In[1]:=
2.2

" Sixth, ... "

Out[1]= 2.2

In[2]:=
```
A1a = x2 + x3 + x4
A1b = x2 + x3 + x4 + x5 + x6 + x7
A2a = Simplify[Integrate[Integrate[Integrate[A1a, {x4, b, x3}], {x3, b, x2}], {x2, b, x1}]]
A2b =
 Integrate[Integrate[Integrate[Integrate[Integrate[Integrate[A1b, {x7, b, x6}], {x6, b, x5}],
      {x5, b, x4}], {x4, b, x3}], {x3, b, x2}], {x2, b, x1}]
```

Out[2]= $x2 + x3 + x4$

Out[3]= $x2 + x3 + x4 + x5 + x6 + x7$

Out[4]= $-\frac{1}{4}(b - x1)^3 (b + x1)$

Out[5]= $\frac{1}{240}(b - x1)^6 (b + x1)$

In[6]:=
3

Out[6]= 3

In[7]:=
```
p1 = (b1^2 - b2^2) / b1^2
p2 = b2^2 / b1^2
e1 = (b1 + a * b2) / 2
e2 = Integrate[x, {x, 0, b1 / 2}] + Integrate[b1 / 2 + a * (x - b1 / 2), {x, b1 / 2, b2}]
A1 = p1 * e1 + p2 * e2 / b2
A2 = D[A1, b2]
A3 = Solve[A2 == 0, b2]
```

Out[7]= $\frac{b1^2 - b2^2}{b1^2}$

Out[8]= $\frac{b2^2}{b1^2}$

Out[9]= $\frac{1}{2}(b1 + a\,b2)$

Out[10]= $-\frac{b1^2}{8} + \frac{a\,b1^2}{8} + \frac{b1\,b2}{2} - \frac{a\,b1\,b2}{2} + \frac{a\,b2^2}{2}$

Out[11]= $\frac{(b1 + a\,b2)\left(b1^2 - b2^2\right)}{2\,b1^2} + \frac{b2\left(-\frac{b1^2}{8} + \frac{a\,b1^2}{8} + \frac{b1\,b2}{2} - \frac{a\,b1\,b2}{2} + \frac{a\,b2^2}{2}\right)}{b1^2}$

Out[12]= $-\frac{b2\,(b1 + a\,b2)}{b1^2} + \frac{b2\left(\frac{b1}{2} - \frac{a\,b1}{2} + a\,b2\right)}{b1^2} + \frac{a\left(b1^2 - b2^2\right)}{2\,b1^2} + \frac{-\frac{b1^2}{8} + \frac{a\,b1^2}{8} + \frac{b1\,b2}{2} - \frac{a\,b1\,b2}{2} + \frac{a\,b2^2}{2}}{b1^2}$

Out[13]= $\left\{\left\{b2 \to \frac{(-1 + 5\,a)\,b1}{8\,a}\right\}\right\}$

In[14]:=
```
A1a = Integrate[Integrate[x1 - x2, {x2, 0, x1}], {x1, 0, b1 / 2}]
A1b = Integrate[Integrate[a * x1 - (a - 1) * b1 / 2 - x2, {x2, 0, b1 / 2}], {x1, b1 / 2, b2}]
A1c = Integrate[Integrate[x1 - x2, {x2, b1 / 2, x1}], {x1, b1 / 2, b2}]
A1d = Integrate[Integrate[x1 - x2, {x2, b2, x1}], {x1, b2, b1}]
A1e = Expand[A1a + A1b + a * A1c + a * A1d]
A2 = D[A1, b2]
A3 = Solve[A2 == 0, b2]
```

Out[14]= $\frac{b1^3}{48}$

Out[15]= $-\frac{b1^3}{16}+\frac{a\,b1^3}{16}+\frac{b1^2\,b2}{8}-\frac{1}{4}a\,b1^2\,b2+\frac{1}{4}a\,b1\,b2^2$

Out[16]= $-\frac{1}{48}(b1-2\,b2)^3$

Out[17]= $\frac{b1^3}{6}-\frac{b1^2\,b2}{2}+\frac{b1\,b2^2}{2}-\frac{b2^3}{6}$

Out[18]= $-\frac{b1^3}{24}+\frac{5\,a\,b1^3}{24}+\frac{b1^2\,b2}{8}-\frac{5}{8}a\,b1^2\,b2+\frac{1}{2}a\,b1\,b2^2$

Out[19]= $-\frac{b2\,(b1+a\,b2)}{b1^2}+\frac{b2\left(\frac{b1}{2}-\frac{a\,b1}{2}+a\,b2\right)}{b1^2}+\frac{a\left(b1^2-b2^2\right)}{2\,b1^2}+\frac{-\frac{b1^2}{8}+\frac{a\,b1^2}{8}+\frac{b1\,b2}{2}-\frac{a\,b1\,b2}{2}+\frac{a\,b2^2}{2}}{b1^2}$

Out[20]= $\left\{\left\{b2\to\frac{(-1+5\,a)\,b1}{8\,a}\right\}\right\}$

In[21]:=
```
4
```

Out[21]= 4

In[22]:=
```
p2 = Integrate[1 / b1 - a, {x, 0, b1 / 2}] + Integrate[1 / b1 + a, {x, b1 / 2, b2}]
e2 = Integrate[x (1 / b1 - a), {x, 0, b1 / 2}] + Integrate[x (1 / b1 + a), {x, b1 / 2, b2}]
A1 = (1 - p2^2) * (b1 + b2) / 2 + p2^2 * e2 / p2
A2 = D[A1, b2]
A3 = Solve[A2 == 0, b2]
```

Out[22]= $\frac{1}{2}\left(-a+\frac{1}{b1}\right) b1+\left(a+\frac{1}{b1}\right)\left(-\frac{b1}{2}+b2\right)$

Out[23]= $-\frac{a\,b1^2}{4}+\frac{a\,b2^2}{2}+\frac{b2^2}{2\,b1}$

Out[24]= $\left(-\frac{a\,b1^2}{4}+\frac{a\,b2^2}{2}+\frac{b2^2}{2\,b1}\right)\left(\frac{1}{2}\left(-a+\frac{1}{b1}\right) b1+\left(a+\frac{1}{b1}\right)\left(-\frac{b1}{2}+b2\right)\right)+$
$\frac{1}{2}(b1+b2)\left(1-\left(\frac{1}{2}\left(-a+\frac{1}{b1}\right) b1+\left(a+\frac{1}{b1}\right)\left(-\frac{b1}{2}+b2\right)\right)^2\right)$

Out[25]= $\left(a+\frac{1}{b1}\right)\left(-\frac{a\,b1^2}{4}+\frac{a\,b2^2}{2}+\frac{b2^2}{2\,b1}\right)-\left(a+\frac{1}{b1}\right)(b1+b2)\left(\frac{1}{2}\left(-a+\frac{1}{b1}\right) b1+\left(a+\frac{1}{b1}\right)\left(-\frac{b1}{2}+b2\right)\right)+$
$\left(a\,b2+\frac{b2}{b1}\right)\left(\frac{1}{2}\left(-a+\frac{1}{b1}\right) b1+\left(a+\frac{1}{b1}\right)\left(-\frac{b1}{2}+b2\right)\right)+\frac{1}{2}\left(1-\left(\frac{1}{2}\left(-a+\frac{1}{b1}\right) b1+\left(a+\frac{1}{b1}\right)\left(-\frac{b1}{2}+b2\right)\right)^2\right)$

Out[26]= $\left\{\left\{b2\to\frac{1}{4}\,b1\,(2+a\,b1)\right\}\right\}$

In[27]:=
```
p1 = (1 / b1 - a2) ^ 2
p2 = (1 / b1 - a2) * (1 / b1 + a2)
p3 = (1 / b1 + a2) ^ 2
ii = (x1 - x2) / 2
A1a = Integrate[Integrate[ii * p1, {x2, 0, x1}], {x1, 0, b1 / 2}]
A1b = Integrate[Integrate[ii * p2, {x2, 0, b1 / 2}], {x1, b1 / 2, b2}]
A1c = Integrate[Integrate[ii * p3, {x2, b1 / 2, x1}], {x1, b1 / 2, b2}]
A1d = Integrate[Integrate[ii * p3, {x2, b2, x1}], {x1, b2, b1}]
A1 = A1a + A1b + A1c + A1d
A2 = D[A1, b2]
A3 = Solve[A2 == 0, b2]
```

Out[27]= $\left(-a2+\frac{1}{b1}\right)^2$

Out[28]= $\left(-a2+\frac{1}{b1}\right)\left(a2+\frac{1}{b1}\right)$

Out[29]= $\left(a2+\frac{1}{b1}\right)^2$

Out[30]= $\frac{x1-x2}{2}$

Out[31]= $\frac{b1}{96}-\frac{a2\,b1^2}{48}+\frac{a2^2\,b1^3}{96}$

Out[32]= $-\frac{b2}{16}+\frac{1}{16}a2^2\,b1^2\,b2+\frac{b2^2}{8\,b1}-\frac{1}{8}a2^2\,b1\,b2^2$

Out[33]= $-\frac{(1+a2\,b1)^2\,(b1-2\,b2)^3}{96\,b1^2}$

Out[34]= $\frac{(1+a2\,b1)^2\,(b1-b2)^3}{12\,b1^2}$

Out[35]= $\frac{b1}{96}-\frac{a2\,b1^2}{48}+\frac{a2^2\,b1^3}{96}-\frac{(1+a2\,b1)^2\,(b1-2\,b2)^3}{96\,b1^2}+$
$\frac{(1+a2\,b1)^2\,(b1-b2)^3}{12\,b1^2}-\frac{b2}{16}+\frac{1}{16}a2^2\,b1^2\,b2+\frac{b2^2}{8\,b1}-\frac{1}{8}a2^2\,b1\,b2^2$

Out[36]= $-\frac{1}{16}+\frac{a2^2\,b1^2}{16}+\frac{(1+a2\,b1)^2\,(b1-2\,b2)^2}{16\,b1^2}-\frac{(1+a2\,b1)^2\,(b1-b2)^2}{4\,b1^2}+\frac{b2}{4\,b1}-\frac{1}{4}a2^2\,b1\,b2$

Out[37]= $\left\{\left\{b2\to\frac{1}{4}b1\,(2+a2\,b1)\right\}\right\}$

In[38]:=
5.2

Out[38]= 5.2

In[39]:=
```
Integrate[Integrate[(x1 - x2) / 2, {x2, 0, x1}], {x1, 0, b}]
```

Out[39]= $\frac{b^3}{12}$

In[40]:= **6.4**

Out[40]= 6.4

In[41]:=
```
Ph = (b1^n - b2^n) / b1^n
Pl = b2^n / b1^n
pil = 1 / b2
pih = 1 / (b1 - b2)
A1 = Ph * pih * Integrate[Log2[pih], {x, b2, b1}] + Pl * pil * Integrate[Log2[pil], {x, 0, b2}]
A2 = D[A1, b2]
```

Out[41]= $b1^{-n}\,(b1^n - b2^n)$

Out[42]= $b1^{-n}\,b2^n$

Out[43]= $\frac{1}{b2}$

Out[44]= $\frac{1}{b1 - b2}$

Out[45]= $\frac{b1^{-n}\,(b1^n - b2^n)\,\text{Log}\left[\frac{1}{b1-b2}\right]}{\text{Log}[2]} + \frac{b1^{-n}\,b2^n\,\text{Log}\left[\frac{1}{b2}\right]}{\text{Log}[2]}$

Out[46]= $-\frac{b1^{-n}\,b2^{-1+n}}{\text{Log}[2]} + \frac{b1^{-n}\,(b1^n - b2^n)}{(b1 - b2)\,\text{Log}[2]} - \frac{b1^{-n}\,b2^{-1+n}\,n\,\text{Log}\left[\frac{1}{b1-b2}\right]}{\text{Log}[2]} + \frac{b1^{-n}\,b2^{-1+n}\,n\,\text{Log}\left[\frac{1}{b2}\right]}{\text{Log}[2]}$

In[47]:=
```
A1 = Ph * pih * Integrate[(x - (b1 + b2) / 2)^2, {x, b2, b1}] +
  Pl * pil * Integrate[(x - b2 / 2)^2, {x, 0, b2}]
A2 = D[A1, b2]
A3 = Simplify[A2 * 6 * b2 * b1^n]
```

Out[47]= $\frac{1}{12}\,b1^{-n}\,b2^{2+n} + \frac{1}{12}\,b1^{-n}\,(b1 - b2)^2\,(b1^n - b2^n)$

Out[48]= $-\frac{1}{6}\,b1^{-n}\,(b1 - b2)\,(b1^n - b2^n) - \frac{1}{12}\,b1^{-n}\,(b1 - b2)^2\,b2^{-1+n}\,n + \frac{1}{12}\,b1^{-n}\,b2^{1+n}\,(2 + n)$

Out[49]= $-b1^{1+n}\,b2 + b1^n\,b2^2 - \frac{1}{2}\,b1^2\,b2^n\,n + b1\,b2^{1+n}\,(1 + n)$

In[50]:=
```
A1a = Integrate[b2 / 2 - x, {x, 0, b2 / 2}] + Integrate[x - b2 / 2, {x, b2 / 2, b2}]
A1b = Integrate[(b1 + b2) / 2 - x, {x, b2, (b1 + b2) / 2}] +
  Integrate[x - (b1 + b2) / 2, {x, (b1 + b2) / 2, b1}]
A1 = Pl * pil * A1a + Ph * pih * A1b
A2 = D[A1, b2]
A3 = Simplify[A2 * 4 * b2 * b1^n]
```

Out[50]= $\frac{b2^2}{4}$

Out[51]= $\frac{b1^2}{4} - \frac{b1\,b2}{2} + \frac{b2^2}{4}$

Out[52]= $\frac{1}{4}\,b1^{-n}\,b2^{1+n} + \frac{b1^{-n}\left(\frac{b1^2}{4} - \frac{b1\,b2}{2} + \frac{b2^2}{4}\right)\left(b1^n - b2^n\right)}{b1 - b2}$

Out[53]= $\frac{b1^{-n}\left(-\frac{b1}{2} + \frac{b2}{2}\right)\left(b1^n - b2^n\right)}{b1 - b2} + \frac{b1^{-n}\left(\frac{b1^2}{4} - \frac{b1\,b2}{2} + \frac{b2^2}{4}\right)\left(b1^n - b2^n\right)}{(b1 - b2)^2} -$
$\frac{b1^{-n}\,b2^{-1+n}\left(\frac{b1^2}{4} - \frac{b1\,b2}{2} + \frac{b2^2}{4}\right)n}{b1 - b2} + \frac{1}{4}\,b1^{-n}\,b2^n\,(1 + n)$

Out[54]= $-b1^n\,b2 - b1\,b2^n\,n + 2\,b2^{1+n}\,(1 + n)$

In[55]:=
```
Solve[6 * b2^2 - 2 * b1 * b2 - b1^2 == 0, b2]
```

Out[55]= $\left\{\left\{b2 \to \frac{1}{6}\left(b1 - \sqrt{7}\ b1\right)\right\}, \left\{b2 \to \frac{1}{6}\left(b1 + \sqrt{7}\ b1\right)\right\}\right\}$