CS 164 Programming Languages and Compilers	**Spring 2019**

Discussion Week of 3/11: Midterm Prepreparation

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1 Regular Expressions

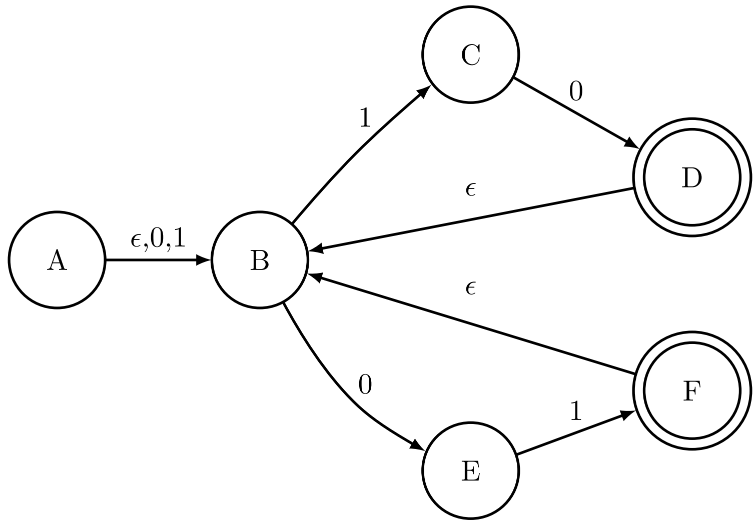
1. (a) Write a regex that matches binary strings divisible by 8.
   Answer: `[01]*000|0|00`

   (b) Provide a *regular* grammar for the regex from part (a).
   Answer:

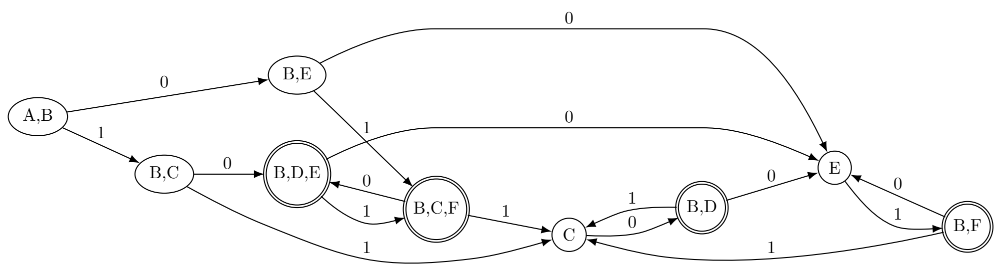
$$
\begin{array}{rcl}
B & : & \text{“0”} \,|\, \text{“1”} \\
P & : & \epsilon \\
  & | & B\ P \\
S & : & P\ \text{“000”} \\
  & | & \text{“0”} \\
  & | & \text{“00”}
\end{array}
$$

2 Finite State Automata

1. (a) Write the corresponding NFA for the regular expression `(0|1)?(10|01)+;`
   Answer:



(b) Convert the NFA from part (a) into a DFA.
Answer:



3 Grammar Rewriting and LL(k) Parsing

Consider the simple ambiguous grammar (which we've seen before):

$$
\begin{array}{rcl}
S & : & E \dashv \\
E & : & E + E \\
 & | & E * E \\
 & | & \texttt{ID}
\end{array}
$$

1. Show that the grammar is ambiguous with two different leftmost derivations of the string `a+b*c`;
Answer:

   - S → E → E * E → E + E * E → a + E * E → a + b * E → a + b * c
   - S → E → E + E → a + E → a + E * E → a + b * E → a + b * c

2. Rewrite this grammar so that it preserves the standard order of operations, is LL(1), and is unambiguous. Draw the resulting tree for the string `a+b*c;`
   Answer:

$$\begin{array}{rcl} S & : & E \dashv \\ E & : & T\ E' \\ E' & : & \epsilon \\ & | & +E \\ T & : & F\ T' \\ T' & : & \epsilon \\ & | & *T \\ F & : & \texttt{ID} \end{array}$$

3. Write down the equivalent unambiguous grammar that enforces both **left** associativity and correct precedence. Why can't this be achieved with an LL(1) grammar?
   Answer: This can't be achieved with an LL(1) because it introduces left recursion.

$$\begin{array}{rcl} S & : & E \dashv \\ E & : & E + T \\ & | & T \\ T & : & T * F \\ & | & F \\ F & : & \texttt{ID} \end{array}$$

4 Earley's Algorithm

Consider the following grammar:

$$\begin{array}{rcl} P & : & E\ \dashv \\ E & : & ID \\ & | & \lambda ID\ .\ E \\ & | & E\ E \end{array}$$

1. Use it to parse the following expression with Earley's algorithm: λ`ID . ID ID` $\dashv$
   Answer: We can construct two different parse trees.

	λ	ID	.	ID	ID	
	0	1	2	3	4	5
a	P: $\bullet$ E $\dashv$, 0	E:λ $\bullet$ ID.E, 0	E:λID $\bullet$ .E, 0	E:λID . $\bullet$ E, 0	E:ID $\bullet$, 3	E:ID $\bullet$, 4
b	E: $\bullet$ ID, 0			E: $\bullet$ ID, 3	E:λID.E$_a$ $\bullet$, 0	E:E E$_a$ $\bullet$, 3
c	E: $\bullet$ λ ID.E, 0			E: $\bullet$ λ ID.E, 3	E:E$_a$ $\bullet$ E, 3	E:E E$_a$ $\bullet$, 0
d	E: $\bullet$ E E, 0			E: $\bullet$ E E, 3	P:E$_b$ $\bullet$ $\dashv$, 0	E:E$_a$ $\bullet$ E, 4
e					E:E$_b$ $\bullet$ E, 0	E:λ ID.E$_b$ $\bullet$, 0
f					E: $\bullet$ ID, 4	E:E$_b$ $\bullet$ E, 3
g					E: $\bullet$ λ ID.E, 4	P:E$_c$ $\bullet$ $\dashv$, 0
h					E: $\bullet$ E E, 4	E:E$_c$ $\bullet$ E, 0
i						P:E$_e$ $\bullet$ $\dashv$, 0
j						E:E$_e$ $\bullet$ E, 0

	λ	ID	.	ID	ID	
	0	1	2	3	4	5
a	P: $\bullet$ E $\dashv$, 0	E:λ $\bullet$ ID.E, 0	E:λID $\bullet$ .E, 0	E:λID . $\bullet$ E, 0	E:ID $\bullet$, 3	E:ID $\bullet$, 4
b	E: $\bullet$ ID, 0			E: $\bullet$ ID, 3	E:λID.E$_a$ $\bullet$, 0	E:E E$_a$ $\bullet$, 3
c	E: $\bullet$ λ ID.E, 0			E: $\bullet$ λ ID.E, 3	E:E$_a$ $\bullet$ E, 3	E:E E$_a$ $\bullet$, 0
d	E: $\bullet$ E E, 0			E: $\bullet$ E E, 3	P:E$_b$ $\bullet$ $\dashv$, 0	E:E$_a$ $\bullet$ E, 4
e					E:E$_b$ $\bullet$ E, 0	E:λ ID.E$_b$ $\bullet$, 0
f					E: $\bullet$ ID, 4	E:E$_b$ $\bullet$ E, 3
g					E: $\bullet$ λ ID.E, 4	P:E$_c$ $\bullet$ $\dashv$, 0
h					E: $\bullet$ E E, 4	E:E$_c$ $\bullet$ E, 0
i						P:E$_e$ $\bullet$ $\dashv$, 0
j						E:E$_e$ $\bullet$ E, 0

2. Is the expression in the language? Is the grammar ambiguous?

Answer: Yes. Yes.