# AIME Problems
2017

Problems

1. Find the number of subsets of $\{1, 2, 3, 4, 5, 6, 7, 8\}$ that are subsets of neither $\{1, 2, 3, 4, 5\}$ nor $\{4, 5, 6, 7, 8\}$.

2. Teams T_1, T_2, T_3, and T_4 are in the playoffs. In the semifinal matches, T_1 plays T_4, and T_2 plays T_3. The winners of those two matches will play each other in the final match to determine the champion. When T_i plays T_j, the probability that T_i wins is $\frac{i}{i+j}$, and the outcomes of all the matches are independent. The probability that T_4 will be the champion is $\frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.

3. A triangle has vertices $A(0, 0)$, $B(12, 0)$, and $C(8, 10)$. The probability that a randomly chosen point inside the triangle is closer to vertex B than to either vertex A or vertex C can be written as $\frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.

4. Find the number of positive integers less than or equal to 2017 whose base-three representation contains no digit equal to 0.

5. A set contains four numbers. The six pairwise sums of distinct elements of the set, in no particular order, are 189, 320, 287, 234, x, and y. Find the greatest possible value of $x + y$.

6. Find the sum of all positive integers n such that $\sqrt{n^2 + 85n + 2017}$ is an integer.

7. Find the number of integer values of k in the closed interval $[-500, 500]$ for which the equation $\log(kx) = 2\log(x + 2)$ has exactly one real solution.

8. Find the number of positive integers n less than 2017 such that

$$1 + n + \frac{n^2}{2!} + \frac{n^3}{3!} + \frac{n^4}{4!} + \frac{n^5}{5!} + \frac{n^6}{6!}$$

   is an integer.

9. A special deck of cards contains 49 cards, each labeled with a number from 1 to 7 and colored with one of seven colors. Each number-color combination appears on exactly one card. Sharon will select a set of eight cards from the deck at random. Given that she gets at least one card of each color and at least one card with each number, the probability that Sharon can discard one of her cards and *still* have at least one card of each color and at least one card with each number is $\frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.

10. Rectangle $ABCD$ has side lengths $AB = 84$ and $AD = 42$. Point M is the midpoint of $\overline{AD}$, point N is the trisection point of $\overline{AB}$ closer to A, and point O is the intersection of $\overline{CM}$ and $\overline{DN}$. Point P lies on the quadrilateral $BCON$, and $\overline{BP}$ bisects the area of $BCON$. Find the area of $\triangle CDP$.

11. Five towns are connected by a system of roads. There is exactly one road connecting each pair of towns. Find the number of ways there are to make all the roads one-way in such a way that it is still possible to get from any town to any other town using the roads (possibly passing through other towns on the way).

12. Circle C_0 has radius 1, and the point A_0 is a point on the circle. Circle C_1 has radius $r < 1$ and is internally tangent to C_0 at point A_0. Point A_1 lies on circle C_1 so that A_1 is located 90° counterclockwise from A_0 on C_1. Circle C_2 has radius r^2 and is internally tangent to C_1 at point A_1. In this way a sequence of circles $C_1, C_2, C_3, \ldots$ and a sequence of points on the circles $A_1, A_2, A_3, \ldots$ are constructed, where circle C_n has radius r^n and is internally tangent to circle C_{n-1} at point A_{n-1}, and point A_n lies on C_n 90° counterclockwise from point A_{n-1}, as shown in the figure below. There is one point B inside all of these circles. When $r = \frac{11}{60}$, the distance from the center C_0 to B is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.
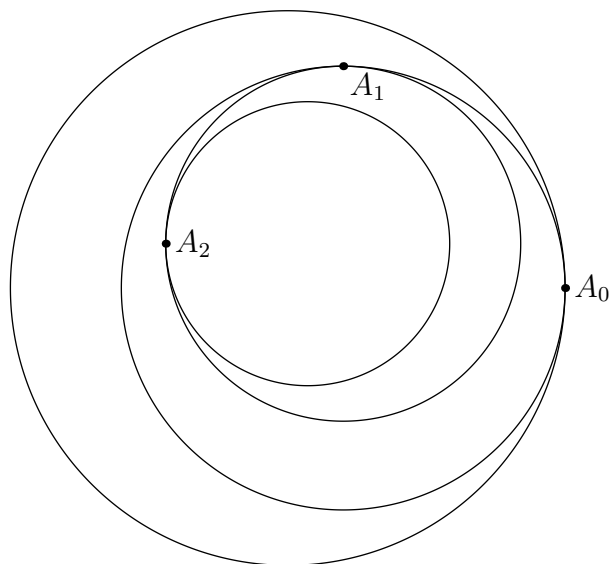
13. For each integer $n \geq 3$, let $f(n)$ be the number of 3-element subsets of the vertices of a regular n-gon that are the vertices of an isosceles triangle (including equilateral triangles). Find the sum of all values of n such that $f(n+1) = f(n) + 78$.

14. A $10 \times 10 \times 10$ grid of points consists of all points in space of the form (i, j, k), where i, j, and k are integers between 1 and 10, inclusive. Find the number of different lines that contain exactly 8 of these points.

15. Tetrahedron $ABCD$ has $AD = BC = 28$, $AC = BD = 44$, and $AB = CD = 52$. For any point X in space, define $f(X) = AX + BX + CX + DX$. The least possible value of $f(X)$ can be expressed as $m\sqrt{n}$, where m and n are positive integers, and n is not divisible by the square of any prime. Find $m + n$.

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