# AMC 8 Problems
# 2010-2024

## Contents

# 2010 AMC 8 Problems

1. At Euclid Middle School, the mathematics teachers are Miss Germain, Mr. Newton, and Mrs. Young. There are 11 students in Mrs. Germain's class, 8 students in Mr. Newton's class, and 9 students in Mrs. Young's class taking the AMC 8 this year. How many mathematics students at Euclid Middle School are taking the contest?

   **(A)** 26 **(B)** 27 **(C)** 28 **(D)** 29 **(E)** 30

2. If $a@b = \frac{a \times b}{a+b}$ for $a, b$ positive integers, then what is 5@10?

   **(A)** $\frac{3}{10}$ **(B)** 1 **(C)** 2 **(D)** $\frac{10}{3}$ **(E)** 50

3. The graph shows the price of five gallons of gasoline during the first ten months of the year. By what percent is the highest price more than the lowest price?

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; import graph; size(16.38cm); real lsf=2; pathpen=linewidth(0.7); pointpen=black; pen fp = fontsize(10); pointfontpen=fp; real xmin=-1.33,xmax=11.05,ymin=-9.01,ymax=-0.44; pen ycycyc=rgb(0.55,0.55,0.55); pair A=(1,-6), B=(1,-2), D=(1,-5.8), E=(1,-5.6), F=(1,-5.4), G=(1,-5.2), H=(1,-5), J=(1,-4.8), K=(1,-4.6), L=(1,-4.4), M=(1,-4.2), N=(1,-4), P=(1,-3.8), Q=(1,-3.6), R=(1,-3.4), S=(1,-3.2), T=(1,-3), U=(1,-2.8), V=(1,-2.6), W=(1,-2.4), Z=(1,-2.2), E_1=(1.4,-2.6), F_1=(1.8,-2.6), O_1=(14,-6), P_1=(14,-5), Q_1=(14,-4), R_1=(14,-3), S_1=(14,-2), C_1=(1.4,-6), D_1=(1.8,-6), G_1=(2.4,-6), H_1=(2.8,-6), I_1=(3.4,-6), J_1=(3.8,-6), K_1=(4.4,-6), L_1=(4.8,-6), M_1=(5.4,-6), N_1=(5.8,-6), T_1=(6.4,-6), U_1=(6.8,-6), V_1=(7.4,-6), W_1=(7.8,-6), Z_1=(8.4,-6), A_2=(8.8,-6), B_2=(9.4,-6), C_2=(9.8,-6), D_2=(10.4,-6), E_2=(10.8,-6), L_2=(2.4,-3.2), M_2=(2.8,-3.2), N_2=(3.4,-4), O_2=(3.8,-4), P_2=(4.4,-3.6), Q_2=(4.8,-3.6), R_2=(5.4,-3.6), S_2=(5.8,-3.6), T_2=(6.4,-3.4), U_2=(6.8,-3.4), V_2=(7.4,-3.8), W_2=(7.8,-3.8), Z_2=(8.4,-2.8), A_3=(8.8,-2.8), B_3=(9.4,-3.2), C_3=(9.8,-3.2), D_3=(10.4,-3.8), E_3=(10.8,-3.8); filldraw(C_1–E_1–F_1–D_1–cycle,ycycyc); filldraw(G_1–L_2–M_2–H_1–cycle,ycycyc); filldraw(I_1–N_2–O_2–J_1–cycle,ycycyc); filldraw(K_1–P_2–Q_2–L_1–cycle,ycycyc); filldraw(M_1–R_2–S_2–N_1–cycle,ycycyc); filldraw(T_1–T_2–U_2–U_1–cycle,ycycyc); filldraw(V_1–V_2–W_2–W_1–cycle,ycycyc); filldraw(Z_1–Z_2–A_3–A_2–cycle,ycycyc); filldraw(B_2–B_3–C_3–C_2–cycle,ycycyc); filldraw(D_2–D_3–E_3–E_2–cycle,ycycyc); D(B–A,linewidth(0.4)); D(H–(8,-5),linewidth(0.4)); D(N–(8,-4),linewidth(0.4)); D(T–(8,-3),linewidth(0.4)); D(B–(8,-2),linewidth(0.4)); D(B–S_1); D(T–R_1); D(N–Q_1); D(H–P_1); D(A–O_1); D(C_1–E_1); D(E_1–F_1); D(F_1–D_1); D(D_1–C_1); D(G_1–L_2); D(L_2–M_2); D(M_2–H_1); D(H_1–G_1); D(I_1–N_2); D(N_2–O_2); D(O_2–J_1); D(J_1–I_1); D(K_1–P_2); D(P_2–Q_2); D(Q_2–L_1); D(L_1–K_1); D(M_1–R_2); D(R_2–S_2); D(S_2–N_1); D(N_1–M_1); D(T_1–T_2); D(T_2–U_2); D(U_2–U_1); D(U_1–T_1); D(V_1–V_2); D(V_2–W_2); D(W_2–W_1); D(W_1–V_1); D(Z_1–Z_2); D(Z_2–A_3); D(A_3–A_2); D(A_2–Z_1); D(B_2–B_3); D(B_3–C_3); D(C_3–C_2); D(C_2–B_2); D(D_2–D_3); D(D_3–E_3); D(E_3–E_2); D(E_2–D_2); label(”0”,(0.88,-5.91),SE*lsf,fp); label(”$ 5$”,(0.3,-4.84),SE*lsf,fp); label(”$ 10$”,(0.2,-3.84),SE*lsf,fp); label(”$ 15$”,(0.2,-2.85),SE*lsf,fp); label(”$ 20$”,(0.2,-1.85),SE*lsf,fp); label(”$\{}mathrm{Price}$”,(0.16,-3.45),SE*lsf,fp); label(”$1$”,(1.54,-5.97),SE*lsf,fp); label(”$2$”,(2.53,-5.95),SE*lsf,fp); label(”$3$”,(3.53,-5.94),SE*lsf,fp); label(”$4$”,(4.55,-5.94),SE*lsf,fp); label(”$5$”,(5 5.95),SE*lsf,fp); label(”$6$”,(6.53,-5.95),SE*lsf,fp); label(”$7$”,(7.55,-5.95),SE*lsf,fp); label(”$8$”,(8.52,-5.95),SE*lsf,fp); label(”$9$”,(9.57,-5.97),SE*lsf,fp); label(”$10$”,(10.56,-5.94),SE*lsf,fp); label(”Month”,(7.14,-6.43),SE*lsf,fp); D(A,linewidth(1pt)); D(B,linewidth(1pt)); D(D,linewidth(1pt)); D(E,linewidth(1pt)); D(F,linewidth(1p D(G,linewidth(1pt)); D(H,linewidth(1pt)); D(J,linewidth(1pt)); D(K,linewidth(1pt)); D(L,linewidth(1pt)); D(M,linewidth(1pt)); D(N,linewidth(1pt)); D(P,linewidth(1pt)); D(Q,linewidth(1pt)); D(R,linewidth(1pt)); D(S,linewidth(1pt)); D(T,linewidth(1pt)); D(U,linewidth(1pt)); D(V,linewidth(1pt)); D(W,linewidth(1pt)); D(Z,linewidth(1pt)); D(E_1,linewidth(1pt)); D(F_1,linewidth(1pt)); D(O_1,linewidth(1pt)); D(P_1,linewidth(1pt)); D(Q_1,linewidth(1pt)); D(R_1,linewidth(1pt)); D(S_1,linewidth(1pt)); D(C_1,linewidth(1pt)); D(D_1,linewidth(1pt)); D(G_1,linewidth(1pt)); D(H_1,linewidth(1pt)); D(I_1,linewidth(1pt)); D(J_1,linewidth(1pt)); D(K_1,linewidth(1pt)); D(L_1,linewidth(1pt)); D(M_1,linewidth(1pt)); D(N_1,linewidth(1pt)); D(T_1,linewidth(1pt)); D(U_1,linewidth(1pt)); D(V_1,linewidth(1pt)); D(W_1,linewidth(1pt)); D(Z_1,linewidth(1pt)); D(A_2,linewidth(1pt)); D(B_2,linewidth(1pt)); D(C_2,linewidth(1pt)); D(D_2,linewidth(1pt)); D(E_2,linewidth(1pt)); D(L_2,linewidth(1pt)); D(M_2,linewidth(1pt)); D(N_2,linewidth(1pt)); D(O_2,linewidth(1pt)); D(P_2,linewidth(1pt)); D(Q_2,linewidth(1pt)); D(R_2,linewidth(1pt)); D(S_2,linewidth(1pt)); D(T_2,linewidth(1pt)); D(U_2,linewidth(1pt)); D(V_2,linewidth(1pt)); D(W_2,linewidth(1pt)); D(Z_2,linewidth(1pt)); D(A_3,linewidth(1pt)); D(B_3,linewidth(1pt)); D(C_3,linewidth(1pt)); D(D_3,linewidth(1pt)); D(E_3,linewidth(1pt)); clip((xmin,ymin)–(xmin,ymax)–(xmax,ymax)–(xmax,ymin)–cycle); \{}end{asy} \{}end{center}

   **(A)** 50 **(B)** 62 **(C)** 70 **(D)** 89 **(E)** 100

4. What is the sum of the mean, median, and mode of the numbers $2, 3, 0, 3, 1, 4, 0, 3$?

   **(A)** 6.5 **(B)** 7 **(C)** 7.5 **(D)** 8.5 **(E)** 9

5. Alice needs to replace a light bulb located 10 centimeters below the ceiling in her kitchen. The ceiling is 2.4 meters above the floor. Alice is 1.5 meters tall and can reach 46 centimeters above the top of her head. Standing on a stool, she can just reach the light bulb. What is the height of the stool, in centimeters?

   **(A)** 32 **(B)** 34 **(C)** 36 **(D)** 38 **(E)** 40

6. Which of the following figures has the greatest number of lines of symmetry?

   **(A)** equilateral triangle **(B)** non-square rhombus **(C)** non-square rectangle **(D)** isosceles trapezoid **(E)** square

7. Using only pennies, nickels, dimes, and quarters, what is the smallest number of coins Freddie would need so he could pay any amount of money less than a dollar?

   **(A)** 6 **(B)** 10 **(C)** 15 **(D)** 25 **(E)** 99

8. As Emily is riding her bicycle on a long straight road, she spots Emerson skating in the same direction $1/2$ mile in front of her. After she passes him, she can see him in her rear mirror until he is $1/2$ mile behind her. Emily rides at a constant rate of 12 miles per hour, and Emerson skates at a constant rate of 8 miles per hour. For how many minutes can Emily see Emerson?

   **(A)** 6 **(B)** 8 **(C)** 12 **(D)** 15 **(E)** 16

9. Ryan got 80% of the problems correct on a 25-problem test, 90% on a 40-problem test, and 70% on a 10-problem test. What percent of all the problems did Ryan answer correctly?

   **(A)** 64 **(B)** 75 **(C)** 80 **(D)** 84 **(E)** 86

10. Six pepperoni circles will exactly fit across the diameter of a 12-inch pizza when placed. If a total of 24 circles of pepperoni are placed on this pizza without overlap, what fraction of the pizza is covered by pepperoni?

    **(A)** $\frac{1}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{5}{6}$ **(E)** $\frac{7}{8}$

11. The top of one tree is 16 feet higher than the top of another tree. The heights of the two trees are in the ratio $3:4$. In feet, how tall is the taller tree?

    **(A)** 48 **(B)** 64 **(C)** 80 **(D)** 96 **(E)** 112

12. Of the 500 balls in a large bag, 80% are red and the rest are blue. How many of the red balls must be removed from the bag so that 75% of the remaining balls are red?

    **(A)** 25 **(B)** 50 **(C)** 75 **(D)** 100 **(E)** 150

13. The lengths of the sides of a triangle in inches are three consecutive integers. The length of the shortest side is 30% of the perimeter. What is the length of the longest side?

    **(A)** 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

14. What is the sum of the prime factors of 2010?

    **(A)** 67 **(B)** 75 **(C)** 77 **(D)** 201 **(E)** 210

15. A jar contains five different colors of gumdrops: 30% are blue, 20% are brown, 15% red, 10% yellow, and the other 30 gumdrops are green. If half of the blue gumdrops are replaced with brown gumdrops, how many gumdrops will be brown?

    **(A)** 35 **(B)** 36 **(C)** 42 **(D)** 48 **(E)** 64

16. A square and a circle have the same area. What is the ratio of the side length of the square to the radius of the circle?

    **(A)** $\frac{\sqrt{\pi}}{2}$ **(B)** $\sqrt{\pi}$ **(C)** $\pi$ **(D)** $2\pi$ **(E)** $\pi^2$

17. The diagram shows an octagon consisting of 10 unit squares. The portion below $\overline{PQ}$ is a unit square and a triangle with base 5. If $\overline{PQ}$ bisects the area of the octagon, what is the ratio $\frac{XQ}{QY}$?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; import graph; size(300); real lsf = 0.5; pen dp = linewidth(0.7) + fontsize(10); defaultpen(dp); pen ds = black; pen xdxdff = rgb(0.49,0.49,1);

draw((0,0)–(6,0),linewidth(1.2pt)); draw((0,0)–(0,1),linewidth(1.2pt)); draw((0,1)–(1,1),linewidth(1.2pt)); draw((1,1)–(1,2),linewidth(1.2pt)); draw((1,2)–(5,2),linewidth(1.2pt)); draw((5,2)–(5,1),linewidth(1.2pt)); draw((5,1)–(6,1),linewidth(1.2pt)); draw((6,1)–(6,0),linewidth(1.2pt)); draw((1,1)–(5,1),linewidth(1.2pt)); draw((1,1)–(1,0),linewidth(1.2pt)); draw((2,2)–(2,0),linewidth(1.2pt)); draw((3,2)–(3,0),linewidth(1.2pt)); draw((4,2)–(4,0),linewidth(1.2pt)); draw((5,1)–(5,0),linewidth(1.2pt)); draw((0,0)–(5,1.5),linewidth(1.2pt)); dot((0,0),ds); label("$P$", (-0.23,-0.26),NE*lsf); dot((0,1),ds); dot((1,1),ds); dot((1,2),ds); dot((5,2),ds); label("$X$", (5.14,2.02),NE*lsf); dot((5,1),ds); label("$Y$", (5.12,1.14),NE*lsf); dot((6,1),ds); dot((6,0),ds); dot((1,0),ds); dot((2,0),ds); dot((3,0),ds); dot((4,0),ds); dot((5,0),ds); dot((2,2),ds); dot((3,2),ds); dot((4,2),ds); dot((5,1.5),ds); label("$Q$", (5.14,1.51),NE*lsf); clip((-4.19,-5.52)–(-4.19,6.5)–(10.08,6.5)–(10.08,-5.52)–cycle); \{}end{asy} \{}end{center}

**(A)** $\frac{2}{5}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{3}{5}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

18. A decorative window is made up of a rectangle with semicircles on either end. The ratio of $AD$ to $AB$ is $3:2$, and $AB$ is 30 inches. What is the ratio of the area of the rectangle to the combined areas of the semicircles?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; import graph; size(5cm); real lsf=0; pen dps=linewidth(0.7)+fontsize(8); defaultpen(dps); pen ds=black; real xmin=-4.27,xmax=14.73,ymin=-3.22,ymax=6.8; draw((0,4)–(0,0)); draw((0,0)–(2.5,0)); draw((2.5,0)–(2.5,4)); draw((2.5,4)–(0,4)); draw(shift((1.25,4))*xscale(1.25)*ysca draw(shift((1.25,0))*xscale(1.25)*yscale(1.25)*arc((0,0),1,-180,0)); dot((0,0),ds); label("$A$",(-0.26,-0.23),NE*lsf); dot((2.5,0),ds); label("$B$",(2.61,-0.26),NE*lsf); dot((0,4),ds); label("$D$",(-0.26,4.02),NE*lsf); dot((2.5,4),ds); label("$C$",(2.64,3.98),NE*lsf); clip((xmin,ymin)–(xmin,ymax)–(xmax,ymax)–(xmax,ymin)–cycle); \{}end{asy} \{}end{center}

**(A)** $2:3$ **(B)** $3:2$ **(C)** $6:\pi$ **(D)** $9:\pi$ **(E)** $30:\pi$

19. The two circles pictured have the same center $C$. Chord $\overline{AD}$ is tangent to the inner circle at $B$, $AC$ is 10, and chord $\overline{AD}$ has length 16. What is the area between the two circles?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; unitsize(45); import graph; size(300); real lsf = 0.5; pen dp = linewidth(0.7) + fontsize(10); defaultpen(dp); pen ds = black; pen xdxdff = rgb(0.49,0.49,1); draw((2,0.15)–(1.85,0.15)–(1.85,0)–(2,0)–cycle); draw(circle((2,1),2.24)); draw(circle((2,1),1)); draw((0,0)–(4,0)); draw((0,0)–(2,1)); draw((2,1)–(2,0)); draw((2,1)–(4,0)); dot((0,0),ds); label("$A$", (-0.19,-0.23),NE*lsf); dot((2,0),ds); label("$B$", (1.97,-0.31),NE*lsf); dot((2,1),ds); label("$C$", (1.96,1.09),NE*lsf); dot((4,0),ds); label("$D$", (4.07,-0.24),NE*lsf); clip((-3.1,-7.72)–(-3.1,4.77)–(11.74,4.77)–(11.74,-7.72)–cycle); \{}end{asy} \{}end{center}

**(A)** $36\pi$ **(B)** $49\pi$ **(C)** $64\pi$ **(D)** $81\pi$ **(E)** $100\pi$

20. In a room, 2/5 of the people are wearing gloves, and 3/4 of the people are wearing hats. What is the minimum number of people in the room wearing both a hat and a glove?

**(A)** 3 **(B)** 5 **(C)** 8 **(D)** 15 **(E)** 20

21. Hui is an avid reader. She bought a copy of the best seller "Math is Beautiful". On the first day, Hui read 1/5 of the pages plus 12 more, and on the second day she read 1/4 of the remaining pages plus 15 pages. On the third day she read 1/3 of the remaining pages plus 18 pages. She then realized that there were only 62 pages left to read, which she read the next day. How many pages are in this book?

**(A)** 120 **(B)** 180 **(C)** 240 **(D)** 300 **(E)** 360

22. The hundreds digit of a three-digit number is 2 more than the units digit. The digits of the three-digit number are reversed, and the result is subtracted from the original three-digit number. What is the units digit of the result?

**(A)** 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

23. Semicircles $POQ$ and $ROS$ pass through the center of circle $O$. What is the ratio of the combined areas of the two semicircles to the area of circle $O$?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; import graph; size(7.5cm); real lsf=0.5; pen dps=linewidth(0.7)+fontsize(10); defaultpen(dps); pen ds=black; real xmin=-6.27,xmax=10.01,ymin=-5.65,ymax=10.98; draw(circle((0,0),2)); draw((-3,0)–(3,0),EndArrow(6)); draw((0,-3)–(0,3),EndArrow(6)); draw(shift((0.01,1.42))*xscale(1.41)*yscale(1.41)*arc((0,0),1,179.76,359.76)); draw(shift((-0.01,-1.42))*xscale(1.41)*ysca 0.38,179.62)); draw((-1.4,1.43)–(1.41,1.41)); draw((-1.42,-1.41)–(1.4,-1.42)); label("$ P(-1,1) $",(-2.57,2.17),SE*lsf);

label("$ Q(1,1) $",(1.55,2.21),SE*lsf); label("$ R(-1,-1) $",(-2.72,-1.45),SE*lsf); label("$S(1,-1)$",(1.59,-1.49),SE*lsf); dot((0,0),ds); label("$O$",(-0.24,-0.35),NE*lsf); dot((1.41,1.41),ds); dot((-1.4,1.43),ds); dot((1.4,-1.42),ds); dot((-1.42,-1.41),ds); clip((xmin,ymin)–(xmin,ymax)–(xmax,ymax)–(xmax,ymin)–cycle); \{}end{asy} \{}end{center}

**(A)** $\frac{\sqrt{2}}{4}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{2}{\pi}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{\sqrt{2}}{2}$

24. What is the correct ordering of the three numbers, $10^8$, $5^{12}$, and $2^{24}$?

**(A)** $2^{24} < 10^8 < 5^{12}$

**(B)** $2^{24} < 5^{12} < 10^8$

**(C)** $5^{12} < 2^{24} < 10^8$

**(D)** $10^8 < 5^{12} < 2^{24}$

**(E)** $10^8 < 2^{24} < 5^{12}$

25. Everyday at school, Jo climbs a flight of 6 stairs. Jo can take the stairs 1, 2, or 3 at a time. For example, Jo could climb 3, then 1, then 2. In how many ways can Jo climb the stairs?

**(A)** 13 **(B)** 18 **(C)** 20 **(D)** 22 **(E)** 24

# 2011 AMC 8 Problems

1. Margie bought 3 apples at a cost of 50 cents per apple. She paid with a 5-dollar bill. How much change did Margie receive?

   **(A)** \$1.50 **(B)** \$2.00 **(C)** \$2.50 **(D)** \$3.00 **(E)** \$3.50

2. Karl's rectangular vegetable garden is 20 feet by 45 feet, and Makenna's is 25 feet by 40 feet. Which of the following statements are true?

   **(A)** Karl's garden is larger by 100 square feet.

   **(B)** Karl's garden is larger by 25 square feet.

   **(C)** The gardens are the same size.

   **(D)** Makenna's garden is larger by 25 square feet.

   **(E)** Makenna's garden is larger by 100 square feet.

3. Extend the square pattern of 8 black and 17 white square tiles by attaching a border of black tiles around the square. What is the ratio of black tiles to white tiles in the extended pattern?\{}begin{center} \{}begin{asy} import olympiad; import cse5; filldraw((0,0)–(5,0)–(5,5)–(0,5)–cycle,white,black); filldraw((1,1)–(4,1)–(4,4)–(1,4)–cycle,mediumgray,black); filldraw((2,2)–(3,2)–(3,3)–(2,3)–cycle,white,black); draw((4,0)–(4,5)); draw((3,0)–(3,5)); draw((2,0)–(2,5)); draw((1,0)–(1,5)); draw((0,4)–(5,4)); draw((0,3)–(5,3)); draw((0,2)–(5,2)); draw((0,1)–(5,1)); \{}end{asy} \{}end{center}

   **(A)** $8:17$ **(B)** $25:49$ **(C)** $36:25$ **(D)** $32:17$ **(E)** $36:17$

4. Here is a list of the numbers of fish that Tyler caught in nine outings last summer:

   $$2, 0, 1, 3, 0, 3, 3, 1, 2.$$

   Which statement about the mean, median, and mode is true?

   **(A)** median < mean < mode **(B)** mean < mode < median

   **(C)** mean < median < mode **(D)** median < mode < mean

   **(E)** mode < median < mean

5. What time was it 2011 minutes after midnight on January 1, 2011?

   **(A)** January 1 at 9:31 PM

   **(B)** January 1 at 11:51 PM

   **(C)** January 2 at 3:11 AM

   **(D)** January 2 at 9:31 AM

   **(E)** January 2 at 6:01 PM

6. In a town of 351 adults, every adult owns a car, motorcycle, or both. If 331 adults own cars and 45 adults own motorcycles, how many of the car owners do not own a motorcycle?

   **(A)** 20 **(B)** 25 **(C)** 45 **(D)** 306 **(E)** 351

7. Each of the following four large congruent squares is subdivided into combinations of congruent triangles or rectangles and is partially bolded. What percent of the total area is partially bolded?

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; import graph; size(7.01cm); real lsf=0.5; pen dps=linewidth(0.7)+fontsize(10); defaultpen(dps); pen ds=black; real xmin=-0.42,xmax=14.59,ymin=-10.08,ymax=5.26; pair A=(0,0), B=(4,0), C=(0,4), D=(4,4), F=(2,0), G=(3,0), H=(1,4), I=(2,4), J=(3,4), K=(0,-2), L=(4,-2), M=(0,-6), O=(0,-4), P=(4,-4), Q=(2,-2), R=(2,-6), T=(6,4), U=(10,0), V=(10,4), Z=(10,2), A_1=(8,4), B_1=(8,0), C_1=(6,-2), D_1=(10,-2), E_1=(6,-6), F_1=(10,-6), G_1=(6,-4), H_1=(10,-4), I_1=(8,-2), J_1=(8,-6), K_1=(8,-4); draw(C–H–(1,0)–A–cycle,linewidth(1.6)); draw(M–O–Q–R–cycle,linewidth(1.6)); draw(A_1–V–Z–cycle,linewidth(1.6)); draw(G_1–K_1–J_1–E_1–cycle,linewidth(1.6)); draw(C–D); draw(D–B); draw(B–A); draw(A–C); draw(H–(1,0)); draw(I–F); draw(J–G); draw(C–H,linewidth(1.6)); draw(H–(1,0),linewidth(1.6)); draw((1,0)–A,linewidth(1.6)); draw(A–C,linewidth(1.6)); draw(K–L); draw((4,-6)–L);

draw((4,-6)–M); draw(M–K); draw(O–P); draw(Q–R); draw(O–Q); draw(M–O,linewidth(1.6)); draw(O–Q,linewidth(1.6)); draw(Q–R,linewidth(1.6)); draw(R–M,linewidth(1.6)); draw(T–V); draw(V–U); draw(U–(6,0)); draw((6,0)–T); draw((6,2)–Z); draw(A_1–B_1); draw(A_1–Z); draw(A_1–V,linewidth(1.6)); draw(V–Z,linewidth(1.6)); draw(Z–A_1,linewidth(1.6)); draw(C_1–D_1); draw(D_1–F_1); draw(F_1–E_1); draw(E_1–C_1); draw(G_1–H_1); draw(I_1–J_1); draw(G_1–K_1,linewidth(1.6)); draw(K_1–J_1,linewidth(1.6)); draw(J_1–E_1,linewidth(1.6)); draw(E_1–G_1,linewidth(1.6)); dot(A,linewidth(1pt)+ds); dot(B,linewidth(1pt)+ds); dot(C,linewidth(1pt)+ds); dot(D,linewidth(1pt)+ds); dot((1,0),linewidth(1pt)+ds); dot(F,linewidth(1pt)+ds); dot(G,linewidth(1pt)+ds); dot(H,linewidth(1pt)+ds); dot(I,linewidth(1pt)+ds); dot(J,linewidth(1pt)+ds); dot(K,linewidth(1pt)+ds); dot(L,linewidth(1pt)+ds); dot(M,linewidth(1pt)+ds); dot((4,-6),linewidth(1pt)+ds); dot(O,linewidth(1pt)+ds); dot(P,linewidth(1pt)+ds); dot(Q,linewidth(1pt)+ds); dot(R,linewidth(1pt)+ds); dot((6,0),linewidth(1pt)+ds); dot(T,linewidth(1pt)+ds); dot(U,linewidth(1pt)+ds); dot(V,linewidth(1pt)+ds); dot((6,2),linewidth(1pt)+ds); dot(Z,linewidth(1pt)+ds); dot(A_1,linewidth(1pt)+ds); dot(B_1,linewidth(1pt)+ds); dot(C_1,linewidth(1pt)+ds); dot(D_1,linewidth(1pt)+ds); dot(E_1,linewidth(1pt)+ds); dot(F_1,linewidth(1pt)+ds); dot(G_1,linewidth(1pt)+ds); dot(H_1,linewidth(1pt)+ds); dot(I_1,linewidth(1pt)+ds); dot(J_1,linewidth(1pt)+ds); dot(K_1,linewidth(1pt)+ds); clip((xmin,ymin)–(xmin,ymax)–(xmax,ymax)–(xmax,ymin)–cycle); \{}end{asy} \{}end{center}

**(A)**$12\frac{1}{2}$ **(B)**20 **(C)**25 **(D)**$33\frac{1}{3}$ **(E)**$37\frac{1}{2}$

8. Bag A has three chips labeled 1, 3, and 5. Bag B has three chips labeled 2, 4, and 6. If one chip is drawn from each bag, how many different values are possible for the sum of the two numbers on the chips?

**(A)** 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 9

9. Carmen takes a long bike ride on a hilly highway. The graph indicates the miles traveled during the time of her ride. What is Carmen's average speed for her entire ride in miles per hour?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; import graph; size(8.76cm); real lsf=0.5; pen dps=linewidth(0.7)+fontsize(10); defaultpen(dps); pen ds=black; real xmin=-3.58,xmax=10.19,ymin=-4.43,ymax=9.63; draw((0,0)–(0,8)); draw((0,0)–(8,0)); draw((0,1)–(8,1)); draw((0,2)–(8,2)); draw((0,3)–(8,3)); draw((0,4)–(8,4)); draw((0,5)–(8,5)); draw((0,6)–(8,6)); draw((0,7)–(8,7)); draw((1,0)–(1,8)); draw((2,0)–(2,8)); draw((3,0)–(3,8)); draw((4,0)–(4,8)); draw((5,0)–(5,8)); draw((6,0)–(6,8)); draw((7,0)–(7,8)); label("$1$",(0.95,-0.24),SE*lsf); label("$2$",(1.92,-0.26),SE*lsf); label("$3$",(2.92,-0.31),SE*lsf); label("$4$",(3.93,-0.26),SE*lsf); label("$5$",(4.92,-0.27),SE*lsf); label("$6$",(5.95,-0.29),SE*lsf); label("$7$",(6.94,-0.27),SE*lsf); label("$5$",(-0.49,1.22),SE*lsf); label("$10$",(-0.59,2.23),SE*lsf); label("$15$",(-0.61,3.22),SE*lsf); label("$20$",(-0.61,4.23),SE*lsf) label("$25$",(-0.59,5.22),SE*lsf); label("$30$",(-0.59,6.2),SE*lsf); label("$35$",(-0.56,7.18),SE*lsf); draw((0,0)–(1,1),linewidth(1.6)); draw((1,1)–(2,3),linewidth(1.6)); draw((2,3)–(4,4),linewidth(1.6)); draw((4,4)–(7,7),linewidth(1.6) label("HOURS",(3.41,-0.85),SE*lsf); label("M",(-1.39,5.32),SE*lsf); label("I",(-1.34,4.93),SE*lsf); label("L",(-1.36,4.51),SE*lsf); label("E",(-1.37,4.11),SE*lsf); label("S",(-1.39,3.7),SE*lsf); clip((xmin,ymin)–(xmin,ymax)–(xmax,ymax)–(xmax,ymin)–cycle); \{}end{asy} \{}end{center}

**(A)** 2 **(B)** 2.5 **(C)** 4 **(D)** 4.5 **(E)** 5

10. The taxi fare in Gotham City is \{}texttt{\{}$2.40} for the first $\frac{1}{2}$ mile and additional mileage charged at the rate \{}texttt{\{}$0.20} for each additional 0.1 mile. You plan to give the driver a \{}texttt{\{}$2} tip. How many miles can you ride for \{}texttt{\{}$10}?

**(A)** 3.0 **(B)** 3.25 **(C)** 3.3 **(D)** 3.5 **(E)** 3.75

11. The graph shows the number of minutes studied by both Asha (black bar) and Sasha (grey bar) in one week. On the average, how many more minutes per day did Sasha study than Asha?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; size(300); real i; defaultpen(linewidth(0.8)); draw((0,140)–origin–(220,0)); for(i=1;i¡13;i=i+1) { draw((0,10*i)–(220,10*i)); } label("$0$",origin,W); label("$20$",(0,20),W); label("$40$",(0,40),W); label("$60$",(0,60),W); label("$80$",(0,80),W); label("$100$",(0,100),W label("$120$",(0,120),W); path MonD=(20,0)–(20,60)–(30,60)–(30,0)–cycle,MonL=(30,0)–(30,70)–(40,70)–(40,0)–cycle,TuesD=(60,0)–(60,90)–(70,90)–(70,0)–cycle,TuesL=(70,0)–(70,80)–(80,80)–(80,0)–cycle,WedD=(100,0)–(100,100)–(110,100)–(110,0)–cycle,WedL=(110,0)–(110,120)–(120,120)–(120,0)–cycle,ThurD=(140,0)–(140,80)–(150,80)–(150,0)–cycle,ThurL=(150,0)–(150,110)–(160,110)–(160,0)–cycle,FriD=(180,0)–(180,70)–(190,70)–(190,0)–cycle,FriL=(190,0)–(190,50)–(200,50)–(200,0)–cycle; fill(MonD,black); fill(MonL,grey); fill(TuesD,black); fill(TuesL,grey); fill(WedD,black); fill(WedL,grey); fill(ThurD,black); fill(ThurL,grey); fill(FriD,black); fill(FriL,grey); draw(MonD^^MonL^^TuesD^^TuesL^^WedD^^WedL^^ThurD^^ThurL^^FriD^^FriL); label("M",(30,-5),S); label("Tu",(70,-5),S); label("W",(110,-5),S); label("Th",(150,-5),S); label("F",(190,-5),S); label("M",(-25,85),W); label("I",(-27,75),W); label("N",(-25,65),W); label("U",(-25,55),W); label("T",(-25,45),W); label("E",(-25,35),W); label("S",(-26,25),W); \{}end{asy} \{}end{center}

**(A)** 6 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 12

12. Angie, Bridget, Carlos, and Diego are seated at random around a square table, one person to a side. What is the probability that Angie and Carlos are seated opposite each other?

**(A)** $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

13. Two congruent squares, $ABCD$ and $PQRS$, have side length 15. They overlap to form the 15 by 25 rectangle $AQRD$ shown. What percent of the area of rectangle $AQRD$ is shaded?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; filldraw((0,0)–(25,0)–(25,15)–(0,15)–cycle,white,black); label(”D”,(0,0),S); label(”R”,(25,0),S); label(”Q”,(25,15),N); label(”A”,(0,15),N); filldraw((10,0)–(15,0)–(15,15)–(10,15)–cycle,mediumgrey,black); label(”S”,(10,0),S); label(”C”,(15,0),S); label(”B”,(15,15),N); label(”P”,(10,15),N); \{}end{asy} \{}end{center}

**(A)** 15 **(B)** 18 **(C)** 20 **(D)** 24 **(E)** 25

14. There are 270 students at Colfax Middle School, where the ratio of boys to girls is $5:4$. There are 180 students at Winthrop Middle School, where the ratio of boys to girls is $4:5$. The two schools hold a dance and all students from both schools attend. What fraction of the students at the dance are girls?

**(A)** $\dfrac{7}{18}$ **(B)** $\dfrac{7}{15}$ **(C)** $\dfrac{22}{45}$ **(D)** $\dfrac{1}{2}$ **(E)** $\dfrac{23}{45}$

15. How many digits are in the product $4^5 \cdot 5^{10}$?

**(A)** 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

16. Let $A$ be the area of the triangle with sides of length $25, 25$, and 30. Let $B$ be the area of the triangle with sides of length $25, 25$, and 40. What is the relationship between $A$ and $B$?

**(A)** $A = \dfrac{9}{16}B$ **(B)** $A = \dfrac{3}{4}B$ **(C)** $A = B$ **(D)** $A = \dfrac{4}{3}B$

**(E)** $A = \dfrac{16}{9}B$

17. Let $w$, $x$, $y$, and $z$ be whole numbers. If $2^w \cdot 3^x \cdot 5^y \cdot 7^z = 588$, then what does $2w + 3x + 5y + 7z$ equal?

**(A)** 21 **(B)** 25 **(C)** 27 **(D)** 35 **(E)** 56

18. A fair 6-sided die is rolled twice. What is the probability that the first number that comes up is greater than or equal to the second number?

**(A)** $\dfrac{1}{6}$ **(B)** $\dfrac{5}{12}$ **(C)** $\dfrac{1}{2}$ **(D)** $\dfrac{7}{12}$ **(E)** $\dfrac{5}{6}$

19. How many rectangles are in this figure?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; pair A,B,C,D,E,F,G,H,I,J,K,L; A=(0,0); B=(20,0); C=(20,20); D=(0,20); draw(A–B–C–D–cycle); E=(-10,-5); F=(13,-5); G=(13,5); H=(-10,5); draw(E–F–G–H–cycle); I=(10,-20); J=(18,-20); K=(18,13); L=(10,13); draw(I–J–K–L–cycle); \{}end{asy} \{}end{center}

**(A)** 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

20. Quadrilateral $ABCD$ is a trapezoid, $AD = 15$, $AB = 50$, $BC = 20$, and the altitude is 12. What is the area of the trapezoid?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; pair A,B,C,D; A=(3,20); B=(35,20); C=(47,0); D=(0,0); draw(A–B–C–D–cycle); dot((0,0)); dot((3,20)); dot((35,20)); dot((47,0)); label(”A”,A,N); label(”B”,B,N); label(”C”,C,S); label(”D”,D,S); draw((19,20)–(19,0)); dot((19,20)); dot((19,0)); draw((19,3)–(22,3)–(22,0)); label(”12”,(21,10),E); label(”50”,(19,22),N); label(”15”,(1,10),W); label(”20”,(41,12),E); \{}end{asy} \{}end{center}

**(A)** 600 **(B)** 650 **(C)** 700 **(D)** 750 **(E)** 800

21. Students guess that Norb's age is $24, 28, 30, 32, 36, 38, 41, 44, 47$, and 49. Norb says, ”At least half of you guessed too low, two of you are off by one, and my age is a prime number.” How old is Norb?

**(A)** 29 **(B)** 31 **(C)** 37 **(D)** 43 **(E)** 48

22. What is the "'tens"' digit of $7^{2011}$?

    **(A)** 0 **(B)** 1 **(C)** 3 **(D)** 4 **(E)** 7

23. How many 4-digit positive integers have four different digits, where the leading digit is not zero, the integer is a multiple of 5, and 5 is the largest digit?

    **(A)** 24 **(B)** 48 **(C)** 60 **(D)** 84 **(E)** 108

24. In how many ways can 10001 be written as the sum of two primes?

    **(A)** 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

25. A circle with radius 1 is inscribed in a square and circumscribed about another square as shown. Which fraction is closest to the ratio of the circle's shaded area to the area between the two squares?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; filldraw((-1,-1)–(-1,1)–(1,1)–(1,-1)–cycle,gray,black); filldraw(Circle((0,0),1), mediumgray,black); filldraw((-1,0)–(0,1)–(1,0)–(0,-1)–cycle,white,black); \{}end{asy} \{}end{center}

    **(A)** $\frac{1}{2}$ **(B)** 1 **(C)** $\frac{3}{2}$ **(D)** 2 **(E)** $\frac{5}{2}$

# 2012 AMC 8 Problems

1. Rachelle uses 3 pounds of meat to make 8 hamburgers for her family. How many pounds of meat does she need to make 24 hamburgers for a neighborhood picnic?

   **(A)** 6 **(B)** $6\frac{2}{3}$ **(C)** $7\frac{1}{2}$ **(D)** 8 **(E)** 9

2. In the country of East Westmore, statisticians estimate there is a baby born every 8 hours and a death every day. To the nearest hundred, how many people are added to the population of East Westmore each year?

   **(A)** 600 **(B)** 700 **(C)** 800 **(D)** 900 **(E)** 1000

3. On February 13 *The Oshkosh Northwester* listed the length of daylight as 10 hours and 24 minutes, the sunrise was $6:57\text{AM}$, and the sunset as $8:15\text{PM}$. The length of daylight and sunrise were correct, but the sunset was wrong. When did the sun really set?

   **(A)** $5:10\text{PM}$ **(B)** $5:21\text{PM}$ **(C)** $5:41\text{PM}$ **(D)** $5:57\text{PM}$ **(E)** $6:03\text{PM}$

4. Peter's family ordered a 12-slice pizza for dinner. Peter ate one slice and shared another slice equally with his brother Paul. What fraction of the pizza did Peter eat?

   **(A)** $\frac{1}{24}$ **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{8}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{4}$

5. In the diagram, all angles are right angles and the lengths of the sides are given in centimeters. Note that the diagram is not drawn to scale. What is the length of $X$, in centimeters?

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; pair A,B,C,D,E,F,G,H,I,J,K,L,M,N,O,P,Q,R; A=(4,0); B=(7,0); C=(7,4); D=(8,4); E=(8,5); F=(10,5); G=(10,7); H=(7,7); I=(7,8); J=(5,8); K=(5,7); L=(4,7); M=(4,6); N=(0,6); O=(0,5); P=(2,5); Q=(2,3); R=(4,3); draw(A–B–C–D–E–F–G–H–I–J–K–L–M–N–O–P–Q–R–cycle); label(”$X$”,(3.4,1.5)); label(”6”,(7.6,1.5)); label(”1”,(7.6,3.5)); label(”1”,(8.4,4.6)); label(”2”,(9.4,4.6)); label(”2”,(10.4,6)); label(”3”,(8.4,7.4)); label(”1”,(7.5,7.8)); label(”2”,(6,8.5)); label(”1”,(4.7,7.8)); label(”1”,(4.3,7.5)); label(”1”,(3.5,6.5)); label(”4”,(1.8,6.5)); label(”1”,(-0.5,5.5)); label(”2”,(0.8,4.5)); label(”2”,(1.5,3.8)); label(”2”,(2.8,2.6)); \{}end{asy} \{}end{center}

   **(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

6. A rectangular photograph is placed in a frame that forms a border two inches wide on all sides of the photograph. The photograph measures 8 inches high and 10 inches wide. What is the area of the border, in square inches?

   **(A)** 36 **(B)** 40 **(C)** 64 **(D)** 72 **(E)** 88

7. Isabella must take four 100-point tests in her math class. Her goal is to achieve an average grade of 95 on the tests. Her first two test scores were 97 and 91. After seeing her score on the third test, she realized she can still reach her goal. What is the lowest possible score she could have made on the third test?

   **(A)** 90 **(B)** 92 **(C)** 95 **(D)** 96 **(E)** 97

8. A shop advertises everything is ”half price in today's sale.” In addition, a coupon gives a 20% discount on sale prices. Using the coupon, the price today represents what percentage off the original price?

   **(A)** 10 **(B)** 33 **(C)** 40 **(D)** 60 **(E)** 70

9. The Fort Worth Zoo has a number of two-legged birds and a number of four-legged mammals. On one visit to the zoo, Margie counted 200 heads and 522 legs. How many of the animals that Margie counted were two-legged birds?

   **(A)** 61 **(B)** 122 **(C)** 139 **(D)** 150 **(E)** 161

10. How many 4-digit numbers greater than 1000 are there that use the four digits of 2012?

    **(A)** 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 12

11. The mean, median, and unique mode of the positive integers 3, 4, 5, 6, 6, 7, and $x$ are all equal. What is the value of $x$?

    **(A)** 5 **(B)** 6 **(C)** 7 **(D)** 11 **(E)** 12

12. What is the units digit (ones place digit) of $13^{2012}$?

    **(A)** 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

13. Jamar bought some pencils costing more than a penny each at the school bookstore and paid \$1.43. Sharona bought some of the same pencils and paid \$1.87. How many more pencils did Sharona buy than Jamar?

    **(A)** 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

14. In the BIG N, a middle school football conference, each team plays every other team exactly once. If a total of 21 conference games were played during the 2012 season, how many teams were members of the BIG N conference?

    **(A)** 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

15. The smallest number greater than 2 that leaves a remainder of 2 when divided by 3, 4, 5, or 6 lies between what numbers?

    **(A)** 40 and 50 **(B)** 51 and 55 **(C)** 56 and 60 **(D)** 61 and 65 **(E)**66 and 99

16. Each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 is used only once to make two five-digit numbers so that they have the largest possible sum. Which of the following could be one of the numbers?

    **(A)** 76531 **(B)** 86724 **(C)** 87431 **(D)** 96240 **(E)** 97403

17. A square with integer side length is cut into 10 squares, all of which have integer side length and at least 8 of which have area 1. What is the smallest possible value of the length of the side of the original square?

    **(A)** 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

18. What is the smallest positive integer that is neither prime nor square and that has no prime factor less than 50?

    **(A)** 3127 **(B)** 3133 **(C)** 3137 **(D)** 3139 **(E)** 3149

19. In a jar of red, green, and blue marbles, all but 6 are red marbles, all but 8 are green, and all but 4 are blue. How many marbles are in the jar?

    **(A)** 6 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 12

20. What is the correct ordering of the three numbers $\frac{5}{19}$, $\frac{7}{21}$, and $\frac{9}{23}$, in increasing order?

    **(A)** $\frac{9}{23} < \frac{7}{21} < \frac{5}{19}$ **(B)** $\frac{5}{19} < \frac{7}{21} < \frac{9}{23}$ **(C)** $\frac{9}{23} < \frac{5}{19} < \frac{7}{21}$
    **(D)** $\frac{5}{19} < \frac{9}{23} < \frac{7}{21}$ **(E)** $\frac{7}{21} < \frac{5}{19} < \frac{9}{23}$

21. Marla has a large white cube that has an edge of 10 feet. She also has enough green paint to cover 300 square feet. Marla uses all the paint to create a white square centered on each face, surrounded by a green border. What is the area of one of the white squares, in square feet?

    **(A)** $5\sqrt{2}$ **(B)** 10 **(C)** $10\sqrt{2}$ **(D)** 50 **(E)** $50\sqrt{2}$

22. Let $R$ be a set of nine distinct integers. Six of the elements are 2, 3, 4, 6, 9, and 14. What is the number of possible values of the median of $R$ ?

    **(A)** 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

23. An equilateral triangle and a regular hexagon have equal perimeters. If the area of the triangle is 4, what is the area of the hexagon?

    **(A)** 4 **(B)** 5 **(C)** 6 **(D)** $4\sqrt{3}$ **(E)** $6\sqrt{3}$

24. A circle of radius 2 is cut into four congruent arcs. The four arcs are joined to form the star figure shown. What is the ratio of the area of the star figure to the area of the original circle?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; size(0,50); draw((-1,1)..(-2,2)..(-3,1)..(-2,0)..cycle); dot((-1,1)); dot((-2,2)); dot((-3,1)); dot((-2,0)); draw((1,0){up}..{left}(0,1)); dot((1,0)); dot((0,1)); draw((0,1){right}..{up}(1,2)); dot((1,2)); draw((1,2){down}..{right}(2,1)); dot((2,1)); draw((2,1){left}..{down}(1,0)); \{}end{asy} \{}end{center}

**(A)** $\frac{4-\pi}{\pi}$ **(B)** $\frac{1}{\pi}$ **(C)** $\frac{\sqrt{2}}{\pi}$ **(D)** $\frac{\pi-1}{\pi}$ **(E)** $\frac{3}{\pi}$

25. A square with area 4 is inscribed in a square with area 5, with one vertex of the smaller square on each side of the larger square. A vertex of the smaller square divides a side of the larger square into two segments, one of length $a$, and the other of length $b$. What is the value of $ab$?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; draw((0,2)–(2,2)–(2,0)–(0,0)–cycle); draw((0,0.3)–(0.3,2)–(2,1.7)–(1.7,0)–cycle); label("$a$",(-0.1,0.15)); label("$b$",(-0.1,1.15)); \{}end{asy} \{}end{center}

**(A)** $\frac{1}{5}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{1}{2}$ **(D)** 1 **(E)** 4

# 2013 AMC 8 Problems

1. Danica wants to arrange her model cars in rows with exactly 6 cars in each row. She now has 23 model cars. What is the smallest number of additional cars she must buy in order to be able to arrange all her cars this way?

   **(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

2. A sign at the fish market says, "50% off, today only: half-pound packages for just \{}$3 per package." What is the regular price for a full pound of fish, in dollars? (Assume that there are no deals for bulk)

   **(A)** 6 **(B)** 9 **(C)** 10 **(D)** 12 **(E)** 15

3. What is the value of $4 \cdot (-1+2-3+4-5+6-7+\cdots+1000)$?

   **(A)** $-10$ **(B)** 0 **(C)** 1 **(D)** 500 **(E)** 2000

4. Eight friends ate at a restaurant and agreed to share the bill equally. Because Judi forgot her money, each of her seven friends paid an extra \{}$2.50 to cover her portion of the total bill. What was the total bill?

   **(A)** \$120 **(B)** \$128 **(C)** \$140 **(D)** \$144 **(E)** \$160

5. Hammie is in the $6^{\text{th}}$ grade and weighs 106 pounds. Her quadruplet sisters are tiny babies and weigh 5, 5, 6, and 8 pounds. Which is greater, the average (mean) weight of these five children or the median weight, and by how many pounds?

   **(A)** median, by 60 **(B)** median, by 20 **(C)** average, by 5 **(D)** average, by 15 **(E)** average, by 20

6. The number in each box below is the product of the numbers in the two boxes that touch it in the row above. For example, $30 = 6 \times 5$. What is the missing number in the top row?

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; unitsize(0.8cm); draw((-1,0)–(1,0)–(1,-2)–(-1,-2)–cycle); draw((-2,0)–(0,0)–(0,2)–(-2,2)–cycle); draw((0,0)–(2,0)–(2,2)–(0,2)–cycle); draw((-3,2)–(-1,2)–(-1,4)–(-3,4)–cycle); draw((-1,2)–(1,2)–(1,4)–(-1,4)–cycle); draw((1,2)–(1,4)–(3,4)–(3,2)–cycle); label("600",(0,-1)); label("30",(-1,1)); label("6",(-2,3)); label("5",(0,3)); \{}end{asy} \{}end{center}

   **(A)** 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

7. Trey and his mom stopped at a railroad crossing to let a train pass. As the train began to pass, Trey counted 6 cars in the first 10 seconds. It took the train 2 minutes and 45 seconds to clear the crossing at a constant speed. Which of the following was the most likely number of cars in the train?

   **(A)** 60 **(B)** 80 **(C)** 100 **(D)** 120 **(E)** 140

8. A fair coin is tossed 3 times. What is the probability of at least two consecutive heads?

   **(A)** $\frac{1}{8}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{3}{8}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{3}{4}$

9. The Incredible Hulk can double the distance it jumps with each succeeding jump. If its first jump is 1 meter, the second jump is 2 meters, the third jump is 4 meters, and so on, then on which jump will it first be able to jump more than 1 kilometer?

   **(A)** $9^{\text{th}}$ **(B)** $10^{\text{th}}$ **(C)** $11^{\text{th}}$ **(D)** $12^{\text{th}}$ **(E)** $13^{\text{th}}$

10. What is the ratio of the least common multiple of 180 and 594 to the greatest common factor of 180 and 594?

    **(A)** 110 **(B)** 165 **(C)** 330 **(D)** 625 **(E)** 660

11. Ted's grandfather used his treadmill on 3 days this week. He went 2 miles each day. On Monday he jogged at a speed of 5 miles per hour. He walked at the rate of 3 miles per hour on Wednesday and at 4 miles per hour on Friday. If Grandfather had always walked at 4 miles per hour, he would have spent less time on the treadmill. How many minutes less?

    **(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

12. At the 2013 Winnebago County Fair a vendor is offering a "fair special" on sandals. If you buy one pair of sandals at the regular price of 50, you get a second pair at a 40% discount, and a third pair at half the regular price. Javier took advantage of the "fair special" to buy three pairs of sandals. What percentage of the 150 dollar regular price did he save?

    **(A)** 25% **(B)** 30% **(C)** 33% **(D)** 40% **(E)** 45%

13. When Clara totaled her scores, she inadvertently reversed the units digit and the tens digit of one score. By which of the following might her incorrect sum have differed from the correct one?

    **(A)** 45 **(B)** 46 **(C)** 47 **(D)** 48 **(E)** 49

14. Abe holds 1 green and 1 red jelly bean in his hand. Bob holds 1 green, 1 yellow, and 2 red jelly beans in his hand. Each randomly picks a jelly bean to show the other. What is the probability that the colors match?

    **(A)** $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{3}{8}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{2}{3}$

15. If $3^p + 3^4 = 90$, $2^r + 44 = 76$, and $5^3 + 6^s = 1421$, what is the product of $p$, $r$, and $s$?

    **(A)** 27 **(B)** 40 **(C)** 50 **(D)** 70 **(E)** 90

16. A number of students from Fibonacci Middle School are taking part in a community service project. The ratio of $8^{\text{th}}$-graders to $6^{\text{th}}$-graders is $5 : 3$, and the ratio of $8^{\text{th}}$-graders to $7^{\text{th}}$-graders is $8 : 5$. What is the smallest number of students that could be participating in the project?

    **(A)** 16 **(B)** 40 **(C)** 55 **(D)** 79 **(E)** 89

17. The sum of six consecutive positive integers is 2013. What is the largest of these six integers?

    **(A)** 335 **(B)** 338 **(C)** 340 **(D)** 345 **(E)** 350

18. Isabella uses one-foot cubical blocks to build a rectangular fort that is 12 feet long, 10 feet wide, and 5 feet high. The floor and the four walls are all one foot thick. How many blocks does the fort contain?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; import three; size(3inch); currentprojection=orthograph 8,15,15); triple A,B,C,D,E,F,G,H,I,J,K,L,M,N,O,P; A = (0,0,0); B = (0,10,0); C = (12,10,0); D = (12,0,0); E = (0,0,5); F = (0,10,5); G = (12,10,5); H = (12,0,5); I = (1,1,1); J = (1,9,1); K = (11,9,1); L = (11,1,1); M = (1,1,5); N = (1,9,5); O = (11,9,5); P = (11,1,5); //outside box far draw(surface(A–B–C–D–cycle),white,nolight); draw(A–B–C–D–cycle); draw(surface(E–A–D–H–cycle),white,nolight); draw(E–A–D–H–cycle); draw(surface(D–C–G–H–cycle),white,nolight); draw(D–C–G–H–cycle); //inside box far draw(surface(I–J–K–L–cycle),white,nolight); draw(I–J–K–L–cycle); draw(surface(I–L–P–M–cycle),white,nolight); draw(I–L–P–M–cycle); draw(surface(L–K–O–P–cycle),white,nolight); draw(L–K–O–P–cycle); //inside box near draw(surface(I–J–N–M–cycle),white,nolight); draw(I–J–N–M–cycle); draw(surface(J–K–O–N–cycle),white,nolight); draw(J–K–O–N–cycle); //outside box near draw(surface(A–B–F–E–cycle),white,nolight); draw(A–B–F–E–cycle); draw(surface(B–C–G–F–cycle),white,nolight); draw(B–C–G–F–cycle); //top draw(surface(E–H–P–M–cycle),white,nolight); draw(surface(E–M–N–F–cycle),white,nolight); draw(surface(F–N–O–G–cycle),white,nolight); draw(surface(O–G–H–P–cycle),white,nolight); draw(M–N–O–P–cycle); draw(E–F–G–H–cycle); label("10",(A–B),SE); label("12",(C–B),SW); label("5",(F–B),W); \{}end{asy} \{}end{center}

    **(A)** 204 **(B)** 280 **(C)** 320 **(D)** 340 **(E)** 600

19. Bridget, Cassie, and Hannah are discussing the results of their last math test. Hannah shows Bridget and Cassie her test, but Bridget and Cassie don't show theirs to anyone. Cassie says, 'I didn't get the lowest score in our class,' and Bridget adds, 'I didn't get the highest score.' What is the ranking of the three girls from the highest score to the lowest score?

    **(A)** Hannah, Cassie, Bridget **(B)** Hannah, Bridget, Cassie
    **(C)** Cassie, Bridget, Hannah **(D)** Cassie, Hannah, Bridget
    **(E)** Bridget, Cassie, Hannah

20. A $1 \times 2$ rectangle is inscribed in a semicircle with longer side on the diameter. What is the area of the semicircle?

    **(A)** $\frac{\pi}{2}$ **(B)** $\frac{2\pi}{3}$ **(C)** $\pi$ **(D)** $\frac{4\pi}{3}$ **(E)** $\frac{5\pi}{3}$

21. Samantha lives 2 blocks west and 1 block south of the southwest corner of City Park. Her school is 2 blocks east and 2 blocks north of the northeast corner of City Park. On school days she bikes on streets to the southwest corner of City Park, then takes a diagonal path through the park to the northeast corner, and then bikes on streets to school. If her route is as short as possible, how many different routes can she take?

    **(A)** 3 **(B)** 6 **(C)** 9 **(D)** 12 **(E)** 18

22. Toothpicks are used to make a grid that is 60 toothpicks long and 32 toothpicks wide. How many toothpicks are used altogether?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; picture corner; draw(corner,(5,0)–(35,0)); draw(corner,(0,-5)–(0,-35)); for (int i=0; i¡3; ++i) { for (int j=0; j¿-2; –j) { if ((i-j)¡3) { add(corner,(50i,50j)); } } } draw((5,-100)–(45,-100)); draw((155,0)–(185,0),dotted); draw((105,-50)–(135,-50),dotted); draw((100,-55)–(100,-85),dotted); draw((55,-100)–(85,-100),dotted); draw((50,-105)–(50,-135),dotted); draw((0,-105)–(0,-135),dotted); \{}end{asy} \{}end{center}

    **(A)** 1920 **(B)** 1952 **(C)** 1980 **(D)** 2013 **(E)** 3932

23. Angle $ABC$ of $\triangle ABC$ is a right angle. The sides of $\triangle ABC$ are the diameters of semicircles as shown. The area of the semicircle on $\overline{AB}$ equals $8\pi$, and the arc of the semicircle on $\overline{AC}$ has length $8.5\pi$. What is the radius of the semicircle on $\overline{BC}$?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; import graph; pair A,B,C; A=(0,8); B=(0,0); C=(15,0); draw((0,8)..(-4,4)..(0,0)–(0,8)); draw((0,0)..(7.5,-7.5)..(15,0)–(0,0)); real theta = aTan(8/15); draw(arc((15/2,4),17/2,-theta,180-theta)); draw((0,8)–(15,0)); dot(A); dot(B); dot(C); label("$A$", A, NW); label("$B$", B, SW); label("$C$", C, SE); \{}end{asy} \{}end{center}

    **(A)** 7 **(B)** 7.5 **(C)** 8 **(D)** 8.5 **(E)** 9

24. Squares $ABCD$, $EFGH$, and $GHIJ$ are equal in area. Points $C$ and $D$ are the midpoints of sides $IH$ and $HE$, respectively. What is the ratio of the area of the shaded pentagon $AJICB$ to the sum of the areas of the three squares?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; pair A,B,C,D,E,F,G,H,I,J;

    A = (0.5,2); B = (1.5,2); C = (1.5,1); D = (0.5,1); E = (0,1); F = (0,0); G = (1,0); H = (1,1); I = (2,1); J = (2,0); draw(A–B); draw(C–B); draw(D–A); draw(F–E); draw(I–J); draw(J–F); draw(G–H); draw(A–J); filldraw(A–B–C–I–J–cycle,grey); draw(E–I); label("$A$", A, NW); label("$B$", B, NE); label("$C$", C, NE); label("$D$", D, NW); label("$E$", E, NW); label("$F$", F, SW); label("$G$", G, S); label("$H$", H, N); label("$I$", I, NE); label("$J$", J, SE); \{}end{asy} \{}end{center}

    **(A)** $\frac{1}{4}$ **(B)** $\frac{7}{24}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{3}{8}$ **(E)** $\frac{5}{12}$

25. A ball with diameter 4 inches starts at point A to roll along the track shown. The track is comprised of 3 semicircular arcs whose radii are $R_1 = 100$ inches, $R_2 = 60$ inches, and $R_3 = 80$ inches, respectively. The ball always remains in contact with the track and does not slip. What is the distance the center of the ball travels over the course from A to B?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; pair A,B; size(8cm); A=(0,0); B=(480,0); draw((0,0)–(480,0),linetype("3 4")); filldraw(circle((8,0),8),black); draw((0,0)..(100,-100)..(200,0)); draw((200,0)..(260,60
    draw((320,0)..(400,-80)..(480,0)); draw((100,0)–(150,-50sqrt(3)),Arrow(size=4)); draw((260,0)–(290,30sqrt(3)),Arrow(si
    draw((400,0)–(440,-40sqrt(3)),Arrow(size=4)); label("$A$", A, SW); label("$B$", B, SE); label("$R_1$", (100,-40), W); label("$R_2$", (260,40), SW); label("$R_3$", (400,-40), W); \{}end{asy} \{}end{center}

    **(A)** $238\pi$ **(B)** $240\pi$ **(C)** $260\pi$ **(D)** $280\pi$ **(E)** $500\pi$

    {{MAA Notice}}

# 2014 AMC 8 Problems

1. Harry and Terry are each told to calculate $8 - (2 + 5)$. Harry gets the correct answer. Terry ignores the parentheses and calculates $8 - 2 + 5$. If Harry's answer is $H$ and Terry's answer is $T$, what is $H - T$?

   **(A)** $-10$ **(B)** $-6$ **(C)** 0 **(D)** 6 **(E)** 10

2. Paul owes Paula 35 cents and has a pocket full of 5-cent coins, 10-cent coins, and 25-cent coins that he can use to pay her. What is the difference between the largest and the smallest number of coins he can use to pay her?

   **(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

3. Isabella had a week to read a book for a school assignment. She read an average of 36 pages per day for the first three days and an average of 44 pages per day for the next three days. She then finished the book by reading 10 pages on the last day. How many pages were in the book?

   **(A)** 240 **(B)** 250 **(C)** 260 **(D)** 270 **(E)** 280

4. The sum of two prime numbers is 85. What is the product of these two prime numbers?

   **(A)** 85 **(B)** 91 **(C)** 115 **(D)** 133 **(E)** 166

5. Margie's car can go 32 miles on a gallon of gas, and gas currently costs \{}$4 per gallon. How many miles can Margie drive on \{}$20?

   **(A)** 64 **(B)** 128 **(C)** 160 **(D)** 320 **(E)** 640

6. Six rectangles each with a common base width of 2 have lengths of 1, 4, 9, 16, 25, and 36. What is the sum of the areas of the six rectangles?

   **(A)** 91 **(B)** 93 **(C)** 162 **(D)** 182 **(E)** 202

7. There are four more girls than boys in Ms. Raub's class of 28 students. What is the ratio of number of girls to the number of boys in her class?

   **(A)** $3:4$ **(B)** $4:3$ **(C)** $3:2$ **(D)** $7:4$ **(E)** $2:1$

8. Eleven members of the Middle School Math Club each paid the same integer amount for a guest speaker to talk about problem solving at their math club meeting. In all, they paid their guest speaker $\$\underline{1A2}$. What is the missing digit A of this 3-digit number?

   **(A)** 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

9. In $\triangle ABC$, $D$ is a point on side $\overline{AC}$ such that $BD = DC$ and $\angle BCD$ measures $70^\circ$. What is the degree measure of $\angle ADB$?

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; size(300); defaultpen(linewidth(0.8)); pair A=(-1,0),C=(1,0),B=dir(40),D=origin; draw(A–B–C–A); draw(D–B); dot("$A$", A, SW); dot("$B$", B, NE); dot("$C$", C, SE); dot("$D$", D, S); label("$70^\{}circ$",C,2*dir(180-35)); \{}end{asy} \{}end{center}

   **(A)** 100 **(B)** 120 **(C)** 135 **(D)** 140 **(E)** 150

10. The first AMC 8 was given in 1985 and it has been given annually since that time. Samantha turned 12 years old the year that she took the seventh AMC 8. In what year was Samantha born?

    **(A)** 1979 **(B)** 1980 **(C)** 1981 **(D)** 1982 **(E)** 1983

11. Jack wants to bike from his house to Jill's house, which is located three blocks east and two blocks north of Jack's house. After biking each block, Jack can continue either east or north, but he needs to avoid a dangerous intersection one block east and one block north of his house. In how many ways can he reach Jill's house by biking a total of five blocks?

    **(A)** 4 **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 10

12. A magazine printed photos of three celebrities along with three photos of the celebrities as babies. The baby pictures did not identify the celebrities. Readers were asked to match each celebrity with the correct

baby pictures. What is the probability that a reader guessing at random will match all three correctly as a fraction?

**(A)** $\frac{1}{9}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

13. If $n$ and $m$ are integers and $n^2 + m^2$ is even, which of the following is impossible?

**(A)** $n$ and $m$ are even **(B)** $n$ and $m$ are odd **(C)** $n+m$ is even **(D)** $n+m$ is odd **(E)** none of these are impossible

14. Rectangle $ABCD$ and right triangle $DCE$ have the same area. They are joined to form a trapezoid, as shown. What is $DE$?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; size(250); defaultpen(linewidth(0.8)); pair A=(0,5),B=origin,C=(6,0),D=(6,5),E=(18,0); draw(A–B–E–D–cycle^^C–D); draw(rightanglemark(D,C,E,30)); label("$A$",A,NW); label("$B$",B,SW); label("$C$",C,S); label("$D$",D,N); label("$E$",E,S); label("$5$",A/2,W); label("$6$",(A+D)/2,N); \{}end{asy} \{}end{center}

**(A)** 12 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 16

15. The circumference of the circle with center $O$ is divided into 12 equal arcs, marked the letters $A$ through $L$ as seen below. What is the number of degrees in the sum of the angles $x$ and $y$?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; size(230); defaultpen(linewidth(0.65)); pair O=origin; pair[] circum = new pair[12]; string[] let = {"$A$","$B$","$C$","$D$","$E$","$F$","$G$","$H$","$I$","$J$ draw(unitcircle); for(int i=0;i¡=11;i=i+1) { circum[i]=dir(120-30*i); dot(circum[i],linewidth(2.5)); label(let[i],circum[i],2 } draw(O–circum[4]–circum[0]–circum[6]–circum[8]–cycle); label("$x$",circum[0],2.75*(dir(circum[0]–circum[4])+dir(cir circum[6]))); label("$y$",circum[6],1.75*(dir(circum[6]–circum[0])+dir(circum[6]–circum[8]))); label("$O$",O,dir(60)); \{}end{asy} \{}end{center}

**(A)** 75 **(B)** 80 **(C)** 90 **(D)** 120 **(E)** 150

16. The "Middle School Eight" basketball conference has 8 teams. Every season, each team plays every other conference team twice (home and away), and each team also plays 4 games against non-conference opponents. What is the total number of games in a season involving the "Middle School Eight" teams?

**(A)** 60 **(B)** 88 **(C)** 96 **(D)** 144 **(E)** 160

17. George walks 1 mile to school. He leaves home at the same time each day, walks at a steady speed of 3 miles per hour, and arrives just as school begins. Today he was distracted by the pleasant weather and walked the first $\frac{1}{2}$ mile at a speed of only 2 miles per hour. At how many miles per hour must George run the last $\frac{1}{2}$ mile in order to arrive just as school begins today?

**(A)** 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

18. Four children were born at City Hospital yesterday. Assume each child is equally likely to be a boy or a girl. Which of the following outcomes is most likely?

**(A)** All 4 are boys **(B)** All 4 are girls **(C)** 2 are girls and 2 are boys **(D)** 3 are of one gender and 1 is of the other gender **(E)** All of these outcomes are equally likely

19. A cube with 3-inch edges is to be constructed from 27 smaller cubes with 1-inch edges. Twenty-one of the cubes are colored red and 6 are colored white. If the 3-inch cube is constructed to have the smallest possible white surface area showing, what fraction of the surface area is white?

**(A)** $\frac{5}{54}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{5}{27}$ **(D)** $\frac{2}{9}$ **(E)** $\frac{1}{3}$

20. Rectangle $ABCD$ has sides $CD = 3$ and $DA = 5$. A circle with a radius of 1 is centered at $A$, a circle with a radius of 2 is centered at $B$, and a circle with a radius of 3 is centered at $C$. Which of the following is closest to the area of the region inside the rectangle but outside all three circles?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; draw((0,0)–(5,0)–(5,3)–(0,3)–(0,0)); draw(Circle((0,0),1) draw(Circle((0,3),2)); draw(Circle((5,3),3)); label("A",(0.2,0),W); label("B",(0.2,2.8),NW); label("C",(4.8,2.8),NE); label("D",(5,0),SE); label("5",(2.5,0),N); label("3",(5,1.5),E); \{}end{asy} \{}end{center}

**(A)** 3.5 **(B)** 4.0 **(C)** 4.5 **(D)** 5.0 **(E)** 5.5

21. The 7-digit numbers $\underline{74A52B1}$ and $\underline{326AB4C}$ are each multiples of 3. Which of the following could be the value of $C$?

    **(A)** 1 **(B)** 2 **(C)** 3 **(D)** 5 **(E)** 8

22. A 2-digit number is such that the product of the digits plus the sum of the digits is equal to the number. What is the units digit of the number?

    **(A)** 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

23. Three members of the Euclid Middle School girls' softball team had the following conversation.

    Ashley: I just realized that our uniform numbers are all 2-digit primes.

    Bethany: And the sum of your two uniform numbers is the date of my birthday earlier this month.

    Caitlin: That's funny. The sum of your two uniform numbers is the date of my birthday later this month.

    Ashley: And the sum of your two uniform numbers is today's date.

    What number does Caitlin wear?

    **(A)** 11 **(B)** 13 **(C)** 17 **(D)** 19 **(E)** 23

24. One day the Beverage Barn sold 252 cans of soda to 100 customers, and every customer bought at least one can of soda. What is the maximum possible median number of cans of soda bought per customer on that day?

    **(A)** 2.5 **(B)** 3.0 **(C)** 3.5 **(D)** 4.0 **(E)** 4.5

25. A straight one-mile stretch of highway, 40 feet wide, is closed. Robert rides his bike on a path composed of semicircles as shown. If he rides at 5 miles per hour, how many hours will it take to cover the one-mile stretch?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; size(10cm); pathpen=black; pointpen=black; D(arc((-2,0),1,300,360)); D(arc((0,0),1,0,180)); D(arc((2,0),1,180,360)); D(arc((4,0),1,0,180)); D(arc((6,0),1,180,240)); D((-1.5,-1)–(5.5,-1)); \{}end{asy} \{}end{center}

    Note: 1 mile = 5280 feet!

    **(A)** $\frac{\pi}{11}$ **(B)** $\frac{\pi}{10}$ **(C)** $\frac{\pi}{5}$ **(D)** $\frac{2\pi}{5}$ **(E)** $\frac{2\pi}{3}$

# 2015 AMC 8 Problems

1. How many square yards of carpet are required to cover a rectangular floor that is 12 feet long and 9 feet wide? (There are 3 feet in a yard.)

   **(A)** 12 **(B)** 36 **(C)** 108 **(D)** 324 **(E)** 972

2. Point $O$ is the center of the regular octagon $ABCDEFGH$, and $X$ is the midpoint of the side $\overline{AB}$. What fraction of the area of the octagon is shaded?

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; pair A,B,C,D,E,F,G,H,O,X; A=dir(45); B=dir(90); C=dir(135); D=dir(180); E=dir(-135); F=dir(-90); G=dir(-45); H=dir(0); O=(0,0); X=midpoint(A–B);

   fill(X–B–C–D–E–O–cycle,rgb(0.75,0.75,0.75)); draw(A–B–C–D–E–F–G–H–cycle);

   dot(”$A$”,A,dir(45)); dot(”$B$”,B,dir(90)); dot(”$C$”,C,dir(135)); dot(”$D$”,D,dir(180)); dot(”$E$”,E,dir(-135)); dot(”$F$”,F,dir(-90)); dot(”$G$”,G,dir(-45)); dot(”$H$”,H,dir(0)); dot(”$X$”,X,dir(135/2)); dot(”$O$”,O,dir(0 draw(E–O–X); \{}end{asy} \{}end{center}

   **(A)** $\frac{11}{32}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{13}{32}$ **(D)** $\frac{7}{16}$ **(E)** $\frac{15}{32}$

3. Jack and Jill are going swimming at a pool that is one mile from their house. They leave home simultaneously. Jill rides her bicycle to the pool at a constant speed of 10 miles per hour. Jack walks to the pool at a constant speed of 4 miles per hour. How many minutes before Jack does Jill arrive?

   **(A)** 5 **(B)** 6 **(C)** 8 **(D)** 9 **(E)** 10

4. The Centerville Middle School chess team consists of two boys and three girls. A photographer wants to take a picture of the team to appear in the local newspaper. She decides to have them sit in a row with a boy at each end and the three girls in the middle. How many such arrangements are possible?

   **(A)** 2 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 12

5. Billy's basketball team scored the following points over the course of the first 11 games of the season:

   $$42, 47, 53, 53, 58, 58, 58, 61, 64, 65, 73$$

   If his team scores 40 in the 12th game, which of the following statistics will show an increase?

   **(A)** range **(B)** median **(C)** mean **(D)** mode **(E)** mid-range

6. In $\triangle ABC$, $AB = BC = 29$, and $AC = 42$. What is the area of $\triangle ABC$?

   **(A)** 100 **(B)** 420 **(C)** 500 **(D)** 609 **(E)** 701

7. Each of two boxes contains three chips numbered 1, 2, 3. A chip is drawn randomly from each box and the numbers on the two chips are multiplied. What is the probability that their product is even?

   **(A)** $\frac{1}{9}$ **(B)** $\frac{2}{9}$ **(C)** $\frac{4}{9}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{5}{9}$

   ==Problem 8==

   What is the smallest whole number larger than the perimeter of any triangle with a side of length 5 and a side of length 19

   **(A)** 24 **(B)** 29 **(C)** 43 **(D)** 48 **(E)** 57

8. On her first day of work, Janabel sold one widget. On day two, she sold three widgets. On day three, she sold five widgets, and on each succeeding day, she sold two more widgets than she had sold on the previous day. How many widgets in total had Janabel sold after working 20 days?

   **(A)** 39 **(B)** 40 **(C)** 210 **(D)** 400 **(E)** 401

9. How many integers between 1000 and 9999 have four distinct digits?

   **(A)** 3024 **(B)** 4536 **(C)** 5040 **(D)** 6480 **(E)** 6561

10. In the small country of Mathland, all automobile license plates have four symbols. The first must be a vowel ($A, E, I, O$, or $U$), the second and third must be two different letters among the 21 non-vowels, and the fourth must be a digit (0 through 9). If the symbols are chosen at random subject to these conditions, what is the probability that the plate will read "$AMC8$"?

    **(A)** $\frac{1}{22{,}050}$ **(B)** $\frac{1}{21{,}000}$ **(C)** $\frac{1}{10{,}500}$ **(D)** $\frac{1}{2{,}100}$ **(E)** $\frac{1}{1{,}050}$

11. How many pairs of parallel edges, such as $\overline{AB}$, and $\overline{GH}$, or $\overline{EH}$, and $\overline{FG}$, does a cube have?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; import three; currentprojection=orthographic(1/2,-1,1/2); /* three - currentprojection, orthographic */ draw((0,0,0)–(1,0,0)–(1,1,0)–(0,1,0)–cycle); draw((0,0,0)–(0,0,1)); draw((0,1,0)–(0,1,1)); draw((1,1,0)–(1,1,1)); draw((1,0,0)–(1,0,1)); draw((0,0,1)–(1,0,1)–(1,1,1)–(0,1,1)–cycle); label("\$D\$",(0,0,0),S); label("\$A\$",(0,0,1),N); label("\$H\$",(0,1,0),S); label("\$E\$",(0,1,1),N); label("\$C\$",(1,0,0),S); label("\$B\$",(1,0,1),N); label("\$G\$",(1,1,0),S); label("\$F\$",(1,1,1),N); \{}end{asy} \{}end{center}

    **(A)** 6 **(B)** 12 **(C)** 18 **(D)** 24 **(E)** 36

12. How many subsets of two elements can be removed from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ so that the mean (average) of the remaining numbers is 6?

    **(A)** 1 **(B)** 2 **(C)** 3 **(D)** 5 **(E)** 6

13. Which of the following integers cannot be written as the sum of four consecutive odd integers?

    **(A)** 16 **(B)** 40 **(C)** 72 **(D)** 100 **(E)** 200

14. At Euler Middle School, 198 students voted on two issues in a school referendum with the following results: 149 voted in favor of the first issue and 119 voted in favor of the second issue. If there were exactly 29 students who voted against both issues, how many students voted in favor of both issues?

    **(A)** 49 **(B)** 70 **(C)** 79 **(D)** 99 **(E)** 149

15. In a middle-school mentoring program, a number of the sixth graders are paired with a ninth-grade student as a buddy. No ninth grader is assigned more than one sixth-grade buddy. If $\frac{1}{3}$ of all the ninth graders are paired with $\frac{2}{5}$ of all the sixth graders, what fraction of the total number of sixth and ninth graders have a buddy?

    **(A)** $\frac{2}{15}$ **(B)** $\frac{4}{11}$ **(C)** $\frac{11}{30}$ **(D)** $\frac{3}{8}$ **(E)** $\frac{11}{15}$

16. Jeremy's father drives him to school in rush hour traffic in 20 minutes. One day, there is no traffic, so his father can drive him 18 miles per hour faster and gets him to school in 12 minutes. How far in miles is it to school?

    **(A)** 4 **(B)** 6 **(C)** 8 **(D)** 9 **(E)** 12

17. An arithmetic sequence is a sequence in which each term after the first is obtained by adding a constant to the previous term. For example, $2, 5, 8, 11, 14$ is an arithmetic sequence with five terms, in which the first term is 2 and the constant added is 3. Each row and each column in this $5 \times 5$ array is an arithmetic sequence with five terms. The square in the center is labelled $X$ as shown. What is the value of $X$?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; size(3.85cm); label("\$X\$",(2.5,2.1),N); for (int i=0; i¡=5; ++i) draw((i,0)–(i,5), linewidth(.5)); for (int j=0; j¡=5; ++j) draw((0,j)–(5,j), linewidth(.5)); void draw_num(pair ll_corner, int num) { label(string(num), ll_corner + (0.5, 0.5), p = fontsize(19pt)); } draw_num((0,0), 17); draw_num((4, 0), 81); draw_num((0, 4), 1); draw_num((4,4), 25); void foo(int x, int y, string n) { label(n, (x+0.5,y+0.5), p = fontsize(19pt)); } foo(2, 4, " "); foo(3, 4, " "); foo(0, 3, " "); foo(2, 3, " "); foo(1, 2, " "); foo(3, 2, " "); foo(1, 1, " "); foo(2, 1, " "); foo(3, 1, " "); foo(4, 1, " "); foo(2, 0, " "); foo(3, 0, " "); foo(0, 1, " "); foo(0, 2, " "); foo(1, 0, " "); foo(1, 3, " "); foo(1, 4, " "); foo(3, 3, " "); foo(4, 2, " "); foo(4, 3, " "); \{}end{asy} \{}end{center}

    **(A)** 21 **(B)** 31 **(C)** 36 **(D)** 40 **(E)** 42

18. A triangle with vertices as $A = (1, 3)$, $B = (5, 1)$, and $C = (4, 4)$ is plotted on a $6 \times 5$ grid. What fraction of the grid is covered by the triangle?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; draw((1,0)–(1,5),linewidth(.5)); draw((2,0)–(2,5),linewidth(.5)); draw((3,0)–(3,5),linewidth(.5)); draw((4,0)–(4,5),linewidth(.5)); draw((5,0)–(5,5),linewidth(.5));

draw((6,0)–(6,5),linewidth(.5)); draw((0,1)–(6,1),linewidth(.5)); draw((0,2)–(6,2),linewidth(.5)); draw((0,3)–(6,3),linewidth(.5)); draw((0,4)–(6,4),linewidth(.5)); draw((0,5)–(6,5),linewidth(.5)); draw((0,0)–(0,6),EndArrow); draw((0,0)–(7,0),EndArrow); draw((1,3)–(4,4)–(5,1)–cycle); label("$y$",(0,6),W); label("$x$",(7,0),S); label("$A$",(1,3),dir(210)); label("$B$",(5,1),SE); label("$C$",(4,4),dir(100)); \{}end{asy} \{}end{center}

**(A)** $\frac{1}{6}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

19. Ralph went to the store and bought 12 pairs of socks for a total of \$24. Some of the socks he bought cost \$1 a pair, some of the socks he bought cost \$3 a pair, and some of the socks he bought cost \$4 a pair. If he bought at least one pair of each type, how many pairs of \$1 socks did Ralph buy?

**(A)** 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

20. In the given figure hexagon $ABCDEF$ is equiangular, $ABJI$ and $FEHG$ are squares with areas 18 and 32 respectively, $\triangle JBK$ is equilateral and $FE = BC$. What is the area of $\triangle KBC$?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; draw((-4,6*sqrt(2))–(4,6*sqrt(2))); draw((-4,-6*sqrt(2))–(4,-6*sqrt(2))); draw((-8,0)–(-4,6*sqrt(2))); draw((-8,0)–(-4,-6*sqrt(2))); draw((4,6*sqrt(2))–(8,0)); draw((8,0)–(4,-6*sqrt(2))); draw((-4,6*sqrt(2))–(4,6*sqrt(2))–(4,8+6*sqrt(2))–(-4,8+6*sqrt(2))–cycle); draw((-8,0)–(-4,-6*sqrt(2))–(-4-6*sqrt(2),-4-6*sqrt(2))–(-8-6*sqrt(2),-4)–cycle); label("$I$",(-4,8+6*sqrt(2)),dir(100)); label("$J$",(4,8+6*sqrt(2)),dir(80)); label("$A$",(-4,6*sqrt(2)),dir(280)); label("$B$",(4,6*sqrt(2)),dir(250)); label("$C$",(8,0),W); label("$D$",(4,-6*sqrt(2)),NW); label("$E$",(-4,-6*sqrt(2)),NE); label("$F$",(-8,0),E); draw((4,8+6*sqrt(2))–(4,6*sqrt(2))–(4+4*sqrt(3),4+6*sqrt(2))–cycle); label("$K$",(4+4*sqrt(3),4+6*sqrt(2)),E); draw((4+4*sqrt(3),4+6*sqrt(2))–(8,0),dashed); label("$H$",(-4-6*sqrt(2),-4-6*sqrt(2)),S); label("$G$",(-8-6*sqrt(2),-4),W); label("$32$",(-10,-8),N); label("$18$",(0,6*sqrt(2)+2),N); \{}end{asy} \{}end{center}

**(A)** $6\sqrt{2}$ **(B)** 9 **(C)** 12 **(D)** $9\sqrt{2}$ **(E)** 32

21. On June 1, a group of students are standing in rows, with 15 students in each row. On June 2, the same group is standing with all of the students in one long row. On June 3, the same group is standing with just one student in each row. On June 4, the same group is standing with 6 students in each row. This process continues through June 12 with a different number of students per row each day. However, on June 13, they cannot find a new way of organizing the students. What is the smallest possible number of students in the group?

**(A)** 21 **(B)** 30 **(C)** 60 **(D)** 90 **(E)** 1080

22. Tom has twelve slips of paper which he wants to put into five cups labeled $A$, $B$, $C$, $D$, $E$. He wants the sum of the numbers on the slips in each cup to be an integer. Furthermore, he wants the five integers to be consecutive and increasing from $A$ to $E$. The numbers on the papers are $2, 2, 2, 2.5, 2.5, 3, 3, 3, 3, 3.5, 4,$ and 4.5. If a slip with 2 goes into cup $E$ and a slip with 3 goes into cup $B$, then the slip with 3.5 must go into what cup?

**(A)** $A$ **(B)** $B$ **(C)** $C$ **(D)** $D$ **(E)** $E$

23. A baseball league consists of two four-team divisions. Each team plays every other team in its division $N$ games. Each team plays every team in the other division $M$ games with $N > 2M$ and $M > 4$. Each team plays a 76-game schedule. How many games does a team play within its own division?

**(A)** 36 **(B)** 48 **(C)** 54 **(D)** 60 **(E)** 72

24. One-inch squares are cut from the corners of this 5 inch square. What is the area in square inches of the largest square that can fit into the remaining space?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; draw((0,0)–(0,5)–(5,5)–(5,0)–cycle); filldraw((0,4)–(1,4)–(1,5)–(0,5)–cycle, gray); filldraw((0,0)–(1,0)–(1,1)–(0,1)–cycle, gray); filldraw((4,0)–(4,1)–(5,1)–(5,0)–cycle, gray); filldraw((4,4)–(4,5)–(5,5)–(5,4)–cycle, gray); \{}end{asy} \{}end{center}

**(A)** 9 **(B)** $12\frac{1}{2}$ **(C)** 15 **(D)** $15\frac{1}{2}$ **(E)** 17

# 2016 AMC 8 Problems

1. The longest professional tennis match lasted a total of 11 hours and 5 minutes. How many minutes is that?

   **(A)** 605 **(B)** 655 **(C)** 665 **(D)** 1005 **(E)** 1105

2. In rectangle $ABCD$, $AB = 6$ and $AD = 8$. Point $M$ is the midpoint of $\overline{AD}$. What is the area of $\triangle AMC$?

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; draw((0,4)–(0,0)–(6,0)–(6,8)–(0,8)–(0,4)–(6,8)–(0,0)); label("$A$", (0,0), SW); label("$B$", (6, 0), SE); label("$C$", (6,8), NE); label("$D$", (0, 8), NW); label("$M$", (0, 4), W); label("$4$", (0, 2), W); label("$6$", (3, 0), S); \{}end{asy} \{}end{center}

   **(A)** 12 **(B)** 15 **(C)** 18 **(D)** 20 **(E)** 24

   [[2016 AMC 8 Problems/Problem 2—Solution ]]

3. Four students take an exam. Three of their scores are 70, 80, and 90. If the average of their four scores is 70, then what is the remaining score??

   **(A)** 40 **(B)** 50 **(C)** 55 **(D)** 60 **(E)** 70

   [[2016 AMC 8 Problems/Problem 3—Solution ]]

4. When Cheenu was a boy, he could run 15 miles in 3 hours and 30 minutes. As an old man, he can now walk 10 miles in 4 hours. How many minutes longer does it take for him to travel a mile now compared to when he was a boy?

   **(A)** 6 **(B)** 10 **(C)** 15 **(D)** 18 **(E)** 30

   [[2016 AMC 8 Problems/Problem 4—Solution ]]

5. The number $N$ is a two-digit number.

   When $N$ is divided by 9, the remainder is 1.

   When $N$ is divided by 10, the remainder is 3.

   What is the remainder when $N$ is divided by 11?

   **(A)** 0 **(B)** 2 **(C)** 4 **(D)** 5 **(E)** 7

   [[2016 AMC 8 Problems/Problem 5—Solution ]]

6. The following bar graph represents the length (in letters) of the names of 19 people. What is the median length of these names?

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; unitsize(0.9cm); draw((-0.5,0)–(10,0), linewidth(1.5)); draw((-0.5,1)–(10,1)); draw((-0.5,2)–(10,2)); draw((-0.5,3)–(10,3)); draw((-0.5,4)–(10,4)); draw((-0.5,5)–(10,5)); draw((-0.5,6)–(10,6)); draw((-0.5,7)–(10,7)); label("frequency",(-0.5,8)); label("0", (-1, 0)); label("1", (-1, 1)); label("2", (-1, 2)); label("3", (-1, 3)); label("4", (-1, 4)); label("5", (-1, 5)); label("6", (-1, 6)); label("7", (-1, 7)); filldraw((0,0)–(0,7)–(1,7)–(1,0)–cycle, black); filldraw((2,0)–(2,3)–(3,3)–(3,0)–cycle, black); filldraw((4,0)–(4,1)–(5,1)–(5,0)–cycle, black); filldraw((6,0)–(6,4)–(7,4)–(7,0)–cycle, black); filldraw((8,0)–(8,4)–(9,4)–(9,0)–cycle, black); label("3", (0.5, -0.5)); label("4", (2.5, -0.5)); label("5", (4.5, -0.5)); label("6", (6.5, -0.5)); label("7", (8.5, -0.5)); label("name length", (4.5, -1)); \{}end{asy} \{}end{center}

   **(A)** 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

   [[2016 AMC 8 Problems/Problem 6—Solution ]]

7. Which of the following numbers is not a perfect square?

   **(A)** $1^{2016}$ **(B)** $2^{2017}$ **(C)** $3^{2018}$ **(D)** $4^{2019}$ **(E)** $5^{2020}$

   [[2016 AMC 8 Problems/Problem 7—Solution ]]

8. Find the value of the expression:

   $$100 - 98 + 96 - 94 + 92 - 90 + \cdots + 8 - 6 + 4 - 2.$$

   **(A)** 20 **(B)** 40 **(C)** 50 **(D)** 80 **(E)** 100

   [[2016 AMC 8 Problems/Problem 8—Solution ]]

9. What is the sum of the distinct prime integer divisors of 2016?

**(A)** 9 **(B)** 12 **(C)** 16 **(D)** 49 **(E)** 63

[[2016 AMC 8 Problems/Problem 9—Solution ]]

10. Suppose that $a * b$ means $3a - b$. What is the value of $x$ if

$$2 * (5 * x) = 1$$

**(A)** $\frac{1}{10}$ **(B)** 2 **(C)** $\frac{10}{3}$ **(D)** 10 **(E)** 14.

[[2016 AMC 8 Problems/Problem 10—Solution ]]

11. Determine how many two-digit numbers satisfy the following property: when the number is added to the number obtained by reversing its digits, the sum is 132.

**(A)** 5 **(B)** 7 **(C)** 9 **(D)** 11 **(E)** 12

[[2016 AMC 8 Problems/Problem 11—Solution ]]

12. Jefferson Middle School has the same number of boys and girls. $\frac{3}{4}$ of the girls and $\frac{2}{3}$ of the boys went on a field trip. What fraction of the students on the field trip were girls?

**(A)** $\frac{1}{2}$ **(B)** $\frac{9}{17}$ **(C)** $\frac{7}{13}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{14}{15}$

[[2016 AMC 8 Problems/Problem 12—Solution ]]

13. Two different numbers are randomly selected from the set $\{-2, -1, 0, 3, 4, 5\}$ and multiplied together. What is the probability that the product is 0?

**(A)** $\dfrac{1}{6}$ **(B)** $\dfrac{1}{5}$ **(C)** $\dfrac{1}{4}$ **(D)** $\dfrac{1}{3}$ **(E)** $\dfrac{1}{2}$

[[2016 AMC 8 Problems/Problem 13—Solution ]]

14. Karl's car uses a gallon of gas every 35 miles, and his gas tank holds 14 gallons when it is full. One day, Karl started with a full tank of gas, drove 350 miles, bought 8 gallons of gas, and continued driving to his destination. When he arrived, his gas tank was half full. How many miles did Karl drive that day?

**(A)** 525 **(B)** 560 **(C)** 595 **(D)** 665 **(E)** 735

[[2016 AMC 8 Problems/Problem 14—Solution ]]

15. What is the largest power of 2 that is a divisor of $13^4 - 11^4$?

**(A)** 8 **(B)** 16 **(C)** 32 **(D)** 64 **(E)** 128

[[2016 AMC 8 Problems/Problem 15—Solution ]]

16. Annie and Bonnie are running laps around a 400-meter oval track. They started at the same time, at the same place, but Annie has pulled ahead because she runs 25% faster than Bonnie. How many laps will Annie have run when she first passes Bonnie?

**(A)** $1\dfrac{1}{4}$ **(B)** $3\dfrac{1}{3}$ **(C)** 4 **(D)** 5 **(E)** 25

[[2016 AMC 8 Problems/Problem 16—Solution ]]

17. An ATM password at Fred's Bank is composed of four digits from 0 to 9, with repeated digits allowable. If no password may begin with the sequence $9, 1, 1$, then how many passwords are possible?

**(A)** 30 **(B)** 7290 **(C)** 9000 **(D)** 9990 **(E)** 9999

[[2016 AMC 8 Problems/Problem 17—Solution ]]

18. In an All-Area track meet, 216 sprinters enter a 100−meter dash competition. The track has 6 lanes, so only 6 sprinters can compete at a time. At the end of each race, the five non-winners are eliminated, and the winner will compete again in a later race. How many races are needed to determine the champion sprinter?

**(A)** 36 **(B)** 42 **(C)** 43 **(D)** 60 **(E)** 72

[[2016 AMC 8 Problems/Problem 18—Solution ]]

19. The sum of 25 consecutive even integers is $10,000$. What is the largest of these 25 consecutive integers?

    **(A)** 360 **(B)** 388 **(C)** 412 **(D)** 416 **(E)** 424

    [[2016 AMC 8 Problems/Problem 19—Solution ]]

20. The least common multiple of $a$ and $b$ is 12, and the least common multiple of $b$ and $c$ is 15. What is the least possible value of the least common multiple of $a$ and $c$?

    **(A)** 20 **(B)** 30 **(C)** 60 **(D)** 120 **(E)** 180

    [[2016 AMC 8 Problems/Problem 20—Solution ]]

21. A top hat contains 3 red chips and 2 green chips. Chips are drawn randomly, one at a time without replacement, until all 3 of the reds are drawn or until both green chips are drawn. What is the probability that the 3 reds are drawn?

    **(A)** $\frac{3}{10}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{7}{10}$

    [[2016 AMC 8 Problems/Problem 21—Solution ]]

22. Rectangle $DEFA$ below is a $3 \times 4$ rectangle with $DC = CB = BA = 1$. What is the area of the "bat wings" (shaded region)? \{}begin{center} \{}begin{asy} import olympiad; import cse5; draw((0,0)–(3,0)–(3,4)–(0,4)–(0,0)–(2,4)–(3,0)); draw((3,0)–(1,4)–(0,0)); fill((0,0)–(1,4)–(1.5,3)–cycle, black); fill((3,0)–(2,4)–(1.5,3)–cycle, black); label("$A$",(3.05,4.2)); label("$B$",(2,4.2)); label("$C$",(1,4.2)); label("$D$",(0,4.2)); label("$E$", (0,-0.2)); label("$F$", (3,-0.2)); \{}end{asy} \{}end{center}

    **(A)** 2 **(B)** $2\frac{1}{2}$ **(C)** 3 **(D)** $3\frac{1}{2}$ **(E)** 4

    [[2016 AMC 8 Problems/Problem 22—Solution ]]

23. Two congruent circles centered at points $A$ and $B$ each pass through the other circle's center. The line containing both $A$ and $B$ is extended to intersect the circles at points $C$ and $D$. The circles intersect at two points, one of which is $E$. What is the degree measure of $\angle CED$?

    **(A)** 90 **(B)** 105 **(C)** 120 **(D)** 135 **(E)** 150

    [[2016 AMC 8 Problems/Problem 23—Solution ]]

24. The digits 1, 2, 3, 4, and 5 are each used once to write a five-digit number $PQRST$. The three-digit number $PQR$ is divisible by 4, the three-digit number $QRS$ is divisible by 5, and the three-digit number $RST$ is divisible by 3. What is $P$?

    **(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

    [[2016 AMC 8 Problems/Problem 24—Solution ]]

    ==Problem 25==

    A semicircle is inscribed in an isosceles triangle with base 16 and height 15 so that the diameter of the semicircle is contained in the base of the triangle as shown. What is the radius of the semicircle?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; draw((0,0)–(8,15)–(16,0)–(0,0)); draw(arc((8,0),7.0588,0
    \{}end{asy} \{}end{center}

    **(A)** $4\sqrt{3}$ **(B)** $\frac{120}{17}$ **(C)** 10 **(D)** $\frac{17\sqrt{2}}{2}$ **(E)** $\frac{17\sqrt{3}}{2}$

    [[2016 AMC 8 Problems/Problem 25—Solution ]]

# 2017 AMC 8 Problems

1. Which of the following values is the largest?

   **(A)** $2+0+1+7$ **(B)** $2\times 0+1+7$ **(C)** $2+0\times 1+7$ **(D)** $2+0+1\times 7$ **(E)** $2\times 0\times 1\times 7$

   [[2017 AMC 8 Problems/Problem 1—Solution ]]

2. Alicia, Brenda, and Colby were the candidates in a recent election for student president. The pie chart below shows how the votes were distributed among the three candidates. If Brenda received 36 votes, then how many votes were cast all together?

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; draw((-1,0)–(0,0)–(0,1)); draw((0,0)–(0.309, -0.951)); filldraw(arc((0,0), (0,1), (-1,0))–(0,0)–cycle, lightgray); filldraw(arc((0,0), (0.309, -0.951), (0,1))–(0,0)–cycle, gray); draw(arc((0,0), (-1,0), (0.309, -0.951))); label("Colby", (-0.5, 0.5)); label("25\{}%", (-0.5, 0.3)); label("Alicia", (0.7, 0.2)); label("45\{}%", (0.7, 0)); label("Brenda", (-0.5, -0.4)); label("30\{}%", (-0.5, -0.6)); \{}end{asy} \{}end{center}

   **(A)** 70 **(B)** 84 **(C)** 100 **(D)** 106 **(E)** 120

   [[2017 AMC 8 Problems/Problem 2—Solution ]]

3. What is the value of the expression $\sqrt{16\sqrt{8\sqrt{4}}}$?

   **(A)** 4 **(B)** $4\sqrt{2}$ **(C)** 8 **(D)** $8\sqrt{2}$ **(E)** 16

   [[2017 AMC 8 Problems/Problem 3—Solution ]]

4. When 0.000315 is multiplied by $7,928,564$ the product is closest to which of the following?

   **(A)** 210 **(B)** 240 **(C)** 2100 **(D)** 2400 **(E)** 24000

   [[2017 AMC 8 Problems/Problem 4—Solution ]]

5. What is the value of the expression $\frac{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8}{1+2+3+4+5+6+7+8}$?

   **(A)** 1020 **(B)** 1120 **(C)** 1220 **(D)** 2240 **(E)** 3360

   [[2017 AMC 8 Problems/Problem 5—Solution ]]

6. If the degree measures of the angles of a triangle are in the ratio $3:3:4$, what is the degree measure of the largest angle of the triangle?

   **(A)** 18 **(B)** 36 **(C)** 60 **(D)** 72 **(E)** 90

   [[2017 AMC 8 Problems/Problem 6—Solution ]]

7. Let $Z$ be a 6-digit positive integer, such as 247247, whose first three digits are the same as its last three digits taken in the same order. Which of the following numbers must also be a factor of $Z$?

   **(A)** 11 **(B)** 19 **(C)** 101 **(D)** 111 **(E)** 1111

   [[2017 AMC 8 Problems/Problem 7—Solution ]]

8. Malcolm wants to visit Isabella after school today and knows the street where she lives but doesn't know her house number. She tells him, "My house number has two digits, and exactly three of the following four statements about it are true."

   (1) It is prime.

   (2) It is even

   (3) It is divisible by 7.

   (4) One of its digits is 9.

   This information allows Malcolm to determine Isabella's house number. What is its units digit?

   **(A)** 4 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

   [[2017 AMC 8 Problems/Problem 8—Solution ]]

9. All of Marcy's marbles are blue, red, green, or yellow. One third of her marbles are blue, one fourth of them are red, and six of them are green. What is the smallest number of yellow marbles?

   **(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

   [[2017 AMC 8 Problems/Problem 9—Solution ]]

10. A box contains five cards, numbered 1, 2, 3, 4, and 5. Three cards are selected randomly without replacement from the box. What is the probability that 4 is the largest value selected?

    **(A)** $\frac{1}{10}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{3}{10}$ **(D)** $\frac{2}{5}$ **(E)** $\frac{1}{2}$

    [[2017 AMC 8 Problems/Problem 10—Solution ]]

11. A square-shaped floor is covered with congruent square tiles. If the total number of tiles that lie on the two diagonals is 37, how many tiles cover the floor?

    **(A)** 148 **(B)** 324 **(C)** 361 **(D)** 1296 **(E)** 1369

    [[2017 AMC 8 Problems/Problem 11—Solution ]]

12. The smallest positive integer greater than 1 that leaves a remainder of 1 when divided by 4, 5, and 6 lies between which of the following pairs of numbers?

    **(A)** 2 and 19 **(B)** 20 and 39 **(C)** 40 and 59 **(D)** 60 and 79 **(E)** 80 and 124

    [[2017 AMC 8 Problems/Problem 12—Solution ]]

13. Peter, Emma, and Kyler played chess with each other. Peter won 4 games and lost 2 games. Emma won 3 games and lost 3 games. If Kyler lost 3 games, how many games did he win?

    **(A)** 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

    [[2017 AMC 8 Problems/Problem 13—Solution ]]

14. Chloe and Zoe are both students in Ms. Demeanor's math class. Last night, they each solved half of the problems in their homework assignment alone and then solved the other half together. Chloe had correct answers to only 80% of the problems she solved alone, but overall 88% of her answers were correct. Zoe had correct answers to 90% of the problems she solved alone. What was Zoe's overall percentage of correct answers?

    **(A)** 89 **(B)** 92 **(C)** 93 **(D)** 96 **(E)** 98

    [[2017 AMC 8 Problems/Problem 14—Solution ]]

15. In the arrangement of letters and numerals below, by how many different paths can one spell AMC8? Beginning at the A in the middle, a path allows only moves from one letter to an adjacent (above, below, left, or right, but not diagonal) letter. One example of such a path is traced in the picture.

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; fill((0.5, 4.5)–(1.5,4.5)–(1.5,2.5)–(0.5,2.5)–cycle,lightgray); fill((1.5,3.5)–(2.5,3.5)–(2.5,1.5)–(1.5,1.5)–cycle,lightgray); label("$8$", (1, 0)); label("$C$", (2, 0)); label("$8$", (3, 0)); label("$8$", (0, 1)); label("$C$", (1, 1)); label("$M$", (2, 1)); label("$C$", (3, 1)); label("$8$", (4, 1)); label("$C$", (0, 2)); label("$M$", (1, 2)); label("$A$", (2, 2)); label("$M$", (3, 2)); label("$C$", (4, 2)); label("$8$", (0, 3)); label("$C$", (1, 3)); label("$M$", (2, 3)); label("$C$", (3, 3)); label("$8$", (4, 3)); label("$8$", (1, 4)); label("$C$", (2, 4)); label("$8$", (3, 4)); \{}end{asy} \{}end{center}

    **(A)** 8 **(B)** 9 **(C)** 12 **(D)** 24 **(E)** 36

    [[2017 AMC 8 Problems/Problem 15—Solution ]]

16. In the figure below, choose point $D$ on $\overline{BC}$ so that $\triangle ACD$ and $\triangle ABD$ have equal perimeters. What is the area of $\triangle ABD$?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; draw((0,0)–(4,0)–(0,3)–(0,0)); label("$A$", (0,0), SW); label("$B$", (4,0), ESE); label("$C$", (0, 3), N); label("$3$", (0, 1.5), W); label("$4$", (2, 0), S); label("$5$", (2, 1.5), NE); \{}end{asy} \{}end{center}

    **(A)** $\frac{3}{4}$ **(B)** $\frac{3}{2}$ **(C)** 2 **(D)** $\frac{12}{5}$ **(E)** $\frac{5}{2}$

    [[2017 AMC 8 Problems/Problem 16—Solution ]]

17. Starting with some gold coins and some empty treasure chests, I tried to put 9 gold coins in each treasure chest, but that left 2 treasure chests empty. So instead I put 6 gold coins in each treasure chest, but then I had 3 gold coins left over. How many gold coins did I have?

    **(A)** 9 **(B)** 27 **(C)** 45 **(D)** 63 **(E)** 81

    [[2017 AMC 8 Problems/Problem 17—Solution ]]

18. In the non-convex quadrilateral $ABCD$ shown below, $\angle BCD$ is a right angle, $AB = 12$, $BC = 4$, $CD = 3$, and $AD = 13$.

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; draw((0,0)–(2.4,3.6)–(0,5)–(12,0)–(0,0)); label(”$B$”, (0, 0), SW); label(”$A$”, (12, 0), ESE); label(”$C$”, (2.4, 3.6), SE); label(”$D$”, (0, 5), N); \{}end{asy} \{}end{center}

    What is the area of quadrilateral $ABCD$?

    **(A)** 12 **(B)** 24 **(C)** 26 **(D)** 30 **(E)** 36

    [[2017 AMC 8 Problems/Problem 18—Solution ]]

19. For any positive integer $M$, the notation $M!$ denotes the product of the integers 1 through $M$. What is the largest integer $n$ for which $5^n$ is a factor of the sum $98! + 99! + 100!$ ?

    **(A)** 23 **(B)** 24 **(C)** 25 **(D)** 26 **(E)** 27

    [[2017 AMC 8 Problems/Problem 19—Solution ]]

20. An integer between 1000 and 9999, inclusive, is chosen at random. What is the probability that it is an odd integer whose digits are all distinct?

    **(A)** $\frac{14}{75}$ **(B)** $\frac{56}{225}$ **(C)** $\frac{107}{400}$ **(D)** $\frac{7}{25}$ **(E)** $\frac{9}{25}$

    [[2017 AMC 8 Problems/Problem 20—Solution ]]

21. Suppose $a$, $b$, and $c$ are nonzero real numbers, and $a + b + c = 0$. What are the possible value(s) for $\frac{a}{|a|} + \frac{b}{|b|} + \frac{c}{|c|} + \frac{abc}{|abc|}$?

    **(A)** 0 **(B)** 1 and $-1$ **(C)** 2 and $-2$ **(D)** $0, 2,$ and $-2$ **(E)** $0, 1,$ and $-1$

    [[2017 AMC 8 Problems/Problem 21—Solution ]]

22. In the right triangle $ABC$, $AC = 12$, $BC = 5$, and angle $C$ is a right angle. A semicircle is inscribed in the triangle as shown. What is the radius of the semicircle?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; draw((0,0)–(12,0)–(12,5)–(0,0)); draw(arc((8.67,0),(12,0) label(”$A$”, (0,0), W); label(”$C$”, (12,0), E); label(”$B$”, (12,5), NE); label(”$12$”, (6, 0), S); label(”$5$”, (12, 2.5), E); \{}end{asy} \{}end{center}

    **(A)** $\frac{7}{6}$ **(B)** $\frac{13}{5}$ **(C)** $\frac{59}{18}$ **(D)** $\frac{10}{3}$ **(E)** $\frac{60}{13}$

    [[2017 AMC 8 Problems/Problem 22—Solution ]]

23. Each day for four days, Linda traveled for one hour at a speed that resulted in her traveling one mile in an integer number of minutes. Each day after the first, her speed decreased so that the number of minutes to travel one mile increased by 5 minutes over the preceding day. Each of the four days, her distance traveled was also an integer number of miles. What was the total number of miles for the four trips?

    **(A)** 10 **(B)** 15 **(C)** 25 **(D)** 50 **(E)** 82

    [[2017 AMC 8 Problems/Problem 23—Solution ]]

24. Mrs. Sanders has three grandchildren, who call her regularly. One calls her every three days, one calls her every four days, and one calls her every five days. All three called her on December 31, 2016. On how many days during the next year did she not receive a phone call from any of her grandchildren?

    **(A)** 78 **(B)** 80 **(C)** 144 **(D)** 146 **(E)** 152

    [[2017 AMC 8 Problems/Problem 24—Solution ]]

25. In the figure shown, $\overline{US}$ and $\overline{UT}$ are line segments each of length 2, and $m\angle TUS = 60°$. Arcs 5.0pt$\overset{\frown}{\mathrm{TR}}$ and 5.0pt$\overset{\frown}{\mathrm{SR}}$ are each one-sixth of a circle with radius 2. What is the area of the region shown?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; draw((1,1.732)–(2,3.464)–(3,1.732)); draw(arc((0,0),(2,0) draw(arc((4,0),(3,1.732),(2,0))); label("$U$", (2,3.464), N); label("$S$", (1,1.732), W); label("$T$", (3,1.732), E); label("$R$", (2,0), S); \{}end{asy} \{}end{center}

    **(A)** $3\sqrt{3}-\pi$ **(B)** $4\sqrt{3}-\frac{4\pi}{3}$ **(C)** $2\sqrt{3}$ **(D)** $4\sqrt{3}-\frac{2\pi}{3}$ **(E)** $4+\frac{4\pi}{3}$

    [[2017 AMC 8 Problems/Problem 25—Solution ]]

# 2018 AMC 8 Problems

1. An amusement park has a collection of scale models, with ratio $1 : 20$, of buildings and other sights from around the country. The height of the United States Capitol is 289 feet. What is the height in feet of its replica to the nearest whole number?

   **(A)** 14 **(B)** 15 **(C)** 16 **(D)** 18 **(E)** 20

2. What is the value of the product

$$\left(1+\frac{1}{1}\right)\cdot\left(1+\frac{1}{2}\right)\cdot\left(1+\frac{1}{3}\right)\cdot\left(1+\frac{1}{4}\right)\cdot\left(1+\frac{1}{5}\right)\cdot\left(1+\frac{1}{6}\right)?$$

   **(A)** $\frac{7}{6}$ **(B)** $\frac{4}{3}$ **(C)** $\frac{7}{2}$ **(D)** 7 **(E)** 8

3. Students Arn, Bob, Cyd, Dan, Eve, and Fon are arranged in that order in a circle. They start counting: Arn first, then Bob, and so forth. When the number contains a 7 as a digit (such as 47) or is a multiple of 7 that person leaves the circle and the counting continues. Who is the last one present in the circle?

   **(A)** Arn **(B)** Bob **(C)** Cyd **(D)** Dan **(E)** Eve

4. The twelve-sided figure shown has been drawn on 1 cm $\times$ 1 cm graph paper. What is the area of the figure in cm$^2$?

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; unitsize(8mm); for (int i=0; i¡7; ++i) { draw((i,0)–(i,7),gray); draw((0,i+1)–(7,i+1),gray); } draw((1,3)–(2,4)–(2,5)–(3,6)–(4,5)–(5,5)–(6,4)–(5,3)–(5,2)–(4,1)–(3,2)–(2,2)–cycle,black+2bp); \{}end{asy} \{}end{center}

   **(A)** 12 **(B)** 12.5 **(C)** 13 **(D)** 13.5 **(E)** 14

5. What is the value of $1 + 3 + 5 + \cdots + 2017 + 2019 - 2 - 4 - 6 - \cdots - 2016 - 2018$?

   **(A)** $-1010$ **(B)** $-1009$ **(C)** 1008 **(D)** 1009 **(E)** 1010

6. On a trip to the beach, Anh traveled 50 miles on the highway and 10 miles on a coastal access road. He drove three times as fast on the highway as on the coastal road. If Anh spent 30 minutes driving on the coastal road, how many minutes did his entire trip take?

   **(A)** 50 **(B)** 70 **(C)** 80 **(D)** 90 **(E)** 100

7. The 5-digit number $\underline{2}\ \underline{0}\ \underline{1}\ \underline{8}\ \underline{U}$ is divisible by 9. What is the remainder when this number is divided by 8?

   **(A)** 1 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 7

8. Mr. Garcia asked the members of his health class how many days last week they exercised for at least 30 minutes. The results are summarized in the following bar graph, where the heights of the bars represent the number of students.

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; size(8cm); void drawbar(real x, real h) { fill((x-0.15,0.5)–(x+0.15,0.5)–(x+0.15,h)–(x-0.15,h)–cycle,gray); } draw((0.5,0.5)–(7.5,0.5)–(7.5,5)–(0.5,5)–cycle); for (real i=1; i¡5; i=i+0.5) { draw((0.5,i)–(7.5,i),gray); } drawbar(1.0,1.0); drawbar(2.0,2.0); drawbar(3.0,1.5); drawbar(4.0,3.5); drawbar(5.0,4.5); drawbar(6.0,2.0); drawbar(7.0,1.5); for (int i=1; i¡8; ++i) { label(”$”+string(i)+”$”,(i,0.25)); } for (int i=1; i¡9; ++i) { label(”$”+string(i)+”$”,(0.5,0.5*(i+1)),W); } label(”Number of Days of Exercise”,(4,-0.1)); label(rotate(90)*”Number of Students”,(-0.1,2.75)); \{}end{asy} \{}end{center}

   What was the mean number of days of exercise last week, rounded to the nearest hundredth, reported by the students in Mr. Garcia's class?

   **(A)** 3.50 **(B)** 3.57 **(C)** 4.36 **(D)** 4.50 **(E)** 5.00

9. Jenica is tiling the floor of her 12 foot by 16 foot living room. She plans to place one-foot by one-foot square tiles to form a border along the edges of the room and to fill in the rest of the floor with two-foot by two-foot square tiles. How many tiles will she use?

   **(A)** 48 **(B)** 87 **(C)** 91 **(D)** 96 **(E)** 120

10. The *harmonic mean* of a set of non-zero numbers is the reciprocal of the average of the reciprocals of the numbers. What is the harmonic mean of 1, 2, and 4?

    **(A)** $\frac{3}{7}$ **(B)** $\frac{7}{12}$ **(C)** $\frac{12}{7}$ **(D)** $\frac{7}{4}$ **(E)** $\frac{7}{3}$

11. Abby, Bridget, and four of their classmates will be seated in two rows of three for a group picture, as shown.

    \{}begin{eqnarray*} \{}text{X}&\{}quad\{}text{X}\{}quad&\{}text{X} \{}\{} \{}text{X}&\{}quad\{}text{X}\{
    \{}end{eqnarray*}

    If the seating positions are assigned randomly, what is the probability that Abby and Bridget are adjacent to each other in the same row or the same column?

    **(A)** $\frac{1}{3}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{7}{15}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{2}{3}$

12. The clock in Sri's car, which is not accurate, gains time at a constant rate. One day as he begins shopping, he notes that his car clock and his watch (which is accurate) both say 12:00 noon. When he is done shopping, his watch says 12:30 and his car clock says 12:35. Later that day, Sri loses his watch. He looks at his car clock and it says 7:00. What is the actual time? **(A)** $5:50$ **(B)** $6:00$ **(C)** $6:30$ **(D)** $6:55$ **(E)** $8:10$

13. Laila took five math tests, each worth a maximum of 100 points. Laila's score on each test was an integer between 0 and 100, inclusive. Laila received the same score on the first four tests, and she received a higher score on the last test. Her average score on the five tests was 82. How many values are possible for Laila's score on the last test?

    **(A)** 4 **(B)** 5 **(C)** 9 **(D)** 10 **(E)** 18

14. Let $N$ be the greatest five-digit number whose digits have a product of 120. What is the sum of the digits of $N$?

    **(A)** 15 **(B)** 16 **(C)** 17 **(D)** 18 **(E)** 20

15. In the diagram below, a diameter of each of the two smaller circles is a radius of the larger circle. If the two smaller circles have a combined area of 1 square unit, then what is the area of the shaded region, in square units?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; size(4cm); filldraw(scale(2)*unitcircle,gray,black); filldraw(shift(-1,0)*unitcircle,white,black); filldraw(shift(1,0)*unitcircle,white,black); \{}end{asy} \{}end{center}

    **(A)** $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** 1 **(E)** $\frac{\pi}{2}$

16. Professor Chang has nine different language books lined up on a bookshelf: two Arabic, three German, and four Spanish. How many ways are there to arrange the nine books on the shelf keeping the Arabic books together and keeping the Spanish books together?

    **(A)** 1440 **(B)** 2880 **(C)** 5760 **(D)** $182,440$ **(E)** $362,880$

17. Bella begins to walk from her house toward her friend Ella's house. At the same time, Ella begins to ride her bicycle toward Bella's house. They each maintain a constant speed, and Ella rides 5 times as fast as Bella walks. The distance between their houses is 2 miles, which is $10,560$ feet, and Bella covers $2\frac{1}{2}$ feet with each step. How many steps will Bella take by the time she meets Ella?

    **(A)** 704 **(B)** 845 **(C)** 1056 **(D)** 1760 **(E)** 3520

18. How many positive factors does $23,232$ have?

    **(A)** 9 **(B)** 12 **(C)** 28 **(D)** 36 **(E)** 42

19. In a sign pyramid a cell gets a "+" if the two cells below it have the same sign, and it gets a "-" if the two cells below it have different signs. The diagram below illustrates a sign pyramid with four levels. How many possible ways are there to fill the four cells in the bottom row to produce a "+" at the top of the pyramid?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; unitsize(2cm); path box = (-0.5,-0.2)–(-0.5,0.2)–(0.5,0.2)–(0.5,-0.2)–cycle; draw(box); label("$+$",(0,0)); draw(shift(1,0)*box); label("$-$",(1,0)); draw(shift(2,0)*box); label("$+$",(2,0)); draw(shift(3,0)*box); label("$-$",(3,0)); draw(shift(0.5,0.4)*box);

label("$-$",(0.5,0.4)); draw(shift(1.5,0.4)*box); label("$-$",(1.5,0.4)); draw(shift(2.5,0.4)*box); label("$-$",(2.5,0.4)); draw(shift(1,0.8)*box); label("$+$",(1,0.8)); draw(shift(2,0.8)*box); label("$+$",(2,0.8)); draw(shift(1.5,1 label("$+$",(1.5,1.2)); \{}end{asy} \{}end{center}

**(A)** 2 **(B)** 4 **(C)** 8 **(D)** 12 **(E)** 16

20. In $\triangle ABC$, a point $E$ is on $\overline{AB}$ with $AE = 1$ and $EB = 2$. Point $D$ is on $\overline{AC}$ so that $\overline{DE} \parallel \overline{BC}$ and point $F$ is on $\overline{BC}$ so that $\overline{EF} \parallel \overline{AC}$. What is the ratio of the area of $CDEF$ to the area of $\triangle ABC$?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; size(7cm); pair A,B,C,DD,EE,FF; A = (0,0); B = (3,0); C = (0.5,2.5); EE = (1,0); DD = intersectionpoint(A–C,EE–EE+(C-B)); FF = intersectionpoint(B–C,EE–EE+(C-A)); draw(A–B–C–A–DD–EE–FF,black+1bp); label("$A$",A,S); label("$B$",B,S); label("$C$",C,N); label("$D$",DD,W); label("$E$",EE,S); label("$F$",FF,NE); label("$1$",(A+EE)/2,S); label("$2$",(EE+B)/2,S); \{}end{asy} \{}end{center}

**(A)** $\frac{4}{9}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{5}{9}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{2}{3}$

21. How many positive three-digit integers have a remainder of 2 when divided by 6, a remainder of 5 when divided by 9, and a remainder of 7 when divided by 11?

**(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

22. Point $E$ is the midpoint of side $\overline{CD}$ in square $ABCD$, and $\overline{BE}$ meets diagonal $\overline{AC}$ at $F$. The area of quadrilateral $AFED$ is 45. What is the area of $ABCD$?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; size(5cm); draw((0,0)–(6,0)–(6,6)–(0,6)–cycle); draw((0,6)–(6,0)); draw((3,0)–(6,6)); label("$A$",(0,6),NW); label("$B$",(6,6),NE); label("$C$",(6,0),SE); label("$D$",(0,0),SW); label("$E$",(3,0),S); label("$F$",(4,2),E); \{}end{asy} \{}end{center}

**(A)** 100 **(B)** 108 **(C)** 120 **(D)** 135 **(E)** 144

23. From a regular octagon, a triangle is formed by connecting three randomly chosen vertices of the octagon. What is the probability that at least one of the sides of the triangle is also a side of the octagon?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; size(3cm); pair A[]; for (int i=0; i¡9; ++i) { A[i] = rotate(22.5+45*i)*(1,0); } filldraw(A[0]–A[1]–A[2]–A[3]–A[4]–A[5]–A[6]–A[7]–cycle,gray,black); for (int i=0; i¡8; ++i) { dot(A[i]); } \{}end{asy} \{}end{center}

**(A)** $\frac{2}{7}$ **(B)** $\frac{5}{42}$ **(C)** $\frac{11}{14}$ **(D)** $\frac{5}{7}$ **(E)** $\frac{6}{7}$

24. In the cube $ABCDEFGH$ with opposite vertices $C$ and $E$, $J$ and $I$ are the midpoints of edges $\overline{FB}$ and $\overline{HD}$, respectively. Let $R$ be the ratio of the area of the cross-section $EJCI$ to the area of one of the faces of the cube. What is $R^2$?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; size(6cm); pair A,B,C,D,EE,F,G,H,I,J; C = (0,0); B = (-1,1); D = (2,0.5); A = B+D; G = (0,2); F = B+G; H = G+D; EE = G+B+D; I = (D+H)/2; J = (B+F)/2; filldraw(C–I–EE–J–cycle,lightgray,black); draw(C–D–H–EE–F–B–cycle); draw(G–F–G–C–G–H); draw(A–B,dashed); draw(A–EE,dashed); draw(A–D,dashed); dot(A); dot(B); dot(C); dot(D); dot(EE); dot(F); dot(G); dot(H); dot(I); dot(J); label("$A$",A,E); label("$B$",B,W); label("$C$",C,S); label("$D$",D,E); label("$E$",EE,N); label("$F$",F,W); label("$G$",G,N); label("$H$",H,E); label("$I$",I,E); label("$J$",J,W); \{}end{asy} \{}end{center}

**(A)** $\frac{5}{4}$ **(B)** $\frac{4}{3}$ **(C)** $\frac{3}{2}$ **(D)** $\frac{25}{16}$ **(E)** $\frac{9}{4}$

25. How many perfect cubes lie between $2^8 + 1$ and $2^{18} + 1$, inclusive?

**(A)** 4 **(B)** 9 **(C)** 10 **(D)** 57 **(E)** 58

# 2019 AMC 8 Problems

1. Ike and Mike go into a sandwich shop with a total of \$30.00 to spend. Sandwiches cost \$4.50 each and soft drinks cost \$1.00 each. Ike and Mike plan to buy as many sandwiches as they can, and use any remaining money to buy soft drinks. Counting both sandwiches and soft drinks, how many items will they buy?

   **(A)** 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

2. Three identical rectangles are put together to form rectangle $ABCD$, as shown in the figure below. Given that the length of the shorter side of each of the smaller rectangles is 5 feet, what is the area in square feet of rectangle $ABCD$?

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; draw((0,0)–(3,0)); draw((0,0)–(0,2)); draw((0,2)–(3,2)); draw((3,2)–(3,0)); dot((0,0)); dot((0,2)); dot((3,0)); dot((3,2)); draw((2,0)–(2,2)); draw((0,1)–(2,1)); label("A",(0,0),S); label("B",(3,0),S); label("C",(3,2),N); label("D",(0,2),N); \{}end{asy} \{}end{center}

   **(A)** 45 **(B)** 75 **(C)** 100 **(D)** 125 **(E)** 150

3. Which of the following is the correct order of the fractions $\frac{15}{11}$, $\frac{19}{15}$, and $\frac{17}{13}$, from least to greatest?

   **(A)** $\frac{15}{11} < \frac{17}{13} < \frac{19}{15}$ **(B)** $\frac{15}{11} < \frac{19}{15} < \frac{17}{13}$ **(C)** $\frac{17}{13} < \frac{19}{15} < \frac{15}{11}$ **(D)** $\frac{19}{15} < \frac{15}{11} < \frac{17}{13}$ **(E)** $\frac{19}{15} < \frac{17}{13} < \frac{15}{11}$

4. Quadrilateral $ABCD$ is a rhombus with perimeter 52 meters. The length of diagonal $\overline{AC}$ is 24 meters. What is the area in square meters of rhombus $ABCD$?

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; draw((-13,0)–(0,5)); draw((0,5)–(13,0)); draw((13,0)–(0,-5)); draw((0,-5)–(-13,0)); dot((-13,0)); dot((0,5)); dot((13,0)); dot((0,-5)); label("A",(-13,0),W); label("B",(0,5),N); label("C",(13,0),E); label("D",(0,-5),S); \{}end{asy} \{}end{center}

   **(A)** 60 **(B)** 90 **(C)** 105 **(D)** 120 **(E)** 144

5. A tortoise challenges a hare to a race. The hare eagerly agrees and quickly runs ahead, leaving the slow-moving tortoise behind. Confident that he will win, the hare stops to take a nap. Meanwhile, the tortoise walks at a slow steady pace for the entire race. The hare awakes and runs to the finish line, only to find the tortoise already there. Which of the following graphs matches the description of the race, showing the distance $d$ traveled by the two animals over time $t$ from start to finish?

6. There are 81 grid points (uniformly spaced) in the square shown in the diagram below, including the points on the edges. Point $P$ is in the center of the square. Given that point $Q$ is randomly chosen among the other 80 points, what is the probability that the line $PQ$ is a line of symmetry for the square?

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; draw((0,0)–(0,8)); draw((0,8)–(8,8)); draw((8,8)–(8,0)); draw((8,0)–(0,0)); dot((0,0)); dot((0,1)); dot((0,2)); dot((0,3)); dot((0,4)); dot((0,5)); dot((0,6)); dot((0,7)); dot((0,8));

   dot((1,0)); dot((1,1)); dot((1,2)); dot((1,3)); dot((1,4)); dot((1,5)); dot((1,6)); dot((1,7)); dot((1,8));

   dot((2,0)); dot((2,1)); dot((2,2)); dot((2,3)); dot((2,4)); dot((2,5)); dot((2,6)); dot((2,7)); dot((2,8));

   dot((3,0)); dot((3,1)); dot((3,2)); dot((3,3)); dot((3,4)); dot((3,5)); dot((3,6)); dot((3,7)); dot((3,8));

   dot((4,0)); dot((4,1)); dot((4,2)); dot((4,3)); dot((4,4)); dot((4,5)); dot((4,6)); dot((4,7)); dot((4,8));

   dot((5,0)); dot((5,1)); dot((5,2)); dot((5,3)); dot((5,4)); dot((5,5)); dot((5,6)); dot((5,7)); dot((5,8));

   dot((6,0)); dot((6,1)); dot((6,2)); dot((6,3)); dot((6,4)); dot((6,5)); dot((6,6)); dot((6,7)); dot((6,8));

   dot((7,0)); dot((7,1)); dot((7,2)); dot((7,3)); dot((7,4)); dot((7,5)); dot((7,6)); dot((7,7)); dot((7,8));

   dot((8,0)); dot((8,1)); dot((8,2)); dot((8,3)); dot((8,4)); dot((8,5)); dot((8,6)); dot((8,7)); dot((8,8)); label("P",(4,4),NE); \{}end{asy} \{}end{center}

   **(A)** $\frac{1}{5}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{2}{5}$ **(D)** $\frac{9}{20}$ **(E)** $\frac{1}{2}$

7. Shauna takes five tests, each worth a maximum of 100 points. Her scores on the first three tests are 76, 94, and 87. In order to average 81 for all five tests, what is the lowest score she could earn on one of the other two tests?

   **(A)** 48 **(B)** 52 **(C)** 66 **(D)** 70 **(E)** 74

8. Gilda has a bag of marbles. She gives 20% of them to her friend Pedro. Then Gilda gives 10% of what is left to another friend, Ebony. Finally, Gilda gives 25% of what is now left in the bag to her brother Jimmy. What percentage of her original bag of marbles does Gilda have left for herself?

   **(A)** 20 **(B)** $33\frac{1}{3}$ **(C)** 38 **(D)** 45 **(E)** 54

9. Alex and Felicia each have cats as pets. Alex buys cat food in cylindrical cans that are 6 cm in diameter and 12 cm high. Felicia buys cat food in cylindrical cans that are 12 cm in diameter and 6 cm high. What is the ratio of the volume of one of Alex's cans to the volume one of Felicia's cans?

   **(A)** 1 : 4 **(B)** 1 : 2 **(C)** 1 : 1 **(D)** 2 : 1 **(E)** 4 : 1

10. The diagram shows the number of students at soccer practice each weekday during last week. After computing the mean and median values, Coach discovers that there were actually 21 participants on Wednesday. Which of the following statements describes the change in the mean and median after the correction is made?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; unitsize(2mm); defaultpen(fontsize(8bp)); real d = 5; real t = 0.7; real r; int[] num = {20,26,16,22,16}; string[] days = {"Monday","Tuesday","Wednesday","Thursd for (int i=0; i¡30; i=i+2) { draw((i,0)–(i,-5*d),gray); }for (int i=0; i¡5; ++i) { r = -1*(i+0.5)*d; fill((0,r-t)–(0,r+t)–(num[i],r+t)–(num[i],r-t)–cycle,gray); label(days[i],(-1,r),W); }for(int i=0; i¡32; i=i+4) { label(string(i),(i,1)); }label("Number of students at soccer practice",(14,3.5)); \{}end{asy} \{}end{center}

    **(A)** The mean increases by 1 and the median does not change.

    **(B)** The mean increases by 1 and the median increases by 1.

    **(C)** The mean increases by 1 and the median increases by 5.

    **(D)** The mean increases by 5 and the median increases by 1.

    **(E)** The mean increases by 5 and the median increases by 5.

11. The third-grade class at Lincoln Elementary School has 93 students. Each student takes a math class or a foreign language class or both. There are 70 third graders taking a math class, and there are 54 third graders taking a foreign language class. How many third graders take "only" a math class and "not" a foreign language class?

    **(A)** 16 **(B)** 23 **(C)** 31 **(D)** 39 **(E)** 70

12. The faces of a cube are painted in six different colors: red ($R$), white ($W$), green ($G$), brown ($B$), aqua ($A$), and purple ($P$). Three views of the cube are shown below. What is the color of the face opposite the aqua face?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; unitsize(2cm); pair x, y, z, trans; int i;

    x = dir(-5); y = (0.6,0.5); z = (0,1); trans = (2,0);

    for (i = 0; i ¡= 2; ++i) { draw(shift(i*trans)*((0,0)–x–(x + y)–(x + y + z)–(y + z)–z–cycle)); draw(shift(i*trans)*((x + z)–x)); draw(shift(i*trans)*((x + z)–(x + y + z))); draw(shift(i*trans)*((x + z)–z)); }

    label(rotate(-3)*"$R$", (x + z)/2); label(rotate(-5)*slant(0.5)*"$B$", ((x + z) + (y + z))/2); label(rotate(35)*slant(0.5) ((x + z) + (x + y))/2);

    label(rotate(-3)*"$W$", (x + z)/2 + trans); label(rotate(50)*slant(-1)*"$B$", ((x + z) + (y + z))/2 + trans); label(rotate(35)*slant(0.5)*"$R$", ((x + z) + (x + y))/2 + trans);

    label(rotate(-3)*"$P$", (x + z)/2 + 2*trans); label(rotate(-5)*slant(0.5)*"$R$", ((x + z) + (y + z))/2 + 2*trans); label(rotate(-85)*slant(-1)*"$G$", ((x + z) + (x + y))/2 + 2*trans); \{}end{asy} \{}end{center}

    **(A)** red **(B)** white **(C)** green **(D)** brown **(E)** purple

13. A "palindrome" is a number that has the same value when read from left to right or from right to left. (For example, 12321 is a palindrome.) Let $N$ be the least three-digit integer which is not a palindrome but which is the sum of three distinct two-digit palindromes. What is the sum of the digits of $N$?

    **(A)** 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

14. Isabella has 6 coupons that can be redeemed for free ice cream cones at Pete's Sweet Treats. In order to make the coupons last, she decides that she will redeem one every 10 days until she has used them all. She knows that Pete's is closed on Sundays, but as she circles the 6 dates on her calendar, she realizes that no circled date falls on a Sunday. On what day of the week does Isabella redeem her first coupon?

**(A)** Monday **(B)** Tuesday **(C)** Wednesday **(D)** Thursday **(E)** Friday

15. On a beach, 50 people are wearing sunglasses and 35 people are wearing caps. Some people are wearing both sunglasses and caps. If one of the people wearing a cap is selected at random, the probability that this person is also wearing sunglasses is $\frac{2}{5}$. If instead, someone wearing sunglasses is selected at random, what is the probability that this person is also wearing a cap?

**(A)** $\frac{14}{85}$ **(B)** $\frac{7}{25}$ **(C)** $\frac{2}{5}$ **(D)** $\frac{4}{7}$ **(E)** $\frac{7}{10}$

16. Qiang drives 15 miles at an average speed of 30 miles per hour. How many additional miles will he have to drive at 55 miles per hour to average 50 miles per hour for the entire trip?

**(A)** 45 **(B)** 62 **(C)** 90 **(D)** 110 **(E)** 135

17. What is the value of the product

$$\left(\frac{1\cdot 3}{2\cdot 2}\right)\left(\frac{2\cdot 4}{3\cdot 3}\right)\left(\frac{3\cdot 5}{4\cdot 4}\right)\cdots\left(\frac{97\cdot 99}{98\cdot 98}\right)\left(\frac{98\cdot 100}{99\cdot 99}\right)?$$

**(A)** $\frac{1}{2}$ **(B)** $\frac{50}{99}$ **(C)** $\frac{9800}{9801}$ **(D)** $\frac{100}{99}$ **(E)** 50

18. The faces of each of two fair dice are numbered 1, 2, 3, 5, 7, and 8. When the two dice are tossed, what is the probability that their sum will be an even number?

**(A)** $\frac{4}{9}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{5}{9}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{2}{3}$

19. In a tournament there are six teams that play each other twice. A team earns 3 points for a win, 1 point for a draw, and 0 points for a loss. After all the games have been played it turns out that the top three teams earned the same number of total points. What is the greatest possible number of total points for each of the top three teams?

**(A)** 22 **(B)** 23 **(C)** 24 **(D)** 26 **(E)** 30

== Problem 20 == How many different real numbers $x$ satisfy the equation

$$(x^2-5)^2=16?$$

**(A)** 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 8

20. What is the area of the triangle formed by the lines $y=5$, $y=1+x$, and $y=1-x$?

**(A)** 4 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 16

21. A store increased the original price of a shirt by a certain percent and then decreased the new price by the same amount. Given that the resulting price was 84% of the original price, by what percent was the price increased and decreased?

**(A)** 16 **(B)** 20 **(C)** 28 **(D)** 36 **(E)** 40

22. After Euclid High School's last basketball game, it was determined that $\frac{1}{4}$ of the team's points were scored by Alexa and $\frac{2}{7}$ were scored by Brittany. Chelsea scored 15 points. None of the other 7 team members scored more than 2 points. What was the total number of points scored by the other 7 team members?

**(A)** 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

23. In triangle $ABC$, point $D$ divides side $\overline{AC}$ so that $AD:DC=1:2$. Let $E$ be the midpoint of $\overline{BD}$ and let $F$ be the point of intersection of line $BC$ and line $AE$. Given that the area of $\triangle ABC$ is 360, what is the area of $\triangle EBF$?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; unitsize(2cm); pair A,B,C,DD,EE,FF; B = (0,0); C = (3,0); A = (1.2,1.7); DD = (2/3)*A+(1/3)*C; EE = (B+DD)/2; FF = intersectionpoint(B–C,A–A+2*(EE-A)); draw(A–B–C–cycle); draw(A–FF); draw(B–DD);dot(A); label("$A$",A,N); dot(B); label("$B$", B,SW);dot(C); label("$C$",C,SE); dot(DD); label("$D$",DD,NE); dot(EE); label("$E$",EE,NW); dot(FF); label("$F$",FF,S); \{}end{asy} \{}end{center}

**(A)** 24 **(B)** 30 **(C)** 32 **(D)** 36 **(E)** 40

24. Alice has 24 apples. In how many ways can she share them with Becky and Chris so that each of the three people has at least two apples?

    **(A)** 105 **(B)** 114 **(C)** 190 **(D)** 210 **(E)** 380

# 2020 AMC 8 Problems

1. Luka is making lemonade to sell at a school fundraiser. His recipe requires 4 times as much water as sugar and twice as much sugar as lemon juice. He uses 3 cups of lemon juice. How many cups of water does he need?

   **(A)** 6 **(B)** 8 **(C)** 12 **(D)** 18 **(E)** 24

2. Four friends do yardwork for their neighbors over the weekend, earning $\$15, \$20, \$25$, and $\$40$, respectively. They decide to split their earnings equally among themselves. In total how much will the friend who earned $\$40$ give to the others?

   **(A)** $\$5$ **(B)** $\$10$ **(C)** $\$15$ **(D)** $\$20$ **(E)** $\$25$

3. Carrie has a rectangular garden that measures 6 feet by 8 feet. She plants the entire garden with strawberry plants. Carrie is able to plant 4 strawberry plants per square foot, and she harvests an average of 10 strawberries per plant. How many strawberries can she expect to harvest? **(A)** 560 **(B)** 960 **(C)** 1120 **(D)** 1920

4. Three hexagons of increasing size are shown below. Suppose the dot pattern continues so that each successive hexagon contains one more band of dots. How many dots are in the next hexagon?

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; // diagram by SirCalcsALot, edited by MRENTHUSIASM size(250); path p = scale(0.8)*unitcircle; pair[] A; pen grey1 = rgb(100/256, 100/256, 100/256); pen grey2 = rgb(183/256, 183/256, 183/256); for (int i=0; i¡7; ++i) { A[i] = rotate(60*i)*(1,0);} path hex = A[0]–A[1]–A[2]–A[3]–A[4]–A[5]–cycle; fill(p,grey1); draw(scale(1.25)*hex,black+linewidth(1.25)); pair S = 6A[0]+2A[1]; fill(shift(S)*p,grey1); for (int i=0; i¡6; ++i) { fill(shift(S+2*A[i])*p,grey2);} draw(shift(S)*scale(3. pair T = 16A[0]+4A[1]; fill(shift(T)*p,grey1); for (int i=0; i¡6; ++i) { fill(shift(T+2*A[i])*p,grey2); fill(shift(T+4*A[i])* fill(shift(T+2*A[i]+2*A[i+1])*p,grey1); } draw(shift(T)*scale(5.25)*hex,black+linewidth(1.25)); \{}end{asy} \{}end{center}

   **(A)** 35 **(B)** 37 **(C)** 39 **(D)** 43 **(E)** 49

5. Three fourths of a pitcher is filled with pineapple juice. The pitcher is emptied by pouring an equal amount of juice into each of 5 cups. What percent of the total capacity of the pitcher did each cup receive?

   **(A)** 5 **(B)** 10 **(C)** 15 **(D)** 20 **(E)** 25

6. Aaron, Darren, Karen, Maren, and Sharon rode on a small train that has five cars that seat one person each. Maren sat in the last car. Aaron sat directly behind Sharon. Darren sat in one of the cars in front of Aaron. At least one person sat between Karen and Darren. Who sat in the middle car?

   **(A)** Aaron **(B)** Darren **(C)** Karen **(D)** Maren **(E)** Sharon

7. How many integers between 2020 and 2400 have four distinct digits arranged in increasing order? (For example, 2347 is one integer.)

   **(A)** 9 **(B)** 10 **(C)** 15 **(D)** 21 **(E)** 28

8. Ricardo has 2020 coins, some of which are pennies (1-cent coins) and the rest of which are nickels (5-cent coins). He has at least one penny and at least one nickel. What is the difference in cents between the greatest possible and least possible amounts of money that Ricardo can have?

   **(A)** 8062 **(B)** 8068 **(C)** 8072 **(D)** 8076 **(E)** 8082

9. Akash's birthday cake is in the form of a $4 \times 4 \times 4$ inch cube. The cake has icing on the top and the four side faces, and no icing on the bottom. Suppose the cake is cut into 64 smaller cubes, each measuring $1 \times 1 \times 1$ inch, as shown below. How many small pieces will have icing on exactly two sides?

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; import three; currentprojection=orthographic(1.75,7,2);

   //++++ edit colors, names are self-explainatory ++++ //pen top=rgb(27/255, 135/255, 212/255); //pen right=rgb(254/255,245/255,182/255); //pen left=rgb(153/255,200/255,99/255); pen top = rgb(170/255, 170/255, 170/255); pen left = rgb(81/255, 81/255, 81/255); pen right = rgb(165/255, 165/255, 165/255); pen edges=black; int max_side = 4; //++++++++++++++++++++++++++++++++++++++++

   path3 leftface=(1,0,0)–(1,1,0)–(1,1,1)–(1,0,1)–cycle; path3 rightface=(0,1,0)–(1,1,0)–(1,1,1)–(0,1,1)–cycle; path3 topface=(0,0,1)–(1,0,1)–(1,1,1)–(0,1,1)–cycle;

for(int i=0; i¡max_side; ++i){ for(int j=0; j¡max_side; ++j){

draw(shift(i,j,-1)*surface(topface),top); draw(shift(i,j,-1)*topface,edges);

draw(shift(i,-1,j)*surface(rightface),right); draw(shift(i,-1,j)*rightface,edges);

draw(shift(-1,j,i)*surface(leftface),left); draw(shift(-1,j,i)*leftface,edges);

} }

picture CUBE; draw(CUBE,surface(leftface),left,nolight); draw(CUBE,surface(rightface),right,nolight); draw(CUBE,sur
draw(CUBE,topface,edges); draw(CUBE,leftface,edges); draw(CUBE,rightface,edges);

int[][] heights = {{4,4,4,4},{4,4,4,4},{4,4,4,4},{4,4,4,4}};

for (int i = 0; i ¡ max_side; ++i) { for (int j = 0; j ¡ max_side; ++j) { for (int k = 0; k ¡ min(heights[i][j], max_side); ++k) { add(shift(i,j,k)*CUBE); } } } \{}end{asy} \{}end{center}

**(A)** 12 **(B)** 16 **(C)** 18 **(D)** 20 **(E)** 24

10. Zara has a collection of 4 marbles: an Aggie, a Bumblebee, a Steelie, and a Tiger. She wants to display them in a row on a shelf, but does not want to put the Steelie and the Tiger next to one another. In how many ways can she do this?

    **(A)** 6 **(B)** 8 **(C)** 12 **(D)** 18 **(E)** 24

11. After school, Maya and Naomi headed to the beach, 6 miles away. Maya decided to bike while Naomi took a bus. The graph below shows their journeys, indicating the time and distance traveled. What was the difference, in miles per hour, between Naomi's and Maya's average speeds?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; // diagram by SirCalcsALot unitsize(1.25cm); dotfactor = 10; pen shortdashed=linetype(new real[] {2.7,2.7});

    for (int i = 0; i ¡ 6; ++i) { for (int j = 0; j ¡ 6; ++j) { draw((i,0)–(i,6), grey); draw((0,j)–(6,j), grey); } }

    for (int i = 1; i ¡= 6; ++i) { draw((-0.1,i)–(0.1,i),linewidth(1.25)); draw((i,-0.1)–(i,0.1),linewidth(1.25)); label(string(5*i), (i,0), 2*S); label(string(i), (0, i), 2*W); }

    draw((0,0)–(0,6)–(6,6)–(6,0)–(0,0)–cycle,linewidth(1.25));

    label(rotate(90) * "Distance (miles)", (-0.5,3), W); label("Time (minutes)", (3,-0.5), S);

    dot("Naomi", (2,6), 3*dir(305)); dot((6,6));

    label("Maya", (4.45,3.5));

    draw((0,0)–(1.15,1.3)–(1.55,1.3)–(3.15,3.2)–(3.65,3.2)–(5.2,5.2)–(5.4,5.2)–(6,6),linewidth(1.35)); draw((0,0)–(0.4,0.1)–(1.15,3.7)–(1.6,3.7)–(2,6),linewidth(1.35)+shortdashed); \{}end{asy} \{}end{center}

    **(A)** 6 **(B)** 12 **(C)** 18 **(D)** 20 **(E)** 24

12. For a positive integer $n$, the factorial notation $n!$ represents the product of the integers from $n$ to 1. (For example, $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$.) What value of $N$ satisfies the following equation?

    $$5! \cdot 9! = 12 \cdot N!$$

    **(A)** 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

13. Jamal has a drawer containing 6 green socks, 18 purple socks, and 12 orange socks. After adding more purple socks, Jamal noticed that there is now a 60% chance that a sock randomly selected from the drawer is purple. How many purple socks did Jamal add?

    **(A)** 6 **(B)** 9 **(C)** 12 **(D)** 18 **(E)** 24

14. There are 20 cities in the County of Newton. Their populations are shown in the bar chart below. The average population of all the cities is indicated by the horizontal dashed line. Which of the following is closest to the total population of all 20 cities?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; // made by SirCalcsALot

    size(300);

    pen shortdashed=linetype(new real[] {6,6});

    for (int i = 2000; i ¡ 9000; i = i + 2000) { draw((0,i)–(11550,i), linewidth(0.5)+1.5*grey); label(string(i), (0,i), W); }

for (int i = 500; i ¡ 9300; i=i+500) { draw((0,i)–(150,i),linewidth(1.25)); if (i % 2000 == 0) { draw((0,i)–(250,i),linewidth(1.25)); } }

int[] data = {8750, 3800, 5000, 2900, 6400, 7500, 4100, 1400, 2600, 1470, 2600, 7100, 4070, 7500, 7000, 8100, 1900, 1600, 5850, 5750}; int data_length = 20;

int r = 550; for (int i = 0; i ¡ data_length; ++i) { fill(((i+1)*r,0)–((i+1)*r, data[i])–((i+1)*r,0)–((i+1)*r, data[i])–((i+1)*r,0)–((i+1)*r, data[i])–((i+2)*r-100, data[i])–((i+2)*r-100,0)–cycle, 1.5*grey); draw(((i+1)*r,0)–((i+1)*r, data[i])–((i+1)*r,0)–((i+1)*r, data[i])–((i+1)*r,0)–((i+1)*r, data[i])–((i+2)*r-100, data[i])–((i+2)*r-100,0)); }

draw((0,4750)–(11450,4750),shortdashed);

label("Cities", (11450*0.5,0), S); label(rotate(90)*"Population", (0,9000*0.5), 10*W);

// axis draw((0,0)–(0,9300), linewidth(1.25)); draw((0,0)–(11550,0), linewidth(1.25)); \{}end{asy} \{}end{center}

**(A)** 65,000 **(B)** 75,000 **(C)** 85,000 **(D)** 95,000 **(E)** 105,000

15. Suppose 15% of $x$ equals 20% of $y$. What percentage of $x$ is $y$?

**(A)** 5 **(B)** 35 **(C)** 75 **(D)** $133\frac{1}{3}$ **(E)** 300

16. Each of the points $A, B, C, D, E,$ and $F$ in the figure below represents a different digit from 1 to 6. Each of the five lines shown passes through some of these points. The digits along each line are added to produce five sums, one for each line. The total of the five sums is 47. What is the digit represented by B?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; // made by SirCalcsALot

size(200); dotfactor = 10;

pair p1 = (-28,0); pair p2 = (-111,213); draw(p1–p2,linewidth(1));

pair p3 = (-160,0); pair p4 = (-244,213); draw(p3–p4,linewidth(1));

pair p5 = (-316,0); pair p6 = (-67,213); draw(p5–p6,linewidth(1));

pair p7 = (0, 68); pair p8 = (-350,10); draw(p7–p8,linewidth(1));

pair p9 = (0, 150); pair p10 = (-350, 62); draw(p9–p10,linewidth(1));

pair A = intersectionpoint(p1–p2, p5–p6); dot("$A$", A, 2*W);

pair B = intersectionpoint(p5–p6, p3–p4); dot("$B$", B, 2*WNW);

pair C = intersectionpoint(p7–p8, p5–p6); dot("$C$", C, 1.5*NW);

pair D = intersectionpoint(p3–p4, p7–p8); dot("$D$", D, 2*NNE);

pair EE = intersectionpoint(p1–p2, p7–p8); dot("$E$", EE, 2*NNE);

pair F = intersectionpoint(p1–p2, p9–p10); dot("$F$", F, 2*NNE); \{}end{asy} \{}end{center}

**(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

17. How many factors of 2020 have more than 3 factors? (As an example, 12 has 6 factors, namely $1, 2, 3, 4, 6,$ and 12.)

**(A)** 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

18. Rectangle $ABCD$ is inscribed in a semicircle with diameter $\overline{FE}$, as shown in the figure. Let $DA = 16$, and let $FD = AE = 9$. What is the area of $ABCD$?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; // diagram by SirCalcsALot draw(arc((0,0),17,180,0)); draw((-17,0)–(17,0)); fill((-8,0)–(-8,15)–(8,15)–(8,0)–cycle, 1.5*grey); draw((-8,0)–(-8,15)–(8,15)–(8,0)–cycle); dot("$A$",(8,0), 1.25*S); dot("$B$",(8,15), 1.25*N); dot("$C$",(-8,15), 1.25*N); dot("$D$",(-8,0), 1.25*S); dot("$E$",(17,0), 1.25*S); dot("$F$",(-17,0), 1.25*S); label("$16$",(0,0),N); label("$9$",(12.5,0),N); label("$9$",(-12.5,0),N); \{}end{asy} \{}end{center}

**(A)** 240 **(B)** 248 **(C)** 256 **(D)** 264 **(E)** 272

19. A number is called flippy if its digits alternate between two distinct digits. For example, 2020 and 37373 are flippy, but 3883 and 123123 are not. How many five-digit flippy numbers are divisible by 15?

**(A)** 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 8

20. A scientist walking through a forest recorded as integers the heights of 5 trees standing in a row. She observed that each tree was either twice as tall or half as tall as the one to its right. Unfortunately some of her data was lost when rain fell on her notebook. Her notes are shown below, with blanks indicating the missing numbers. Based on her observations, the scientist was able to reconstruct the lost data. What was the average height of the trees, in meters?

| | |
|---|---|
| Tree 1 | ___ meters |
| Tree 2 | 11 meters |
| Tree 3 | ___ meters |
| Tree 4 | ___ meters |
| Tree 5 | ___ meters |
| Average height | ___ .2 meters |

**(A)** 22.2 **(B)** 24.2 **(C)** 33.2 **(D)** 35.2 **(E)** 37.2

21. A game board consists of 64 squares that alternate in color between black and white. The figure below shows square $P$ in the bottom row and square $Q$ in the top row. A marker is placed at $P$. A step consists of moving the marker onto one of the adjoining white squares in the row above. How many 7-step paths are there from $P$ to $Q$? (The figure shows a sample path.)

\{}begin{center} \{}begin{asy} import olympiad; import cse5; // diagram by SirCalcsALot size(200); int[] x = {6, 5, 4, 5, 6, 5, 6}; int[] y = {1, 2, 3, 4, 5, 6, 7}; int N = 7; for (int i = 0; i ¡ 8; ++i) { for (int j = 0; j ¡ 8; ++j) { draw((i,j)–(i+1,j)–(i+1,j+1)–(i,j+1)–(i,j)); if ((i+j) % 2 == 0) { filldraw((i,j)–(i+1,j)–(i+1,j+1)–(i,j+1)–(i,j)–cycle,black); } } } for (int i = 0; i ¡ N; ++i) { draw(circle((x[i],y[i])+(0.5,0.5),0.35),grey); } label("$P$", (5.5, 0.5)); label("$Q$", (6.5, 7.5)); \{}end{asy} \{}end{center}

**(A)** 28 **(B)** 30 **(C)** 32 **(D)** 33 **(E)** 35

22. When a positive integer $N$ is fed into a machine, the output is a number calculated according to the rule shown below.

\{}begin{center} \{}begin{asy} import olympiad; import cse5; size(300); defaultpen(linewidth(0.8)+fontsize(13)); real r = 0.05; draw((0.9,0)–(3.5,0),EndArrow(size=7)); filldraw((4,2.5)–(7,2.5)–(7,-2.5)–(4,-2.5)–cycle,gray(0.65)); fill(circle((5.5,1.25),0.8),white); fill(circle((5.5,1.25),0.5),gray(0.65)); fill((4.3,-r)–(6.7,-r)–(6.7,-1-r)–(4.3,-1-r)–cycle,white); fill((4.3,-1.25+r)–(6.7,-1.25+r)–(6.7,-2.25+r)–(4.3,-2.25+r)–cycle,white); fill((4.6,-0.25-r)–(6.4,-0.25-r)–(6.4,-0.75-r)–(4.6,-0.75-r)–cycle,gray(0.65)); fill((4.6,-1.5+r)–(6.4,-1.5+r)–(6.4,-2+r)–(4.6,-2+r)–cycle,gray(0.65)); label("$N$",(0.45,0)); draw((7.5,1.25)–(11.25,1.25),EndArrow(size=7)); draw((7.5,-1.25)–(11.25,-1.25),EndArrow(size=7)); label("if $N$ is even",(9.25,1.25),N); label("if $N$ is odd",(9.25,-1.25),N); label("$\{}frac N2$",(12,1.25)); label("$3N+1$",(12.6,-1.25)); \{}end{asy} \{}end{center}

For example, starting with an input of $N = 7$, the machine will output $3 \cdot 7 + 1 = 22$. Then if the output is repeatedly inserted into the machine five more times, the final output is 26.

$$7 \to 22 \to 11 \to 34 \to 17 \to 52 \to 26$$

When the same 6-step process is applied to a different starting value of $N$, the final output is 1. What is the sum of all such integers $N$?

$$N \to \underline{\quad} \to \underline{\quad} \to \underline{\quad} \to \underline{\quad} \to \underline{\quad} \to 1$$

**(A)** 73 **(B)** 74 **(C)** 75 **(D)** 82 **(E)** 83

23. Five different awards are to be given to three students. Each student will receive at least one award. In how many different ways can the awards be distributed?

**(A)** 120 **(B)** 150 **(C)** 180 **(D)** 210 **(E)** 240

24. A large square region is paved with $n^2$ gray square tiles, each measuring $s$ inches on a side. A border $d$ inches wide surrounds each tile. The figure below shows the case for $n = 3$. When $n = 24$ , the 576 gray tiles cover 64% of the area of the large square region. What is the ratio $\frac{d}{s}$ for this larger value of $n$?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; draw((0,0)–(13,0)–(13,13)–(0,13)–cycle); filldraw((1,1)–(4,1)–(4,4)–(1,4)–cycle, mediumgray); filldraw((1,5)–(4,5)–(4,8)–(1,8)–cycle, mediumgray); filldraw((1,9)–(4,9)–(4,12)–(1,12)–cycle, mediumgray); filldraw((5,1)–(8,1)–(8,4)–(5,4)–cycle, mediumgray); filldraw((5,5)–(8,5)–(8,8)–(5,8)–cycle, mediumgray); filldraw((5,9)–(8,9)–(8,12)–(5,12)–cycle, mediumgray); filldraw((9,1)–(12,1)–(12,4)–(9,4)–cycle, mediumgray); filldraw((9,5)–(12,5)–(12,8)–(9,8)–cycle, mediumgray); filldraw((9,9)–(12,9)–(12,12)–(9,12)–cycle, mediumgray); \{}end{asy} \{}end{center}

**(A)** $\frac{6}{25}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{9}{25}$ **(D)** $\frac{7}{16}$ **(E)** $\frac{9}{16}$

25. Rectangles $R_1$ and $R_2$, and squares $S_1$, $S_2$, and $S_3$, shown below, combine to form a rectangle that is 3322 units wide and 2020 units high. What is the side length of $S_2$ in units?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; draw((0,0)–(5,0)–(5,3)–(0,3)–(0,0)); draw((3,0)–(3,1)–(0,1)); draw((3,1)–(3,2)–(5,2)); draw((3,2)–(2,2)–(2,1)–(2,3)); label("\$R_1\$",(3/2,1/2)); label("\$S_3\$",(4,1)); label("\$S_2\$",(5/2,3/2)); label("\$S_1\$",(1,2)); label("\$R_2\$",(7/2,5/2)); \{}end{asy} \{}end{center}

**(A)** 651 **(B)** 655 **(C)** 656 **(D)** 662 **(E)** 666

# 2022 AMC 8 Problems

1. The Math Team designed a logo shaped like a multiplication symbol, shown below on a grid of 1-inch squares. What is the area of the logo in square inches?

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; defaultpen(linewidth(0.5)); size(5cm); defaultpen(fontsize(14pt)); label("$\{}textbf{Math}$", (2.1,3.7)–(3.9,3.7)); label("$\{}textbf{Team}$", (2.1,3)–(3.9,3)); filldraw((1,2)–(2,1)–(3,2)–(4,1)–(5,2)–(4,3)–(5,4)–(4,5)–(3,4)–(2,5)–(1,4)–(2,3)–(1,2)–cycle, mediumgray*0.5 + lightgray*0.5);

   draw((0,0)–(6,0), gray); draw((0,1)–(6,1), gray); draw((0,2)–(6,2), gray); draw((0,3)–(6,3), gray); draw((0,4)–(6,4), gray); draw((0,5)–(6,5), gray); draw((0,6)–(6,6), gray);

   draw((0,0)–(0,6), gray); draw((1,0)–(1,6), gray); draw((2,0)–(2,6), gray); draw((3,0)–(3,6), gray); draw((4,0)–(4,6), gray); draw((5,0)–(5,6), gray); draw((6,0)–(6,6), gray); \{}end{asy} \{}end{center}

   **(A)** 10 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

2. Consider these two operations:

   $$a \blacklozenge b = a^2 - b^2$$
   $$a \bigstar b = (a-b)^2$$

   What is the value of $(5 \blacklozenge 3) \bigstar 6$?

   **(A)** $-20$ **(B)** 4 **(C)** 16 **(D)** 100 **(E)** 220

3. When three positive integers $a$, $b$, and $c$ are multiplied together, their product is 100. Suppose $a < b < c$. In how many ways can the numbers be chosen?

   **(A)** 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

4. The letter \{}textbf{M} in the figure below is first reflected over the line $q$ and then reflected over the line $p$. What is the resulting image?

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; // pog diagram usepackage("newtxtext"); size(3cm); draw((-1,0)–(1,0)); draw((0,-1)–(0,1)); label("$\{}textbf{\{}textsf{M}}$",(0.25,0.6)); draw((-0.8,-0.8)–(0.8,0.8),linewidth(1.1)); label("$p$", (-1,0),NE); label("$q$", (-0.75,-0.75), N*1.5); \{}end{asy} \{}end{center}

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; // pog diagram usepackage("newtxtext"); size(12.5cm); draw((-1,0)–(1,0)); draw((0,-1)–(0,1)); label(rotate(90)*"$\{}textbf{\{}textsf{M}}$",(0.6,-0.25)); draw((-0.8,-0.8)–(0.8,0.8),linewidth(1.1)); label("$\{}textbf{(A)}$",(-1,1),W); draw((2,0)–(4,0)); draw((3,-1)–(3,1)); label(rotate(270)*"$\{}textbf{\{}textsf{M}}$",(2.8,0.7)); draw((2.2,-0.8)–(3.8,0.8),linewidth(1.1)); label("$\{}textbf{(B)}$",(2,1),W); draw((5,0)–(7,0)); draw((6,-1)–(6,1)); label(rotate(90)*"$\{}textbf{\{}textsf{M}}$ draw((5.2,-0.8)–(6.8,0.8),linewidth(1.1)); label("$\{}textbf{(C)}$",(5,1),W); draw((-1,-2.5)–(1,-2.5)); draw((0,-3.5)–(0,-1.5)); label(rotate(180)*"$\{}textbf{\{}textsf{M}}$",(-0.25,-3.1)); draw((-0.8,-3.3)–(0.8,-1.7),linewidth(1.1)); label("$\{}textbf{(D)}$",(-1,-1.5),W); draw((2,-2.5)–(4,-2.5)); draw((3,-3.5)–(3,-1.5)); label(rotate(270)*"$\{}textbf{\ 2.75)); draw((2.2,-3.3)–(3.8,-1.7),linewidth(1.1)); label("$\{}textbf{(E)}$",(2,-1.5),W); \{}end{asy} \{}end{center}

5. Anna and Bella are celebrating their birthdays together. Five years ago, when Bella turned 6 years old, she received a newborn kitten as a birthday present. Today the sum of the ages of the two children and the kitten is 30 years. How many years older than Bella is Anna?

   **(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

6. Three positive integers are equally spaced on a number line. The middle number is 15, and the largest number is 4 times the smallest number. What is the smallest of these three numbers?

   **(A)** 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

7. When the World Wide Web first became popular in the 1990s, download speeds reached a maximum of about 56 kilobits per second. Approximately how many minutes would the download of a 4.2-megabyte song have taken at that speed? (Note that there are 8000 kilobits in a megabyte.)

   **(A)** 0.6 **(B)** 10 **(C)** 1800 **(D)** 7200 **(E)** 36000

8. What is the value of
$$\frac{1}{3}\cdot\frac{2}{4}\cdot\frac{3}{5}\cdots\frac{18}{20}\cdot\frac{19}{21}\cdot\frac{20}{22}?$$
**(A)** $\frac{1}{462}$ **(B)** $\frac{1}{231}$ **(C)** $\frac{1}{132}$ **(D)** $\frac{2}{213}$ **(E)** $\frac{1}{22}$

9. A cup of boiling water (212°F) is placed to cool in a room whose temperature remains constant at 68°F. Suppose the difference between the water temperature and the room temperature is halved every 5 minutes. What is the water temperature, in degrees Fahrenheit, after 15 minutes?
**(A)** 77 **(B)** 86 **(C)** 92 **(D)** 98 **(E)** 104

10. One sunny day, Ling decided to take a hike in the mountains. She left her house at 8 AM, drove at a constant speed of 45 miles per hour, and arrived at the hiking trail at 10 AM. After hiking for 3 hours, Ling drove home at a constant speed of 60 miles per hour. Which of the following graphs best illustrates the distance between Lings car and her house over the course of her trip?
\{}begin{center} \{}begin{asy} import olympiad; import cse5; unitsize(12); usepackage(”mathptmx”); defaultpen(fontsize(8)+linewidth(.7)); int mod12(int i) {if (i¡13) {return i;} else {return i-12;}} void drawgraph(pair sh,string lab) { for (int i=0;i¡11;++i) { for (int j=0;j¡6;++j) { draw(shift(sh+(i,j))*unitsquare,mediumgray); } } draw(shift(sh)*((-1,0)–(11,0)),EndArrow(angle=20,size=8)); draw(shift(sh)*((0,-1)–(0,6)),EndArrow(angle=20,size= for (int i=1;i¡10;++i) { draw(shift(sh)*((i,-.2)–(i,.2))); } label(”8\{}tiny{\{}textsc{am}}”,sh+(1,-.2),S);
for (int i=2;i¡9;++i) { label(string(mod12(i+7)),sh+(i,-.2),S); } label(”4\{}tiny{\{}textsc{pm}}”,sh+(9,-.2),S); for (int i=1;i¡6;++i) { label(string(30*i),sh+(0,i),2*W); } draw(rotate(90)*”Distance (miles)”,sh+(-2.1,3),fontsize(10)); label(”$\{}textbf{(”+lab+”)}$”,sh+(-2.1,6.8),fontsize(12)); } drawgraph((0,0),”A”); drawgraph((15,0),”B”); drawgraph((0,-10),”C”); drawgraph((15,-10),”D”); drawgraph((0,-20),”E”); dotfactor=6; draw((1,0)–(3,3)–(6,3)–(8,0),linewidth(.9)); dot((1,0)^^(3,3)^^(6,3)^^(8,0)); pair sh = (15,0); draw(shift(sh)*((1,0)–(3,1.5)–(6,1.5)–(8,0)),linewidth(.9)); dot(sh+(1,0)^^sh+(3,1.5)^^sh+(6,1.5)^^sh+(8,0)); pair sh = (0,-10); draw(shift(sh)*((1,0)–(3,1.5)–(6,1.5)–(7.5,0)),linewidth(.9)); dot(sh+(1,0)^^sh+(3,1.5)^^sh+(6,1.5)^^s pair sh = (15,-10); draw(shift(sh)*((1,0)–(3,4)–(6,4)–(9.3,0)),linewidth(.9)); dot(sh+(1,0)^^sh+(3,4)^^sh+(6,4)^^sh+(9. pair sh = (0,-20); draw(shift(sh)*((1,0)–(3,3)–(6,3)–(7.5,0)),linewidth(.9)); dot(sh+(1,0)^^sh+(3,3)^^sh+(6,3)^^sh+(7.5 \{}end{asy} \{}end{center}

11. Henry the donkey has a very long piece of pasta. He takes a number of bites of pasta, each time eating 3 inches of pasta from the middle of one piece. In the end, he has 10 pieces of pasta whose total length is 17 inches. How long, in inches, was the piece of pasta he started with?
**(A)** 34 **(B)** 38 **(C)** 41 **(D)** 44 **(E)** 47

12. The arrows on the two spinners shown below are spun. Let the number $N$ equal 10 times the number on Spinner A, added to the number on Spinner B. What is the probability that $N$ is a perfect square number?
\{}begin{center} \{}begin{asy} import olympiad; import cse5; //diagram by pog give me 1 billion dollars for this size(6cm); usepackage(”mathptmx”); filldraw(arc((0,0), r=4, angle1=0, angle2=90)–(0,0)–cycle,mediumgray*0.5+gray*0.5); filldraw(arc((0,0), r=4, angle1=90, angle2=180)–(0,0)–cycle,lightgray); filldraw(arc((0,0), r=4, angle1=180, angle2=270)–(0,0)–cycle,mediumgray); filldraw(arc((0,0), r=4, angle1=270, angle2=360)–(0,0)–cycle,lightgray*0.5+mediumgray*0.5); label(”$5$”, (-1.5,1.7)); label(”$6$”, (1.5,1.7)); label(”$7$”, (1.5,-1.7)); label(”$8$”, (-1.5,-1.7)); label(”Spinner A”, (0, -5.5)); filldraw(arc((12,0), r=4, angle1=0, angle2=90)–(12,0)–cycle,mediumgray*0.5+gray*0.5); filldraw(arc((12,0), r=4, angle1=90, angle2=180)–(12,0)–cycle,lightgray); filldraw(arc((12,0), r=4, angle1=180, angle2=270)–(12,0)–cycle,mediumgray); filldraw(arc((12,0), r=4, angle1=270, angle2=360)–(12,0)–cycle,lightgray*0.5+mediumgray*0.5); label(”$1$”, (10.5,1.7)); label(”$2$”, (13.5,1.7)); label(”$3$”, (13.5,-1.7)); label(”$4$”, (10.5,-1.7)); label(”Spinner B”, (12, -5.5)); \{}end{asy} \{}end{center}
**(A)** $\frac{1}{16}$ **(B)** $\frac{1}{8}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{3}{8}$ **(E)** $\frac{1}{2}$

13. How many positive integers can fill the blank in the sentence below?
One positive integer is \{}underline{\{}hspace{2cm}} more than twice another, and the sum of the two numbers is 28.
**(A)** 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

14. In how many ways can the letters in **BEEKEEPER** be rearranged so that two or more **E**s do not appear together?

**(A)** 1 **(B)** 4 **(C)** 12 **(D)** 24 **(E)** 120

15. Laszlo went online to shop for black pepper and found thirty different black pepper options varying in weight and price, shown in the scatter plot below. In ounces, what is the weight of the pepper that offers the lowest price per ounce?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; //diagram by pog size(5.5cm); usepackage("mathptmx"); defaultpen(mediumgray*0.5+gray*0.5+linewidth(0.63)); add(grid(6,6)); label(scale(0.7)*"$1$", (1,-0.3), black); label(scale(0.7)*"$2$", (2,-0.3), black); label(scale(0.7)*"$3$", (3,-0.3), black); label(scale(0.7)*"$4$", (4,-0.3), black); label(scale(0.7)*"$5$", (5,-0.3), black); label(scale(0.7)*"$1$", (-0.3,1), black); label(scale(0.7)*"$2$", (-0.3,2), black); label(scale(0.7)*"$3$", (-0.3,3), black); label(scale(0.7)*"$4$", (-0.3,4), black); label(scale(0.7)*"$5$", (-0.3,5), black); label(scale(0.8)*rotate(90)*"Price (dollars)", (-1,3.2), black); label(scale(0.8)*"Weight (ounces)", (3.2,-1), black); dot((1,1.2),black); dot((1,1.7),black); dot((1,2),black); dot((1,2.8),black);

dot((1.5,2.1),black); dot((1.5,3),black); dot((1.5,3.3),black); dot((1.5,3.75),black);

dot((2,2),black); dot((2,2.9),black); dot((2,3),black); dot((2,4),black); dot((2,4.35),black); dot((2,4.8),black);

dot((2.5,2.7),black); dot((2.5,3.7),black); dot((2.5,4.2),black); dot((2.5,4.4),black);

dot((3,2.5),black); dot((3,3.4),black); dot((3,4.2),black);

dot((3.5,3.8),black); dot((3.5,4.5),black); dot((3.5,4.8),black);

dot((4,3.9),black); dot((4,5.1),black);

dot((4.5,4.75),black); dot((4.5,5),black);

dot((5,4.5),black); dot((5,5),black); \{}end{asy} \{}end{center}

**(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

16. Four numbers are written in a row. The average of the first two is 21, the average of the middle two is 26, and the average of the last two is 30. What is the average of the first and last of the numbers?

**(A)** 24 **(B)** 25 **(C)** 26 **(D)** 27 **(E)** 28

17. If $n$ is an even positive integer, the *double factorial* notation $n!!$ represents the product of all the even integers from 2 to $n$. For example, $8!! = 2 \cdot 4 \cdot 6 \cdot 8$. What is the units digit of the following sum?

$$2!! + 4!! + 6!! + \cdots + 2018!! + 2020!! + 2022!!$$

**(A)** 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

18. The midpoints of the four sides of a rectangle are $(-3,0), (2,0), (5,4)$, and $(0,4)$. What is the area of the rectangle?

**(A)** 20 **(B)** 25 **(C)** 40 **(D)** 50 **(E)** 80

19. Mr. Ramos gave a test to his class of 20 students. The dot plot below shows the distribution of test scores.

\{}begin{center} \{}begin{asy} import olympiad; import cse5; //diagram by pog . give me 1,000,000,000 dollars for this diagram size(5cm); defaultpen(0.7); dot((0.5,1)); dot((0.5,1.5)); dot((1.5,1)); dot((1.5,1.5)); dot((2.5,1)); dot((2.5,1.5)); dot((2.5,2)); dot((2.5,2.5)); dot((3.5,1)); dot((3.5,1.5)); dot((3.5,2)); dot((3.5,2.5)); dot((3.5,3)); dot((4.5,1)); dot((4.5,1.5)); dot((5.5,1)); dot((5.5,1.5)); dot((5.5,2)); dot((6.5,1)); dot((7.5,1)); draw((0,0.5)–(8,0.5),linewidth(0.7)); defaultpen(fontsize(10.5pt)); label("$65$", (0.5,-0.1)); label("$70$", (1.5,-0.1)); label("$75$", (2.5,-0.1)); label("$80$", (3.5,-0.1)); label("$85$", (4.5,-0.1)); label("$90$", (5.5,-0.1)); label("$95$", (6.5,-0.1)); label("$100$", (7.5,-0.1)); \{}end{asy} \{}end{center}

Later Mr. Ramos discovered that there was a scoring error on one of the questions. He regraded the tests, awarding some of the students 5 extra points, which increased the median test score to 85. What is the minimum number of students who received extra points?

(Note that the \{}textit{median} test score equals the average of the 2 scores in the middle if the 20 test scores are arranged in increasing order.)

**(A)** 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

20. The grid below is to be filled with integers in such a way that the sum of the numbers in each row and the sum of the numbers in each column are the same. Four numbers are missing. The number $x$ in the lower left corner is larger than the other three missing numbers. What is the smallest possible value of $x$?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; unitsize(0.5cm); draw((3,3)–(-3,3)); draw((3,1)–(-3,1)); draw((3,-3)–(-3,-3)); draw((3,-1)–(-3,-1)); draw((3,3)–(3,-3)); draw((1,3)–(1,-3)); draw((-3,3)–(-3,-3)); draw((-1,3)–(-1,-3)); label((-2,2),"$-2$"); label((0,2),"$9$"); label((2,2),"$5$"); label((2,0),"${-}1$"); label((2,-2),"$8$"); label((-2,-2),"$x$"); \{}end{asy} \{}end{center}

**(A)** $-1$ **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 9

21. Steph scored 15 baskets out of 20 attempts in the first half of a game, and 10 baskets out of 10 attempts in the second half. Candace took 12 attempts in the first half and 18 attempts in the second. In each half, Steph scored a higher percentage of baskets than Candace. Surprisingly they ended with the same overall percentage of baskets scored. How many more baskets did Candace score in the second half than in the first?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; size(7cm); draw((-8,27)–(72,27)); draw((16,0)–(16,35)); draw((40,0)–(40,35)); label("12", (28,3)); draw((25,6.5)–(25,12)–(31,12)–(31,6.5)–cycle); draw((25,5.5)–(31,5.5)); label("18", (56,3)); draw((53,6.5)–(53,12)–(59,12)–(59,6.5)–cycle); draw((53,5.5)–(59,5.5)); draw((53,5.5)–(59,5.5)); label("20", (28,18)); label("15", (28,24)); draw((25,21)–(31,21)); label("10", (56,18)); label("10", (56,24)); draw((53,21)–(59,21)); label("First Half", (28,31)); label("Second Half", (56,31)); label("Candace", (2.35,6)); label("Steph", (0,21)); \{}end{asy} \{}end{center}

    **(A)** 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

22. A bus takes 2 minutes to drive from one stop to the next, and waits 1 minute at each stop to let passengers board. Zia takes 5 minutes to walk from one bus stop to the next. As Zia reaches a bus stop, if the bus is at the previous stop or has already left the previous stop, then she will wait for the bus. Otherwise she will start walking toward the next stop. Suppose the bus and Zia start at the same time toward the library, with the bus 3 stops behind. After how many minutes will Zia board the bus?

    **(A)** 17 **(B)** 19 **(C)** 20 **(D)** 21 **(E)** 23

23. A $\triangle$ or $\bigcirc$ is placed in each of the nine squares in a 3-by-3 grid. Shown below is a sample configuration with three $\triangle$s in a line.

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; //diagram by kante314 size(3.3cm); defaultpen(linewidth(1)); real r = 0.37; path equi = r * dir(-30) – (r+0.03) * dir(90) – r * dir(210) – cycle; draw((0,0)–(0,3)–(3,3)–(3,0)–cycle); draw((0,1)–(3,1)–(3,2)–(0,2)–cycle); draw((1,0)–(1,3)–(2,3)–(2,0)–cycle); draw(circle((3/2,5/2),1/3)); draw(circle((5/2,1/2),1/3)); draw(circle((3/2,3/2),1/3)); draw(shift(0.5,0.38) * equi); draw(shift(1.5,0.38) * equi); draw(shift(0.5,1.38) * equi); draw(shift(2.5,1.38) * equi); draw(shift(0.5,2.38) * equi); draw(shift(2.5,2.38) * equi); \{}end{asy} \{}end{center}

    How many configurations will have three $\triangle$s in a line and three $\bigcirc$s in a line?

    **(A)** 39 **(B)** 42 **(C)** 78 **(D)** 84 **(E)** 96

24. The figure below shows a polygon $ABCDEFGH$, consisting of rectangles and right triangles. When cut out and folded on the dotted lines, the polygon forms a triangular prism. Suppose that $AH = EF = 8$ and $GH = 14$. What is the volume of the prism?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; usepackage("mathptmx"); size(275); defaultpen(linewidth(0.8)); real r = 2, s = 2.5, theta = 14; pair G = (0,0), F = (r,0), C = (r,s), B = (0,s), M = (C+F)/2, I = M + s/2 * dir(-theta); pair N = (B+G)/2, J = N + s/2 * dir(180+theta); pair E = F + r * dir(- 45 - theta/2), D = I+E-F; pair H = J + r * dir(135 + theta/2), A = B+H-J; draw(A–B–C–I–D–E–F–G–J–H–cycle^^rightanglemark(F,I,C)^^rightanglemark(G,J,B)); draw(J–B–G^^C–F–I,linetype ("4 4")); dot("$A$",A,N); dot("$B$",B,1.2*N); dot("$C$",C,N); dot("$D$",D,dir(0)); dot("$E$",E,S); dot("$F$",F,1.5*dir(-100)); dot("$G$",G,S); dot("$H$",H,W); dot("$I$",I,NE); dot("$J$",J,1.5*S); \{}end{asy} \{}end{center}

    **(A)** 112 **(B)** 128 **(C)** 192 **(D)** 240 **(E)** 288

25. A cricket randomly hops between 4 leaves, on each turn hopping to one of the other 3 leaves with equal probability. After 4 hops, what is the probability that the cricket has returned to the leaf where it started?

    **(A)** $\frac{2}{9}$ **(B)** $\frac{19}{80}$ **(C)** $\frac{20}{81}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{7}{27}$

# 2023 AMC 8 Problems

1. What is the value of $(8 \times 4 + 2) - (8 + 4 \times 2)$?

   **(A)** 0 **(B)** 6 **(C)** 10 **(D)** 18 **(E)** 24

2. A square piece of paper is folded twice into four equal quarters, as shown below, then cut along the dashed line. When unfolded, the paper will match which of the following figures?

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; //Restored original diagram. Alter it if you would like, but it was made by TheMathGuyd, // Diagram by TheMathGuyd. I even put the lined texture :) // Thank you Kante314 for inspiring thicker arrows. They do look much better size(0,3cm); path sq = (-0.5,-0.5)–(0.5,-0.5)–(0.5,0.5)–(-0.5,0.5)–cycle; path rh = (-0.125,-0.125)–(0.5,-0.5)–(0.5,0.5)–(-0.125,0.875)–cycle; path sqA = (-0.5,-0.5)–(-0.25,-0.5)–(0,-0.25)–(0.25,-0.5)–(0.5,-0.5)–(0.5,-0.25)–(0.25,0)–(0.5,0.25)–(0.5,0.5)–(0.25,0.5)–(0,0.25)–(-0.25,0.5)–(-0.5,0.5)–(-0.5,0.25)–(-0.25,0)–(-0.5,-0.25)–cycle; path sqB = (-0.5,-0.5)–(-0.25,-0.5)–(0,-0.25)–(0.25,-0.5)–(0.5,-0.5)–(0.5,0.5)–(0.25,0.5)–(0,0.25)–(-0.25,0.5)–(-0.5,0.5)–cycle; path sqC = (-0.25,-0.25)–(0.25,-0.25)–(0.25,0.25)–(-0.25,0.25)–cycle; path trD = (-0.25,0)–(0.25,0)–(0,0.25)–cycle; path sqE = (-0.25,0)–(0,-0.25)–(0.25,0)–(0,0.25)–cycle; filldraw(sq,mediumgrey,black); draw((0.75,0)–(1.25,0),currentpen+1,Arrow(size=6)); //folding path sqside = (-0.5,-0.5)–(0.5,-0.5); path rhside = (-0.125,-0.125)–(0.5,-0.5); transform fld = shift((1.75,0))*scale(0.5); draw(fld*sq,black); int i; for(i=0; i¡10; i=i+1) { draw(shift(0,0.05*i)*fld*sqside,deepblue); } path rhedge = (-0.125,-0.125)–(-0.125,0.8)–(-0.2,0.85)–cycle; filldraw(fld*rhedge,grey); path sqedge = (-0.5,-0.5)–(-0.5,0.4475)–(-0.575,0.45)–cycle; filldraw(fld*sqedge,grey); filldraw(fld*rh,white,black); int i; for(i=0; i¡10; i=i+1) { draw(shift(0,0.05*i)*fld*rhside,deepblue); } draw((2.25,0)–(2.75,0),currentpen+1,Arrow(size=6)); //cutting transform cut = shift((3.25,0))*scale(0.5); draw(shift((-0.01,+0.01))*cut*sq); draw(cut*sq); filldraw(shift((0.01,-0.01))*cut*sq,white,black); int j; for(j=0; j¡10; j=j+1) { draw(shift(0,0.05*j)*cut*sqside,deepblue); } draw(shift((0.01,-0.01))*cut*(0,-0.5)–shift((0.01,-0.01))*cut*(0.5,0 //Answers Below, but already Separated //filldraw(sqA,grey,black); //filldraw(sqB,grey,black); //filldraw(sq,grey,black) //filldraw(sqC,white,black); //filldraw(sq,grey,black); //filldraw(trD,white,black); //filldraw(sq,grey,black); //filldraw(sqE,white,black); \{}end{asy} \{}end{center}

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; // Diagram by TheMathGuyd. size(0,7.5cm); path sq = (-0.5,-0.5)–(0.5,-0.5)–(0.5,0.5)–(-0.5,0.5)–cycle; path rh = (-0.125,-0.125)–(0.5,-0.5)–(0.5,0.5)–(-0.125,0.875)–cycle; path sqA = (-0.5,-0.5)–(-0.25,-0.5)–(0,-0.25)–(0.25,-0.5)–(0.5,-0.5)–(0.5,-0.25)–(0.25,0)–(0.5,0.25)–(0.5,0.5)–(0.25,0.5)–(0,0.25)–(-0.25,0.5)–(-0.5,0.5)–(-0.5,0.25)–(-0.25,0)–(-0.5,-0.25)–cycle; path sqB = (-0.5,-0.5)–(-0.25,-0.5)–(0,-0.25)–(0.25,-0.5)–(0.5,-0.5)–(0.5,0.5)–(0.25,0.5)–(0,0.25)–(-0.25,0.5)–(-0.5,0.5)–cycle; path sqC = (-0.25,-0.25)–(0.25,-0.25)–(0.25,0.25)–(-0.25,0.25)–cycle; path trD = (-0.25,0)–(0.25,0)–(0,0.25)–cycle; path sqE = (-0.25,0)–(0,-0.25)–(0.25,0)–(0,0.25)–cycle;

   //ANSWERS real sh = 1.5; label("$\{}textbf{(A)}$",(-0.5,0.5),SW); label("$\{}textbf{(B)}$",shift((sh,0))*(-0.5,0.5),SW); label("$\{}textbf{(C)}$",shift((2sh,0))*(-0.5,0.5),SW); label("$\{}textbf{(D)}$",shift((0,-sh))*(-0.5,0.5),SW); label("$\{}textbf{(E)}$",shift((sh,-sh))*(-0.5,0.5),SW); filldraw(sqA,mediumgrey,black); filldraw(shift((sh,0))*sqB,mediumgrey,black); filldraw(shift((2*sh,0))*sq,mediumgrey,black); filldraw(shift((2*sh,0))*sqC,w filldraw(shift((0,-sh))*sq,mediumgrey,black); filldraw(shift((0,-sh))*trD,white,black); filldraw(shift((sh,-sh))*sq,medium filldraw(shift((sh,-sh))*sqE,white,black); \{}end{asy} \{}end{center}

3. \{}textit{Wind chill} is a measure of how cold people feel when exposed to wind outside. A good estimate for wind chill can be found using this calculation

   $$(\text{wind chill}) = (\text{air temperature}) - 0.7 \times (\text{wind speed}),$$

   where temperature is measured in degrees Fahrenheit (°F) and the wind speed is measured in miles per hour (mph). Suppose the air temperature is 36°F and the wind speed is 18 mph. Which of the following is closest to the approximate wind chill?

   **(A)** 18 **(B)** 23 **(C)** 28 **(D)** 32 **(E)** 35

4. The numbers from 1 to 49 are arranged in a spiral pattern on a square grid, beginning at the center. The first few numbers have been entered into the grid below. Consider the four numbers that will appear in the shaded squares, on the same diagonal as the number 7. How many of these four numbers are prime?

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; /* Made by MRENTHUSIASM */ size(175);

   void ds(pair p) { filldraw((0.5,0.5)+p–(-0.5,0.5)+p–(-0.5,-0.5)+p–(0.5,-0.5)+p–cycle,mediumgrey); }

   ds((0.5,4.5)); ds((1.5,3.5)); ds((3.5,1.5)); ds((4.5,0.5));

   add(grid(7,7,grey+linewidth(1.25)));

int adj = 1; int curUp = 2; int curLeft = 4; int curDown = 6;

label(”$1$”,(3.5,3.5));

for (int len = 3; len¡=3; len+=2) { for (int i=1; i¡=len-1; ++i) { label(”$”+string(curUp)+”$”,(3.5+adj,3.5-adj+i)); label(”$”+string(curLeft)+”$”,(3.5+adj-i,3.5+adj)); label(”$”+string(curDown)+”$”,(3.5-adj,3.5+adj-i)); ++curDown; ++curLeft; ++curUp; } ++adj; curUp = lenˆ2 + 1; curLeft = lenˆ2 + len + 2; curDown = lenˆ2 + 2*len + 3; }

draw((4,4)–(3,4)–(3,3)–(5,3)–(5,5)–(2,5)–(2,2)–(6,2)–(6,6)–(1,6)–(1,1)–(7,1)–(7,7)–(0,7)–(0,0)–(7,0),linewidth(2)); \{}end{asy} \{}end{center}

**(A)** 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

5. A lake contains 250 trout, along with a variety of other fish. When a marine biologist catches and releases a sample of 180 fish from the lake, 30 are identified as trout. Assume that the ratio of trout to the total number of fish is the same in both the sample and the lake. How many fish are there in the lake?

   **(A)** 1250 **(B)** 1500 **(C)** 1750 **(D)** 1800 **(E)** 2000

6. The digits $2, 0, 2,$ and $3$ are placed in the expression below, one digit per box. What is the maximum possible value of the expression?

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; // Diagram by TheMathGuyd. I can compress this later size(5cm); real w=2.2; pair O,I,J; O=(0,0);I=(1,0);J=(0,1); path bsqb = O–I; path bsqr = I–I+J; path bsqt = I+J–J; path bsql = J–O; path lsqb = shift((1.2,0.75))*scale(0.5)*bsqb; path lsqr = shift((1.2,0.75))*scale(0.5)*bsqr; path lsqt = shift((1.2,0.75))*scale(0.5)*bsqt; path lsql = shift((1.2,0.75))*scale(0.5)*bsq draw(bsqb,dashed); draw(bsqr,dashed); draw(bsqt,dashed); draw(bsql,dashed); draw(lsqb,dashed); draw(lsqr,dashed); draw(lsqt,dashed); draw(lsql,dashed); label(scale(3)*”$\{}times$”,(w,1/3)); draw(shift(1.3w,0)*bsqb,dashed); draw(shift(1.3w,0)*bsqr,dashed); draw(shift(1.3w,0)*bsqt,dashed); draw(shift(1.3w,0)*bsql,dashed); draw(shift(1.3w,0)* draw(shift(1.3w,0)*lsqr,dashed); draw(shift(1.3w,0)*lsqt,dashed); draw(shift(1.3w,0)*lsql,dashed); \{}end{asy} \{}end{center}

   **(A)** 0 **(B)** 8 **(C)** 9 **(D)** 16 **(E)** 18

7. A rectangle, with sides parallel to the $x$-axis and $y$-axis, has opposite vertices located at $(15, 3)$ and $(16, 5)$. A line is drawn through points $A(0, 0)$ and $B(3, 1)$. Another line is drawn through points $C(0, 10)$ and $D(2, 9)$. How many points on the rectangle lie on at least one of the two lines?

   \{}begin{center} \{}begin{asy} import olympiad; import cse5; usepackage(”mathptmx”); size(9cm); draw((0,-.5)–(0,11),EndArrow(size=.15cm)); draw((1,0)–(1,11),mediumgray); draw((2,0)–(2,11),mediumgray); draw((3,0)–(3,11),mediumgray); draw((4,0)–(4,11),mediumgray); draw((5,0)–(5,11),mediumgray); draw((6,0)–(6,11),mediumgray); draw((7,0)–(7,11),mediumgray); draw((8,0)–(8,11),mediumgray); draw((9,0)–(9,11),mediumgray); draw((10,0)–(10,11),mediumgray); draw((11,0)–(11,11),mediumgray); draw((12,0)–(12,11),mediumgray); draw((13,0)–(13,11),mediumgray); draw((14,0)–(14,11),mediumgray); draw((15,0)–(15,11),mediumgray); draw((16,0)–(16,11),mediumgray);

   draw((-.5,0)–(17,0),EndArrow(size=.15cm)); draw((0,1)–(17,1),mediumgray); draw((0,2)–(17,2),mediumgray); draw((0,3)–(17,3),mediumgray); draw((0,4)–(17,4),mediumgray); draw((0,5)–(17,5),mediumgray); draw((0,6)–(17,6),mediumgray); draw((0,7)–(17,7),mediumgray); draw((0,8)–(17,8),mediumgray); draw((0,9)–(17,9),mediumgray); draw((0,10)–(17,10),mediumgray);

   draw((-.13,1)–(.13,1)); draw((-.13,2)–(.13,2)); draw((-.13,3)–(.13,3)); draw((-.13,4)–(.13,4)); draw((-.13,5)–(.13,5)); draw((-.13,6)–(.13,6)); draw((-.13,7)–(.13,7)); draw((-.13,8)–(.13,8)); draw((-.13,9)–(.13,9)); draw((-.13,10)–(.13,10));

   draw((1,-.13)–(1,.13)); draw((2,-.13)–(2,.13)); draw((3,-.13)–(3,.13)); draw((4,-.13)–(4,.13)); draw((5,-.13)–(5,.13)); draw((6,-.13)–(6,.13)); draw((7,-.13)–(7,.13)); draw((8,-.13)–(8,.13)); draw((9,-.13)–(9,.13)); draw((10,-.13)–(10,.13)); draw((11,-.13)–(11,.13)); draw((12,-.13)–(12,.13)); draw((13,-.13)–(13,.13)); draw((14,-.13)–(14,.13)); draw((15,-.13)–(15,.13)); draw((16,-.13)–(16,.13));

   label(scale(.7)*”$1$”, (1,-.13), S); label(scale(.7)*”$2$”, (2,-.13), S); label(scale(.7)*”$3$”, (3,-.13), S); label(scale(.7)*”$4$”, (4,-.13), S); label(scale(.7)*”$5$”, (5,-.13), S); label(scale(.7)*”$6$”, (6,-.13), S); label(scale(.7)*”$7$”, (7,-.13), S); label(scale(.7)*”$8$”, (8,-.13), S); label(scale(.7)*”$9$”, (9,-.13), S); label(scale(.7)*”$10$”, (10,-.13), S); label(scale(.7)*”$11$”, (11,-.13), S); label(scale(.7)*”$12$”, (12,-.13), S); label(scale(.7)*”$13$”, (13,-.13), S); label(scale(.7)*”$14$”, (14,-.13), S); label(scale(.7)*”$15$”, (15,-.13), S); label(scale(.7)*”$16$”, (16,-.13), S);

   label(scale(.7)*”$1$”, (-.13,1), W); label(scale(.7)*”$2$”, (-.13,2), W); label(scale(.7)*”$3$”, (-.13,3), W); label(scale(.7)*”$4$”, (-.13,4), W); label(scale(.7)*”$5$”, (-.13,5), W); label(scale(.7)*”$6$”, (-.13,6), W);

label(scale(.7)*"$7$", (-.13,7), W); label(scale(.7)*"$8$", (-.13,8), W); label(scale(.7)*"$9$", (-.13,9), W); label(scale(.7)*"$10$", (-.13,10), W);

dot((0,0),linewidth(4)); label(scale(.75)*"$A$", (0,0), NE); dot((3,1),linewidth(4)); label(scale(.75)*"$B$", (3,1), NE);

dot((0,10),linewidth(4)); label(scale(.75)*"$C$", (0,10), NE); dot((2,9),linewidth(4)); label(scale(.75)*"$D$", (2,9), NE);

draw((15,3)–(16,3)–(16,5)–(15,5)–cycle,linewidth(1.125)); dot((15,3),linewidth(4)); dot((16,3),linewidth(4)); dot((16,5),linewidth(4)); dot((15,5),linewidth(4)); \{}end{asy} \{}end{center}

**(A)** 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

8. Lola, Lolo, Tiya, and Tiyo participated in a ping pong tournament. Each player competed against each of the other three players exactly twice. Shown below are the win-loss records for the players. The numbers 1 and 0 represent a win or loss, respectively. For example, Lola won five matches and lost the fourth match. What was Tiyos win-loss record?

| Player | Result |
|---|---|
| Lola | 111011 |
| Lolo | 101010 |
| Tiya | 010100 |
| Tiyo | ?????? |

**(A)** 000101 **(B)** 001001 **(C)** 010000 **(D)** 010101 **(E)** 011000

==Problem 9== Malaika is skiing on a mountain. The graph below shows her elevation, in meters, above the base of the mountain as she skis along a trail. In total, how many seconds does she spend at an elevation between 4 and 7 meters?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; // Diagram by TheMathGuyd. Found cubic, so graph is perfect. import graph; size(8cm); int i; for(i=1; i¡9; i=i+1) { draw((-0.2,2i-1)–(16.2,2i-1), mediumgrey); draw((2i-1,-0.2)–(2i-1,16.2), mediumgrey); draw((-0.2,2i)–(16.2,2i), grey); draw((2i,-0.2)–(2i,16.2), grey); } Label f; f.p=fontsize(6); xaxis(-0.5,17.8,Ticks(f, 2.0),Arrow()); yaxis(-0.5,17.8,Ticks(f, 2.0),Arrow()); real f(real x) { return -0.03125 x^(3) + 0.75x^(2) - 5.125 x + 14.5; } draw(graph(f,0,15.225),currentpen+1); real dpt=2; real ts=0.75; transform st=scale(ts); label(rotate(90)*st*"Elevation (meters)",(-dpt,8)); label(st*"Time (seconds)",(8,-dpt)); \{}end{asy} \{}end{center}

**(A)** 6 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 14

9. Harold made a plum pie to take on a picnic. He was able to eat only $\frac{1}{4}$ of the pie, and he left the rest for his friends. A moose came by and ate $\frac{1}{3}$ of what Harold left behind. After that, a porcupine ate $\frac{1}{3}$ of what the moose left behind. How much of the original pie still remained after the porcupine left?

**(A)** $\frac{1}{12}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{5}{12}$

10. NASAs Perseverance Rover was launched on July 30, 2020. After traveling 292,526,838 miles, it landed on Mars in Jezero Crater about 6.5 months later. Which of the following is closest to the Rovers average interplanetary speed in miles per hour?

**(A)** 6,000 **(B)** 12,000 **(C)** 60,000 **(D)** 120,000 **(E)** 600,000

11. The figure below shows a large white circle with a number of smaller white and shaded circles in its interior. What fraction of the interior of the large white circle is shaded?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; // Diagram by TheMathGuyd size(6cm); draw(circle((3,3),3)); filldraw(circle((2,3),2),lightgrey); filldraw(circle((3,3),1),white); filldraw(circle((1,3),1),white); filldraw(circle((5.5,3),0.5),lightgrey); filldraw(circle((4.5,4.5),0.5),lightgrey); filldraw(circle((4.5,1.5),0.5),lightgrey); int i, j; for(i=0; i¡7; i=i+1) { draw((0,i)–(6,i), dashed+grey); draw((i,0)–(i,6), dashed+grey); } \{}end{asy} \{}end{center}

**(A)** $\frac{1}{4}$ **(B)** $\frac{11}{36}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{19}{36}$ **(E)** $\frac{5}{9}$

12. Along the route of a bicycle race, 7 water stations are evenly spaced between the start and finish lines, as shown in the figure below. There are also 2 repair stations evenly spaced between the start and finish lines. The 3rd water station is located 2 miles after the 1st repair station. How long is the race in miles?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; // Credits given to Themathguyd and Kante314 usepackage(”mathptmx”); size(10cm); filldraw((11,4.5)–(171,4.5)–(171,17.5)–(11,17.5)–cycle,mediumgray*0.4 + lightgray*0.6); draw((11,11)–(171,11),linetype(”2 2”)+white+linewidth(1.2)); draw((0,0)–(11,0)–(11,22)–(0,22)–cycle); draw((171,0)–(182,0)–(182,22)–(171,22)–cycle);

draw((31,4.5)–(31,0)); draw((51,4.5)–(51,0)); draw((151,4.5)–(151,0));

label(scale(.85)*rotate(45)*”Water 1”, (23,-13.5)); label(scale(.85)*rotate(45)*”Water 2”, (43,-13.5)); label(scale(.85)*rotate(45)*”Water 7”, (143,-13.5));

filldraw(circle((103,-13.5),.2)); filldraw(circle((98,-13.5),.2)); filldraw(circle((93,-13.5),.2)); filldraw(circle((88,-13.5),.2)); filldraw(circle((83,-13.5),.2));

label(scale(.85)*rotate(90)*”Start”, (5.5,11)); label(scale(.85)*rotate(270)*”Finish”, (176.5,11)); \{}end{asy} \{}end{center}

**(A)** 8 **(B)** 16 **(C)** 24 **(D)** 48 **(E)** 96

13. Nicolas is planning to send a package to his friend Anton, who is a stamp collector. To pay for the postage, Nicolas would like to cover the package with a large number of stamps. Suppose he has a collection of 5-cent, 10-cent, and 25-cent stamps, with exactly 20 of each type. What is the greatest number of stamps Nicolas can use to make exactly \$7.10 in postage? (Note: The amount \$7.10 corresponds to 7 dollars and 10 cents. One dollar is worth 100 cents.)

**(A)** 45 **(B)** 46 **(C)** 51 **(D)** 54 **(E)** 55

14. Viswam walks half a mile to get to school each day. His route consists of 10 city blocks of equal length and he takes 1 minute to walk each block. Today, after walking 5 blocks, Viswam discovers he has to make a detour, walking 3 blocks of equal length instead of 1 block to reach the next corner. From the time he starts his detour, at what speed, in mph, must he walk, in order to get to school at his usual time?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; // Diagram by TheMathGuyd size(13cm); // this is an important stickman to the left of the origin pair C=midpoint((-0.5,0.5)–(-0.6,0.05)); draw((-0.5,0.5)–(-0.6,0.05)); // Head to butt draw((-0.64,0.16)–(-0.7,0.2)–C–(-0.47,0.2)–(-0.4,0.22)); // LH-C-RH draw((-0.6,0.05)–(-0.55,-0.1)–(-0.57,-0.25)); draw((-0.6,0.05)–(-0.68,-0.12)–(-0.8,-0.20));

filldraw(circle((-0.5,0.5),0.1),white,black);

int i; real d,s; // gap and side d=0.2; s=1-2*d; for(i=0; i¡10; i=i+1) { //dot((i,0), red); //marks to start filldraw((i+d,d)–(i+1-d,d)–(i+1-d,1-d)–(i+d,1-d)–cycle, lightgrey, black); filldraw(conj((i+d,d))–conj((i+1-d,d))–conj((i+1-d,1-d))–conj((i+d,1-d))–cycle,lightgrey,black); }

fill((5+d,-d/2)–(6-d,-d/2)–(6-d,d/2)–(5+d,d/2)–cycle,lightred);

draw((0,0)–(5,0)–(5,1)–(6,1)–(6,0)–(10.1,0),deepblue+linewidth(1.25)); //Who even noticed label(”School”, (10,0),E, Draw()); \{}end{asy} \{}end{center}

**(A)** 4 **(B)** 4.2 **(C)** 4.5 **(D)** 4.8 **(E)** 5

15. The letters $P, Q,$ and $R$ are entered into a $20 \times 20$ table according to the pattern shown below. How many Ps, Qs, and Rs will appear in the completed table?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; /* Made by MRENTHUSIASM, Edited by Kante314 */ usepackage(”mathdots”); size(5cm); draw((0,0)–(6,0),linewidth(1.5)+mediumgray); draw((0,1)–(6,1),linewidth(1.5)+mediumgray); draw((0,2)–(6,2),linewidth(1.5)+mediumgray); draw((0,3)–(6,3),linewidth(1.5)+me draw((0,4)–(6,4),linewidth(1.5)+mediumgray); draw((0,5)–(6,5),linewidth(1.5)+mediumgray);

draw((0,0)–(0,6),linewidth(1.5)+mediumgray); draw((1,0)–(1,6),linewidth(1.5)+mediumgray); draw((2,0)–(2,6),linewidth(1.5)+mediumgray); draw((3,0)–(3,6),linewidth(1.5)+mediumgray); draw((4,0)–(4,6),linewidth(1.5)+me draw((5,0)–(5,6),linewidth(1.5)+mediumgray);

label(scale(.9)*”\{}textsf{P}”, (.5,.5)); label(scale(.9)*”\{}textsf{Q}”, (.5,1.5)); label(scale(.9)*”\{}textsf{R}”, (.5,2.5)); label(scale(.9)*”\{}textsf{P}”, (.5,3.5)); label(scale(.9)*”\{}textsf{Q}”, (.5,4.5)); label(”\$\{}vdots\$”, (.5,5.6));

label(scale(.9)*”\{}textsf{Q}”, (1.5,.5)); label(scale(.9)*”\{}textsf{R}”, (1.5,1.5)); label(scale(.9)*”\{}textsf{P}”, (1.5,2.5)); label(scale(.9)*”\{}textsf{Q}”, (1.5,3.5)); label(scale(.9)*”\{}textsf{R}”, (1.5,4.5)); label(”\$\{}vdots\$”, (1.5,5.6));

label(scale(.9)*”\{}textsf{R}”, (2.5,.5)); label(scale(.9)*”\{}textsf{P}”, (2.5,1.5)); label(scale(.9)*”\{}textsf{Q}”, (2.5,2.5)); label(scale(.9)*”\{}textsf{R}”, (2.5,3.5)); label(scale(.9)*”\{}textsf{P}”, (2.5,4.5)); label(”\$\{}vdots\$”, (2.5,5.6));

label(scale(.9)*"\{}textsf{P}", (3.5,.5)); label(scale(.9)*"\{}textsf{Q}", (3.5,1.5)); label(scale(.9)*"\{}textsf{R}", (3.5,2.5)); label(scale(.9)*"\{}textsf{P}", (3.5,3.5)); label(scale(.9)*"\{}textsf{Q}", (3.5,4.5)); label("$\{}vdots$", (3.5,5.6));

label(scale(.9)*"\{}textsf{Q}", (4.5,.5)); label(scale(.9)*"\{}textsf{R}", (4.5,1.5)); label(scale(.9)*"\{}textsf{P}", (4.5,2.5)); label(scale(.9)*"\{}textsf{Q}", (4.5,3.5)); label(scale(.9)*"\{}textsf{R}", (4.5,4.5)); label("$\{}vdots$", (4.5,5.6));

label(scale(.9)*"$\{}dots$", (5.5,.5)); label(scale(.9)*"$\{}dots$", (5.5,1.5)); label(scale(.9)*"$\{}dots$", (5.5,2.5)); label(scale(.9)*"$\{}dots$", (5.5,3.5)); label(scale(.9)*"$\{}dots$", (5.5,4.5)); label(scale(.9)*"$\{}iddots$", (5.5,5.6)); \{}end{asy} \{}end{center}

**(A)** 132 Ps, 134 Qs, 134 Rs

**(B)** 133 Ps, 133 Qs, 134 Rs

**(C)** 133 Ps, 134 Qs, 133 Rs

**(D)** 134 Ps, 132 Qs, 134 Rs

**(E)** 134 Ps, 133 Qs, 133 Rs

16. A \{}textit{regular octahedron} has eight equilateral triangle faces with four faces meeting at each vertex. Jun will make the regular octahedron shown on the right by folding the piece of paper shown on the left. Which numbered face will end up to the right of $Q$?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; // Diagram by TheMathGuyd import graph; // The Solid // To save processing time, do not use three (dimensions) // Project (roughly) to two size(15cm); pair Fr, Lf, Rt, Tp, Bt, Bk; Lf=(0,0); Rt=(12,1); Fr=(7,-1); Bk=(5,2); Tp=(6,6.7); Bt=(6,-5.2); draw(Lf–Fr–Rt); draw(Lf–Tp–Rt); draw(Lf–Bt–Rt); draw(Tp–Fr–Bt); draw(Lf–Bk–Rt,dashed); draw(Tp–Bk–Bt,dashed); label(rotate(-8.13010235)*slant(0.1)*"$Q$", (4.2,1.6)); label(rotate(21.8014095)*slant(-0.2)*"$?$", (8.5,2.05)); pair g = (-8,0); // Define Gap transform real a = 8; draw(g+(-a/2,1)–g+(a/2,1), Arrow()); // Make arrow // Time for the NET pair DA,DB,DC,CD,O; DA = (4*sqrt(3),0); DB = (2*sqrt(3),6); DC = (DA+DB)/3; CD = conj(DC); O=(0,0); transform trf=shift(3g+(0,3)); path NET = O–(-2*DA)–(-2DB)–(-DB)–(2DA-DB)–DB–O–DA–(DA-DB)–O–(-DB)–(-DA)–(-DA-DB)–(-DB); draw(trf*NET); label("$7$",trf*DC); label("$Q$",trf*DC+DA-DB); label("$5$",trf*DC-DB); label("$3$",trf*DC-DA-DB); label("$6$",trf*CD); label("$4$",trf*CD-DA); label("$2$",trf*CD-DA-DB); label("$1$",trf*CD-2DA); \{}end{asy} \{}end{center}

**(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

17. Greta Grasshopper sits on a long line of lily pads in a pond. From any lily pad, Greta can jump 5 pads to the right or 3 pads to the left. What is the fewest number of jumps Greta must make to reach the lily pad located 2023 pads to the right of her starting point?

**(A)** 405 **(B)** 407 **(C)** 409 **(D)** 411 **(E)** 413

18. An equilateral triangle is placed inside a larger equilateral triangle so that the region between them can be divided into three congruent trapezoids, as shown below. The side length of the inner triangle is $\frac{2}{3}$ the side length of the larger triangle. What is the ratio of the area of one trapezoid to the area of the inner triangle?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; // Diagram by TheMathGuyd pair A,B,C; A=(0,1); B=(sqrt(3)/2,-1/2); C=-conj(B); fill(2B–3B–3C–2C–cycle,grey); dot(3A); dot(3B); dot(3C); dot(2A); dot(2B); dot(2C); draw(2A–2B–2C–cycle); draw(3A–3B–3C–cycle); draw(2A–3A); draw(2B–3B); draw(2C–3C); \{}end{asy} \{}end{center}

**(A)** $1:3$ **(B)** $3:8$ **(C)** $5:12$ **(D)** $7:16$ **(E)** $4:9$

19. Two integers are inserted into the list $3, 3, 8, 11, 28$ to double its range. The mode and median remain unchanged. What is the maximum possible sum of the two additional numbers?

**(A)** 56 **(B)** 57 **(C)** 58 **(D)** 60 **(E)** 61

20. Alina writes the numbers $1, 2, \ldots, 9$ on separate cards, one number per card. She wishes to divide the cards into 3 groups of 3 cards so that the sum of the numbers in each group will be the same. In how many ways can this be done?

**(A)** 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

21. In a sequence of positive integers, each term after the second is the product of the previous two terms. The sixth term is 4000. What is the first term?

    **(A)** 1 **(B)** 2 **(C)** 4 **(D)** 5 **(E)** 10

22. Each square in a $3 \times 3$ grid is randomly filled with one of the 4 gray and white tiles shown below on the right.

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; size(5.663333333cm); draw((0,0)–(3,0)–(3,3)–(0,3)–cycle,gray); draw((1,0)–(1,3)–(2,3)–(2,0),gray); draw((0,1)–(3,1)–(3,2)–(0,2),gray);

    fill((6,.33)–(7,.33)–(7,1.33)–cycle,mediumgray); draw((6,.33)–(7,.33)–(7,1.33)–(6,1.33)–cycle,gray); fill((6,1.67)–(7,2.67)–(6,2.67)–cycle,mediumgray); draw((6,1.67)–(7,1.67)–(7,2.67)–(6,2.67)–cycle,gray); fill((7.33,.33)–(8.33,.33)–(7.33,1.33)–cycle,mediumgray); draw((7.33,.33)–(8.33,.33)–(8.33,1.33)–(7.33,1.33)–cycle,gray); fill((8.33,1.67)–(8.33,2.67)–(7.33,2.67)–cycle,mediumgray); draw((7.33,1.67)–(8.33,1.67)–(8.33,2.67)–(7.33,2.67)–cycle,gray); \{}end{asy} \{}end{center}

    What is the probability that the tiling will contain a large gray diamond in one of the smaller $2 \times 2$ grids? Below is an example of such tiling.

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; size(2cm);

    fill((1,0)–(0,1)–(0,2)–(1,1)–cycle,mediumgray); fill((2,0)–(3,1)–(2,2)–(1,1)–cycle,mediumgray); fill((1,2)–(1,3)–(0,3)–cycle,mediumgray); fill((1,2)–(2,2)–(2,3)–cycle,mediumgray); fill((3,2)–(3,3)–(2,3)–cycle,mediumgray);

    draw((0,0)–(3,0)–(3,3)–(0,3)–cycle,gray); draw((1,0)–(1,3)–(2,3)–(2,0),gray); draw((0,1)–(3,1)–(3,2)–(0,2),gray); \{}end{asy} \{}end{center}

    **(A)** $\frac{1}{1024}$ **(B)** $\frac{1}{256}$ **(C)** $\frac{1}{64}$ **(D)** $\frac{1}{16}$ **(E)** $\frac{1}{4}$

23. Isosceles triangle $ABC$ has equal side lengths $AB$ and $BC$. In the figures below, segments are drawn parallel to $\overline{AC}$ so that the shaded portions of $\triangle ABC$ have the same area. The heights of the two unshaded portions are 11 and 5 units, respectively. What is the height $h$ of $\triangle ABC$?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; //Diagram by TheMathGuyd size(12cm); real h = 2.5; // height real g=4; //c2c space real s = 0.65; //Xcord of Hline real adj = 0.08; //adjust line diffs pair A,B,C; B=(0,h); C=(1,0); A=-conj(C); pair PONE=(s,h*(1-s)); //Endpoint of Hline ONE pair PTWO=(s+adj,h*(1-s-adj)); //Endpoint of Hline ONE path LONE=PONE–(-conj(PONE)); //Hline ONE path LTWO=PTWO–(-conj(PTWO)); path T=A–B–C–cycle; //Triangle

    fill (shift(g,0)*(LTWO–B–cycle),mediumgrey); fill (LONE–A–C–cycle,mediumgrey);

    draw(LONE); draw(T); label("\$A\$",A,SW); label("\$B\$",B,N); label("\$C\$",C,SE);

    draw(shift(g,0)*LTWO); draw(shift(g,0)*T); label("\$A\$",shift(g,0)*A,SW); label("\$B\$",shift(g,0)*B,N); label("\$C\$",shift(g,0)*C,SE);

    draw(B–shift(g,0)*B,dashed); draw(C–shift(g,0)*A,dashed); draw((g/2,0)–(g/2,h),dashed); draw((0,h*(1-s))–B,dashed); draw((g,h*(1-s-adj))–(g,0),dashed); label("\$5\$", midpoint((g,h*(1-s-adj))–(g,0)),UnFill); label("\$h\$", midpoint((g/2,0)–(g/2,h)),UnFill); label("\$11\$", midpoint((0,h*(1-s))–B),UnFill); \{}end{asy} \{}end{center}

    **(A)** 14.6 **(B)** 14.8 **(C)** 15 **(D)** 15.2 **(E)** 15.4

24. Fifteen integers $a_1, a_2, a_3, \dots, a_{15}$ are arranged in order on a number line. The integers are equally spaced and have the property that

    $$1 \le a_1 \le 10, \ 13 \le a_2 \le 20, \ \text{ and } \ 241 \le a_{15} \le 250.$$

    What is the sum of digits of $a_{14}$?

    **(A)** 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

# 2024 AMC 8 Problems

1. What is the unit's digit of:

$$222{,}222 - 22{,}222 - 2{,}222 - 222 - 22 - 2?$$

**(A)** 0 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 10

2. What is the value of this expression in decimal form?

$$\frac{44}{11} + \frac{110}{44} + \frac{44}{1100}$$

**(A)** 6.4 **(B)** 6.504 **(C)** 6.54 **(D)** 6.9 **(E)** 6.94

3. Four squares of side length $4, 7, 9,$ and $10$ are arranged in increasing size order so that their left edges and bottom edges align. The squares alternate in color white-gray-white-gray, respectively, as shown in the figure. What is the area of the visible gray region in square units?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; size(150); filldraw((0,0)–(10,0)–(10,10)–(0,10)–cycle,gray(0.7),linewidth(1)); filldraw((0,0)–(9,0)–(9,9)–(0,9)–cycle,white,linewidth(1)); filldraw((0,0)–(7,0)–(7,7)–(0,7)–cycle,gray(0.7),linewidth(1)); filldraw((0,0)–(4,0)–(4,4)–(0,4)–cycle,white,linewidth(1)); draw((11,0)–(11,4),linewidth(1)); draw((11,6)–(11,10),linewidth(1)); label(”$10$”,(11,5),fontsize(14pt)); draw((10.75,0)–(11.25,0),linewidth(1)); draw((10.75,10)–(11.25,10),linewidth(1)); draw((0,11)–(3,11),linewidth(1)); draw((5,11)–(9,11),linewidth(1)); draw((0,11.25)–(0,10.75),linewidth(1)); draw((9,11.25)–(9,10.75),linewidth(1)); label(”$9$”,(4,11),f draw((-1,0)–(-1,1),linewidth(1)); draw((-1,3)–(-1,7),linewidth(1)); draw((-1.25,0)–(-0.75,0),linewidth(1)); draw((-1.25,7)–(-0.75,7),linewidth(1)); label(”$7$”,(-1,2),fontsize(14pt)); draw((0,-1)–(1,-1),linewidth(1)); draw((3,-1)–(4,-1),linewidth(1)); draw((0,-1.25)–(0,-.75),linewidth(1)); draw((4,-1.25)–(4,-.75),linewidth(1)); label(”$4$”,(2,-1),fontsize(14pt)); \{}end{asy} \{}end{center}

**(A)** 42 **(B)** 45 **(C)** 49 **(D)** 50 **(E)** 52

4. When Yunji added all the integers from 1 to 9, she mistakenly left out a number. Her sum turned out to be a square number. What number did Yunji leave out?

**(A)** 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

5. Aaliyah rolls two standard 6-sided dice. She notices that the product of the two numbers rolled is a multiple of 6. Which of the following integers cannot be the sum of the two numbers?

**(A)** 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

6. Sergei skated around an ice rink, gliding along different paths. The gray lines in the figures below show four of the paths labeled $P$, $Q$, $R$, and $S$. What is the sorted order of the four paths from shortest to longest?

**(A)** $P, Q, R, S$ **(B)** $P, R, S, Q$ **(C)** $Q, S, P, R$ **(D)** $R, P, S, Q$ **(E)** $R, S, P, Q$

7. A $3 \times 7$ rectangle is covered without overlap by 3 shapes of tiles: $2 \times 2$, $1 \times 4$, and $1 \times 1$, shown below. What is the minimum possible number of $1 \times 1$ tiles used?

**(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

8. On Monday Taye has \$2. Every day, he either gains \$3 or doubles the amount of money he had on the previous day. How many different dollar amounts could Taye have on Thursday, 3 days later?

**(A)** 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

9. All the marbles in Maria's collection are red, green, or blue. Maria has half as many red marbles as green marbles and twice as many blue marbles as green marbles. Which of the following could be the total number of marbles in Maria's collection?

**(A)** 24 **(B)** 25 **(C)** 26 **(D)** 27 **(E)** 28

10. In January 1980 the Moana Loa Observation recorded carbon dioxide ($CO_2$) levels of 338 ppm (parts per million). Over the years the average $CO_2$ reading has increased by about 1.515 ppm each year. What is the expected $CO_2$ level in ppm in January 2030? Round your answer to the nearest integer.

**(A)** 399 **(B)** 414 **(C)** 420 **(D)** 444 **(E)** 459

11. The coordinates of $\triangle ABC$ are $A(5,7)$, $B(11,7)$, and $C(3,y)$, with $y > 7$. The area of $\triangle ABC$ is 12. What is the value of $y$?

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; // Diagram inaccurate to prevent measuring with ruler. size(10cm); draw((3,10)–(11,7)–(5,7)–(3,10));

    dot((5,7)); label("$A(5,7)$",(5,7),S); dot((11,7)); label("$B(11,7)$",(11,7),S);

    dot((3,10)); label("$C(3,y)$",(3,10),NW);

    // Problem 11: put on here by Andrei.martynau \{}end{asy} \{}end{center}

    **(A)** 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

12. Rohan keeps a total of 90 guppies in 4 fish tanks.

    *There is 1 more guppy in the 2nd tank than the 1st tank.

    *There are 2 more guppies in the 3rd tank than the 2nd tank.

    *There are 3 more guppies in the 4th tank than the 3rd tank.

    How many guppies are in the 4th tank?

    **(A)** 20 **(B)** 21 **(C)** 23 **(D)** 24 **(E)** 26

13. Buzz Bunny is hopping up and down a set of stairs, one step at a time. In how many ways can Buzz Bunny start on the ground, make a sequence of 6 hops, and end up back on the ground? (For example, one sequence of hops is up-up-down-down-up-down.)

    \{}begin{center} \{}begin{asy} import olympiad; import cse5; /* AMC8 P13 2024, revised by Teacher David */ /** * This Geometry Artwork/Graph is designed using GeoSketch v1.0, * a free software tool created by Tina Yan, William Zhong, and * Teacher David. * * For more information, please refer to * https://geosketch.org (under construction) */ defaultpen(linewidth(1pt)); unitsize(0.3pt); import graph; /** * Define a quadratic bezier curve function. */ typedef pair quad_bezier(real t); quad_bezier fungen (pair a, pair b, pair c) { return new pair (real t) { real x = (1-t)*(1-t)*a.x + 2*(1-t)*t*b.x + t*t*c.x; real y = (1-t)*(1-t)*a.y + 2*(1-t)*t*b.y + t*t*c.y; return (x,y); }; }

    quad_bezier qb0 = fungen((293,243),(237,276),(239,310)); draw(graph(qb0, 0,1)); quad_bezier qb1 = fungen((239,310),(274,301),(295,254)); draw(graph(qb1, 0,1)); quad_bezier qb2 = fungen((266,294),(260,309),(266,323)); draw(graph(qb2, 0,1)); quad_bezier qb3 = fungen((266,323),(294,311),(302,257)); draw(graph(qb3, 0,1)); quad_bezier qb4 = fungen((333,258),(341,249),(348,244)); draw(graph(qb4, 0,1)); quad_bezier qb5 = fungen((348,244),(355,241),(351,234)); draw(graph(qb5, 0,1)); quad_bezier qb6 = fungen((351,234),(348,226),(338,226)); draw(graph(qb6, 0,1)); quad_bezier qb7 = fungen((351,234),(350,219),(321,208)); draw(graph(qb7, 0,1)); quad_bezier qb8 = fungen((260,247),(135,293),(137,170)); draw(graph(qb8, 0,1)); quad_bezier qb9 = fungen((122,161),(132,147),(148,144)); draw(graph(qb9, 0,1)); quad_bezier qb10 = fungen((148,144),(176,155),(204,146)); draw(graph(qb10, 0,1)); quad_bezier qb11 = fungen((204,146),(216,141),(235,137)); draw(graph(qb11, 0,1)); quad_bezier qb12 = fungen((228,156),(208,160),(188,161)); draw(graph(qb12, 0,1)); quad_bezier qb13 = fungen((319,214),(313,174),(283,168)); draw(graph(qb13, 0,1)); quad_bezier qb14 = fungen((228,156),(242,158),(247,171 draw(graph(qb14, 0,1)); quad_bezier qb15 = fungen((245,181),(250,158),(266,143)); draw(graph(qb15, 0,1)); quad_bezier qb16 = fungen((266,143),(287,134),(298,135)); draw(graph(qb16, 0,1)); quad_bezier qb17 = fungen((298,135),(309,143),(300,148)); draw(graph(qb17, 0,1)); quad_bezier qb18 = fungen((300,148),(272,150),(270,175 draw(graph(qb18, 0,1)); quad_bezier qb19 = fungen((282,177),(274,158),(300,148)); draw(graph(qb19, 0,1));

    draw(arc((317.8948497854077,245.25965665236052), 19.760615163024095, 143.54947493250435, 40.14574559948477)); draw(arc((282.65584415584414,295.7857142857143), 53.78971270402217, -78.91253214600312, -114.90992209204622)); draw(arc((127.7,168.5), 9.420191080864546, 9.162347045721713, 232.7651660184253)); draw(arc((229.125,145.625), 10.435815732370902, -55.73889710090544, 96.1886159632416)); draw(arc((186.26470588235293,181.5), 20.5733139205802 -85.1615330431756, 85.1615330431756)); filldraw(ellipse((314,235), 13.0, 10.04987562112089), rgb(254,255,255), black); filldraw(rotate(14.036243467926468,(315,251))*ellipse((315,235), 9.219544457292887, 8.246211251235321), rgb(0,0,0), black);

    pair o = (400,190); real len=80; real height=56; for (int i=0; i¡4; ++i) { pair a = (i*len, i*height); path p = a – a+(len,0) – a+(len, height); draw(shift(o)*p); } path p = (0,0)–(0,-height)–(4*len,-height); draw(shift(o)*p); \{}end{asy} \{}end{center}

    **(A)** 4 **(B)** 5 **(C)** 6 **(D)** 8 **(E)** 12

14. What is the distance of the shortest route from A to Z?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; /* AMC8 P14 2024, by NUMANA: BUI VAN HIEU */ import graph; unitsize(2cm); real r=0.25; // Define the nodes and their positions pair[] nodes = { (0,0), (2,0), (1,1), (3,1), (4,0), (6,0) }; string[] labels = { "A", "M", "X", "Y", "C", "Z" };

// Draw the nodes as circles with labels for(int i = 0; i ¡ nodes.length; ++i) { draw(circle(nodes[i], r)); label("$" + labels[i] + "$", nodes[i]); } // Define the edges with their node indices and labels int[][] edges = { {0, 1}, {0, 2}, {2, 1}, {2, 3}, {1, 3}, {1, 4}, {3, 4}, {4, 5}, {3, 5} }; string[] edgeLabels = { "8", "5", "2", "10", "6", "14", "5", "10", "17" }; pair[] edgeLabelsPos = { S, SE, SW, S, SE, S, SW, S, NE}; // Draw the edges with labels for (int i = 0; i ¡ edges.length; ++i) { pair start = nodes[edges[i][0]]; pair end = nodes[edges[i][1]]; draw(start + r*dir(end-start) – end-r*dir(end-start), Arrow); label("$" + edgeLabels[i] + "$", midpoint(start – end), edgeLabelsPos[i]); } // Draw the curved edge with label draw(nodes[1]+r * dir(-45)..controls (3, -0.75) and (5, -0.75)..nodes[5]+r * dir(-135), Arrow); label("$25$", midpoint(nodes[1]..controls (3, -0.75) and (5, -0.75)..nodes[5]), 2S); \{}end{asy} \{}end{center}

**(A)** 28 **(B)** 29 **(C)** 30 **(D)** 31 **(E)** 32

15. Let the letters $F, L, Y, B, U, G$ represent distinct digits. Suppose $\underline{F}\ \underline{L}\ \underline{Y}\ \underline{F}\ \underline{L}\ \underline{Y}$ is the GREATEST number that satisfies the equation

$$8 \cdot \underline{F}\ \underline{L}\ \underline{Y}\ \underline{F}\ \underline{L}\ \underline{Y} = \underline{B}\ \underline{U}\ \underline{G}\ \underline{B}\ \underline{U}\ \underline{G}.$$

What is the value of $\underline{F}\ \underline{L}\ \underline{Y} + \underline{B}\ \underline{U}\ \underline{G}$?

**(A)** 1089 **(B)** 1098 **(C)** 1107 **(D)** 1116 **(E)** 1125

16. Minh enters the numbers 1 through 81 into the cells of a $9 \times 9$ grid in some order. She calculates the product of the numbers in each row and column. What is the least number of rows and columns that could have a product divisible by 3?

**(A)** 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

17. A chess king is said to "attack" all squares one step away from it (basically any square right next to it in any direction), horizontally, vertically, or diagonally. For instance, a king on the center square of a 3 x 3 grid attacks all 8 other squares, as shown below. Suppose a white king and a black king are placed on different squares of 3 x 3 grid so that they do not attack each other. In how many ways can this be done?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; /* AMC8 P17 2024, revised by Teacher David */ unitsize(29pt); import math; add(grid(3,3));

pair [] a = {(0.5,0.5), (0.5, 1.5), (0.5, 2.5), (1.5, 2.5), (2.5,2.5), (2.5,1.5), (2.5,0.5), (1.5,0.5)};

for (int i=0; i¡a.length; ++i) { pair x = (1.5,1.5) + 0.4*dir(225-45*i); draw(x – a[i], arrow=EndArrow()); }

label("$K$", (1.5,1.5)); \{}end{asy} \{}end{center}

**(A)** 20 **(B)** 24 **(C)** 27 **(D)** 28 **(E)** 32

18. Three concentric circles centered at $O$ have a radius of $1, 2,$ and $3$. Points $B$ and $C$ lie on the largest circle. The region between the two smaller circles is shaded, as is the portion of the region between the two larger circles bounded by central angle $BOC$, as shown in the figure below. Suppose the shaded and unshaded regions are equal in area. What is the measure of $\angle BOC$ in degrees?

\{}begin{center} \{}begin{asy} import olympiad; import cse5; size(100); import graph;

draw(circle((0,0),3)); real radius = 3; real angleStart = -54; // starting angle of the sector real angleEnd = 54; // ending angle of the sector label("$O$",(0,0),W); pair O = (0, 0); filldraw(arc(O, radius, angleStart, angleEnd)–O–cycle, lightgray); filldraw(circle((0,0),2),lightgray); filldraw(circle((0,0),1),white); draw((1.763,2.427)–(0,0)–(1.763,-2.427)); label("$B$",(1.763,2.427),NE); label("$C$",(1.763,-2.427),SE); \{}end{asy} \{}end{center}

**(A)** 108 **(B)** 120 **(C)** 135 **(D)** 144 **(E)** 150

19. Jordan owns 15 pairs of sneakers. Three fifths of the pairs are red and the rest are white. Two thirds of the pairs are high-top and the rest are low-top. The red high-top sneakers make up a fraction of the collection. What is the least possible value of this fraction?

**(A)** 0 **(B)** $\frac{1}{5}$ **(C)** $\frac{4}{15}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{2}{5}$

20. Any three vertices of the cube $PQRSTUVW$, shown in the figure below, can be connected to form a triangle. (For example, vertices $P, Q$, and $R$ can be connected to form $\triangle PQR$.) How many of these triangles are equilateral and contain $P$ as a vertex?

 \{}begin{center} \{}begin{asy} import olympiad; import cse5; unitsize(4); pair P,Q,R,S,T,U,V,W; P=(0,30); Q=(30,30); R=(40,40); S=(10,40); T=(10,10); U=(40,10); V=(30,0); W=(0,0); draw(W–V); draw(V–Q); draw(Q–P); draw(P–W); draw(T–U); draw(U–R); draw(R–S); draw(S–T); draw(W–T); draw(P–S); draw(V–U); draw(Q–R); dot(P); dot(Q); dot(R); dot(S); dot(T); dot(U); dot(V); dot(W); label("$P$",P,NW); label("$Q$",Q,NW); label("$R$",R,NE); label("$S$",S,N); label("$T$",T,NE); label("$U$",U,NE); label("$V$",V,SE); label("$W$",W,SW); \{}end{asy} \{}end{center}

 **(A)** 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 6

21. A group of frogs (called an "army") is living in a tree. A frog turns green when in the shade and turns yellow when in the sun. Initially, the ratio of green to yellow frogs was $3:1$. Then 3 green frogs moved to the sunny side and 5 yellow frogs moved to the shady side. Now the ratio is $4:1$. What is the difference between the number of green frogs and the number of yellow frogs now?

 **(A)** 10 **(B)** 12 **(C)** 16 **(D)** 20 **(E)** 24

22. A roll of tape is 4 inches in diameter and is wrapped around a ring that is 2 inches in diameter. A cross section of the tape is shown in the figure below. The tape is 0.015 inches thick. If the tape is completely unrolled, approximately how long would it be? Round your answer to the nearest 100 inches.

 \{}begin{center} \{}begin{asy} import olympiad; import cse5; /* AMC8 P22 2024, revised by Teacher David */ size(150);

 pair o = (0,0); real r1 = 1; real r2 = 2;

 filldraw(circle(o, r2), mediumgray, linewidth(1pt)); filldraw(circle(o, r1), white, linewidth(1pt));

 draw((-2,-2.6)–(-2,-2.4)); draw((2,-2.6)–(2,-2.4)); draw((-2,-2.5)–(2,-2.5), L=Label("4 in."));

 draw((-1,0)–(1,0), L=Label("2 in.", align=(0,1)), arrow=Arrows());

 draw((2,0)–(2,-1.3), linewidth(1pt)); \{}end{asy} \{}end{center}

 **(A)** 300 **(B)** 600 **(C)** 1200 **(D)** 1500 **(E)** 1800

23. Rodrigo has a very large sheet of graph paper. First he draws a line segment connecting point $(0,4)$ to point $(2,0)$ and colors the 4 cells whose interiors intersect the segment, as shown below. Next Rodrigo draws a line segment connecting point $(2000,3000)$ to point $(5000,8000)$. How many cells will he color this time?

 \{}begin{center} \{}begin{asy} import olympiad; import cse5; filldraw((0,4)–(1,4)–(1,3)–(0,3)–cycle, gray(.75), gray(.5)+linewidth(1)); filldraw((0,3)–(1,3)–(1,2)–(0,2)–cycle, gray(.75), gray(.5)+linewidth(1)); filldraw((1,2)–(2,2)–(2,1)–(1,1)–cycle, gray(.75), gray(.5)+linewidth(1)); filldraw((1,1)–(2,1)–(2,0)–(1,0)–cycle, gray(.75), gray(.5)+linewidth(1));

 draw((-1,5)–(-1,-1),gray(.9)); draw((0,5)–(0,-1),gray(.9)); draw((1,5)–(1,-1),gray(.9)); draw((2,5)–(2,-1),gray(.9)); draw((3,5)–(3,-1),gray(.9)); draw((4,5)–(4,-1),gray(.9)); draw((5,5)–(5,-1),gray(.9));

 draw((-1,5)–(5, 5),gray(.9)); draw((-1,4)–(5,4),gray(.9)); draw((-1,3)–(5,3),gray(.9)); draw((-1,2)–(5,2),gray(.9)); draw((-1,1)–(5,1),gray(.9)); draw((-1,0)–(5,0),gray(.9)); draw((-1,-1)–(5,-1),gray(.9));

 dot((0,4)); label("$(0,4)$",(0,4),NW);

 dot((2,0)); label("$(2,0)$",(2,0),SE);

 draw((0,4)–(2,0));

 draw((-1,0) – (5,0), arrow=Arrow); draw((0,-1) – (0,5), arrow=Arrow); \{}end{asy} \{}end{center}

 **(A)** 6000 **(B)** 6500 **(C)** 7000 **(D)** 7500 **(E)** 8000

24. Jean has made a piece of stained glass art in the shape of two mountains, as shown in the figure below. One mountain peak is 8 feet high while the other peak is 12 feet high. Each peak forms a $90^\circ$ angle, and the straight sides form a $45^\circ$ angle with the ground. The artwork has an area of 183 square feet. The sides of the mountain meet at an intersection point near the center of the artwork, $h$ feet above the ground. What is the value of $h$?

 \{}begin{center} \{}begin{asy} import olympiad; import cse5; unitsize(.3cm); filldraw((0,0)–(8,8)–(11,5)–(18,12)–(30,0)–cycle,gray(0.7),linewidth(1)); draw((-1,0)–(-1,8),linewidth(.75)); draw((-1.4,0)–(-.6,0),linewidth(.75));

draw((-1.4,8)–(-.6,8),linewidth(.75)); label("$8$",(-1,4),W); label("$12$",(31,6),E); draw((-1,8)–(8,8),dashed); draw((31,0)–(31,12),linewidth(.75)); draw((30.6,0)–(31.4,0),linewidth(.75)); draw((30.6,12)–(31.4,12),linewidth(.75)); draw((31,12)–(18,12),dashed); label("$45^{\{}circ}$",(.75,0),NE,fontsize(10pt)); label("$45^{\{}circ}$",(29.25,0),NW, draw((8,8)–(7.5,7.5)–(8,7)–(8.5,7.5)–cycle); draw((18,12)–(17.5,11.5)–(18,11)–(18.5,11.5)–cycle); draw((11,5)–(11,0),dashed); label("$h$",(11,2.5),E); \{}end{asy} \{}end{center}

**(A)** 4 **(B)** 5 **(C)** $4\sqrt{2}$ **(D)** 6 **(E)** $5\sqrt{2}$

25. A airplane has 4 rows of seats with 3 seats in each row. Eight passengers have boarded the plane and are distributed randomly among the seats. A married couple is next to board. What is the probability there will be 2 adjacent seats in the same row for the couple?

**(A)** $\frac{8}{15}$ **(B)** $\frac{32}{55}$ **(C)** $\frac{20}{33}$ **(D)** $\frac{34}{55}$ **(E)** $\frac{8}{11}$