PD Tracking of a RR Manipulator Arm

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November 10, 2017

1 Problem Statement

The goal is to track a desired position for the end-effector of a manipulator arm. The manipulator arm is essentially a double pendulum, where the torque on each joint can be controlled independently to move the end-effector to a desired position.

2 Forward Kinematics

The position of the end-effector can be expressed as a function of the two joint angles as

$$\vec{p} = \begin{bmatrix} (l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2))\hat{i} \\ (l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2))\hat{j} \end{bmatrix} \tag{1}$$

where l_1 and l_2 are the lengths and θ_1 and θ_2 are the angles of the first and second linkages, respectively. The first linkage is attached to the ground and its angle is defined with respect to the ground plane. The second linkage is attached to the first linkage and its angle is defined with respect to the orientation of the first linkage. The free end of the second linkage is the position of the end-effector.

3 Controller

The controller has the form

$$u = -k_p J^T (p - r(t)) - k_d \dot{q}, \tag{2}$$

where $q = [\theta_1, \theta_2]^T$ and $r(t)$ is the reference trajectory in cartesian x and y coordinates. $J = D_\theta p$ is the manipulator Jacobian, where D_θ is calculated as

$$D_\theta = \frac{\partial \vec{p}}{\partial \theta} = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} \end{bmatrix} \tag{3}$$

Each of the partial derivatives are calculated as

$$\frac{\partial x}{\partial \theta_1} = -l_1 \sin \theta_1 \tag{4}$$

$$\frac{\partial x}{\partial \theta_2} = -l_2 \sin (\theta_1 + \theta_2) \tag{5}$$

$$\frac{\partial y}{\partial \theta_1} = l_1 \cos \theta_1 \tag{6}$$

$$\frac{\partial y}{\partial \theta_2} = l_2 \cos (\theta_1 + \theta_2) \tag{7}$$