Towards an Automatic On-Line Signature Verifier Using Only One Reference Per Signer

Moises Diaz*, Andreas Fischer†‡§, Réjean Plamondon† and Miguel A. Ferrer*
* IDeTIC Institute, Universidad de Las Palmas de Gran Canaria, Spain.
Email: {mdiaz, mferrer}@idetic.eu
†École Polytechnique de Montréal, Laboratoire Scribens, Montréal, Canada
Email: {andreas.fischer, rejean.plamondon}@polymtl.ca
‡DIUF Department, University of Fribourg, Switzerland.
Email: andreas.fischer@unifr.ch
§iCoSys Institute, University of Applied Sciences and Arts Western, Switzerland.
Email: andreas.fischer@hefr.ch

***Abstract*—What can be done with only one enrolled real handwritten signature in Automatic Signature Verification (ASV)? Using 5 or 10 signatures for training is the most common case to evaluate ASV. In the scarcely addressed case of only one available signature for training, we propose to use modified duplicates. Our novel technique relies on a fully neuromuscular representation of the signatures based on the Kinematic Theory of rapid human movements and its Sigma-Lognormal model. This way, a real on-line signature is converted into the Sigma-Lognormal model domain. The model parameters are then varied to generate new duplicated signatures.**

I. Introduction

Verifying the identity of an individual based on handwritten signatures without boring the client by asking for many signatures is a big challenge for both commercial and research issues. An important constraint in ASV is the number of available signatures enrolled in the systems. The recent systems are able to achieve an acceptable performance for handwritten signatures when 5 or more signatures characterize the signers.

One enrolled signature per user would be the optimal condition in some critical cases. For instance, acquiring a signature of a wealthy business person could have a decisive value for a bank. In this context, what would be the performance of some current ASV? This question opens a new challenge which has not been completely studied in the literature.

A new trend to get more signatures at no human cost is duplicating the real available signatures [1]–[5]. The duplicator engine takes a real signature to apply some distortions and get an artificial duplicated specimen. This way, we could design as many signature as desired could be produced. It has been demonstrated that the use of duplicated signature can improve the performance of ASV systems [1], [2]. To the best of our knowledge, for the recent state-of-the-art systems, there is only one study about the use of a single reference signature, namely in [1]. Authors used duplicated signatures to enlarge the training by using geometrical and affine transformations. Nevertheless, such transformation are not completely human-like. To produce more realistic variabilities in the signatures, a cognitive or neuromuscular approach would be more appropriate

Concisely, this paper proposes the use of the Kinematic theory of rapid human movements to duplicate signatures. The proposed method represents analytically the given real signature in the Sigma-Lognormal domain. A so called reconstructed signature is thus generated from the original signal. Then, the new duplicated signatures are produced by fine-tuning the Sigma-Lognormal parameters. A main advantage of this approach with respect to ASV is that the reconstructed and the duplicated specimens are more hardware independent than the original ones, reducing the constraints of the acquisition protocol.

This paper addresses the proposed procedure in two scenarios: In the *Scenario 1* we analyze the consequences of replacing the original signatures by the reconstructed ones, when only one signature composes the reference set. In the *Scenario 2* we study the impact of using duplicated signatures in the reference set. In both scenarios a DTW-based state-of-the-art verifier is used as matcher.

The outline of this paper is organized as follows. The neuromuscular model is briefly revised in Sect. II; Sect. III presents the model implementation; the experimental protocol is discussed in Sect. IV; the results are reported in Sect. V and; the Sect. VI concludes the paper.

II. Neuromuscular model

The neuromuscular model [6] is able to represent a high complex fine motor control based on the the kinematic theory of rapid human movements and its Sigma-Lognormal model. This model decomposes the handwriting movements into a sequence $s_1, \ldots, s_n$ of elementary strokes. Stroke means the most elemental hand movement during the handwriting at a specific time under the effects of a well-learned spatial action plan executed by the neuromuscular system.

A. Signature reconstruction from the Sigma-Lognormal domain

The Sigma-Lognormal ($\Sigma\Lambda$) stroke model [7] is used for decomposing complex handwriting movements. Each stroke $\{P_i\}_{i=1}^{i=n}$ is represented by one parametrized lognormal, n being the number of total strokes during the execution of

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one component. The theory assumes that a signer executes the movement at time t_{0_i} covering a distance proportional to D_i. The parameters μ_i and σ_i are related to the neuromuscular execution of the motor command. Also, it is assumed that the movement of a single stroke happens along a pivot with respect to the start angle θ_{s_i} and the end angle θ_{e_i}. In total, each stroke is described in 2D space by six Sigma-Lognormal parameters: $P_i = (D_i, t_{0_i}, \mu_i, \sigma_i, \theta_{s_i}, \theta_{e_i})$. This way, the velocity of the complete handwriting movement is considered as the vectorial summation of the stroke velocities.

$$\vec{v}(t) = \sum_{i=1}^{n} \vec{v_i}(t) \tag{1}$$

where the module and direction of each stroke is described as:

$$|\vec{v}_i(t)| = \frac{D_i}{\sqrt{2\pi}\sigma_i(t - t_{0_i})} \exp\left(-\frac{(\ln(t - t_{0_i}) - \mu_i)^2}{2\sigma_i^2}\right) \tag{2}$$

$$\phi_i(t) = \theta_{s_i} + \frac{\theta_{e_i} - \theta_{s_i}}{D_i} \int_0^t |\vec{v}_i(\tau)| d\tau \tag{3}$$

B. Estimation of the Sigma-Lognormal parameters

One of the main aspects of this model is the fully automatic extraction of the stroke sequence P_i from an original trajectory. The algorithm [6] is based on two main steps.

In the 1st Step, it localizes the number of strokes P_i using the original velocity $\vec{v}(t)$. A local maximum is identified in the speed profile alongside with neighboring inflexion points and minima. To guarantee the stroke identification, the maximum speed and the area under the curve have to be greater than a certain threshold. The 2nd Step extracts the analytical parameter of each identified stroke based on zero crossings of the first and second derivatives of the lognormal equation. The result of this Robust XZERO (RX_0) estimator is further improved with non-linear least squares curve fitting.

These two steps are repeated until the quality of the stroke sequence cannot be further improved. Such a quality of the extraction process is estimated with respect to the squared Euclidean distance between the original velocity $\vec{v}_o(t)$ and the reconstructed velocity $\vec{v}_r(t)$ expressed as signal-to-noise ratio (SNR), defined as:

$$SNR = 10 \log \left(\frac{\int_{t_s}^{t_e} |\vec{v}_o(\tau)|^2 d\tau}{\int_{t_s}^{t_e} |\vec{v}_o(\tau) - \vec{v}_r(\tau)|^2 d\tau} \right) \tag{4}$$

The time t_s is the start time and t_e is the end time of the trajectory. High SNR values indicate high reconstruction quality. The reader could refer to [6] for more details on the neuromuscular model.

III. Generation of Duplicated Signatures

The duplicator engine takes as input the original sequences of a signature $\{x, y, t\}_o$. Then it provides the sequences of a duplicated signature $\{x, y, t\}_d$. The implementation of the duplicator engine requires three processes.

A. Preprocessing

The preprocessing consists of three consecutive steps for each component. We use here the term component to describe the trajectory between a pen-down and pen-up [8]. i) Each component *(pen-down)* is calculated through the discontinuities in the approximate derivative of the dynamic time $\{t\}_o$. Then the time sequence is artificially substituted for a new one timing sequence $\{t'\}_o$, sampled at 200 Hz according to these limits. Later, the sampling points of each sequence $\{x, y\}_o$ are worked out through cubic splines, where $\{t'\}_o$ is the interpolant. ii) Next, a Chebyshev filter is applied to the interpolated trajectory removing the particular noise introduced often by the capturing devices. iii) During 200 ms at 200 Hz at the very beginning and at the end of the signal, the initial $\{x_1, y_1\}_o$ and final trajectory points $\{x_n, y_n\}_o$ are repeated. This last step introduces during the first and last 200 ms a null velocity, which improves the reconstruction of the signatures with the $\Sigma\Lambda$ model.

B. Reconstructed signature

Using the $\Sigma\Lambda$ stroke model described in Sect. II, each component is worked out with the Robust XZERO (RX_0) algorithm. At this point, we obtain an analytical representation of a signature as a sequence of strokes: $P_i = (D_i, t_{0_i}, \mu_i, \sigma_i, \theta_{s_i}, \theta_{e_i})$. Then, the signature is generated under the Sigma-Lognormal domain through the Equations (2) and (3). Finally, the first and final 200 ms, which were used to enhance the reconstruction, are removed from each components.

C. Duplicated signature generation

The Sigma-Lognormal representation offers the opportunity to design duplicated signatures under a neuromuscular writing perspective. This way, we could add intra-personal variability working with the parametrized signatures. Following the experience of Fischer et al. [9], the generation of duplicated signatures has been performed by uncoupled deformations of t_{0_i}, μ_i, σ_i. The time parameter allows to modify the duration of each individual stroke and, consequently, the duration of the total signature by enlarging or contracting it. The other two parameters are related to the muscular ability of the signer's motor control and introduce variability enough to create new specimens like in [9]. This way, for each stroke we produce a new stroke: $P_i \rightarrow \hat{P}_i = (D_i, \hat{t}_{0_i}, \hat{\mu}_i, \hat{\sigma}_i, \theta_{s_i}, \theta_{e_i})$.

A normal distribution is used to introduce distortion in the new strokes. This way, the new values are computed according to: $\hat{\mu}_i = \mu_i \cdot \mathcal{N}(1, \lambda)$, $\hat{\sigma}_i = \sigma \cdot \mathcal{N}(1, \lambda)$ and, $\hat{t}_{0_i} = t_{0i} + \mathcal{N}(0, \kappa)$, being λ and κ values used to give variability to the signatures. The range of the random values is limited to twice standard deviations.

Once the new sequence of strokes $\hat{P}_i$ is worked out, we generate the new duplicated component as a real one, according to Sect. III-B. Finally, the duplicated signature $\{x, y, t\}_d$ is obtained by joining each duplicated component starting at the same time and position that each component was executed in the original signature.

IV. Experimental Protocol

The details about the database, the verification scheme and the verifier are presented throughout this section.

A. Database

The on-line SUSIG database, collected at the Sabanci University in Turkey, consists of two sub-corpus [10]: the *Visual* and *Blind sub-corpus*. As the majority of recent works [11]–[17], the *Visual sub-corpus* has been considered in this study. This sub-corpus contains the signatures of 94 users, 20 genuine specimens per user, acquired in two sessions, and 10 forged signatures: $94 \cdot 20 = 1880$ and $94 \cdot 10 = 940$ genuine and forged files.

B. Verification methodology

The Visual sub-corpus has been divided into two parts in our experimentation. The first part consists of the 30 first users and it has been used to fine-tune the λ and κ deformation range of values. The second part is composed by the whole database. All results reported in Sect. V are obtained with the second part in order to compare with other works.

As reference set, we have chosen *only the first signature in the first session* provided by each signer. Using the first signatures as reference set mimics a more challenging and realistic scenario in a verification application: no intra-writer variability is known at the beginning.

The genuine scores were calculated matching the rest of genuine specimens with the respective reference set. For the database used, we computed 19 scores for each signer, in total 19 · 94 = 1786 scores for the FRR curve. Two tests were carried out in all experiments: The zero-effort attack or *Random Impostors - (RI)* mimics a signer trying to verify the identity of the others using his/her own signature and without previous knowledge of the originals. For this attack the scores were calculated matching one genuine signature of the other signers against the other different signer's reference set, in total 94 · 94 = 8836 scores for its FAR curve. The second test computes the deliberated attack or *Skilled Impostor (SI)* scores. They are calculated by matching the impostor specimens of each signer against their own reference set: 10 · 94 = 940 scores in total for its FAR curve. To evaluate the verification performance, we adopt the Equal Error Rate (ERR), which measures the error rate at which both FAR and FRR are equal using a global threshold for the whole database.

C. Dynamic Time Warping configuration

The Dynamic Time Warping is an elastic technique to match two sequences of different lengths, well-known in on-line signature verification. In the recent literature, many authors have coped with automatic on-line signature verification with different variations of DTW [11]–[13], [18], [19]. In this work, this function-based local approach uses only the spatial reconstructed or duplicated coordinates x, y, aligning the center of mass. To parametrize the matrix with the features, we have associated a sub-matrix to each component. This way, each sub-matrix has been built using the first and second-order regression: $f^i = [x_n^i, y_n^i, \dot{x}_n^i, \dot{y}_n^i, \ddot{x}_n^i, \ddot{y}_n^i]$. Since some devices do not capture the pressure signal, it has been omitted. Then the final parametrized matrix $\mathcal{F}$ is obtained by concatenating all individual sub-matrices f^i and computing the z-score normalization using the mean and the diagonal covariance matrix: $\mathcal{F} = \text{z-score}(f^1 : f^2 : \ldots : f^\ell)$, being ℓ the number of components. Readers could refer to [20] for further details.

Then all features are matched using a standard version of the Dynamic Time Warping algorithm to find the best non-linear alignment between two normalized feature matrix $\mathcal{F}_A, \mathcal{F}_B$. Then, the final score $\mathcal{S}$ quantifies the membership of a questioned signature Q to the signer model. It is computed as the minimum distance between a questioned signature against the enrolled signatures per signer: $\mathcal{S} = \{\underset{\mathcal{R}}{\text{argmin}}[DTW(\mathcal{R}, Q)]\}/\mu_\mathcal{R}$. Where $\mathcal{R}$ means all signatures included in the reference set and $\mu_\mathcal{R}$ a weighted factor calculated by the average DTW distance ($\mu_\mathcal{R}$) from the reference signatures (real and duplicated).

V. Results

The experiments are designed to solve the following issues. Firstly, we test our verifier with original and reconstructed signatures. Secondly, we generate duplicated signatures exploring how we can fine-tune the Sigma-Lognormal parameters related to the motor control. Then, the impact of adding these duplicated signatures in the reference set is analyzed by using the same verifier.

A. Performance evaluation replacing real signatures by their corresponding reconstructed.

The reconstructed signatures need to be evaluated to understand if they could be considered suitable enough in human movement analysis. We use the SNR (equation (4)) for this purpose. Previous studies [21] have shown that a SNR greater than 15 dB is sufficient for that purpose. In average, 96.77 % of all signatures in the Visual sub-corpus were reconstructed with a SNR above this threshold. Specifically, the average SNR is 19.87 ± 2.40 dB, which means that the reconstructed signatures from the Visual sub-corpus are adequate for our analyses.

The results are shown in Table I for both RI and SI tests. It can be seen that the reconstructed signatures offer a better performance than real ones when only the first signature is enrolled in our system. The benefits in both RI and SI tests are quite acceptable, especially for RI where the EER reduction is decreased by half.

Working with reconstructed signatures also leads to create a framework independent of the hardware: i.e. the preprocessing carried out to extract the parameters establish a specific constant sampling rate as well as a filter which eliminate the specific details of the particular hardware. Moreover, the reconstructed signatures appear also as an interesting opportunity to work with interoperability in handwriting signatures.

B. Performance evaluation adding duplicated signatures.

The second contribution of this work is to test the utility of using duplicated signatures when only one real signature is

TABLE I: EER for real and reconstructed signatures.

	Enroll Sign.	**RI - EER**	**SI - EER**
Real Signatures	1st sign.	14.43%	17.12%
Reconstructed Signatures	1st sign.	7.44%	15.53%

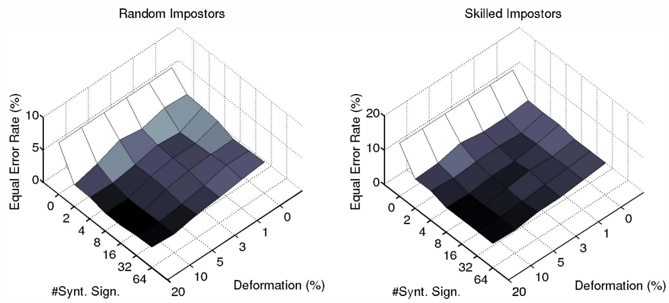

Fig. 1: EER results enrolling different duplicated signatures for $\lambda = 20\%$ and different deformations for κ.

TABLE II: EER results enrolling different duplicated signatures for $\kappa = 5\%$ and $\lambda = 20\%$.

Real	1	1	1	1	1	1	1
Dup.	0	2	4	8	16	32	64
RI	8.09 %	5.71 %	3.83 %	3.83 %	3.79 %	3.68 %	3.61 %
SI	15.54 %	10.54 %	8.62 %	7.97 %	7.97 %	7.90 %	7.87 %

available for verification purposes. Considering the improvement shown in Sect. V-A, all signatures used in this experiment were produced by the $\Sigma\Lambda$ model.

The duplicated signatures used in this experiment have been performed for uncoupled variations of $\hat{\mu}_i$ and $\hat{\sigma}_i$ parameters where $\lambda = 20\,\%$. Visually, we have analyzed that this value offers a good performance without damaging the appearance of the signature. The different values of the deformation for κ, which affect to $\hat{t}_0$ parameter, were also taken into account for the duplicated generation. In the experiments, the range of κ variation is: $(0, 1, 3, 5, 10, 20)\,\%$. We noted that larger values than $\lambda = 20\,\%$ still provides good trade-off between illegibility and performance if $\kappa = 0$. However, when such variations were combined with the t_0 variations, the legibility was rapidly lost without obtaining much more performance benefits.

In this framework, each reference signature provided by one signer could be duplicated as many times as we wish at no human cost. In our experiments, we have duplicated each signature 64 times. To analyze their effect, we have added the duplicated signatures to the reference set following an exponential notation with base 2: $\{2^n\}_{n=1}^{n=6}$. Additionally, the performance when the $\hat{t}_0$ parameter changes while λ was fixed to $20\,\%$ is also analyzed.

Fig. 1 shows the results obtained for both RI and SI. Three aspects should be highlighted. Firstly, a considerable drop in the EER is observed when only 2 duplicated signatures are added to the reference set. This impressive result is perceived both in RI and SI. Secondly, the performance achieved with the DTW system seems to saturate for 16, 32 and 64 signatures for the maximum deformation in κ. However, for medium values of κ, it is not clear which is the best number of duplicated signatures that we could introduce in our DTW system. Thirdly, it is clearly demonstrated in the surfaced graphics that the larger κ deforms, the better performance gets. Nevertheless, the best trade-off found in this work, between human-like duplicated trajectories and velocity, preserving the signature legibility and performance is for the value $\kappa = 5\,\%$ and $\lambda = 20\,\%$. Table II shows the performance vs the number of duplicated samples for the selected deformation values. This way, in terms of EER, the performance for RI and SI are 3.61 % and 7.87 % respectively.

TABLE III: EER for state-of-the-art ASV.

MCYT-330		
Contribution	RI - EER	SI - EER
Duplicated + HMM. [1]	6.60 %	15.60 %
SUSIG Visual Sub-corpus		
Contribution	RI - EER	SI - EER
Duplicated + DTW (this work)	**3.61 %**	**7.87 %**

To illustrate the challenge of designing single reference signature verification system, we compare in table III the results reported in [1] with our results. We aware such comparison is biased because both studies were not relying upon the same database. However, we illustrate the difference to locate our methodology in the current state-of-the-art.

In Fig. 2, the left hand column shows typical references signatures collected from the user who provides the top specimen, while the right hand column shows typical duplicated specimens generated in this study. As one can see, our methodology is able to generate duplicated samples that looks pretty similar to the original specimens.

VI. Conclusion

This paper has presented a novel solution to cope with automatic on-line signature verification when only one signature is available in the reference set. The duplicated on-line signatures are produced under a neuromuscular perspective based on the Kinematic Theory of rapid human movements and its Sigma-Lognormal model. These artificial duplicated signatures have demonstrated their utility in a reference set improving the performance of DTW. Note that, the duplicated signatures are generated using only one signature as seed.

DTW is one of the best state-of-the-art verifiers for on-line signatures. In this work, we have proved that our duplicated signatures are able to introduce new information in order to enhance the characterization of the signer and thus the performance of the verifier.

The promising results shown in this work encourage to follow researching in this model. In our future works, we will try to combine the present model with a cognitive inspired model [4] to perform new specimens in order to improve the current results. Moreover, we are aware that a further research is needed to analyze our model with other verification systems as well as other databases. Introducing physiological neuromuscular models to build duplicated specimens opens the possibility to mimic signatures acquired in more than one session as well as aging processes and behavioral disorders.

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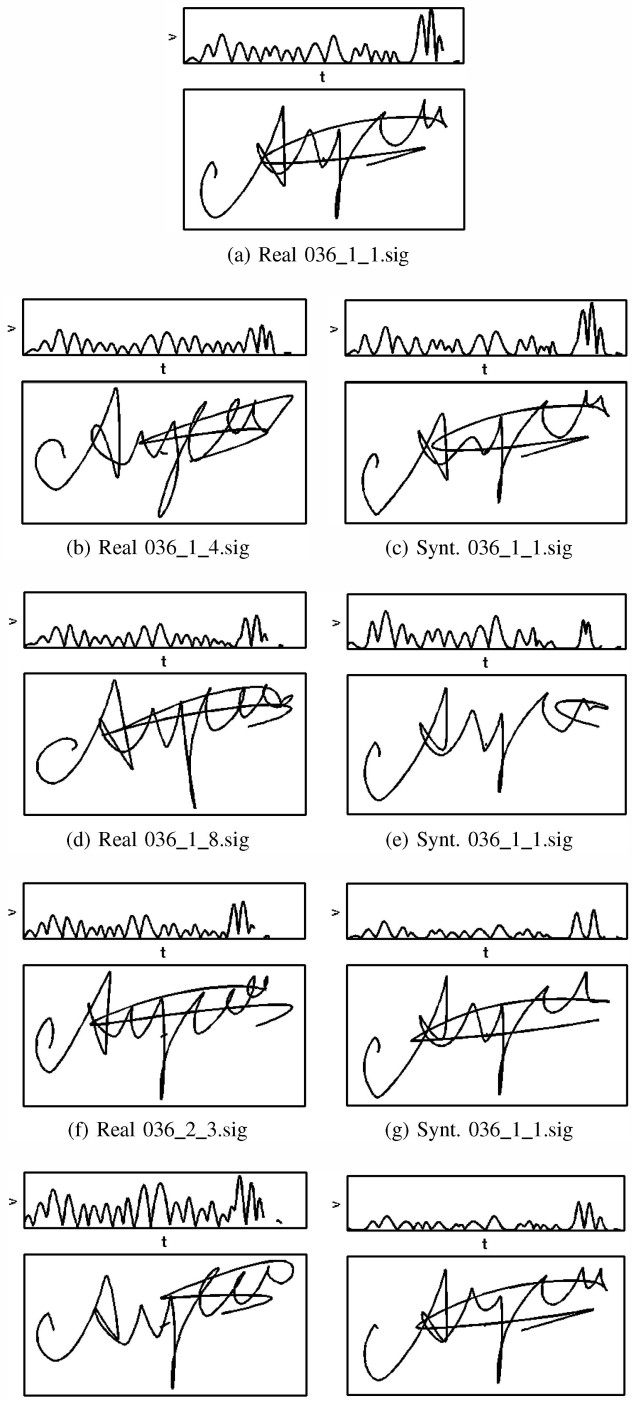

(a) Real 036_1_1.sig

(b) Real 036_1_4.sig

(c) Synt. 036_1_1.sig

(d) Real 036_1_8.sig

(e) Synt. 036_1_1.sig

(f) Real 036_2_3.sig

(g) Synt. 036_1_1.sig

(h) Real 036_2_5.sig

(i) Synt. 036_1_1.sig

Fig. 2: Real signatures (left column) and duplicates (right column) from user 036: velocities and trajectories.

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