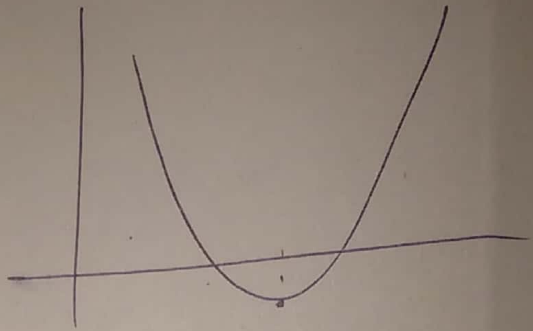
30-Oct-19

Exercise. I

Q3.

$$\min_{x} \quad x^2 - 2x = f(x)$$



$\min_{x} f(x) \Rightarrow$

(a) $f'(x)\Big|_{x = x_{min}} \Rightarrow 2x - 2 = 0$

$\Rightarrow \boxed{x = 1;}$

$\boxed{\text{Same result observed at both.}}$ (Ans)

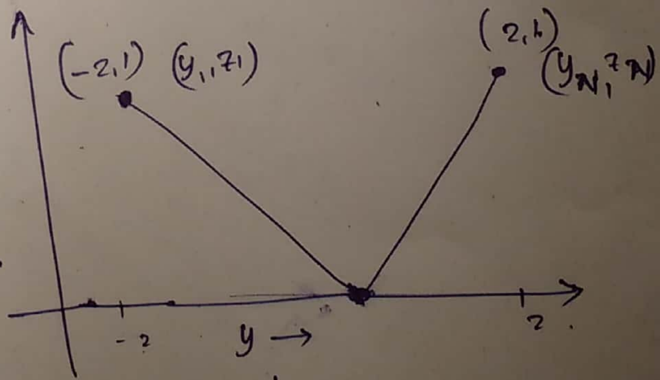
(b) Opt. subject to $(x >= 1.5)$

$\boxed{x = 1.5 \Rightarrow \text{intuitively matches with graph.}}$ (Ans)

(c) $\boxed{\text{Casadi returns } x = 1; y = 1;}$ (Ans)

Q4).

(a) $\boxed{\text{a soln exist}}$ satisfying the given ground constraints.



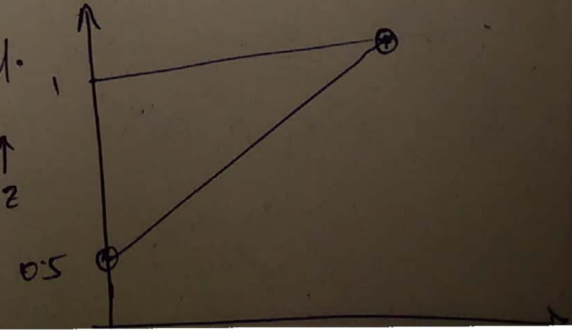
(b) error returned with given ground constraints : $z_i \geq 0.5$ & $z_i - 0.1y \geq 0.5$.

$\Rightarrow$ No solutions obtained.

(b) constraints, y_N

$z_i > 0.5$

$\Rightarrow$ soln exist.



Part - III. : Convex Optimization

Q5.

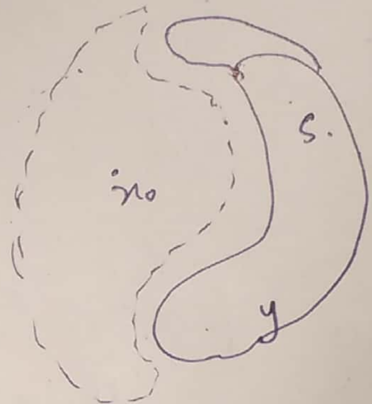
(a) A wedge

$\{x \in R^n \mid a_1^T x \le b, a_2^T x \le b_2\}$.

$\Rightarrow$ $\boxed{\text{convex set}}$

(b) set of points closer to given point than a given set.

$\Rightarrow$ $\boxed{\text{Not convex}}$



c

(c) set of points closer to 1 set than another.

$\Rightarrow$ $\boxed{\text{Not convex}}$

(d) $f(x_1, x_2) = 1/x_1 x_2$ $\quad x_1, x_2 \in R^2_{++}$.

$f(x_1, x_2) = \frac{1}{x_1 x_2}$.

$$\nabla f = \begin{bmatrix} \frac{\partial f(x_1, x_2)}{\partial x_1} \\ \\ \frac{\partial f(x_1, x_2)}{\partial x_2} \end{bmatrix} \in R^{2\times1} = \begin{bmatrix} -1/x_2 x_1^2 \\ \\ \frac{-1}{x_1 x_2^2} \end{bmatrix}$$

Hessian, $H = \nabla^2 f = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix} \in R^{2\times2}$

$$H = \begin{bmatrix} \underset{>0}{\frac{2}{x_2 x_1^3}} & \underset{>0}{\frac{1}{x_1^2 x_2^2}} \\ \underset{>0}{\frac{1}{x_2^2 x_1^2}} & \underset{>0}{\frac{2}{x_1 x_2^3}} \end{bmatrix}$$

$\Rightarrow H \geq 0 \quad \forall (x_1, x_2) \in R^{++}$

$\Rightarrow$ Hence $f(x_1,x_2)$: $\boxed{\text{convex}}$ in given domain (Ans)

(e) $f(x_1,x_2) = \frac{x_1}{x_2}$ on R^2_{++}

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = \begin{bmatrix} \underset{>0}{1/x_2} \\ \underset{<0}{-x_1/x_2^2} \end{bmatrix} \in R^2$$

$$H = \nabla^2 f = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} 0 & -1/x_2^2 \\ -1/x_2^2 & 2x_1/x_2^3 \end{bmatrix} \in R^{2\times2}.$$

H is ~~posi~~ not positive semi-definite. in $(x_1, x_2) \in R^2_{++}$.

$\Rightarrow \boxed{f(x_1, x_2) : \text{ not convex}}$ (Ans)

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Q6). To prove: $\min_{x \in R^n} f(x)$ has global min^m point.

where $f(x)$: continous, coercive function.

Acc to Weierstrass th^m

" If $\Omega \in R^n$ is non-empty & compact (bounded & closed) and $f: \Omega \to R$ is continous, then there exist a global minimizer of optim^n problem

$$\min_{x \in R^n} f(x) \quad \text{subject to } x \in \Omega.$$

Here, consider $\Omega \in R^n$, a subset of domain (bounded & closed).

Using WT th,

$\min_{x \in R^n} f(x)$ has global minimizer

subject to $f(x): R^n \to R$:= continous.

$\Rightarrow$ $\boxed{\text{Hence proved}}$ (Ans.

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Q7) a) $V_{chain}(y,z) = \frac{1}{2}\sum_{i=1}^{N-1} D\left((y_i - y_{i+1})^2 + (z_i - z_{i+1})^2\right)$

$+ g_0 \sum_{i=1}^{N} m\, z_i$

subject to $z_i \geq -0.2 + 0.1 y_i^2$.

$V_{chain}(y,z)\Big|_{\cancel{z=z_{min}}} \geq V_{chain_{min}}(y,z)\Big|_{z=z_{min}} = -0.2 + 0.1 y_i^2.$

$V_{chain_{min}}(y,z) = \frac{1}{2}\sum_{i=1}^{N-1}\left[D\left((y_i - y_{i+1})^2 + \left[0.1 y_i^2 - 0.1 y_{i+1}^2\right]^2\right)\right.$

$\left. + g_0 \cancel{\Sigma} m \left(-0.2 + 0.1 y_i^2\right)\right]$

+ const.

$\boxed{\text{.....cont}}$

$\boxed{\ldots \text{ cont}}$

Q7) a). $Y = D\left(y_i - y_{i+1}\right)^2 + \left(0.1 y_i^2 - 0.1 y_{i+1}^2\right)^2 + g_o m\left(-0.2 + 0.1 . y_i^2\right)$

$$\frac{\partial Y}{\partial y_i} = 2D\left(y_i - y_{i+1}\right) + 2\left(0.1y_i^2 - 0.1y_{i+1}^2\right) 0.2y_i + g_o m * 0.2 y_i$$

$$\frac{\partial Y}{\partial y_{i+1}} = 2D\left(y_i - y_{i+1}\right)(-1) + 2\left(0.1y_i^2 - 0.1y_{i+1}^2\right)\left(-0.2y_{i+1}\right) + 0;$$

$$\nabla^2 Y = \begin{bmatrix} \frac{\partial^2 Y}{\partial y_i^2} & \frac{\partial^2 Y}{\partial y_i \partial y_{i+1}} \\ \frac{\partial^2 Y}{\partial y_{i+1} \partial y_i} & \frac{\partial^2 Y}{\partial y_{i+1}^2} \end{bmatrix} \in R^{2\times 2} = H$$

$$= \begin{bmatrix} 2D + 2\left(0.2\left(0.1y_i^2 - 0.1y_{i+1}^2\right) + 0.2y_i\left(0.2y_i\right)\right) + g_o m * 0.2; & \\ -2D + 2 * 0.2y_i\left(-0.2y_{i+1}\right) & \\ 2D + 2\left(0.2y_i\right) - 0.2y_{i+1}, & 2D + 2\left(-0.2\left(0.1y_i^2 - 0.1y_{i+1}^2\right) + 0.2y_{i+1} * 0.2y_{i+1}\right) \end{bmatrix}$$

$\Rightarrow H = \nabla^2 Y$ is not positive semi definite

$\Rightarrow$ Hence function is <u>not</u> convex

Q8)

a) $V_{el}^{i} = \frac{1}{2} D d_i^2, \quad i = 1, \ldots, N-1$

$$d_i^2 = (y_i - y_{i+1})^2 + (z_i - z_{i+1})^2 + L_i^2 - 2\sqrt{(y_i - y_{i+1})^2 + (z_i - z_{i+1})^2}\, L_i$$

Since $\frac{d^2 V_{el}^i}{\partial y_i^2}, \frac{\partial^2 V_{el}^i}{\partial y_{i+1}^2}$ not ≥ 0

$\Rightarrow$ function is non-convex (Ans).

(b). $V_{el}^i = \frac{1}{2} D \max(0, d_i^2)$.

$$\frac{\partial^2 V_{el}^i}{\partial y_i^2} = \frac{1}{2} D \max\left(0, \frac{\partial^2 d_i^2}{d y_i^2}\right).$$

≥ 0

$\Rightarrow$ Convex function.

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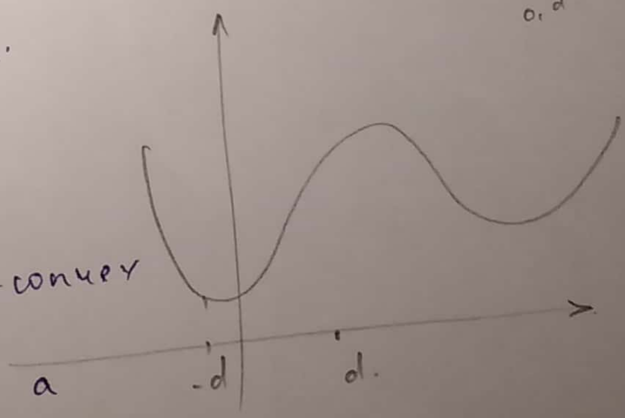

Q8) c).

The problem will be non-convex and do not generate unique solution.