

AMC 8 Problems
2024

2024 AMC 8

1. What is the unit's digit of:

$$222,222 - 22,222 - 2,222 - 222 - 22 - 2?$$

(A) 0 (B) 2 (C) 4 (D) 8 (E) 10

2. What is the value of this expression in decimal form?

$$\frac{44}{11} + \frac{110}{44} + \frac{44}{1100}$$

(A) 6.4 (B) 6.504 (C) 6.54 (D) 6.9 (E) 6.94

3. Four squares of side length 4, 7, 9, and 10 are arranged in increasing size order so that their left edges and bottom edges align. The squares alternate in color white-gray-white-gray, respectively, as shown in the figure. What is the area of the visible gray region in square units?

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\{\}\begin{center} \{\}\begin{asy} import olympiad; import cse5; size(150); filldraw((0,0)--(10,0)--(10,10)--(0,10)--cycle,gray(0.7),linewidth(1)); filldraw((0,0)--(9,0)--(9,9)--(0,9)--cycle,white,linewidth(1)); filldraw((0,0)--(7,0)--(7,7)--(0,7)--cycle,gray(0.7),linewidth(1)); filldraw((0,0)--(4,0)--(4,4)--(0,4)--cycle,white,linewidth(1)); draw((11,0)--(11,4),linewidth(1)); draw((11,6)--(11,10),linewidth(1)); label("$10$", (11,5), fontsize(14pt)); draw((10.75,0)--(11.25,0),linewidth(1)); draw((10.75,10)--(11.25,10),linewidth(1)); draw((0,11)--(3,11),linewidth(1)); draw((5,11)--(9,11),linewidth(1)); draw((0,11.25)--(0,10.75),linewidth(1)); draw((9,11.25)--(9,10.75),linewidth(1)); label("$9$", (4,11), fontsize(14pt)); draw((-1,0)--(-1,1),linewidth(1)); draw((-1,3)--(-1,7),linewidth(1)); draw((-1.25,0)--(-0.75,0),linewidth(1)); draw((-1.25,7)--(-0.75,7),linewidth(1)); label("$7$", (-1,2), fontsize(14pt)); draw((0,-1)--(1,-1),linewidth(1)); draw((3,-1)--(4,-1),linewidth(1)); draw((0,-1.25)--(0,-.75),linewidth(1)); draw((4,-1.25)--(4,-.75),linewidth(1)); label("$4$", (2,-1), fontsize(14pt)); \{\}\end{asy} \{\}\end{center}
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(A) 42 (B) 45 (C) 49 (D) 50 (E) 52

4. When Yunji added all the integers from 1 to 9, she mistakenly left out a number. Her sum turned out to be a square number. What number did Yunji leave out?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

5. Aaliyah rolls two standard 6-sided dice. She notices that the product of the two numbers rolled is a multiple of 6. Which of the following integers cannot be the sum of the two numbers?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

6. Sergei skated around an ice rink, gliding along different paths. The gray lines in the figures below show four of the paths labeled P , Q , R , and S . What is the sorted order of the four paths from shortest to longest?

(A) P, Q, R, S (B) P, R, S, Q (C) Q, S, P, R (D) R, P, S, Q (E) R, S, P, Q

7. A 3×7 rectangle is covered without overlap by 3 shapes of tiles: 2×2 , 1×4 , and 1×1 , shown below. What is the minimum possible number of 1×1 tiles used?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

8. On Monday Taye has \$2. Every day, he either gains \$3 or doubles the amount of money he had on the previous day. How many different dollar amounts could Taye have on Thursday, 3 days later?

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

9. All the marbles in Maria's collection are red, green, or blue. Maria has half as many red marbles as green marbles and twice as many blue marbles as green marbles. Which of the following could be the total number of marbles in Maria's collection?

(A) 24 (B) 25 (C) 26 (D) 27 (E) 28

10. In January 1980 the Moana Loa Observation recorded carbon dioxide (CO_2) levels of 338 ppm (parts per million). Over the years the average CO_2 reading has increased by about 1.515 ppm each year. What is the expected CO_2 level in ppm in January 2030? Round your answer to the nearest integer.

(A) 399 (B) 414 (C) 420 (D) 444 (E) 459

11. The coordinates of $\triangle ABC$ are $A(5, 7)$, $B(11, 7)$, and $C(3, y)$, with $y > 7$. The area of $\triangle ABC$ is 12. What is the value of y ?

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\begin{center} \begin{asy} import olympiad; import cse5; // Diagram inaccurate to prevent measuring with ruler. size(10cm); draw((3,10)--(11,7)--(5,7)--(3,10));
dot((5,7)); label("$A(5,7)$",(5,7),S); dot((11,7)); label("$B(11,7)$",(11,7),S);
dot((3,10)); label("$C(3,y)$",(3,10),NW);
// Problem 11: put on here by Andrei.martynau \end{asy} \end{center}
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(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

12. Rohan keeps a total of 90 guppies in 4 fish tanks.

*There is 1 more guppy in the 2nd tank than the 1st tank.

*There are 2 more guppies in the 3rd tank than the 2nd tank.

*There are 3 more guppies in the 4th tank than the 3rd tank.

How many guppies are in the 4th tank?

(A) 20 (B) 21 (C) 23 (D) 24 (E) 26

13. Buzz Bunny is hopping up and down a set of stairs, one step at a time. In how many ways can Buzz Bunny start on the ground, make a sequence of 6 hops, and end up back on the ground? (For example, one sequence of hops is up-up-down-down-up-down.)

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\begin{center} \begin{asy} import olympiad; import cse5; /* AMC8 P13 2024, revised by Teacher David */ /** * This Geometry Artwork/Graph is designed using GeoSketch v1.0, * a free software tool created by Tina Yan, William Zhong, and * Teacher David. * * For more information, please refer to * https://geosketch.org (under construction) */ defaultpen(linewidth(1pt)); unitsize(0.3pt); import graph;
/** * Define a quadratic bezier curve function. */ typedef pair quad_bezier(real t); quad_bezier funge(pair a, pair b, pair c) { return new pair (real t) { real x = (1-t)*(1-t)*a.x + 2*(1-t)*t*b.x + t*t*c.x; real y = (1-t)*(1-t)*a.y + 2*(1-t)*t*b.y + t*t*c.y; return (x,y); }; }
quad_bezier qb0 = funge((293,243),(237,276),(239,310)); draw(graph(qb0, 0,1)); quad_bezier qb1 = funge((239,310),(274,301),(295,254)); draw(graph(qb1, 0,1)); quad_bezier qb2 = funge((266,294),(260,309),(266,323)); draw(graph(qb2, 0,1)); quad_bezier qb3 = funge((266,323),(294,311),(302,257)); draw(graph(qb3, 0,1));
quad_bezier qb4 = funge((333,258),(341,249),(348,244)); draw(graph(qb4, 0,1)); quad_bezier qb5 = funge((348,244),(355,241),(351,234)); draw(graph(qb5, 0,1)); quad_bezier qb6 = funge((351,234),(348,226),(338,226)); draw(graph(qb6, 0,1)); quad_bezier qb7 = funge((351,234),(350,219),(321,208)); draw(graph(qb7, 0,1));
quad_bezier qb8 = funge((260,247),(135,293),(137,170)); draw(graph(qb8, 0,1)); quad_bezier qb9 = funge((122,161),(132,147),(148,144)); draw(graph(qb9, 0,1)); quad_bezier qb10 = funge((148,144),(176,155),(204,146)); draw(graph(qb10, 0,1)); quad_bezier qb11 = funge((204,146),(216,141),(235,137)); draw(graph(qb11, 0,1));
quad_bezier qb12 = funge((228,156),(208,160),(188,161)); draw(graph(qb12, 0,1)); quad_bezier qb13 = funge((319,214),(313,174),(283,168)); draw(graph(qb13, 0,1)); quad_bezier qb14 = funge((228,156),(242,158),(247,171)); draw(graph(qb14, 0,1)); quad_bezier qb15 = funge((245,181),(250,158),(266,143)); draw(graph(qb15, 0,1));
quad_bezier qb16 = funge((266,143),(287,134),(298,135)); draw(graph(qb16, 0,1)); quad_bezier qb17 = funge((298,135),(309,143),(300,148)); draw(graph(qb17, 0,1)); quad_bezier qb18 = funge((300,148),(272,150),(270,175)); draw(graph(qb18, 0,1)); quad_bezier qb19 = funge((282,177),(274,158),(300,148)); draw(graph(qb19, 0,1));
draw(arc((317.8948497854077,245.25965665236052), 19.760615163024095, 143.54947493250435, 40.14574559948477));
draw(arc((282.65584415584414,295.7857142857143), 53.78971270402217, -78.91253214600312, -114.90992209204622));
draw(arc((127.7,168.5), 9.420191080864546, 9.162347045721713, 232.7651660184253)); draw(arc((229.125,145.625), 10.435815732370902, -55.73889710090544, 96.1886159632416)); draw(arc((186.26470588235293,181.5), 20.573313920580, -85.1615330431756, 85.1615330431756)); filldraw(ellipse((314,235), 13.0, 10.04987562112089), rgb(254,255,255), black); filldraw(rotate(14.036243467926468,(315,251))*ellipse((315,235), 9.219544457292887, 8.246211251235321), rgb(0,0,0), black);
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pair o = (400,190); real len=80; real height=56; for (int i=0; i<4; ++i) { pair a = (i*len, i*height);
path p = a - a+(len,0) - a+(len, height); draw(shift(o)*p); } path p = (0,0)-(0,-height)-(4*len,-height);
draw(shift(o)*p); \end{asy} \end{center}
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(A) 4 (B) 5 (C) 6 (D) 8 (E) 12

14. What is the distance of the shortest route from A to Z?

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\{\}\begin{center} \{\}\begin{asy} import olympiad; import cse5; /* AMC8 P14 2024, by NUMANA: BUI
VAN HIEU */ import graph; unitsize(2cm); real r=0.25; // Define the nodes and their positions pair[]
nodes = { (0,0), (2,0), (1,1), (3,1), (4,0), (6,0) }; string[] labels = { "A", "M", "X", "Y", "C", "Z" };
// Draw the nodes as circles with labels for(int i = 0; i < nodes.length; ++i) { draw(circle(nodes[i], r));
label("$" + labels[i] + "$", nodes[i]); } // Define the edges with their node indices and labels int[][] edges
= { {0, 1}, {0, 2}, {2, 1}, {2, 3}, {1, 3}, {1, 4}, {3, 4}, {4, 5}, {3, 5} }; string[] edgeLabels = { "8",
"5", "2", "10", "6", "14", "5", "10", "17" }; pair[] edgeLabelsPos = { S, SE, SW, S, SE, S, SW, S,
NE}; // Draw the edges with labels for (int i = 0; i < edges.length; ++i) { pair start = nodes[edges[i][0]];
pair end = nodes[edges[i][1]]; draw(start + r*dir(end-start) - end-r*dir(end-start), Arrow); label("$" +
edgeLabels[i] + "$", midpoint(start - end), edgeLabelsPos[i]); } // Draw the curved edge with label
draw(nodes[1]+r * dir(-45)..controls (3, -0.75) and (5, -0.75)..nodes[5]+r * dir(-135), Arrow); label("$25$",
midpoint(nodes[1]..controls (3, -0.75) and (5, -0.75)..nodes[5]), 2S); \}\end{asy} \}\end{center}
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(A) 28 (B) 29 (C) 30 (D) 31 (E) 32

15. Let the letters F, L, Y, B, U, G represent distinct digits. Suppose $\underline{F} \underline{L} \underline{Y} \underline{F} \underline{L} \underline{Y}$ is the GREATEST number that satisfies the equation

$$8 \cdot \underline{F} \underline{L} \underline{Y} \underline{F} \underline{L} \underline{Y} = \underline{B} \underline{U} \underline{G} \underline{B} \underline{U} \underline{G}.$$

What is the value of $\underline{F} \underline{L} \underline{Y} + \underline{B} \underline{U} \underline{G}$?

(A) 1089 (B) 1098 (C) 1107 (D) 1116 (E) 1125

16. Minh enters the numbers 1 through 81 into the cells of a 9×9 grid in some order. She calculates the product of the numbers in each row and column. What is the least number of rows and columns that could have a product divisible by 3?

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

17. A chess king is said to "attack" all squares one step away from it (basically any square right next to it in any direction), horizontally, vertically, or diagonally. For instance, a king on the center square of a 3×3 grid attacks all 8 other squares, as shown below. Suppose a white king and a black king are placed on different squares of 3×3 grid so that they do not attack each other. In how many ways can this be done?

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\{\}\begin{center} \{\}\begin{asy} import olympiad; import cse5; /* AMC8 P17 2024, revised by Teacher
David */ unitsize(29pt); import math; add(grid(3,3));
pair [] a = {(0.5,0.5), (0.5, 1.5), (0.5, 2.5), (1.5, 2.5), (2.5,2.5), (2.5,1.5), (2.5,0.5), (1.5,0.5)};
for (int i=0; i<a.length; ++i) { pair x = (1.5,1.5) + 0.4*dir(225-45*i); draw(x - a[i], arrow=EndArrow()); }
label("$K$", (1.5,1.5)); \}\end{asy} \}\end{center}
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(A) 20 (B) 24 (C) 27 (D) 28 (E) 32

18. Three concentric circles centered at O have a radius of 1, 2, and 3. Points B and C lie on the largest circle. The region between the two smaller circles is shaded, as is the portion of the region between the two larger circles bounded by central angle BOC , as shown in the figure below. Suppose the shaded and unshaded regions are equal in area. What is the measure of $\angle BOC$ in degrees?

```
\{\}\begin{center} \{\}\begin{asy} import olympiad; import cse5; size(100); import graph;
draw(circle((0,0),3)); real radius = 3; real angleStart = -54; // starting angle of the sector real angleEnd
= 54; // ending angle of the sector label("$O$", (0,0), W); pair O = (0, 0); filldraw(arc(O, radius, an-
gleStart, angleEnd)--O--cycle, lightgray); filldraw(circle((0,0),2),lightgray); filldraw(circle((0,0),1),white);
draw((1.763,2.427)--(0,0)--(1.763,-2.427)); label("$B$", (1.763,2.427), NE); label("$C$", (1.763,-2.427), SE); \}\end{asy}
\}\end{center}
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(A) 108 (B) 120 (C) 135 (D) 144 (E) 150

19. Jordan owns 15 pairs of sneakers. Three fifths of the pairs are red and the rest are white. Two thirds of the pairs are high-top and the rest are low-top. The red high-top sneakers make up a fraction of the collection. What is the least possible value of this fraction?

(A) 0 (B) $\frac{1}{5}$ (C) $\frac{4}{15}$ (D) $\frac{1}{3}$ (E) $\frac{2}{5}$

20. Any three vertices of the cube $PQRSTUVW$, shown in the figure below, can be connected to form a triangle. (For example, vertices P , Q , and R can be connected to form $\triangle PQR$.) How many of these triangles are equilateral and contain P as a vertex?

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\{\}\begin{center} \{\}\begin{asy} import olympiad; import cse5; unitsize(4); pair P,Q,R,S,T,U,V,W; P=(0,30);
Q=(30,30); R=(40,40); S=(10,40); T=(10,10); U=(40,10); V=(30,0); W=(0,0); draw(W-V); draw(V-Q);
draw(Q-P); draw(P-W); draw(T-U); draw(U-R); draw(R-S); draw(S-T); draw(W-T); draw(P-S);
draw(V-U); draw(Q-R); dot(P); dot(Q); dot(R); dot(S); dot(T); dot(U); dot(V); dot(W); label("$P$",P,NW);
label("$Q$",Q,NW); label("$R$",R,NE); label("$S$",S,N); label("$T$",T,NE); label("$U$",U,NE); label("$V$",V,SE);
label("$W$",W,SW); \{\}\end{asy} \{\}\end{center}
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(A) 0 (B) 1 (C) 2 (D) 3 (E) 6

21. A group of frogs (called an "army") is living in a tree. A frog turns green when in the shade and turns yellow when in the sun. Initially, the ratio of green to yellow frogs was 3 : 1. Then 3 green frogs moved to the sunny side and 5 yellow frogs moved to the shady side. Now the ratio is 4 : 1. What is the difference between the number of green frogs and the number of yellow frogs now?

(A) 10 (B) 12 (C) 16 (D) 20 (E) 24

22. A roll of tape is 4 inches in diameter and is wrapped around a ring that is 2 inches in diameter. A cross section of the tape is shown in the figure below. The tape is 0.015 inches thick. If the tape is completely unrolled, approximately how long would it be? Round your answer to the nearest 100 inches.

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\{\}\begin{center} \{\}\begin{asy} import olympiad; import cse5; /* AMC8 P22 2024, revised by Teacher
David */ size(150);
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pair o = (0,0); real r1 = 1; real r2 = 2;
filldraw(circle(o, r2), mediumgray, linewidth(1pt)); filldraw(circle(o, r1), white, linewidth(1pt));
draw((-2,-2.6)-(-2,-2.4)); draw((2,-2.6)-(2,-2.4)); draw((-2,-2.5)-(2,-2.5), L=Label("4 in.));
draw((-1,0)-(1,0), L=Label("2 in.", align=(0,1)), arrow=Arrows());
draw((2,0)-(2,-1.3), linewidth(1pt)); \{\}\end{asy} \{\}\end{center}
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(A) 300 (B) 600 (C) 1200 (D) 1500 (E) 1800

23. Rodrigo has a very large sheet of graph paper. First he draws a line segment connecting point $(0, 4)$ to point $(2, 0)$ and colors the 4 cells whose interiors intersect the segment, as shown below. Next Rodrigo draws a line segment connecting point $(2000, 3000)$ to point $(5000, 8000)$. How many cells will he color this time?

```
\{\}\begin{center} \{\}\begin{asy} import olympiad; import cse5; filldraw((0,4)-(1,4)-(1,3)-(0,3)-cycle, gray(.75),
gray(.5)+linewidth(1)); filldraw((0,3)-(1,3)-(1,2)-(0,2)-cycle, gray(.75), gray(.5)+linewidth(1)); filldraw((1,2)-
(2,2)-(2,1)-(1,1)-cycle, gray(.75), gray(.5)+linewidth(1)); filldraw((1,1)-(2,1)-(2,0)-(1,0)-cycle, gray(.75),
gray(.5)+linewidth(1));
draw((-1,5)-(-1,-1),gray(.9)); draw((0,5)-(0,-1),gray(.9)); draw((1,5)-(1,-1),gray(.9)); draw((2,5)-(2,-1),gray(.9));
draw((3,5)-(3,-1),gray(.9)); draw((4,5)-(4,-1),gray(.9)); draw((5,5)-(5,-1),gray(.9));
draw((-1,5)-(5,5),gray(.9)); draw((-1,4)-(5,4),gray(.9)); draw((-1,3)-(5,3),gray(.9)); draw((-1,2)-(5,2),gray(.9));
draw((-1,1)-(5,1),gray(.9)); draw((-1,0)-(5,0),gray(.9)); draw((-1,-1)-(5,-1),gray(.9));
dot((0,4)); label("$ (0,4) $", (0,4), NW);
dot((2,0)); label("$ (2,0) $", (2,0), SE);
draw((0,4)-(2,0));
draw((-1,0) - (5,0), arrow=Arrow); draw((0,-1) - (0,5), arrow=Arrow); \{\}\end{asy} \{\}\end{center}
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(A) 6000 (B) 6500 (C) 7000 (D) 7500 (E) 8000

24. Jean has made a piece of stained glass art in the shape of two mountains, as shown in the figure below. One mountain peak is 8 feet high while the other peak is 12 feet high. Each peak forms a 90° angle, and the straight sides form a 45° angle with the ground. The artwork has an area of 183 square feet. The sides of the mountain meet at an intersection point near the center of the artwork, h feet above the ground. What is the value of h ?

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\{\}\begin{center} \{\}\begin{asy} import olympiad; import cse5; unitsize(.3cm); filldraw((0,0)-(8,8)-(11,5)-
(18,12)-(30,0)-cycle,gray(0.7),linewidth(1)); draw((-1,0)-(-1,8),linewidth(.75)); draw((-1.4,0)-(-.6,0),linewidth(.75));
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draw((-1.4,8)-(-.6,8),linewidth(.75)); label("$8$",(-1,4),W); label("$12$",(31,6),E); draw((-1,8)-(8,8),dashed);
draw((31,0)-(31,12),linewidth(.75)); draw((30.6,0)-(31.4,0),linewidth(.75)); draw((30.6,12)-(31.4,12),linewidth(.75));
draw((31,12)-(18,12),dashed); label("$45^\circ$",(.75,0),NE,fontsize(10pt)); label("$45^\circ$",(29.25,0),NW,
draw((8,8)-(7.5,7.5)-(8,7)-(8.5,7.5)-cycle); draw((18,12)-(17.5,11.5)-(18,11)-(18.5,11.5)-cycle); draw((11,5)-
(11,0),dashed); label("$h$", (11,2.5),E); \end{asy} \end{center}

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(A) 4 (B) 5 (C) $4\sqrt{2}$ (D) 6 (E) $5\sqrt{2}$

25. A airplane has 4 rows of seats with 3 seats in each row. Eight passengers have boarded the plane and are distributed randomly among the seats. A married couple is next to board. What is the probability there will be 2 adjacent seats in the same row for the couple?

(A) $\frac{8}{15}$ (B) $\frac{32}{55}$ (C) $\frac{20}{33}$ (D) $\frac{34}{55}$ (E) $\frac{8}{11}$