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Theory of Chaos and Its Application to the Crisis of Debts and the Origin of Inflation

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Abstract. The paper is mainly devoted to mathematical background explaining the current crisis in economics.

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The word “chaos” ($\chi\acute{\alpha}o\varsigma$) is a category of cosmology, the initial state of the Universe. In Greek mythology, this is the original state of the world, the state of a kind of “yawning abyss,” from which the first gods originated, including Helios, the god of the sun, the natural nuclear reactor which works without renewal. The problem of how it can be destroyed and what are its properties engrossed the author in 1986, when it was necessary to solve, in the shortest possible time, a series of problems related to the destroyed Chernobyl nuclear power plant [1].[1]

Since then, the author faces this problem, and he tries to find general laws related to mathematics, physics, biology, economics, and linguistics. It should be noted that not only physical problems help to find economic laws but also, conversely, economic statistical observations help to solve physical problems [2].

In the present paper, which is mainly devoted to the crisis in economics, we begin with the foundations of mathematics, namely, with number theory.

1. THEOREMS ON CHAOS

For the main theory of classical chaos,[2] one can take the theory of partition of an integer n into a sum of k summands. This problem is related to financial problems in the most immediate way. Let us recall the famous scene from M. Bulgakov’s novel “The Master and Margarita” in which Koroviev, the assistant to Voland (Satan), during their ensorcered magical performance at the Moscow circus, creates a rain of identical high-denomination banknotes that fall from the ceiling on the audience. Assume that there were n banknotes and k persons in the audience, where $k \leqslant \sqrt{n}$. We certainly assume that all versions of distributing n banknotes among k persons are equiprobable. Let us state a theorem of number theory.

Let n be a positive integer. By a *partition* of n we mean a way to represent a natural number n as a sum of natural numbers. Let $p(n)$ be the total number of partitions of n, where the order of the summands is not taken into account, i.e., partitions that differ only in the order of summands are assumed to be the same. The number $p_k(n)$ of partitions of a positive integer n into k positive integer summands is one of the fundamental objects of investigation in number theory. If S is the number of partitions in some subset of the set of all partitions of a positive integer n into k summands, then the ratio $S/p_k(n)$ is referred to as the *relative number of variants* or the *portion* of variants. Denote by S^a the number of summands (entering a given sum of k summands representing n) that belong to the interval from $(n-a)$ to n.

In a given partition, denote the number of summands (in the sum) equal to 1 by N_1, the number of summands equal to 2 by N_2, etc., and the number of summands equal to i by N_i. Then $\sum N_i = k$ is the number of summands, and the sum $\sum iN_i$ is obviously equal to the partitioned

[1] Academician V. P. Myasnikov, now deceased, said that the intensive computational work carried out for the design of the sarcophagus over the Chernobyl reactor leaves the same trace in a mathematician’s world view as the participation in a war in that of a writer.

[2] For the point of view of the author at the quantum chaos, see [3].

positive integer. Thus, we have

$$\sum_{i=1}^{\infty} iN_i = n, \qquad \sum_{i=1}^{\infty} N_i = k, \tag{1}$$

where the N_i are natural numbers not exceeding n. These formulas can readily be verified for the above example. Here all the families $\{N_i\}$ are equiprobable.

Let us define constants b and κ by the following two relations:

$$\int_0^\infty \frac{\xi d\xi}{e^{b\xi+b\kappa}-1} = n, \tag{2}$$

$$\int_0^\infty \frac{d\xi}{e^{b\xi+b\kappa}-1} = k. \tag{3}$$

It can readily be seen that, for $k < \sqrt{n}$, a solution of these equations exists and is unique, say,

$$b = b(k,n), \qquad \kappa = \kappa(k,n).$$

What is a condensate? This is the phenomenon under which the number of summands in the sum that take the least possible value is very large.

Theorem 1. *Consider the number $S_{a,g}$ of partitions of n into sums of k summands that contain numbers belonging to the interval $\left[a, g+\sqrt{k}\log k\right]$, where $a \geqslant 0$ and $g \geqslant a$.*

Let $k \leqslant \sqrt{n}$. Then the portion of versions for which $S_{a,g}$ is outside the interval

$$\left[\int_a^{g+\sqrt{k}\log k} \frac{dx}{e^{b(x+\kappa)}-1} - \sqrt{k\log k\log\log k},\ \int_a^{g+\sqrt{k}\log k} \frac{dx}{e^{b(x+\kappa)}-1} + \sqrt{k\log k\log\log k}\right] \tag{4}$$

tends to zero more rapidly than any power of $1/n$.

In other words, the ratio of the number of variants for which $S_{a,g}$ does not belong to the interval (4) *to the total number $p_k(n)$ of all partitions of n into k summands tends to zero more rapidly than an arbitrarily large* (*fixed*) *power of $1/n$.*

Here one can replace $\log\log k$ by any other slowly increasing function.

Roughly speaking, we can estimate the percentage of the audience obtaining a number of banknotes from a given interval with accuracy up to

$$O\Big(\frac{1}{\sqrt{k}}\sqrt{\log k\log\log k}\Big).$$

In essence, these theorems are special cases of a general theorem proved by the author in [4]. In physics, the distribution of the above form is referred to as the *Bose–Einstein distribution.* The relationship between the theory of partitions and the Bose–Einstein distribution was noticed at the mid forties of the last century [5]. The theorem on weak convergence in partition theory (in a sense) was first proved under a similar assumption ($k \leqslant \sqrt{n}$) by Vershik [6] at the end of the last century.

It is natural to regard the rain of banknotes that fell to the audience as a chaos. The Bose–Einstein distribution for the number of particles dN of an ideal Bose gas has the form

$$dN = \frac{dv_x dv_y dv_z}{e^{(mv^2/2-\mu)/(kT)}-1}, \qquad v = \sqrt{v_x^2+v_y^2+v_z^2}, \tag{5}$$

where μ, $\mu \leqslant 0$, stands for the chemical potential, k for the Boltzmann constant, m for the mass, and T for the temperature.

For this distribution to coincide with that presented above, we must consider the two-dimensional density. In this case,

$$dN = \frac{dv_x dv_y}{e^{mv^2/2-\mu} - 1}. \tag{6}$$

Therefore, we define the chaos of the partition of a positive integer n into a sum of k summands as a two-dimensional chaos. In our opinion, this is an essential characteristic of chaos. In this case, the chaos of Bose particles of a three-dimensional ideal gas is a chaos of dimension three.

Note that, if $k \ll \sqrt{n}$ (k is much less than n), then κ is large, and the distribution (4) becomes the Maxwell distribution (for estimates related to Maxwell and Gibbs distributions, see [7] and [2]).

This means that the number of ones is very large if k is large; this is a phenomenon which is similar to that known in physics as the Bose condensate. Thus, it was rigorously proved that a condensate exists in the two-dimensional case as well (for this topic, see also [5]), contrary to the opinion prevailing among physicists [8] that there is no Bose condensate in the two-dimensional case.

Let us state a version of the above theorem.

Theorem 2. *The ratio of the number of versions in which the number of persons* (*in the audience*) *who did not get any money is less than the number*

$$l = k - c\sqrt{n}\log(n) - o\big(\sqrt{n}\log(n)\big), \qquad c = \frac{1}{\pi}\sqrt{3/2}, \tag{7}$$

to the total number of variants of distribution of n *banknotes among* k *persons tends to zero more rapidly than* $e^{-l^{1/2-\delta}}$, *where* δ, $0 < \delta < 1/2$, *is independent of* n *and as small as desired.*

Thus, we include the number 0 in the list of numbers used as summands in partitions.

It follows from this theorem that, if

$$k > 2\frac{1}{\pi}\sqrt{3/2}\sqrt{n}\log(n),$$

then the probability of the event that the number of persons in the audience who did not get any money is not less than $k - (1/\pi)\sqrt{3/2}\sqrt{n}\log(n)$ is close to one.

Assume that Koroviev showered one million banknotes and there were 10000 persons in the audience. Then, by Theorem 2, approximately 4500 persons got no money. The other persons got more or less distinct sums. However, 4500 persons in the audience are solidary and united in that each of them had got 0 banknotes. This is the condensate.

Note that the dimension about which physicists write in this context [9] is two times greater than the dimension introduced by the author in [10]. Here we shall follow the physicists. As is said in physics textbooks, there is no Bose condensate in the two-dimensional case (i.e., according to our old terminology, for $d = 1$). Vershik [6] claims the same, and this is correct from the point of view of terminology. However, the same condensation phenomenon takes place only for a parastatistics close to the Bose statistics.

Let us return to the proof of our Theorem 2 in number theory. Consider the sums

$$\sum iN_i = M, \qquad \sum N_i = N \tag{8}$$

and assume again that all variants $\{N_i\}$ are equiprobable. We must also take into account the condition that $N_i \leqslant N$. This very condition means that the numbers N_i are not arbitrary (as is the case for the Bose statistics) and are bounded, as is the case for a parastatistics (for instance, $N_i \leqslant 1$ for the Fermi statistics).

It can readily be seen that these relations hold for the partition of $M = n$ into $N = k$ summands.

The distribution for the parastatistics $N_i \leqslant N$ and $\sum N_i = N$ is determined from the relations

$$\sum_{i=1}^{n} \frac{1}{e^{b(i+\kappa)} - 1} - \frac{N}{e^{bN(i+\kappa)} - 1} = N, \qquad \sum_{i=1}^{n} \Big(\frac{i}{e^{b(i+\kappa)} - 1} - \frac{iN}{e^{bN(i+\kappa)} - 1}\Big) = M, \tag{9}$$

where $b > 0$ and $\kappa > 0$ are constants defined from relations (9), $n = M/N$ is sufficiently large, and the numbers M and N are also large, and we can pass (by using the Euler–Maclaurin summation formula) to the integrals (for the estimates for this passage, see [11]),

$$\int_0^\infty \Big(\frac{1}{e^{b(x+\kappa)}-1} - \frac{N}{e^{bN(x+\kappa)}-1}\Big)dx \cong N, \tag{10}$$

$$\int_0^\infty \Big(\frac{x}{e^{b(x+\kappa)}-1} - \frac{Nx}{e^{bN(x+\kappa)}-1}\Big)dx \simeq M. \tag{11}$$

It can be proved that $\kappa = 0$ gives the number $N_{\max}$ with satisfactory accuracy. Hence,

$$N_{\max} = \int_0^\infty \Big(\frac{1}{e^{bx}-1} - \frac{N_{\max}}{e^{bN_{\max}x}-1}\Big)dx. \tag{12}$$

Denote $N_{\max}$ by k_0, which is customary in number theory.

Consider the value of the integral (with the same integrand) taken from ε to ∞ and then pass to the limit as $\varepsilon \to 0$. After making the change $bx = \xi$ in the first term and $bk_0x = \xi$ in the second term, we obtain

$$\begin{aligned} k_0 &= \frac{1}{b}\int_{\varepsilon b}^\infty \frac{d\xi}{e^\xi - 1} - \int_{\varepsilon b k_0}^\infty \frac{d\xi}{e^\xi - 1} = \frac{1}{b}\int_{\varepsilon b}^{\varepsilon b k_0} \frac{d\xi}{e^\xi - 1} \\ &\sim \frac{1}{b}\int_{\varepsilon b}^{\varepsilon b k_0} \frac{d\xi}{\xi} = \frac{1}{b}\{\log(\varepsilon b k_0) - \log(\varepsilon b)\} = \frac{1}{b}\log k_0. \end{aligned} \tag{13}$$

On the other hand, making the change $bx = \xi$ in (11) and using the notation $M = n$ and $N = k$, which is customary in number theory, we obtain

$$\frac{1}{b^2}\int_0^\infty \frac{\xi d\xi}{e^\xi - 1} = n. \tag{14}$$

This gives

$$b = \left(\sqrt{n}\Big/\sqrt{\int_0^\infty \frac{\xi d\xi}{e^\xi - 1}}\right)^{-1}, \qquad k_0 = \frac{1}{2}\frac{\sqrt{n}}{\sqrt{\pi^2/6}}\log n(1+o(1)). \tag{15}$$

Let us now find the next term of the asymptotics by setting

$$k_0 = c^{-1}n^{1/2}\log c^{-1}n^{1/2} + \alpha n^{1/2} + o(n^{1/2}), \tag{16}$$

where

$$c = \frac{2\pi}{\sqrt{6}}.$$

Furthermore, using the formula

$$k_0 = c^{-1}n^{1/2}\log k_0$$

and expanding $\log k_0$ in

$$\frac{\alpha}{c^{-1}\log c^{-1}n^{1/2}},$$

we find

$$\alpha = -2\log\frac{c}{2}.$$

We thus obtain the Erdős formula [12].

Let us show that, if $k > k_0$, then $\kappa < 0$.

Lemma 1. *Let $n > k \gg k_0$. Suppose that $\kappa = -\mu$ and $\mu > 0$. In this case, equations* (2) *and* (3) *have solutions with $\mu \sim k^{-1/2-\delta}$.*

Indeed, consider relations (2) and (3). Make the change $\xi - \mu = \eta$, and then $b\eta = \varphi$. We obtain

$$n = \frac{1}{b^2}\int_0^\mu \frac{k\xi d\xi}{1-e^{-k\xi}} - \frac{1}{b^2}\int_0^\mu \frac{\xi d\xi}{1-e^{-\xi}} + \frac{1}{b^2}\int_0^\infty \Big(\frac{\xi}{e^\xi - 1} - \frac{k\xi}{e^{k\xi}-1}\Big)d\xi, \tag{17}$$

$$k = \frac{1}{b}\int_\varepsilon^\mu \Big(\frac{k}{1-e^{-k\xi}} - \frac{1}{1-e^{-\xi}}\Big)d\xi + \frac{1}{b}\int_\varepsilon^\infty \Big(\frac{1}{e^\xi-1} - \frac{k}{e^{\xi k}-1}\Big)d\xi. \tag{18}$$

After making the change $k\xi = x$ in the corresponding integrals, we see that

$$n = \frac{1}{b^2}\frac{1}{k}\int_0^{\mu k} \frac{xdx}{1-e^{-x}} + O\Big(\frac{1}{b^2}\Big), \tag{19}$$

$$k = \frac{1}{b}\int_0^{\mu k} \frac{dx}{1-e^{-x}} + O\Big(\frac{\log k}{b}\Big), \tag{20}$$

as $k \to \infty$. Therefore, $\mu k \ll \sqrt{k}$; for instance, $\mu k = \sqrt{k^{1-\delta}}$ for any $1 > \delta > 0$.

This relation can be satisfied provided that $\mu \sim (\sqrt{k^{1+\delta}})^{-1}$. This proves the lemma.

Therefore, as $l \to \infty$, the number of variants decreases[3] more rapidly than $e^{-\sqrt{l^{1-\delta}}}$.

Let us now consider the one-dimensional case of a Bose condensate, which is of importance in physical problems. The notation is customary for statistical physics, namely, $n = \mathcal{E}$ is the energy and $k = N$ is the number of particles.

Introduce constants b and κ by the following relations:

$$\int_0^\infty \frac{\xi d\sqrt{\xi}}{e^{b(\xi+\kappa)}-1} = \mathcal{E}, \tag{21}$$

$$\int_0^\infty \frac{d\sqrt{\xi}}{e^{b(\xi+\kappa)}-1} = N. \tag{22}$$

Theorem 3. *Let the dimension d be equal to* 1*. In this case, the portion of the number of variants in which zero occurs less than*

$$N - 4c^2\mathcal{E}^{2/3} - o(\mathcal{E}^{2/3}) \tag{23}$$

times, where

$$c = \int_0^\infty \Big(\frac{1}{e^{\xi^2}-1} - \frac{1}{\xi^2}\Big)d\xi\Big(\int_0^\infty \frac{\sqrt{\xi}d\xi}{e^\xi - 1}\Big)^{2/3},$$

tends to zero more rapidly than

$$e^{-(N-4c^2\mathcal{E}^{2/3})^{1/3-\delta}},$$

where δ, $0 < \delta < 1/3$, is independent of n and as small as desired.

Proof. Let us first determine b from the relation

$$\mathcal{E} = \frac{1}{2}\int_0^\infty \frac{\sqrt{\xi}d\xi}{e^{b\xi}-1} = \frac{1}{2b^{3/2}}\int_0^\infty \frac{\sqrt{\xi}d\xi}{e^\xi - 1}. \tag{24}$$

[3]It is certainly possible to determine the number of variants for $k > k_0$ more precisely by using the standard properties of Kolmogorov complexity [13]; however, the estimate we present is sufficient for our purposes.

This gives

$$b = (2\mathcal{E})^{-2/3}\Big(\int_0^\infty \frac{\sqrt{\xi}d\xi}{e^\xi - 1}\Big)^{2/3}. \tag{25}$$

We use the following identity:

$$\int_0^\infty \left(\frac{1}{x^2} - \frac{1}{x^2(1+\alpha x^2)}\right) dx = \sqrt{\alpha}\int_0^\infty \frac{d\xi}{1+\xi^2}. \tag{26}$$

Let us now find N_{cr} from the relation

$$N_{\mathrm{cr}} = \int_0^\infty \Big(\frac{1}{\sqrt{\xi}(e^{b\xi}-1)} - \frac{N_{\mathrm{cr}}}{\sqrt{\xi}e^{bN_{\mathrm{cr}}\xi} - 1}\Big)d\xi = \frac{2}{\sqrt{b}}\int_0^\infty \Big(\frac{1}{e^{\xi^2}-1} - \frac{N_{\mathrm{cr}}}{e^{\xi^2 N_{\mathrm{cr}}} - 1}\Big)d\xi. \tag{27}$$

Subtracting $1/\xi^2$ from the minuend and subtrahend in the integrand, we obtain $N_{\max} = N_{\mathrm{cr}}$, which determines the passage to the Bose condensate,

$$\begin{aligned} N_{\mathrm{cr}} = \frac{2}{\sqrt{b}}\int \Big(\frac{1}{e^{\xi^2}-1} - \frac{2}{\xi^2}\Big)d\xi + \frac{2}{\sqrt{b}}\int \Big(\frac{1}{\xi^2} - \frac{1}{\xi^2(1+\frac{N_{\mathrm{cr}}}{2}\xi^2)}\Big)d\xi \\ - \frac{2}{\sqrt{b}}\int \Big(\frac{N_{\mathrm{cr}}}{e^{N_{\mathrm{cr}}\xi^2}-1} - \frac{N_{\mathrm{cr}}}{N_{\mathrm{cr}}\xi^2(1+\frac{N_{\mathrm{cr}}}{2}\xi^2)}\Big)d\xi. \end{aligned} \tag{28}$$

Since

$$\frac{1}{\sqrt{b}}\int_0^\infty \Big(\frac{N_{\mathrm{cr}}}{e^{N_{\mathrm{cr}}\xi^2}-1} - \frac{N_{\mathrm{cr}}}{N_{\mathrm{cr}}\xi^2(1+\frac{N_{\mathrm{cr}}}{2}\xi^2)}\Big)d\xi = \frac{\sqrt{N_{\mathrm{cr}}}}{\sqrt{b}}\int_0^\infty \Big(\frac{1}{e^{\eta^2}-1} - \frac{1}{\eta^2(1+\eta^2/2)}\Big)d\eta \tag{29}$$

and

$$\frac{1}{\eta^2(1+\eta^2/2)} = \frac{1}{\eta^2} - \frac{1}{2}\frac{1}{(1+\eta^2/2)}, \tag{30}$$

after solving the quadratic equation for $x = \sqrt{N_{\mathrm{cr}}}$, we obtain the assertion of the theorem by presenting a relation similar to that in Lemma 1.

Lemma 2. *Let $n > k \gg k_0$. Consider $\kappa = -\mu$ and $\mu > 0$. In this case, equations* (2) *and* (3) *have solutions $\mu \sim k^{-1/2-\delta}$.*

Proof. Consider relations (2) and (3). Make the change $\xi - \mu = \eta$ and then the change $b\eta = \varphi$. We obtain

$$n = \frac{1}{b^2}\int_0^\mu \frac{k\xi d\sqrt{\xi}}{1-e^{-k\xi}} - \frac{1}{b^2}\int_0^\mu \frac{\xi d\sqrt{\xi}}{1-e^{-\xi}} + \frac{1}{b^2}\int_0^\infty \Big(\frac{\xi}{e^\xi - 1} - \frac{k\xi}{e^{k\xi}-1}\Big)d\sqrt{\xi}, \tag{31}$$

$$k = \frac{1}{b}\int_\varepsilon^\mu \Big(\frac{k}{1-e^{-k\xi}} - \frac{1}{1-e^{-\xi}}\Big)d\sqrt{\xi} + \frac{1}{b}\int_\varepsilon^\infty \Big(\frac{1}{e^\xi-1} - \frac{k}{e^{\xi k}-1}\Big)d\sqrt{\xi}. \tag{32}$$

After the change $k\xi = x$ in the corresponding integrals, we see that, as $k \to \infty$,

$$n = \frac{1}{b^2}\frac{1}{\sqrt{k}}\int_0^{\mu k} \frac{xd\sqrt{x}}{1-e^{-x}} + O\Big(\frac{1}{b^2}\Big), \tag{33}$$

$$k = \frac{1}{b}\int_\mu^{\mu k} \frac{d\sqrt{x}}{1-e^{-x}} + O\Big(\frac{\log k}{b}\Big). \tag{34}$$

This yields $(\mu k)^{3/2} \ll \sqrt{k}$. For instance, $(\mu k)^3 \cong k^{1-\delta}$ for any δ with $1 > \delta > 0$.

This relation can be satisfied if $\mu k \sim (\sqrt[3]{k^{1-\delta}})$.

This implies that the number of variants decreases as $l \to \infty$ more rapidly than $e^{-\sqrt[3]{k^{1-\delta}}}$.

The case $d = 1$ is especially important because, by the original definition of white noise in physical literature, it corresponds to the spectrum of the audibility interval with uniform distribution of power. In other words, the spectral components of white noise are uniformly distributed over the entire interval of audible frequencies; in our case, the interval is determined by the quantity n, which we represented by the letter $\mathcal{E}$ in Theorem 3 (for better agreement with the physical notation). For this reason, the author refers to a chaos of dimension 1 as a white noise, which agrees with other definitions of other mathematicians [14].

We have determined the value k_0 (above which the condensate occurs) for white noise in the above sense.

In his famous work on complexity theory, Kolmogorov introduced some concepts and posed the problem of matching these concepts with the corresponding notions of probability theory. He wrote: "The only point of importance is to realize that, addressing probability theory, we use a significantly rougher relativization" [15]. The old-fashioned probability theory became as obsolete for solving complex economical problems as the classical mechanics was at the beginning of the 20th century for solving physical problems.

In turn, since the theory of chaos developed here is quite close to Kolmogorov's approach in complexity theory,[4] the notion of white noise considered has been related by Shafer and Vovk to the concept of white noise in probability theory (see [14]).

Let the number M represent the Gross National Product (GNP). There are specific efficient ways to find this quantity. Let N be the money supply of the given currency; the state bank can indicate this value exactly. The ratio M/N is the number of times the money supply turned over, the so-called turnover rate or the *turnover number* of the money supply N.

The GNP of a country is composed of the GNP of different regions, industries, etc. If we denote by N_i the number of roubles (dollars) and by i their turnover number, then we can say that $\sum iN_i = M$ (or $\sum iN_i \leqslant M$ if not everything is taken into account).

A condensate occurs if the number N_0 is large; that part of the money supply has no turnover. Either it fell out of circulation over under an inflation or is kept "under the pillows" or in safes. Let i be the turnover rate of the money supply N_i. Then

$$M = \sum_{i=0}^{\infty} iN_i.$$

Let N be the total money supply, $N = \sum N_i$, where N_i is the money supply making i turnovers, and thus $N_i = N(x)$, where $x = i$. Let $N(x)$ satisfy the conditions of one of the above theorems (Theorems 2 and 3) or of Theorem 4 below (i.e., the Hausdorff fractal dimension for the sequence of statistical data is greater than zero or the Pareto index does not exceed two).

In this case, if the mean turnover rate $v_0 = \mathrm{GNP}/N$ is less than M/N, i.e.,

$$v_0 < M/N_{\mathrm{cr}} + o(b^{-1}), \tag{35}$$

then a part of the money supply falls out of circulation (doesn't turn over).

Remark 1. This is possible under inflation (in which case, for instance, copecks (coins of the value of 0.01 rouble) fall out of circulation), when a part of money is kept in safes, or, as is the case in our country, when foreign currency is put aside for a rainy day, especially because of distrust to banks, which do not adhere to the principle of secrecy of accounts.

Certainly, the above theorems do not take into account a lot of accompanying economic factors. The dependence $N(x)$ can take into account both the Baumol–Tobin relation on the turnover rate j of an individual ($j \cong \sqrt{jN_j}$), where jN_j is the income ("the GNP of an individual"), and the distribution of individuals with respect to their incomes. However, all this enters the notion of multiplicative measure introduced by Vershik in [6] (see also [17]).

We shall discuss debts and credits below.

[4] And also to prediction theory originating from Ray J. Solomonoff [16].

Consider now the case of dimension d satisfying the condition $0 < d < 2$. Let the dimension of chaos be $d = 2 - 2\alpha$, where $0 < \alpha < 1$.

Define constants b and κ from the following relations:

$$\int_0^\infty \frac{\xi d\xi^\alpha}{e^{b(\xi+\kappa)} - 1} = n, \tag{36}$$

$$\int_0^\infty \Big(\frac{1}{e^{b(\xi+\kappa)} - 1} - \frac{k}{e^{bk(\xi+\kappa)} - 1}\Big) d\xi^\alpha = k. \tag{37}$$

For $\kappa = 0$, we have

$$n = \int \frac{\xi d\xi^\alpha}{e^{b\xi} - 1} = \frac{1}{b^{1+\alpha}} \int_0^\infty \frac{\eta d\eta^\alpha}{e^\eta - 1}. \tag{38}$$

Hence,

$$b = \frac{1}{n^{1/(1+\alpha)}} \Big(\int_0^\infty \frac{\xi d\xi^\alpha}{e^\xi - 1}\Big)^{1/(1+\alpha)}. \tag{39}$$

In this case, for the critical number k_0 we obtain (by setting the probability limit to be equal to n)

$$\begin{aligned}
k_0 &= \int_0^\infty \Big\{\frac{1}{e^{b\xi} - 1} - \frac{k_0}{e^{k_0 b\eta} - 1}\Big\} d\xi^\alpha \\
&= \frac{1}{b^\alpha} \int_0^\infty \Big(\frac{1}{e^\xi - 1} - \frac{1}{\xi}\Big) d\xi^\alpha + \frac{1}{b^\alpha} \int_0^\infty \Big(\frac{1}{\xi} - \frac{1}{\xi(1 + (k_0/2)\xi)}\Big) d\xi^\alpha \\
&\quad - \frac{k_0^{1-\alpha}}{b^\alpha} \int_0^\infty \Big\{\frac{k_0^\alpha}{e^{k_0\xi} - 1} - \frac{k_0^\alpha}{k_0\xi(1 + (k_0/2)\xi)}\Big\} d\xi^\alpha.
\end{aligned}$$

Write

$$c = \int_0^\infty \Big(\frac{1}{\xi} - \frac{1}{e^\xi - 1}\Big) d\xi^\alpha.$$

After the change $k_0\xi = \eta$, we obtain

$$\begin{aligned}
\frac{k_0^{1-\alpha}}{b^\alpha} \int_0^\infty \Big\{\frac{k_0^\alpha}{e^\eta - 1} - \frac{k_0^\alpha}{\eta(1 + \eta/2)}\Big\} d\xi^\alpha &= \frac{k_0^{1-\alpha}}{b^\alpha} \int_0^\infty \Big\{\frac{1}{e^\eta - 1} - \frac{1}{\eta(1 + \eta/2)}\Big\} d\eta^\alpha \\
= \frac{k_0^{1-\alpha}}{b^\alpha} \Big\{\int_0^\infty \Big(\frac{1}{e^\eta - 1} - \frac{1}{\eta}\Big) + \int_0^\infty \frac{d\eta^\alpha}{2(1 + \frac{\eta}{2})}\Big\} &= -c\frac{k_0^{1-\alpha}}{b^\alpha} + c_1 \frac{k_0^{1-\alpha}}{b^\alpha}.
\end{aligned} \tag{41}$$

Since $1/(\eta(1 + \eta/2)) = 1/\eta - 1/(2(1 + \eta/2))$, denoting $c_1 = \int_0^\infty \frac{d\eta^\alpha}{2(1+\eta/2)}$, we see that

$$\int_0^\infty \Big(\frac{1}{\xi} - \frac{1}{\xi(1 + \frac{k_0}{2}\xi)}\Big) d\xi^\alpha = \frac{k_0}{2} \int_0^\infty \frac{d\xi^\alpha}{1 + \frac{k_0}{2}\xi} = \Big(\frac{k_0}{2}\Big)^{1-\alpha} \int_0^\infty \frac{d\eta^\alpha}{1 + \eta} = c_1 \Big(\frac{k_0}{2}\Big)^{1-\alpha}. \tag{42}$$

Therefore,

$$\begin{aligned}
k_0 &= -\frac{1}{b^\alpha} c_1 + \frac{1}{b^\alpha} c\Big(\frac{k_0}{2}\Big)^{1-\alpha} - \frac{k_0^{1-\alpha}}{b^\alpha} \int_0^\infty \Big\{\frac{1}{e^\eta - 1} - \frac{1}{\eta(1 + \eta/2)}\Big\} d\eta^\alpha - \frac{1}{2}\frac{k_0^{1-\alpha}}{b^\alpha} \int \frac{d\eta^\alpha}{1 + \eta/2} \\
&= -\frac{1}{b^\alpha} c + \frac{k_0^{1-\alpha}}{b^\alpha} c.
\end{aligned} \tag{43}$$

This gives

$$k_0 = k_0(\alpha) = c^{1/\alpha} n^{\alpha/(1+\alpha)} \Big(\int_0^\infty \frac{\xi d\xi^\alpha}{e^\xi - 1}\Big)^{-\alpha} + o(n^{\alpha/(1+\alpha)}). \tag{44}$$

Using relations similar to those presented in Lemmas 1 and 2, we find the value $\mu = -\kappa$.

Thus, we have proved the following theorem.

Theorem 4. *Let the dimension of chaos be d, where $0 < d < 2$. Then the ratio of the number of variants in which zero occurs less than*

$$l = k - k_0(\alpha) - o(k_0(\alpha)) \tag{45}$$

times, where k_0 is defined in (43), to the total number of variants tends to zero more rapidly than

$$e^{-l^{\alpha/(\alpha+1)-\delta}}, \qquad \alpha = 1 - d/2,$$

where δ, $0 < \delta < \alpha/(\alpha+1)$, is independent of n and as small as desired.

The case of turbulence (and the Kolmogorov law) is of special interest. In this case, $\alpha = 2/3$, and the dimension d is $d = 2 - 4/3 = 2(1 - 2/3) = 2/3$. The frequencies above which a condensate occurs (i.e., a restructuring to the lowest frequencies) are determined by the frequency

$$k_0(2/3) = c^{3/2}\mathcal{E}^{2/5}\Big(\int_0^\infty \frac{\xi d\xi^{2/3}}{e^\xi - 1}\Big)^{-2/3} \qquad \Big(c = \int_0^\infty \Big(\frac{1}{\xi} - \frac{1}{e^\xi - 1}\Big) d\xi^{2/3}\Big). \tag{46}$$

The case $\alpha = 0$ will be considered in a further publication.

Let $k < k_0(\alpha)/2$ for $d > 0$. Then the following theorem holds.

Theorem 5. *Let S^a, where $a \geqslant \sqrt{k}\log k$, be equal to $\sum_{i=n-a}^n N_i$. Then the portion of the variants for which S^a does not belong to the interval*

$$\Big[\int_a^\infty \frac{dx^\alpha}{e^{b(x+\kappa)} - 1} - \sqrt{k\log k \log\log k},\ \int_a^\infty \frac{dx^\alpha}{e^{b(x+\kappa)} - 1} + \sqrt{k\log k\log\log k}\Big] \tag{47}$$

tends to zero more rapidly than any power of $1/n$.

For $d = 0$ and $\kappa > 0$, the portion of the variants for which S^a does not belong to the interval

$$\Big[\int_a^\infty \frac{dx}{b(x+\kappa)x} - \sqrt{k\log k\log\log k},\ \int_a^\infty \frac{dx}{b(x+\kappa)x} + \sqrt{k\log k\log\log k}\Big] \tag{48}$$

tends to zero more rapidly than any power of $1/n$ [18].

In the last case, it is more convenient to express b in terms of k and κ rather than n by using the relation

$$\frac{1}{b}\int_b^\infty \frac{dx}{x(x+\kappa)} = k, \tag{49}$$

which gives

$$\frac{1}{b} = k\kappa\left(\frac{1 + \log\kappa^2 k}{2}\right)\left(\sqrt{1 + \frac{4}{1 + \log\kappa^2 k}} - 1\right). \tag{50}$$

This means (see [18]) that the words occurring once in the dictionary (the condensate) are not taken into account. These words must be deleted from the total number of words in the book as well.

The parameter κ is chosen according to experimental statistical measurements.

In forthcoming papers, we intend to consider the case of negative dimension, also without the condensate.

Let us consider the case of white noise separately: $d = 1$. If $N/N_{\mathrm{mean}} \to 0$ (see above), then one can neglect the one in the denominator, and the distributions (49) and (51) acquire Gaussian form. This very phenomenon is called Gaussian white noise.

Consider the integers

$$S_{a,g} = \sum_{i=a}^{g+\sqrt{k}\log k} N_i,$$

where $a \geqslant 0$ and $g \geqslant a$. How often do these numbers occur in all possible variants?

Theorem 6. *Let $k \leqslant k_0(\alpha)/2$. The portion of variants for which the number $S_{a,g}$ does not belong to the interval*

$$\left[\int_a^{g+\sqrt{k}\log k} \frac{dx^\alpha}{e^{b(x+\kappa)}-1} - \sqrt{k\log k\log\log k},\ \int_a^{g+\sqrt{k}\log k} \frac{dx^\alpha}{e^{b(x+\kappa)}-1} + \sqrt{k\log k\log\log k}\right] \tag{51}$$

tends to zero more rapidly than any power of $1/n$.

In other words, the ratio of the number of variants for which the number $S_{a,g}$ does not belong to the interval (51) *to the total number of corresponding variants tends to zero more rapidly than any chosen power* (*as large as desired*) *of $1/n$.*

Let us formulate the last theorems in a form more customary for probability theory.

Theorem 7. *Let $k < k_0(\alpha)/2$. Define constants b and κ from relations* (36)–(37). *Let $a \geqslant 0$, $g \geqslant a$, and let the dimension be $d = (2-2\alpha)$, $0 < \alpha < 1$. In this case,*

$$\mathbf{P}\left\{\left|\sum_{i=[a]}^{i=[g+\sqrt{k}\log k]} N_i - \int_a^{g+\sqrt{k}\log k} \frac{d\xi^\alpha}{e^{b(x+\kappa)}-1}\right| \geqslant \sqrt{k\log k\log\log k}\right\} = O(n^{-m}), \tag{52}$$

where m is finite and as large as desired and $\mathbf{P}$ stands for the ratio of the number of variants satisfying the condition in the curly braces to the total number of corresponding variants.

Recall that, for $\sum_{i=0}^\infty iN_i = n$ and $\sum_{i=0}^\infty N_i = k$, the number of variants is the number of all possible families of $\{N_i\}$ satisfying these relations.

Let l be the number of particles in the condensate (in other words, the number of persons in the audience who got no banknotes, or the amount of money that does not turn over in the evaluation of the GNP, or, as we shall see below, the amount of debts to be restructured into long-term or bankrupt ones).

Theorem 8. *Let the dimension of chaos be d, $0 < d < 2$, let*

$$\widetilde{k}_0(\alpha) = c^{1/\alpha} n^{\alpha/(1+\alpha)}\left(\int_0^\infty \frac{\xi d\xi^\alpha}{e^\xi - 1}\right)^{-\alpha}, \quad \textit{where} \quad c = \int_0^\infty \left(\frac{1}{\xi} - \frac{1}{e^\xi - 1}\right) d\xi^\alpha,$$

and let $k > k_0(\alpha)$. Then

$$\mathbf{P}\Big\{\big|l - [k - \widetilde{k}_0(\alpha)]\big| \geqslant O\big(n^{\alpha/(1+\alpha)}/\log n\big)\Big\} \leqslant e^{-\left(k-\widetilde{k}_0(\alpha)\right)^{(\alpha/(\alpha+1)-\delta)}}, \tag{53}$$

where δ, $0 < \delta < \alpha/(\alpha+1)$, is as small as desired and independent of n and $\mathbf{P}$ stands for the ratio of the number of variants satisfying the condition in the brackets to the total number of versions corresponding to the value $k = \widetilde{k}_0(\alpha)$ (*cf.* (44)). *If $d = 2$, then*

$$\widetilde{k}_0 = \frac{1}{\pi}\sqrt{3/2}\sqrt{n}\log n.$$

In order to avoid the situation in which $k - k_0$ persons remain without banknotes, the audience can organize itself into $[k/k_0]$ groups (families) and distribute the money in each of these groups according to some established rules (for instance, in equal parts). This is an analog of combining the market and state dispensations (two different arithmetics, [24]). Thus, the value $[k/k_0]$ determines the correlation between the state and market sections in a state of crisis.

To what area of mathematics one can refer the problem in question?

First, this is the theory of partitions as a part of number theory. Indeed, for $k < \sqrt{n}$, we can say how many terms in a sum of k summands contain summands with values between $[a]$ and $[b + \sqrt{k}\log k]$, assuming that all partitions of n into a sum of k summands are equally possible.

Second, this is a branch of science related to Kolmogorov's complexity because, for $k > 2k_0$, we add several zeros (as summands) to a sum of k summands and find out the point at which the complexity becomes maximal. This gives the number of members of the audience having no banknotes at all if all variants with added zeros are assumed to be equally possible.

Third, this is related to tropical mathematics because

1) we use logarithmic paper to calculate the entropy;

2) to calculate the Hausdorff dimension for a number sequence, we also use the ratio of logarithms.

Fourth, this is related to quantum physics, because the Bose–Einstein distribution and the Bose–Einstein condensate can be regarded as famous quantum phenomena.

Fifth, this is related to statistics because, in particular, this describes the white noise.

Sixth, to probability theory, because inequalities established here are inequalities of Chebyshev type.

On the other hand, as far as most experts in each of the areas listed above are concerned, the theory developed in this way is rejected by them. Firs of all, of quantum physicists. The above theory encroaches upon the basic principle formulated in the classical book "Statistical Physics" by Landau and Lifshits. We read there: "In the Bose statistics, the chemical potential is always negative" ([8, §53; see also §59]).

One of essential points of our theory is the very existence of positive chemical potentials $\mu = -\kappa$ (Lemmas 1 and 2).

Our theory causes perplexity among experts in number theory because we allow adding zeros to an expansion into a sum of summands. The point is that one can add arbitrarily many zeros. Nevertheless, complexity theory shows its abilities here: zeros can be added indeed; however, in such a way that the complexity becomes maximal.

The above theory causes the perplexity of experts in probability theory because it has no continuous analog, namely, the condition that the family $\{N_i\}$ is integral is essential here. In the same way, in tropical mathematics, one passes to the limit in the course of "dequantization," whereas here we consider the asymptotics near an essential singular point.

In complexity theory, the Bose–Einstein entropy was first considered in joint papers of V. V. V'yugin and the author, and it is not yet customary for the experts.

Therefore, although the above conception is simple and widely applicable, it is so unusual that this is a hard task to explain it to experts. Fist of all, this is the case for the postulate in the famous Landau–Lifshits handbook. Therefore, as was noted at the beginning of the paper, economic problems play a connecting role for all these areas of mathematics and quantum physics when solving the corresponding problems immediately and often individually (as was the case, in particular, in the author's private life).[5] The very economic problems help to tie up together the diverse components of the above conception.

2. DIMENSION AS AN INVARIANT OF CHAOS

The author noticed the following fact in the Chernobyl catastrophe. The fractal dimension of the alloy composed of diverse components and solidified in the form of an "elephant leg" going down to the concrete floor of the station coincided in the computation (possibly occasionally) with the dimension corresponding to the frequency of rejects of the contaminated gas through the heap of the destroyed reactor to the atmosphere.[6] This strange coincidence impressed the author greatly.

The very "porousness of the medium" and its fractal dimension, along with the application of the Darcy law [1], enabled us to make heuristically, and sometimes intuitively, several correct predictions concerning the behavior and the cooling of the heap as a whole. In the above notion of dimension of chaos, the most important role is played by the quantity b, which is the inverse temperature in physics, and the dimension can be defined by the dependence of the energy $\mathcal{E}$ (which

[5]The author is not a businessman; however, at critical moments, he took minimal measures for a catastrophe not to befall his family.

[6]Note that all computational data were placed on secret lists and deleted from our book [1].

is also denoted in the present paper by the symbols n and M) on b, i.e., on the temperature. And this very quantity turns out to be determining and constant. For an absolutely black body, it is equal to 6 (see [19]), to 1 for white noise, and to 2/3 for turbulent motions.

Thus, in this connection, one can pose the following scholastic question: What is the dimension of chaos in its original meaning mentioned at the beginning of the paper?[7]

As is said in the Bible, before the creation of the world, there was chaos. Here are the exact words: "The earth was without form and void, and darkness was upon the face of the deep." Theologists, beginning with St. Basil the Great (Ἅγιος Βασίλειος ὁ Μέγας) who systematized earlier opinions in the book "Six Days,"[8] already treated this as chaos from which the world could be created. In what followed, theologists even used the analogy with a chaos of mixed words from which holy books are created.

However, Zipf (using immensely many experiments) and all further investigations showed that, in the definition of the author, the dimension of this chaos is equal to zero. If one expands the photos of nature into colors, then, as can be seen from [18], the dimension of this chaos is also equal to 0.

Some theologists present the example of colored plates used to create tesselation patterns in temples. It can be proved that the dimension is also equal to zero in this case.

If we follow these considerations, then the words "without form and void" can be treated as the claim that the original dimension of chaos was zero, all the more, because "it casts no shadow."[9]

Professor I. G. Pospelov supplied me with statistical data concerning the dimension of American banks enumerated in the descending order of their assets. Beginning with the index 200, these banks have zero dimension, and these very banks must be supported most of all in the state of crisis.

3. CRISIS OF DEBTS

We shall characterize debts by two numbers:

(1) the credit period s after which the debt must be returned, in units corresponding to the least period $s_{\min}$ and the greatest period $s_{\max}$;

(2) the size of the debt N_s evaluated in some currency, say, in dollars.

Let n_s be the number of subjects having debts of period s, and let each of these subjects, k ($k = 1, \dots, n_s$), have the debt N_{sk}, respectively. Thus,

$$N_s = \sum_{k=1}^{n_s} N_{sk}, \qquad M = \sum_{s=s_{\min}}^{s_{\max}} \frac{\sum_{k=1}^{n_s} N_{sk}}{s}, \qquad N = \sum_{s=s_{\min}}^{s_{\max}} \sum_{k=1}^{n_s} N_{sk}, \qquad N \gg 1. \tag{54}$$

The main condition:

we assume that all families $\{N_{sk}\}$ are equiprobable.

The fact that only the numbers N_{sk}, N_s, and N are integer is inessential. For the problem to be related to integers completely, we measure $1/s$ in integers, i.e., write $i = [s_{\max}/s]$. To avoid gaps with respect to i, without loss of generality, we count time by using the quantities approximately equal to the ratios $s = s_{\max}/i$, $i = 1, \dots, s_{\max}$, rather than months. Then all sums will be taken in reverse order. Namely,

$$M = \sum_{i=1}^{s_{\max}/s_{\min}} iN_i, \qquad \sum_{i=1}^{s_{\max}/s_{\min}} N_i = N. \tag{55}$$

Let k_i be the number of subjects having debts of period i and let, for each of them ($k = 1, \dots, n_i$),

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[7]The rest of this section was written together with T. V. Maslova.

[8]In his writing "Homiliae in Hexaemeron" (Talks on the Six Days), St. Basil the Great validated creationism (a religious theory on the creation of the world by God out of nothing, which is a specific feature of theistic religions (Judaism, Christianity, and Islam)) using the Aristotelian physics and Neoplatonic philosophy.

[9]M. A. Bulgakov, "The Master and Margarita," §14.

the debt be equal to N_{ik}, respectively. Thus,

$$N_i = \sum_{k=1}^{n_i} N_{ik}, \qquad M = \sum_{i=1}^{s_{\max}/s_{\min}} i \sum_{k=1}^{n_i} N_{ik}, \qquad N = \sum_{i=1}^{s_{\max}/s_{\min}} \sum_{k=1}^{n_i} N_{ik}, \quad N \gg 1. \tag{56}$$

Assume that the fractal dimension of the family N_i is greater than zero or that the Pareto index is less than two.

If the payoff of all debts with regard to the GNP, debt services, etc., evaluated by business analysts, exceeds the number $M/N_{\rm cr}$ in Theorems 2, 3, and 4, then a condensate necessarily occurs, i.e., either the debts become insolvent or they go down to the lowest level, namely, to the longest debt, to the mortgage, which will then collapse. Thus, the debts (possibly short-term and virtual) will condense on real economics as long-term credit obligations. This is a mathematical law of number theory.

In order to increase the dangerous numbers which, one exceeded, cause dangerous consequences, one can introduce diverse new currencies in the country. For instance, for the Pareto index equal to 2, if the number of currencies becomes k times larger, then the above dangerous numbers become k times larger as well. For instance, the previous currencies should be preserved in EU, along with the euro [20]. Custom restrictions increase these numbers. Special benefits to the workers of a given firm (surplus dividends by goods rather than money), exchange, reduction of prices of goods made in the given country (for instance, the price of gasoline in Saudi Arabia), rent-free apartments, hostels for workers of a given firm, coupons for taxi fares, – all these tools improve the mathematical situation, the situation of the "implacable number law" for a Bose system.

It is of special importance to separate "currencies" in liabilities, in credits, in debts. As we have seen above, the turnover rate of assets, turnovers of money, plays an essential role in the GNP. In the case of debts, only the lengths of debts are of importance. *The rate of the turnover of debts is not within the framework of the above scheme.* And this fact is not accidental. The rate of turnover of debts plays a negative role in the above mathematical conception of self-organizing chaos. Therefore, to improve the mathematical part of the problem, the credits must be divided into credits of different level of privilege (a firm can give credits to its staff workers with diverse versions of debt service, diverse count rules for floating percentages) which could enable one to classify the debts as diverse "currencies." In this case, the action of the mathematical law that describes the passage to a catastrophe caused by exceeding the number k_0 is weakened because the very quantity k_0 drastically increases.

Thus, the turnover of debts is hindered due to their different cost depending on their percentage.

These thoroughgoing reforms can possibly somewhat hinder the blood circulation of the economic system. However, the danger of occurrence a zero-genus phase condition ([10, 21, 22]) must not be ignored.

4. ON THE PREVENTION OF THE CRISIS OF DEBTS

Many people understand that the cause of the catastrophe, the global slump, is in the tremendous debts of the USA. For instance, if somebody has fallen seriously ill after becoming too cold and the illness began, say, with gastritis, then a good practitioner knows that the patient has an immune deficiency, and the cause of the disease is in becoming too cold. However, a bad physician asks: "What did you have for lunch?" And he regards this as the source of the deterioration in health. Similarly, some economists assume that the cause of the crisis is a wrong mortgage lending rather than huge short-term debts and "empty vessels."

In order to decide how to struggle with the disease, the crisis, one must understand its true cause. We claim that the true cause of the crisis is mathematical rather than economical, and therefore we can say how this cause can be eliminated. It is quite probable that this will be incomprehensible for economists. Without our interference, life will correct these mistakes by itself; however, with significant losses. This had already happened in Russia in 1989–91 (see the addenda).

During the time of slavery, a person not returning his debt to the creditor became a slave of the creditor. Thus, the pledge was personal freedom. In the early Middle Ages, the Catholic Church forbade its members to lend money at a rate of interest. The money-lenders, the so-called usurers, were non-Christians or Lombardians, as a rule (hence the Russian word "lombard", the pawnshop).

In tsarist Russia, a bankrupt person, i.e., someone who did not pay his debt, was put into a "yama" (a hole), i.e., into debtor's prison. However, to a lender, the fact that his insolvent is sitting in a "hole" could give moral satisfaction only. This gave no profit to the lender, in contrast to the creditor who was a slave-owner, because his debtor worked for him. Therefore, in Russia, the debtors declaring themselves as insolvents could often make a good profit out of their creditors, which was described in detail by A. N. Ostrovsky in his plays.

Credit is needed both to promote trade and even to make an advantageous marriage (as was described by Sukhovo-Kobylin in his plays). A credit given for education, for raising the level of one's skill, for a lodging (creating conditions for normal work), for transport reducing the time of transfer (and time is money!) forms conditions necessary for the development of economics, to increase the productivity of labor, the degree of efficiency of all actual creators of all welfare.

Nowadays, correspondence is out of fashion, people phone one another by mobile phone or send an SMS and, instead of epistolary novels (like, for instance, "Poor People" by Dostoevsky), the writers will compose short comics in the form of an SMS correspondence.

To avoid the "hole," one must anticipate the time at which to pay numerous debts of diverse payout period. How much a given person (an enterprise) earns, how much money is spent to service the debts, etc. In the modern computer world, there are experts in this area, the so-called business analysts. The rate of globalization increases the rate of blood circulation of the economic system, and thus increases the role of global business analysts and top-managers. On the modern market, a great role is played by trust, a brand. The distrust to the ruble is an example, and this distrust cannot be prevented even by supporting it by considerable funds. This very distrust is the result of the 1998 devaluation. Similarly, if the USA will devaluate the dollar, they will lose the trust ("honest merchant's word," as described in A. N. Ostrovsky's "Portionless") gained by the dollar during many years, which enabled one to speak of a unipolar world. At the same time, this very world speaks English as an international language. This unipolar system was so unwisely used by the previous USA administration when hanging Saddam Hussein and violating a certain balance among Muslim tendencies. The fact that Barack Hussein Obama was elected as the new President of the USA can possibly ensure that the system constructed in America with great difficulty and blood during long years is capable to correct catastrophic mistakes.

It is not too late now to put into a "hole" the creators of pyramids and "empty vessels" to give a moral satisfaction to everybody who lost in this way, to make bankrupt the bosses without admitting that, by virtue of the above theorems, the debts would fall to long-term credits and shake real economics. It is not too late to recover the national currencies (without abolishing the euro) in the countries of EU before it will happen spontaneously and to give the possibility to make a complicated exchange between these countries without recalculation to euro and dollars. These measures will at least free the economy from the "implacable number law" and will move problems from the mathematical area into a purely economical one.

If these thoroughgoing reforms will not be made, then, according to the mathematical law, they will happen spontaneously, and hence will cause great losses. In this case, the dollar will be most probably devaluated, and USA will go bankrupt. And, if the initiative of this change will not come from the top, then this will happen as a result of even deeper catastrophe, when the implacable number law will ruin middle class people. Let me present some paradoxical analogies.

As is known, the remarkable metamorphosis which happens to locusts is similar to percolation; the group of green separately living locusts eaten by birds and some other animals may become a swarm, and, under rapid propagation, their color changes, and they become yellow. This swarm devastates fields completely.

The author also explained percolation by Bose condensation (see [20]). Since the distribution into biological species and families also satisfies Zipf-type laws, this phase transition of the locust can be explained by the mathematical laws of number theory, namely, the condensation into a swarm.

A similar phenomenon takes place for some species of caterpillars. In his famous play "Rhinoceros," Ionesco described the conversion of Germans into the ideology of fascism and their consolidation in it after the defeat in the First World War.

However, does the "implacable number law" act here as well? The revolutions, which were compared by the author with zero-order phase transition [23], lead to a beautiful passage from the egoistic individualistic ideology to the altruistic one and wonderful shifts in culture. Thus, during the revolution, futurism appeared in our country — Mayakovsky, Pasternak, Khlebnikov, Tsvetayeva, Prokofiev, Shostakovich, Malevich, Kandinsky, Chagall, e.a.

Revolutions begin with terror — according to a Russian prison rule: "I'll burn myself but incinerate the bugs." Moreover, "revolutions devour their children." Look at the death figures of personalities of the Great French Revolution. A revolution is a liberation of the oppressed, and this is beautiful. However, it is not desirable that it will devour my own children as well as its own.

For my children, it would be better if processes will develop in an evolutionary way, i.e., as it did from the time of uncle Tom's cabin to the time of Barack Obama. And, to this end, it would be better (instead of spontaneous transition, instead of killing Martin Luther King, Jr.) if the authorities would apply the scientific mathematical approach and at least would not violate the "implacable number law."

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REFERENCES

1. V. P. Maslov, V. P. Myasnikov, and V. G. Danilov, *Mathematical Modeling of the Damaged Block of the Chernobyl Nuclear Power Plant* (Nauka, Moscow, 1988).
2. V. P .Maslov, "New Look at Thermodynamics of Gas and Clusterization," Russ. J. Math. Phys. **15** (4), 493–510 (2008).
3. V. P. Maslov, *Quantization of Thermodynamics and Ultrasecond Quantization* (Sci. Res. Center "Regular and Chaotic Dynamics," Moscow, 2001) [in Russian].
4. V. P. Maslov, *Quantum Economics* (Nauka, Moscow, 2005).
5. M. J. Coons, "General Moment Theorems with Applications," Master's Thesis (Baylor University, 2005).
6. A. M. Vershik, "Statistical Mechanics of Combinatorial Partitions, and Their Limit Configurations," Funktsional. Anal. i Prilozhen. **30** (2), 19–39 (1996) [Funct. Anal. Appl. **30** (2), 90–105 (1996)].
7. V. P. Maslov, *Mathematical Conception of the Gas Theory*, `arXiv:0812.4669 29 Dec 2008`.
8. I. A. Kvasnikov, *Thermodynamics and Statistical Physics. Theory of Equilibrium Systems* (Moscow: URSS, 2002) [in Russian].
9. L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics,* Vol. 5: *Statistical Physics* (Nauka, Moscow, 1964; Pergamon Press, Oxford, 1968).
10. V. P. Maslov, "Negative Asymptotic Dimension, a New Condensate, and Their Relation to the Quantized Zipf Law," Mat. Zametki **80** (6), 856–863 (2006) [Math. Notes **80** (5–6), 806–813 (2006)].
11. V. P. Maslov and V. E. Nazaikinskii, "On the Distribution of Integer Random Variables Related by Two Linear Inequalities: II," Mat. Zametki **84** (1), 69–98 (2008) [Math. Notes **84** (1–2), 73–99 (2008)].
12. P. Erdős, "On Some Asymptotic Formulas in the Theory of Partitions," Bull. Amer. Math. Soc. **52**, 185–188 (1946).
13. V. V. V'yugin and V. P. Maslov, "Concentration Theorems for Entropy and Free Energy," Problemy Peredachi Informatsii **41** (2), 72–88 (2005) [Probl. Inf. Transm. **41** (2), 134–149 (2005)].
14. G. Shafer and V. Vovk, *Probability and Finance. It's Only a Game!* Wiley Ser. Probab. Stat. Financial Eng. Sect. (Wiley, New York, 2001).
15. A. N. Kolmogorov, "On the Logical Foundations of Information Theory and Probability Theory," Problemy Peredachi Informatsii **5** (3), 3–7 (1969) [Probl. Inf. Transm. **5** (3), 1–4 (1969)].
16. R. J. Solomonoff, "A Formal Theory of Inductive Inference. I," Information and Control **7** (1), 1–22 (1964).
17. V. P. Maslov, "Theorems on the Debt Crisis and the Occurrence of Inflation," Math. Notes **85** (1–2), 146–150 (2009).
18. V. P. Maslov and T. V. Maslova, "On Zipf's Law and Rank Distributions in Linguistics and Semiotics," Mat. Zametki **80** (5), 718–732 (2006) [Math. Notes **80** (5-6), 679–691 (2006)].
19. V. P. Maslov, "Quasithermodynamics and a Correction to the Stefan–Boltzmann Law," Mat. Zametki **83** (1), 77–85 (2008) [Math. Notes **83** (1–2), 72–79 (2008)].
20. V. P. Maslov, *Economic Law of Increase of Kolmogorov Complexity. Transition from Financial Crisis 2008 to the Zero-Order Phase Transition* (*Social Explosion*), `arXiv:0812.4737 29 Dec 2008`.
21. V. P. Maslov, *Quantum Economics*, 2nd Ed. (Nauka, Moscow, 2006).
22. V. P. Maslov, "Quasistable Economics and Its Relationship to the Thermodynamics of Superfluids. Default as a Zero Order Phase Transition," Russ. J. Math. Phys. **11** (3, 4), 308–334, 429–455 (2004).
23. V. P. Maslov, "Phase Transitions of Genus Zero," Mat. Zametki **76** (5), 748–761 (2004) [Math. Notes **76** (5-6), 697–710 (2004)].
24. V. P. Maslov, "On the nonlinearity of averages in financial mathematics," Mat. Zametki **74** (6), 944–947 (2003) [Math. Notes **74** (5–6), 893–896 (2003)].

ADDENDA

“ . . . it is a terrible thing for a man to find out suddenly that all his life he has been speaking nothing but the truth.”
Oscar Wilde, “The Importance of Being Earnest,” 1895.

In the years 1989–1991, the author spent a great deal of time trying explain to the country’s leading economists and politicians the relevant mathematical results concerning the coming catastrophe in the USSR economy.

The author’s articles in the Soviet press (1989–1991)

The situation at that time, when the cost of one personal computer was about the same as that of 1000 cubic meters of timber, was reminiscent of the beginning of commerce in tropical Africa, when European traders would exchange worthless little mirrors for ebony and gold. Having in mind economists and politicians (not mathematicians), I called the relevant mathematics “tropical,”[10] since I felt that the term “idempotent analysis” used by mathematicians for this branch of mathematics would sound disharmonious to the ear of an economist or a politician.

Although at that time I was able to publish articles in newspapers and even talk to some of the highest political leaders, I did not succeed in convincing anyone. Before the 1998 default, I also tried, but not as hard.

Here is a list of my publications in the Soviet press in 1989-1991.

1. “Are we destined to foretell?”, *Kommunist*, No. 13, 1989, pp. 89–91.
2. “Who has what prices?”, *Pravda*, March 8, 1991.
3. “The ruble under the dollar’s thumb (how to restore the blood flow in our economy),” *Nezavisimaya Gazeta*, March 26, 1991.
4. “And all the goods will flow abroad,” *Torgovaya Gazeta*, No. 68, June 6, 1991.
5. “The law of the broken thermos bottle,” *Poisk*, No. 3 (89), March 11, 1991.
6. “How to avoid a total catastrophe,” *Izvestiya*, No. 187, August 7,1991. [On August 19, 1991, the putsch began, and on December 8, 1991, The Belovezhskoe agreement on the disintegration of the Soviet Union was signed.]
7. “How does the state intend to pay its debts?”, *Torgovaya Gazeta*, March 22, 1995.
8. “How does a pound of rubles fare against one of their pound sterling?”, *Literaturnaya Gazeta*, November 27, 1996.
9. “The authors of the catastrophe are those who strive to become the saviors,” *Izvestiya*, September 24, 1998.

In the paper [17], I touched upon the question of how hard it was to explain to our leadership the mathematical laws which indicated the way out of the crisis cycle. It is even more difficult, I think, to explain them to foreign politicians.

Let me quote an excerpt from an interview I gave to the *Minnesota Daily* (published June 1, 1990, page 11, during Gorbachev’s visit to the US). “Victor Maslov $\langle\cdots\rangle$ said the biggest problem is that the current Soviet leaders have no idea about the average citizen’s struggles. $\langle\cdots\rangle$ He said a large portion of the real economy in the Soviet Union is dependent on the black market – a fact that goes unrecognized by the government. “The government has lots of good ideas but when they try to put these ideas to our people, it is not good because they have not investigated the real situation”, Maslov said. $\langle\cdots\rangle$ Maslov said it might be beneficial for Soviet leaders to speak with economists here, but added that the economists might be unfamiliar with the unique problems facing the Soviet Union.”

I had in mind the Nobel Prize winner V. V. Leontiev and Leo Hurwicz (the future Nobel Prize winner), whom I succeeded in convincing in the inexpediency of measures proposed to alleviate the “tropical” situation in the USSR. But they both said that they cannot intervene because they are not experts in the Soviet economy.

Excerpta from “Expertise and Experiments” (*Novy Mir*, no. 1, 1991)

As an addition to what was explained in [17], let me present some excerpts from my article in the once popular journal *Novy Mir*. The article was submitted to the journal in 1989, but since *Novy Mir* did not appear at all in 1990 (because of financial difficulties), the article was published only in January 1991. In it, I developed the conclusions that follow from the application of idempotent

[10]See the author’s article in the journal *Kommunist*, No. 13, 1989.

analysis to economic problems of the USSR.

Since it is impossible to avoid mathematical rules, a second currency, the dollar, began functioning in the country, the USSR disintegrated, and the Islamic revolution spread to several countries. The Pol Pot regime in Cambodia was crushed by my father-in-law, Le Duan [Le Duc Anh],[11] but as an attractor (see below), it did not disappear.

EXPERTISE AND EXPERIMENTS

Excerpts from the author's article in "Novy Mir", No. 1, 1991. The heading and comments in square brackets [·] were added by the editor of the translation.

[**Computer-aided panel of experts**]

It is natural [to save the Soviet economy] not to act by trial and error, but to begin by picking a panel of experts, representing different sections of the population, to search for compromise solutions, to analyze a huge number of variants, and choose the optimal one among them. ⟨· · ·⟩ It is precisely a panel of experts, expressing the averaged opinion of various strata of society, their possible reaction, the means used by various sections of the population to circumvent certain laws and regulations, that can replace statistical data and the study of the rules, partially market-oriented, partially semi-legal, which people employ in practice. ⟨· · ·⟩

The expert system must be conductive to an optimal and careful intervention in the existing systems of social relationships, taking into account the psychology of the Soviet person. ⟨· · ·⟩ It is only the interaction of a panel of experts with a computer that can succeed in establishing a stable equilibrium between the centralized, the regional, and the market segments of the economy in their complex mutual relations. The expert system allows testing out various versions of rules and regulations not on the living population, as had so unfortunately been done during the partial prohibition of vodka, but by means of computer modeling aided by the panel of experts. The result of such testing must be made public and be verifiable.

If by means of effective measures we will succeed in stopping the USSR economy from falling apart, which may lead to the actual disintegration of the Soviet Union, then according to a simple calculation, the optimal solution would not be integration with Europe, but with Japan, South Korea, with countries where the development of electronics in many years ahead of ours and far in front, say, of that of heavy industry.

[**Three currencies**]

As an example, let me describe the solution of the problem of making the ruble convertible proposed by the expert system. ⟨· · ·⟩

The impossibility of spending rubles to buy needed goods and the instability of the situation in the country, as well as the instinctive equalitarian attitude of various segments of the population, impelled people who had amassed large sums in rubles to exchange them for dollars and deposit the dollars in foreign banks. These circumstances lead to the depreciation of Soviet made goods, their flow out of the country, and thus to colossal economic losses for the state. ⟨· · ·⟩

The presently existing [in 1989] huge discrepancy in the exchange rate of the ruble against the dollar for different types of goods and services contributes to the deepening of the economic crisis, and, as computer modeling has shown, it cannot be overcome in the framework of the two-currency system without a significant increase of social tension. ⟨· · ·⟩

We are familiar with the three-currency system introduced during the NEP period [the New Economic Policy in Soviet Russia in the 1920ies] involving the ruble, the chervonets [banknote equivalent to the 10 ruble gold coin], and the dollar, as well as the three-currency system presently working in different regions of Russia and involving the ruble, the dollar, and the "certificate for specific goods" (the latter can be exchanged for the type of merchandise specified on it).

The expert panel proposed a rather paradoxical compromise version: rubles, dollars, "general-purpose certificates." The latter are coupons or cards that can be exchanged for a wide choice of goods. This version is the one that won the contest supervised by computers together with the panel of experts. Thus the system most manageable by means of price regulation is described by the following model.

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[11]See V. Maslov, "Daring to touch Radha", Lviv, Academic Express, 1993, pp. 42–43; available at the website `http://www.viktor-maslov.narod.ru` in the Literature section; see the discussion concerning the Pol Pot regime on pp. 27–28.

The salaries in rubles remain the same, but in addition to the salary, employees are given all-purpose certificates of nominal value equal to the salary for the same position in the 1980ies. Using these certificates, one would be able to purchase essential goods for rubles, as well as certain items needed for comfort (automobiles, vacuum cleaners, TV sets, refrigerators). Besides, these certificates or coupons could be used jointly with rubles for services, rent, travel expenses. $\langle\cdots\rangle$

The fundamental question of what goods can be sold for dollars, or for certificates plus rubles, and what priorities should be chosen cannot be answered without the help of the panel of experts (and ratified by the Supreme Soviet). $\langle\cdots\rangle$

The buying and selling of certificates on the black market should not be forbidden or hampered.

The advantage of the proposed system is its flexibility and stability, the weakness of its reaction to unexpected changes such as strikes, popular uprisings, etc. The latter can only lead to a higher inflation rate of the ruble, which will only accelerate the passage to a convertible currency.

[**Long-term forecast and attractors**]

The difference between an expert system giving the optimal recipe at any given moment and a system providing a long-term recipe (taking into account the far away future), is similar to the difference between tactics and strategy. One can be a good tactician but a poor strategist. The mathematical part of the system that I have described is tactical. Its implementation is laborious, but possible. But when it is necessary to give long-term predictions while influencing their evolution, the problem becomes significantly more complex.

The situation may be compared with the diagnosis and the therapy of an illness. The diagnostic expert system, which assists the group of doctors in choosing the best therapy for the patient at the given moment, crucially differs from the expert system predicting the course of the illness in the long term while influencing its development. The patient has several options: to be totally cured, or becoming a chronic sufferer of the disease, and so on. Any of the options that may be realized in the long term will be called an attractor. Clearly, in order to create such an expert system, it is necessary to input into the computer's memory all the possible pathways for evolving to the chronic state. Besides the optimal trajectories of the changes in the patient's health resulting from the prescribed therapy, the computer must show which of these trajectories lead to one of the various attractors. $\langle\cdots\rangle$

A similar, but much more complicated mathematical problem arises in creating an expert system with strategic goals that would model social and economic phenomena. Such a system must take into account social, political, and economic regimes that existed in the past, and those that arose recently. These can be the regimes implemented in Western countries, the national-socialist systems of Spain, Italy, or Germany, and, finally, "unique" regimes (the Khomeny regime in Iran, the "communist" regime of Pol Pot in Cambodia), which have no analogs, but are theoretically possible. Their symptoms must be studied and input in the computer's memory. If the trajectories of solutions tend towards such attractors, this must be taken in consideration. $\langle\cdots\rangle$

Such a strange attractor arises in the situation around Germany, where East German citizens either emigrated to West Germany in order to earn hard currency there, or didn't work at all, since their salaries in the West would be 10 times larger. They were awaiting reunification with West Germany as a kind of emigration together with their homes and factories. The computer extrapolates this into the following "strange" attractor: at first, East Germany will rejoin West Germany, later so will Poland (under less favorable conditions), then the Baltic states, then the Russian Federation. The result will be an economic conquest of the territories that Hitler dreamed of. $\langle\cdots\rangle$

In studying long-term processes, we essentially lose the possibility of efficiently using the experts, since they will have to describe the evolution of the world outlook of the corresponding social stratum in twenty-thirty years time on the basis of their present reaction. Nevertheless, the experts should still be called upon to predict the changes in the basic viewpoint of the given segment of society. $\langle\cdots\rangle$

If for the initial axiom we accept the existence of evolution laws not yet understood, it is just as absurd to complain about them and condemn their functions as it would be to accuse Archimedes' principle of the responsibility of somebody drowning. The laws must be studied and one must act within a sufficiently wide framework of these laws. $\langle\cdots\rangle$

This would be a first step towards the understanding of the actual laws of human society. It is extremely difficult to give a mathematical formalization of long-term evolution (unlike the case of a tactical expert system), nevertheless we should strive to such an understanding. At least in order to give a forecast from the outside.