

$$4) M(2,2) \quad U = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M(2 \times 2) \mid 2a + 3b + d = 0 \text{ e } a + b + c + d = 0 \right\}$$

$$W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M(2 \times 2) \mid 2a + b + d = 0 \right\}$$

Seja $A \in U \cap W$, deve satisfazer

$$\begin{cases} 2a + 3b + d = 0 \\ a + b + c + d = 0 \\ 2a + b + d = 0 \end{cases}$$

$$\begin{bmatrix} 2 & 3 & 0 & 1 & | & 0 \\ 1 & 1 & 1 & 1 & | & 0 \\ 2 & 1 & 0 & 1 & | & 0 \end{bmatrix}$$

$$\xrightarrow{L_2 \leftrightarrow L_1} \begin{bmatrix} 1 & 1 & 1 & 1 & | & 0 \\ 2 & 3 & 0 & 1 & | & 0 \\ 2 & 1 & 0 & 1 & | & 0 \end{bmatrix} \xrightarrow{\begin{matrix} L_2 \rightarrow L_2 - 2L_1 \\ L_3 \rightarrow L_3 - 2L_1 \end{matrix}}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & | & 0 \\ 0 & 1 & -2 & -1 & | & 0 \\ 0 & -1 & -2 & -1 & | & 0 \end{bmatrix}$$

$$\xrightarrow{L_3 \rightarrow L_3 + L_2} \begin{bmatrix} 1 & 1 & 1 & 1 & | & 0 \\ 0 & 1 & -2 & -1 & | & 0 \\ 0 & 0 & -4 & -2 & | & 0 \end{bmatrix}$$

$$\xrightarrow{L_3 \rightarrow -\frac{1}{4}L_3} \begin{bmatrix} 1 & 1 & 1 & 1 & | & 0 \\ 0 & 1 & -2 & -1 & | & 0 \\ 0 & 0 & 1 & \frac{1}{2} & | & 0 \end{bmatrix}$$

$$\begin{cases} a + b + c + d = 0 \\ b - d = 0 \\ c + \frac{1}{2}d = 0 \end{cases}$$

$$\boxed{c = -\frac{1}{2}d}$$

$$\boxed{b = d}$$

$$a + d - \frac{1}{2}d + d = 0$$

$$\frac{2a + 2d - 1d + 2d}{2} = 0$$

$$2a - d = 0$$

$$2a = d$$

$$\boxed{a = \frac{d}{2}}$$

$$\frac{2a - d}{2} = 0$$