QC101μ Introduction to Quantum Computing with IBM Q Experience

Session 1

Quantum Computing, Qubits and Operations

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Objectives

- Understand the difference between bit and qubit

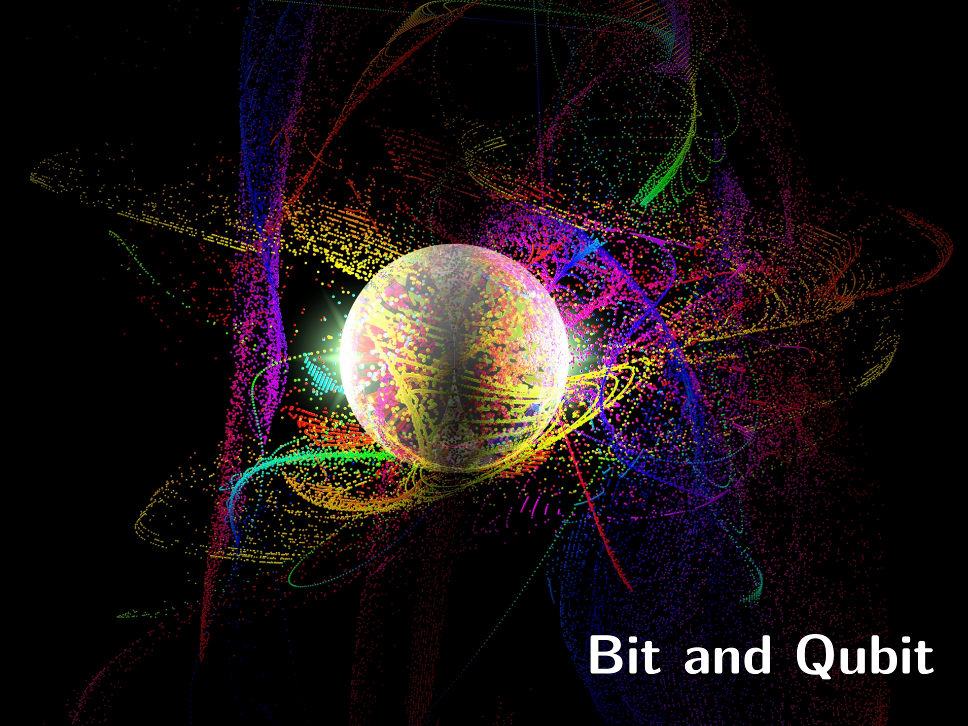
  In relation with quantum physics and the properties of electrons

- Understand operations that can be performed with qubits

  Superposition, entanglement, and quantum logic gates

- Express quantum computing computations with linear algebra

  Based on a vector representation of qubits and operations


Bit and Qubit

Quantum Mechanics

- Superpowers for electrons or other subatomic particles
  - **Superposition**: wave and a particle at the same time
  - **Quantum tunnelling**: bypass barriers, move through walls
  - **Entanglement**: having a twin acting in the same way
- Do not affect matter in the classical physics world

  Unfortunately, we cannot teleport or be at two places at once...

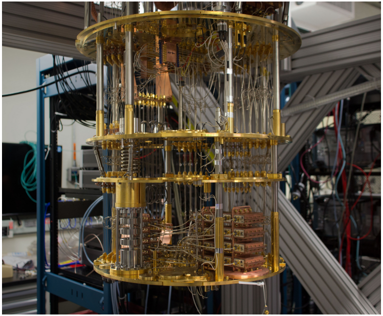
Quantum Computer

- Quantum computer can use superposition and entanglement

  Using quantum phenomena of subatomic particles

- Solve complex problems out of reach of classical computers

  Typically used for search problems with very large search space



Quantum Supremacy

- Many big companies are working on quantum computers

  IBM, Google, Intel, Microsoft, Quantum Circuits, ionQ, Bell Labs, Quantum Diamond Technologies, etc.

- Global race to reach the quantum supremacy

  Quantum computers will outperform classical supercomputers

Bit versus Qubit

- A bit is the basic information unit of classical computer

  Can only have one single value which is zero (0) or one (1)

- A qubit is the basic information unit of quantum computer
  - Only has a single value when it is measured/observed
  - Is a combination of both 0 and 1 by superposition

Superposition

- Quantum state of a qubit as a linear superposition

  Two orthonormal basis states $|0\rangle$ *("ket 0") and* $|1\rangle$ *("ket 1")*

- A qubit can be written as follows:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad \text{with } |\alpha|^2 + |\beta|^2 = 1$$

- Weights of quantum state are probability amplitudes
  - Probability of outcome $|0\rangle$ with value "0" is $|\alpha|^2$
  - Probability of outcome $|1\rangle$ with value "1" is $|\beta|^2$

Qubit System

- One qubit is in a superposition of two possible states

  The qubit is at the same time both 1 *and* 0

- Possible to build a complex system with n qubits
  - Is in a superposition of 2^n states at the same time
  - Can exhibit quantum entanglement between qubits

- A 2-qubit system can exist in any superposition of 4 states

$$|\psi\rangle = \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle \qquad \text{with } \sum_{i=0}^{3} |\alpha_i|^2 = 1$$

Entanglement

- Entangled qubits are correlated with one another

  Information about one qubit reveals information about the other

- Entangled qubits form a system as a whole

  Measuring one provides information about the other ones

- Simplest case with the Bell state for 2 qubits

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$


Quantum Logic Gate

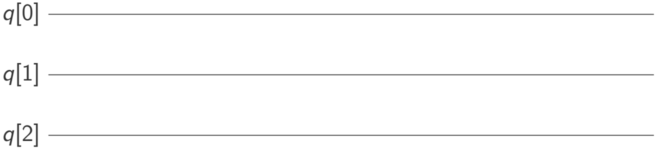
Quantum Circuit

- Computations represented with quantum circuit model

  Operations performed on a n-qubit register q

- Quantum logic gates used as the building blocks

  Similar to logic gates for conventional digital circuit



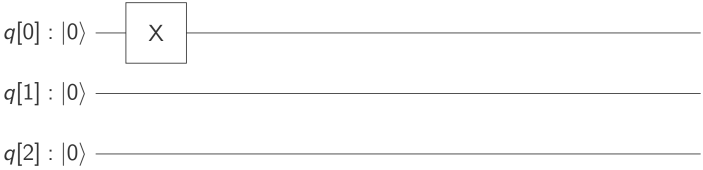
X Gate

- The X Gate flips the value of a qubit

  It acts on a single qubit and maps $|0\rangle$ to $|1\rangle$ and conversely

- Flipping one qubit of a 3-qubit register initialised with $|000\rangle$

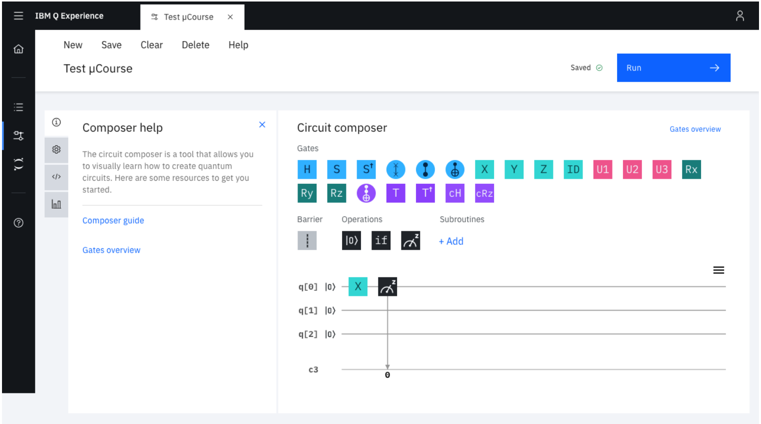
  The final value of the register is $|001\rangle$



IBM Q Experience

- Quantum cloud services and software platform by IBM

Available for free at https://quantum-computing.ibm.com



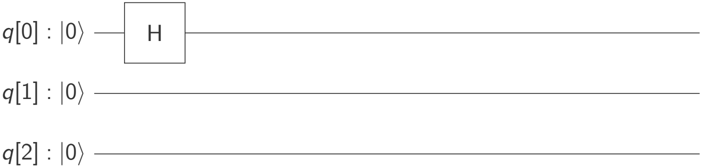
Hadamard Gate

- The H Gate creates a superposition on a single qubit

  It maps $|0\rangle$ *to* $\dfrac{|0\rangle + |1\rangle}{\sqrt{2}}$ *and* $|1\rangle$ *to* $\dfrac{|0\rangle - |1\rangle}{\sqrt{2}}$

- Introducing superposition in a 3-qubit register

  The final value of the register is $\dfrac{|000\rangle + |001\rangle}{\sqrt{2}}$



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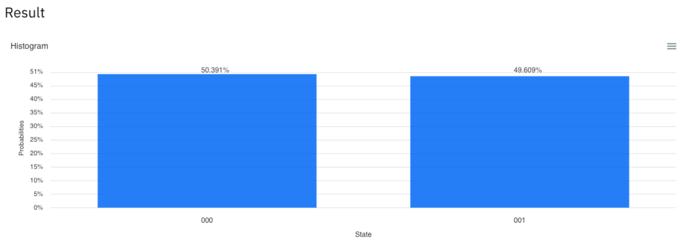


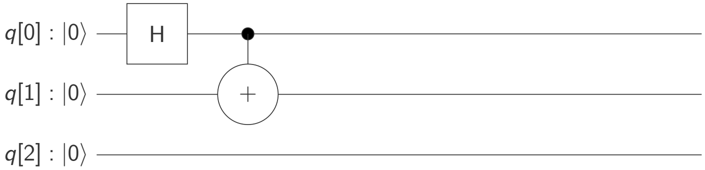
Controlled NOT Gate

- The CNOT Gate performs X on a qubit if another is $|1\rangle$

  It acts on two or more qubits and introduces entanglement

- Introducing entanglement in a 3-qubit register

  The final value of the register is $\dfrac{|000\rangle + |011\rangle}{\sqrt{2}}$



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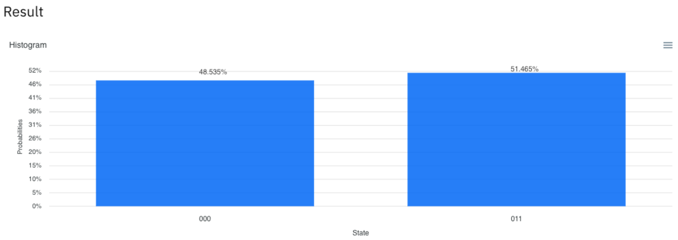
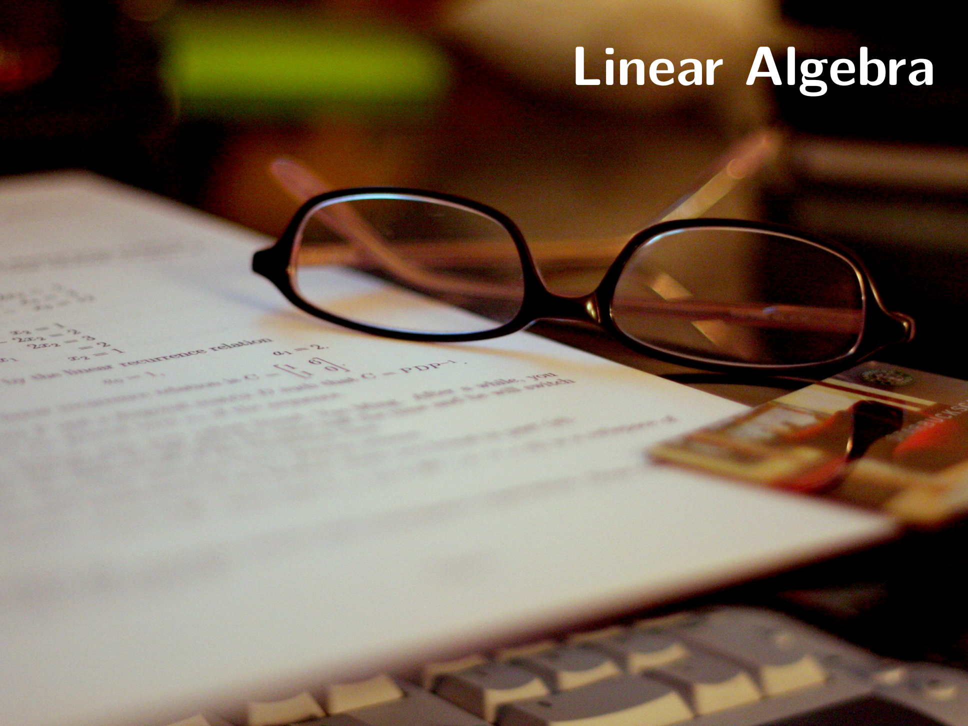


Linear Algebra



Vector Representation

- A single qubit can be represented as a vector

  Only taking into account the probability amplitudes

- A quantum state is a linear combination of possible outcomes

  That is $|0\rangle := \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ *and* $|1\rangle := \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ *for a single qubit*

- The vector representation of as single qubit is:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

Unitary Matrices

- Quantum logic gates are represented by unitary matrices

  With dimension $2^n \times 2^n$ for a gate acting on a n-qubit register

- Result of a gate obtained by multiplying U matrix by the state

  Each gate is associated to a matrice U

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Example (1)

- Starting with a 1-qubit register initialised with $|0\rangle$:

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

- Applying a H Gate gives $\dfrac{|0\rangle + |1\rangle}{\sqrt{2}}$:

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Example (2)

- Starting with a 2-qubit register initialised with $|10\rangle$:

$$\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

- Applying a CNOT Gate on the qubit register gives $|11\rangle$:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

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Credits

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