# A Real Implementation for Constructive Negation

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# Motivation

- Negation role at Logic Programming
- Problems of the proposals:
  - Complexity
  - Expressiveness
  - Semantics
- Limited implementations:
  - Negation as failure
  - Delay technique

# Constructive Negation

- Papers about Semantical aspects
- Practical Chan's proposal (coroutining)
- Implementation problems (Eclipse)
- We provide:
  - A complete theoretical algorithm (refining and extending to the constructive negation method)
  - A discussion about implementation issues
  - A preliminary implementation

# Frontier

```
odd(s(0)).
odd(s(s(X))):- odd(X).
```

```
?- cneg(odd(Y)).
```

$$Frontier(odd(Y)) = C_1 \vee C_2 =$$

$$(Y = s(0)) \vee (\exists X \;\; Y = s(s(X)) \wedge odd(X))$$

$$\neg\, Frontier(odd(Y)) = \neg\, C_1 \wedge \neg\, C_2 =$$

$$(Y \neq s(0)) \wedge ((\forall X1 \;\; Y \neq \; s(s(X1)))\vee$$

$$(\exists X2 \;\; Y = s(s(X2)) \wedge \neg\, odd(X2)))$$

# Preparation

- Simplification of the conjunction
- Organization of the conjunction

$$C_i \equiv \overline{I} \wedge \overline{D} \wedge \overline{R}$$

- Normalization of the conjunction (GoalVars - RelVars - ImpVars)
    - Elimination of redundant variables and equalities
    - Elimination of irrelevant disequalities

# Negation of subformulas(I)

- Negation of $\overline{I}$

$$\overline{I} \equiv I_1 \wedge \ldots \wedge I_{NI} \equiv$$

$$\underbrace{\exists\, \overline{Z}_1\; X_1 = t_1}_{I_1} \wedge \ldots \wedge \underbrace{\exists\, \overline{Z}_{NI}\; X_{NI} = t_{NI}}_{I_{NI}}$$

$$\neg\, C_i \equiv \neg\, \overline{I} \equiv \bigvee_{i=1}^{NI} \forall\, \overline{Z}_i X_i \neq t_i \equiv$$

$$\underbrace{\forall\, \overline{Z}_1\; X_1 \neq t_1}_{\neg\, I_1} \vee \ldots \vee \underbrace{\forall\, \overline{Z}_{NI}\; X_{NI} \neq t_{NI}}_{\neg\, I_{NI}}$$

# Negation of subformulas(II)

- Negation of $\overline{D}_{imp}$

$$\overline{D}_{imp} \equiv D_1 \wedge \ldots \wedge D_{N_{D_{imp}}}$$

$$D_i \equiv \forall\, \overline{W}_i\ \exists\, \overline{Z}_i\ Y_i \neq s_i$$

$$\neg\, D_i \equiv \exists\, \overline{W}_i\ Y_i = s_i$$

$$\begin{aligned} \neg\, C_i \equiv\ & \overline{I} \wedge \neg\, D_1 \vee \\ & \overline{I} \wedge D_1 \wedge \neg\, D_2 \vee \\ & \ldots \\ & \overline{I} \wedge D_1 \wedge \ldots \wedge D_{N_{D_{imp}}-1} \wedge \neg\, D_{N_{D_{imp}}} \end{aligned}$$

# Negation of subformulas(III)

- Negation of $\overline{R}_{imp}$

$$\overline{R}_{imp} \equiv R_1 \land \ldots \land R_{N_{R_{imp}}}$$

$$\begin{aligned} \neg\, C_i \equiv\ & \overline{I} \land \overline{D}_{imp} \land \neg\, R_1 \lor \\ & \overline{I} \land \overline{D}_{imp} \land R_1 \land \neg\, R_2 \lor \\ & \ldots \\ & \overline{I} \land \overline{D}_{imp} \land R_1 \land \ldots \land R_{N_{R_{imp}}-1} \land \neg\, R_{N_{R_{imp}}} \end{aligned}$$

# Negation of subformulas(IV)

- Negation of $\overline{D}_{exp} \wedge \overline{R}_{exp}$

$$\neg\, (\exists\, \overline{V}_{exp}\; \overline{D}_{exp} \wedge \overline{R}_{exp}) \equiv \forall\, \overline{V}_{exp}\; \neg\, (\overline{D}_{exp} \wedge \overline{R}_{exp})$$

$$\neg\, C_i \equiv \overline{I} \wedge \overline{D}_{imp} \wedge \overline{R}_{imp} \wedge \forall\, \overline{V}_{exp}\; \neg\, (\overline{D}_{exp} \wedge \overline{R}_{exp})$$

# Implementation Issues (I)

- Code expansion

```
:- module(mod1,[odd/1,not_odd/1],[cneg]).

odd(s(0)).
odd(s(s(X))) :- odd(X).

not_odd(X) :- cneg(odd(X)).

stored_clause(odd(s(0)),[]).
stored_clause(odd(s(s(X))),[odd(X)]).
```

# Implementation Issues (II)

- Disequality constraints (Attributed variables)

$$\underbrace{\bigwedge_i (X_i = t_i)}_{\text{positive information}} \qquad \wedge$$

$$\underbrace{(\bigwedge_j \forall\, \overline{Z}_j^1\, (Y_j^1 \neq s_j^1) \vee \ldots \vee \bigwedge_l \forall\, \overline{Z}_l^n\, (Y_l^n \neq s_l^n))}_{\text{negative information}}$$

# Optimizations

- Compact information (disjunction of conjunction of disequalities)

$$(X \neq 0 \ \vee \ \exists Y \ \ X \neq Y) \ \wedge \ (\forall Z \ \ X \neq s(Z)) \Rightarrow$$

$$(X = / = 0, X = / = Y); (X = / = s(fA(Z)))$$

- Pruning subgoals (equivalent to true / fail)
- Constraint simplification

$$F \equiv \bigvee_i \bigwedge_j \forall \overline{Z}_j^i \ (Y_j^i \neq s_j^i)$$

# Examples (I)

```
boole(0).
boole(1).
```

```
?- cneg(boole(X)).
X=/=1,X=/=0 ? ;
no
```

```
positive(0).
positive(s(X)):-
        positive(X).
```

```
?- cneg(positive(X)).
X=/=s(fA(_A)),X=/=0 ? ;
X = s(_A),
_A=/=s(fA(_B)),_A=/=0 ? ;
X = s(s(_A)),
_A=/=s(fA(_B)),_A=/=0 ?
yes
```

# Examples (II)

```
number(0).
number(s(X)):-
        number(X).

greater(s(X),0):-
        number(X).
greater(s(X),s(Y)):-
        greater(X,Y).
```

```
?- cneg(greater(X,Y)).
Y=/=0,Y=/=s(fA(_A)) ?;

Y=/=s(fA(_A)),
X=/=s(fA(_B)) ?;

X = s(_A),Y = 0,
_A=/=s(fA(_B)),_A=/=0 ?;

X = s(s(_A)),Y = 0,
_A=/=s(fA(_B)),_A=/=0 ?
yes
```

# Experimental results

| goals | Goal | naf(Goal) | cneg(Goal) | ratio |
|---|---|---|---|---|
| boole(1) | 2049 | 2099 | 2069 | 0.98 |
| positive(s(s(s(s(s(s(0))))))) | 2079 | 1600 | 2159 | 1.3 |
| greater(s(s(s(0))),s(0)) | 2110 | 2099 | 2100 | 1.00 |
| **average** | | | | 1.06 |
| positive($s^{500000}$(0)) | 2930 | 2949 | 41929 | 14.21 |
| positive($s^{1000000}$(0)) | 3820 | 3689 | 81840 | 22.18 |
| greater($s^{500000}$(0),$s^{500000}$(0)) | 3200 | 3339 | 22370 | 7.70 |
| **average** | | | | 14.69 |
| positive(X) | 2020 | - | 7189 | |
| greater(s(s(s(0))),X) | 2099 | - | 6990 | |
| queens(s(s(0)),Qs) | 6939 | - | 9119 | |

# Conclusion and Future Work

- Detailed description of the modified algorithm
- Complete and consistent implementation
- Efficiency problem
  - WAM level (future work)
  - Negation subsystem for Prolog:
    Static + Dynamic analyses to choose
    among the different negation techniques