# AIME Problems
2024

# Problems

1. Every morning Aya goes for a 9-kilometer-long walk and stops at a coffee shop afterwards. When she walks at a constant speed of $s$ kilometers per hour, the walk takes her 4 hours, including $t$ minutes spent in the coffee shop. When she walks at $s+2$ kilometers per hour, the walk takes her 2 hours and 24 minutes, including $t$ minutes spent in the coffee shop. Suppose Aya walks at $s+\frac{1}{2}$ kilometers per hour. Find the number of minutes the walk takes her, including the $t$ minutes spent in the coffee shop.

2. There exist real numbers $x$ and $y$, both greater than 1, such that

$$\log_x(y^x) = \log_y(x^{4y}) = 10.$$

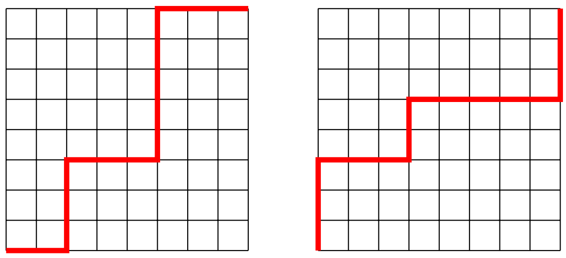
Find $xy$.

3. Alice and Bob play the following game. A stack of $n$ tokens lies before them. The players take turns with Alice going first. On each turn, the player removes 1 token or 4 tokens from the stack. The player who removes the last token wins. Find the number of positive integers $n$ less than or equal to 2024 such that there is a strategy that guarantees that Bob wins, regardless of Alices moves.

4. Jen enters a lottery by picking 4 distinct numbers from $S = \{1, 2, 3, \cdots, 9, 10\}$. 4 numbers are randomly chosen from $S$. She wins a prize if at least two of her numbers were 2 of the randomly chosen numbers, and wins the grand prize if all four of her numbers were the randomly chosen numbers. The probability of her winning the grand prize given that she won a prize is $\frac{m}{n}$ where $m$ and $n$ are relatively prime positive integers. Find $m+n$.

5. Rectangle $ABCD$ has dimensions $AB = 107$ and $BC = 16$, and rectangle $EFGH$ has dimensions $EF = 184$ and $FG = 17$. Points $D$, $E$, $C$, and $F$ lie on line $DF$ in that order, and $A$ and $H$ lie on opposite sides of line $DF$, as shown. Points $A$, $D$, $H$, and $G$ lie on a common circle. Find $CE$.


6. Consider the paths of length 16 that follow the lines from the lower left corner to the upper right corner on an $8 \times 8$ grid. Find the number of such paths that change direction exactly four times, like in the examples shown below.

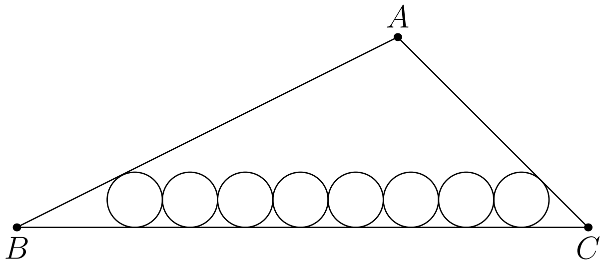


7. Find the largest possible real part of

$$(75+117i)z + \frac{96+144i}{z}$$

where $z$ is a complex number with $|z| = 4$.

8. Eight circles of radius 34 can be placed tangent to $\overline{BC}$ of $\triangle ABC$ so that the circles are sequentially tangent to each other, with the first circle being tangent to $\overline{AB}$ and the last circle being tangent to $\overline{AC}$, as shown. Similarly, 2024 circles of radius 1 can be placed tangent to $\overline{BC}$ in the same manner. The inradius of $\triangle ABC$ can be expressed as $\frac{m}{n}$, where $m$ and $n$ are relatively prime positive integers. Find $m+n$.




9. Let $ABCD$ be a rhombus whose vertices all lie on the hyperbola $\frac{x^2}{20} - \frac{y^2}{24} = 1$ and are in that order. If its diagonals intersect at the origin, find the largest number less than $BD^2$ for all rhombuses $ABCD$.

10. Let $ABC$ be a triangle inscribed in circle $\omega$. Let the tangents to $\omega$ at $B$ and $C$ intersect at point $D$, and let $\overline{AD}$ intersect $\omega$ at $P$. If $AB = 5$, $BC = 9$, and $AC = 10$, $AP$ can be written as the form $\frac{m}{n}$, where $m$ and $n$ are relatively prime integers. Find $m+n$.

11. Each vertex of a regular octagon is independently colored either red or blue with equal probability. The probability that the octagon can then be rotated so that all of the blue vertices end up at positions where there were originally red vertices is $\frac{m}{n}$, where $m$ and $n$ are relatively prime positive integers. What is $m+n$?

12. Define $f(x) = ||x| - \frac{1}{2}|$ and $g(x) = ||x| - \frac{1}{4}|$. Find the number of intersections of the graphs of $y = 4g(f(\sin(2\pi x)))$ and $x = 4g(f(\cos(3\pi y)))$.

13. Let $p$ be the least prime number for which there exists a positive integer $n$ such that $n^4 + 1$ is divisible by $p^2$. Find the least positive integer $m$ such that $m^4 + 1$ is divisible by $p^2$.

14. Let $ABCD$ be a tetrahedron such that $AB = CD = \sqrt{41}$, $AC = BD = \sqrt{80}$, and $BC = AD = \sqrt{89}$. There exists a point $I$ inside the tetrahedron such that the distances from $I$ to each of the faces of the tetrahedron are all equal. This distance can be written in the form $\frac{m\sqrt{n}}{p}$, when $m$, $n$, and $p$ are positive integers, $m$ and $p$ are relatively prime, and $n$ is not divisible by the square of any prime. Find $m+n+p$.

15. Let $\mathcal{B}$ be the set of rectangular boxes with surface area 54 and volume 23. Let $r$ be the radius of the smallest sphere that can contain each of the rectangular boxes that are elements of $\mathcal{B}$. The value of $r^2$ can be written as $\frac{p}{q}$, where $p$ and $q$ are relatively prime positive integers. Find $p+q$.