

2.2

$$A1 = \left((1-d)^3 + (2+2/3) * d (1-d)^2 + (1+2/3) * (1-d) * d^2 \right) * (b1+b2)/2 + \left((1/3) * d (1-d)^2 + (1+1/3) * (1-d) * d^2 + d^3 \right) * b2/2$$

$$A2 = \left((1-d)^3 + (1+1/3) * d (1-d)^2 + (1/3) * (1-d) * d^2 \right) * (b1+b2)/2 + \left((1+2/3) * d (1-d)^2 + (2+2/3) * (1-d) * d^2 + d^3 \right) * b2/2$$

$$R = (b1-b2)^3 * (b1+b2)/2 + 3 * (b1-b2)^2 * b2 * A1 + 3 * (b1-b2) * b2^2 * A2 + b2^4/2$$

Simplify[D[R, b2]/b1]

$$\frac{1}{2} (b1+b2) \left((1-d)^3 + \frac{8}{3} (1-d)^2 d + \frac{5}{3} (1-d) d^2 \right) + \frac{1}{2} b2 \left(\frac{1}{3} (1-d)^2 d + \frac{4}{3} (1-d) d^2 + d^3 \right)$$

$$\frac{1}{2} (b1+b2) \left((1-d)^3 + \frac{4}{3} (1-d)^2 d + \frac{1}{3} (1-d) d^2 \right) + \frac{1}{2} b2 \left(\frac{5}{3} (1-d)^2 d + \frac{8}{3} (1-d) d^2 + d^3 \right)$$

$$\frac{b2^4}{2} + \frac{1}{2} (b1-b2)^3 (b1+b2) +$$

$$3 (b1-b2)^2 b2 \left(\frac{1}{2} (b1+b2) \left((1-d)^3 + \frac{8}{3} (1-d)^2 d + \frac{5}{3} (1-d) d^2 \right) + \frac{1}{2} b2 \left(\frac{1}{3} (1-d)^2 d + \frac{4}{3} (1-d) d^2 + d^3 \right) \right) +$$

$$3 (b1-b2) b2^2 \left(\frac{1}{2} (b1+b2) \left((1-d)^3 + \frac{4}{3} (1-d)^2 d + \frac{1}{3} (1-d) d^2 \right) + \frac{1}{2} b2 \left(\frac{5}{3} (1-d)^2 d + \frac{8}{3} (1-d) d^2 + d^3 \right) \right)$$

$$- \frac{1}{2} (-1+2d) (-6b1b2d + b1^2(1+d) + 3b2^2(-1+2d))$$

$$N[1/\sqrt{3}]$$

$$NSolve[-.3 b2 + 1.05 + -2.7 * b2^2 == 0, b2]$$

$$0.57735$$

$$\{\{b2 \rightarrow -0.681635\}, \{b2 \rightarrow 0.570524\}\}$$

3.1

$$Hc = \text{Integrate}[\text{Log2}[1/(b3-b4)], \{x, b4, b3\}]/b1 + \text{Integrate}[\text{Log2}[1/(b2-b3)], \{x, b3, b2\}]/b1$$

$$dHc = D[Hc, b3]$$

$$\text{Solve}[dHc == 0, b3]$$

$$\frac{(b2-b3) \text{Log}\left[\frac{1}{b2-b3}\right]}{b1 \text{Log}[2]} + \frac{(b3-b4) \text{Log}\left[\frac{1}{b3-b4}\right]}{b1 \text{Log}[2]}$$

$$- \frac{\text{Log}\left[\frac{1}{b2-b3}\right]}{b1 \text{Log}[2]} + \frac{\text{Log}\left[\frac{1}{b3-b4}\right]}{b1 \text{Log}[2]}$$

$$\left\{ \left\{ b3 \rightarrow \frac{b2+b4}{2} \right\} \right\}$$

4.1

$$A1a = \text{Integrate}\left[x (1 - a) / b1, \{x, 0, b1 / 2\}\right] + \text{Integrate}\left[x (1 + a) / b1, \{x, b1 / 2, b2\}\right]$$

$$A1b = \text{Integrate}\left[(1 - a) / b1, \{x, 0, b1 / 2\}\right] + \text{Integrate}\left[(1 + a) / b1, \{x, b1 / 2, b2\}\right]$$

$$R = (1 - A1b^2) * (b2 + b1) / 2 + A1a * A1b$$

$$dR = D[R, b2]$$

$$\text{Solve}[dR == 0, b2]$$

$$-\frac{a b1}{4} + \frac{b2^2}{2 b1} + \frac{a b2^2}{2 b1}$$

$$\frac{1 - a}{2} + \frac{(1 + a) \left(-\frac{b1}{2} + b2\right)}{b1}$$

$$\left(-\frac{a b1}{4} + \frac{b2^2}{2 b1} + \frac{a b2^2}{2 b1}\right) \left(\frac{1 - a}{2} + \frac{(1 + a) \left(-\frac{b1}{2} + b2\right)}{b1}\right) + \frac{1}{2} (b1 + b2) \left(1 - \left(\frac{1 - a}{2} + \frac{(1 + a) \left(-\frac{b1}{2} + b2\right)}{b1}\right)^2\right)$$

$$\frac{(1 + a) \left(-\frac{a b1}{4} + \frac{b2^2}{2 b1} + \frac{a b2^2}{2 b1}\right)}{b1} - \frac{(1 + a) (b1 + b2) \left(\frac{1 - a}{2} + \frac{(1 + a) \left(-\frac{b1}{2} + b2\right)}{b1}\right)}{b1} +$$

$$\left(\frac{b2}{b1} + \frac{a b2}{b1}\right) \left(\frac{1 - a}{2} + \frac{(1 + a) \left(-\frac{b1}{2} + b2\right)}{b1}\right) + \frac{1}{2} \left(1 - \left(\frac{1 - a}{2} + \frac{(1 + a) \left(-\frac{b1}{2} + b2\right)}{b1}\right)^2\right)$$

$$\left\{\left\{b2 \rightarrow \frac{1}{4} (2 + a) b1\right\}\right\}$$

4.2

$$\text{Solve}[(b1 - b2) (1 + a) == .5 b1, b2]$$

$$\text{Solve}[2 a (b1 - b2) + b1 - 2 b2 == 0, b2]$$

$$\left\{\left\{b2 \rightarrow -\frac{1. (0.5 b1 - 1. (1. + a) b1)}{1. + a}\right\}\right\}$$

$$\left\{\left\{b2 \rightarrow \frac{b1 + 2 a b1}{2 (1 + a)}\right\}\right\}$$

5.2

Find B3/B2

$$c1 = 10^{(1/9)}$$

$$A1 = \text{Function}[\{b2\}, (b2^{10} - (b3/c1)^{10}) / (b2 - b3/c1) - (c1 * b3)^9]$$

$$A2 = A1[1]$$

$$\text{NSolve}[A2 == 0, b3]$$

$$10^{1/9}$$

$$\text{Function}[\{b2\}, \frac{b2^{10} - \left(\frac{b3}{c1}\right)^{10}}{b2 - \frac{b3}{c1}} - (c1 b3)^9]$$

$$-10 b3^9 + \frac{1 - \frac{b3^{10}}{10 \cdot 10^{1/9}}}{1 - \frac{b3}{10^{1/9}}}$$

Solve::verif : Potential solution {b3 → 1.291549665014883875410075546472} (possibly discarded by verifier)
should be checked by hand. May require use of limits. >>

$$\{\{b3 \rightarrow -0.696554 - 0.242985 i\}, \{b3 \rightarrow -0.696554 + 0.242985 i\}, \{b3 \rightarrow -0.398351 - 0.631118 i\}, \\ \{b3 \rightarrow -0.398351 + 0.631118 i\}, \{b3 \rightarrow 0.0859914 - 0.761163 i\}, \{b3 \rightarrow 0.0859914 + 0.761163 i\}, \\ \{b3 \rightarrow 0.577308 - 0.561078 i\}, \{b3 \rightarrow 0.577308 + 0.561078 i\}, \{b3 \rightarrow 0.876259\}\}$$

Find B2

$$c1 = .876$$

$$A1 = \text{Function}[\{b1\}, (b1^{10} - (b2 * c1)^{10}) / (b1 - b2 * c1) - 10 * b2^9]$$

$$A2 = A1[1]$$

$$\text{NSolve}[A2 == 0, b2]$$

$$0.876$$

$$\text{Function}[\{b1\}, \frac{b1^{10} - (b2 c1)^{10}}{b1 - b2 c1} - 10 b2^9]$$

$$-10 b2^9 + \frac{1 - 0.266098 b2^{10}}{1 - 0.876 b2}$$

$$\{\{b2 \rightarrow 0.914725\}, \{b2 \rightarrow 0.571269 + 0.567068 i\}, \{b2 \rightarrow 0.571269 - 0.567068 i\}, \\ \{b2 \rightarrow 0.081705 + 0.75927 i\}, \{b2 \rightarrow 0.081705 - 0.75927 i\}, \{b2 \rightarrow -0.398518 + 0.627648 i\}, \\ \{b2 \rightarrow -0.398518 - 0.627648 i\}, \{b2 \rightarrow -0.693937 + 0.241426 i\}, \{b2 \rightarrow -0.693937 - 0.241426 i\}\}$$

Find B3 and B4

$$A1 = c1 * .914725$$

$$A2 = A1 / \sqrt[9]{10}$$

$$0.801299$$

$$0.620417$$

5.3

$$A1a = (b1 - b2)^{10}$$

$$A1b = (b1^{10} - b2^{10}) * (b1 + b2) + (b2^{10} - (b1/2)^{10}) * (b2 + b1/2)$$

$$A1c = ((b1/2)^{10} - A1a) * (3 b1/2 - b2) + A1a * (b1 - b2)$$

$$R = A1b + A1c$$

$$dR = D[R, b2]$$

$$(b1 - b2)^{10}$$

$$(b1 + b2) (b1^{10} - b2^{10}) + \left(\frac{b1}{2} + b2 \right) \left(-\frac{b1^{10}}{1024} + b2^{10} \right)$$

$$(b1 - b2)^{11} + \left(\frac{b1^{10}}{1024} - (b1 - b2)^{10} \right) \left(\frac{3 b1}{2} - b2 \right)$$

$$(b1 - b2)^{11} + \left(\frac{b1^{10}}{1024} - (b1 - b2)^{10} \right) \left(\frac{3 b1}{2} - b2 \right) + (b1 + b2) (b1^{10} - b2^{10}) + \left(\frac{b1}{2} + b2 \right) \left(-\frac{b1^{10}}{1024} + b2^{10} \right)$$

$$\frac{511 b1^{10}}{512} - 10 (b1 - b2)^{10} + 10 (b1 - b2)^9 \left(\frac{3 b1}{2} - b2 \right) + 10 b2^9 \left(\frac{b1}{2} + b2 \right) - 10 b2^9 (b1 + b2)$$

$$A2 = \text{Function}[\{b1\},$$

$$\frac{511 b1^{10}}{512} - 10 (b1 - b2)^{10} + 10 (b1 - b2)^9 \left(\frac{3 b1}{2} - b2 \right) + 10 b2^9 \left(\frac{b1}{2} + b2 \right) - 10 b2^9 (b1 + b2)]$$

$$A3 = A2[1]$$

$$\text{NSolve}[A3 == 0, b2]$$

$$\text{Function}[\{b1\}, \frac{511 b1^{10}}{512} - 10 (b1 - b2)^{10} + 10 (b1 - b2)^9 \left(\frac{3 b1}{2} - b2 \right) + 10 b2^9 \left(\frac{b1}{2} + b2 \right) - 10 b2^9 (b1 + b2)]$$

$$\frac{511}{512} - 10 (1 - b2)^{10} + 10 (1 - b2)^9 \left(\frac{3}{2} - b2 \right) + 10 b2^9 \left(\frac{1}{2} + b2 \right) - 10 b2^9 (1 + b2)$$

$$\{\{b2 \rightarrow 0.214345 - 0.285996 i\}, \{b2 \rightarrow 0.214345 + 0.285996 i\}, \{b2 \rightarrow 0.478591 - 0.869207 i\}, \\ \{b2 \rightarrow 0.478591 + 0.869207 i\}, \{b2 \rightarrow 0.500014 - 2.83564 i\}, \{b2 \rightarrow 0.500014 + 2.83564 i\}, \\ \{b2 \rightarrow 0.639015 - 0.528847 i\}, \{b2 \rightarrow 0.639015 + 0.528847 i\}, \{b2 \rightarrow 0.836069\}\}$$

5.4

$$\text{Integrate}[x, \{x, .85 b1, b1\}] + \text{Integrate}[x, \{x, b2 + .15 b1, b1\}]$$

$$0.6275 b1^2 - 0.15 b1 b2 - 0.5 b2^2$$

$$A1b = (b1^{10} - b2^{10}) * (b1 + b2) + (b2^{10} - (b1/2)^{10}) * (b2 + b1/2)$$

$$A1c = ((b1/2)^{10} - A1a) * (3 b1/2 - b2) + A1a * (b1 - b2)$$

$$A1d = 2 (b1^{10} - b2^{10}) * (0.6275 b1^2 - 0.15 b1 * b2 - 0.5 b2^2) / (b1 - b2) + (b2^{10} - (b1/2)^{10}) * (b2 + b1/2 + .15 b1)$$

$$A1e = ((b1/2)^{10} - A1a) * (3 b1/2 - b2 + .15 b1) + A1a * (b1 - b2 + .15 b1)$$

$$R = A1d + A1e$$

$$dR = D[R, b2]$$

$$(b1 + b2) (b1^{10} - b2^{10}) + \left(\frac{b1}{2} + b2 \right) \left(-\frac{b1^{10}}{1024} + b2^{10} \right)$$

$$(b1 - b2)^{11} + \left(\frac{b1^{10}}{1024} - (b1 - b2)^{10} \right) \left(\frac{3 b1}{2} - b2 \right)$$

$$\frac{2 (0.6275 b1^2 - 0.15 b1 b2 - 0.5 b2^2) (b1^{10} - b2^{10})}{b1 - b2} + (0.65 b1 + b2) \left(-\frac{b1^{10}}{1024} + b2^{10} \right)$$

$$(b1 - b2)^{10} (1.15 b1 - b2) + \left(\frac{b1^{10}}{1024} - (b1 - b2)^{10} \right) (1.65 b1 - b2)$$

$$(b1 - b2)^{10} (1.15 b1 - b2) + \left(\frac{b1^{10}}{1024} - (b1 - b2)^{10} \right) (1.65 b1 - b2) +$$

$$\frac{2 (0.6275 b1^2 - 0.15 b1 b2 - 0.5 b2^2) (b1^{10} - b2^{10})}{b1 - b2} + (0.65 b1 + b2) \left(-\frac{b1^{10}}{1024} + b2^{10} \right)$$

$$- \frac{b1^{10}}{512} - 10 (b1 - b2)^9 (1.15 b1 - b2) + 10 (b1 - b2)^9 (1.65 b1 - b2) +$$

$$b2^{10} + 10 b2^9 (0.65 b1 + b2) - \frac{20 b2^9 (0.6275 b1^2 - 0.15 b1 b2 - 0.5 b2^2)}{b1 - b2} +$$

$$\frac{2 (-0.15 b1 - 1. b2) (b1^{10} - b2^{10})}{b1 - b2} + \frac{2 (0.6275 b1^2 - 0.15 b1 b2 - 0.5 b2^2) (b1^{10} - b2^{10})}{(b1 - b2)^2}$$

$$A2 = \text{Function}\left[\{b1\}, -\frac{b1^{10}}{512} - 10(b1 - b2)^9(1.15 - b1 - b2) + 10(b1 - b2)^9(1.65 - b1 - b2) + \right. \\ \left. b2^{10} + 10b2^9(0.65 - b1 + b2) - \frac{20b2^9(0.6275 - b1^2 - 0.15 - b1b2 - 0.5 - b2^2)}{b1 - b2} + \right. \\ \left. \frac{2(-0.15 - b1 - 1. - b2)(b1^{10} - b2^{10})}{b1 - b2} + \frac{2(0.6275 - b1^2 - 0.15 - b1b2 - 0.5 - b2^2)(b1^{10} - b2^{10})}{(b1 - b2)^2} \right]$$

$$A3 = A2[1]$$

$$\text{NSolve}[A3 == 0, b2]$$

$$\text{Function}\left[\{b1\}, -\frac{b1^{10}}{512} - 10(b1 - b2)^9(1.15 - b1 - b2) + 10(b1 - b2)^9(1.65 - b1 - b2) + \right. \\ \left. b2^{10} + 10b2^9(0.65 - b1 + b2) - \frac{20b2^9(0.6275 - b1^2 - 0.15 - b1b2 - 0.5 - b2^2)}{b1 - b2} + \right. \\ \left. \frac{2(-0.15 - b1 - 1. - b2)(b1^{10} - b2^{10})}{b1 - b2} + \frac{2(0.6275 - b1^2 - 0.15 - b1b2 - 0.5 - b2^2)(b1^{10} - b2^{10})}{(b1 - b2)^2} \right] \\ - \frac{1}{512} - 10(1 - b2)^9(1.15 - b2) + 10(1 - b2)^9(1.65 - b2) + \\ b2^{10} + 10b2^9(0.65 + b2) - \frac{20b2^9(0.6275 - 0.15 - b2 - 0.5 - b2^2)}{1 - b2} + \\ \frac{2(-0.15 - 1. - b2)(1 - b2^{10})}{1 - b2} + \frac{2(0.6275 - 0.15 - b2 - 0.5 - b2^2)(1 - b2^{10})}{(1 - b2)^2} \\ \left\{ \{b2 \rightarrow 0.260848 + 2.8356 i\}, \{b2 \rightarrow 0.260848 - 2.8356 i\}, \{b2 \rightarrow 0.455793 + 0.877287 i\}, \right. \\ \left. \{b2 \rightarrow 0.455793 - 0.877287 i\}, \{b2 \rightarrow 0.629203 + 0.51929 i\}, \{b2 \rightarrow 0.629203 - 0.51929 i\}, \right. \\ \left. \{b2 \rightarrow 0.745934\}, \{b2 \rightarrow 0.220102 + 0.288306 i\}, \{b2 \rightarrow 0.220102 - 0.288306 i\} \right\}$$

7

$$R = (b1^2 - b2^2)(b1 + b2)/2 + b2((b2 - 10)(10 + (b2 - 10)/2) - 3000)$$

$$dR = D[R, b2]$$

$$\text{Solve}[dR == 0, b2]$$

$$\left(-3000 + \left(10 + \frac{1}{2}(-10 + b2)\right)(-10 + b2)\right)b2 + \frac{1}{2}(b1 + b2)(b1^2 - b2^2) \\ - 3000 + \left(10 + \frac{1}{2}(-10 + b2)\right)(-10 + b2) + b2^2 - b2(b1 + b2) + \frac{1}{2}(b1^2 - b2^2) \\ \left\{ \left\{ b2 \rightarrow \frac{-6100 + b1^2}{2b1} \right\} \right\}$$

```

Rnum =
  Function[{b1, b2}, ((b1^2 - b2^2) (b1 + b2) / 2 + b2 ((b2 - 10) (10 + (b2 - 10) / 2) - 3000)) / 10^4]
Rnum[100, 19.5]
N[Rnum[100, 10]]

Function[{b1, b2},  $\frac{\frac{1}{2} (b1^2 - b2^2) (b1 + b2) + b2 ((b2 - 10) (10 + \frac{b2-10}{2}) - 3000)}{10^4}$ ]

51.9013

51.45

```

8

```

In[9]:= R22 = P / 2 + 11 * 21 / 32 * (1 - P)
R12 = 5 / 8 * (P + 11 * (1 - P))
R11 = 21 / 32 * P + 11 * (1 - P) / 2

Out[9]=

$$\frac{231 (1 - P)}{32} + \frac{P}{2}$$


Out[10]=

$$\frac{5}{8} (11 (1 - P) + P)$$


Out[11]=

$$\frac{11 (1 - P)}{2} + \frac{21 P}{32}$$


In[12]:= Solve[R22 == R12, P]
Solve[R11 == R12, P]
Solve[R11 == R22, P]

Out[12]=

$$\left\{ \left\{ P \rightarrow \frac{11}{15} \right\} \right\}$$


Out[13]=

$$\left\{ \left\{ P \rightarrow \frac{44}{45} \right\} \right\}$$


Out[14]=

$$\left\{ \left\{ P \rightarrow \frac{11}{12} \right\} \right\}$$


```