A Real Implementation for Constructive Negation

Susana Muñoz Juan José Moreno

Facultad de Informática

Universidad Politécnica de Madrid

28660 Madrid, Spain

susana@fi.upm.es

jjmoreno@fi.upm.es

Motivation

- Negation role at Logic Programming
- Problems of the proposals:
  - Complexity
  - Expressiveness
  - Semantics
- Limited implementations:
  - Negation as failure
  - Delay technique

Negation as failure

- SLDNF resolution $\begin{cases} naf(G) :- & call(G), !, \\ & fail. \\ naf(G). \end{cases}$

- Execution:

```
?- naf(even(s(s(0)))).
no

?- naf(even(s(0))).
yes
```

```
?- naf(even(X)).
no

?- naf(even(s(Y))).
no
```

- Problem: *naf* is not sound and complete.

Interpretation of Quantifications

$$naf\,(p(\overline{X})) \;\equiv\; \neg\exists\overline{X}.\; p(\overline{X})$$

- $naf\,(p(\overline{X})$ checks if $p(\overline{X})$ is “true” or “false” $\Rightarrow$ **No variable instantiation**

$$cneg(p(\overline{X})) \;\equiv\; \exists\overline{X}.\; \neg p(\overline{X})$$

- $cneg(p(\overline{X})$ provides the values of $\overline{X}$ that make false $p(\overline{X}) \Rightarrow$ **Constructive answer**

Constructive Answers

```
?- cneg(even(X)).
X = s(0) ?;
X = s(s(s(0))) ?;
...
```

```
?- cneg(even(X)).
X=/=0, X=/=s(s(fA(Y))) ?;
X=s(s(Y)),
Y=/=0, Y=/=s(s(fA(Z))) ?;
...
```

```
?- cneg(null(X)).
X = s(0) ?;
X = s(s(0)) ?;
X = s(s(s(0))) ?;
...
```

```
?- cneg(null(X)).
X =/= 0 ?;
no
```

Constructive Negation

- Papers about Semantical aspects
- Practical Chan's proposal (coroutining)
- Implementation problems (Eclipse)
- We provide:
  - A complete theoretical algorithm (refining and extending to the constructive negation method)
  - A discussion about implementation issues
  - A preliminary implementation

Negation of the Frontier

```
even(0).
even(s(s(X))):- even(X).
```

```
?- cneg(even(Y)).
```

Frontier$(even(Y)) = C_1 \vee C_2 =$

$$(Y = 0) \vee (\exists X \;\; Y = s(s(X)) \wedge even(X))$$

$\neg$ ***Frontier***$(even(Y)) = \neg\, C_1 \wedge \neg\, C_2 = \;\; [Y \neq 0] \wedge$

$$[(\forall X1.\, Y \neq \;\; s(s(X1))) \vee ((\exists X2.\, Y = s(s(X2)) \wedge \neg\, even(X2))]$$

Implementation Issues

- Disequality constraints (Attributed variables)
  Constraints Normal Form

$$\underbrace{\bigwedge_i (X_i = t_i)}_{\text{positive information}} \quad \wedge$$

$$\underbrace{(\bigwedge_j \forall\, \overline{Z}_j^1\ (Y_j^1 \neq s_j^1) \vee \ldots \vee \bigwedge_l \forall\, \overline{Z}_l^n\ (Y_l^n \neq s_l^n))}_{\text{negative information}}$$

Examples

```
boole(0).
boole(1).
```

```
?- cneg(boole(X)).
X=/=1, X=/=0 ? ;
no
```

```
positive(0).
positive(s(X)):-
      positive(X).
```

```
?- cneg(positive(X)).
X=/=s(fA(_A)), X=/=0 ?;
X = s(_A),
_A=/=s(fA(_B)), _A=/=0 ?;
X = s(s(_A)),
_A=/=s(fA(_B)), _A=/=0 ?
yes
```

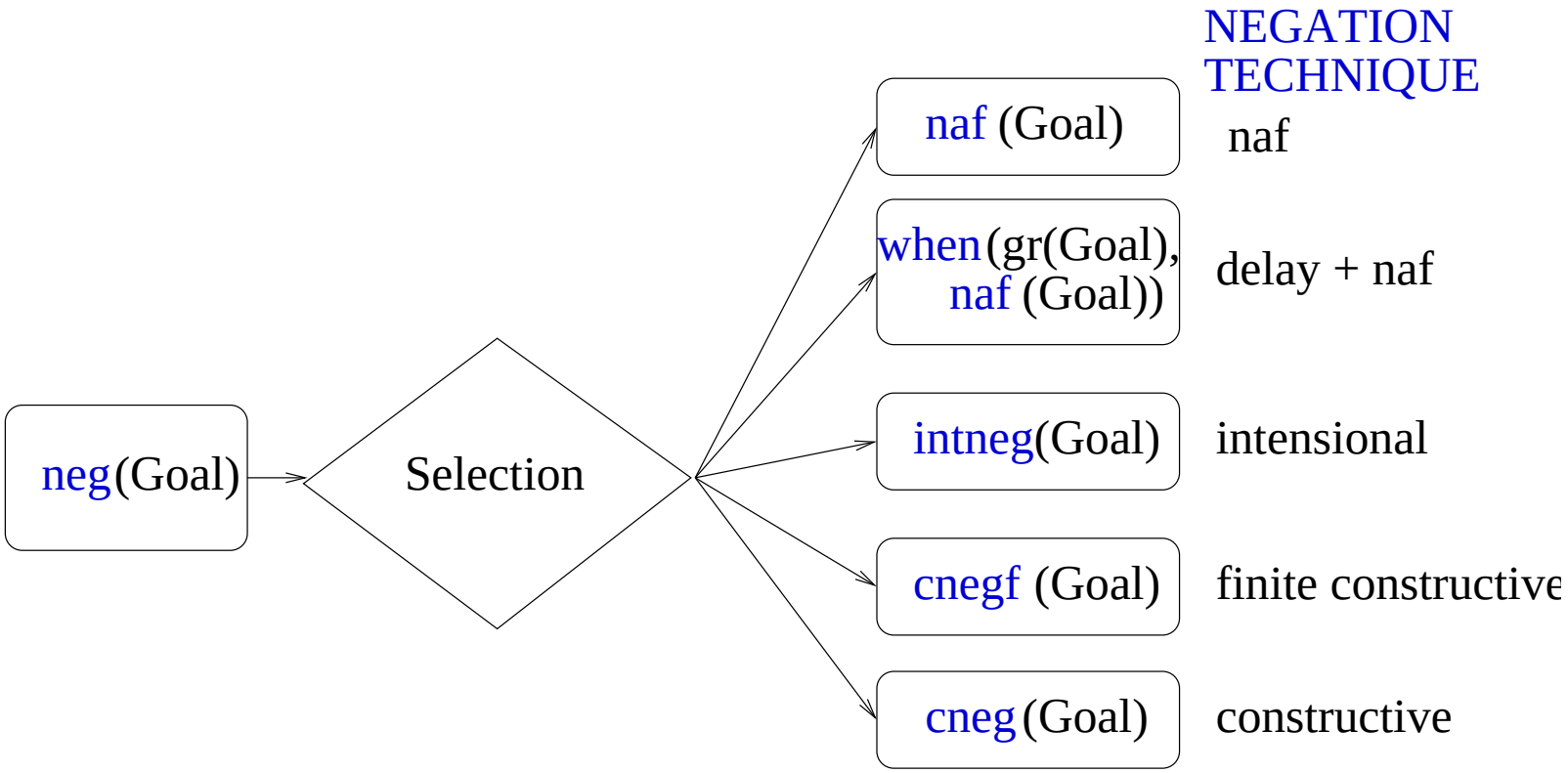
Experimental results

goals	Goal	naf(Goal)	cneg(Goal)	ratio
boole(1)	2049	2099	2069	0.98
positive(s(s(s(s(s(s(0))))))	2079	1600	2159	1.3
greater(s(s(s(0))),s(0))	2110	2099	2100	1.00
average				1.06
positive($s^{500000}(0)$)	2930	2949	41929	14.21
positive($s^{1000000}(0)$)	3820	3689	81840	22.18
greater($s^{500000}(0)$,$s^{500000}(0)$)	3200	3339	22370	7.70
average				14.69
positive(X)	2020	-	7189	
greater(s(s(s(0))),X)	2099	-	6990	
queens(s(s(0)),Qs)	6939	-	9119	

Conclusion and Future Work

- Detailed description of the modified algorithm (w.r.t. Chan's proposal)
- Complete and consistent implementation
- Optimizations
  - Compact information
  - Pruning subgoals
  - Constraint simplifi cation
  - Finite constructive negation (cnegf)
- Efficiency problem
  - WAM level (future work)
  - Negation System for Prolog

Negation System for Prolog



- Static phase + Dynamic phase

A Real Implementation for Constructive Negation

Susana Muñoz Juan José Moreno

Facultad de Informática

Universidad Politécnica de Madrid

28660 Madrid, Spain

susana@fi.upm.es

jjmoreno@fi.upm.es