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Abstract

Purpose – The value chain of the Norwegian meat production industry has recently been through major structural changes resulting in increased flows and transportation needs at all levels. The purpose of this paper is to present results of the initial stage of a five-year research project between the Norwegian Meat Research Centre, Norwegian meat companies and Molde University College. The main goal of the project is to develop a decision support system for the transport of live animals to a slaughterhouse to reduce transportation costs while maintaining high level of livestock welfare and meat quality, as these are three main factors for the profitability of both farmers and industry.

Design/methodology/approach – The paper presents a mixed integer programming model that combines vehicle routing and inventory control. We introduce the possibility for multiple routes for a given vehicle on a given day in a multiple-period planning perspective. Arrival times of the loaded vehicles to the slaughterhouse are controlled by production (slaughter) rate and inventory level at the abattoirs so that the supply of animals for slaughter is steady and production breaks are avoided. Livestock welfare is secured by the route duration constraints.

Findings – The model has been formulated and tested on small data sets. The major future challenge is to solve real-life problems from the involved companies.

Research limitations/implications – The main limitation is the present inability to solve large cases.

Originality/value – The model combining transportation and inventory control in a setting of animal welfare constraints is original.

Keywords Livestock, Transportation, Optimization techniques, Production planning, Meat, Norway

Paper type Research paper

Introduction

This paper reports on an MIP model created for a project that has been set up with The Norwegian Meat Research Centre (Fagsenteret for kjøtt), Molde University College, and the largest cooperative and private slaughterhouses in Norway, Gilde Norsk Kjøtt and Fatland Jæren. The purpose of the project is to study how we possibly can obtain cost effective transportation while at the same time taking care of animal welfare.

Transportation of live animals to slaughterhouses is a very classical logistics problem. A number of vehicles with limited capacity must collect animals at diverse farms and bring them in to provide a steady flow of animals at the abattoir. As the number of slaughterhouses is being reduced, the need for cost effective transportation increases substantially. Also the fact that many slaughterhouses are now specialized in terms of animal types has the same effect.

Combined with these effects stemming from increased competition in the business, there are also relevant regulations taking care of animal welfare. Some of these are

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national, some are imposed by the EU. The most prominent of these regulations is that no animal may spend more than eight hours on a vehicle. There are also regulations on how much space must be allotted to each animal. In a sparsely populated country like Norway, also strong regulations on mixing animal types (as well as animals from different herds) on a vehicle point in the same direction: increased need for transportation, and hence increased needs for effective transportation.

There is a connection between financial gains and animal welfare. The better an animal is treated before being slaughtered, the more valuable meat results. The strength of this connection varies, however, among animal types and among the different regulations (time, mix of types, and mix of herds). So even though the welfare constraints may appear to produce only costs in an optimization model, there is also a positive effect, although hard to quantify. Also the incentive structure within the business hides this fact. There is no feedback from bad meat quality (for example, from a stressed animal) to the potential savings in the treatment of the animal that led to the stress. It is not clear how to handle this, but the effect is not to be neglected.

The driving force behind the transportation planning is a slaughtering plan, which again is a function of expected demand and general knowledge about available animals. Of course, these two processes should be more integrated, but that is something left for later work. Here, we use the industry standard of letting production plans determine transportation.

The model presented deals with multiple-period planning of transportation of live animals to a slaughterhouse according to a slaughtering plan – with emphasis on routing of vehicles. This involves decisions on when the vehicles should arrive to the farms and to the slaughterhouse and simultaneous design of vehicle routes for each period. The model objective is to minimize the total transportation costs.

The model is interesting because it differs from known vehicle routing models and their extensions. It combines time scheduling of visits on each route a vehicle performs on a day with a careful orchestration of the vehicle's arrivals to the abattoir to secure continuous production. The model also includes animal welfare constraints. The vehicle routing nature of the model requires the use of continuous time variables, while from the production planning side the usage of discrete time periods is essential. The model links both types of time variables thus representing a combination of a vehicle routing and a multi-period production planning problem.

The rest of this paper is organised as follows: we first present a detailed problem description, including problem characteristics and constraints, problem size, current daily planning routines and a review of existing literature on the relevant optimization problems. Next follows a specification of problem characteristics and assumptions for the basic model. The optimization model formulation is then presented, followed by directions for further research, including possible solution approaches.

Problem description

The decision problem has two integrated parts: a vehicle routing component and a production planning component. In the vehicle routing problem (VRP), the scheduler needs to generate routes for vehicles that collect animals from supply farms and deliver them to the slaughterhouse. In the production planning problem, the scheduler needs to schedule the vehicles' returns to the slaughterhouse after the collection according to the slaughtering plan. Our goal is to develop a specific sequence of suppliers (farms) for each vehicle to visit on each route in such a way that the total transportation cost is minimized while honoring the operational constraints on the routes and the slaughtering plan.

The scheduler has to take many factors in account, including general characteristics of the routing problem like demand, vehicle fleet and working rules, as well as production process characteristics (original for our problem) such as slaughtering plan and inventory level. On the other hand, the operational characteristics of the slaughterhouse routing activity will be specified, i.e. the constraints that govern the route configuration. In an operational setting, the routes driven on any day are subject to a number of constraints reflecting restrictions imposed by the vehicles, the drivers' schedules and animal welfare requirement.

Nature of supply and demand

The farmers have a wide range of animals they want to have transported to the slaughterhouse. The livestock is divided into types and categories. The types are the different animal species, like bovine, ovine and pigs. They are divided into categories by age/size and gender. The main categories of bovine are calf, veal calf, young bull, heifer and cow. For pigs, there are slaughter hog and sow. Categories of ovine are lamb, sheep, ram and goat. For some categories, there is a need for further splitting. Bovine of the categories young bull, heifer and cow may have horns and are thus split into two categories. The ovine categories lamb, sheep and ram are also split into two categories, one for animals with wool and one for those without. The categories require different amounts of space per animal in the vehicle. It should be noted that the space requirements are based on more detailed information about the weight of the animals than we have access to by just knowing the category. The slaughterhouse needs to know the number of each animal type and category to be collected from each farmer in order to plan both the collection of animals and the production process at the abattoir.

The farmer notifies the slaughterhouse some time in advance when he wants to deliver animals for slaughter, depending on a given time limit. The time limit for ordinary registration is usually some hour on Tuesday. The animals are then supposed to be collected and slaughtered during the following week. For bovine and ovine, a second type of registration, long-term registration, is encouraged by means of a premium. The time limit is then usually Friday afternoon. The farmer chooses a basis week for collection, and the slaughterhouse may collect bovine in a five-week period and ovine in a three-week period, in both cases with the basis week in the middle. All animals of the same category should be picked up at a farm in a single visit.

The relevant time horizon for the problem starts when the farmer registers animals for slaughtering and ends when the animals are ready for slaughter at the abattoir. The time for collection may be decided in several different ways:

- depending on the type of registration (ordinary or long-term), animals may be collected during one specific week or during a period of three or five weeks;
- some animals need to be collected on a specific day or during a two or three day period;
- some slaughterhouses have fixed days when they collect animals from different areas;

- when registration is done by telephone, the day for collection is often decided at once; and
- for the long-term registered animals, it must be decided during which week they should be collected.

This is usually done before any routes are constructed.

Vehicle fleet

Some slaughterhouses have their own fleet of vehicles, but can also hire private drivers with own vehicles located at their home places one by one if necessary. Some slaughterhouses only hire vehicles from third-party transport companies located in different areas.

The slaughterhouses and the transporters have vehicles of varying capacities. The vehicles have a fixed number of compartments (usually three) with a fixed area in each section. Most sections may be split in two tiers; some may be split in three. Bovine will in most cases need the full height of a compartment, while pigs and ovine may be stacked in two or three tiers. Some of the vehicles have trailers. It is not allowed to leave the trailer behind and collect it later if there are animals on board, so the trailers may be viewed merely as an extension of the vehicle capacity. Vehicle with and without a trailer may need different amounts of time to load and unload the same amount of animals due to different manoeuvring characteristics.

Restrictions on vehicle load, and loading constraints

The various vehicles' capacities require restrictions on the total load carried by the vehicle due to the orders with high volume. The fact that many animal categories are considered makes the capacity constraints more complicated. For most of the vehicles it is possible to split the sections into tiers, which means that the vehicle capacity is dynamic and may be adjusted to the animal categories. The driver performs this adjustment during tours by lifting and lowering the floors in the sections. Some vehicles may not be able to visit all farms due to their size, weight or maneuverability.

Multiple routes per vehicle

The vehicles are normally used for several routes per day, usually two or three. For the first route of the day the vehicle may originate at the slaughterhouse or at the driver's home, the following routes start from the slaughterhouse. All vehicles terminate their routes at the slaughterhouse except possibly the last route of the day, which may end at the driver's home. The vehicles need to be cleaned between routes.

Animal welfare

Some preliminary results of the ongoing study, conducted at the Norwegian Meat Research Centre (Midtveit, 2004), show that the handling of animals during transportation both influences the welfare of the animals and the quality of the meat, and thereby the value of the final product. Animal welfare can be measured in different ways: the number of casualties during transport, the number and types of injuries, and observation of animal behavior during transport. Meat quality is measured by analyzing the pH value of the meat and other physiological tests. Naturally, the animal welfare is affected by numerous factors such as the time animals spend on the vehicle, the way the animals are mixed during transportation and the time animals are waiting until they are slaughtered.

There is a general rule saying that no animal is allowed to stay on the vehicle for more than eight hours. In addition, it is beneficial to keep the time on the vehicle as low as possible for certain categories of animals. The importance of how long the animal stays on the vehicle varies over the categories of animals. For some animal categories, it is beneficial to keep the lair time (a period between animal's arrival at the abattoir and slaughter) as short as possible, or at least avoid staying overnight, in order to keep the animal welfare, and thereby the meat quality, at a high level. Mixing of certain animal categories on a vehicle or in the same compartment of a vehicle is not allowed for animal welfare reasons. It is not allowed to mix animals with and without horns, even if they belong to the same category. Pigs from different herds should not be in the same compartment to avoid fighting.

The different farms to be visited may have different health statuses, which imposes restrictions on the visiting order. Some farms have a health status that requires them to be visited by an empty and clean vehicle (like breeding herds), and thus they have to be the first farm on the tour. Other farms may have disease in the herd that requires that no other farms are visited by the same vehicle afterwards until the vehicle is cleaned and disinfected, meaning that they have to be the last farm on the tour.

Crew requirements

Drivers who transport living animals are not comprised by regulations for driving period and rest period. However, when multiple routes per day are performed, it may be necessary to put restrictions on the route duration to satisfy the crew requirements about work day and to balance the routes.

Slaughtering plan

Each slaughterhouse processes one or more animal types. If more than one type is processed, there is a separate production line for each animal type. For each production line, its capacity measured in animals per hour is known. One or more of the production lines may operate simultaneously. Using information from farmers about animals registered for collection in a certain week and the capacity of each production line, the slaughterhouse creates a slaughtering plan for this week, giving the numbers of the different animal types to be slaughtered each day. The slaughter plan tells what production lines to operate and at what rate during certain time period during the day. For the slaughterhouses it is important to have a continuous production process throughout the day, which means that there must always be a sufficient number of the different animal types available. The cost if you run out of animals and must stop the production line is large, so it is crucial that this situation does not occur. The slaughterhouses handle this problem by holding and replenishing the stock of animals in the lair so that the storage never empties. The routes thus have to be planned so as both to avoid stock out situations at the slaughterhouse and to avoid situations where animals are waiting too long before slaughter.

Inventory

When the animals arrive at the abattoir, they are unloaded and laired until the time of slaughter. Lair serves as a temporary store between farm and slaughter. The capacity of the lair is limited and differs for different animal types. Generally, there is room for more ovine than bovine, not only because of their size but also because the bovine requires stalls instead of pens. The animals may be kept in the lair overnight, but not for more than one night, and with as few animals as possible. Inventory must be large enough to ensure a stable production the morning of the following day. In order to have enough animals to start the slaughter process on Monday morning, some livestock collection is done on Sunday afternoon. No animals can be laired over the weekend.

Criteria for cost-effectiveness

Criteria for the cost-effectiveness of live stock transportation can be different: the monetary global transportation cost, dependent on the global distance traveled (or on the global travel time); the capacity utilization of vehicles, dependent on the number of vehicles required to serve all the farms; the quality of meat; or any weighted combination of these.

Problem size

The size of the problem, measured in number of farms and number of routes, may vary a lot for different slaughterhouses at different times of the year. The size also depends on the length of the planning horizon, which varies from one day to a week. For Gilde Hed-Opp at Rudshøgda and Fatland Jæren at Hommersåk, which are used as examples in the project, the number of farms visited per day will typically be between 40 and 200. The number of visits per route is usually from two to six, giving the number of routes per day between 8 and 40. This means that problem instances including data for one week may have up to 1,000 farms, and thus need up to about 200 routes.

Daily planning routine

Now the routes are constructed in several ways. If transport companies are hired to collect animals, they are given all relevant information about what farmers to visit during a week and the number of animals to collect from each of them. Then they are told how many animals of the different types to bring to the slaughterhouse during different time periods each day. The transport companies themselves decide how to collect the animals during the week to meet the demand from the slaughterhouse.

If the slaughterhouse has its own vehicles and drivers, or if drivers with vehicles are hired one by one, the drivers are told what farmers to visit on each tour. The drivers are free to decide the order of visits on each tour, as they usually have the necessary local knowledge.

When the planner at the slaughterhouse constructs the tours, he takes several considerations in account:

- avoid interruptions in the production process and provide smooth supply of animals during the day;
- visit the farmers that are close on the same tour;
- collect only one animal type per tour if possible; and
- each driver/vehicle has his own district to operate in.

The route planning is usually done for one day at the time, one or two days in advance. Visiting days for all farmers are planned before the first routes are set up, but adjustments are made if this is necessary to have better routes.

Relevant optimization problems in the literature

The part of the problem dealing with allocations of transportation tasks to a fleet of vehicles with simultaneous routing for each vehicle represents the classical VRP (Dantzig and Ramser, 1959). The VRP is a computationally hard optimization problem with high industrial relevance. In the basic version of the VRP, the capacitated VRP (CVRP), all the customers correspond to deliveries and the demands are deterministic, known in advance, and may not be split. The vehicles are identical and based at a single depot, and only capacity restrictions for the vehicles are imposed. The objective is to minimize the total cost (i.e. a weighted function of the number of routes and their length or travel time) to serve all the customers. Earlier (Toth and Vigo, 2002) three different basic modeling approaches have been proposed for the CVRP in the literature: vehicle flow models, commodity flow models and set-partitioning formulations.

The vehicle flow formulations may be adapted to model some variants of the CVRP, for example, may take into account the case of a non-homogeneous fleet, where each vehicle may have a different capacity, or incorporate the maximum distance/route duration constraints. The binary variables used in these models are associated with each road section in the road network and indicate the number of times the road section between two specific customers is traversed by the specific vehicle. These formulations have also been used to model such extension of the VRP as VRP with time windows (VRPTW), where the service at each customer must start within an associated time window and the vehicle must remain at the customer location during service (Cordeau *et al.*, 2002). In the mathematical formulation for the VRPTW, the continuous time variables were first introduced, specifying the start of service at the customer when serviced by a certain vehicle.

The literature on the VRP with multiple use of vehicles (VRPM) is very scant. But we find a heuristic algorithm (Taillard *et al.*, 1986), and a modification of the VRPTW model (Hajri-Gabouj and Darmoul, 2003), taking into account multiple uses of vehicles with double time windows for the depot.

In the literature on the numerous and diverse applications of vehicle routing, two industries can be of interest to us concerning the interaction between routing and production: dairy milk collection and newspaper delivery.

In the dairy milk collection problem, there is an important similarity with the livestock collection problem: health regulations require that the time elapsed from the first milk pickup on the route until the milk tanker is emptied should not exceed a season-specific limit, thus protecting the quality of the milk. In Sankaran and Ubgade (1994) we find incorporated a constraint on the time between the first stop and return to the dairy for every tanker trip for the daily routing problem, mentioning that improvement in milk quality would more than offset the increase in the global transportation cost. The queuing situation that occurs when the tankers return to unload milk at a processing plant and there are not enough pumps to empty the tankers has also been discussed earlier (Basnet *et al.*, 1997). The decision aid designed helps the schedulers in the allocation of tankers to the routes in order to alleviate pumping bay congestion by minimizing the make-span.

The studies of the newspaper delivery problem (NDP) (Holt and Watts, 1988; Hurter and Van Buer, 1996) explicitly considered the interaction between the routing and production decisions. The challenge of newspaper delivery is to work with tight time windows, since newspapers are a perishable commodity. For a large newspaper, printing presses generally run for several hours, and as the printing progresses, vehicles are being loaded and dispatched. Generally, the newspapers for the remote transfer points (which have the tightest due times) must be printed and dispatched first. This suggests sequencing production by the geographical locations of the transfer points. In Holt and Watts' heuristic algorithm the critical start times for the routes (equal to the latest time the route can start and still meet its deadlines) are used to determine the loading sequence at the dock. Hurter and Van Buer considered the routing problem studied as a VRPTW, where the route start time depends on the completion time for the loading of the vehicle, which is computed from the production schedule. In their heuristic solution, the production schedule is altered if infeasibility (with respect to delivery time windows) arises for the route constructed.

These NDP studies sought to coordinate production and vehicle schedules governed by demand and time windows of the transfer points. The rate of production (number of papers per minute) in the NDP is an important parameter in determining the planning horizon. In the livestock collection problem the situation is somewhat opposite: the delivery point is a slaughterhouse, and the termination time for the specific route cannot be associated with some time window, stating that "this route must be back at the depot between time e and time l in order to be feasible." We will not have a situation where a single route is feasible or infeasible because of time windows constraints, it will rather be a question if the whole solution, viewed as a set of routes, is feasible or not with respect to production plan and the lair capacity. If the slaughterhouse is processing pigs during an eight hours day at a rate of 100 animals per hour, fetching 50 from the lair every half an hour, then a feasible solution must secure that at least another 50 pigs are available every half an hour.

It should be noted from the previous review that, also several similar to our problem aspects were integrated to the VRP, solution methodologies are all heuristic, due to high complexity of the problem. It should be also mentioned that there are publications where mathematical models are used to combine the original nature of the problem with production/inventory management, for example, in ship routing (Christiansen, 1999) or in material handling scheduling (Khayat *et al.*, 2003).

Assumptions and simplifications

As described, the original problem is too complex to attack directly. On the first stage of the study (Gullberg and Hovden, 2004) we did not attempt to include all factors and restrictions described above in the optimization model. Rather, we attempted to determine what characteristics and operational constraints are essential for the problem, and to formulate a simplified basic optimization model in order to be able to test it on small instances, while leaving extension and detailed elaboration of this basic model for the future stages of the project. The assumptions and simplifications made reflect our experience with operations of the slaughterhouses on which our study is based.

Specification of general characteristics

We consider the decision problem analyzed as a cost minimization deterministic problem in which the discrete decisions: when to visit the farms for collection of animals and when to unload the vehicles after arrival at the slaughterhouse, are the most important ones.

From the problem study we have concluded that information on the collection demand becomes available at least some days in advance (early registration) or several weeks in advance (long term registration). As the farms normally expect to be visited within a week, and because several days' routing has larger potential for cost reduction than daily routing, we consider a several days' planning horizon.

In the basic version of our formulation we assume that only a single animal category can be transported in each vehicle, thus reducing the number of commodities to one animal category. The initial rationale for this assumption was to reduce the number of variables in the model thus simplifying the solution, as well as to relax the animal welfare restriction on prohibited mixture of different animal categories in the same vehicle. We also react on the recent specialization of slaughterhouses on only one animal type (category) and on our interviews with the transporters reporting that each vehicle is almost always loaded with a single animal category. Under this assumption, the original problem decomposes by animal category into similar problems of smaller size. Thus, in our model we consider one specialized slaughterhouse as a depot where demands in a certain geographical zone should be collected and at the same time as a production line for the corresponding animal category.

It is assumed that the demand data is known with certainty by the start of the planning period, i.e. the location of the farms to be visited and the number of animals to be picked up during this period. The service time per animal can vary at different farms, but the total service time is proportional to the number of animals. We assume that each farm with registered demand is served by one visit of one vehicle during one day of the planning horizon.

For the simplification purpose we assume that the vehicle fleet belongs to the slaughterhouse and is located there. Vehicles of different sizes and capacities (measured in maximal number of animals to carry) will be considered. The capacity of each vehicle is constant, i.e. could not be exceeded during transportation. We assume that the demand to be picked up at a farm never exceeds the capacity of every available vehicle, i.e. it is possible to pick up all the animals from the same farm (herd) in a single visit. It is also assumed that the total capacity of the vehicle fleet is large enough to serve all those farms who have declared their demand to be picked up during the planning horizon.

Honored operational constraints

In this sub-section we specify what operational constraints are considered in our basic model. From the first group of restrictions the capacity constraints remain essential for our model, while restrictions on access to farms will be not considered, i.e. all vehicles can visit all farms.

We will assume that overnight trips are not possible, and each vehicle can make at most three round tours during the day. All vehicles originate the first route of a day from the slaughterhouse; all vehicles terminate every route at the slaughterhouse and should be discharged for the day by the end of the working day. The starting time from the slaughterhouse for the first route of a day is the same for all vehicles. There is some time required for cleaning the vehicle between routes.

From the group of constraints named as animal welfare, the following are taken into consideration. There will be a limitation on the time elapsed from the first animal is collected on the route until the vehicle returns to the slaughterhouse for unloading. After being unloaded, the animals can either be send immediately for slaughter or wait in the lair a certain time interval until being slaughtered. The amount of animals waiting in the lair at the end of the day must not exceed the amount that is going to be slaughtered in the first time interval of the next day. During transportation, animals from the different herds can be mixed on the same vehicle. Requirements on the visiting order due to the different health status for farms will be not considered on this stage. For the time being, it is not known how the animal welfare depends on the sequence in which farms are visited, i.e. the sequence in which animals are loaded on the vehicle. Incorporation of precedence restrictions on the animals loaded on a vehicle and on the farms visited is part of in our ongoing research.

In vehicle routing terminology, the assumptions and simplifications presented above permit us to formulate the routing part of the livestock collection problem as a multi-period, single depot, deterministic demand VRPM, capacity restrictions and route duration constraints.

Production

Owing to the specialization, only one production line is assumed at the slaughterhouse. The production (slaughtering) plan, given as the number of animals to be slaughtered in every specific time interval of every specific day, is assumed to be known in advance. In this way, each day is divided into discrete time intervals. We assume all time intervals of the same duration, and a known exogenous slaughtering plan. The minimal number of animals required to be kept in the lair at any time interval of any day is given (as a safety stock to avoid stock-outs in production). The number of animals in the lair before the start of the planning horizon is known, representing the initial inventory.

In production planning terminology we have a dynamic production planning problem, with the planning horizon of one day divided into a certain number of time intervals, where the total number of animals delivered to the slaughterhouse in a particular time interval is supposed to be handled in that interval, through slaughtering or held over in the lair for later use.

Objective

The objective we declare for the basic model is to minimize the global travel time (as it is related to the global transportation cost). The distance matrix is assumed to be available so that the travel times between farms and slaughterhouse would be estimated. Travel times are assumed not vehicle- or load-dependent. At the initial stage of the study we are not concerned with minimization of vehicles utilized and designing the optimal fleet configuration. Estimation of values of possible fixed costs, applied if a vehicle is send out on at least one route for a day, and incorporating them in the model is left for the next stage of the study. Minimization of the penalties associated with bad treatment of animals during transportation and prolonged waiting time in the lair, and thus maximization of meat quality, is a subject of ongoing research, as for the time being we are just in the beginning of the study of this effect.

Mathematical formulation of the basic problem

We formulate the basic variant of the livestock collection problem as a mixed integer linear programming (MILP) problem. First, we present the arc flow constraints: the spatial and the temporal aspects, including multiple usages of vehicles per day and welfare requirements on the travel time. Second, the constraints linking the incoming flow of animals and production, and the inventory constraints are introduced. Then, the objective function is presented, and the results of testing.

Routing

For the routing part of the model, we have chosen to use notation similar to that what has been presented earlier (Toth and Vigo, 2002) for the vehicle flow formulation of CVRP and its VRPTW extension. To impose the multi-period (multi-day) nature of our problem and the assumption about multiple routes per vehicle per day, we explicitly indicate on which day and on which route during that day a vehicle travels directly between two points. In this way, we introduce the five-index flow variables in our basic model. The variables will be defined in order of their appearance in the model description. Sets describing which subscripts to sum over in various constraints for simplicity are omitted.

Sequencing of farmers on routes/spatial aspects

The livestock collection problem is defined on the road network, where the nodes correspond to the farms and the slaughterhouse (further referred as the depot). The farms will be denoted by i or j, and the depot node by 0. For each pair of nodes i and j of the network, an arc (i, j) is defined, associated with the shortest path starting from node i and arriving at node j in the road graph. We let k denote the vehicle. Since every vehicle may be used on every day of the planning horizon and on the same day can make a given number of round trips, we let p denote the day of the planning horizon and r the number of the route during the same day. The number of days in the planning period and the number of routes per day are given as the fixed parameters of the problem. To describe in what sequence farms should be visited on a route, we use binary variables x which indicate if one farm is visited immediately after another farm by certain vehicle on certain day on certain route number. In other words, variable x_{ijkpr} takes value one if vehicle k travels from node i directly to node j on its route number r on day p of the planning horizon, and takes value 0 otherwise. The constraints below impose the spatial characteristics of the routes.

One visit

The following constraint impose that each farm is visited exactly once during the planning horizon. In other words, each farm appears exactly in one route, performed by some vehicle on some day of the planning horizon.

$$\sum_j \sum_k \sum_p \sum_r x_{ijkpr} = 1, \quad \forall i \tag{1}$$

Route structure

As a rule, each route performed by each vehicle starts and ends at the depot, i.e. is a round tour. To express this property, the depot is represented by two nodes: starting depot 0 and returning (fictitious) depot $n + 1$, where n is a number of farms to be served within the planning horizon. The following restrictions state that each vehicle's route on every day of the planning horizon starts from the starting depot and ends in the returning depot. Not to press all vehicles to perform the definite number of routes per day, i.e. to permit vehicles to remain at the depot, the virtual round trips between the starting depot and the returning depot are allowed. To make it possible, the travel time between the starting and the ending depots is set to zero.

$$\sum_j x_{0jkpr} = 1, \quad \forall k, p, r \tag{2}$$

$$\sum_i x_{i,n+1,kpr} = 1, \quad \forall k, p, r \tag{3}$$

The next restriction ensures connectivity of every route, or, in vehicle routing terminology, continuity of the flow on the path to be followed by certain vehicle on certain day on certain route number. It says that the same vehicle enters and leaves a given farm if the farm is in its route on a day.

$$\sum_j x_{ijkpr} - \sum_j x_{jikpr} = 0, \quad \forall k, i, p, r \tag{4}$$

Load feasibility

We let C_k denote the vehicle k's capacity, expressed in the maximal number of animals the vehicle can load. We associate with farm i the nonnegative demand d_i, which represents the load (the number of animals) that must be picked up at farm i's location. Both the starting and the ending depot have fictitious demands equal to zero. To ensure that along each route on every day, the current load of the associated vehicle cannot exceed the vehicle's capacity, the following restriction states that the total load of vehicle k, equal to the sum of the demands of all farms served on route number r the vehicle performs on day p should not exceed the capacity of the vehicle.

$$\sum_i d_i \sum_j x_{ijkpr} \leq C_k \quad \forall k, p, r \tag{5}$$

Temporal aspects

The main issue of our study was to link the routing problem and the production process into one integrated model. Production requirements determine scheduling of vehicles' returns from the routes and, accordingly, scheduling of visits along the routes. Thus, we need to specify the time when farms are visited on routes, which leads to a mixed routing and scheduling problem. To describe the time of the visit, the continuous time variable w is used, specifying the arrival time (the start of service) at farms/depot. In other words, variable w_{ikpr} specifies the arrival time at farm i when serviced by vehicle k on day p on route number r (note that $w_{n+1,kpr}$ specifies the end time for route number r performed on day p by vehicle k). The check of the operational time constraints, imposed on the routes (and evaluation of the global travel time) requires knowledge of the travel time between each pair of farms and between the depot and the farms. We define a nonnegative parameter t_{ij} as a travel time for arc (i, j), computed as the sum of travel times of the arcs belonging to the shortest path from node i to j in the road graph. Moreover, we denote by l_i the loading time per one animal for each farm i, given in the number of time instants used to load one animal on the vehicle. The same parameter for each depot is set to zero. Further, we introduce constraints that take into account the temporal aspects.

Compatibility requirements between routes and schedules

To ensure feasibility of the schedules for all routes and compute the arrival time at every node, we impose the condition:

$$x_{ijkpr} = 1 \Leftrightarrow w_{jkpr} = w_{ikpr} + l_i d_i + t_{ij} \quad \forall (i,j), k, p, r \tag{6}$$

Here we assumed that no waiting is permitted at any node. Condition (6) can be rewritten as follows:

$$x_{ijkpr}(w_{ikpr} + l_i d_i + t_{ij} - w_{jkpr}) = 0, \quad \forall (i,j), k, p, r$$

The binary conditions on x-variables allow constraints above to be linearized as:

$$w_{ikpr} + l_i d_i + t_{ij} \leq w_{jkpr} + (1 - x_{ijkpr})M, \quad \forall (i,j), k, p, r \tag{6a}$$

$$w_{ikpr} + l_i d_i + t_{ij} \geq w_{jkpr} - (1 - x_{ijkpr})M, \quad \forall (i,j), k, p, r \tag{6b}$$

where M denotes a large number. We omit the explanation how to set a value for M.

Starting and ending times

The following constraint sets the starting time from the depot for all vehicles on their first route of the day. For simplicity, we assume that all vehicles leave the slaughterhouse for the first route every day at time instant zero, that is:

$$w_{0,k,p,1} = 0, \quad \forall k, p \tag{7}$$

The next constraint states that every route should be terminated by the end of the working day, i.e.

$$w_{n+1,k,p,r} \leq L, \quad \forall k, p, r \tag{8}$$

where constant L represents the latest possible arrival at the depot.

Multiple use of vehicles

By using end time variables for the routes we can implement a multiple use of vehicles within a day, which is essential for our problem.

To ensure that every current vehicle's route on a day should not start before the same vehicle has arrived from its immediate preceding route of the day, and to compute the starting time for the subsequent route, the following constraint is used:

$$w_{n+1,k,p,r} + s(1 - x_{0,n+1,k,p,r}) = w_{0,k,p,r+1}, \quad \forall k, p, \forall r = 1, \ldots, R-1 \tag{9}$$

Here R denotes the number of routes a vehicle can make per day, and parameter s represents the service time at the depot between the tours measured in time instants, used for unloading, washing and setting the vehicle ready for its next tour. Constraint (9) computes the starting time for every subsequent route of a vehicle by adding the service time to the time of arrival at the depot from the preceding route. Vehicles that retain at the depot are forced to perform virtual routes between the depot nodes and for these the service time is not added. When the virtual route between depots takes place, variable $x_{0,n+1,k,p,r}$ takes value 1 and the term with the service time is removed. The starting time of the next trip is thereby defined.

Welfare requirements

Constraint related to the legislation concerning the animals' total time in transport can be written as:

$$\sum_i \sum_j (l_i d_i + t_{ij}) x_{ijkpr} - t_{0j} x_{0jkpr} \leq Q, \quad \forall k, p, r, \ \forall j \tag{10}$$

For every route, the time elapsed from the first stop on a route for pickup until the vehicle terminates the route at the depot, calculated as the total duration of the route minus the travel time between the depot and the first visited farm, should not exceed the certain limit Q, defined by animal welfare regulations.

Linking routing with production/inventory

The production scheduling requirements (the slaughtering plan) and the routing scheduling decisions interact in two ways. First, the amount to be slaughtered in a certain time interval governs the total amount to be delivered (input flow of animals) within this interval. Second, the time interval when the vehicle should be unloaded determines the end time for the route. Thus, the daily slaughtering plan should orchestrate the vehicles' arrivals at the depot and determine their total loads at returns.

To link routing and production, production time intervals are introduced for each day. It is assumed that these time intervals are of the same duration (for example, one hour). We enumerate intervals by natural numbers, and denote by t the time window that elapses from the end of time interval $(t-1)$ to the end of time interval t. To describe the incoming flow of animals within each time interval, binary time variable z is used, indicating if vehicle arrives at the depot within a certain interval on a certain day. In other words, variable z_{kprt} takes value 1 if vehicle k ends its route number r on day p within interval t, and takes value 0 otherwise. To measure inventory level, variable I_{pt} is introduced, specifying the level of inventory (number of animals in the lair) at the end of interval t on day p. The parameter S_{pt} gives the number of animals to be slaughtered during interval t on day p. The following constraint calculate the inventory level at the end of interval t from the inventory level at the end of interval $(t-1)$, the total quantity delivered/discharged by all vehicles in interval t, adjusted for the production/slaughtering in t, i.e.

$$I_t = I_{t-1} + \sum_k \sum_r z_{kprt} \sum_i \sum_j d_i x_{ijkpr} - S_{pt}, \quad \forall p, t \tag{11}$$

We replace the quadratic term by introducing a binary variable v_{ijkprt} and adding three new restrictions which force v_{ijkprt} to take value 1 if and only if both z_{kprt} and x_{ijkpr} are equal to 1:

$$v_{ijkprt} \leq z_{kprt}, \quad \forall i, j, k, p, r, t \tag{12a}$$

$$v_{ijkprt} \leq x_{ijkpr}, \quad \forall i, j, k, p, r, t \tag{12b}$$

$$z_{kprt} + x_{ijkpr} - v_{ijkprt} \leq 1, \quad \forall i, j, k, p, r, t \tag{12c}$$

The inventory level should guide the vehicles' arrivals to avoid halts in the production, i.e. so that it will be always a safety stock of F animals in the lair. It is imposed by constraint:

$$I_{pt} \geq F \quad \forall p, t \tag{13}$$

We state that inventory level at the beginning of next day will be the same as at the end of the previous day by:

$$I_{pT} = I_{p+1,0} \quad \forall p = 1, \ldots, P - 1 \tag{14}$$

Constant P denotes the number of days in the planning horizon. The initial inventory at the start of the planning horizon is supposed to be given.

To calculate inventory level for time interval t in constraint (11) above, it is necessary to know whether the arrival time at the depot $w_{n+1,kpr}$ for vehicle k on its trip number r on day p is within interval $(t-1, t]$ or not. The most difficult part of modeling was to link the continuous arrival-time variable w and the binary route-end variable z for each route.

Let $W_{n+1,kpr}$ be a value for the arrival-time variable $w_{n+1,kpr}$. We impose the condition:

$$W_{n+1,kpr} \in (t-1, t] \Rightarrow z_{kprt} = 1 \tag{15}$$

for route number r vehicle k terminates on day p, by introducing new indicator variables α, β and five coupling constraints. Variable α is used to indicate if $w > t - 1$, and β to indicate if $w \leq t$, i.e.

$$\alpha_{kprt} = 1 \Leftrightarrow w_{n+1,kpr} > t - 1$$

$$\beta_{kprt} = 1 \Leftrightarrow w_{n+1,kpr} \leq t.$$

Each of these logical conditions is translated into two linear constraints, here omitted. To fulfil the binding procedure, it should be required that:

$$\alpha_{kprt} = 1 \text{ and } \beta_{kprt} = 1 \Rightarrow z_{kprt} = 1,$$

or

$$\alpha_{kprt} + \beta_{kprt} - 1 = z_{kprt}.$$

The objective function

The problem is to minimize the global travel time (related to the global transportation cost), and no production/inventory cost exist. The objective function to minimize is:

$$\sum_i \sum_j \sum_k \sum_p \sum_r t_{ij} x_{ijkpr} \tag{16}$$

Concluding remarks and further research

In this paper, we have presented a real livestock collection problem that can be modeled as a mixed integer programming problem. The model integrates two components of supply chain management: routing of supply vehicles and production scheduling at the abattoir, where production plans determine transportation. Because of the complexity of the problem, some assumptions and simplifications have been made at the initial stage of our study to formulate the basic version of the model. The interest of the formulation is that it lifts any possible ambiguity about the precise problem definition. As a result researches who will work on this problem at the future stages of the project will have a precise characterization of it.

The model can be fed directly to standard commercial optimization software for mixed linear programming. However, due to its complexity, only small sized data instances have been solved to optimality. Previously (Gullberg and Hovden, 2004) used CPLEX to solve the model for seven farms for a two days planning horizon, where each day is split into four production time intervals, for two vehicles with maximum three routes per vehicle per day, and found that adding the eighth farm made the problem too hard for CPLEX to find an optimal solution in reasonable time. We are aware that the proposed type of formulation is mostly descriptive in the sense that it cannot be solved directly for instances of the size that arise in practice. To be able to solve real-life sized instances of the livestock collection problem we hope to develop efficient approximation algorithms and heuristic methods at the next stages of the project. However, the MILP formulation could be used to assess the quality of heuristics on small size instances.

There are several directions of our future study of the problem. At first, we continue the modeling process, reversing some of the simplifications in order to obtain a problem description that corresponds more closely to the real-life problem. The issues to be modeled are: considering different animal categories; including constraints prohibiting mixing of different animal types/categories on the same vehicle, precedence constraints, dynamic vehicle capacity, minimization of the fleet size, load-dependent costs.

Later, a goal programming approach could be used to introduce some of the issues affecting the quality of the meat (such as the time animals spend on the vehicle or/and kept in the lair and the number of animals kept overnight) as additional goals in the problem.

For the time being all the factors in the livestock collection problem are treated as deterministic. In the future, we may take the stochastic nature of some of these factors (for example, travel time) into consideration in order to have solutions that are more robust. However, such factors as planning horizon (normally one week), collection demands and production plan, according to our examination of planning routines at the two example abattoirs, are supposed to be known with certainty.

Another direction of the study is to try to solve the model using constraint programming, or apply Dantzig-Wolfe decomposition approach with some model adjustments. However, the main direction is to develop, on the basis of the extended model, effective solution methods to solve instances from the real world. It is planned to develop a tabu search based heuristic for the problem, which is in itself a major undertaking. Later, column generation based approach will be tried. The latter approach could be exact, heuristic, or both. Obtaining a complete data set on a real-life instance is another challenging task of the project.

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