A) $\frac{5}{4}$ **(B)** $\frac{8}{7}$ **(C)** $\frac{5\sqrt{2}}{4}$ **(D)** $\frac{3}{2}$ **(E)** $\frac{3\sqrt{3}}{4}$

12. For complex numbers $u = a + bi$ and $v = c + di$, define the binary operation $\otimes$ by

$$u \otimes v = ac + bdi.$$

Suppose z is a complex number such that $z \otimes z = z^2 + 40$. What is $|z|$?

(A) 2 **(B)** 5 **(C)** $\sqrt{5}$ **(D)** $\sqrt{10}$ **(E)** $5\sqrt{2}$

13. A rectangular box P has distinct edge lengths a, b, and c. The sum of the lengths of all 12 edges of P is 13, the sum of the areas of all 6 faces of P is $\frac{11}{2}$, and the volume of P is $\frac{1}{2}$. What is the length of the longest interior diagonal connecting two vertices of P?

(A) 2 **(B)** $\frac{3}{8}$ **(C)** $\frac{9}{8}$ **(D)** $\frac{9}{4}$ **(E)** $\frac{3}{2}$

14. For how many ordered pairs (a, b) of integers does the polynomial $x^3 + ax^2 + bx + 6$ have 3 distinct integer roots?

(A) 5 **(B)** 6 **(C)** 8 **(D)** 7 **(E)** 4

15. Suppose a, b, and c are positive integers such that

$$\frac{a}{14} + \frac{b}{15} = \frac{c}{210}.$$

Which of the following statements are necessarily true?

I. If $\gcd(a, 14) = 1$ or $\gcd(b, 15) = 1$ or both, then $\gcd(c, 210) = 1$.

II. If $\gcd(c, 210) = 1$, then $\gcd(a, 14) = 1$ or $\gcd(b, 15) = 1$ or both.

III. $\gcd(c, 210) = 1$ if and only if $\gcd(a, 14) = \gcd(b, 15) = 1$.

(A) I, II, and III **(B)** I only **(C)** I and II only **(D)** III only **(E)** II and III only

16. In Coinland, there are three types of coins, each worth $6, 10$, and 15. What is the sum of the digits of the maximum amount of money that is impossible to have?

(A) 8 **(B)** 10 **(C)** 7 **(D)** 11 **(E)** 9

17. Triangle ABC has side lengths in arithmetic progression, and the smallest side has length 6. If the triangle has an angle of $120°$, what is the area of ABC?

(A) $12\sqrt{3}$ **(B)** $8\sqrt{6}$ **(C)** $14\sqrt{2}$ **(D)** $20\sqrt{2}$ **(E)** $15\sqrt{3}$

18. Last academic year Yolanda and Zelda took different courses that did not necessarily administer the same number of quizzes during each of the two semesters. Yolanda's average on all the quizzes she took during the first semester was 3 points higher than Zelda's average on all the quizzes she took during the first semester. Yolanda's average on all the quizzes she took during the second semester was 18 points higher than her average for the first semester and was again 3 points higher than Zelda's average on all the quizzes Zelda took during her second semester. Which one of the following statements cannot possibly be true?

(A) Yolanda's quiz average for the academic year was 22 points higher than Zelda's.

(B) Zelda's quiz average for the academic year was higher than Yolanda's.

(C) Yolanda's quiz average for the academic year was 3 points higher than Zelda's.

(D) Zelda's quiz average for the academic year equaled Yolanda's.

(E) If Zelda had scored 3 points higher on each quiz she took, then she would have had the same average for the academic year as Yolanda.

19. Each of 2023 balls is placed in one of 3 bins. Which of the following is closest to the probability that each of the bins will contain an odd number of balls?

(A) $\frac{2}{3}$ **(B)** $\frac{3}{10}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{4}$

20. Cyrus the frog jumps 2 units in a direction, then 2 more in another direction. What is the probability that he lands less than 1 unit away from his starting position?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{\sqrt{3}}{8}$ **(D)** $\frac{\arctan\frac{1}{2}}{\pi}$ **(E)** $\frac{2\arcsin\frac{1}{4}}{\pi}$

21. A lampshade is made in the form of the lateral surface of the frustum of a right circular cone. The height of the frustum is $3\sqrt{3}$ inches, its top diameter is 6 inches, and its bottom diameter is 12 inches. A bug is at the bottom of the lampshade and there is a glob of honey on the top edge of the lampshade at the spot farthest from the bug. The bug wants to crawl to the honey, but it must stay on the surface of the lampshade. What is the length in inches of its shortest path to the honey?

(A) $6+3\pi$ **(B)** $6+6\pi$ **(C)** $6\sqrt{3}$ **(D)** $6\sqrt{5}$ **(E)** $6\sqrt{3}+\pi$

22. A real-valued function f has the property that for all real numbers a and b,

$$f(a+b)+f(a-b)=2f(a)f(b).$$

Which one of the following cannot be the value of $f(1)$?

(A) 0 **(B)** 1 **(C)** -1 **(D)** 2 **(E)** -2

23. When n standard six-sided dice are rolled, the product of the numbers rolled can be any of 936 possible values. What is n?

(A) 11 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 9

24. Suppose that a, b, c and d are positive integers satisfying all of the following relations.

$$abcd = 2^6\cdot 3^9\cdot 5^7$$

$$\text{lcm}(a,b) = 2^3\cdot 3^2\cdot 5^3$$

$$\text{lcm}(a,c) = 2^3\cdot 3^3\cdot 5^3$$

$$\text{lcm}(a,d) = 2^3\cdot 3^3\cdot 5^3$$

$$\text{lcm}(b,c) = 2^1\cdot 3^3\cdot 5^2$$

$$\text{lcm}(b,d) = 2^2\cdot 3^3\cdot 5^2$$

$$\text{lcm}(c,d) = 2^2\cdot 3^3\cdot 5^2$$

What is $\gcd(a,b,c,d)$?

(A) 30 **(B)** 45 **(C)** 3 **(D)** 15 **(E)** 6

25. A regular pentagon with area $\sqrt{5}+1$ is printed on paper and cut out. The five vertices of the pentagon are folded into the center of the pentagon, creating a smaller pentagon. What is the area of the new pentagon?

(A) $4-\sqrt{5}$ **(B)** $\sqrt{5}-1$ **(C)** $8-3\sqrt{5}$ **(D)** $\frac{\sqrt{5}+1}{2}$ **(E)** $\frac{2+\sqrt{5}}{3}$

2024 AMC1212A

1. What is the value of $9901 \cdot 101 - 99 \cdot 10101$?

   (A) 2 **(B)** 20 **(C)** 200 **(D)** 202 **(E)** 2020

2. A model used to estimate the time it will take to hike to the top of the mountain on a trail is of the form $T = aL + bG$, where a and b are constants, T is the time in minutes, L is the length of the trail in miles, and G is the altitude gain in feet. The model estimates that it will take 69 minutes to hike to the top if a trail is 1.5 miles long and ascends 800 feet, as well as if a trail is 1.2 miles long and ascends 1100 feet. How many minutes does the model estimates it will take to hike to the top if the trail is 4.2 miles long and ascends 4000 feet?

   (A) 240 **(B)** 246 **(C)** 252 **(D)** 258 **(E)** 264

3. The number 2024 is written as the sum of not necessarily distinct two-digit numbers. What is the least number of two-digit numbers needed to write this sum?

   (A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

4. What is the least value of n such that $n!$ is a multiple of 2024?

   (A) 11 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 253

5. A data set containing 20 numbers, some of which are 6, has mean 45. When all the 6s are removed, the data set has mean 66. How many 6s were in the original data set?

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

6. The product of three integers is 60. What is the least possible positive sum of the three integers?

   (A) 2 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 13

7. In ΔABC, $\angle ABC = 90^\circ$ and $BA = BC = \sqrt{2}$. Points $P_1, P_2, \ldots, P_{2024}$ lie on hypotenuse $\overline{AC}$ so that $AP_1 = P_1P_2 = P_2P_3 = \cdots = P_{2023}P_{2024} = P_{2024}C$. What is the length of the vector sum

$$\overrightarrow{BP_1} + \overrightarrow{BP_2} + \overrightarrow{BP_3} + \cdots + \overrightarrow{BP_{2024}}?$$

   (A) 1011 **(B)** 1012 **(C)** 2023 **(D)** 2024 **(E)** 2025

8. How many angles θ with $0 \le \theta \le 2\pi$ satisfy $\log(\sin(3\theta)) + \log(\cos(2\theta)) = 0$?

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

9. Let M be the greatest integer such that both $M + 1213$ and $M + 3773$ are perfect squares. What is the units digit of M?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 8

10. Let α be the radian measure of the smallest angle in a $3-4-5$ right triangle. Let β be the radian measure of the smallest angle in a $7-24-25$ right triangle. In terms of α, what is β?

    (A) $\frac{\alpha}{3}$ **(B)** $\alpha - \frac{\pi}{8}$ **(C)** $\frac{\pi}{2} - 2\alpha$ **(D)** $\frac{\alpha}{2}$ **(E)** $\pi - 4\alpha$

11. There are exactly K positive integers b with $5 \le b \le 2024$ such that the base-b integer 2024_b is divisible by 16 (where 16 is in base ten). What is the sum of the digits of K?

    (A) 16 **(B)** 17 **(C)** 18 **(D)** 20 **(E)** 21

12. The first three terms of a geometric sequence are the integers a, 720, and b, where $a < 720 < b$. What is the sum of the digits of the least possible value of b?

    (A) 9 **(B)** 12 **(C)** 16 **(D)** 18 **(E)** 21

13. The graph of $y = e^{x+1} + e^{-x} - 2$ has an axis of symmetry. What is the reflection of the point $(-1, \frac{1}{2})$ over this axis?

    (A) $\left(-1, -\frac{3}{2}\right)$ **(B)** $(-1, 0)$ **(C)** $\left(-1, \frac{1}{2}\right)$ **(D)** $\left(0, \frac{1}{2}\right)$ **(E)** $\left(3, \frac{1}{2}\right)$

14. The numbers, in order, of each row and the numbers, in order, of each column of a 5×5 array of integers form an arithmetic progression of length 5. The numbers in positions $(5,5)$, $(2,4)$, $(4,3)$, and $(3,1)$ are $0, 48, 16$, and 12, respectively. What number is in position $(1,2)$?

$$\begin{bmatrix} . & ? & . & . & . \\ . & . & . & 48 & . \\ 12 & . & . & . & . \\ . & . & 16 & . & . \\ . & . & . & . & 0 \end{bmatrix}$$

(A) 19 **(B)** 24 **(C)** 29 **(D)** 34 **(E)** 39

15. The roots of $x^3 + 2x^2 - x + 3$ are p, q, and r. What is the value of

$$(p^2 + 4)(q^2 + 4)(r^2 + 4)?$$

(A) 64 **(B)** 75 **(C)** 100 **(D)** 125 **(E)** 144

16. A set of 12 tokens — 3 red, 2 white, 1 blue, and 6 black — is to be distributed at random to 3 game players, 4 tokens per player. The probability that some player gets all the red tokens, another gets all the white tokens, and the remaining player gets the blue token can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

(A) 387 **(B)** 388 **(C)** 389 **(D)** 390 **(E)** 391

17. Integers a, b, and c satisfy $ab + c = 100$, $bc + a = 87$, and $ca + b = 60$. What is $ab + bc + ca$?

(A) 212 **(B)** 247 **(C)** 258 **(D)** 276 **(E)** 284

18. On top of a rectangular card with sides of length 1 and $2 + \sqrt{3}$, an identical card is placed so that two of their diagonals line up, as shown ($\overline{AC}$, in this case).

Continue the process, adding a third card to the second, and so on, lining up successive diagonals after rotating clockwise. In total, how many cards must be used until a vertex of a new card lands exactly on the vertex labeled B in the figure?

(A) 6 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** No new vertex will land on B.

19. Cyclic quadrilateral $ABCD$ has lengths $BC = CD = 3$ and $DA = 5$ with $\angle CDA = 120^\circ$. What is the length of the shorter diagonal of $ABCD$?

(A) $\frac{31}{7}$ **(B)** $\frac{33}{7}$ **(C)** 5 **(D)** $\frac{39}{7}$ **(E)** $\frac{41}{7}$

20. Points P and Q are chosen uniformly and independently at random on sides $\overline{AB}$ and $\overline{AC}$, respectively, of equilateral triangle ΔABC. Which of the following intervals contains the probability that the area of $\triangle APQ$ is less than half the area of $\triangle ABC$?

(A) $\left[\frac{3}{8}, \frac{1}{2}\right]$ **(B)** $\left(\frac{1}{2}, \frac{2}{3}\right]$ **(C)** $\left(\frac{2}{3}, \frac{3}{4}\right]$ **(D)** $\left(\frac{3}{4}, \frac{7}{8}\right]$ **(E)** $\left(\frac{7}{8}, 1\right]$

21. Suppose that $a_1 = 2$ and the sequence (a_n) satisfies the recurrence relation

$$\frac{a_n - 1}{n - 1} = \frac{a_{n-1} + 1}{(n-1) + 1}$$

for all $n \geq 2$. What is the greatest integer less than or equal to

$$\sum_{n=1}^{100} a_n^2?$$

(A) 338,550 **(B)** 338,551 **(C)** 338,552 **(D)** 338,553 **(E)** 338,554

22. The figure below shows a dotted grid 8 cells wide and 3 cells tall consisting of $1'' \times 1''$ squares. Carl places 1-inch toothpicks along some of the sides of the squares to create a closed loop that does not intersect itself. The numbers in the cells indicate the number of sides of that square that are to be covered by toothpicks, and any number of toothpicks are allowed if no number is written. In how many ways can Carl place the toothpicks?

(A) 130 **(B)** 144 **(C)** 146 **(D)** 162 **(E)** 196

23. What is the value of

$$\tan^2\frac{\pi}{16}\cdot\tan^2\frac{3\pi}{16} + \tan^2\frac{\pi}{16}\cdot\tan^2\frac{5\pi}{16} + \tan^2\frac{3\pi}{16}\cdot\tan^2\frac{7\pi}{16} + \tan^2\frac{5\pi}{16}\cdot\tan^2\frac{7\pi}{16}?$$

(A) 28 **(B)** 68 **(C)** 70 **(D)** 72 **(E)** 84

24. A *disphenoid* is a tetrahedron whose triangular faces are congruent to one another. What is the least total surface area of a disphenoid whose faces are scalene triangles with integer side lengths?

(A) $\sqrt{3}$ **(B)** $3\sqrt{15}$ **(C)** 15 **(D)** $15\sqrt{7}$ **(E)** $24\sqrt{6}$

25. A graph is *symmetric* about a line if the graph remains unchanged after reflection in that line. For how many quadruples of integers (a, b, c, d), where $|a|, |b|, |c|, |d| \le 5$ and c and d are not both 0, is the graph of

$$y = \frac{ax+b}{cx+d}$$

symmetric about the line $y = x$?

(A) 1282 **(B)** 1292 **(C)** 1310 **(D)** 1320 **(E)** 1330

2024 AMC1212B

1. In a long line of people arranged left to right, the 1013th person from the left is also the 1010th person from the right. How many people are in the line?

   (A) 2021 **(B)** 2022 **(C)** 2023 **(D)** 2024 **(E)** 2025

2. What is $10! - 7! \cdot 6!$?

   (A) -120 **(B)** 0 **(C)** 120 **(D)** 600 **(E)** 720

3. For how many integer values of x is $|2x| \leq 7\pi$?

   (A) 16 **(B)** 17 **(C)** 19 **(D)** 20 **(E)** 21

4. Balls numbered $1, 2, 3, \ldots$ are deposited in 5 bins, labeled A, B, C, D, and E, using the following procedure. Ball 1 is deposited in bin A, and balls 2 and 3 are deposited in B. The next three balls are deposited in bin C, the next 4 in bin D, and so on, cycling back to bin A after balls are deposited in bin E. (For example, $22, 23, \ldots, 28$ are deposited in bin B at step 7 of this process.) In which bin is ball 2024 deposited?

   (A) A **(B)** B **(C)** C **(D)** D **(E)** E

5. In the following expression, Melanie changed some of the plus signs to minus signs:

   $$1 + 3 + 5 + 7 + \cdots + 97 + 99$$

   When the new expression was evaluated, it was negative. What is the least number of plus signs that Melanie could have changed to minus signs?

   (A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

6. The national debt of the United States is on track to reach $5 \cdot 10^{13}$ dollars by 2033. How many digits does this number of dollars have when written as a numeral in base 5? (The approximation of $\log_{10} 5$ as 0.7 is sufficient for this problem.)

   (A) 18 **(B)** 20 **(C)** 22 **(D)** 24 **(E)** 26

7. In the figure below $WXYZ$ is a rectangle with $WX = 4$ and $WZ = 8$. Point M lies $\overline{XY}$, point A lies on $\overline{YZ}$, and $\angle WMA$ is a right angle. The areas of $\triangle WXM$ and $\triangle WAZ$ are equal. What is the area of $\triangle WMA$?

   (A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

8. What value of x satisfies

   $$\frac{\log_2 x \cdot \log_3 x}{\log_2 x + \log_3 x} = 2?$$

   (A) 25 **(B)** 32 **(C)** 36 **(D)** 42 **(E)** 48

9. A dartboard is the region B in the coordinate plane consisting of points (x, y) such that $|x| + |y| \leq 8$. A target T is the region where $(x^2 + y^2 - 25)^2 \leq 49$. A dart is thrown and lands at a random point in B. The probability that the dart lands in T can be expressed as $\frac{m}{n} \cdot \pi$, where m and n are relatively prime positive integers. What is $m + n$?

   (A) 39 **(B)** 71 **(C)** 73 **(D)** 75 **(E)** 135

10. A list of 9 real numbers consists of 1, 2.2, 3.2, 5.2, 6.2, and 7, as well as x, y, z with $x \leq y \leq z$. The range of the list is 7, and the mean and median are both positive integers. How many ordered triples (x, y, z) are possible?

    (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** infinitely many

11. Let $x_n = \sin^2(n^\circ)$. What is the mean of $x_1, x_2, x_3, \cdots, x_{90}$?

    (A) $\frac{11}{45}$ **(B)** $\frac{22}{45}$ **(C)** $\frac{89}{180}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{91}{180}$

12. Suppose z is a complex number with positive imaginary part, with real part greater than 1, and with $|z| = 2$. In the complex plane, the four points 0, z, z^2, and z^3 are the vertices of a quadrilateral with area 15. What is the imaginary part of z?

    (A) $\frac{3}{4}$ **(B)** 1 **(C)** $\frac{4}{3}$ **(D)** $\frac{3}{2}$ **(E)** $\frac{5}{3}$

13. There are real numbers x, y, h and k that satisfy the system of equations

$$x^2 + y^2 - 6x - 8y = h$$

$$x^2 + y^2 - 10x + 4y = k$$

    What is the minimum possible value of $h + k$?

    (A) -54 **(B)** -46 **(C)** -34 **(D)** -16 **(E)** 16

14. How many different remainders can result when the 100th power of an integer is divided by 125?

    (A) 1 **(B)** 2 **(C)** 5 **(D)** 25 **(E)** 125

15. A triangle in the coordinate plane has vertices $A(\log_2 1, \log_2 2)$, $B(\log_2 3, \log_2 4)$, and $C(\log_2 7, \log_2 8)$. What is the area of $\triangle ABC$?

    (A) $\log_2 \frac{\sqrt{3}}{7}$ **(B)** $\log_2 \frac{3}{\sqrt{7}}$ **(C)** $\log_2 \frac{7}{\sqrt{3}}$ **(D)** $\log_2 \frac{11}{\sqrt{7}}$ **(E)** $\log_2 \frac{11}{\sqrt{3}}$

16. A group of 16 people will be partitioned into 4 indistinguishable 4-person committees. Each committee will have one chairperson and one secretary. The number of different ways to make these assignments can be written as $3^r M$, where r and M are positive integers and M is not divisible by 3. What is r?

    (A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

17. Integers a and b are randomly chosen without replacement from the set of integers with absolute value not exceeding 10. What is the probability that the polynomial $x^3 + ax^2 + bx + 6$ has 3 distinct integer roots?

    (A) $\frac{1}{240}$ **(B)** $\frac{1}{221}$ **(C)** $\frac{1}{105}$ **(D)** $\frac{1}{84}$ **(E)** $\frac{1}{63}$

18. The Fibonacci numbers are defined by $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. What is

$$\frac{F_2}{F_1} + \frac{F_4}{F_2} + \frac{F_6}{F_3} + \cdots + \frac{F_{20}}{F_{10}}?$$

    (A) 318 **(B)** 319 **(C)** 320 **(D)** 321 **(E)** 322

19. Equilateral $\triangle ABC$ with side length 14 is rotated about its center by angle θ, where $0 < \theta < 60^\circ$, to form $\triangle DEF$. See the figure. The area of hexagon $ADBECF$ is $91\sqrt{3}$. What is $\tan\theta$?

    (A) $\frac{3}{4}$ **(B)** $\frac{5\sqrt{3}}{11}$ **(C)** $\frac{4}{5}$ **(D)** $\frac{11}{13}$ **(E)** $\frac{7\sqrt{3}}{13}$

20. Suppose A, B, and C are points in the plane with $AB = 40$ and $AC = 42$, and let x be the length of the line segment from A to the midpoint of $\overline{BC}$. Define a function f by letting $f(x)$ be the area of $\triangle ABC$. Then the domain of f is an open interval (p, q), and the maximum value r of $f(x)$ occurs at $x = s$. What is $p + q + r + s$?

    (A) 909 **(B)** 910 **(C)** 911 **(D)** 912 **(E)** 913

21. The measures of the smallest angles of three different right triangles sum to 90°. All three triangles have side lengths that are primitive Pythagorean triples. Two of them are $3-4-5$ and $5-12-13$. What is the perimeter of the third triangle?

    (A) 40 **(B)** 126 **(C)** 154 **(D)** 176 **(E)** 208

22. Let $\triangle ABC$ be a triangle with integer side lengths and the property that $\angle B = 2\angle A$. What is the least possible perimeter of such a triangle?

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

23. A right pyramid has regular octagon $ABCDEFGH$ with side length 1 as its base and apex V. Segments $\overline{AV}$ and $\overline{DV}$ are perpendicular. What is the square of the height of the pyramid?

(A) 1 **(B)** $\frac{1+\sqrt{2}}{2}$ **(C)** $\sqrt{2}$ **(D)** $\frac{3}{2}$ **(E)** $\frac{2+\sqrt{2}}{3}$

24. What is the number of ordered triples (a, b, c) of positive integers, with $a \leq b \leq c \leq 9$, such that there exists a (non-degenerate) triangle $\triangle ABC$ with an integer inradius for which a, b, and c are the lengths of the altitudes from A to $\overline{BC}$, B to $\overline{AC}$, and C to $\overline{AB}$, respectively? (Recall that the inradius of a triangle is the radius of the largest possible circle that can be inscribed in the triangle.)

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

25. Pablo will decorate each of 6 identical white balls with either a striped or a dotted pattern, using either red or blue paint. He will decide on the color and pattern for each ball by flipping a fair coin for each of the 12 decisions he must make. After the paint dries, he will place the 6 balls in an urn. Frida will randomly select one ball from the urn and note its color and pattern. The events "the ball Frida selects is red" and "the ball Frida selects is striped" may or may not be independent, depending on the outcome of Pablo's coin flips. The probability that these two events are independent can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is m? (Recall that two events A and B are independent if $P(A \text{ and } B) = P(A) \cdot P(B)$.)

(A) 243 **(B)** 245 **(C)** 247 **(D)** 249 **(E)** 251