Synthetic Off-line Signature Image Generation

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Abstract

This paper proposes a novel methodology to generate static/off-line signatures of new identities. The signature of the new synthetic identity is obtained particularizing the random variables of a statistical distribution of global signature properties. The results mimic real signature shapes and writing style properties, which are estimated from static signature databases. New instances, as well as forgeries, from the synthetic identities are obtained introducing a natural variability from the synthetic individual properties. As additional novelty, an ink deposition model based on a ballpoint is developed for realistic static signature image generation. The range of the static signature generator has been established matching the performance obtained with the synthetic databases and those obtained with two public databases.

1. Introduction

Personal identification using unique physiological and behavioral characteristics is now increasingly employed in a variety of commercial and forensic applications. A very challenging biometric trait is the handwritten signature due to its intra personal variability and the ease of forgery. Nowadays several research papers are still focused on improving recognition accuracy, although topics such as interoperability, template protection, scalability, performance evaluation, and biometric sample synthesis, are receiving much attention. Statistical reliable evaluation of the performance requires the availability of large databases and common benchmarks which is time costly and expensive. Additionally, legal issues regarding data protection hamper the sharing and distribution of biometric dates [1].

As an alternative for evaluation performance, biometric sample synthesis matter arising from the feasible way to easily provide large datasets for common evaluation frameworks. Training and testing algorithms avoiding legal considerations are really interesting for the scientific community. In fact, models and methods to generate biometric samples have been recently proposed for various traits, such as fingerprint [2], face [3], iris [4], speech [5], handwriting [6] and also signature [7, 8, 9, 10]. The advantage of synthetic samples is not only performance assessment. They could be used for increasing the training data, vulnerability assessment, etc.

The generation of synthetic handwritten signature has emerged as an attractive alternative for the researchers during the last years. The different proposal found in the literature can be presented as follows [9]:

Generation of duplicated samples [8, 11]. In this case the synthesis algorithm generates new samples of an existing user. Therefore, the number of samples per user is increased but no the number of users.

Generation of new synthetic identities [7, 9, 10]. In this case, global characteristic from a signature database are developed in order to generate samples of a new synthetic identity. Popel [7] describes this sort of approach for signature generation using a model based on visual characteristics extracted from the time domain. After a visual validation, no clear quantitative results are given. A novel methodology has been proposed by Galbally et al. [9, 10] for the generation of synthetic on-line handwritten signatures combining the advantages of both spectral analysis for signature generation and kinematic theory of human movements.

The above works propose methodologies to synthesize genuine signatures while forged synthetic signatures have usually left aside. Furthermore, the gray level distribution of the ink on the paper has not been considered. Taking into account the mentioned drawback, our proposal can be summarized as: i) A novel methodology to generate handmade realistic off-line signature of new identities. The proposal method allows the generation of both genuine and forged signatures. The method is able to simulate different forger skills, which is the False Acceptance Rates of signature recognition systems. ii) An ink deposition model to generate realistic signatures simulating the most common

handwriting tools. Modelling the deposition of ink has not been addressed in on-line signature generation but it is crucial to obtain realistic off-line signatures.

The individual constants related to the signature envelope, variability and writing style are set up comparing the performance results of the synthetic database with two free access real signatures databases as MCYT and GPDS960Gray. Furthermore, an example of database synthesized in this study is made publicly available for further research in forensics and civilian biometrics applications.

The outline of the paper is as follows: the section 2 describes the methodology for static signature generation in both cases: genuine and forged signatures. The ink deposition model is presented at the section 3. The section 4 is dedicated to experiments assessing the synthetic database and comparing with other real databases. The fifth section closes the paper with conclusions and future research lines.

2. Static Signature Generation

The proposed method models the signature as a distribution of parameters related to the signature envelope, inner variability and writing style. The forgeries of an individual can be provided generating signatures with higher inner variability than the genuine users and different writing style.

Concisely, the procedure to generate the signature model of a new individual is based on three basic assumptions: i) Generate a new signature envelope; ii) Locate randomly the signature reference stroke corners inside the envelope and; iii) Draw a ball around each reference stroke corner representing the stroke corner variability of the synthetic signer.

Then each instance is generated following these steps: iv) Randomly selecting a dot inside each corner ball and; v) Draw a line between the corners obtaining the signature path. A smooth filter based on polynomial cubic interpolation is applied to the signature path to obtain the signature stroke. The parameters of the polynomial smoothing will fix the synthetic individual style of writing. An example of the above procedure can be seen in Fig. 1 and Fig. 2. Finally, the ink deposition model is applied.

2.1. Synthetic signature envelopes

The signature envelope is modelled by means of Point Distribution Models (PDMs) or Active Shape Model (ASM). It consists of a mean signature envelope and a number of eigenvectors which describe the main modes of shape variation [12].

The PDM is built as follows: Let N training signatures, the first one of every MCYT off-line database signer, i.e. N=75. Each signature is converted to black and white by means of the Otsu's threshold and the salt and pepper noise is removed. Each signature is morphologically dilated 14 times by using a 3×3 mask. The signature envelope is the contour of the dilated image. All the envelopes are aligned

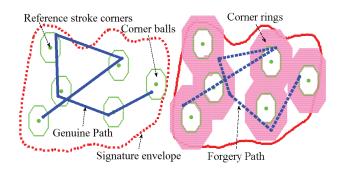


Figure 1. Example of signature path generation for genuine (on the left) and forged signature (on the right).

moving their geometrical center to the origin of a cartesian coordinate system.

From each envelope, 320 equidistant points called landmarks are selected in order to get the vector $\mathbf{x}^s = (x_1^s, x_2^s, \dots, x_n^s, y_1^s, y_2^s, \dots, y_n^s)$, being x_i^s, y_i^s the coordinates of the i^{th} landmark of the s^{th} envelope, n=320 and $s=1,\dots,N=75$. The first landmark x_1^s, y_1^s is the one that satisfies $y_1^s=0$ and $x_1^s>0$. The average signature envelope is calculated as: $\bar{\mathbf{x}}=\frac{1}{N}\sum_{i=1}^N \mathbf{x}^s$.

The PDM captures the statistics assuming that the point cloud \mathbf{x}^s is a 2N dimensional ellipsoid which is calculated by applying Principal Component Analysis (PCA). The $2n \times 2n$ covariance matrix is calculated as $\mathbf{S} = \sum_{S=1}^{N} (\mathbf{x}^s - \bar{\mathbf{x}}) \cdot (\mathbf{x}^s - \bar{\mathbf{x}})^T / N$. The principal axes of the ellipsoid are described by the eigenvectors $\{\mathbf{p}_k\}_{k=1}^{k=2n}$ of \mathbf{S} and the length of its axis is related with the eigenvalues $\lambda_k \geq \lambda_{k+1}, k=1,\ldots,2n$.

A signature envelope is synthesized using the mean envelope and a weighted sum of these deviations obtained from the first l modes as follows: $\mathbf{x}^f = \bar{\mathbf{x}} + \mathbf{P} \cdot \mathbf{b}$, being $\mathbf{P} = (\mathbf{p}_1, \dots, \mathbf{p}_l)$ where l is such that $\sum_{k=1}^l \lambda_k \approx 0.98 \sum_{k=1}^{2n} \lambda_k$, and $\mathbf{b} = \{b_1, \dots, b_l\}$ is a vector of weights which are obtained randomly with a uniform distribution of mean zero and deviation equal to $|b| < 4\sqrt{\lambda_k}$ for each vector component. Fig. 1 shows an example of synthesized envelope.

2.2. Signature path

In this paper, the proposed signature stroke generation, as in the case of [9, 10], is not aimed to produce signatures with handwriting or real name. This experiment is addressed to generate signatures consisting in a sort of complex flourish but sometimes isolated characters have been produced.

To generate the signer signature model, first the number and position of the reference stroke corners is defined. The number of reference corners, which define the complexity of the signature, is randomly selected in the range $N_c = [8, \ldots, 20]$. The location of the reference corners

 $\{xrc_i, yrc_i\}_{i=1}^{Nc}$ is worked out randomly inside the signature envelope. They are ordered randomly as a sequence. The within signer variability is introduced defining a ball around each reference stroke corner, which are called ball corners and shown at Fig. 1. As each writer has its own variability, the corner balls radius are modelled as a random variable. Our signature path is obtained locating a stroke corner randomly inside each ball corner and connecting them by straight lines, it is shown in Fig. 1.

2.3. Signature trajectory

Obviously, the sequence of straight lines with corners is not a signature flourish produced by a writer with a natural way of writing. Two approaches to produce handwriting trajectories have been found [13]: a) The space oriented models which approaches the trajectory formation mechanism based on the capability of expressing and controlling the trajectory of the hand in space and; b) The muscle oriented models which try to relate the trajectory formation with the muscle geometry and properties.

To produce a static signature trajectory, we rely on space oriented model. This agrees with the hypothesis that handwritten trajectories are divided into simple strokes which can be composite chaining them and smoothing the transition between consecutive segments. To preserve the manmade features of the composite planar trajectory, the expressed strokes in parametric terms must be represented by at least a cubic polynomial function of time. Also the jointing technique between strokes must guarantee continuity up to the second time derivative [13].

As the spline satisfies the above conditions, the signature trajectory was approximated using them, being the signature corners the spline control points. However, it was found that unnatural eccentric trajectories appear occasionally. As alternative to avoid such drawback, in this paper we propose forming the signature trajectory by a polynomial smoothing of the signature path. It is specifically done by using a Savitsky-Golay [14] filter which has been also used for bioengineering smoothing problems such as ECG.

The Savitzky-Golay smoothing filter essentially performs a local polynomial regression of degree k on a series of values of at least k=2f+2 points to determine the smoothed value for each point. This approach tends to preserve features of the distribution such as relative maxima, minima and width. So, it is able to smooth the signature trajectory keeping some natural writing features as very close loops.

Concisely, consider a group of 2f+1 samples of the signal x[n] to be smoothed centred at n=0, we obtain the coefficients of a polynomial $p(n)=\sum_{l=1}^k a_l n^l$, that minimize the mean squared approximation error $\varepsilon_f=\sum_{n=-f}^f \left(p(n)-x(n)\right)^2$ for the group of input samples centred at n=0. The smoothed output value is obtained by

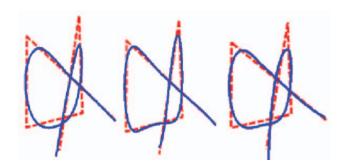


Figure 2. Signature path example (dashed line) and handwriting styles (solid line) according to different k and f values. From left to right k = 3, 4, 4 and f = 70, 70, 90 respectively

evaluating p(n) at the central point n=0. The output of the next sample is obtained by shifting the analysis interval to the right by one sample and redefining the origin to the position of the next sample.

Experimentally, a natural writing is approximately obtained filtering the signature path with polynomial degrees between $3 \le k \le 5$ and $60 \le f \le 60 + 10k$. To generate different instance of the same writer k should be kept the same and f could be slightly changed, for instant as $f(1+q\cdot 0.05)$ being q a normal random variable with mean and variance equal to 0 and 1 respectively. Fig. 2 shows an example of how the writing style is modified by k and f parameters.

2.4. Generation of forgeries

A forgery is a signature from an impostor writer, different than the original owner, trying to imitate supplanted identity. The forger skills and his knowledges about the genuine samples define the similarity between the original and forged signatures. The above procedure to synthesize signatures allows the generation of forgeries based on the next principles: i) As forgers imitate the genuine signatures, they follow the same signature path that the genuine one. So the forgeries will be generated using the genuine signature path; ii) Obviously, keeping the inner signer variability is difficult for forgers. Therefore, we increase the corner ball converting it into a corner ring (see Fig. 1). iii) As forgers have a different neuromuscular system, the k and f variables will be randomly selected again for every forgery.

3. Ink characterization

The off-line signatures are usually written by using a ballpoint. Therefore, a ballpoint model has been design in order to produce a realistic image. Previous works about ink deposition model have been found in the literature. For instance, in [15] a virtual brush steam was performed to write synthetic Chinese letters.

The proposed ballpoint model in this paper is based on

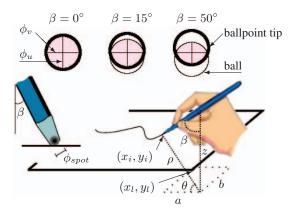


Figure 3. Ballpoint spot modelling depending on the position

a ball inside a circle space (ballpoint tip). When the internal ball rolls along the stroke, the ink is deposited on the paper. In this case the spot of the ballpoint on the paper is expected to be a circle. However, since the ballpoint does not used to be perpendicular to the paper, the contact surface is approximate as an ellipse. This approaches the intersection between the spherical ball and the ballpoint tip circle when the ballpoint is tilted to write as can be seen in Fig. 3. The minor and major axes of the ellipse (ϕ_u, ϕ_v) are directly proportional to the ballpoint tip circle diameter ϕ_{spot} and the ballpoint tilt. Concisely, the minor axe of the ellipse could be approximated by $\phi_u = \phi_{spot} \cdot \cos \beta$, being β the ballpoint tilt, and the major axe ϕ_v is, given ϕ_u , the distance between the intersection of both circles.

Angle β is calculated for a right handed writer as $\tan^{-1}(\rho/z)$, being z the height of the ballpoint tilt axis and ρ the Euclidean distance between the writing hand toehold approximate by (x_l-a,y_l-b) and the written dot (x_i,y_i) . It supposes that (x_l,y_l) is the lower right corner of the signature and the distance between the hand toehold and the lower right signature corner offset estimated as $a=b=2.5\,cm$. Once worked out β , we are able to obtain the elliptical spot corresponding to the signature dot (x_i,y_i) . As the ellipse is perpendicular to the ballpoint, it is rotate $\frac{\pi}{2}-\theta$, being $\theta=\tan^{-1}\left(x_i/y_i\right)$.

In our signature image generation with the ink model, each pixel of the signature trajectory is replaced by its elliptical spot adding the overlap of consecutive spots. Pixel within a single spot correspond to a two-dimensional Gaussian function, whose standard deviations (σ_x, σ_y) are directly proportional to ellipse axis. As the resultant image presents no realistic dark areas where the stroke crosses due to the overlap add algorithm, the maximum dark value is set to $2\phi_v$, which saturates the ink level. Additionally, the obtained image is too regular, however the deposited ink by the ballpoint is usually noisy due to the irregularities of the ballpoint, as in real cases. Hence, the spot is multiplied by a random spot to simulate the noise.

The last step to improve the realism of the off-line signature image consist of approximate the stroke gray level histogram to a real ink histogram distribution. Franke $\it et al.$ [16] studied the ink histograms of the three more usual inks: fluid, viscose and solid. So, three curves approximating the histograms given by Franke $\it et al.$ are generated. The histogram of the generated signature is modified to fix the randomly selected ink. A final smooth stage is carried out by using a $\it 3 \times \it 3$ Gaussian mask.

Therefore, as 3 different ink types are simulated and the usual commercial diameters are $\phi_{spot} = 0.20, 0.25, 0.30, 0.35, 0.40, 0.45 \,mm$, the available number of ballpoints to generate an off-line signature is 18. An example of the result can be seen in Fig. 4.

Additionally, a simple paper model was developed to improve the realism of the images. The granularity paper was modelled processing morphologically a random image with a disk structuring element of radius 10.

4. Experiments

The experiments are aimed to obtain: i) The relation among the parameter values of the synthesizer and the performance of the verifier and; ii) The variability values which give a similar performance than real data as GPDS960Gray and MCYT database.

The following parameters control the synthesis procedure: a) The number of signature corners; b) The corner ball radius; c) The corner ring radiuses and; d) The polynomial and smoothing degree (k and f values) of the Savitzky-Golay filter. As this paper just generate signature like a complex flourish, the number of signature corners is selected randomly in the range [8, 20], being 8 the simplest signatures and 20 the most complex ones. As the signature is as least a cubic line, the value of k is randomly selected between 3 and 5. The value of the smoothing parameter f is taken as a random value $60 \le f \le 60 + 10k$. This value increase with k in order to attenuate the inflection points of the greater polynomial degree and keep natural trajectories. Therefore, only remains to calculate the radius of the corner balls and rings and their relation with real databases.

4.1. Verifier

Synthetic signatures have been evaluated in terms of performance, trying to emulate the real database results. This has been done in order to adjust the intra and among class variability constants from the databases. The performance evaluation is computed by using the verifier proposed in [17]. It is based on texture feature such as local binary pattern (LBP) and local derivative pattern (LDP). The signature is transformed to the LBP and LDP image which is divided in 12 sectors. The histogram of each sector is worked out, concatenated and its dimension reduced with a Discrete Cosine Transform (DCT) obtaining, in this way, the feature



Figure 4. Real signature stroke detail from GPDS960Gray signature database (on the left), detail of synthetic stroke (on the right)

vector. The combination at score level is evaluated through a weighted sum as in [17]. The classifier is based on a least square support vector machine (LSSVM) [18].

4.2. Databases

Two publicly available off-line signature corpuses were used. The first database is the off-line sub corpus of the MCYT database [19]. It includes 75 signers, 15 genuine signatures and 15 simulated forgeries for each signer. All signature data were acquired with the same ink type and the same paper templates. The second database is the GPDS960Gray corpus [17]. This corpus contains 24 genuine signatures and 30 simulated forgeries from 881 individuals. The repetitions of each genuine signature and forged specimen were collected allowing each user his or her own ballpoint on sheets of white A4 paper. Each sheet provided two different box sizes for the signature. Both databases were scanned at 600 dpi with 256 gray levels.

4.3. Experimental methodology

Following a well-known experimental protocol for the MCYT database, the training set consists 10 randomly selected genuine signatures. The remaining genuine signatures are used as testing samples. When testing a specific signer, other genuine test scores are computed by using the test genuine samples from all the remaining users. Real impostor test scores are computed considering the simulated forgeries of each signer. All the experiments are repeated 10 times and the averaged results in term of Equal Error Rate (EER) are provided. Both GPDS960Gray and MCYT database were experimentally exploited by using the same methodology.

4.4. Baseline: results with MCYT and GPDS960Gray corpuses

Table 1 and 2 show the results obtained with MCYT and GPDS960Gray databases and the proposed verifier in terms of Equal Error Rate (EER). As can be seen in the case of 75 users, the performance with the genuine in both databases is similar but the forgeries of the MCYT database look easier to detect than in the case of the GPDS960Gray database. These values are the baseline to adjust the parameters of the signature synthesizer.

Table 1. EER (%) with LS-SVM Classifier for MCYT database

Gen	uine Si	gners	For	Forged Signers				
LDP	LBP	Score fusion	LDP	LBP	Score fusion			
0.62	0.41	0.35	11.90	11.48	11.54			

Table 2. EER (%) with LS-SVM Classifier for GPDS960Gray database

	Genuine Signers			Forged Signers			
Users	LDP	LBP	Score fusion	LDP	LBP	Score fusion	
75	0.53	0.54	0.37	14.11	14.44	13.78	
150	0.63	0.66	0.44	16.78	16.04	15.90	
881	1.18	1.13	0.88	24.67	23.49	23.42	

Table 3. EER (%) for synthetic database of 150 users training 10 and different variability

Ball radius (mm)	LDP	LBP	Score fusion
2.6	0.16	0.33	0.09
2.8	0.30	0.60	0.19
3.0	0.54	0.77	0.34
3.2	0.66	0.98	0.44
3.4	1.36	1.90	0.94
3.6	1.86	2.21	1.27

4.5. Adjusting Corner ball and Ring radius

The corner ball and ring are adjusted in two steps: Firstly, the ball radius for genuine test is adjusted; Secondly the corner ring radius for generating forgeries is adjusted as well. Table 3 shows the results of generating 24 genuine signatures of 150 signers with different corner ball radius. As can be seen, the most similar results to GPDS960Gray database are obtained with corner ball radius around 3.2 mm. The results suggest that the LBP operator is more sensitive to ink variability than the LDP. This fact is greater in the synthetic database than in the real databases because of the diverse modelled inks. The scarce ink variability of the real databases is considered a limitation because it does not usually ask to sign with different ballpoints. The dependence of the verifiers with the ink can be carried out with the synthetic database as can be seen in Fig. 6.

Table 4. EER (%) for synthetic forgeries of 150 users and different forged signers variability

Ring radius (mm)	LDP	LBP	Score fusion
6.4	18.23	22.83	18.60
6.6	17.84	22.85	18.33
6.8	15.98	20.19	16.28
7.0	15.57	19.70	15.81
7.2	14.58	19.15	14.90
7.4	14.77	18.63	14.74

Regarding to the generation of forgeries, varying the corner ring radius generates 30 forgeries per user, see Table 4. The inner ring radius is equal to 0.9 multiplied by the user corner ball radius. Those results define the detection error for forgers with different skills. As can be seen, for the first 150 users of the GPDS960Gray database, the forgers' skills could be considered equivalent to generate forgeries with a ring radius around 7 mm.

4.6. Synthetic Database

According to the variability experimentally obtained, a synthetic database has been generated as follows: corner ball radius range is calculated for each signer randomly between 2.8 to 3.2 mm. The inner ring radius equal to 0.9 multiplied by the corner ball radius and the ring radius is equal to 7.0 mm. 24 genuine and 30 forged signatures were generated for each synthetic identity. An example of different instances of several synthetic users with forgeries can be seen in Fig. 5. The database can be free download from our web page www.gpds.ulpgc.es/download.

Table 5 analyses the results obtained with such database for different number of users. As can be seen the results are very similar to those obtained with the real databases except in the case of the forgeries. In the case of the synthetic database, the results are mainly dependent of the external corner ring radius which define the forgers' skill and do not depend on the number of users. Instead, in the case of the GPDS960Gray database the result with forgeries depends on the number of users. It could be related to the different steps this database was built. Fig. 6 shows obtained DET curves from GPDS960Gray and two synthetic databases: a) Synthetic signatures generated simulating the same ballpoint for each identity (Synthetic DB); b) Synthetic signatures generated simulating different ballpoints per identity (Synthetic DB Variable Ink). The experiments show the strong influence of the ink deposition model. Hence, the obtained results with the synthetic database (Synthetic DB Variable Ink) suggest that the LDP operator outperform the LBP technique in terms of robustness in presence of ink variations.

Table 5. EER (%) with LS-SVM Classifier for Synthetic database

		Genuine Signers			For	ged Sig	gners
Users	Train	LDP	LBP	Score fusion	LDP	LBP	Score fusion
75	10	0.47	0.80	0.29	13.80	17.44	14.04
150	10	0.53	0.81	0.34	14.68	17.22	15.33
881	10	1.03	1.53	0.73	15.08	17.42	14.20

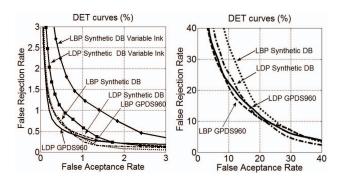


Figure 6. DET Curves for genuine (left) and forged (right) signers using 150 users and training with 10 genuine signatures.

5. Conclusion

This paper has proposed a novel method for the generation of synthetic off-line handwritten signatures. The proposed method generates entirely new signers. As additional novelty, it is included the generation of forged signatures and the modelling of deposited ink on the paper with realistic appearance. The validation protocol is based on performance by using a state of the art signature verifier. The obtained results with the synthetic signatures have shown a very high degree of similarity with the obtained results using other two public databases.

The novel synthetic generation algorithm presents a great potential for many applications. For instance performance estimation, security evaluation, scalability studies, ink effect studies, etc. Research on the inclusion of synthetic readable names is being done with encouraging results.

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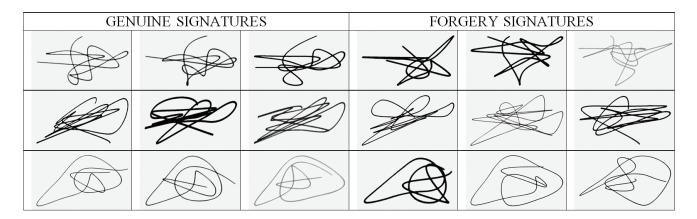


Figure 5. Three synthetic signatures with different number of corners and ballpoints, showing 3 genuine and 3 forged signatures.

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