QUESTION

Over all real numbers, find the minimum value of a positive real number, y such that

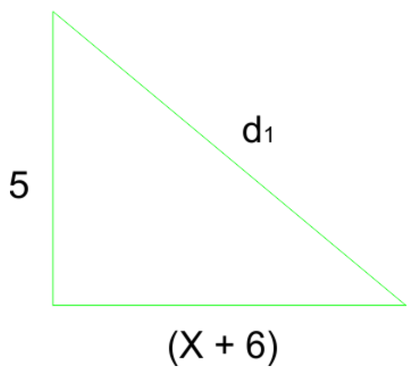
y = $\sqrt{(x+6)2+25}$ + $\sqrt{(x-6)2+121}$

ANSWER

Let $\sqrt{(x+6)2+25}$ represent one side of a right-angle triangle of length d_1 whose other sides are:

X+6 and 5 respectively, such that

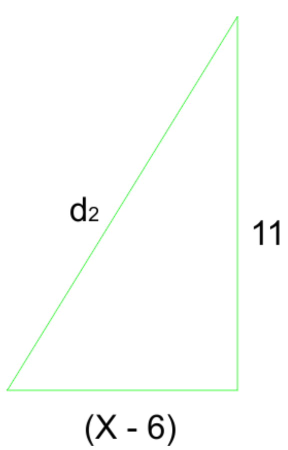
$d_1^2 = (x + 6)^2 + 5^2$



Let $\sqrt{(x-6)2+121}$ represent one side of another right-angle triangle of length d_2 whose other sides are:

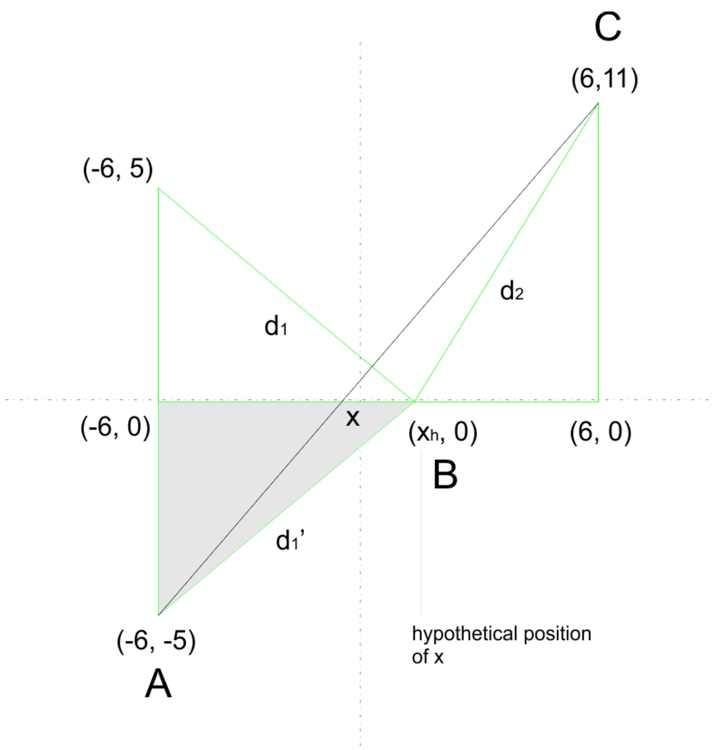
X+6 and 11 respectively, such that

$d_2^2 = (x - 6)^2 + 11^2$



Hence, $y = d_1 + d_2$

Laying the triangles on a Cartesian plane, we have:



Let there be a triangle bound by coordinates (-6, -5), (x_h, 0) and (-6, 0) that is similar to the triangle bound by (-6, 5), (x_h, 0), and (-6, 0), such that:

Distance between (-6, -5) and ($x_{h,}$ 0) = $d_1' = d_1$

$y = d_1' + d_2$

Let points:

(-6, 5) = A

($x_{h,}$ 0) = B

(6, 11) = C

According to triangular inequalities law,

AB + BC $\geq$ AC

The smallest value for y is the shortest distance between A and C, which is a straight line, whose equation is $4x - 3y + 9 = 0$

WORKINGS

Gradient $m = y_2 - y_1 / X_2 - X_1$

$m = \frac{16}{12} = \frac{4}{3}$

y = $\frac{4}{3}x + c$ if y = 11 and x = 6, c will be equal to 3

3y = 4x + 9

4x – 3y + 9 = 0

The x-intercept (which is the position of x for the smallest value of y) is equal to:

4x = 3y – 9

x = $\frac{3}{4}y - \frac{9}{4}$

x intercept = - $\frac{9}{4}$

Plugging back into the equation,

y = $\sqrt{(x+6)2+25}$ + $\sqrt{(x-6)2+121}$

y = $\sqrt{\left(\frac{-9}{4}+6\right)^2+25}$ + $\sqrt{\left(\frac{-9}{4}-6\right)^2+121}$

y = $\frac{25}{4} + \frac{55}{4}$

y = 20, this is the minimum value of y.