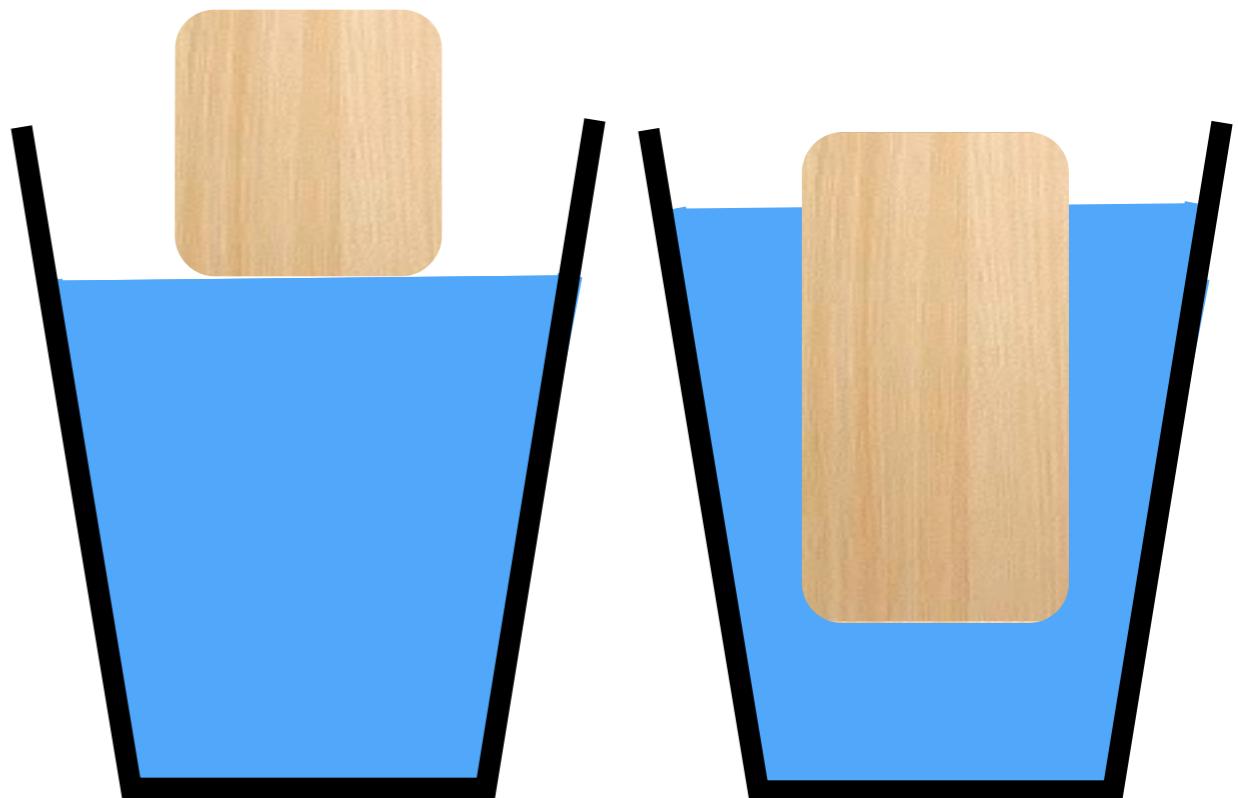
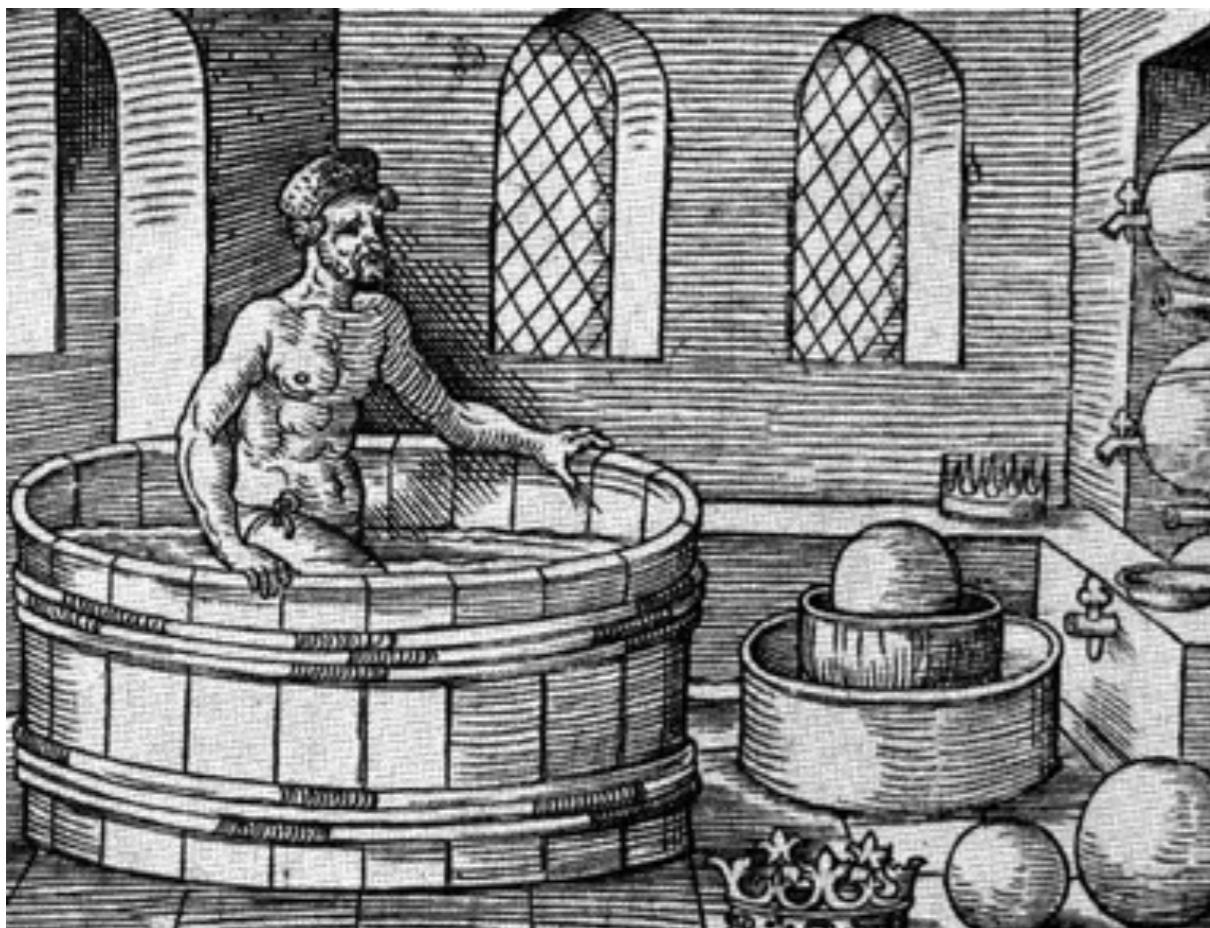


Modelos quantitativos de bacias sedimentares

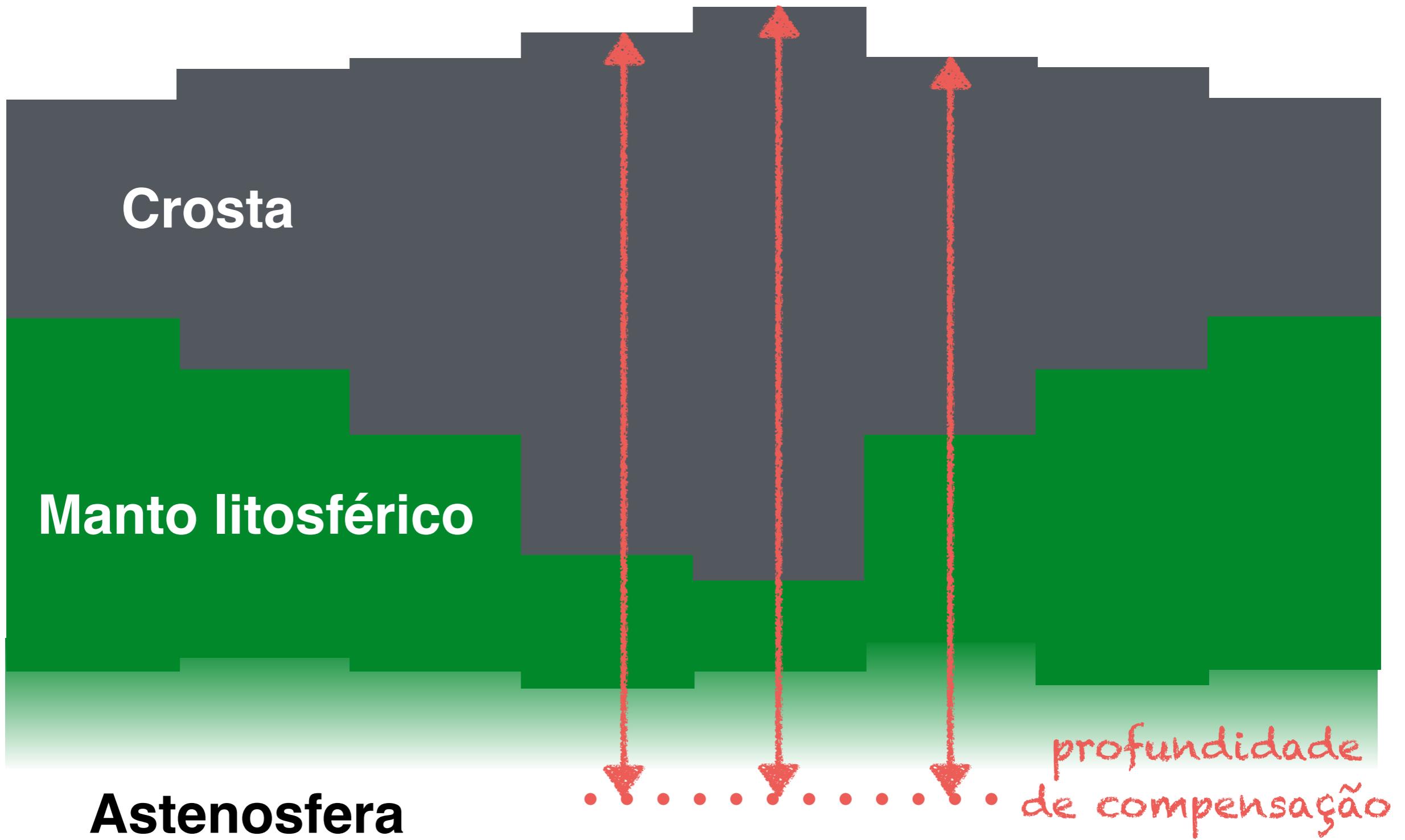
Isostasia e Flexura da Litosfera

Princípio de Arquimedes

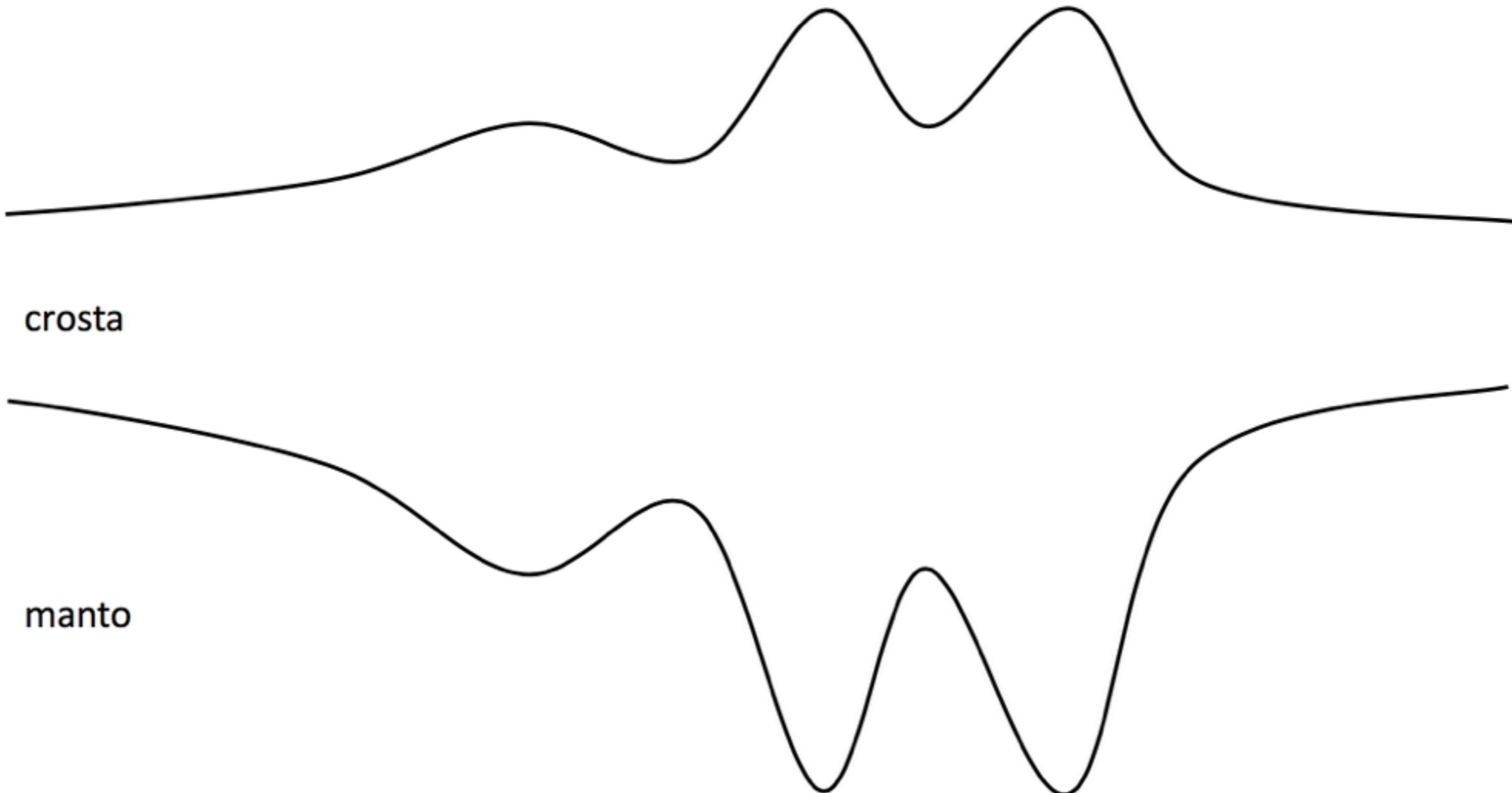


Any floating object displaces its own weight of fluid.
— Arquimedes de Siracusa

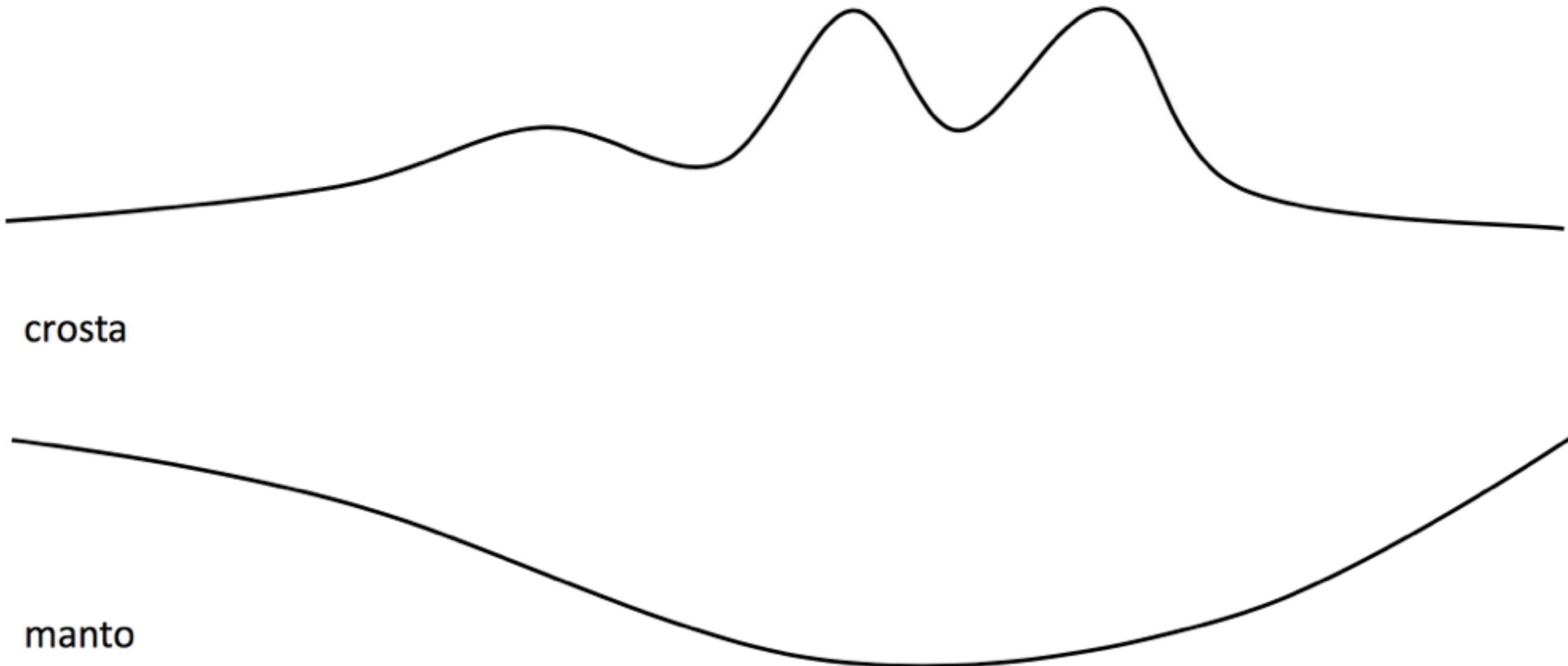
Isostasia da litosfera



Isostasia local



Isostasia regional



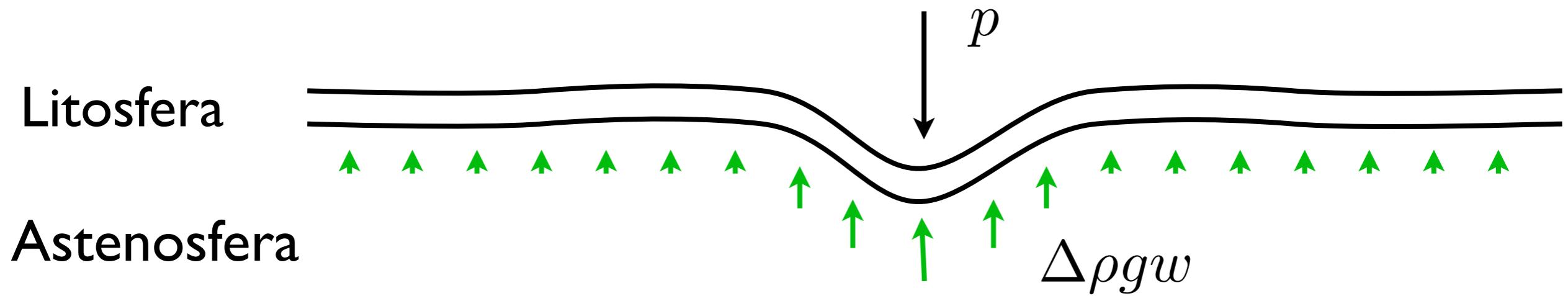
Flexura da litosfera: os primórdios

Para grandes blocos – por exemplo, um continente ou uma bacia oceânica inteira – a teoria da isostasia deve ser aceita sem dúvida; mas onde há feições menores, como montanhas individuais, a lei perde a sua validade. Tais feições podem ser sustentadas pela elasticidade do bloco todo.

Alfred Wegener (1929) *tradução livre*



https://en.wikipedia.org/wiki/Alfred_Wegener#/media/File:Wegener_Expedition-1930_008.jpg



$$D \frac{d^4 w}{dx^4} + \Delta\rho gw = p$$

Annotations in red explain the terms:

- deslocamento vertical da placa** (vertical displacement of the plate) points to the deflection term $d^4 w / dx^4$.
- carga** (load) points to the load term p .
- Rigidez da placa elástica** (elastic plate rigidity) points to the constant D .
- coordenada** (coordinate) points to the horizontal axis x .
- gravidade** (gravity) and **diferença de densidade** (density difference) both point to the term $\Delta\rho gw$.

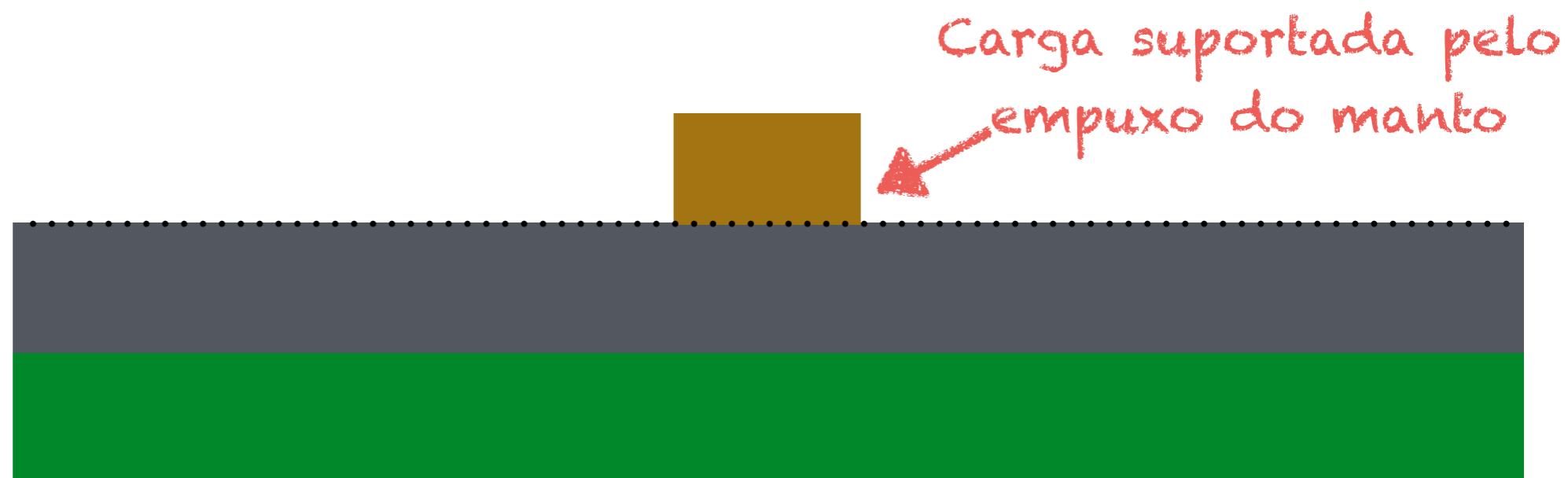
$$D \frac{d^4 w}{dx^4} + \Delta \rho g w = p$$

deslocamento vertical da placa
 carga
 gravidade
 diferença de densidade
 coordenada
 Rrigidez da placa elástica
 Módulo de elasticidade
 Espessura elástica efetiva
 Coeficiente de Poisson

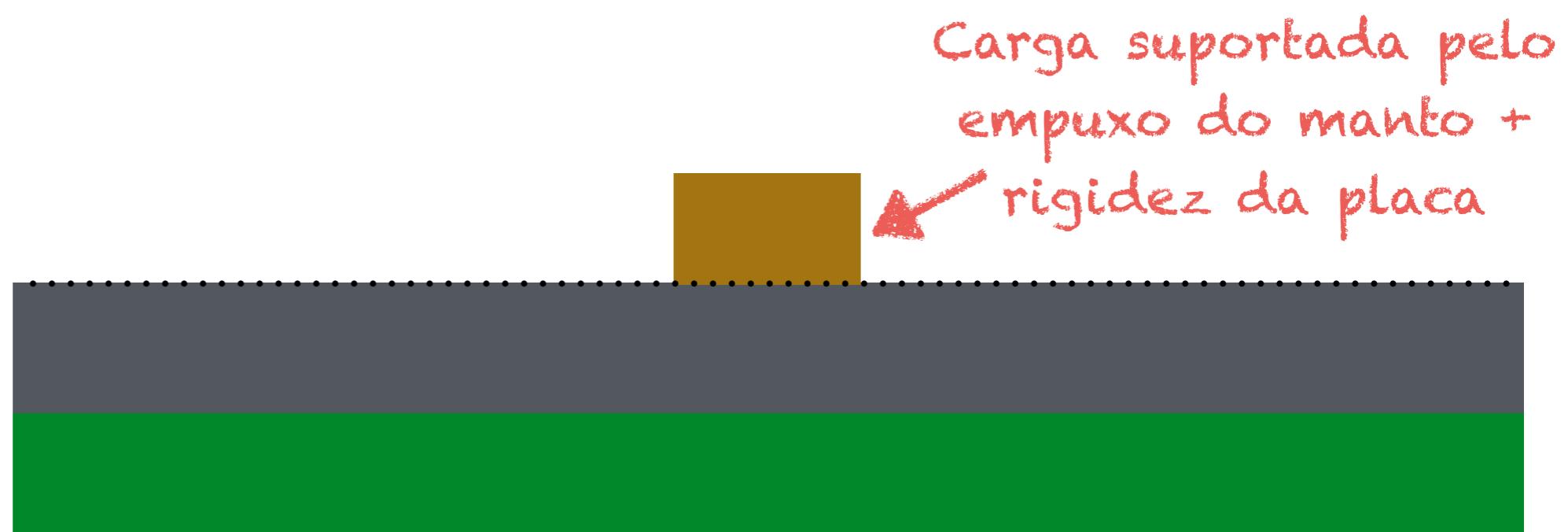
$$D = \frac{ET_e^3}{12(1 - \nu^2)}$$

Isostasia e Flexura da Litosfera

Isostasia
Local



Isostasia
Flexural



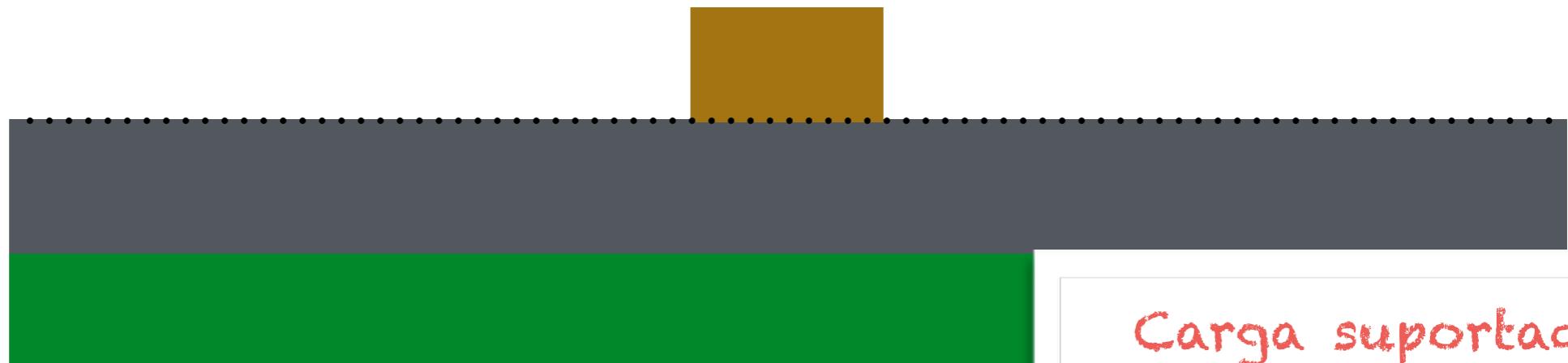
Espessura da placa elástica

T_e : Espessura elástica efetiva

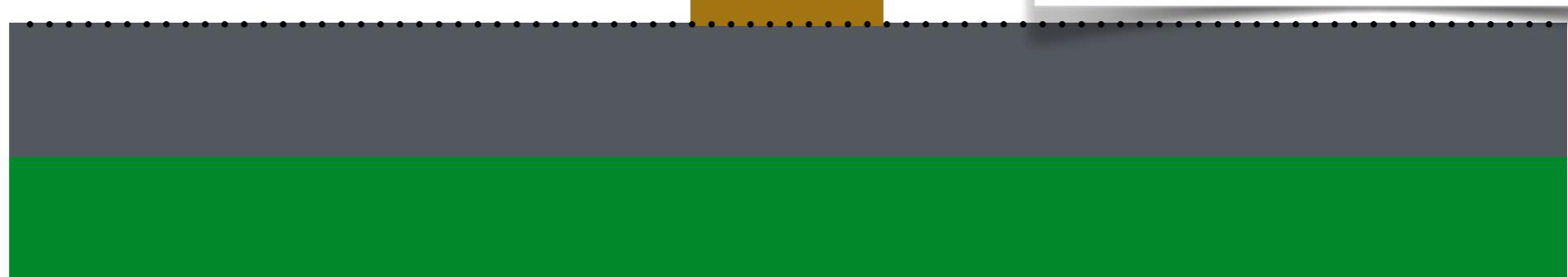
$T_e = 0$



T_e finite



$T_e \rightarrow \infty$



Carga suportada
apenas pela rigidez
da placa

Size of the load $\times T_e$

Para o mesmo T_e :

Carga
larga



Carga

intermediária



Carga estreita



Contribuições da isostasia e flexura para a tectônica de placas

- Existência de um fluido (astenosfera) que permite a compensação isostática e o deslocamento lateral das placas litosféricas.
- A camada externa da Terra (litosfera) preserva a sua rigidez ao longo do tempo geológico.

Cinemática das placas



<http://www.bl.uk/voices-of-science/interviewees/dan-mckenzie>

McKenzie & Parker (1967)



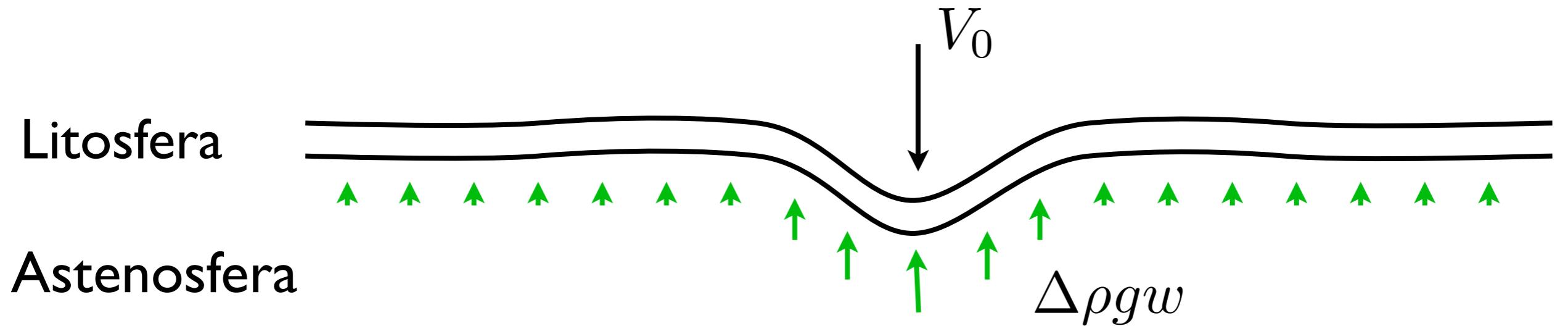
<https://www.onbeing.org/program/fragility-and-evolution-our-humanity/101>

Le Pichon (1968)



https://en.wikipedia.org/wiki/W._Jason_Morgan#/media/File:Morgan,_W._Jason.jpg

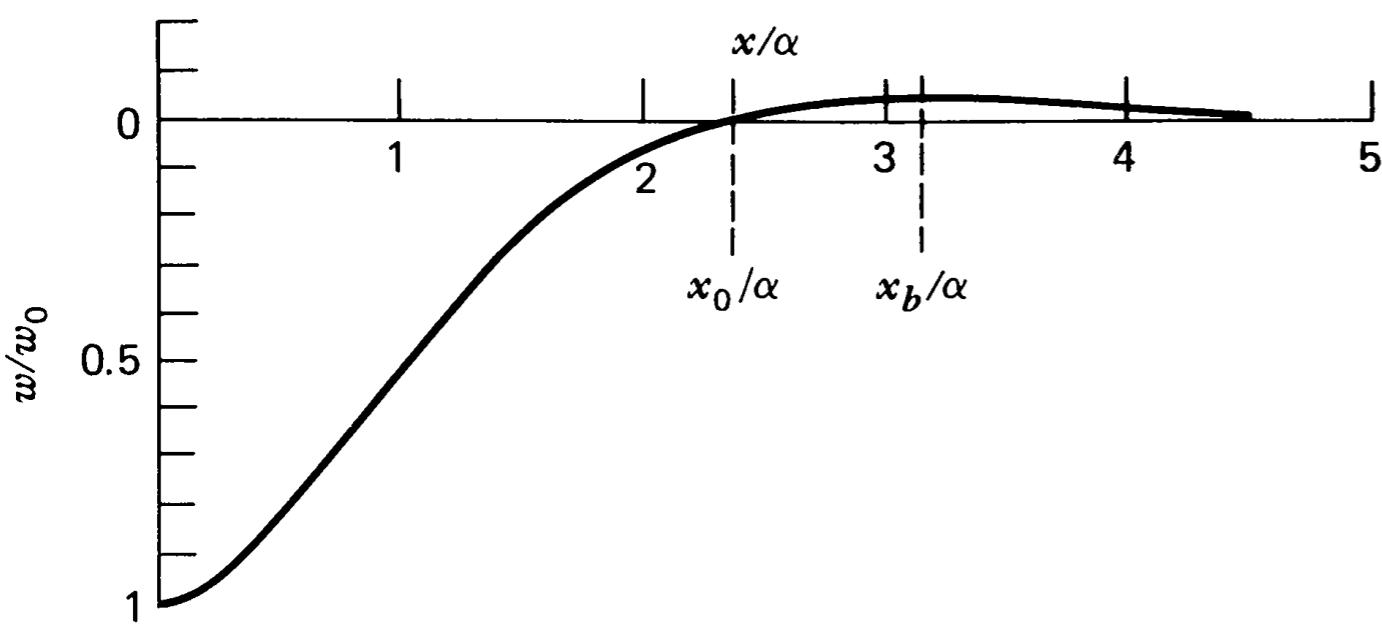
Morgan (1968)



$$D \frac{d^4 w}{dx^4} + \Delta \rho g w = p$$

Solução analítica para o caso de uma carga pontual

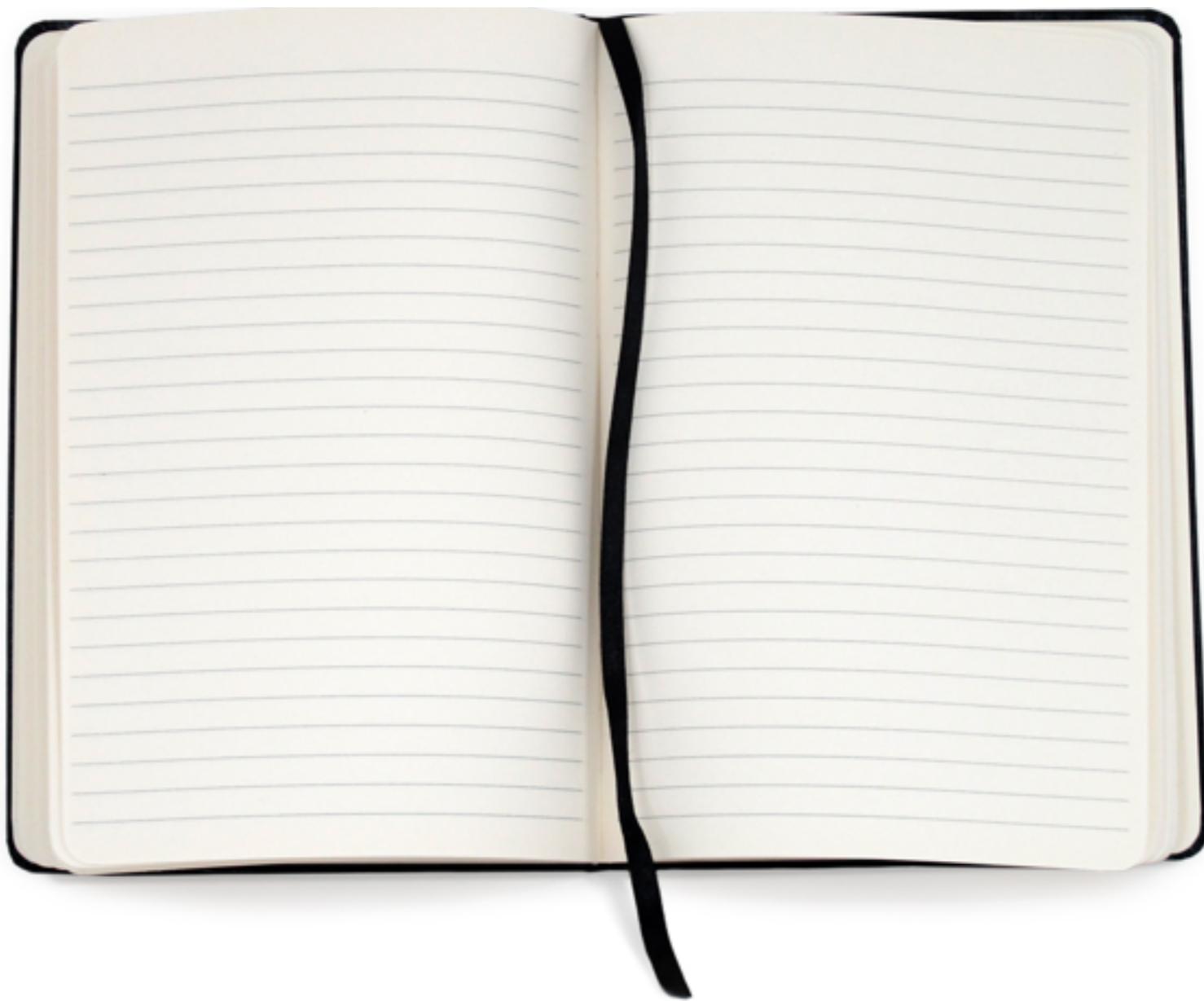
$$w = \frac{V_0 \alpha^3}{8D} e^{-x/\alpha} \left(\cos \frac{x}{\alpha} + \sin \frac{x}{\alpha} \right)$$



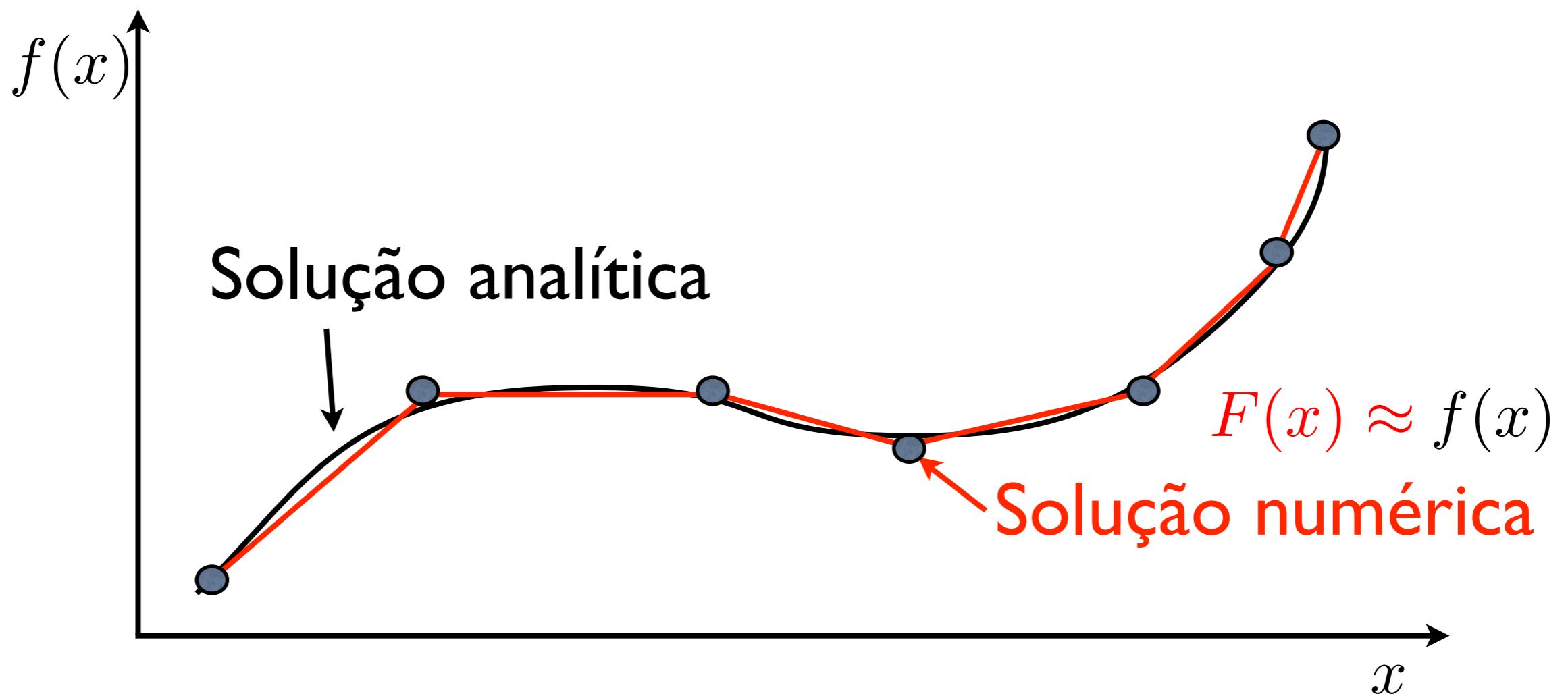
$$D = \frac{ET_e^3}{12(1 - \nu^2)}$$

$$\alpha = \left[\frac{4D}{(\rho_m - \rho_w)g} \right]^{1/4}$$

Prática: Flex_analitico



Aproximação numérica

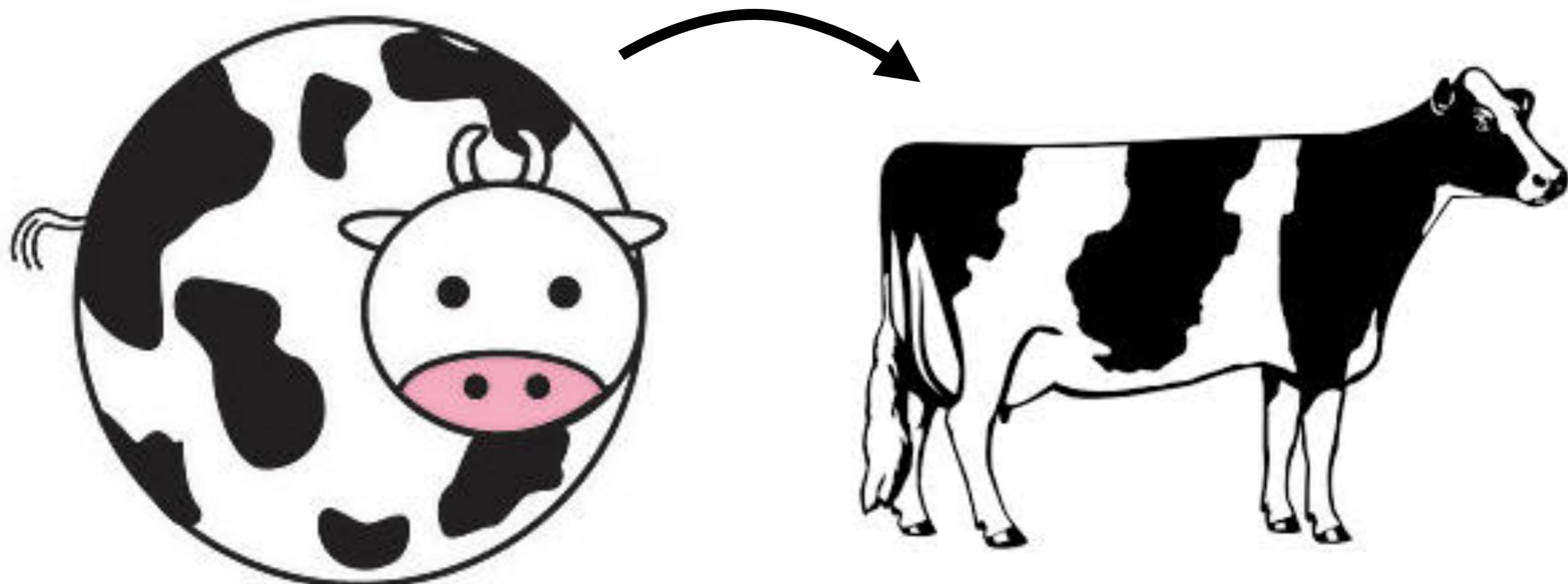


Sol. analítica x Sol. numérica

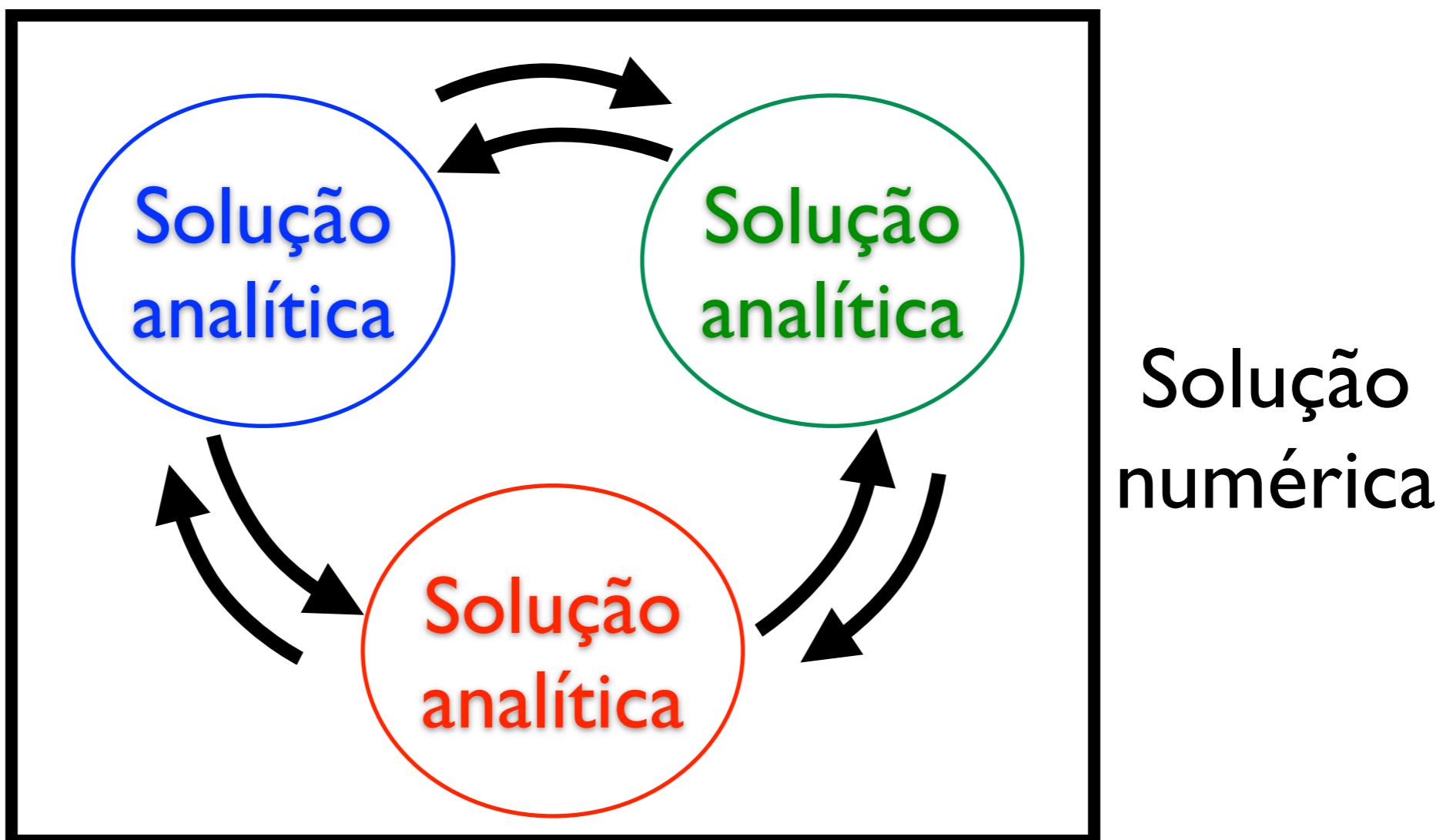
- Solução exata
- Contínua
- Nada é escondido
- Todos os cenários
- Não exige validação
- Solução aproximada
- Discreta
- Pode ser uma caixa preta
- 1 simulação por cenário
- Exige validação

Ora bolas! Para que serve a modelagem numérica?

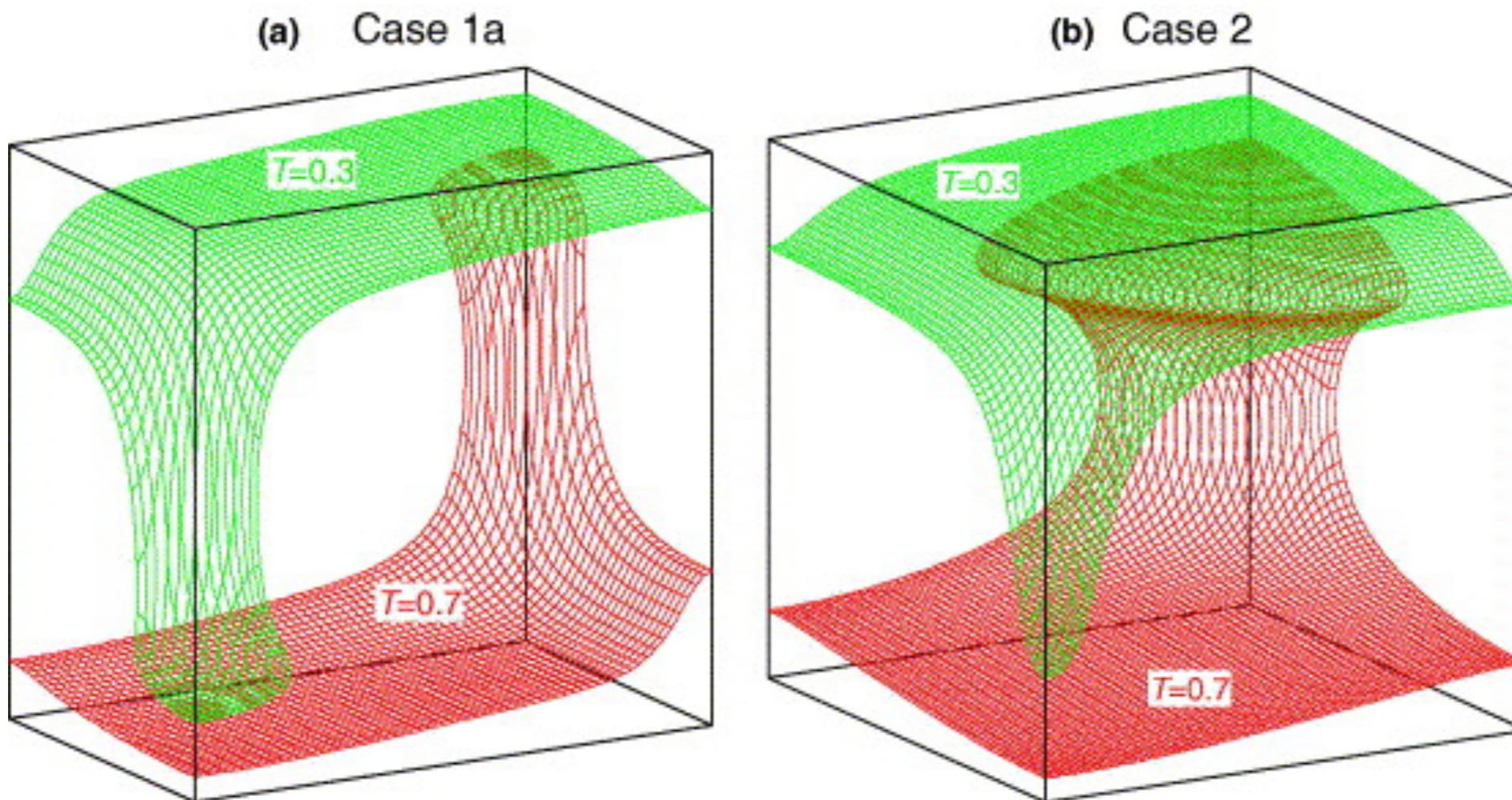
I. Geometria complicada



2. Acoplamento entre diferentes processos



3. Não existência de solução analítica



Método numérico: Diferenças finitas

$$\begin{aligned}\frac{dw(a)}{dx} &= \lim_{\Delta x \rightarrow 0} \frac{w(a + \Delta x) - w(a)}{\Delta x} \\ &\approx \frac{w(a + \Delta x) - w(a)}{\Delta x} = \frac{\Delta w}{\Delta x}\end{aligned}$$

w_{i-2} Δx w_{i-1} w_i w_{i+1} w_{i+2} 

$$w_{i-2}$$

$$\Delta x$$

$$w_{i-1}$$

$$w_i$$

$$w_{i+1}$$

$$w_{i+2}$$

$$D\frac{d^4 w}{dx^4} + \Delta \rho g w = p$$

w_{i-2} Δx w_{i-1} w_i w_{i+1} w_{i+2}

$$D \frac{d^4 w}{dx^4} + \Delta \rho g w = p$$



w_{i-2} Δx w_{i-1} w_i w_{i+1} w_{i+2}

$$D \frac{d^4 w}{dx^4} + \Delta \rho g w = p$$

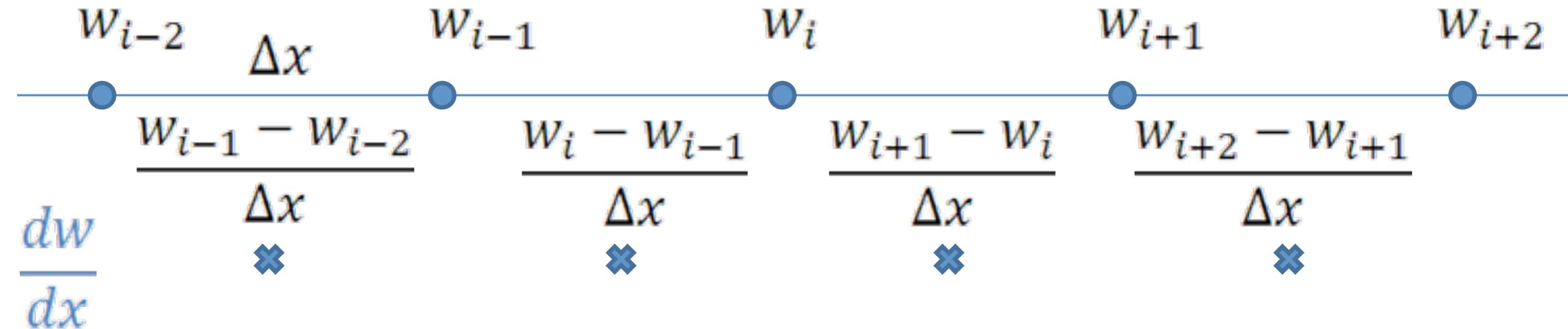


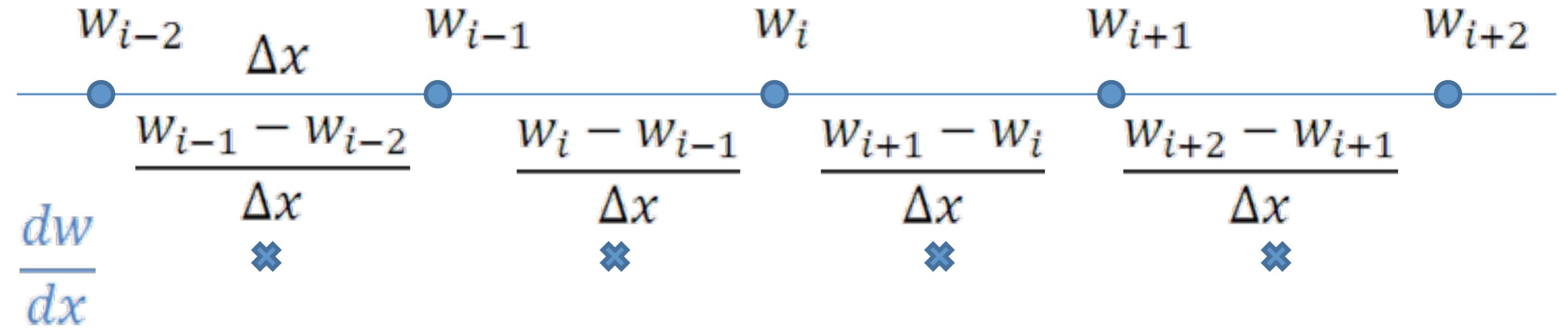
$$\text{?} + \Delta \rho g w_i = p_i$$

w_{i-2} Δx w_{i-1} w_i w_{i+1} w_{i+2}

$$\frac{dw}{dx}$$







$$\begin{array}{cccccc} w_{i-2} & \Delta x & w_{i-1} & w_i & w_{i+1} & w_{i+2} \\ \bullet & & \bullet & & \bullet & & \bullet \\ \frac{w_{i-1} - w_{i-2}}{\Delta x} & & \frac{w_i - w_{i-1}}{\Delta x} & & \frac{w_{i+1} - w_i}{\Delta x} & & \frac{w_{i+2} - w_{i+1}}{\Delta x} \\ \text{※} & & \text{※} & & \text{※} & & \text{※} \\ \frac{dw}{dx} & & & & & & \end{array}$$

$$\frac{d^2w}{dx^2}$$

※ ※ ※

w_{i-2} Δx w_{i-1} w_i w_{i+1} w_{i+2}

$$\frac{w_{i-1} - w_{i-2}}{\Delta x}$$

$$\frac{w_i - w_{i-1}}{\Delta x}$$

$$\frac{w_{i+1} - w_i}{\Delta x}$$

$$\frac{w_{i+2} - w_{i+1}}{\Delta x}$$

$$\frac{dw}{dx}$$

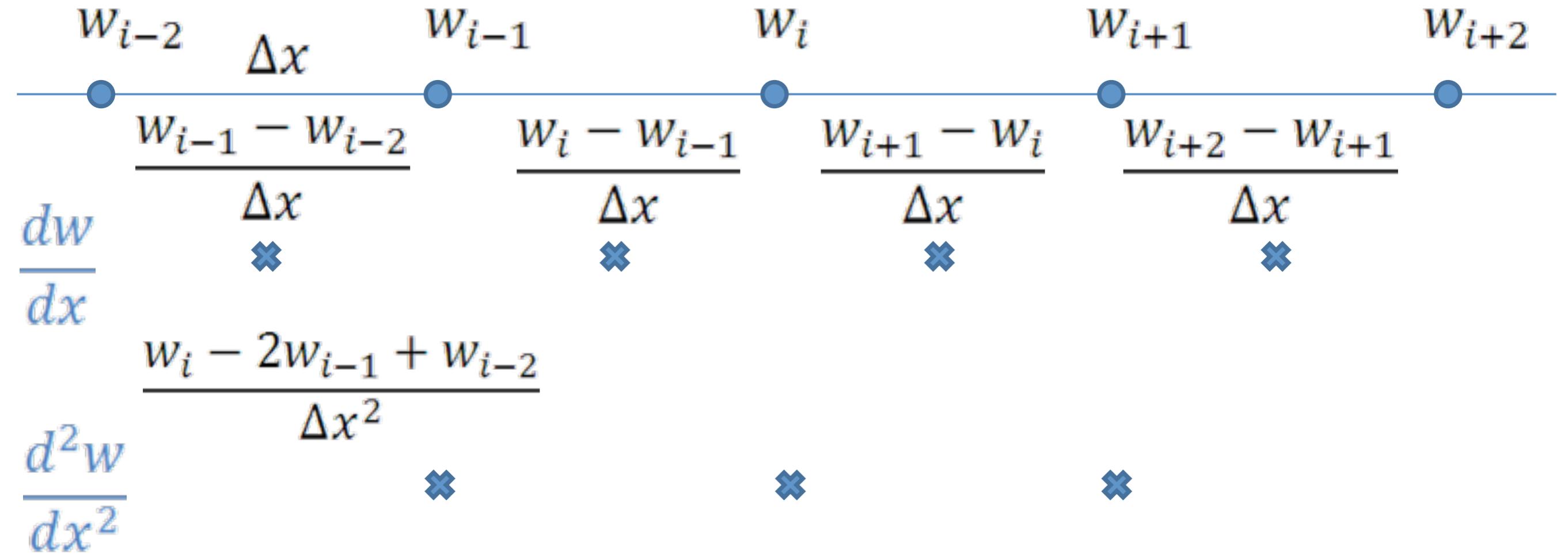
$$\frac{w_i - w_{i-1}}{\Delta x} - \frac{w_{i-1} - w_{i-2}}{\Delta x}$$

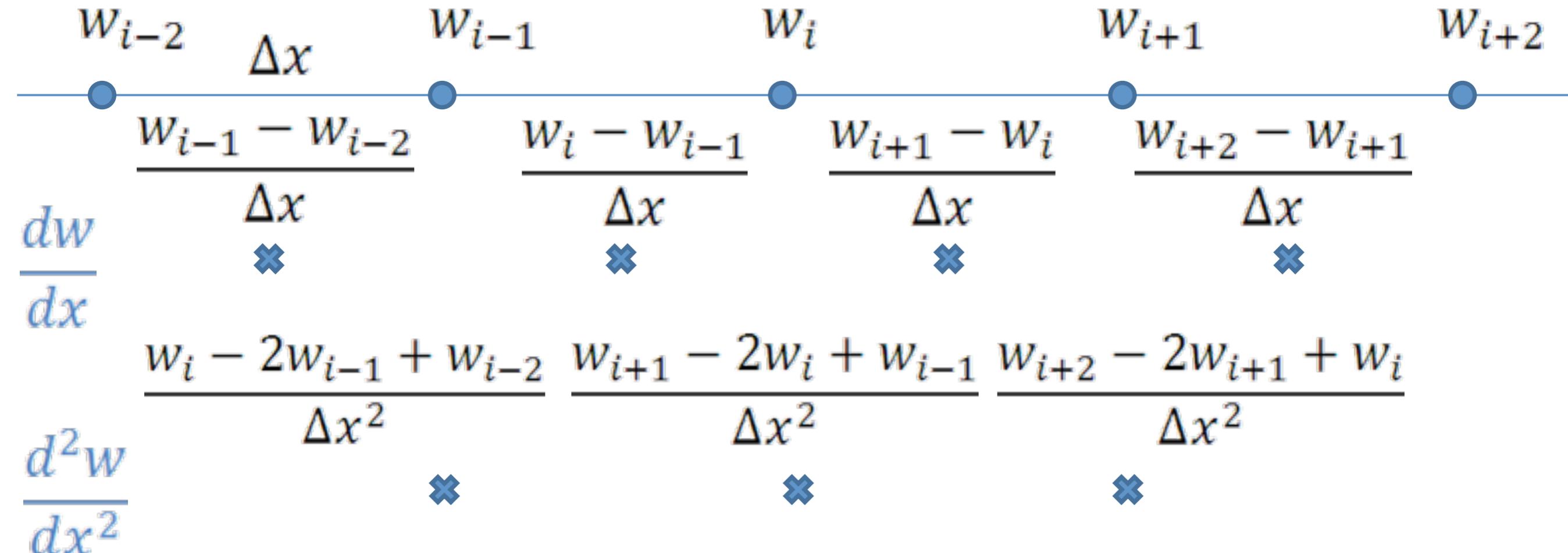
$$\frac{d^2w}{dx^2}$$

$$\frac{\Delta x}{\Delta x}$$

$$\times$$

$$\times$$





✖

✖

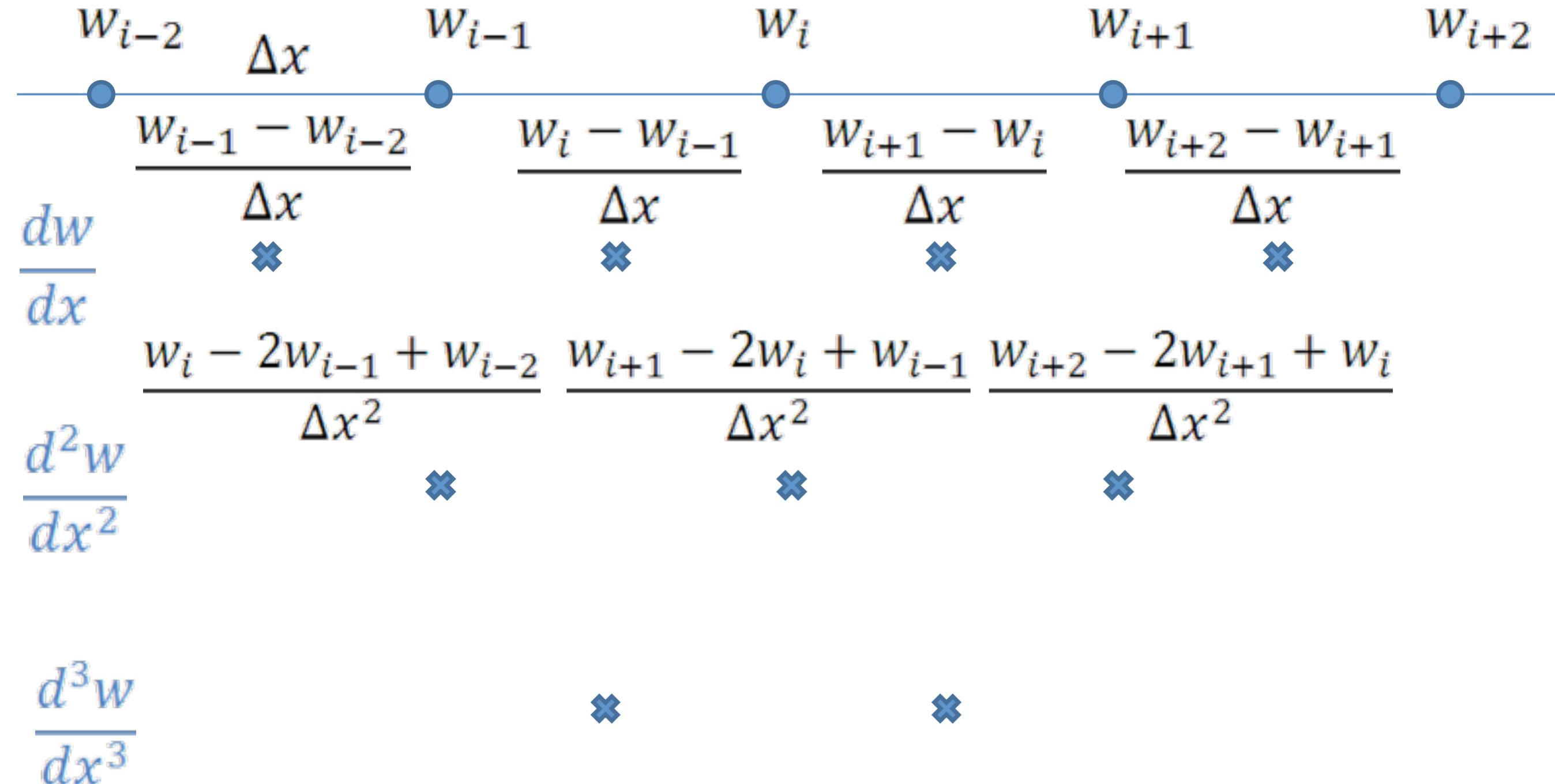
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w_{i-2} Δx w_{i-1} w_i w_{i+1} w_{i+2}

$\frac{w_{i-1} - w_{i-2}}{\Delta x}$ $\frac{w_i - w_{i-1}}{\Delta x}$ $\frac{w_{i+1} - w_i}{\Delta x}$ $\frac{w_{i+2} - w_{i+1}}{\Delta x}$

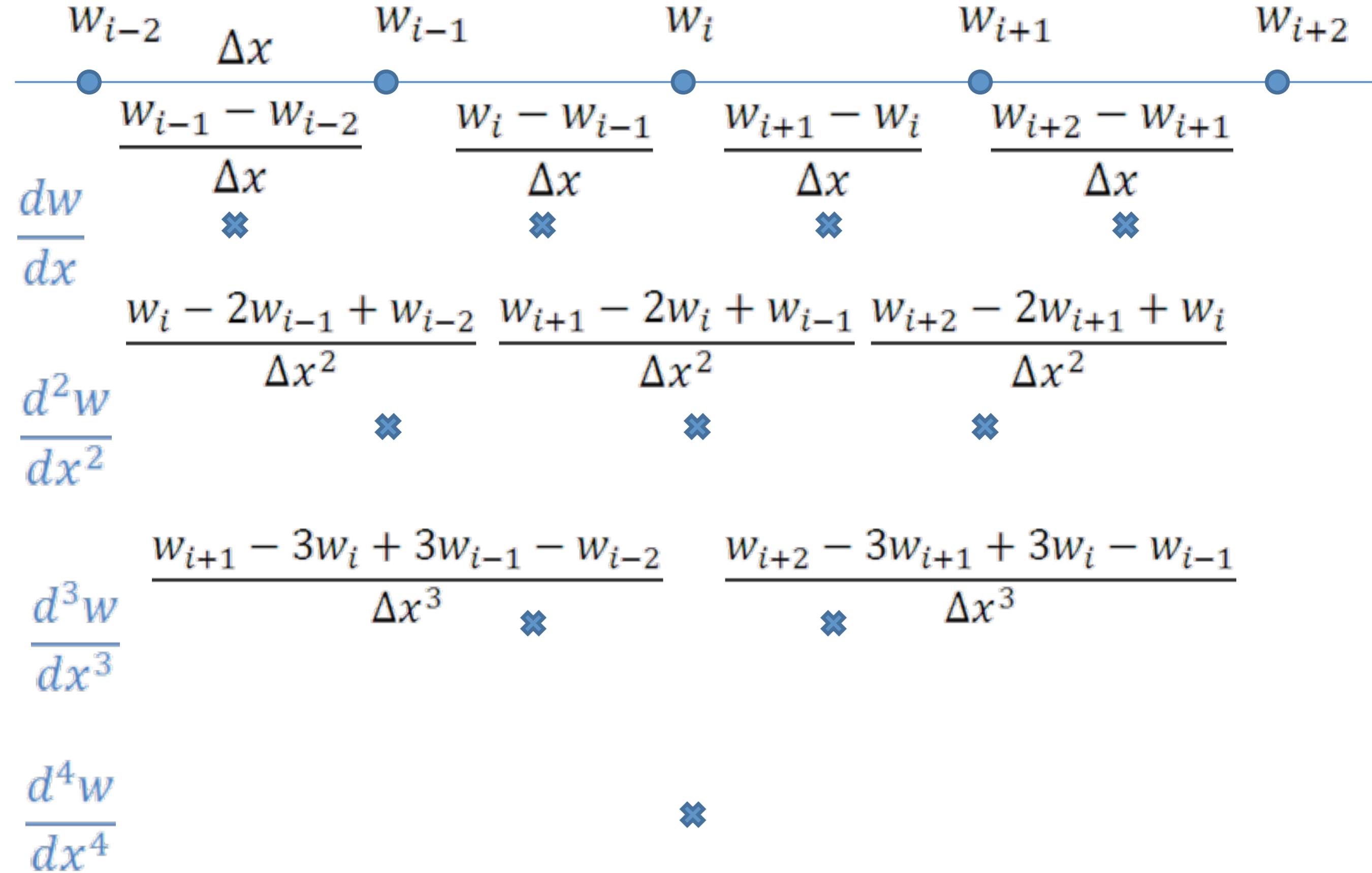
$\frac{dw}{dx}$ \otimes \otimes \otimes \otimes

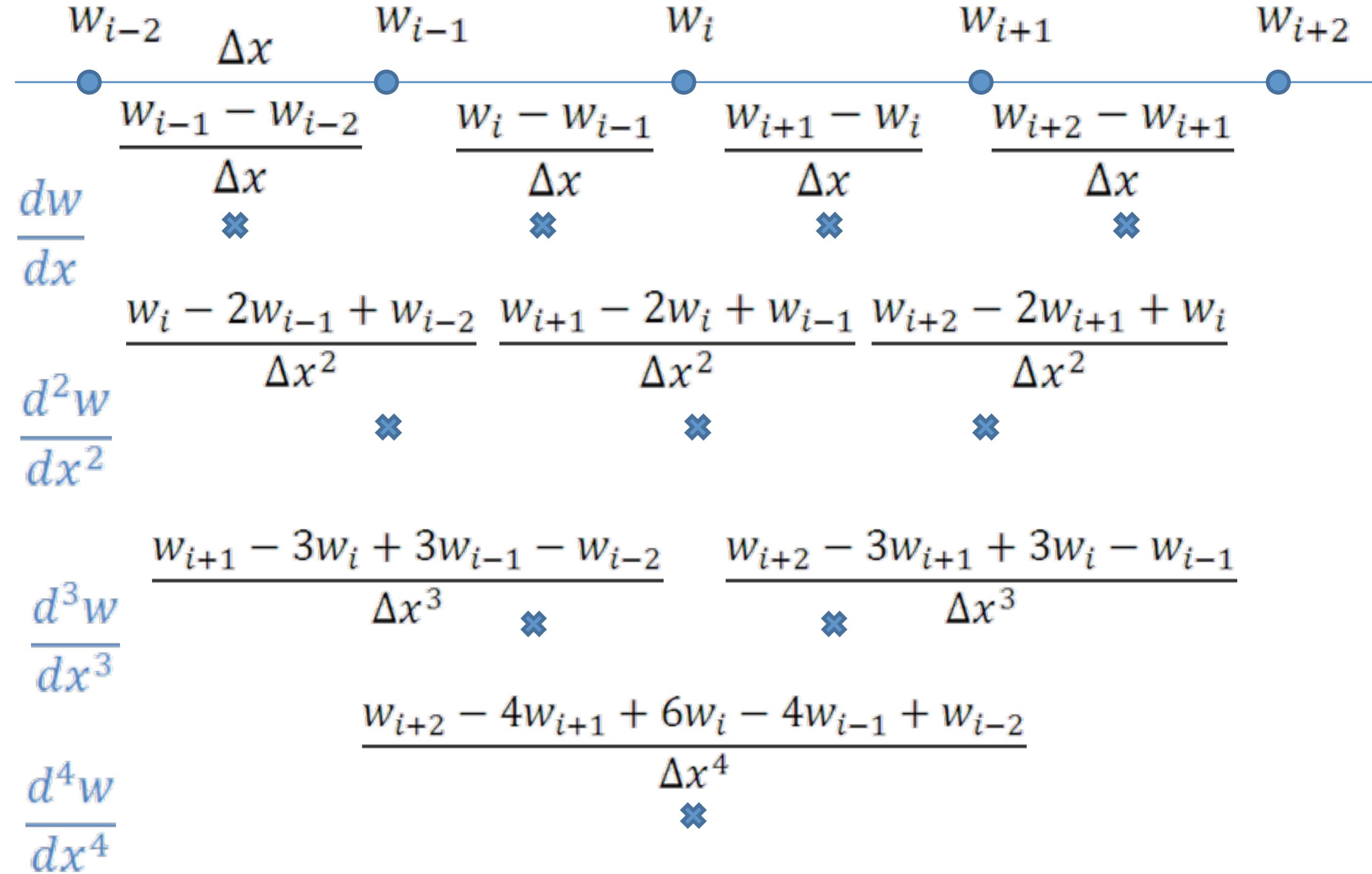
$\frac{w_i - 2w_{i-1} + w_{i-2}}{\Delta x^2}$ $\frac{w_{i+1} - 2w_i + w_{i-1}}{\Delta x^2}$ $\frac{w_{i+2} - 2w_{i+1} + w_i}{\Delta x^2}$

$\frac{d^2w}{dx^2}$ \otimes \otimes \otimes

$\frac{w_{i+1} - 3w_i + 3w_{i-1} - w_{i-2}}{\Delta x^3}$ $\frac{w_{i+2} - 3w_{i+1} + 3w_i - w_{i-1}}{\Delta x^3}$

$\frac{d^3w}{dx^3}$ \otimes \otimes





$$D\frac{d^4w}{dx^4}+\Delta\rho gw=p$$

$$D\frac{d^4w}{dx^4}+\Delta\rho gw=p$$

$$D\frac{w_{i+2}-4w_{i+1}+6w_i-4w_{i-1}+w_{i-2}}{\Delta x^4}\!+\!\Delta\rho gw_i=p_i$$

$$D\frac{d^4w}{dx^4}+\Delta\rho gw=p$$

$$D\frac{w_{i+2}-4w_{i+1}+6w_i-4w_{i-1}+w_{i-2}}{\Delta x^4}+\Delta\rho gw_i=p_i$$

$$D[w_{i+2}-4w_{i+1}+6w_i-4w_{i-1}+w_{i-2}]+\Delta x^4\Delta\rho gw_i=\Delta x^4p_i$$

$$D \frac{d^4 w}{dx^4} + \Delta \rho g w = p$$

$$D \frac{w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}}{\Delta x^4} + \Delta \rho g w_i = p_i$$

$$D[w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}] + \Delta x^4 \Delta \rho g w_i = \Delta x^4 p_i$$

$$D \frac{d^4 w}{dx^4} + \Delta \rho g w = p$$

$$D \frac{w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}}{\Delta x^4} + \Delta \rho g w_i = p_i$$

$$D[w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}] + \Delta x^4 \Delta \rho g w_i = \Delta x^4 p_i$$

$$Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta \rho g]w_i - 4Dw_{i+1} + Dw_{i+2} = \\ = \Delta x^4 p_i$$

$$D \frac{d^4 w}{dx^4} + \Delta \rho g w = p$$

$$D \frac{w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}}{\Delta x^4} + \Delta \rho g w_i = p_i$$

$$D[w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}] + \Delta x^4 \Delta \rho g w_i = \Delta x^4 p_i$$

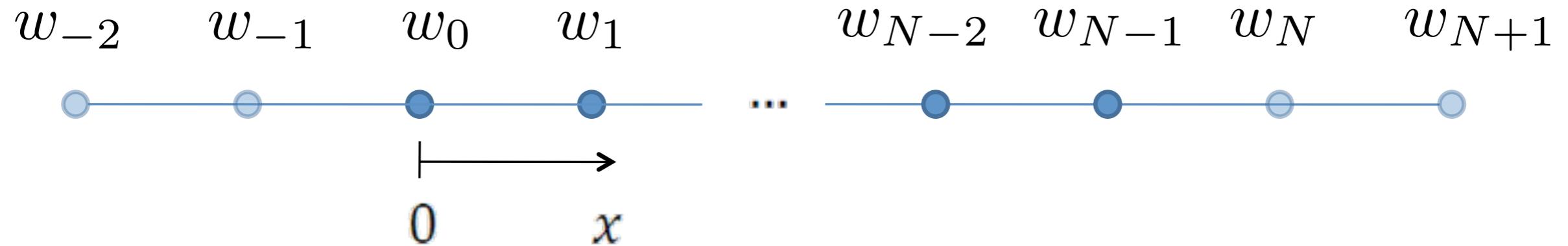
$$\boxed{Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta \rho g]w_i - 4Dw_{i+1} + Dw_{i+2} = \\ = \Delta x^4 p_i}$$

Condição de contorno

Placa contínua:

$$w \rightarrow 0 \text{ para } x \rightarrow 0$$

$$w \rightarrow 0 \text{ para } x \rightarrow x_n$$



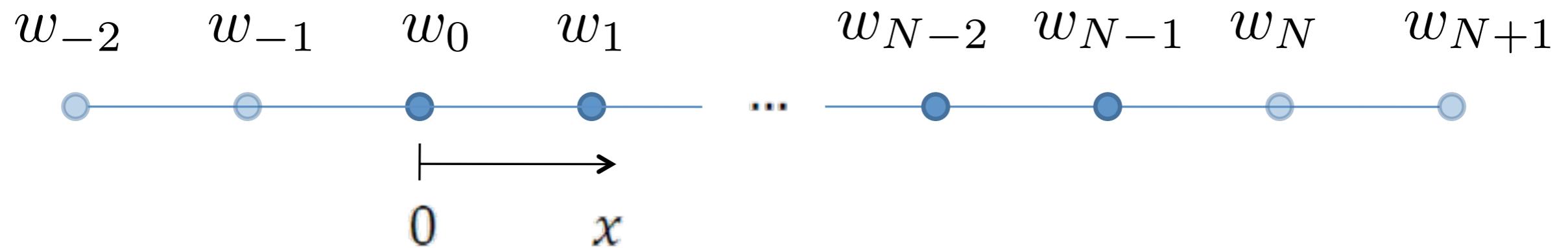
Condição de contorno

Placa contínua:

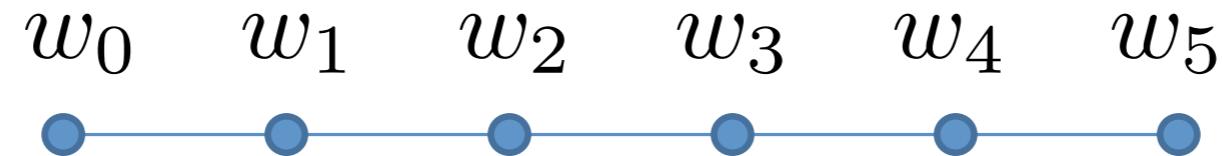
$$w \rightarrow 0 \text{ para } x \rightarrow 0$$

$$w \rightarrow 0 \text{ para } x \rightarrow x_n$$

$$w_{-2}, w_{-1}, w_N, w_{N-1} = 0$$

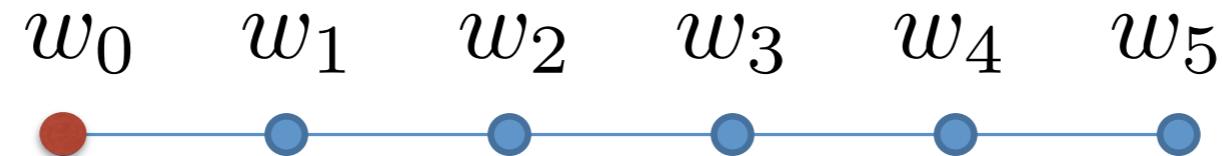


Exemplo



$$Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta \rho g]w_i - 4Dw_{i+1} + Dw_{i+2} = \\ = \Delta x^4 p_i$$

Exemplo

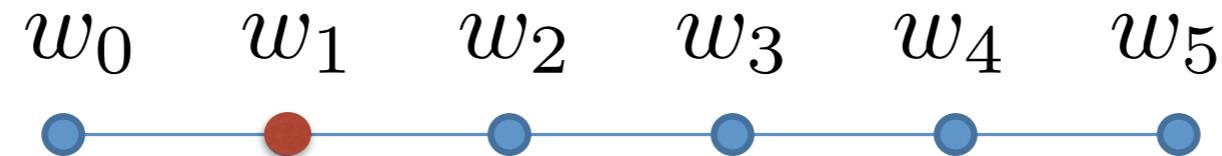


$w_0 :$

$$[6D + \Delta x^4 \Delta \rho g] w_0 - 4Dw_1 + Dw_2 = \Delta x^4 p_0$$

$$\begin{aligned} D w_{i-2} - 4 D w_{i-1} + [6D + \Delta x^4 \Delta \rho g] w_i - 4 D w_{i+1} + D w_{i+2} = \\ = \Delta x^4 p_i \end{aligned}$$

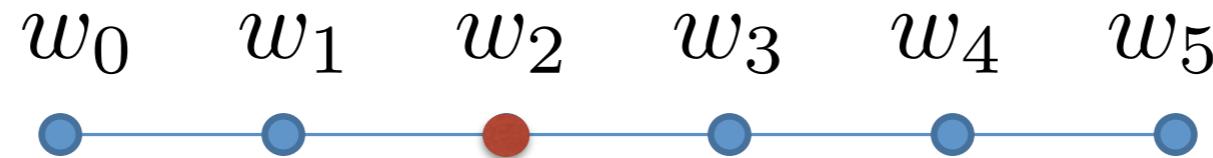
Exemplo



$$w_0 : [6D + \Delta x^4 \Delta \rho g] w_0 - 4Dw_1 + Dw_2 = \Delta x^4 p_0$$
$$w_1 : -4Dw_0 + [6D + \Delta x^4 \Delta \rho g] w_1 - 4Dw_2 + Dw_3 = \Delta x^4 p_1$$

$$Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta \rho g]w_i - 4Dw_{i+1} + Dw_{i+2} = \\ = \Delta x^4 p_i$$

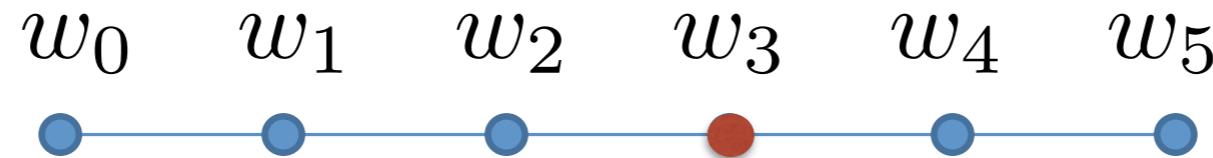
Exemplo



$$\begin{aligned}w_0 : \quad & [6D + \Delta x^4 \Delta \rho g] w_0 - 4Dw_1 + Dw_2 = \Delta x^4 p_0 \\w_1 : \quad & -4Dw_0 + [6D + \Delta x^4 \Delta \rho g] w_1 - 4Dw_2 + Dw_3 = \Delta x^4 p_1 \\w_2 : \quad & Dw_0 - 4Dw_1 + [6D + \Delta x^4 \Delta \rho g] w_2 - 4Dw_3 + Dw_4 = \Delta x^4 p_2\end{aligned}$$

$$\begin{aligned}Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta \rho g] w_i - 4Dw_{i+1} + Dw_{i+2} = \\= \Delta x^4 p_i\end{aligned}$$

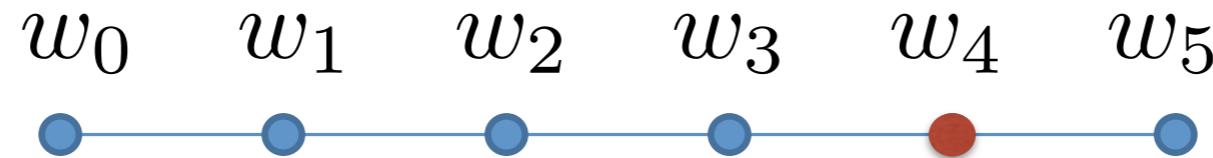
Exemplo



$$\begin{aligned}w_0 : \quad & [6D + \Delta x^4 \Delta \rho g] w_0 - 4Dw_1 + Dw_2 = \Delta x^4 p_0 \\w_1 : \quad & -4Dw_0 + [6D + \Delta x^4 \Delta \rho g] w_1 - 4Dw_2 + Dw_3 = \Delta x^4 p_1 \\w_2 : \quad & Dw_0 - 4Dw_1 + [6D + \Delta x^4 \Delta \rho g] w_2 - 4Dw_3 + Dw_4 = \Delta x^4 p_2 \\w_3 : \quad & Dw_1 - 4Dw_2 + [6D + \Delta x^4 \Delta \rho g] w_3 - 4Dw_4 + Dw_5 = \Delta x^4 p_3\end{aligned}$$

$$\begin{aligned}Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta \rho g] w_i - 4Dw_{i+1} + Dw_{i+2} = \\= \Delta x^4 p_i\end{aligned}$$

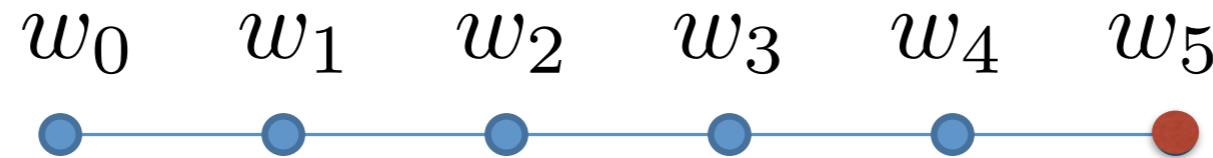
Exemplo



$$\begin{aligned}w_0 : \quad & [6D + \Delta x^4 \Delta \rho g] w_0 - 4Dw_1 + Dw_2 = \Delta x^4 p_0 \\w_1 : \quad & -4Dw_0 + [6D + \Delta x^4 \Delta \rho g] w_1 - 4Dw_2 + Dw_3 = \Delta x^4 p_1 \\w_2 : \quad & Dw_0 - 4Dw_1 + [6D + \Delta x^4 \Delta \rho g] w_2 - 4Dw_3 + Dw_4 = \Delta x^4 p_2 \\w_3 : \quad & Dw_1 - 4Dw_2 + [6D + \Delta x^4 \Delta \rho g] w_3 - 4Dw_4 + Dw_5 = \Delta x^4 p_3 \\w_4 : \quad & Dw_2 - 4Dw_3 + [6D + \Delta x^4 \Delta \rho g] w_4 - 4Dw_5 = \Delta x^4 p_4\end{aligned}$$

$$\begin{aligned}Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta \rho g] w_i - 4Dw_{i+1} + Dw_{i+2} = \\= \Delta x^4 p_i\end{aligned}$$

Exemplo



$$\begin{aligned}w_0 : \quad & [6D + \Delta x^4 \Delta \rho g] w_0 - 4Dw_1 + Dw_2 = \Delta x^4 p_0 \\w_1 : \quad & -4Dw_0 + [6D + \Delta x^4 \Delta \rho g] w_1 - 4Dw_2 + Dw_3 = \Delta x^4 p_1 \\w_2 : \quad & Dw_0 - 4Dw_1 + [6D + \Delta x^4 \Delta \rho g] w_2 - 4Dw_3 + Dw_4 = \Delta x^4 p_2 \\w_3 : \quad & Dw_1 - 4Dw_2 + [6D + \Delta x^4 \Delta \rho g] w_3 - 4Dw_4 + Dw_5 = \Delta x^4 p_3 \\w_4 : \quad & Dw_2 - 4Dw_3 + [6D + \Delta x^4 \Delta \rho g] w_4 - 4Dw_5 = \Delta x^4 p_4 \\w_5 : \quad & Dw_3 - 4Dw_4 + [6D + \Delta x^4 \Delta \rho g] w_5 = \Delta x^4 p_5\end{aligned}$$

$$\begin{aligned}Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta \rho g] w_i - 4Dw_{i+1} + Dw_{i+2} = \\= \Delta x^4 p_i\end{aligned}$$

$$[6D + \Delta x^4 \Delta \rho g] w_0 - 4Dw_1 + Dw_2 = \Delta x^4 p_0$$

$$-4Dw_0 + [6D + \Delta x^4 \Delta \rho g] w_1 - 4Dw_2 + Dw_3 = \Delta x^4 p_1$$

$$Dw_0 - 4Dw_1 + [6D + \Delta x^4 \Delta \rho g] w_2 - 4Dw_3 + Dw_4 = \Delta x^4 p_2$$

$$Dw_1 - 4Dw_2 + [6D + \Delta x^4 \Delta \rho g] w_3 - 4Dw_4 + Dw_5 = \Delta x^4 p_3$$

$$Dw_2 - 4Dw_3 + [6D + \Delta x^4 \Delta \rho g] w_4 - 4Dw_5 = \Delta x^4 p_4$$

$$Dw_3 - 4Dw_4 + [6D + \Delta x^4 \Delta \rho g] w_5 = \Delta x^4 p_5$$

$$\begin{aligned}
& [6D + \Delta x^4 \Delta \rho g] w_0 - 4Dw_1 + Dw_2 = \Delta x^4 p_0 \\
& -4Dw_0 + [6D + \Delta x^4 \Delta \rho g] w_1 - 4Dw_2 + Dw_3 = \Delta x^4 p_1 \\
& Dw_0 - 4Dw_1 + [6D + \Delta x^4 \Delta \rho g] w_2 - 4Dw_3 + Dw_4 = \Delta x^4 p_2 \\
& Dw_1 - 4Dw_2 + [6D + \Delta x^4 \Delta \rho g] w_3 - 4Dw_4 + Dw_5 = \Delta x^4 p_3 \\
& Dw_2 - 4Dw_3 + [6D + \Delta x^4 \Delta \rho g] w_4 - 4Dw_5 = \Delta x^4 p_4 \\
& Dw_3 - 4Dw_4 + [6D + \Delta x^4 \Delta \rho g] w_5 = \Delta x^4 p_5
\end{aligned}$$

$$\left[\begin{array}{cccccc}
6D + \Delta x^4 \Delta \rho g & -4D & D & 0 & 0 & 0 \\
-4D & 6D + \Delta x^4 \Delta \rho g & -4D & D & 0 & 0 \\
D & -4D & 6D + \Delta x^4 \Delta \rho g & -4D & D & 0 \\
0 & D & -4D & 6D + \Delta x^4 \Delta \rho g & -4D & D \\
0 & 0 & D & -4D & 6D + \Delta x^4 \Delta \rho g & -4D \\
0 & 0 & 0 & D & -4D & 6D + \Delta x^4 \Delta \rho g
\end{array} \right] \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{bmatrix} =$$

$$= \Delta x^4 \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{bmatrix}$$

$$\begin{bmatrix} 6D + \Delta x^4 \Delta \rho g & -4D & D & 0 & 0 & 0 \\ -4D & 6D + \Delta x^4 \Delta \rho g & -4D & D & 0 & 0 \\ D & -4D & 6D + \Delta x^4 \Delta \rho g & -4D & D & 0 \\ 0 & D & -4D & 6D + \Delta x^4 \Delta \rho g & -4D & D \\ 0 & 0 & D & -4D & 6D + \Delta x^4 \Delta \rho g & -4D \\ 0 & 0 & 0 & D & -4D & 6D + \Delta x^4 \Delta \rho g \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{bmatrix} = \Delta x^4 \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{bmatrix}$$

$$\mathbf{Aw} = \mathbf{p}$$

$$A(i, j = i - 2) = D$$

$$A(i, j = i - 1) = -4D$$

$$A(i, j = i) = 6D + \Delta x^4 \Delta \rho g$$

$$A(i, j = i + 1) = -4D$$

$$A(i, j = i + 2) = D$$

Prática: Flex_numerico

