Simulation Experiment Results

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Load in the results

```
library(knitr)
library(tidyverse)

library(gridExtra)
library(latex2exp)

load("../data/synthetic-data.RData")
attach(synthetic.data.config)
```

Cleaning up and renaming things

```
estimates = readRDS("../data/sim_exp-estimate_extinction_results.RDS")
estimates = estimates %>% filter(!(method %in% c("SI-RM", "GB-RM")))
estimates = estimates %>% mutate(across(method, str_replace, 'SI-RM-corrected', 'SI-RM'),
                                 method_cat = ifelse(method %in% c("SI-RM", "MINMI"),
                                                     "Proposed",
                                                     "Existing"),
                                 method = ifelse(method == "GRIWM",
                                                 "GRIWM (q=0.05)",
                                                 ifelse(method == "GRIWM-corrected",
                                                        "GRIWM-BA (q=0.5)",
                                                        ifelse(method == "STRAUSS",
                                                          "Strauss",
                                                          method
                                                        ))))
estimates = estimates %>% filter(error_factor != 4)
head(estimates)
```

```
##   id error_factor method     lower     point    upper point_runtime
## 1  1          0.0    MLE        NA 12660.896       NA  1.907349e-05
## 2  1          0.0 BA-MLE        NA 12293.940       NA  1.682997e-03
## 3  1          0.0 SI-UGM 11262.804 12422.265 12681.61  3.777524e+00
## 4  2          0.5    MLE        NA  9871.056       NA  1.279831e-03
## 5  2          0.5 BA-MLE        NA  9364.609       NA  4.145861e-03
## 6  2          0.5 SI-UGM  7789.998  9518.421 10035.51  2.695084e+00
##   conf_int_runtime B.point B.lower B.upper method_cat
```

```
## 1              NA       NA       NA       NA   Existing
## 2              NA       NA       NA       NA   Existing
## 3        3.777524       NA       NA       NA   Existing
## 4              NA       NA       NA       NA   Existing
## 5              NA       NA       NA       NA   Existing
## 6        2.695084       NA       NA       NA   Existing
```

```
# Point estimates
performance.point_estimates = estimates %>%
  filter(!is.na(point)) %>%
  group_by(error_factor, method, method_cat) %>%
  summarise(MSE_000 = mean((point - theta.true)^2)/1000,
            bias = mean(point)-theta.true,
            variance_000 = var(point)/1000,
            avg_runtime = round(mean(point_runtime), 5))
```

```
## `summarise()` has grouped output by 'error_factor', 'method'. You can override
## using the `.groups` argument.
```

```
kable(performance.point_estimates)
```

error_factor	method	method_cat	MSE_000	bias	variance_000	avg_runtime
0.0	BA-MLE	Existing	234.5607	-0.9379571	234.7946	0.00002
0.0	GRIWM-BA (q=0.5)	Existing	246.1900	133.7070000	228.5410	13.89881
0.0	GRIWM (q=0.05)	Existing	1183.5359	-949.8850000	281.5360	2.33548
0.0	MINMI	Proposed	247.4576	139.4092643	228.2509	0.00001
0.0	MLE	Existing	438.6601	475.2971837	212.9656	0.00002
0.0	SI-RM	Proposed	438.6601	475.2971837	212.9656	0.05641
0.0	SI-UGM	Existing	248.9648	150.6138962	226.5068	4.70657
0.0	Strauss	Existing	234.8453	-0.7152842	235.0799	0.00002
0.5	BA-MLE	Existing	244.0353	-22.0990141	243.7908	0.00003
0.5	GRIWM-BA (q=0.5)	Existing	244.7798	95.1730000	235.9578	13.88254
0.5	GRIWM (q=0.05)	Existing	1275.8894	-992.6550000	290.8162	2.35887
0.5	MINMI	Proposed	253.0606	118.8454210	239.1755	0.00047
0.5	MLE	Existing	428.0602	455.1437961	221.1254	0.00002
0.5	SI-RM	Proposed	428.0602	455.1437961	221.1254	0.06072
0.5	SI-UGM	Existing	250.7756	117.2579537	237.2634	2.33059
0.5	Strauss	Existing	245.5802	-22.8493286	245.3034	0.00002
1.0	BA-MLE	Existing	365.6242	-45.7554617	363.8946	0.00002
1.0	GRIWM-BA (q=0.5)	Existing	345.2420	34.4120000	344.4022	13.90095
1.0	GRIWM (q=0.05)	Existing	1547.7470	-1060.0020000	424.5673	18.10725
1.0	MINMI	Proposed	373.3208	103.7670782	362.9161	0.00057
1.0	MLE	Existing	516.8878	432.6138460	330.0631	0.00002
1.0	SI-RM	Proposed	516.8878	432.6138460	330.0631	0.05992
1.0	SI-UGM	Existing	371.3002	115.2129004	358.3845	1.93741

error_factor	method	method_cat	MSE_000	bias	variance_000	avg_runtime
1.0	Strauss	Existing	366.4727	-50.5383309	364.2829	0.00002
2.0	BA-MLE	Existing	542.9867	-233.5215118	488.9434	0.00002
2.0	GRIWM-BA (q=0.5)	Existing	504.9335	-278.4774775	427.8120	13.94220
2.0	GRIWM (q=0.05)	Existing	2506.9717	-1407.5250000	526.3714	2.36410
2.0	MINMI	Proposed	491.9000	27.3303294	491.6447	0.00071
2.0	MLE	Existing	507.4514	253.7890364	443.4860	0.00002
2.0	SI-RM	Proposed	507.4514	253.7890364	443.4860	0.05993
2.0	SI-UGM	Existing	501.5358	65.3780494	497.7593	1.68008
2.0	Strauss	Existing	553.8156	-247.9251968	492.8416	0.00002

```
# Confidence Intervals
performance.conf_int_estimates = estimates %>%
  filter(!is.na(conf_int_runtime)) %>%
  mutate(width = upper - lower,
         contains_theta = ifelse(theta.true > lower & theta.true < upper, 1, 0)) %>%
  group_by(error_factor, method, method_cat) %>%
  summarise(Coverage = round(mean(contains_theta) * 100, 1),
            `Average Width` = round(mean(width), 2),
            `Average Runtime` = round(mean(conf_int_runtime), 4)) %>%
  ungroup() %>%
  arrange(desc(Coverage), `Average Width`, `Average Runtime`)
```

```
## `summarise()` has grouped output by 'error_factor', 'method'. You can override
## using the `.groups` argument.
```

```
kable(performance.conf_int_estimates)
```

error_factor	method	method_cat	Coverage	Average Width	Average Runtime
0.0	SI-RM	Proposed	97.4	2005.35	0.0564
0.0	SI-UGM	Existing	97.3	1961.14	4.7066
0.5	SI-RM	Proposed	96.3	2097.43	0.0607
0.5	SI-UGM	Existing	96.2	2091.09	2.3306
0.5	MINMI	Proposed	95.2	2066.21	0.0013
2.0	SI-RM	Proposed	95.1	2960.26	0.0599
2.0	MINMI	Proposed	95.0	2945.03	0.0021
2.0	SI-UGM	Existing	94.9	2964.30	1.6801
0.0	MINMI	Proposed	94.5	1917.16	0.0000
1.0	SI-RM	Proposed	93.8	2346.11	0.0599
1.0	SI-UGM	Existing	93.7	2351.53	1.9374
1.0	MINMI	Proposed	93.0	2325.84	0.0014
2.0	GRIWM-BA (q=0.5)	Existing	80.6	1949.67	13.9422
1.0	GRIWM-BA (q=0.5)	Existing	69.5	1047.99	13.9010
0.5	GRIWM-BA (q=0.5)	Existing	49.9	548.08	13.8825
2.0	GRIWM (q=0.05)	Existing	22.6	2163.50	2.3641

error_factor	method	method_cat	Coverage	Average Width	Average Runtime
1.0	GRIWM (q=0.05)	Existing	13.9	1162.65	18.1072
0.5	GRIWM (q=0.05)	Existing	7.0	608.33	2.3589
0.0	GRIWM (q=0.05)	Existing	0.0	0.00	2.3355
0.0	GRIWM-BA (q=0.5)	Existing	0.0	0.00	13.8988

Point Estimates

```
library(kableExtra)
```

```
## Warning: package 'kableExtra' was built under R version 4.2.2
```

```
##
## Attaching package: 'kableExtra'
```

```
## The following object is masked from 'package:dplyr':
##
##     group_rows
```

```
for (err in error_factors) {
  experiment.results = performance.point_estimates %>%
    filter(error_factor == err) %>%
    ungroup() %>%
    mutate(across(!c(method, avg_runtime, method_cat), round)) %>%
    mutate(avg_runtime = round(avg_runtime, digits = 5)) %>%
    arrange(MSE_000) %>%
    select(-c(error_factor, method_cat))

  print(kable(experiment.results))

  experiment.results.kbl = experiment.results %>%
    kable(
      booktabs = T,
      col.names = c("", "(000's years)", "(years)", "(000's years)", "(seconds)"),
      format = "latex"
    ) %>%
    add_header_above(
      c(
        "Method" = 1,
        "MSE" = 1,
        "Bias" = 1,
        "Variance" = 1,
        "Average Runtime" = 1
      ),
      line = F,
      align = c("l", "c", "c", "c", "c")
    )

  writeLines(
```

```
    experiment.results.kbl,
    paste0("../figures/table-sim-exp-point-error", err, ".tex")
  )
}
```

method	MSE_000	bias	variance_000	avg_runtime
BA-MLE	235	-1	235	0.00002
Strauss	235	-1	235	0.00002
GRIWM-BA (q=0.5)	246	134	229	13.89881
MINMI	247	139	228	0.00001
SI-UGM	249	151	227	4.70657
MLE	439	475	213	0.00002
SI-RM	439	475	213	0.05641
GRIWM (q=0.05)	1184	-950	282	2.33548

method	MSE_000	bias	variance_000	avg_runtime
BA-MLE	244	-22	244	0.00003
GRIWM-BA (q=0.5)	245	95	236	13.88254
Strauss	246	-23	245	0.00002
SI-UGM	251	117	237	2.33059
MINMI	253	119	239	0.00047
MLE	428	455	221	0.00002
SI-RM	428	455	221	0.06072
GRIWM (q=0.05)	1276	-993	291	2.35887

method	MSE_000	bias	variance_000	avg_runtime
GRIWM-BA (q=0.5)	345	34	344	13.90095
BA-MLE	366	-46	364	0.00002
Strauss	366	-51	364	0.00002
SI-UGM	371	115	358	1.93741
MINMI	373	104	363	0.00057
MLE	517	433	330	0.00002
SI-RM	517	433	330	0.05992
GRIWM (q=0.05)	1548	-1060	425	18.10725

method	MSE_000	bias	variance_000	avg_runtime
MINMI	492	27	492	0.00071
SI-UGM	502	65	498	1.68008
GRIWM-BA (q=0.5)	505	-278	428	13.94220
MLE	507	254	443	0.00002
SI-RM	507	254	443	0.05993
BA-MLE	543	-234	489	0.00002
Strauss	554	-248	493	0.00002
GRIWM (q=0.05)	2507	-1408	526	2.36410

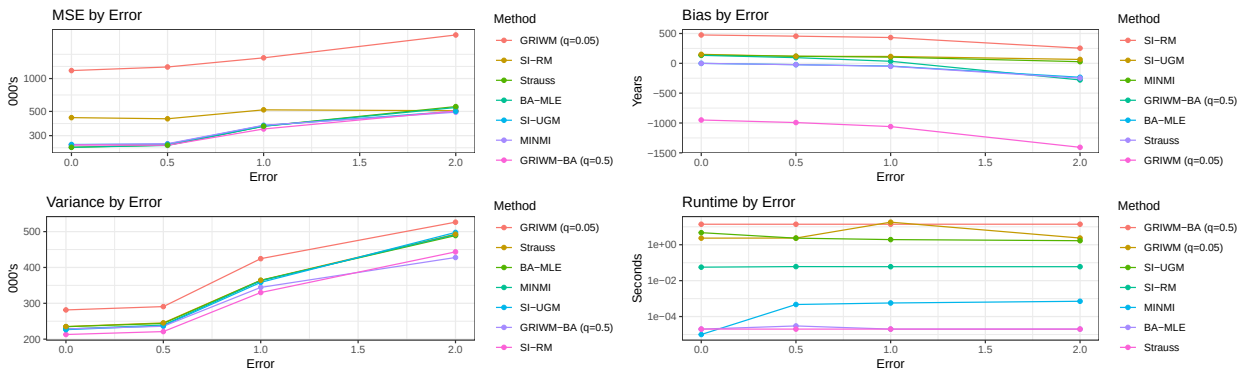
method	MSE_000	bias	variance_000	avg_runtime

```
performance.point_estimates.long = performance.point_estimates %>%
  rename(Error = error_factor,
         Method = method,
         Category = method_cat,
         MSE = MSE_000,
         Bias = bias,
         Variance = variance_000,
```

```
        Runtime = avg_runtime) %>%
  pivot_longer(cols=c(MSE, Bias, Variance, Runtime),
               names_to = "Metric") %>%
  mutate(Method = factor(Method),
         Category = factor(Category),
         Metric = factor(Metric)) %>%
  filter(Method != "MLE")

head(performance.point_estimates.long)
```

```
## # A tibble: 6 x 5
## # Groups:   Error, Method [2]
##   Error Method           Category Metric       value
##   <dbl> <fct>            <fct>    <fct>        <dbl>
## 1     0 BA-MLE           Existing MSE      235.
## 2     0 BA-MLE           Existing Bias      -0.938
## 3     0 BA-MLE           Existing Variance 235.
## 4     0 BA-MLE           Existing Runtime    0.00002
## 5     0 GRIWM-BA (q=0.5) Existing MSE      246.
## 6     0 GRIWM-BA (q=0.5) Existing Bias     134.
```

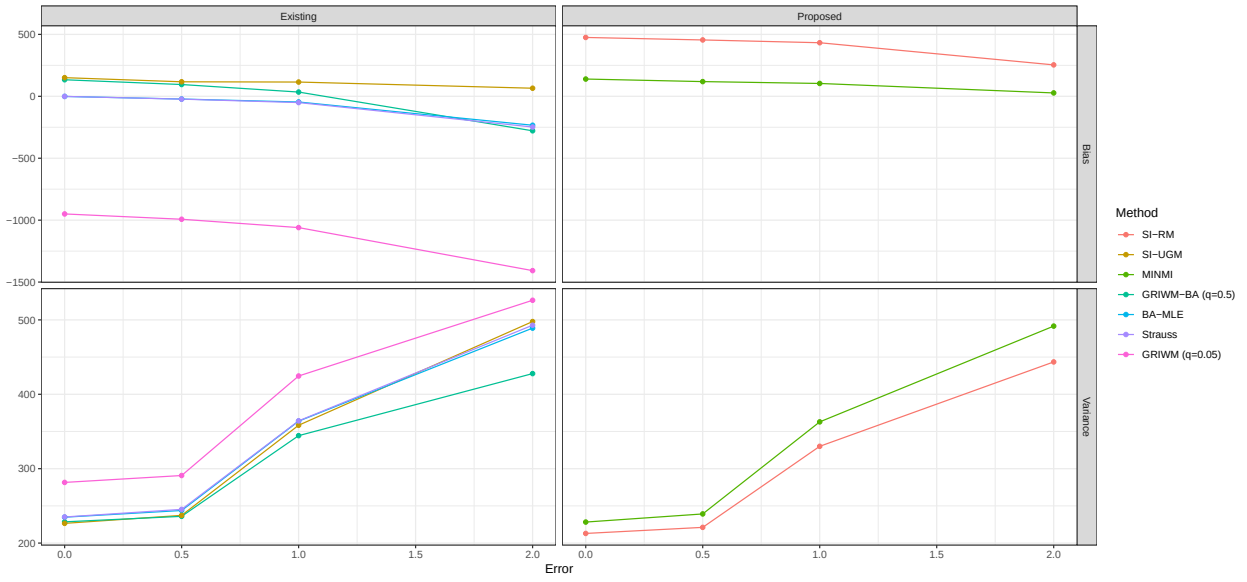
```
metrics = unique(performance.point_estimates.long$Metric)
performance.point_estimates.plots = lapply(metrics, function(met) {
  p = ggplot(data = performance.point_estimates.long %>% filter(Metric == met),
             aes(x = Error, y = value, colour = reorder(Method, value, decreasing = T))) +
    geom_line() +
    geom_point() +
    theme_bw() +
    labs(title = paste(met, "by Error"), ylab=NULL, colour = "Method")
  if (met %in% c("MSE", "Runtime")) {
    p = p+scale_y_log10()
  }
  p
})

performance.point_estimates.plots[[1]] = performance.point_estimates.plots[[1]] + ylab("000's")
performance.point_estimates.plots[[2]] = performance.point_estimates.plots[[2]] + ylab("Years")
performance.point_estimates.plots[[3]] = performance.point_estimates.plots[[3]] + ylab("000's")
performance.point_estimates.plots[[4]] = performance.point_estimates.plots[[4]] + ylab("Seconds")

do.call(grid.arrange, performance.point_estimates.plots)
```



```
for (i in 1:length(metrics)) {
  ggsave(plot = performance.point_estimates.plots[[i]],
         file = paste0("../figures/plot-sim-exp-point-est-", metrics[i], ".svg"),
         dpi=320)
}
```

```
## Saving 6.5 x 4.5 in image
## Saving 6.5 x 4.5 in image
## Saving 6.5 x 4.5 in image
## Saving 6.5 x 4.5 in image
```

```
perf.point_estimates.bias_var.plot = performance.point_estimates.long %>%
  filter(Metric %in% c("Bias", "Variance")) %>%
  ggplot(aes(x=Error, y=value, colour=reorder(Method, value, decreasing = T))) +
  geom_line() +
  geom_point() +
  facet_grid(Metric ~ Category, scale="free_y") +
  labs(colour = "Method", y=NULL) +
  theme_bw()

perf.point_estimates.bias_var.plot
```




```
ggsave(plot = perf.point_estimates.bias_var.plot,
       file = "../figures/plot-sim-exp-point-est-Bias-Variance.svg",
       width=15, height=7,
       dpi = 320)
```

Confidence Intervals

```
options(scipen = 9)
for (metric in c("Coverage", "Average Width", "Average Runtime")) {
```

```
  experiment.results.conf_int = performance.conf_int_estimates %>%
    select(c(method, error_factor, one_of(metric))) %>%
    pivot_wider(
      id_cols = method,
      names_from = error_factor,
      values_from = one_of(metric),
      names_prefix = paste(metric, "| error = sigma*")
    ) %>%
    arrange(!!syms(paste(metric, "| error = sigma*0")))
  print(kable(experiment.results.conf_int))

  experiment.results.kbl = experiment.results.conf_int %>%
    kable(
      col.names = c("", paste0(c(0, 0.5, 1, 2), r"{*$\sigma$}")),
      booktabs = T,
      format = "latex",
      escape = FALSE
    ) %>%
    add_header_above(unlist(lst("Method" = 1,!!metric := 4)), line = F)
  writeLines(
    experiment.results.kbl,
    paste0(
      "../figures/table-sim-exp-conf-int-",
      str_replace(tolower(metric), " ", "-"),
      ".tex"
    )
  )
}
```

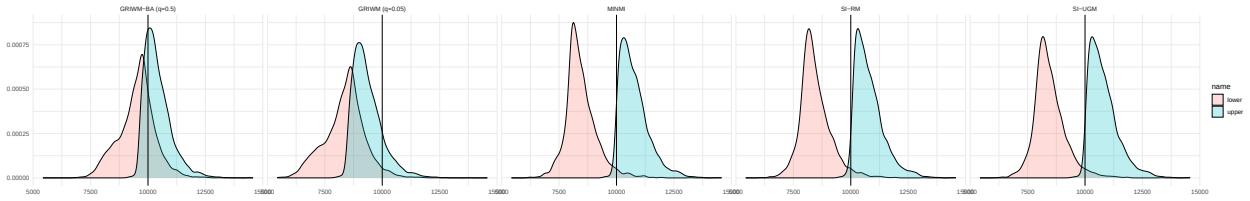
method	Coverage | error = sigma*0	Coverage | error = sigma*0.5	Coverage | error = sigma*2	Cover
SI-RM	97.4	96.3	95.1	
SI-UGM	97.3	96.2	94.9	
MINMI	94.5	95.2	95.0	
GRIWM-BA (q=0.5)	0.0	49.9	80.6	
GRIWM (q=0.05)	0.0	7.0	22.6	

method	Average Width | error = sigma*0	Average Width | error = sigma*0.5	Average Width | error =
SI-RM	2005.35	2097.43	
SI-UGM	1961.14	2091.09	
MINMI	1917.16	2066.21	
GRIWM-BA (q=0.5)	0.00	548.08	
GRIWM (q=0.05)	0.00	608.33	

method	Average Runtime | error = sigma*0	Average Runtime | error = sigma*0.5	Average Runtime | e
SI-RM	0.0564	0.0607	
SI-UGM	4.7066	2.3306	
MINMI	0.0000	0.0013	
GRIWM-BA (q=0.5)	13.8988	13.8825	
GRIWM (q=0.05)	2.3355	2.3589	

```
estimates %>%
  filter(!is.na(lower)) %>%
  select(method, lower, upper) %>%
  pivot_longer(cols=c(lower, upper)) %>%
```

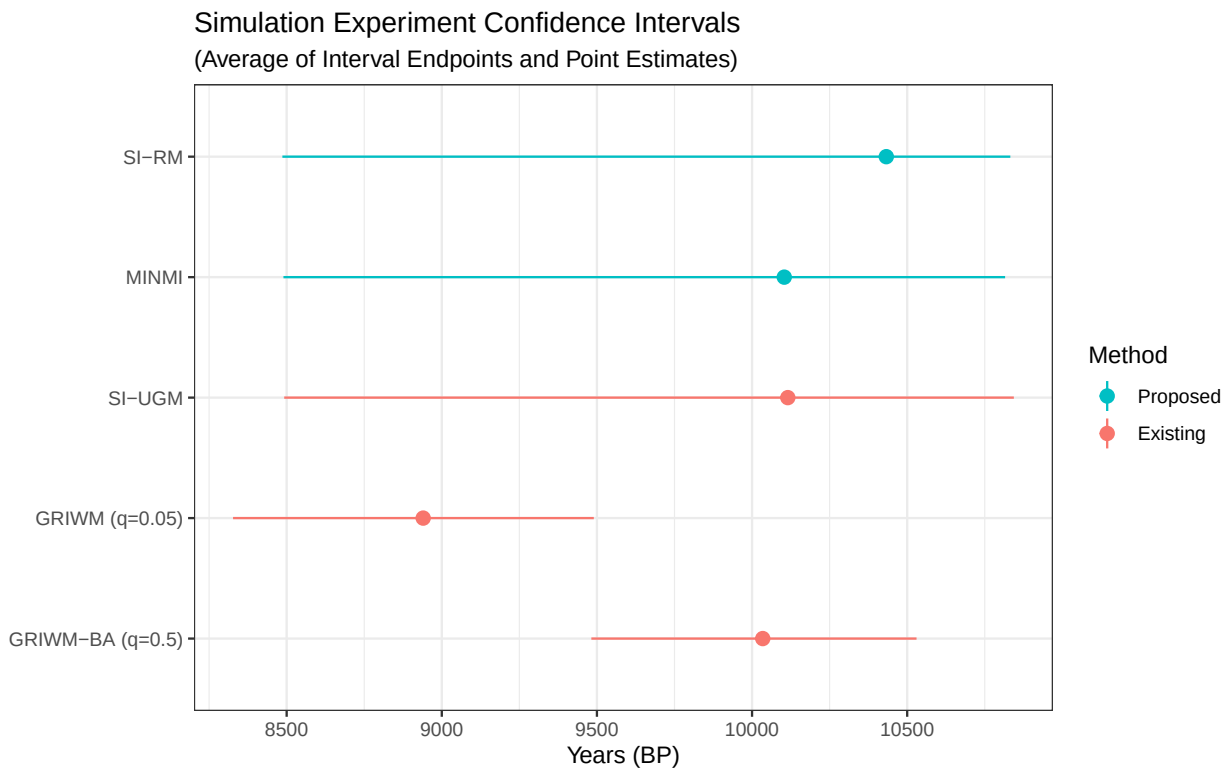
```
  filter(!is.na(value)) %>%
  ggplot(aes(x=value, fill=name)) +
  geom_density(alpha=0.25) +
  geom_vline(aes(xintercept=theta.true)) +
  facet_wrap(method ~ ., nrow=1) +
  theme_minimal() +
  labs(x=NULL, y=NULL)
```




```
p = estimates %>%
  filter(!is.na(lower) & error_factor == 1) %>%
  group_by(method_cat, method) %>%
  summarise(lower = mean(lower), upper=mean(upper), point=mean(point)) %>%
  mutate(width=upper-lower) %>%
  mutate(method_cat_int = ifelse(method_cat == "Existing", 0, 1)) %>%
  ggplot(aes(colour=method_cat)) +
  geom_pointrange(aes(xmin=lower, xmax=upper, x=point, y=reorder(method, method_cat_int))) +
  guides(colour = guide_legend(reverse=TRUE)) +
  labs(y=NULL, x="Years (BP)",
       title="Simulation Experiment Confidence Intervals",
       colour="Method",
       subtitle="(Average of Interval Endpoints and Point Estimates)") +
  scale_x_continuous(breaks = seq(from=8000, to=11500, by=500)) +
  theme_bw()
```

```
## `summarise()` has grouped output by 'method_cat'. You can override using the
## `.groups` argument.
```

```
p
```




```
ggsave(filename="../figures/sim-exp-intervals.svg", plot=p, height=5, width=8)
```

Monte Carlo Samples for MINMI

```
B.minmi = readRDS("../data/sim_exp-B-minmi.RDS")
B.minmi = B.minmi %>% arrange(error_factor) %>% mutate(error_factor = paste0(error_factor, r"{*$\sigma$}"))

B.minmi.kbl = B.minmi %>%
  filter(error_factor != 0) %>%
  kable(col.names = c("Variation", "$q = 0.025$", "$q = 0.5$", "$q = 0.975$"),
        booktabs=T, format="latex", escape = FALSE) %>%
  add_header_above(c(`Measurement Error`=1, `$B$`=3), line=F)

print(B.minmi.kbl)
```

```
##
## \begin{tabular}{lrrr}
## \toprule
## \multicolumn{1}{c}{Measurement Error} & \multicolumn{3}{c}{\$B\$} \\
## Variation & $q = 0.025$ & $q = 0.5$ & $q = 0.975$\\
## \midrule
## 0*$\sigma$ & 2 & 2 & 2\\
## 0.5*$\sigma$ & 5 & 6 & 3\\
## 1*$\sigma$ & 14 & 24 & 9\\
## 2*$\sigma$ & 42 & 91 & 33\\
## 4*$\sigma$ & 136 & 243 & 119\\
```

```
## \bottomrule
## \end{tabular}
```

```
writeLines(B.minmi.kbl, "../figures/table-sim-exp-minmi-Bs.tex")
```