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On the integrated production, inventory, and distribution routing problem

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The integrated Production, Inventory, and Distribution Routing Problem (PIDRP) is concerned with coordinating production, inventory, and delivery operations to meet customer demand with the objective of minimizing the cost. The PIDRP considered in this paper also involves heterogeneous transporters with noninstantaneous traveling times and multiple customer demand centers each with its own inventory capacity. Optimally solving such an integrated problem is in general difficult due to its combinatorial nature, especially when transporter routing is involved. We propose a two-phase solution approach to this problem. Phase I solves a mixed-integer program which includes all the constraints in the original model but with the transporter routings being restricted to *direct shipments* between facilities and customer demand centers. The resulting optimal solution to phase I is always feasible to the original model. Phase II solves an associated consolidation problem to handle the potential inefficiency of direct shipment. The delivery consolidation problem is formulated as a capacitated transportation problem with additional constraints and is solved heuristically. Unlike the classical decoupled approach, this two-phase approach does not separate the optimization for the production lot sizes and the transportation schedules. Its main advantage lies in its ability to *simultaneously* coordinate the production, inventory, and transportation operations of the entire planning horizon, without the need to aggregate the demand or relax the constraints on transportation capacities. This enables us to quickly identify a quality suboptimal solution to the original complex problem. The suboptimality is, however, due to the simplified assumption that in phase I only direct shipment is deployed, which is then partially corrected for by the effort of phase II. We evaluate the performance of this proposed two-phase approach and report on its application to a real-life supply network.

1. Introduction

Companies are becoming increasingly aware of the importance of improving their supply chains. The coordination and integration of the production (supply), inventory, and distribution (demand) operations is widely perceived to be a route to obtain a competitive advantage (Thomas and Griffin, 1996; Fumero and Vercellis, 1999; Brown *et al.*, 2001; Lee and Whang, 2001; Bloomquist *et al.*, 2002; and Gupta *et al.*, 2002). This issue becomes especially critical after companies merge since system redundancy (in terms of a low capacity utilization) and inefficiency (in terms of a high distribution cost) are inevitably introduced into the combined supply/distribution network. A representative example of this is the recent merger of Nabisco and Kraft Food Inc. who had to expend a large amount of effort to create a new joint supply network (Harps, 2003).

In this study, we are interested in the following integrated Production, Inventory, and Distribution Routing Problem (PIDRP). We are given a single product and a set of plants, each with its own production capacity, inventory capacity, raw material supply contract, inventory holding cost, and production cost. Associated with each plant is a heterogeneous fleet of transporters, each with its own operation cost, required loading/unloading time, loading capacity, available time, and traveling speed. We are also given a set of customer demand centers (DCs) located over a wide

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geographical region, each with its own demand per time period in the planning horizon, its own inventory capacity, holding cost, and safety stock requirement. The problem is *to determine the operation schedules to coordinate the production, inventory, and transportation routing operations so that the customer demand, transporter travel time and capacity constraints, plant production, and inventory constraints are all satisfied, while the resulting operation cost (i.e., the sum of production, inventory, and transportation cost) over a given planning horizon is minimized*.

Our research on this problem was motivated by the post-merger demand-supply coordination problem encountered by a leading chemical company in North America. After acquiring a large manufacturing facility from one of its competitors, the company experienced operational difficulties including: (i) there being no guideline as how much should be produced at an individual facility since the production costs, material costs, supply contract, logistics partner contract, production costs, and inventory capacities are different at each facility; and (ii) a large number of DCs (most of them being ocean terminals) to be supplied. The DCs have individual inventory capacities, geographical locations, and demand patterns and consequently different safety stock requirements over a year. Due to the finite nature of production capacity, plants often use any excess capacity to build up inventories. Similarly, the DCs build up inventories whenever arrivals exceed consumption over a period. Very often products are directly moved from the delivery vessel to a waiting ship for export to a foreign market. Any surplus stock is held in the customer's inventory and can be used at a later time often saving a delivery from the producer. It is unclear which DC should be supplied by which facility or group of facilities, and how much inventory DC should hold. The ships used to transport the chemicals are owned by the individual facilities. These vessels have different capacities, operation costs, travel speeds, loading/unloading times, and ability to access particular DCs in different seasons (the shipping lanes may freeze over in winter). Whenever the company's own fleet capacity is insufficient, it charters vessels from a third-party company whose rates depend on the tonnage and travel distance. These rates are generally higher than in-house shipping costs and thus failing to optimally solve the integrated planning/scheduling problem can result in significant additional expenditure.

Motivated by this application, we consider in this study a hypothetical PIDRP that contains *multiple production plants*, all of which are able to produce the same product, and *many customers*, each with a specific demand for the product in each time period over a T-period planning horizon. Each plant has a limited production capacity and owns a fleet of transporters each with a different capacity, speed, and availability. The customer demands are assumed to be deterministically known over the periods in T. For the in-house transporters, the operational costs are proportional to the travel time rather than the shipping quantity. However, for chartered transporters, the industry practice is that costs are based on both the quantity shipped and the distance traveled. We will assume that there is no fixed cost associated with transporter usage and that each transporter can make multiple trips during each time period (a trip is defined as a sequence of DC locations a transporter visits beginning and ending at the same plant.) Different transporters (from either the same plant or different plants) can deliver to an individual DC in a period. It is also assumed that the plant inventory cost is based on an average inventory level, a DC's inventory cost is based on the ending inventory in each period, and the loading and unloading times are quantity-independent. Our aim is to develop an integrated operations plan, which minimizes production, inventory, and transportation cost, subject to all the constraints involved.

We use the following notation in our mathematical analyses.

Model parameters

I = set of plants;
J = set of DCs;
T = set of time periods in the planning horizon;
$V(i)$ = set of transporters owned by plant i;
$N(v)$ = maximum number of trips made by transporter v during each period;
$a_i(t)$ = production cost at plant i during time t;
$h_i(t)$ = inventory holding cost at plant i during time t;
$\hbar_j(t)$ = inventory holding cost at DC_j during time t;
c_v = variable shipping cost for transporter v (per hour);
$c^c_{i,j}$ = shipping rate (per unit load) for chartered transporters to go from location i to location j;
C_v = maximum loading capacity for transporter v;
$T_v(t)$ = available operation time (travel and loading/unloading) for transporter v during period t;
$t^v_{j,k}$ = traveling time from location j to location k by transporter v (includes loading time if location j is a plant, unloading time if location k is a DC);
$D_j(t)$ = demand of DC_j in time period t that must be satisfied by either the inventory at DC_j, or by the shipment that arrives during t, or by both (i.e., no backlogs);
$p^{\max}_i$ = maximum production capacity of plant i;
$s^{\max}_i$, $s^{\min}_i$ = maximum ending inventory capacity and safety stock (i.e., the minimum inventory) requirement, respectively, at plant i;
$z^{\max}_i$, $z^{\min}_i$ = maximum ending inventory capacity and safety stock (i.e., the minimum inventory) requirement, respectively, at DC_i.

Variables

$p_i(t)$ = production quantity by plant i during time t;
$s_i(t)$ = ending inventory of plant i at time t;

$z_j(t)$ = ending inventory of DC_j at time t;
$x^{v,n}_{i,j,k}(t)$ = equal to one, if transporter v of plant i visits DC_k immediately after visiting DC_j during its nth trip in time period t, $\forall i \in \mathrm{I}$, $v \in V(i)$, $n \in N(v)$, $j \in \{i\} \cup J$, $k \in \{i\} \cup J$, $j \neq k$, $t \in T$;
$g^{v,n}_{i,j,k}(t)$ = quantity carried by transporter v of plant i while it is traveling from DC_j to DC_k during its nth trip in period t, $\forall i \in \mathrm{I}$, $v \in V(i)$, $n \in N(v)$, $j \in \{i\} \cup J$, $k \in \{i\} \cup J$, $j \neq k$, $t \in T$;
$q^{v,n}_{i,j}(t)$ = quantity delivered by transporter v, $v \in V(i)$, to DC_j from plant i during its nth trip in period t;
$Q_{i,j}(t)$ = quantity shipped from plant i to DC_j by chartered transporters in period t.

Our integrated optimization problem is to minimize the sum of the plants' fleet transportation costs, chartered transporter shipping costs, production costs plus inventory costs, and DC inventory costs. The following mixed-integer program gives a formal definition of PIDRP with a single product (which we denote as problem (**P**) in the remainder of this paper):

$$\begin{aligned}\mathrm{P}: \min \sum_{t\in T}\sum_{i\in I}\sum_{v\in V(i)}\sum_{n\in N(v)}\sum_{j,k\in\{i\}\cup J, j\neq k} c_v t^v_{j,k} x^{v,n}_{i,j,k}(t) \\ + \sum_{t\in T}\sum_{i\in I}\sum_{j\in J} c^{\mathrm{c}}_{i,j} Q_{i,j}(t) + \sum_{t\in T}\sum_{i\in I} a_i(t) p_i(t) \\ + \sum_{t\in T}\sum_{i\in I} h_i(t)(p_i(t)/2 + s_i(t)) + \sum_{t\in T}\sum_{j\in J} \hbar_j(t) z_j(t),\end{aligned} \tag{1}$$

subject to
plant inventory balance constraints:

$$\begin{aligned} s_i(t) = s_i(t-1) + p_i(t) - \sum_{j\in J}\sum_{v\in V(i)}\sum_{n\in N(v)} q^{v,n}_{i,j}(t) \\ - \sum_{j\in J} Q_{i,j}(t) \quad \forall i \in \mathrm{I},\ t \in T, \end{aligned} \tag{2}$$

customer inventory balance constraints:

$$\begin{aligned} z_j(t) = z_j(t-1) + \sum_{i\in I}\sum_{v\in V(i)}\sum_{n\in N(v)} q^{v,n}_{i,j}(t) \\ + \sum_{i\in I} Q_{i,j}(t) - D_j(t) \quad \forall j \in J,\ t \in T, \end{aligned} \tag{3}$$

storage capacity and safety stock requirement constraints:

$$s^{\min}_i \le s_i(t) \le s^{\max}_i \quad \forall i \in I,\ t \in T, \tag{4}$$

$$z^{\min}_j \le z_j(t) \le z^{\max}_j \quad \forall j \in J,\ t \in T, \tag{5}$$

production capacity constraints:

$$0 \le p_i(t) \le p^{\max}_i \quad \forall i \in I,\ t \in T \tag{6}$$

trip integrity constraints:

$$\begin{aligned} \sum_{\substack{j \in \{i\}\cup J \\ j\neq k}} x^{v,n}_{i,j,k}(t) = \sum_{\substack{j \in \{i\}\cup J \\ j\neq k}} x^{v,n}_{i,k,j}(t) \\ \forall i \in I,\ v \in V(i), n \in N(v),\ k \in \{i\}\cup J,\ t \in T, \end{aligned} \tag{7}$$

$$\sum_{j\in J} x^{v,n}_{i,i,j}(t) \le 1 \quad \forall i \in I,\ v \in V(i),\ n \in N(v),\ t \in T, \tag{8}$$

transporter capacity constraints:

$$\begin{aligned} g^{v,n}_{i,j,k}(t) \le C_v x^{v,n}_{i,j,k}(t) \qquad \forall i \in I,\ j \in \{i\}\cup J,\ k \in \{i\}\cup J, \\ j \neq k, v \in V(i),\ n \in N(v),\ t \in T, \end{aligned} \tag{9}$$

commodity flow conservation constraints:

$$\begin{aligned} \sum_{\substack{j \in \{i\}\cup J \\ j\neq k}} g^{v,n}_{i,j,k}(t) - \sum_{\substack{l \in \{i\}\cup J \\ l\neq k}} g^{v,n}_{i,k,l}(t) = q^{v,n}_{i,k}(t) \\ \forall i \in \mathrm{I},\ v \in V(i),\ k \in J,\ n \in N(v),\ t \in T, \end{aligned} \tag{10a}$$

$$\begin{aligned} \sum_{j\in J} g^{v,n}_{i,j,i}(t) - \sum_{l\in J} g^{v,n}_{i,i,l}(t) = -\sum_{j\in J} q^{v,n}_{i,j}(t) \\ \forall i \in I,\ v \in V(i),\ n \in N(v),\ t \in T, \end{aligned} \tag{10b}$$

transportation duration constraints:

$$\begin{aligned} \sum_{n\in N(v)}\sum_{j\in\{i\}\cup J}\sum_{k\in\{i\}\cup J} t^v_{j,k} x^{v,n}_{i,j,k}(t) \le T_v(t) \\ \forall i \in I,\ v \in V(i),\ t \in T, \end{aligned} \tag{11}$$

non-negativily and integer requirement:

$$\begin{aligned} q^{v,n}_{j,k}(t) \ge 0,\ g^{v,n}_{i,j,k}(t) \ge 0, \qquad \forall i \in I,\ j \in \{i\}\cup J, \\ k \in \{i\}\cup J,\ j \neq k,\ v \in V(i),\ n \in N(v),\ t \in T, \end{aligned} \tag{12a}$$

$$Q_{i,j}(t) \ge 0 \quad \forall i \in I, \quad j \in J, \quad t \in T, \tag{12b}$$

$$\begin{aligned} x^{v,n}_{i,j,k}(t) = \text{binary} \qquad \forall i \in I,\ j \in \{i\}\cup J,\ k \in \{i\}\cup J, \\ j \neq k,\ v \in V(i),\ n \in N(v), \quad t \in T. \end{aligned} \tag{12c}$$

The first term in objective function (1) of the model defines the traveling cost of the company's own transporters, where $c_v t^v_{j,k} x^{v,n}_{i,j,k}(t)$ estimates the cost of assigning transporter v of plant i to travel from customer j to customer k. The second term of Equation (1) represents the shipping cost due to hiring chartered transporters, where $c^{\mathrm{c}}_{i,j} Q_{i,j}(t)$ defines the resulting cost if quantity $Q_{i,j}(t)$ is delivered by a chartered transporter from plant i to customer j. The third term denotes the total production cost by all the plants over T periods. The last two terms denote the plant inventory cost charged against average inventory level and the customer inventory cost charged against ending inventory, in each time period. While here the inventory cost structures are different, they could be the same in a model, depending on the assumptions.

Model (P) has two main families of constraints: product flow balance constraints (Equations (2)–(6)), and transporter routing constraints (Equations (7)–(11)). Constraints (2) ensure the inventory flow balance at plants and require each plant to have a sufficient supply (via production and/or inventory) to meet the distribution needs (via own and/or chartered transporters) in each time period. Constraints (3) ensure the inventory flow balance at DCs and require each DC to have enough supply (from either inventory and/or the quantity arrived in that period) to meet the demand. Constraints (4) and (5) ensure the storage capacity and minimum inventory requirements at both plants and DCs to be satisfied, and constraints (6) ensure the quantity produced by each plant is within its maximum capacity. Constraints (7) and (8) are trip integrity constraints. Constraints (7) impose the requirement that a transporter after delivering to a DC must leave that DC. Constraints (8) limit each transporter to leave its home/base plant at most once per trip. Constraints (9) and (10) are the transporter load balance constraints, where $g_{i,j,k}^{v,n}(t)$ defines the total quantity carried by transporter v while it travels between locations j and k, and includes the quantity scheduled to be delivered to location k, $q_{i,k}^{v,n}(t)$. Constraints (9) ensure that the quantity carried by a transporter does not exceed its maximum loading capacity. Constraints (10a) and (10b) are the commodity flow conservation constraints. For each DC, constraints (10a) require the quantity unloaded at that DC to be equal to the difference between the quantity on the transporter as it enters the DC and that on the same transporter as it leaves the DC. Constraints (10b) are similar to constraints (10a) but defined for each plant. These two sets of constraints eliminate the formation of subtours during the search process (Fumero *et al.*, 1999). Constraints (11) ensure that the total operation time of each transporter (including traveling plus loading and unloading times) in each time period does not exceed its limit.

In a similar way, we can extend model (P) to a formulation with multiple products.

Directly solving model (P) is not easy. The required computation time to verify the optimal solution can often become excessive because of the large number of integer variables involved. For example, with a problem of two plants, eight owned transporters, at most 12 trips per transporter per period, 20 DCs, and a 12-period/month planning horizon, one can verify that the number of integer variables gets close to 1000 000 (967 680 to be exact). The main reason for such a large number of integer variables is that the model attempts to construct the optimal transporter routing, production, and inventory schedules all at the same time.

There are many studies in the literature on either optimizing the inventory and distribution policies, or optimizing the production and inventory plans (e.g., see Williams (1981), Dror and Ball (1987), Cohen and Lee (1988, 1989), Dror and Trudeau (1988, 1996), Goyal and Gupta (1989), Blumenfeld *et al.* (1991), Anily and Federgruen (1993), Chandra (1993), Pyke and Cohen (1993, 1994), Slats *et al.* (1995), and Federgruen and Simchi-Levi (1995)). Reviews on these and other related works can be found in Bhatnangar *et al.* (1993), Thomas and Griffin (1996), Vidal and Goetschalckx (1997), Baita *et al.* (1998), and Erengüç *et al.* (1999). However, results that may be directly applied to solve model (P) are limited. Available approaches are either based on problem decomposition or heuristics. A seminal work in this regard involving multiple plants, identical transporters, and multiple products was contributed by Chandra and Fisher (1994). They analyzed and evaluated the performance of two approaches. The decoupled approach determines the production lot size to minimize the setup and inventory cost subject to the total demand per time period, and then schedules the transporter deliveries to meet customer demand subject to inventory availability determined by the given production lot size. The delivery schedules are based on known vehicle routing heuristics, and improved by combining the delivery to a customer at a later period with delivery to the same customer at an earlier period. This approach, however, does not allow modification of the given production lot sizes. The coordinated approach follows essentially the same process as the decoupled approach except that transportation decisions may entail changes in the production plans. Their computational results show a consistent improvement on the total cost by the coordinated approach.

Fumero and Vercellis (1999) proposed an integrated optimization model solved via Lagrangean relaxation for the PIDRP involving multiple products, multiple periods, a single plant, and identical transporters. They used Lagrangean relaxation to produce four separate subproblems, while at the same time preserving a global optimization perspective through the dual master problems, and compared the performance of the proposed approach with a decoupled approach based on the one in Chandra and Fisher (1994). A consistent and significant improvement in cost savings by the integrated optimization approach over that by the decoupled approach was observed. Van Buer *et al.* (1999) studied a special version of PIDRP with no inventories. Several heuristic algorithms, such as tabu search and simulated annealing, were used to solve the problem. They found no significant performance difference among these heuristics, and noted that reusing the trucks that had completed earlier routes could be an important way to achieve low-cost solutions. Özdamar and Yazgac (1999) considered a PIDRP involving a central factory, a set of warehouses, and identical vehicles. They adopted a hierarchical approach to make use of medium-range aggregate information and to satisfy weekly fluctuating demand. Their study focused on optimizing the fleet size instead of transporter routing. Applications of optimization techniques for the integrated PIDRP without the involvement of transporter routing were reported by Martin *et al.* (1993), Brown *et al.* (2001), and Gupta *et al.* (2002).

In this paper, we propose a two-phase approach to solve (P). In phase I, we solve a restricted version of the problem, called (P_1), which contains all the original constraints in (P) except that the transporter routings are limited to *direct shipments*. We show that the resulting optimal solution to (P_1) is always feasible to (P), and thus gives an upper bound solution to (P). Successfully solving this restricted problem determines the feasible quantity to be produced, inventoried, and transported by each transporter (per time period) from each plant to each DC. It also determines the feasible number of transporter trips (per time period) performed by each individual transporter, in terms of its own capacity, cost, speed, and availability. Note that for all DCs with a demand higher than the transporter capacity, especially when there is an overhead cost associated with a delivery (e.g., loading/unloading operations), direct shipment of a full load makes good sense (Note, however, that even when the demand is higher than the capacity of a transporter, direct shipment may not be optimal.) The potential deficiency of this phase I solution, however, comes from the fact that sometimes a consolidation of Less-than Transporter Load (LTL) assignments can further reduce the transportation cost. Therefore, in phase II, we propose a heuristic transporter routing algorithm, the *Load Consolidation (LC)* algorithm, that removes all the LTL assignments from the phase I solution and consolidates these assignments into transporter routing schedules subject to the transporter capacity and available time (remaining after performing the full load assignment determined by the phase I solution) constraints. In addition to evaluating the performance of this two-phase approach using a set of randomly generated test cases, we also apply this approach to a real-life supply network coordination problem.

The main advantage of this proposed approach, over the classical decoupled approach, is its ability to simultaneously coordinate the production, inventory, and transportation operations over the entire planning horizon, without the need to aggregate the demand or relax the constraints on transportation capacities. It enables us to quickly identify a quality suboptimal solution to the original complex problem. The suboptimality is, however, due to the simplified assumption that in phase I only direct shipment is deployed. The sacrificed optimality is then partially made up by the efforts of phase II which improves the suboptimal solutions by further exploring its neighborhood areas in the feasible region.

Section 2 of this paper presents the proposed two-phase approach to the PIDRP. We formulate the phase I problem to explicitly include the demand from each DC in each time period and the heterogeneity of transporters into the production and inventory optimization assuming direct shipment between plants and DCs. We then, for the phase II problem, propose a heuristic search algorithm, the LC algorithm, to consolidate the LTL assignments caused by the potential inefficiency of direct shipment. Section 3 presents our performance evaluation for the two-phase approach. Section 4 reports the application of this approach to a real-life supply network with additional practical constraints. Finally, we conclude the study and discuss future research directions in Section 5.

2. The two-phase approach to the PIDRP

Let the transporter routing be restricted to direct shipment between plants and DCs. That is, a transporter may only visit a single DC per trip (i.e., leaving a plant, visiting a DC, and then returning to the plant). Let $y_{i,j}^{v}(t)$ denote the number of direct shipment trips that transporter v, $v \in V(i)$, makes between plant i and DC$_j$ in time period t. Let $q_{i,j}^{v}(t)$ be the total quantity delivered from plant i to DC$_j$ by transporter v in period t in $y_{i,j}^{v}(t)$ trips, and replace $\sum_{n\in N(v)} q_{i,j}^{v,n}(t)$ in Equations (2) and (3) by $q_{i,j}^{v}(t)$. Then, we can reformulate (P) into a more restricted problem, (P$_1$), as follows:

$$\begin{aligned} \mathrm{P}_1 : \min \sum_{t\in T}\sum_{i\in I}\sum_{v\in V(i)}\sum_{j\in J} c_v t_{i,j}^{v} y_{i,j}^{v}(t) \\ + \sum_{t\in T}\sum_{i\in I}\sum_{j\in J} c_{i,j}^{\mathrm{c}} Q_{i,j}(t) + \sum_{t\in T}\sum_{i\in I} a_i(t) p_i(t) \\ + \sum_{t\in T}\sum_{i\in I} h_i(t)(p_i(t)/2 + s_i(t)) + \sum_{t\in T}\sum_{j\in J} \hbar_j(t) z_j(t), \end{aligned} \tag{13}$$

subject to Equations (2), (3), (4), (5), (6) and transporter capacity constraints:

$$q_{i,j}^{v}(t) \le C_v y_{i,j}^{v}(t) \qquad \forall i \in I,\ v \in V(i),\ j \in J,\ t \in T, \tag{14}$$

transportation duration constraints:

$$\sum_{j\in J} t_{i,j}^{v} y_{i,j}^{v}(t) \le T_v(t) \qquad \forall i \in I,\ v \in V(i),\ t \in T, \tag{15}$$

$$y_{i,j}^{v}(t) \text{ integers.} \tag{16}$$

Optimally solving model (P$_1$) determines, for each time period, the production quantity for each plant, the quantity to be delivered from a plant to a DC, the sizes of the inventories, and the optimal number of trips made by an individual transporter. Note that model (P$_1$) contains substantially fewer integer variables than model (P), and is computationally more tractable because of the elimination of transporter routing constraints. For the same-sized problem with two plants, eight owned vessels, 20 DCs, and 12 time periods, the number of integer variables is now reduced from 967 680 to 3840.

Let $G^*(\mathrm{P}_1)$ and $G^*(\mathrm{P})$ be the optimal objective function values of (P$_1$), and (P), respectively. Then it is easy to see that a feasible solution to (P$_1$) is always a feasible solution to (P), and $G^*(\mathrm{P}) \le G^*(\mathrm{P}_1)$.

The gap between $G^*(\mathrm{P})$ and $G^*(\mathrm{P}_1)$ depends on the magnitude of the cost savings that may be achieved by using tours to consolidate LTL assignments in the transporter routing schedules. For this reason, we solve a follow-up phase II problem to consolidate the LTL assignments. To do so, let $H(t)$ denote the subset of DCs that receive LTL shipments in period t from the optimal solution to (P$_1$). Each $DC_j,\ DC_j \in H(t)$, has a LTL delivery quantity $d_j(t) < C_v$, where C_v is the capacity of transporter v that was assigned to perform the delivery by the phase I solution. Let $\tau_v(t),\ \tau_v(t) \le T_v(t)$ be the available time for transporter v to perform LTL assignments in period t, where $\tau_v(t)$ is equal to $T_v(t)$ minus the time needed by transporter v to perform its full load transportation assignments determined by the optimal solution to (P$_1$).

Let a transporter *trip* be a *route* leaving a plant, visiting a sequence of DCs, and then returning to that plant. The problem in phase II is then to *determine for each plant the set of trips for each transporter, in each period t, that minimizes the total transportation cost subject to the satisfaction of LTL demand $d_j(t)$, $j \in H(t)$, the available transporter capacity (C_v), and the transporter availability ($\tau_v(t)$) constraints*. We call this problem (P_2).

Problem (P_2) is defined for each plant in each period. Hence, in the following formulation the plant index and the period index are dropped.

Let 0 denote the plant, and N be an upper bound on the maximum number of trips a transporter can make in a time period. Also

$$x_{i,j}^{v,n} = \begin{cases} 1 & \text{if during its } n\text{th trip transporter} \\ & \quad v \text{ visits location } j \text{ immediately after location } i, \\ 0 & \text{otherwise.} \end{cases}$$

Then we can formulate problem (P_2) in the following manner.

$$P_2 : \min \sum_{i,j \in H \cup \{0\}} \sum_{v \in V} \sum_{n \in N} c_v t_{i,j}^v x_{i,j}^{v,n}, \tag{17}$$

subject to
each customer must be visited exactly once:

$$\sum_{i \in H \cup \{0\}} \sum_{v \in V} \sum_{n \in N} x_{i,j}^{v,n} = 1 \qquad \forall j \in H, \tag{18}$$

trip integrity constraints:

$$\sum_{i \in H \cup \{0\}} x_{i,p}^{v,n} - \sum_{j \in H \cup \{0\}} x_{p,j}^{v,n} = 0$$
$$\forall v \in V,\ p \in H \cup \{0\}, n \in N \tag{19}$$

each trip can start from the plant at most once:

$$\sum_{j \in H} x_{0,j}^{v,n} \leq 1 \qquad \forall v \in V,\ n \in N \tag{20}$$

vehicle capacity constraints:

$$\sum_{i \in H} \left(d_i \sum_{j \in H \cup \{0\}} x_{i,j}^{v,n} \right) \leq C_v \qquad \forall v \in V, n \in N \tag{21}$$

subtour elimination constraints:

$$y_i - y_j + \|H\| \sum_{v \in V} \sum_{n \in N} x_{i,j}^{v,n} \leq \|H\| - 1$$
$$\forall i \in H,\ j \in H, i \neq j \tag{22}$$

transporter maximum traveling time constraints:

$$\sum_{i,j \in H \cup \{0\}} \sum_{n \in N} t_{i,j}^v x_{i,j}^{v,n} \leq \tau_v \qquad \forall v \in V, \tag{23}$$

other constraints:

$$x_{i,j}^{v,n} \text{ binary} \qquad \forall i, \quad j \in H \cup \{0\},\ v \in V,\ n \in N \tag{24a}$$
$$y_i \text{ arbitrary} \qquad \forall i \in H. \tag{24b}$$

In this formulation, the objective is to minimize the total cost of delivering LTL qualities. Constraints (18), (19), and (20) are the degree constraints. Constraints (18) ensure that each DC is visited once and only once. Constraints (19) impose the requirement that if a transporter enters a location, it must leave that location. Constraints (20) require that a transporter leaves a plant at most once during each trip. Constraints (21) are the capacity constraints defined for each trip, and ensure that no trip has a total demand exceeding the capacity C_v. Constraints (22) are the classical subtour elimination constraints. Constraints (23) enforce the maximum time a transporter can travel. Constraints (24a) are integer constraints.

Problem (P_2) is similar to the capacitated vehicle routing problem with multiple use of vehicles. A comprehensive review on the literature about this problem can be found in Taillard *et al.* (1996) who highlight that most studies on multitrip routing problems assume identical transporters and consider tabu-search-based heuristics. For the problems with heterogeneous transporters, as in our case, the results are very limited, to our knowledge.

Let $G^*(\lfloor P_1 \rfloor)$ be equal to $G^*(P_1)$ minus the traveling and holding cost associated with the LTL assignments. Then we have the following result (proof omitted).

Proposition 1. *For any given feasible solution to* (P_1)*, a feasible solution to the associated* (P_2) *plus the full-load assignments defined by that feasible solution to* (P_1) *gives a feasible solution to* (P)*, and* $G*(P) \leq G(P_2) + G(\lfloor P_1 \rfloor) \leq G(P_1)$.

To develop a heuristic solution procedure to solve (P_2), we start with the following subproblem (S_2).

(S_2): We are given a single plant, a single transporter with capacity C, and a set of H DCs, each with a demand d_i, $d_i < C, 1 \leq i \leq H, 1 \leq H \leq ||J||$. The cost (or the time) for the transporter to travel from DC_i to DC_j is known as $t_{i,j}$. Let π be a permutation of DCs in H and, without loss of generality, let:

$$\pi = \langle DC_1, DC_2, \ldots, DC_H \rangle.$$

The problem is to partition π into a sequence of transporter trips (see Fig. 1) to minimize the total cost subject to C and $\{d_j | 1 \leq j \leq H\}$.

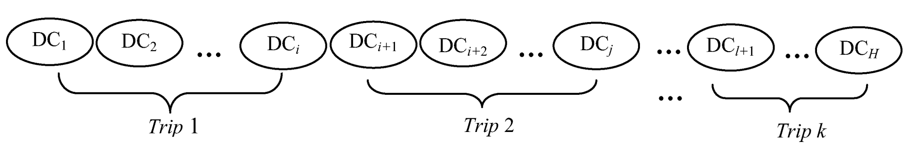

Fig. 1. A sequence of trips.

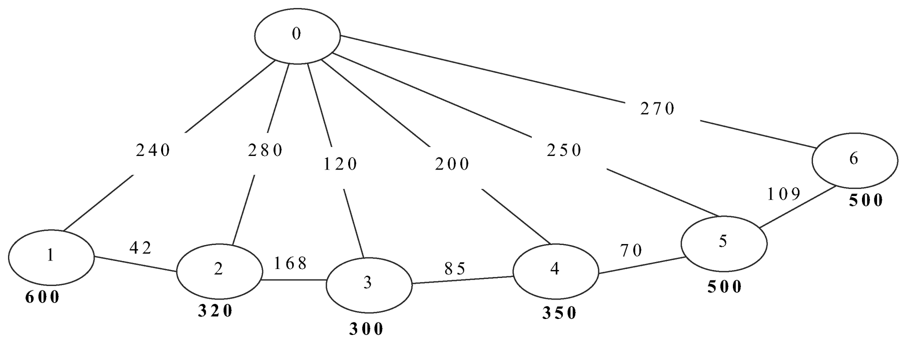

Fig. 2. A network with six customer DCs and a given route $\langle DC_1, DC_2, \ldots, DC_6\rangle$.

Let $\pi^+ = \langle\{0\}||\pi\rangle$, where node 0 denotes the plant. If we assume that each DC may be visited only once, then (S_2) can be solved in strongly polynomial time by a straightforward extension from the known optimal partitioning procedure (Beasley, 1983; Altinkemer and Gavish, 1987; Li and Simchi-Levi, 1990). For the completeness of the paper, we present this *Extended Optimal Partitioning* (EOP) procedure as follows:

Procedure EOP

Construct a directed acyclic graph G with $V(G) = \{i|0 \leq i \leq H\}$, where node i denotes DC_i, $0 \leq i \leq H$. Let $E(G)$ be the set of directed arcs on G, where $(i, j) \in E(G)$ iff:

$$d_{i+1} + d_{i+2} + \cdots + d_{j-1} + d_j \leq C,$$

that is, the transporter has enough capacity to serve all the demands from node $i+1$ to node j. Each arc represents a feasible trip for the transporter. Add all such feasible arcs, (i, j), $0 \leq i < j$, $1 \leq j \leq H$, to $G(V,E)$, and let the arc length be the resulting trip cost, $\tau_{i,j} = t_{i,0} + t_{0,i+1} + \sum_{h=i+1\cdots j-1} t_{h,h+1}$.

Figure 2 illustrates a network with $H = 6$ and $\pi = \langle DC_1, DC_2, \ldots, DC_6\rangle$, where the number on arc (i, j) denotes $t_{i,j}$, and the number on node j denotes d_j. Assuming $C = 1200$ units, then the resulting $G(V,E)$ is given in Fig. 3. As we can see, each path on graph $G(V,E)$ from node 0 to node H defines a feasible sequence of transporter trips that satisfies all the DC demands and the transporter capacity constraints. For example, an arc from node 1 to node 4 represents the trip from node 1 to node 0 (to get loaded), node 0 to node 2, node 2 to node 3, and then node 3 to node 4, with an arc cost of $240 + 280 + 168 + 85 = 773$. The shortest path on $G(V,E)$ is 0-2-3-6 with a total cost of 1181 and defines three trips: 0-1-2-0, 0-3-0, and 0-4-5-6-0.

Proposition 2. *If each DC can be visited only once, then the shortest path on $G(V,E)$ solves (S_2) optimally in $O(H^3)$.*

Proof. The shortest path on $G(V,E)$ minimizes the total cost while satisfying the demand and transporter capacity constraints. Since $G(V,E)$ is acyclic and contains at most $O(H^2)$ arcs and $H+1$ nodes, the shortest path on $G(V,E)$ can be determined in $O(H^3)$ according to Ahuja *et al.* (1993). ■

The EOP procedure is used as a subroutine in our proposed heuristic routing algorithm (for heterogeneous capacitated transporters) in phase II, called the LC algorithm. This algorithm requires, for each period t, a predetermined tour that connects all DCs in set $H(t)$, π^0, and a predetermined tour for each transporter, $\pi(v)$, $1 \leq v \leq V$, that connects all DCs with LTLs assigned to transporter v from the phase I solution. Given such tours, the proposed LC algorithm calls two heuristic procedures, H_1 and H_2, each being able to generate a feasible solution to (P_2). The best feasible solution by H_1 and H_2 is then used as the solution of the LC algorithm.

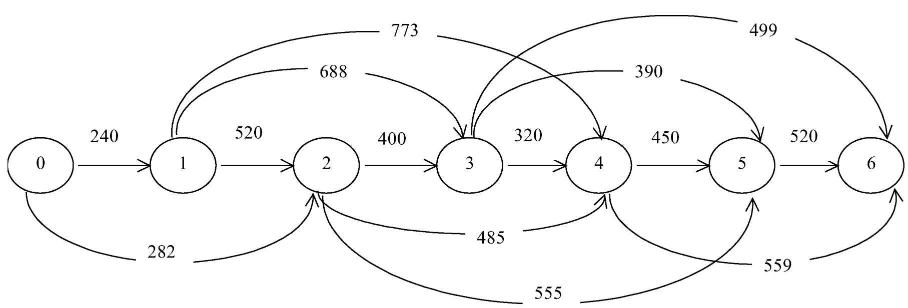

Fig. 3. The directed acyclic graph $G(V, E)$ for the network in Fig. 2.

Let $H(t)$ be a union of all the DCs that have a LTL delivery in period t (we do not differentiate between transporters), the shortest path covering $\pi(v)$ be $P(\pi(v))$, and $\tau_v(t)$ be the total available time that transporter v has to perform the LTL assignments for the time period. For each given period t, heuristic H_1 calls the EOP procedure to identify $P(\pi(v))$, $\forall v \in V$. Since $\pi(v)$ consists of feasible direct shipment assignments for transporter v from the phase I solution, the length of any path on $\pi(v)$, including that of $P(\pi(v))$, is no more than $\tau_v(t)$. Given $P(\pi(v))$, $\forall v \in V$, heuristic H_1 evaluates the potential savings (in terms of transportation cost) of shifting node j, $j \in \pi(v)$, from $P(\pi(v))$ to $P(\pi(v'))$, $v' \neq v$, subject to $\tau_v(t)$ and $\tau_{v'}(t)$. If taking node j does not violate $\tau_{v'}(t)$ for any given path $P(\pi(v'))$, $v' \neq v$, then the net saving is equal to the savings obtained by eliminating j from path $P(\pi(v))$ minus the cost of adding j to$P(\pi(v'))$. Otherwise, this net saving is equal to that achieved by exchanging node j on $P(\pi(v))$ with some node j' on $P(\pi(v'))$. If this swap also fails, node j will remain on $P(\pi(v))$. By comparing the resulting net savings from all $P(\pi(v'))$, $v' \neq v$, the path that yields the maximum net savings is selected to take node j (and node j is then permanently eliminated from path $P(\pi(v))$). Heuristic H_1 repeats this process until all nodes $j, j \in H(t)$, are evaluated.

Heuristic H_2 applies two transporter ranking criteria, the *fastest transporter first* and the *lowest cost-per-mile-per-ton transporter first*, and starts with tour π^0 that connects all the DCs in $H(t)$. For each given criterion, H_2 selects the highest ranking transporter and applies the EOP procedure to π^0 to construct the shortest path for the selected transporter while relaxing $\tau_v(t)$. If the resulting length of the shortest path exceeds $\tau_v(t)$ for the transporter being considered, then either a trip (when transporters are identical) or a node (when transporters are heterogeneous) is removed until the resulting length of the shortest path satisfies $\tau_v(t)$. Let $H'(t)$ be the set of removed nodes. Heuristic H_2 repeats the same procedure to the next highest ranking transporter until $H'(t)$ becomes empty.

Proposition 3. *For* (P_2) *with two heterogeneous transporters, if the transporters must travel along a given tour* π^0 *and the assignments to each transporter must be based*

Table 1. Parameters of the test problems in data set 1 for the LC algorithm

Test cases	$\|J\|$	*Transporter 1*		*Transporter 2*		*Transporter 3*		*Transporter 4*		*Transporter 5*		*Available time* (τ_v)
		C	*c*	*C*	*c*	*C*	*c*	*C*	*c*	*C*	*c*	
P1	8	20	1	30	1.4	40	1.6	70	2.5	120	5.0	80
P2	8	60	1	80	1.4	150	2.6					140
P3	8	20	1	30	1.4	40	1.6	70	2.5	120	5.0	140
P4	8	60	1	80	1.4	150	2.6					200
P1-1	8	20	1	30	1	40	1	70	1	120	1	80
P2-1	8	60	1	80	1	150	1					140
P3-1	8	20	1	30	1	40	1	70	1	120	1	140
P4-1	8	60	1	80	1	150	1					200
P1-2	8	20	1	30	1	40	1	70	1	120	1	160
P2-2	8	60	1	80	1	150	1					220
P3-2	8	20	1	30	1	40	1	70	1	120	1	160
P4-2	8	60	1	80	1	150	1					220
P1-3	8	20	1	30	1.4	40	1.6	70	2.5	120	5.0	160
P2-3	8	60	1	80	1.4	150	2.6					220
P3-3	8	20	1	30	1.4	40	1.6	70	2.5	120	5.0	160
P4-3	8	60	1	80	1.4	150	2.6					220
P5	12	20	1	30	1.4	40	1.6	70	2.5	120	5.0	100
P6	12	60	1	80	1.4	150	2.6					180
P7	12	20	1	30	1.4	40	1.6	70	2.5	120	5.0	160
P8	12	60	1	80	1.4	150	2.6					280
P5-1	12	20	1	30	1	40	1	70	1	120	1	100
P6-1	12	60	1	80	1	150	1					180
P7-1	12	20	1	30	1	40	1	70	1	120	1	160
P8-1	12	60	1	80	1	150	1					280
P5-2	12	20	1	30	1	40	1	70	1	120	1	240
P6-2	12	60	1	80	1	150	1					280
P7-2	12	20	1	30	1	40	1	70	1	120	1	240
P8-2	12	60	1	80	1	150	1					280
P5-3	12	20	1	30	1.4	40	1.6	70	2.5	120	5.0	240
P6-3	12	60	1	80	1.4	150	2.6					280
P7-3	12	20	1	30	1.4	40	1.6	70	2.5	120	5.0	240
P8-3	12	60	1	80	1.4	150	2.6					280

on consecutive DCs along π^0, *then* (P$_2$) *is solvable in* $O(H^4)$.

Proof. Let $\pi^0 = \langle \mathrm{DC}_1, \mathrm{DC}_2 \ldots, \mathrm{DC}_i, \ldots \mathrm{DC}_H \rangle$ and let i, $1 \le i \le H$, be the DC that partitions π^0 into two subsequences of DCs:

$$\sigma_1 = \langle \mathrm{DC}_1, \mathrm{DC}_2 \ldots, \mathrm{DC}_i \rangle \quad \text{and}$$
$$\sigma_2 = \langle \mathrm{DC}_{i+1}, \mathrm{DC}_{i+2} \ldots \mathrm{DC}_H \rangle.$$

Assign transporter 1 to σ_1, transporter 2 to σ_2, and apply the EOP procedure to solve for the shortest path on σ_1, and σ_2, respectively. Let $L(P(\sigma_1))$ and $L(P(\sigma_2))$ be the resulting path lengths. Partition i is feasible iff $L(P(\sigma_1)) \le \tau_1(t)$ and $L(P(\sigma_2)) \le \tau_2(t)$. Let $G(i, 1, 2) = L(P(\sigma_1)) + L(P(\sigma_2))$ be the resulting length/cost if partition i is feasible or $G(i, 1, 2) = \infty$ otherwise. Reverse the assignment of transporters (i.e., assign transporter 2 to σ_1 and transporter 1 to σ_2), apply the above process to obtain $G(i, 2, 1)$. Let $G(i) = \min\{G(i, 1, 2), G(i, 2, 1)\}$ and i^* be the partition that $G(i^*) = \min\{G(i)|\forall i\}$. The transporter trips under $G(i^*)$ give the optimal solution to the problem. Since there are at most H alternative values for parameter i, and $G(i)$ can be determined in $O(H^3)$ for each give partition i, the total computational time is thus bounded by $O(H^4)$. ■

3. Performance evaluation

In this section we present empirical results on the computational performance of: (i) the LC algorithm; and (ii) the two-phase approach which uses a CPLEX MIP solver to solve (P$_1$) and then the LC algorithm to solve (P$_2$). The LC algorithm was coded in *C* and run on a 750 MHz Pentium III computer. The CPLEX MIP solver was run on a 1.2 GMHz Dell Latitude. We evaluate the LC algorithm so as to be able to compare its performance with the optimal solutions of the capacitated routing problem that allow multiple use of transporters, for which we adopted test data from the existing literature for related problems. We evaluate the two-phase approach in order to be able to obtain empirical observations on its solution quality when applied to the PIDRP. Our test data for this part of the evaluation were generated based on close-to-real ratios between the values of data encountered in practice. Examples of such data ratios include the ratio of safety stock level at each customer site to the maximum demand anticipated for that customer over the multiperiod planning horizon, the ratio of average transporter trip length to the period length, the ratio of holding cost to the production variable cost, and the ratio of average transporter capacity to the average period demand level.

3.1. *Computational performance of the LC algorithm*

The performance of the LC algorithm was evaluated based on two sets of test problems, where set 1 contains 32 relatively small problems and set 2 contains 24 relatively large test problems. Tables 1 and 2 summarize the parameters used to generate these test problems, where $||J||$ stands for

Table 2. Parameters of the test problems in data set 2 for the LC algorithm

Test cases	$\|J\|$	*Transporter 1*		*Transporter 2*		*Transporter 3*		*Transporter 4*		*Transporter 5*		*Transporter 6*		Available time
		C	*c*	*C*	*c*	*C*	*c*	*C*	*c*	*C*	*c*	*C*	*c*	
P9	16	20	1	30	1.4	40	1.6	70	2.5	120	5.0			150
P10	16	60	1	80	1.4	150	2.6							220
P11	16	20	1	30	1.4	40	1.6	70	2.5	120	5.0			200
P12	16	60	1	80	1.4	150	2.6							320
G-p3	20	20	1	30	1	40	1	70	1	120	1			200
G-p4	20	60	1	80	1	150	1							300
G-p5	20	20	1	30	1	40	1	70	1	120	1			250
G-p6	20	60	1	80	1	150	1							400
P13	25	20	1	40	1.1	50	1.2	70	1.7	120	3.0	200	5.2	200
P14	25	120	1	160	1.1	300	2.4							400
P15	25	50	1	100	1.8	160	3.2							400
P16	25	40	1	80	1.6	140	2.1							400
P17	30	20	1	40	1.1	50	1.2	70	1.7	120	3.0	200	5.2	220
P18	30	120	1	160	1.1	300	2.4							440
P19	30	50	1	100	1.8	160	3.2							450
P20	30	40	1	80	1.6	140	2.1							450
P21	40	20	1	40	1.1	50	1.2	70	1.7	120	3.0	200	5.2	400
P22	40	120	1	160	1.1	300	2.4							640
P23	40	50	1	100	1.8	160	3.2							650
P24	40	40	1	80	1.6	140	2.1							650
T-p13	50	20	1	30	1.1	40	1.2	70	1.7	120	2.5	200	3.2	400
T-p14	50	120	1	160	1.1	300	1.4							900
T-p15	50	50	1	100	1.6	160	2.0							800
T-p16	50	40	1	80	1.6	140	2.1							800

Table 3. The LC algorithm solutions and CPLEX solutions for data set 1

Test cases	$\|J\|$	*LC algorithm solution (all in less than 0.2 seconds)*	*CPLEX*		$[(G_{LC} - G_{CPLEX})/G_{CPLEX}] \times 100\%$
			Solution	*CPU time (seconds)*	
P1	8	370.1	370.1	7	0.00
P2	8	203.4	203.4	10	0.00
P3	8	608.6	557	120	9.26
P4	8	299	270.2	64	10.81
P1-1	8	183	183	26	0.00
P2-1	8	156	154	1.2	1.30
P3-1	8	213	213	22	0.00
P4-1	8	146	146	0.13	0.00
P1-2	8	159	154	0.07	3.25
P2-2	8	145	145	0.05	0.00
P3-2	8	188	185	1.32	1.62
P4-2	8	146	146	0.15	0.00
P1-3	8	337.4	337.4	9.68	0.00
P2-3	8	183	183	2.36	0.00
P3-3	8	561.6	557	95.2	0.83
P4-3	8	270.2	270.2	120	0.00
P5	12	502.8	483.1	7200	4.08
P6	12	260.2	260.2	780	0.00
P7	12	734.3	731.3	7200	0.41
P8	12	353	348.8	6100	1.20
P5-1	12	257	250	1000	2.80
P6-1	12	200	198	5.26	1.01
P7-1	12	280	262	850	6.87
P8-1	12	229	220	510	4.09
P5-2	12	198	193	2.13	2.59
P6-2	12	198	193	3.33	2.59
P7-2	12	238	232	200	2.59
P8-2	12	229	220	620	4.09
P5-3	12	458.2	446.8	7200	2.55
P6-3	12	235	235	750	0.00
P7-3	12	723.4	720	7200	0.47
P8-3	12	353	348.8	6000	1.20

the number of DCs, C refers to the transporter capacity, c refers to the unit shipping cost ($/mile), and τ_v stands for the maximum traveling time available for a transporter to perform LTL assignments in phase II. In Table 2, the instances from G-p3 to G-p6 are based on the test problems from Golden *et al.* (1984), and the instances from T-p13 to T-p16 are based on the benchmarking problems from Taillard *et al.* (1996). Since the intention of using these test data in this study differs from that in Golden *et al.* (1984) and Taillard *et al.* (1996), we removed the transporter fixed cost and added the maximum transporter available time for LTL assignments (τ_v), while all the other parameters (including node locations) from Golden *et al.* (1984) and Taillard *et al.* (1996) remain unchanged. For all the remaining test problems, we sampled the distance data from the study of Christofides and Eilon (1969) (i.e., the data for our 8-node problems are based on those for the first 8 nodes in their 100-node problem, and the data for our 12-node problems are based on their first 12-node problems, etc.).

Tables 3 and 4 compare the solutions of the LC algorithm with those obtained using the CPLEX MIP solver in a 2-hour CPU time. For relatively small problems ($8 \leq N \leq 12$), the average deviation from the CPLEX solutions is 1.98%. For larger problems ($16 \leq N \leq 50$), the LC algorithm found the same or better solutions, in 13 out of 16 cases, than the best effort using the CPLEX MIP solver in 2 hours. For all the 56 test problems, the LC algorithm terminated in less than 0.2 seconds.

Out of these 56 test problems, there were two instances in which the LC algorithm terminated with large error gaps (10.81 and 11.14%). The main reason for this is that the LC algorithm starts its search based on a given route and prespecified assignment of DCs to each transporter, whereas the CPLEX solver that solves (P_2) directly optimizes the route and DC assignments simultaneously during the search.

Note that even though our 2-hour CPU time limit on the CPLEX solution was arbitrarily set, we did notice that, for

Table 4. LC algorithm solutions and CPLEX solutions for data set 2

Test cases	$\|J\|$	*LC algorithm solution (all in less than 0.2 seconds)*	*CPLEX solution (in 2 hours)*	$[(G_{LC} - G_{CPLEX})/G_{CPLEX}] \times 100\%$
P9	16	682.4	713.2	−4.3
P10	16	381.4	351.6	+8.48
P11	16	947.8	951.7	−0.41
P12	16	433.4	453.6	−4.5
G-P3	20	673.7	742.9	−9.3
G-P4	20	311	295	+5.42
G-P5	20	857.5	867.3	−1.1
G-P6	20	344	344	0.00
P13	25	931.6	—	—
P14	25	413.9	415.4	−0.4
P15	25	874	911.6	−0.41
P16	25	818.9	736.8	+11.14
P17	30	1153.6	—	—
P18	30	463.1	481.8	−3.9
P19	30	1056.4	—	—
P20	30	1008.7	1062.7	−5.1
P21	40	1501.7	—	—
P22	40	569.5	838	−32
P23	40	1034	—	—
P24	40	1073.5	—	—
T-P13	50	1846.8	—	—
T-P14	50	683.2	847	−19.3
T-P15	50	1135	1335	−15
T-P16	50	1264.2	—	—

— No feasible solution within 2 hours by CPLEX.

most test problems with similar sizes to our experiments, the solutions do not improve significantly after a 2-hour search, and the problem of running out of memory starts to occur frequently. Also note that for future studies we do have an option of using CPLEX to solve the phase II problem for smaller problems and leave the heuristic as a tool for handling larger phase II problems.

3.2. *Computational performance of the two-phase approach*

We randomly generated 48 test problems to compare the solutions of the proposed two-phase approach with those obtained using the CPLEX MIP solver to solve the original model (P) directly. For the proposed two-phase approach, the optimization problem in phase I (P_1) was solved using the CPLEX MIP solver and the LTL consolidation problem in phase II (P_2) was solved using the LC algorithm.

To generate the test problem, the distances between DCs and the distance between each DC and the (single) plant were randomly sampled from a uniform distribution over [0 100]. The travel cost between each pair of locations was given by a constant (c) multiplied by the respective distance. We used a fleet of two heterogeneous transporters with the same traveling speed but with different loading capacities, C_i, $i = 1, 2$, randomly sampled from uniform [8, 16]. The number of time periods in the planning horizon ranged from two to four, and the parameter $\mu = (C_1 + C_2)/2$ was used to generate the period demand for DCs from uniform [1 μ, 1.5 μ], [1.5 μ, 2 μ], [2 μ, 2.5 μ], and [2.5 μ, 3 μ]. The safety stock level at each DC was set to be one-third of the maximum period demand over all time periods in the planning horizon and the storage capacity was set to be the maximum period demand over the planning horizon. The starting inventory at each DC had two levels. The lower level was set equal to the DC safety stock requirement, while the high level was set to be the average of the safety stock requirement and DC storage capacity. For all the test cases, we set $c = 2$, $a = 3$, and $h = 0.50$. Among the 48 test cases, cases 1 to 16 had $||J|| = 5$ and $T_v(t) = 360$, cases 17 to 32 had $||J|| = 6$ and $T_v(t) = 640$, cases 33 to 40 had $||J|| = 10$ and $T_v(t) = 1280$, and cases 41 to 48 had $||J|| = 12$ and $T_v(t) = 1600$, for all transporters v.

Table 5 compares solutions of the two-phase approach with those obtained by applying the CPLEX MIP solver to model (P) directly. As we can see, in 34 out of 48 cases, the two-phase approach found, in much less time, either the same/better solution than the best solution by the CPLEX MIP solver in a 4-hour CPU time, or a feasible solution that the CPLEX MIP solver cannot find within the given time limit. We also see that, in 14 out of 48 cases, the two-phase approach failed to get a better solution. The largest gap from the CPLEX MIP solution is 10.22%. Nevertheless, the performance of the two-phase approach seems promising and requires less than 1 minute of CPU time for most test problems.

4. An application of the two-phase approach

We have applied the two-phase approach to a real-life supply network planning problem encountered at a leading chemical company that produces various highway/road maintenance chemicals at its multiple plant sites, and distributes finished goods to a large number of DCs in North America. Due to steady increases in customer demand, the company has been continuously expanding the market and the distribution network by adding new distribution channels. The transportation between plants and DCs is by heterogeneous ocean ships and water barges. Due to high marine transportation costs, any inefficiency in scheduling may easily lead to a significant increase (in the magnitude of millions of dollars) in the operation costs.

The specific data set we used in this section (note that some of the data are modified to maintain confidentiality) includes 12 time periods, two plants (the Michigan plant and the Ontario plant), 13 DCs, and three heterogeneous vessels with loading capacities and traveling speed of (9050 tons, 7 knots), (8000 tons, 13 knots), and (9050 tons, 7 knots), respectively. In addition to transporting finished goods to DCs, the vessels are also used to transport raw material from the Michigan plant to

Table 5. Two-phase approach solutions and CPLEX MIP solutions

Test cases	$G^*(\lfloor P_1 \rfloor)$	$G(P_2)$	Two-phase approach: Solution	Two-phase approach: Time (in seconds)	CPLEX solution[a] for (P) (best in 4 hours)	$[(G_{LC} - G_{CPLEX})/G_{CPLEX}] \times 100\%$
1	1249.25	202	1451.25	0.27	1451.25	0.00
2	1466.25	628	2094.25	0.34	2051.75	2.07
3	2286.25	404	2690.25	0.27	2690.25	0.00
4	3255.75	280	3535.75	0.29	3536.25	−0.01
5	704.5	488	1192.5	0.26	1192.5	0.00
6	1060	618	1678	0.28	1678	0.00
7	1763.5	490	2253.5	0.27	2224	1.33
8	2387.5	348	2735.5	0.33	2713	0.83
9	2290.25	622	2912.25	0.45	2912.25	0.00
10	3443.25	444	3887.25	0.47	3891.25	−0.10
11	4239.25	888	5127.25	8.75	5147.75	−0.40
12	6105.25	220	6325.25	6.76	6327.25	−0.03
13	1869.25	608	2477.25	0.46	2477.25	0.00
14	2726.75	914	3640.75	940.4	3635.75	0.14
15	4066.75	444	4510.75	0.78	4531.75	−0.46
16	5202.25	444	5646.25	25.26	5649.25	−0.05
17	1348.75	950	2298.75	0.29	2298.75	0.00
18	2354.75	980	3334.75	0.3	3269.75	1.99
19	2706.25	1504	4210.25	0.27	4216.25	−0.14
20	4428.75	828	5256.75	0.2	5202.75	1.04
21	515.75	1504	2019.75	0.27	1970.75	2.49
22	1649.25	886	2535.25	0.27	2300.25	10.22
23	2694.25	658	3352.25	0.29	3352.75	−0.01
24	3498.75	676	4174.75	0.22	4181.25	−0.16
25	2636.5	1990	4626.5	0.71	4533	2.06
26	5214	1228	6442	54.27	6463	−0.32
27	7356.5	772	8128.5	13.26	8313.5	−2.23
28	9414	676	10 090	350.2	10233	−1.40
29	2266.5	1916	4182.5	0.31	4033.5	3.69
30	4779	850	5629	6.36	5603.5	0.46
31	5613	1836	7449	940.48	7461.5	−0.17
32	8313	676	8989	320.29	9001	−0.13
33	3619.75	874	4493.75	0.27	4661.75	−3.60
34	4292.25	2242	6534.25	0.29	—	—
35	6360.25	1556	7916.25	0.56	8266.75	−4.24
36	9146.75	576	9722.75	6.13	—	—
37	2105.25	1596	3701.25	0.27	3494.75	5.91
38	3388.75	1842	5230.75	0.28	5964.75	−12.31
39	5332.75	1090	6422.75	0.29	7417.25	−13.41
40	6753.75	1080	7833.75	0.29	8027.25	−2.41
41	4733.25	912	5645.25	0.28	5712.25	−1.17
42	5578.75	2780	8358.75	0.31	9760.75	−14.36
43	7904.75	2168	10 072.75	0.31	—	—
44	10584.75	1616	12 200.75	0.33	11430.25	6.74
45	2547.75	2216	4763.75	0.28	4448.75	7.08
46	4138.75	2558	6696.75	0.29	—	—
47	6795.25	1286	8081.25	0.3	—	—
48	8090.25	2116	10 206.25	0.29	—	—

[a]Best CPLEX solution in 4 hours of CPU time.

the Ontario plant. Whenever the company faces a shortage in vessel capacity, chartered vessels are available but at a higher cost. In addition to those standard constraints modeled in (P), additional constraints that ensure the practical feasibility/implementation of our solution were also included. First, each vessel requires a 1-month maintenance time in each planning year. To do so, we expanded model (P_1) by introducing binary (logical) variables:

$$\delta^v(t) = \begin{cases} 1 & \text{if vessel } v \text{ is available in month } t, \\ 0 & \text{if vessel } v \text{ undergoes maintenance in month } t, \end{cases}$$

and represented the maintenance requirement as:

$$\sum_{t=1}^{12} \delta^v(t) = 11.$$

To limit the number of trips in month t, we added constraints of the form of:

$$y^v_{i,j}(t) \leq M\delta^v(t),$$

where M is an upper bound on the maximum number of trips that vessel v can make in month t. Second, no vessel may travel northeast of Cleveland in winter, because the chemicals may freeze during transportation and the ship lanes may close during the winter season. This is enforced by specifying a set of DCs that cannot be accessed during the winter season (December, January, February, and March), J', and then setting *a priori* the respective trip variables $y^v_{i,j}(t) = 0, \forall v \in V(i), t \in \{1, 2, 3, 12\}, i \in I, j \in J'$. By applying the CPLEX MIP solver to solve model (P_1) and then the LC algorithm to solve model (P_2), our two-phase approach found the solution to the following issues:

1. monthly production schedule and ending inventory plans at each plant;
2. monthly raw material transportation plan;
3. distribution plans and vessel monthly routing schedules;
4. monthly ending inventory plan for each DC; and
5. recommended annual vessel maintenance schedules.

Tables 6, 7, and 8 report the itemized annual cost projections, facility production/inventory plans, and vessel routing schedules constructed by the two-phase approach. Table 6 indicates that the vessel transportation cost is the major cost component in the company's annual operation budget (62.58% of the annual operation cost). With the support of optimization tools, the company is able to fully rely on its own fleet to distribute the product in the planning horizon without the need for any chartered service. Tables 7 and 8 present the resulting production/inventory plans and vessel routing schedules that together minimize the annual production, inventory, and transportation cost.

Table 6. Itemized annual operation cost projection

Cost factor	*Cost ($)*	*Percentage of total cost (%)*
Transportation	6697 141	62.58
Chartered transportation	0	0
Production cost	4056 680	32.18
Inventory	686 766	5.23
Total	11 440 587	100

Table 7. Twelve-month production plan for the Ontario plant

	Production quantity (tons)	*Product ending inventory (tons)*	*Raw material ending inventory (tons)*
Initial		81 087	40 000
July 2001	46 330.8	81 086.6	40 000
August 2001	7699.2	81 086.6	40 000
September 2001	24 758.4	99 929.7	50 000
October 2001	10 984.6	87 872.8	50 000
November 2001	36 582.0	78 863.9	50 000
December 2001	8 136.5	20 271.6	26 855
January 2002	34 096.9	20 271.6	26 324
February 2002	37 554.0	40 543.3	26 258
March 2002	24 764.6	40 543.3	32 024
April 2002	5252.15	42 791.2	50 000
May 2002	10 790.5	50 208.5	50 000
June 2002	41 745.7	81 086.6	40 000

Figure 4 shows the strategic plant-DC relations recommended by the two-phase approach. As we can see from the figure, nine DCs (including three DCs near the Lake Superior and Lake Huron areas) should be strategically supplied by the Michigan plant, and five DCs (which are near and to the east of Lake Erie and Lake Ontario areas) should be served by the Ontario plant. One particular DC (DC 13) should be supplied jointly by both plants. In addition to the operation plans, the solution by the two-phase approach also offers senior management at the company valuable information on building direct and long-term relationships between plants and customers at the DCs and

Table 8. Sampled vessel routing schedules (vessel 2 from the Ontario plant)

Date	*Shipment*
July 2001	Five direct shipment trips: Ontario ⇔ Michigan
August 2001	Consolidated trip: Ontario ⇒ DC 11 ⇒ DC 13 ⇒ Ontario
September 2001	Three direct shipment trips: Ontario ⇔ Michigan One direct shipment trip: Ontario ⇔ DC 5 One direct shipment trip: Ontario ⇔ DC 7
October 2001	Four direct shipment trips: Ontario ⇔ Michigan
November 2001	Four direct shipment trips: Ontario ⇔ Michigan
December 2001	Four direct shipment trips: Ontario ⇔ Michigan, One direct shipment trip: Ontario ⇔ DC 5
January 2002	Four direct shipment trips: Ontario ⇔ Michigan
February 2002	One direct shipment trip: Ontario⇔ DC 5
March 2002	Three direct shipment trips: Ontario ⇔ Michigan One direct shipment trip: Ontario ⇔ DC 5
April 2002	Three direct shipment trips: Ontario ⇔ Michigan One direct shipment trip: Ontario ⇔ DC 7
May 2002	Five direct shipment trips: Ontario ⇔ Michigan
June 2002	Consolidated trip: Ontario ⇒ DC 7 ⇒ DC 12 ⇒ Ontario One direct shipment trip: Ontario ⇔ DC 5

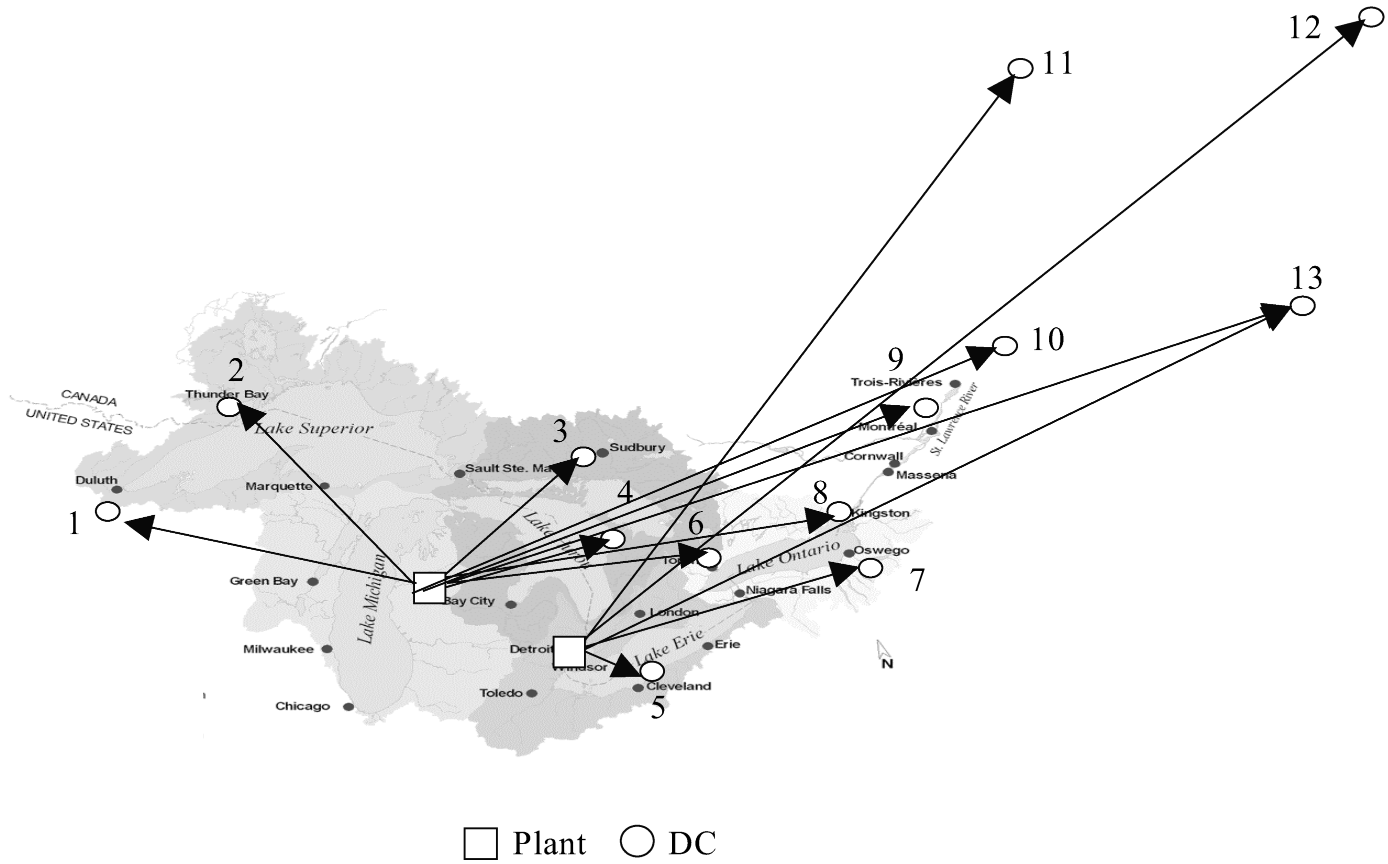

Fig. 4. Strategic relations between production facilities and DCs.

improving the forecasting accuracy because of the known target customers.

The solution to this problem obtained using the two-phase approach was obtained in 20 minutes CPU time on a laptop with a 512 MB RAM and a 900 MHz CPU. For most other applications of the two-phase approach (i.e., either as a decision support tool for operations planning or as a tool for answering what-if questions) at the company, feasible solutions started to stabilize (i.e., the gap between solutions from consecutive iterations remains within 0.1–0.5%) within 1 hour.

5. Conclusions and future studies

We have proposed a two-phase solution approach to the integrated production, inventory, and distribution problem where the transporter routing must be optimized together with the production lot sizes and the inventory policies. The phase I model is solved as a mixed-integer programming problem subject to all the constraints in the original model except that the transporter routings are restricted to direct shipments. The resulting phase I solution is always feasible to the original problem and determines the production quantity for each DC, the ending inventory for both plants and DCs, and the load and number of trips by each transporter from each plant to each DC in each time period. To handle the potential inefficiency of the direct shipment, phase II applies a heuristic procedure (the LC algorithm) to solve an associated consolidation problem. The associated delivery consolidation problem is formulated as a capacitated transportation problem with additional constraints. The computational performance of this proposed two-phase approach and its application to a real-life supply network are reported.

There are several potential extensions of this work. First, from a practical point of view, models that allow the DC demands to be random variables and some DCs to be used as transshipments points could be of great value to real-world problems. Second, from an academic research point of view, algorithms that can effectively solve the integrated production, inventory, and distribution routing problem subject to direct shipment for phase I, other than using the CPLEX MIP solver to solve (P_1) directly, will be of interest. For a noticeable number of test cases we performed, the time required for CPLEX to verify the optimal solution to (P_1) was excessive (see test problems 14, 31, and 32 in Table 5). One possible approach to solve this problem is to use Lagrangian relaxation. Third, we assumed in this study that each plant owns a fixed fleet of heterogeneous transporters. Relaxing this assumption and allowing the assignment of transporters to plants to be optimized is likely to lead to a better solution. Finally, there is an extensive literature on the capacitated vehicle routing problem and thus a comparative study of the LC algorithm used in phase II of this study with existing heuristics results is possible. Such a study has the potential to further improve the solution quality of the approaches used to solve integrated production, inventory, and distribution routing problems.

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