Questão 8

1) $\cosh^2 x - \sinh^2 x = 1$ → Mostrar

$$\underbrace{\left(\frac{1}{2}(e^x + e^{-x})\right)^2}_{\cosh^2} - \underbrace{\left(\frac{1}{2}(e^x - e^{-x})\right)^2}_{\sinh^2}$$

$$= \left(\frac{1}{2}\right)^2 (e^x + e^{-x})^2 - \left(\frac{1}{2}\right)^2 (e^x - e^{-x})^2$$

$$= \frac{1}{4}\left(e^x(1+e^{-2x})\right)^2 - \frac{1}{4}\left(e^x(1-e^{-2x})\right)^2$$

$$= \frac{1}{4}\left((e^x)^2(1+e^{-2x})^2\right) - \frac{1}{4}\left((e^x)^2(1-e^{-2x})^2\right)$$

$$= \frac{1}{4}\left(e^{2x}(1+e^{-2x})^2\right) - \frac{1}{4}\left(e^{2x}(1-e^{-2x})^2\right)$$

$$= \frac{e^{2x}(1+e^{-2x})^2}{4} - \frac{e^{2x}(1-e^{-2x})^2}{4}$$

$$\frac{e^{2x}(1+2e^{-2x}+e^{-4x^2}) - e^{2x}(1-2e^{-2x}+e^{-4x^2})}{4}$$

$$\frac{(e^{2x}+2e^{-2x+2x}+e^{-4x^2+2x}) - (e^{2x}-2e^{-2x+2x}+e^{-4x^2+2x})}{4}$$

$$\frac{\cancel{e^{2x}}+2e^{-2x+2x}+\cancel{e^{-4x^2+2x}}-\cancel{e^{2x}}+2e^{-2x+2x}-\cancel{e^{-4x^2+2x}}}{4}$$

$$\frac{2+2}{4}=\frac{4}{4}=1$$

b) Mostrar que $\sinh(\cosh^{-1}(x)) = \sqrt{x^2-1} \quad \forall\, x \in [1,+\infty)$

Para qualquer $f(x)$

$\sinh^2(f(x)) = \cosh^2(f(x)) - 1 \quad \forall\, x \in \mathrm{Dom}(f)$

Então

$\sinh^2(\cosh^{-1}(x)) = \cosh^2(\cosh^{-1}(x)) - 1$

$\sinh^2(\cosh^{-1}(x)) = (\cancel{\cosh}(\cancel{\cosh^{-1}}(x))^2 - 1$

$\sinh^2(\cosh^{-1}(x)) = x^2 - 1$

$(\sinh(\cosh^{-1}(x))^2 = x^2 - 1$

$\sinh(\cosh^{-1}(x)) = \pm\sqrt{x^2-1}$. Como o domínio é $[1,+\infty)$, a solução será apenas a parte positiva ou seja $\sinh(\cosh^{-1}(x)) = \sqrt{x^2-1}$