

## 2.2

" Sixth, ... "

$$A1a = x^2 + x^3 + x^4$$

$$A1b = x^2 + x^3 + x^4 + x^5 + x^6 + x^7$$

$$A2a = \text{Simplify}[\text{Integrate}[\text{Integrate}[\text{Integrate}[A1a, \{x^4, b, x^3\}], \{x^3, b, x^2\}], \{x^2, b, x^1\}]]$$

$$A2b =$$

$$\text{Integrate}[\text{Integrate}[\text{Integrate}[\text{Integrate}[\text{Integrate}[\text{Integrate}[A1b, \{x^7, b, x^6\}], \{x^6, b, x^5\}], \{x^5, b, x^4\}], \{x^4, b, x^3\}], \{x^3, b, x^2\}], \{x^2, b, x^1\}]$$

$$x^2 + x^3 + x^4$$

$$x^2 + x^3 + x^4 + x^5 + x^6 + x^7$$

$$-\frac{1}{4} (b - x^1)^3 (b + x^1)$$

$$\frac{1}{240} (b - x^1)^6 (b + x^1)$$

## 3

$$p1 = (b1^2 - b2^2) / b1^2$$

$$p2 = b2^2 / b1^2$$

$$e1 = (b1 + a * b2) / 2$$

$$e2 = \text{Integrate}[x, \{x, 0, b1 / 2\}] + \text{Integrate}[b1 / 2 + a * (x - b1 / 2), \{x, b1 / 2, b2\}]$$

$$A1 = p1 * e1 + p2 * e2 / b2$$

$$A2 = D[A1, b2]$$

$$A3 = \text{Solve}[A2 == 0, b2]$$

$$\frac{b1^2 - b2^2}{b1^2}$$

$$\frac{b2^2}{b1^2}$$

$$\frac{1}{2} (b1 + a b2)$$

$$-\frac{b1^2}{8} + \frac{a b1^2}{8} + \frac{b1 b2}{2} - \frac{a b1 b2}{2} + \frac{a b2^2}{2}$$

$$\frac{(b1 + a b2) (b1^2 - b2^2)}{2 b1^2} + \frac{b2 \left( -\frac{b1^2}{8} + \frac{a b1^2}{8} + \frac{b1 b2}{2} - \frac{a b1 b2}{2} + \frac{a b2^2}{2} \right)}{b1^2}$$

$$-\frac{b2 (b1 + a b2)}{b1^2} + \frac{b2 \left( \frac{b1}{2} - \frac{a b1}{2} + a b2 \right)}{b1^2} + \frac{a (b1^2 - b2^2)}{2 b1^2} + \frac{-\frac{b1^2}{8} + \frac{a b1^2}{8} + \frac{b1 b2}{2} - \frac{a b1 b2}{2} + \frac{a b2^2}{2}}{b1^2}$$

$$\left\{ \left\{ b2 \rightarrow \frac{(-1 + 5 a) b1}{8 a} \right\} \right\}$$

```

A1a = Integrate[Integrate[x1 - x2, {x2, 0, x1}], {x1, 0, b1 / 2}]
A1b = Integrate[Integrate[a * x1 - (a - 1) * b1 / 2 - x2, {x2, 0, b1 / 2}], {x1, b1 / 2, b2}]
A1c = Integrate[Integrate[x1 - x2, {x2, b1 / 2, x1}], {x1, b1 / 2, b2}]
A1d = Integrate[Integrate[x1 - x2, {x2, b2, x1}], {x1, b2, b1}]
A1e = Expand[A1a + A1b + a * A1c + a * A1d]
A2 = D[A1, b2]
A3 = Solve[A2 == 0, b2]

```

$$\begin{aligned}
& \frac{b_1^3}{48} \\
& - \frac{b_1^3}{16} + \frac{a b_1^3}{16} + \frac{b_1^2 b_2}{8} - \frac{1}{4} a b_1^2 b_2 + \frac{1}{4} a b_1 b_2^2 \\
& - \frac{1}{48} (b_1 - 2 b_2)^3 \\
& \frac{b_1^3}{6} - \frac{b_1^2 b_2}{2} + \frac{b_1 b_2^2}{2} - \frac{b_2^3}{6} \\
& - \frac{b_1^3}{24} + \frac{5 a b_1^3}{24} + \frac{b_1^2 b_2}{8} - \frac{5}{8} a b_1^2 b_2 + \frac{1}{2} a b_1 b_2^2 \\
& - \frac{b_2 (b_1 + a b_2)}{b_1^2} + \frac{b_2 \left( \frac{b_1}{2} - \frac{a b_1}{2} + a b_2 \right)}{b_1^2} + \frac{a (b_1^2 - b_2^2)}{2 b_1^2} + \frac{-\frac{b_1^2}{8} + \frac{a b_1^2}{8} + \frac{b_1 b_2}{2} - \frac{a b_1 b_2}{2} + \frac{a b_2^2}{2}}{b_1^2} \\
& \left\{ \left\{ b_2 \rightarrow \frac{(-1 + 5 a) b_1}{8 a} \right\} \right\}
\end{aligned}$$

**4**

```

p2 = Integrate[1 / b1 - a, {x, 0, b1 / 2}] + Integrate[1 / b1 + a, {x, b1 / 2, b2}]
e2 = Integrate[x (1 / b1 - a), {x, 0, b1 / 2}] + Integrate[x (1 / b1 + a), {x, b1 / 2, b2}]
A1 = (1 - p2^2) * (b1 + b2) / 2 + p2^2 * e2 / p2
A2 = D[A1, b2]
A3 = Solve[A2 == 0, b2]

```

$$\frac{1}{2} \left( -a + \frac{1}{b1} \right) b1 + \left( a + \frac{1}{b1} \right) \left( -\frac{b1}{2} + b2 \right)$$

$$- \frac{a b1^2}{4} + \frac{a b2^2}{2} + \frac{b2^2}{2 b1}$$

$$\left( -\frac{a b1^2}{4} + \frac{a b2^2}{2} + \frac{b2^2}{2 b1} \right) \left( \frac{1}{2} \left( -a + \frac{1}{b1} \right) b1 + \left( a + \frac{1}{b1} \right) \left( -\frac{b1}{2} + b2 \right) \right) +$$

$$\frac{1}{2} (b1 + b2) \left( 1 - \left( \frac{1}{2} \left( -a + \frac{1}{b1} \right) b1 + \left( a + \frac{1}{b1} \right) \left( -\frac{b1}{2} + b2 \right) \right)^2 \right)$$

$$\left( a + \frac{1}{b1} \right) \left( -\frac{a b1^2}{4} + \frac{a b2^2}{2} + \frac{b2^2}{2 b1} \right) - \left( a + \frac{1}{b1} \right) (b1 + b2) \left( \frac{1}{2} \left( -a + \frac{1}{b1} \right) b1 + \left( a + \frac{1}{b1} \right) \left( -\frac{b1}{2} + b2 \right) \right) +$$

$$\left( a b2 + \frac{b2}{b1} \right) \left( \frac{1}{2} \left( -a + \frac{1}{b1} \right) b1 + \left( a + \frac{1}{b1} \right) \left( -\frac{b1}{2} + b2 \right) \right) + \frac{1}{2} \left( 1 - \left( \frac{1}{2} \left( -a + \frac{1}{b1} \right) b1 + \left( a + \frac{1}{b1} \right) \left( -\frac{b1}{2} + b2 \right) \right)^2 \right)$$

$$\left\{ \left\{ b2 \rightarrow \frac{1}{4} b1 (2 + a b1) \right\} \right\}$$

```

p1 = (1 / b1 - a2) ^ 2
p2 = (1 / b1 - a2) * (1 / b1 + a2)
p3 = (1 / b1 + a2) ^ 2
ii = (x1 - x2) / 2
A1a = Integrate[Integrate[ii * p1, {x2, 0, x1}], {x1, 0, b1 / 2}]
A1b = Integrate[Integrate[ii * p2, {x2, 0, b1 / 2}], {x1, b1 / 2, b2}]
A1c = Integrate[Integrate[ii * p3, {x2, b1 / 2, x1}], {x1, b1 / 2, b2}]
A1d = Integrate[Integrate[ii * p3, {x2, b2, x1}], {x1, b2, b1}]
A1 = A1a + A1b + A1c + A1d
A2 = D[A1, b2]
A3 = Solve[A2 == 0, b2]

```

$$\begin{aligned}
& \left( -a2 + \frac{1}{b1} \right)^2 \\
& \left( -a2 + \frac{1}{b1} \right) \left( a2 + \frac{1}{b1} \right) \\
& \left( a2 + \frac{1}{b1} \right)^2 \\
& \frac{x1 - x2}{2} \\
& \frac{b1}{96} - \frac{a2 b1^2}{48} + \frac{a2^2 b1^3}{96} \\
& - \frac{b2}{16} + \frac{1}{16} a2^2 b1^2 b2 + \frac{b2^2}{8 b1} - \frac{1}{8} a2^2 b1 b2^2 \\
& - \frac{(1 + a2 b1)^2 (b1 - 2 b2)^3}{96 b1^2} \\
& \frac{(1 + a2 b1)^2 (b1 - b2)^3}{12 b1^2} \\
& \frac{b1}{96} - \frac{a2 b1^2}{48} + \frac{a2^2 b1^3}{96} - \frac{(1 + a2 b1)^2 (b1 - 2 b2)^3}{96 b1^2} + \\
& \frac{(1 + a2 b1)^2 (b1 - b2)^3}{12 b1^2} - \frac{b2}{16} + \frac{1}{16} a2^2 b1^2 b2 + \frac{b2^2}{8 b1} - \frac{1}{8} a2^2 b1 b2^2 \\
& - \frac{1}{16} + \frac{a2^2 b1^2}{16} + \frac{(1 + a2 b1)^2 (b1 - 2 b2)^2}{16 b1^2} - \frac{(1 + a2 b1)^2 (b1 - b2)^2}{4 b1^2} + \frac{b2}{4 b1} - \frac{1}{4} a2^2 b1 b2 \\
& \left\{ \left\{ b2 \rightarrow \frac{1}{4} b1 (2 + a2 b1) \right\} \right\}
\end{aligned}$$

## 5.2

```
Integrate[Integrate[(x1 - x2) / 2, {x2, 0, x1}], {x1, 0, b}]
```

$$\frac{b^3}{12}$$

## 6.4

$$\text{Ph} = (b1^n - b2^n) / b1^n$$

$$\text{Pl} = b2^n / b1^n$$

$$\text{pil} = 1 / b2$$

$$\text{pih} = 1 / (b1 - b2)$$

$$\text{A1} = \text{Ph} * \text{pih} * \text{Integrate}[\text{Log2}[\text{pih}], \{x, b2, b1\}] + \text{Pl} * \text{pil} * \text{Integrate}[\text{Log2}[\text{pil}], \{x, 0, b2\}]$$

$$\text{A2} = \text{D}[\text{A1}, b2]$$

$$b1^{-n} (b1^n - b2^n)$$

$$b1^{-n} b2^n$$

$$\frac{1}{b2}$$

$$\frac{1}{b1 - b2}$$

$$\frac{b1^{-n} (b1^n - b2^n) \text{Log}\left[\frac{1}{b1-b2}\right]}{\text{Log}[2]} + \frac{b1^{-n} b2^n \text{Log}\left[\frac{1}{b2}\right]}{\text{Log}[2]}$$

$$-\frac{b1^{-n} b2^{-1+n}}{\text{Log}[2]} + \frac{b1^{-n} (b1^n - b2^n)}{(b1 - b2) \text{Log}[2]} - \frac{b1^{-n} b2^{-1+n} n \text{Log}\left[\frac{1}{b1-b2}\right]}{\text{Log}[2]} + \frac{b1^{-n} b2^{-1+n} n \text{Log}\left[\frac{1}{b2}\right]}{\text{Log}[2]}$$

$$\text{A1} = \text{Ph} * \text{pih} * \text{Integrate}[(x - (b1 + b2) / 2)^2, \{x, b2, b1\}] +$$

$$\text{Pl} * \text{pil} * \text{Integrate}[(x - b2 / 2)^2, \{x, 0, b2\}]$$

$$\text{A2} = \text{D}[\text{A1}, b2]$$

$$\text{A3} = \text{Simplify}[\text{A2} * 6 * b2 * b1^n]$$

$$\frac{1}{12} b1^{-n} b2^{2+n} + \frac{1}{12} b1^{-n} (b1 - b2)^2 (b1^n - b2^n)$$

$$-\frac{1}{6} b1^{-n} (b1 - b2) (b1^n - b2^n) - \frac{1}{12} b1^{-n} (b1 - b2)^2 b2^{-1+n} n + \frac{1}{12} b1^{-n} b2^{1+n} (2 + n)$$

$$-b1^{1+n} b2 + b1^n b2^2 - \frac{1}{2} b1^2 b2^n n + b1 b2^{1+n} (1 + n)$$

```

A1a = Integrate[b2 / 2 - x, {x, 0, b2 / 2}] + Integrate[x - b2 / 2, {x, b2 / 2, b2}]
A1b = Integrate[(b1 + b2) / 2 - x, {x, b2, (b1 + b2) / 2}] +
      Integrate[x - (b1 + b2) / 2, {x, (b1 + b2) / 2, b1}]
A1 = Pl * pil * A1a + Ph * pih * A1b
A2 = D[A1, b2]
A3 = Simplify[A2 * 4 * b2 * b1^n]

```

$$\begin{aligned}
& \frac{b2^2}{4} \\
& \frac{b1^2}{4} - \frac{b1 b2}{2} + \frac{b2^2}{4} \\
& \frac{1}{4} b1^{-n} b2^{1+n} + \frac{b1^{-n} \left( \frac{b1^2}{4} - \frac{b1 b2}{2} + \frac{b2^2}{4} \right) (b1^n - b2^n)}{b1 - b2} \\
& \frac{b1^{-n} \left( -\frac{b1}{2} + \frac{b2}{2} \right) (b1^n - b2^n)}{b1 - b2} + \frac{b1^{-n} \left( \frac{b1^2}{4} - \frac{b1 b2}{2} + \frac{b2^2}{4} \right) (b1^n - b2^n)}{(b1 - b2)^2} - \\
& \frac{b1^{-n} b2^{-1+n} \left( \frac{b1^2}{4} - \frac{b1 b2}{2} + \frac{b2^2}{4} \right) n}{b1 - b2} + \frac{1}{4} b1^{-n} b2^n (1 + n) \\
& - b1^n b2 - b1 b2^n n + 2 b2^{1+n} (1 + n)
\end{aligned}$$

```

Solve[6 * b2^2 - 2 * b1 * b2 - b1^2 == 0, b2]

```

$$\left\{ \left\{ b2 \rightarrow \frac{1}{6} \left( b1 - \sqrt{7} b1 \right) \right\}, \left\{ b2 \rightarrow \frac{1}{6} \left( b1 + \sqrt{7} b1 \right) \right\} \right\}$$