Questão 8

1) $\cosh^2 x - \sinh^2 x = 1$ → mostrar

$$\left(\frac{1}{2}(e^x + e^{-x})\right)^2 - \left(\frac{1}{2}(e^x - e^{-x})\right)^2$$

↳ $\cosh^2$ ↳ $\sinh^2$

$$= \left(\frac{1}{2}\right)^2 (e^x + e^{-x})^2 - \left(\frac{1}{2}\right)^2 (e^x - e^{-x})^2$$

$$= \frac{1}{4}\left(e^x(1+e^{-2x})\right)^2 - \frac{1}{4}\left(e^x(1-e^{-2x})\right)^2$$

$$= \frac{1}{4}\left((e^x)^2(1+e^{-2x})^2\right) - \frac{1}{4}\left((e^x)^2(1-e^{-2x})^2\right)$$

$$= \frac{1}{4}\left(e^{2x}(1+e^{-2x})^2\right) - \frac{1}{4}\left(e^{2x}(1-e^{-2x})^2\right)$$

$$= \frac{e^{2x}(1+e^{-2x})^2}{4} - \frac{e^{2x}(1-e^{-2x})^2}{4}$$

$$\frac{e^{2x}(1+2e^{-2x}+e^{-4x^2}) - e^{2x}(1-2e^{-2x}+e^{-4x^2})}{4}$$

$$\frac{(e^{2x}+2e^{-2x+2x}+e^{-4x^2+2x}) - (e^{2x}-2e^{-2x+2x}+e^{-4x^2+2x})}{4}$$

$$\frac{\cancel{e^{2x}}+2e^{-2x+2x}+e^{-4x^2+2x}-\cancel{e^{2x}}+2e^{-2x+2x}-e^{-4x^2+2x}}{4}$$

$$\frac{2+2}{4} = \frac{4}{4} = 1$$

b) Mostrar que $\cosh(\sin^{-1}(x)) = \sqrt{x^2+1}$

Para qualquer função $f(x)$:

$\cosh^2(f(x)) = \sinh^2(f(x)) + 1$

Então

$\cosh^2(\sinh^{-1}(x)) = \sinh^2(\sinh^{-1}(x)) + 1$

$\cosh^2(\sinh^{-1}(x)) = (\cancel{\sinh}(\cancel{\sinh^{-1}}(x))^2 + 1$

$\cosh^2(\sinh^{-1}(x)) = x^2 + 1$

$\rightarrow \cosh^2(f(x)) = x^2 + 1$

$(\cosh(f(x)))^2 = x^2 + 1$

$\cosh(f(x)) = \sqrt{x^2+1}$