Exercício

a) A: $T(-1,-1) \to (-4,-4)$

B: $T(1,-1) \to (2,-2)$

I. $(x,y) = a(A) + b(B)$

II. $T(x,y) = aT(A) + bT(B)$

I.) $(x,y) = a(-1,-1) + b(1,-1)$

$$+\begin{cases} -a + b = x \\ -a - b = y \end{cases}$$

$$-2a = x + y$$

$$\boxed{a = \frac{x+y}{-2}}$$

$$\frac{x+y}{2} + b = x \qquad b = x - \frac{x+y}{2}$$

$$b = \frac{2x - x + y}{2} \qquad \boxed{b = \frac{x-y}{2}}$$

II) $T(x,y) = \left(\frac{x+y}{-2}\right)(-4,-4) + \left(\frac{x-y}{2}\right)(2,-2)$

$$T(x,y) = \left(\frac{-4x-4y}{-2} + \frac{2x-2y}{2}, \frac{-4x-4y}{-2} + \frac{-2x+2y}{2}\right)$$

$$T(x,y) = \left(\frac{4x+4y+2x-2y}{2}, \frac{4x+4y-2x+2y}{2}\right)$$

$$T(x,y) = \left(\frac{6x+2y}{2}, \frac{2x+6y}{2}\right) \qquad T(x,y) = \left(\frac{2(3x+y)}{2}, \frac{2(x+3y)}{2}\right)$$

$$T(x,y) = (3x+y; x+3y)$$

TESTANDO A EQUAÇÃO NOS PONTOS

$T(x,y) = (3x + y,\ x + 3y)$

A: $T(-1,-1) = (3(-1) + (-1),\ -1 + 3(-1))$

$T(-1,-1) = (-3-1,\ -1-3)$

$T(-1,-1) = (-4,-4)$

B: $T(1,-1) = (3-1,\ 1-3)$

$T(1,-1) = (2,-2)$

C: $T(1,1) = (3+1,\ 1+3)$

$T(1,1) = (4,4)$

d: $T(1.5,1) = (4.5+1,\ 1.5+3)$

$T(1.5,1) = (5.5,\ 4.5$

e: $T(0,2) = (2,6)$

F: $T(-1.5,1) = (3(-1.5)+1, -1.5+3)$

$T(-1.5,1) = (-3.5,\ 1.5)$

G: $T(-1,1) = (-3+1,\ -1+3)$

$T(-1,1) = (-2,2)$

b) I) $S: \mathbb{R}^2 \to M_{(2x2)}$ $S(x,y) = \begin{bmatrix} x-y & 2y \\ 2y-x & x \end{bmatrix}$

$T(x,y) = (3x+y,\ x+3y)$

$(S \circ T)(x,y) = S(T(x,y))$

$(S \circ T)(x,y) = S(3x+y,\ x+3y)$

$(S \circ T)(x,y) = \begin{bmatrix} 3x+y-x-3y & 2x+6y \\ 2x+6y-3x-y & 3x+y \end{bmatrix}$

$(S \circ T)(x,y) = \begin{bmatrix} 2x-2y & 2x+6y \\ -x+5y & 3x+y \end{bmatrix}$

$$(S \circ T)(x,y) = \begin{bmatrix} 2x-2y & 2x+6y \\ -x+5y & 3x+y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{I}$$

Verificando se são LI's

$$x\begin{bmatrix} 2 & 2 \\ -1 & 3 \end{bmatrix} + y\begin{bmatrix} -2 & 6 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{cases} 2x-2y=0 \\ 2x+6y=0 \\ -x+5y=0 \\ 3x+y=0 \end{cases}$$

$2x = 2y \quad \boxed{x=y}$ I

$4y+6y=0 \quad 10y=0 \quad \boxed{y=0}$ II

Substituindo II em I $\boxed{x=0}$

Como x e y são 0, logo são LI's

Como $\dim \mathbb{R}^2 = \dim N(S \circ T) + \dim Im(S \circ T)$

$$2 = 0 + 2$$

Logo o núcleo do domínio será o vetor nulo do $\mathbb{R}^2$

E a imagem será a própria matriz transformação

A dimensão do núcleo é 0 e da imagem é 2

$$\begin{bmatrix} 2 & -2 \\ 2 & 6 \\ -1 & 5 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x-2y \\ 2x+6y \\ -x+5y \\ 3x+y \end{bmatrix}$$

↳ matriz transformação do $\mathbb{R}^2 \to M_{2\times2}$

II

(111)

$\alpha = \{(-1,1), (1,1)\}$

$\qquad\quad\ \uparrow u_1 \qquad\ \ \uparrow u_2$

$\beta = \left\{ \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix} \right\}$

* Transformação do α para matriz utilizando a regra $(S \circ T)(x,y) = \begin{bmatrix} 2x-2y & 2x+6y \\ -x+5y & 3x+y \end{bmatrix}$

$\alpha: (-1,1) \rightarrow \begin{bmatrix} -4 & 4 \\ 6 & -2 \end{bmatrix} \leftarrow u_1$

$\alpha: (1,1) \rightarrow \begin{bmatrix} 0 & 8 \\ 4 & 4 \end{bmatrix} \leftarrow u_2$

$$[S \circ T]_{\beta}^{\alpha} \rightarrow u_1 = a_{11}(w_1) + a_{21}(w_2) + \dots$$

$$u_2 = a_{12}(w_1) + a_{22}(w_2) + \dots$$

$$\begin{bmatrix} -4 & 4 \\ 6 & 2 \end{bmatrix} = a\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + d\begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix}$$

$$\begin{cases} a + d = -4 \\ -a - b + 2d = 4 \\ b + c + 2d = 6 \\ -3d = -2 \end{cases}$$

$\boxed{d = \frac{2}{3}}$

$a + \frac{2}{3} = -4 \quad a = -4 - \frac{2}{3} \quad a = \frac{-12-2}{3}$

$\boxed{a = \frac{-14}{3}}$

$2 + c + \frac{4}{3} = 6$

$c + \frac{4}{3} = 4$

$c = 4 - \frac{4}{3}$

$c = \frac{12-4}{3}$

$\boxed{c = \frac{8}{3}}$

$\frac{10}{3} - b + \frac{4}{3} = 4$

$b = \frac{10}{3} + \frac{4}{3} - 4$

$b = \frac{10+4-12}{3}$

$b = \frac{6}{3} \quad \boxed{b = 2}$

$$\begin{bmatrix} 0 & 8 \\ 4 & 4 \end{bmatrix} = e\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} + F\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} + G\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + h\begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix}$$

$$\begin{cases} e + h = 0 \\ -e - F + 2h = 8 \\ F + G + 2h = 4 \\ -3h = 4 \end{cases}$$

$\rightarrow e - \frac{4}{3} = 0 \quad \boxed{e = \frac{4}{3}}$

$-3h = 4 \rightarrow \boxed{h = -\frac{4}{3}}$

$$-\frac{4}{3} - F - \frac{8}{3} = 8$$

$$F = -8 - \frac{4}{3} - \frac{8}{3}$$

$$F = \frac{-24-4-8}{3}$$

$$F = \frac{-36}{3}$$

$$\boxed{F = -12}$$

$$-12 + G - \frac{8}{3} = 4$$

$$G = 4 + 12 + \frac{8}{3}$$

$$G = \frac{12+36+8}{3}$$

$$\boxed{G = \frac{56}{3}}$$

$$[S \circ T]_{\beta}^{\alpha} = \begin{bmatrix} a & e \\ b & F \\ c & G \\ d & H \end{bmatrix}$$

$$[S \circ T]_{\beta}^{\alpha} = \begin{bmatrix} -14/3 & 4/3 \\ 2 & -12 \\ 8/3 & 56/3 \\ 2/3 & -4/3 \end{bmatrix}$$