# Implementation Results in
# Classical Constructive Negation

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**Abstract.** Logic Programming has been advocated as a language for system specification, especially for those involving logical behaviours, rules and knowledge. However, modeling problems involving negation, which is quite natural in many cases, is somewhat limited if Prolog is used as the specification / implementation language. These restrictions are not related to theory viewpoint, where users can find many different models with their respective semantics; they concern practical implementation issues. The negation capabilities supported by current Prolog systems are rather constrained, and there is no a correct and complete implementation. In this paper, we refine and propose some extensions to the classical method of constructive negation, providing the complete theoretical algorithm. Furthermore, we also discuss implementation issues providing a preliminary implementation and an optimized implementation to negate predicates with a finite number of solutions.

**Keywords**  Constructive Negation, Negation in Logic Programming, Constraint Logic Programming, Implementations of Logic Programming, Optimization.

## 1   Introduction

From its very beginning Logic Programming has been advocated to be both a programming language and a specification language. It is natural to use Logic Programming for specifying/programming systems involving logical behaviours, rules and knowledge. However, this idea has a severe limitation: the use of negation. Negation is probably the most significant aspect of logic that was not included from the outset. This is due to the fact that dealing with negation involves significant additional complexity. Nevertheless, the use of negation is very natural and plays an important role in many cases, for instance, constraints management in databases, program composition, manipulation and transformation, default reasoning, natural language processing, etc.

Although this restriction cannot be perceived from the theoretical point of view (because there are many alternative ways to understand and incorporate negation into

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Logic Programming), the problems really start at the semantic level, where the different proposals (negation as failure -*naf*-, stable models, well-founded semantics, explicit negation, etc.) differ not only as to expressiveness but also as to semantics. However, the negation techniques supported by current Prolog compilers are rather limited, restricted to negation as failure under Fitting/Kunen semantics [9] (sound only under some circumstances usually not checked by compilers) which is a built-in or library in most Prolog compilers (Quintus, SICStus, Ciao, BinProlog, etc.), and the "delay technique" (applying negation as failure only *when* the variables of the negated goal become ground, which is sound but incomplete due to the possibility of floundering), which is present in Nu-Prolog, Gödel, and Prolog systems that implement delays (most of the above).

Of all the proposals, constructive negation [4, 5] (that we will call *classical* constructive negation) is probably the most promising because it has been proven to be sound and complete, and its semantics is fully compatible with Prolog's. Constructive negation was, in fact, announced in early versions of the Eclipse Prolog compiler, but was removed from the latest releases. The reasons seem to be related to some technical problems with the use of coroutining (risk of floundering) and the management of constrained solutions.

The goal of this paper is to give an algorithmic description of constructive negation, i.e. explicitly stating the details needed for an implementation. We also intend to discuss the pragmatic ideas needed to provide a concrete and real implementation. Early results for a concrete implementation extending the Ciao Prolog compiler are presented. We assume some familiarity with constructive negation techniques and Chan's papers.

The remainder of the paper is organized as follows. Section 2 details our constructive negation algorithm. It explains how to obtain the *frontier* of a goal (Section 2.1), how to prepare the goal for negation (Section 2.2) and, finally, how to negate the goal (Section 2.3). Section 3 discusses implementation issues: code expansion (Section 3.1), required disequality constraints (Section 3.2), optimizations (Section 3.3), examples (Section 4.1) and some experimental results (Section 4). Finally, we conclude and outline some future work.

## 2 Constructive Negation

Most of the papers addressing constructive negation deal with semantic aspects. In fact, only the original papers by Chan gave some hints about a possible implementation based on coroutining, but the technique was only outlined. When we tried to reconstruct this implementation we came across several problems, including the management of constrained answers and floundering (which appears to be the main reason why constructive negation was removed from recent versions of Eclipse). It is our belief that this problems cannot be easily and efficiently overcome. Therefore, we decided to design an implementation from scratch. One of our additional requirements is that we want to use a standard Prolog implementation (to be able to reuse thousands of existing Prolog lines and maintain their efficiency), so we will avoid implementation-level manipulations that would delay simple programs without negations.

We start with the definition of a frontier and how it can be managed to negate the respective formula.

## 2.1 Frontier

Firstly, we present Chan's definition of frontier (we actually owe the formal definition to Stuckey [18]).

**Definition 1.** Frontier

*A frontier of a goal G is the disjunction of a finite set of nodes in the derivation tree such that every derivation of G is either finitely failed or passes through exactly one frontier node.*

What is missing is a method to generate the frontier. So far we have used the simplest possible frontier: the frontier of depth 1 obtained by taking all the possible single SLD resolution steps. This can be done by a simple inspection of the clauses of the program[1]. Additionally, built-in based goals receive a special treatment (moving conjunctions into disjunctions, disjunctions into conjunction, eliminating double negations, etc.)

**Definition 2.** Depth-one frontier

- *If $G \equiv (G_1; G_2)$ then $Frontier(G) \equiv Frontier(G_1) \vee Frontier(G_2)$.*
- *If $G \equiv (G_1, G_2)$ then $Frontier(G) \equiv Frontier(G_1) \wedge Frontier(G_2)$ and then we have to apply DeMorgan's distributive property to retain the disjunction of conjunctions format.*
- *If $G \equiv p(\overline{X})$ and predicate $p/m$ is defined by N clauses:*

$$p(\overline{X}^1) : -C_1'.$$
$$p(\overline{X}^2) : -C_2'.$$
$$\dots$$
$$p(\overline{X}^3) : -C_N'.$$

*The frontier of the goal has the format: $Frontier(G) \equiv \{C_1 \vee C_2 \vee \dots \vee C_N\}$, where each $C_i$ is the union of the conjunction of subgoals $C_i'$ plus the equalities that are needed to unify the variables of $\overline{X}$ and the respective terms of $\overline{X}^i$.*

Consider, for instance, the following code:

```
odd(s(0)).
odd(s(s(X)))  :- odd(X).
```

The frontier for the goal $odd(Y)$ is as follows:

$$Frontier(odd(Y)) = \{(Y = s(0)) \vee (Y = s(s(X)) \wedge odd(X))\}$$

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[1] Nevertheless, we plan to improve the process by using abstract interpretation and detecting the degree of evaluation of a term that the execution will generate.

To get the negation of $G$ it suffices to negate the frontier formula. This is done by negating each component of the disjunction of all implied clauses (that form the frontier) and combining the results.

The solutions of $cneg(G)$ are the solutions of the combination (conjunction) of one solution of each of the N conjunctions $C_i$. Now we are going to explain how to negate a single conjunction $C_i$. This is done in two phases: *Preparation* and *Negation of the formula*.

## 2.2 Preparation

Before negating a conjunction obtained from the frontier, we have to simplify, organize, and normalize this conjunction:

– **Simplification of the conjunction**. If one of the terms of $C_i$ is trivially equivalent to *true* (e.g. $X = X$), we can eliminate this term from $C_i$. Symmetrically,if one of the terms is trivially *fail* (e.g. $X \neq X$), we can simplify $C_i \equiv fail$. The simplification phase can be carried out during the generation of frontier terms.
– **Organization of the conjunction**. Three groups are created containing the components of $C_i$, which are divided into equalities ($\overline{I}$), disequalities ($\overline{D}$), and other subgoals ($\overline{R}$). Then, we get $C_i \equiv \overline{I} \wedge \overline{D} \wedge \overline{R}$.
– **Normalization of the conjunction**. Let us classify the variables in the formula. The set of variables of the goal is called *GoalVars*. The set of free variables of $\overline{R}$ is called *RelVars*.
  • **Elimination of redundant variables and equalities**. If $I_i \equiv X = Y$, where $Y \notin GoalVars$, then we now have the formula $(I_1 \wedge \ldots \wedge I_{i-1} \wedge I_{i+1} \wedge \ldots \wedge I_{NI} \wedge \overline{D} \wedge \overline{R}\,)\sigma$, where $\sigma = \{Y/X\}$, i.e. the variable $Y$ is substituted by $X$ in the entire formula.
  • **Elimination of irrelevant disequalities**. *ImpVars* is the set of variables of *GoalVars* and the variables that appear in $\overline{I}$. The disequalities $D_i$ that contain any variable that was neither in *ImpVars* nor in *RelVars* are irrelevant and should be eliminated.

## 2.3 Negation of the formula

It is not feasible, to get all solutions of $C_i$ and to negate their disjunction because $C_i$ can have an infinite number of solutions. So, we have to use the classical constructive negation algorithm.

We consider that *ExpVars* is the set of variables of $\overline{R}$ that are not in *ImpVars*, i.e. *RelVars*, except the variables of $\overline{I}$ in the normalized formula.

*First step:* **Division of the formula**
$C_i$ is divided into:

$$C_i \equiv \overline{I} \wedge \overline{D}_{imp} \wedge \overline{R}_{imp} \wedge \overline{D}_{exp} \wedge \overline{R}_{exp}$$

where $\overline{D}_{exp}$ are the disequalities in $\overline{D}$ with variables in *ExpVars* and $\overline{D}_{imp}$ are the other disequalities, $\overline{R}_{exp}$ are the goals of $\overline{R}$ with variables in *ExpVars* and $\overline{R}_{imp}$ are the other goals, and $\overline{I}$ are the equalities.

Therefore, the constructive negation of the divided formula is:

$$\neg\, C_i \equiv \neg\, \overline{I}\vee$$
$$(\overline{I}\wedge\neg\,\overline{D}_{imp})\vee$$
$$(\overline{I}\wedge\overline{D}_{imp}\wedge\neg\,\overline{R}_{imp})\vee$$
$$(\overline{I}\wedge\overline{D}_{imp}\wedge\overline{R}_{imp}\wedge\neg\,(\overline{D}_{exp}\wedge\overline{R}_{exp}))$$

It is not possible to separate $\overline{D}_{exp}$ and $\overline{R}_{exp}$ because they contain free variables and they cannot be negated separately. The answers of the negations will be the answers of the negation of the equalities, the answers of the negation of the disequalities without free variables, the answers of the negation of the subgoals without free variables and the answers of the negation of the other subgoals of the conjunctions (the ones with free variables). Each of them will be obtained as follows:

*Second step:* **Negation of subformulas**

– **Negation of $\overline{I}$.** We have $\overline{I}\equiv I_1\wedge\ldots\wedge I_{NI}\equiv$

$$\exists\,\overline{Z}_1\ X_1=t_1\wedge\ldots\wedge\exists\,\overline{Z}_{NI}\ X_{NI}=t_{NI}$$

where $\overline{Z}_i$ are the variables of the equality $I_i$ that are not included in *GoalVars* (i.e. that are not quantified and are therefore free variables). When we negate this conjunction of equalities we get the constraint

$$\underbrace{\forall\,\overline{Z}_1\ X_1\neq t_1}_{\neg\,I_1}\vee\ldots\vee\underbrace{\forall\,\overline{Z}_{NI}\ X_{NI}\neq t_{NI}}_{\neg\,I_{NI}}\equiv\bigvee_{i=1}^{NI}\forall\,\overline{Z}_iX_i\neq t_i$$

This constraint is the first answer of the negation of $G$ that contains $NI$ components.

– **Negation of $\overline{D}_{imp}$.** If we have $N_{D_{imp}}$ disequalities $\overline{D}_{imp}\equiv D_1\wedge\ldots\wedge D_{N_{D_{imp}}}$ where $D_i\equiv\forall\,\exists\,\overline{Z}_i\ \overline{W}_i\ Y_i\neq s_i$ where $Y_i$ is a variable of *ImpVars*, $s_i$ is a term without variables in *ExpVars*, $\overline{W}_i$ are universally quantified variables that are neither in the equalities [2], nor in the other goals of $\overline{R}$ because otherwise $\overline{R}$ would be a disequality of $\overline{D}_{exp}$. Then we will get $N_{D_{imp}}$ new solutions with the format:

$\overline{I}\wedge\neg\,D_1$
$\overline{I}\wedge D_1\wedge\neg\,D_2$
$\ldots$
$\overline{I}\wedge D_1\wedge\ldots\wedge D_{N_{D_{imp}}-1}\wedge\neg\,D_{N_{D_{imp}}}$

where $\neg\,D_i\equiv\exists\,\overline{W}_i\ Y_i=s_i$. The negation of a universal quantification turns into an existential quantification and the quantification of free variables of $\overline{Z}$ gets lost, because the variables are unified with the evaluation of the equalities of $\overline{I}$. Then, we will get $N_{D_{imp}}$ new answers.

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[2] There are, of course, no universally quantified variables in an equality

– **Negation of $\overline{R}_{imp}$.** If we have $N_{R_{imp}}$ subgoals $\overline{R}_{imp} \equiv R_1 \wedge \ldots \wedge R_{N_{R_{imp}}}$. Then we will get new answers from each of the conjunctions:

$\overline{I} \wedge \overline{D}_{imp} \wedge \neg\, R_1$
$\overline{I} \wedge \overline{D}_{imp} \wedge R_1 \wedge \neg\, R_2$
$\ldots$
$\overline{I} \wedge \overline{D}_{imp} \wedge R_1 \wedge \ldots \wedge R_{N_{R_{imp}}-1} \wedge \neg\, R_{N_{R_{imp}}}$

where $\neg\, R_i \equiv cneg(R_i)$. Constructive negation is again applied over $R_i$ recursively using this operational semantics.

– **Negation of $\overline{D}_{exp} \wedge \overline{R}_{exp}$.** This conjunction cannot be disclosed because of the negation of $\exists\, \overline{V}_{exp}\ \overline{D}_{exp} \wedge \overline{R}_{exp}$, where $\overline{V}_{exp}$ gives universal quantifications:
$\forall\, \overline{V}_{exp}\ cneg(\overline{D}_{exp} \wedge \overline{R}_{exp})$. The entire constructive negation algorithm must be applied again. Note that the new set *GoalVars* is the former set *ImpVars*. Variables of $\overline{V}_{exp}$ are considered as free variables. When solutions of $cneg(\overline{D}_{exp} \wedge \overline{R}_{exp})$ are obtained some can be rejected: solutions with equalities with variables in $\overline{V}_{exp}$. If there is a disequality with any of these variables, e.g. $V$, the variable will be universally quantified in the disequality. This is the way to negate the negation of a goal, but there is a detail that was not considered in former approaches and that is necessary to get a sound implementation: the existence of universally quantified variables in $\overline{D}_{exp} \wedge \overline{R}_{exp}$ by the iterative application of the method. So, what we are really negating is a subgoal of the form: $\exists\, \overline{V}_{exp}\ \overline{D}_{exp} \wedge \overline{R}_{exp}$. Here we will provide the last group of answers that come from:

$$\overline{I} \wedge \overline{D}_{imp} \wedge \overline{R}_{imp} \wedge \forall\, \overline{V}_{exp} \neg\, (\overline{D}_{exp} \wedge \overline{R}_{exp})$$

## 3 Implementation Issues

Having described the theoretical algorithm, including important details, we now discuss important aspects for a practical implementation, including how to compute the frontier and manage answer constraints.

### 3.1 Code Expansion

The first issue is how to get the frontier of a goal. It is possible to handle the code of clauses during the execution thanks to the Ciao package system [3], which allows the code to be expanded at run time. The expansion is implemented in the *cneg.pl* package which is included in the declaration of the module that is going to be expanded (i.e. where there are goals that are negations).

Note that a similar, but less efficient, behaviour can be emulated using metaprogramming facilities, available in most Prolog compilers.

### 3.2 Disequality constraints

An instrumental step for managing negation is to be able to handle disequalities between terms such as $t_1 \neq t_2$. The typical Prolog resources for handling these disequalities are

limited to the built-in predicate `/==` `/2`, which needs both terms to be ground because it always succeeds in the presence of free variables. It is clear that a variable needs to be bound with a disequality to achieve a "constructive" behaviour. Moreover, when an equation $X = t(\overline{Y})$ is negated, the free variables in the equation must be universally quantified, unless affected by a more external quantification, i.e. $\forall \overline{Y}\ X \neq t(\overline{Y})$ is the correct negation. As we explained in [13], the inclusion of disequalities and constrained answers has a very low cost. It incorporates negative normal form constraints instead of bindings and the decomposition step can produce disjunctions.

A Prolog predicate `=/=` `/2` [13] has been defined, used to check disequalities, similarly to explicit unification (=). Each constraint is a disjunction of conjunctions of disequalities. When a universal quantification is used in a disequality (e.g., $\forall Y\ X \neq c(Y)$), the new constructor `fA/1` is used (e.g., `X` `/` `c(fA(Y))`).

### 3.3 Optimizing the algorithm and the implementation

Our constructive negation algorithm and the implementation techniques admit some additional optimizations that can improve the runtime behaviour of the system. Basically, the optimizations rely on the compact representation of information, as well as the early detection of successful or failing branches.

**Compact information**. In our system, negative information is represented quite compactly, providing fewer solutions from the negation of $\overline{I}$. The advantage is twofold. On the one hand constraints contain more information and failing branches can be detected earlier (i.e. the search space could be smaller). On the other hand, if we ask for all solutions using backtracking, we are cutting the search tree by offering all the solutions together in a single answer. For example, we can offer a simple answer for the negation of a predicate $p$ (the code for $p$ is skipped):

```
?- cneg(p(X,Y,Z,W)).

(X=/=0, Y=/=s(Z)) ; (X=/=Y) ; (X=/=Z) ;
(X=/=W) ; (X=/=s(0), Z=/=0) ? ;
no
```

(which is equivalent to $(X \neq 0 \wedge Y \neq s(Z)) \vee X \neq Y \vee X \neq Z \vee X \neq W \vee X \neq s(0) \vee Z \neq 0)$, instead of returning six answers upon backtracking:

```
?- cneg(p(X,Y,Z,W)).

X=/=0, Y=/=s(Z) ? ;
X=/=Y ? ;
X=/=Z ? ;
X=/=W ? ;
X=/=s(0) ? ;
Z=/=0 ? ;
no
```

**Pruning subgoals**. The frontiers generation search tree can be cut with a double action over the ground subgoals: removing the subgoals whose failure we are able to detect early on, and simplifying the subgoals that can be reduced to true. Suppose we have a predicate $p/2$ defined as

```
p(X,Y):- greater(X,Y),
         q(X,Y,Z),
         r(Z).
```

where $q/3$ and $r/1$ are predicates defined by several clauses with a complex computation. To negate the goal $p(s(0), s(s(0)))$, its frontier is computed:

$$Frontier(p(s(0), s(s(0)))) \equiv$$
$$X = s(0) \wedge Y = s(s(0)) \wedge greater(X, Y) \wedge q(X, Y, Z) \wedge r(Z) \equiv$$
$$greater(s(0), s(s(0))) \wedge q(s(0), s(s(0)), Z) \wedge r(Z) \equiv$$
$$fail \wedge q(s(0), s(s(0)), Z) \wedge r(Z) \equiv$$
$$fail$$

The next step is to expand the code of the subgoals of the frontier to the combination (disjunction) of the code of all their clauses, and the result will be a very complicated and hard to check frontier. However, the process is optimized by evaluating ground terms. In this case, $greater(s(0), s(s(0))$ fails and, therefore, it is not necessary to continue with the generation of the frontier, because the result is reduced to fail (i.e. the negation of $p(s(0), s(s(0)))$ will be trivially true). The opposite example is a simplification:

$$Frontier(p(s(s(0)), s(0))) \equiv$$
$$X = s(s(0)) \wedge Y = s(0) \wedge greater(X, Y) \wedge q(X, Y, Z) \wedge r(Z) \equiv$$
$$greater(s(s(0)), s(0)) \wedge q(s(s(0)), s(0), Z) \wedge r(Z) \equiv$$
$$true \wedge q(s(s(0)), s(0), Z) \wedge r(Z) \equiv$$
$$q(s(s(0)), s(0), Z) \wedge r(Z)$$

**Constraint simplification**. During the whole process for negating a goal, the frontier variables are constrained. In cases where the constraints are satisfiable, they can be eliminated and where the constraints can be reduced to fail, the evaluation can be stopped with result *true*.

We focus on the negative information of a normal form constraint $F$:

$$F \equiv \bigvee_i \bigwedge_j \forall \overline{Z}_j^i \ (Y_j^i \neq s_j^i)$$

Firstly, the Prenex form [16] can be obtained by extracting the universal variables with different names to the head of the formula, applying logic rules:

$$F \equiv \forall \overline{x} \bigvee_i \bigwedge_j (Y_j^i \neq s_j^i)$$

and using the distributive property:

$$F \equiv \forall \overline{x} \bigwedge_k \bigvee_l (Y_l^k \neq s_l^k)$$

The formula can be separated into subformulas that are simple disjunctions of disequalities :

$$F \equiv \bigwedge_k \forall \overline{x} \bigvee_l (Y_l^k \neq s_l^k) \equiv F_1 \wedge \ldots \wedge F_n$$

Each single formula $F_k$ can be evaluated. The first step will be to substitute the existentially quantified variables (variables that do not belong to $\overline{x}$) by Skolem constants $s_j^i$ that will keep the equivalence without losing generality:

$$F_k \equiv \forall \overline{x} \bigvee_l (Y_l^k \neq s_l^k) \equiv \forall \overline{x} \bigvee_l (Y_{Skl}^k \neq s_{Skl}^k)$$

Then it can be transformed into:

$$F_k \equiv \neg \exists \overline{x} \neg (\bigvee_l (Y_{Skl}^k \neq s_{Skl}^k)) \equiv \neg Fe_K$$

The meaning of $F_k$ is the negation of the meaning of $Fe_k$;

$$Fe_k \equiv \exists \overline{x} \neg (\bigvee_l (Y_{Skl}^k \neq s_{Skl}^k))$$

Solving the negations, the result is obtained through simple unifications of the variables of $\overline{x}$:

$$Fe_k \equiv \exists \overline{x} \bigwedge \neg (Y_{Skl}^k \neq s_{Skl}^k) \equiv \exists \overline{x} \bigwedge (Y_{Skl}^k = s_{Skl}^k)$$

Therefore, we get the truth value of $F_k$ from the negation of the value of $Fe_k$ and, finally, the value of $F$ is the conjunction of the values of all $F_k$. If $F$ succeeds, then the constraint is removed because it is redundant and we continue with the negation process. If it fails, then the negation directly succeeds.

## 4 Experimental results

Our prototype is a simple library that is added to the set of libraries of Ciao Prolog. Indeed, it is easy to port the library to other Prolog compilers. The only requirement is that attributed variables should be available.

This section reports some experimental results from our prototype implementation. First of all, we show the behaviour of the implementation in some simple examples.

### 4.1 Examples

The interesting side of this implementation is that it returns constructive results from a negative question. Let us start with a simple example involving predicate $boole/1$.

```
boole(0).                        ?- cneg(boole(X)).
boole(1).                        X=/=1, X=/=0 ? ;
                                 no
```

Another simple example obtained from [18] gives us the following answers:

```
p(a,b,c).                        ?- proof1(X,Y,Z).
p(b,a,c).
p(c,a,b).                        Z = c,
                                 X=/=b, X=/=a ? ;
proof1(X,Y,Z):-
    X =/= a,                     Z = c,
    Z = c,                       Y=/=a, X=/=a ? ;
    cneg(p(X,Y,Z)).
                                 no
```

[18] contains another example showing how a constructive answer ($\forall T \; X \neq s(T)$) is provided for the negation of an undefined goal in Prolog:

```
p(X):- X = s(T), q(T).          ?- r(X).

q(T):- q(T).                     X=/=s(fA(_A)) ?

r(X):- cneg(p(X)).              yes
```

Notice that if we would ask for a second answer, then it will loop according to the Prolog resolution. An example with an infinite number of solutions is more interesting.

```
                                 ?- cneg(positive(X)).

                                 X=/=s(fA(_A)), X=/=0 ? ;
positive(0).                     X = s(_A),
positive(s(X)):-                 (_A=/=s(fA(_B)), _A=/=0) ? ;
        positive(X).            X = s(s(_A)),
                                 (_A=/=s(fA(_B)), _A=/=0) ? ;
                                 X = s(s(s(_A))),
                                 (_A=/=s(fA(_B)), _A=/=0) ?
                                 yes
```

## 4.2 Implementation measures

We have firstly measured the execution times in milliseconds for the above examples when using negation as failure (*naf*/1) and constructive negation (*cneg*/1). A '-' in a cell means that negation as failure is not applicable. Some goals were executed a number of times to get a significant measurement. All of them were made using Ciao Prolog[3] 1.5 on a Pentium II at 350 MHz. The results are shown in Table 1. We have added a first column with the runtime of the evaluation of the positive goal that is negated in

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[3] The negation system is coded as a library module ('package" [3]), which includes the respective syntactic and semantic extensions (i.e. Ciao's attributed variables). Such extensions apply locally within each module which uses this negation library.

| goals | Goal | naf(Goal) | cneg(Goal) | ratio |
|---|---|---|---|---|
| boole(1) | 2049 | 2099 | 2069 | 0.98 |
| boole(8) | 2070 | 2170 | 2590 | 1.19 |
| positive(s(s(s(s(s(s(0)))))))) | 2079 | 1600 | 2159 | 1.3 |
| positive(s(s(s(s(s(0))))))) | 2079 | 2139 | 2060 | 0.96 |
| greater(s(s(s(0))),s(0)) | 2110 | 2099 | 2100 | 1.00 |
| greater(s(0),s(s(s(0)))) | 2119 | 2129 | 2089 | 0.98 |
| **average** | | | | 1.06 |
| positive(500000) | 2930 | 2949 | 41929 | 14.21 |
| positive(1000000) | 3820 | 3689 | 81840 | 22.18 |
| greater(500000,500000) | 3200 | 3339 | 22370 | 7.70 |
| **average** | | | | 14.69 |
| boole(X) | 2080 | - | 3109 | |
| positive(X) | 2020 | - | 7189 | |
| greater(s(s(s(0))),X) | 2099 | - | 6990 | |
| greater(X,Y) | 7040 | - | 7519 | |
| queens(s(s(0)),Qs) | 6939 | - | 9119 | |

**Table 1.** Runtime comparation

the other columns and a last column with the ratio that measures the speedup of the *naf* technique w.r.t. constructive negation.

Using **naf** instead of **cneg** results in small ratios around 1.06 on average for ground calls with few recursive calls. So, the possible slow-down for constructive negation is not so high as we might expect for these examples. Furthermore, the results are rather similar. But the same goals with data that involve many recursive calls yield ratios near 14.69 on average w.r.t **naf**, increasing exponentially with the number of recursive calls. There are, of course, many goals that cannot be negated using the *naf* technique and that are solved using constructive negation.

## 5 Finite Constructive Negation

The problem with the constructive negation algorithm is of course efficiency. It is the price that it has to be paid for a powerful mechanism that negates any kind of goal. Thinking of Prolog programs, many goals have a finite number of solutions. There is a simplification of the constructive negation algorithm that we use to negate these goals. It is very simple in the sense that if we have a goal $\neg G$ where the solution of the positive subgoal $G$ is a set of $n$ solutions like $\{S_1, S_2, ..., S_n\}$, then we can consider these equivalences:

$$G \equiv S_1 \vee S_2 \vee ... \vee S_n$$

$$\neg G \equiv \neg(S_1 \vee S_2 \vee ... \vee S_n)$$

$$\neg G \equiv (\neg S_1 \wedge \neg S_2 \wedge ... \wedge \neg S_n)$$

Of course, these solutions are a conjunction of unifications (equalities) and disequality constraints. As described in section 3.2, we know how to handle and negate this kind of information. The implementation of the predicate $cnegf/1$ is something akin to

```
cnegf(Goal):-
  varset(Goal,GVars), % Getting variables of the Goal
  setof(GVars,Goal,LValores),!, % Getting the solutions
  cneg_solutions(GVars,LValores). % Negating solutions

cnegf(_Goal). % Without solutions, the negation succeeds
```

where $cneg\_solutions/2$ is the predicate that negates the disjunction of conjunctions of solutions of the goal that we are negating. It works as described in section 2.3, but it is simpler, because here we are only negating equalities and disequalities.

We get the set of variables, *GVars*, of the goal, *Goal*, that we want to negate (we use the predicate $varset/2$). Then we use the $setof/3$ predicate to get the values of the variables of *GVars* for each solution of *Goal*. For example, if we want to evaluate $cnegf(boole(X))$, then we get $varset(boole(X),[X])$, $setof([X],boole(X),[[0],[1]])$ (i.e. $X = 0 \vee X = 1$) and $cneg\_solutions/2$ will return $X \neq 0 \wedge X \neq 1$.

If we have the goal $p(X,Y)$, which, has two solutions $X = a$, $Y = b$ and $X = c$, $Y = d$, then, in the evaluation of $cnegf(p(X,Y))$, we will get $varset(p(X,Y),[X,Y])$, $setof([X,Y],p(X,Y),[[a,b],[c,d]])$ (i.e. $(X = a \wedge Y = b) \vee (X = c \wedge Y = d)$) and $cneg\_solutions/2$ will return the four solutions $(X \neq a \wedge X \neq c) \vee (X \neq a \wedge Y \neq d) \vee (Y \neq b \wedge X \neq c) \vee (Y \neq b \wedge Y \neq d)$.

### 5.1 Analysis of the number of solutions

The below optimization is very intuitive but, perhaps, the main problem is to detect when a goal is going to have a finite number of solutions. To get sound results, we are going to use this technique (finite constructive negation) just to negate the goals that, we are sure do not have infinite solutions. So, our analysis is conservative.

We use a combination of two analyses: the non-failure analysis [10] and the analysis of upper cost [11]. Both are implemented in the Ciao Prolog precompiler [8]. We test these analyses at compilation time and then, when possible, we directly execute the optimized version of constructive negation at compilation time.

If we get for the subgoal *G* that:

 – the nonfailure analysis yields "*G* does not fails" and
 – the cost analysis yields "*G* has an upper cost inferior to infinite",

then we cab be sure about that *G* has a finite number of solutions.

It is more complicated to check this at execution time although we could provide a rough approximation. First, we get a maximun number *N* of solutions of *G* (we can use the library predicate $findnsols/4$) and then we check the number of solutions that we have obtained. If it is less than *N*, we can assure that *G* has a finite number of solutions and otherwise we do not know.

## 5.2 Experimental results

We have implemented a predicate $cnegf/1$ to negate the disjunction of all the solutions of its argument. The implementation of this predicate takes advantage of backtracking to obtain only the information that we need to get the first answer. Then, if the user asks for another answer, then the backtracking gets information enough to provide it. Accordingly, we avoid the complete evaluation of the negation of all the solutions first time round. We negate the subterms only when we need to provide the next solution. In this sense, if we have the goal $G$ (where each $S_i$ is the conjunction of $Ni$ equalities or disequalities)

$$G \equiv S_1 \vee ... \vee S_n$$

$$G \equiv (S_1^1 \wedge ... \wedge S_1^{N1}) \vee ... \vee (S_n^1 \wedge ... \wedge S_n^{Nn})$$

and we then want to obtain $\neg G$, then we have

$$\neg G \equiv \neg(S_1 \vee S_2 \vee ... \vee S_n)$$

$$\neg G \equiv (\neg S_1 \wedge \neg S_2 \wedge ... \wedge \neg S_n)$$

$$\neg G \equiv \neg(S_1^1 \wedge ... \wedge S_1^{N1}) \wedge ... \wedge \neg(S_n^1 \wedge ... \wedge S_n^{Nn})$$

$$\neg G \equiv (\neg S_1^1 \vee ... \vee \neg S_1^{N1}) \wedge ... \wedge (\neg S_n^1 \vee ... \vee \neg S_n^{Nn})$$

$$\neg G \equiv (\neg S_1^1 \wedge ... \wedge \neg S_n^1) \vee ... \vee (\neg S_1^{N1} \wedge ... \wedge \neg S_n^{Nn})$$

we begin calculating just the first answer of the negation that will be $(\neg S_1^1 \wedge ... \wedge \neg S_n^1)$, and the rest will be calculated if necessary using backtracking.

Let us present a simple example:

```
?- member(3,[X,Y,Z]).          ?- cnegf(member(3,[X,Y,Z])).
X = 3 ? ;                       X=/=3, Y=/=3, Z=/=3 ?;
Y = 3 ? ;                       no
Z = 3 ? ;
no
```

We get the symmetric behavior for the negation of the negation of the initial query

```
?- cnegf(cnegf(member(3,[X,Y,Z]))).
X = 3 ? ;
Y = 3 ? ;
Z = 3 ? ;
no
```

We checked some time results in Table 2. The results are much more significant for more complicated goals. Indeed, the more complicated the code of a predicate is, the more inefficient its classical constructive negation (*cneg*) is. However, finite constructive negation (*cnegf*) is independent of code complexity. Finite constructive negation depends on the complexity of the solutions obtained for the positive goal and, of course, the number of solutions of this goal.

| goals | Goal | cneg(Goal) | cnegf(Goal) | ratio |
|---|---|---|---|---|
| boole(1) | 821 | 831 | 822 | 1,01 |
| positive(s(s(s(0)))) | 811 | 1351 | 860 | 1,57 |
| greater(s(0),s(s(0))) | 772 | 1210 | 840 | 1,44 |
| positive(s$^{500000}$(0)) | 1564 | 8259 | 3213 | 2,57 |
| positive(s$^{7500000}$(0)) | 2846 | 12445 | 3255 | 3,82 |
| greater(s$^{50000}$(0),s$^{50000}$(0)) | 1240 | 66758 | 30112 | 2,21 |
| boole(X) | 900 | 1321 | 881 | 1,49 |
| greater(s(s(s(0))),X) | 990 | 1113 | 1090 | 1,02 |
| queens(s(s(s(0))),Qs) | 1481 | 50160 | 1402 | 35,77 |

**Table 2.** Runtime comparison of finite constructive negation

## 6 Conclusion and Future Work

After running some preliminary experiments with the classical constructive negation technique following Chan's description, we realized that the algorithm needed some additional explanations and modifications.

Having given a detailed specification of algorithm in a detailed way we proceed to provide a real, complete and consistent implementation. The result, we have reported are very encouraging, because we have proved that it is possible to extend Prolog with a constructive negation module relatively inexpensively and overall without any delay in Prolog programs that are not using this negation. Nevertheless, it is quite important to address possible optimizations, and we are working to improve the efficiency of the implementation. These include a more accurate selection of the frontier based on the demanded form of argument in the vein of [12]). Another possible future work is to incorporate our algorithm at the WAM machine level.

In any case, we will probably not be able to provide an efficient enough implementation of constructive negation, because the algorithm is inherently inefficient. This is why we do not intend to use it either for all cases of negation or for negating goals directly.

Our goal is to design and implement a practical negation operator and incorporate it into a Prolog compiler. In [13, 14] we systematically studied what we understood to be the most interesting existing proposals: negation as failure (*naf*) [6], use of delays to apply *naf* securely [15], intensional negation [1, 2], and constructive negation [4, 5, 7, 17, 18]. As none of them can satisfy our requirements of completeness and efficiency, we propose to use a combination of these techniques, where the information from static program analyzers could be used to reduce the cost of selecting techniques [14]. So, in many cases, we avoid the inefficiency of classical constructive negation. However, we still need it because it is the only method that is sound and complete for all kind of goals. For example, looking at the goals in Table 1, the strategy will obtain all ground negations using the *naf* technique and it would only use classical constructive negation for the goals with variables where it is impossible to use *naf*. Otherwise, the strategy will use finite constructive negation (*cnegf*) for the three last goals of Table 2 because the positive goals have a finite number of solutions.

We are testing the implementation and trying to improve the code, and our intention is to include it in the next version of Ciao Prolog [4].

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